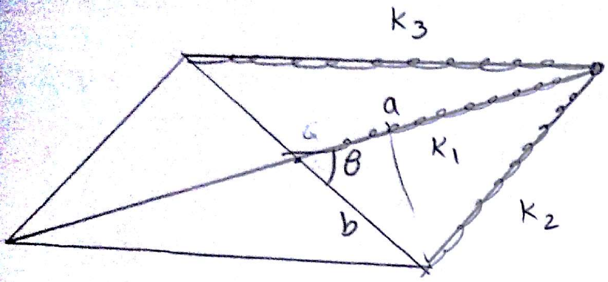
آزمون اسل ۹۱

مسئله ۱



(a1

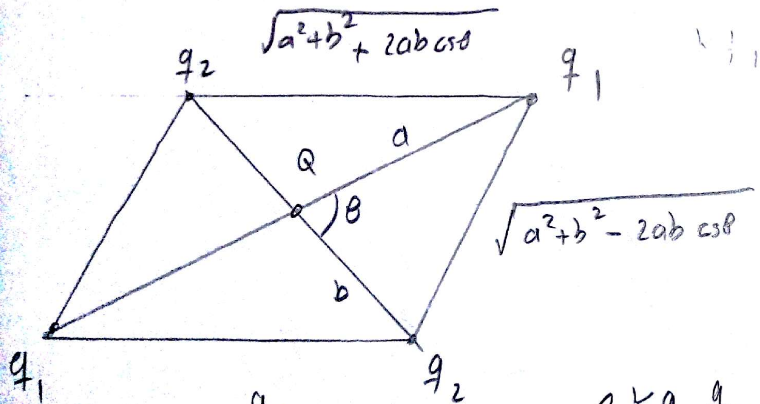
$$k_2 b \sin\theta = k_3 b \sin\theta \rightarrow \boxed{k_2 = k_3}$$

$$k_2 (a - b\cos\theta) + k_3 (a + b\cos\theta) + k_1 a = m a \omega^2$$

$$(k_1 + k_2 + k_3) \cancel{a} + \underbrace{\cancel{b\cos\theta (k_3 - k_2)}}_{0} = m \cancel{a} \omega^2 \rightarrow \boxed{m\omega^2 = k_1 + k_2 + k_3}$$

(a2

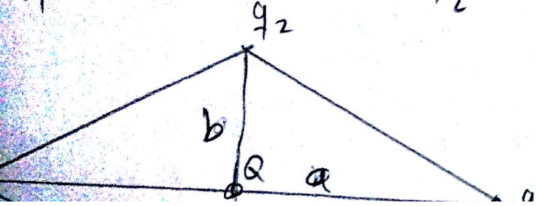
$$\omega^2 = \frac{k_1}{m} f\left(\frac{k_2}{k_1}, \frac{k_3}{k_1}\right) \leftarrow \text{تغییری نخواهد کرد}$$



(b1

$$\frac{kq_2 b \sin\theta}{(a^2+b^2-2ab\cos\theta)^{3/2}} = \frac{kq_2 b \sin\theta}{(a^2+b^2+2ab\cos\theta)^{3/2}}$$

$$\rightarrow \cos\theta = 0 \rightarrow \boxed{\theta = \frac{\pi}{2}}$$



$$\frac{2kq_1 q_2}{(a^2+b^2)^{3/2}} a + \frac{kq_1 Q}{a^2} + \frac{kq_1^2}{4a^2} = m a \omega^2$$

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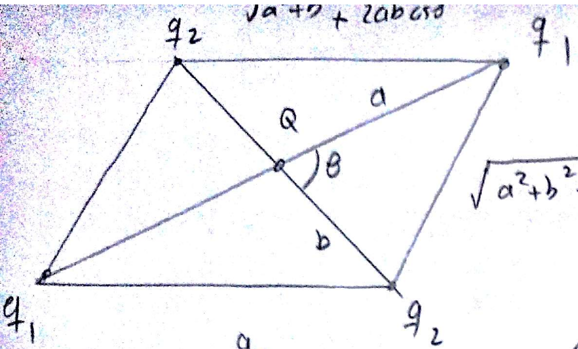
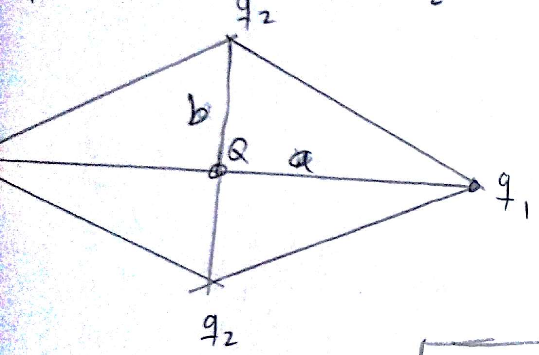
$\sqrt{a^2+b^2+2ab\cos\theta}$



$$\frac{k q_2 b \sin\theta}{(a^2+b^2-2ab\cos\theta)^{3/2}} = \frac{k q_2 b \sin\theta}{(a^2+b^2+2ab\cos\theta)^{3/2}} \quad \text{(b1}$$

$\sqrt{a^2+b^2-2ab\cos\theta}$ $\rightarrow \cos\theta = 0 \rightarrow \boxed{\theta = \frac{\pi}{2}}$



$$\frac{2kq_1q_2}{(a^2+b^2)^{3/2}}\,a + \frac{kq_1Q}{a^2} + \frac{kq_1^2}{4a^2} = ma\omega^2$$

$$\frac{2kq_1q_2\,b}{(a^2+b^2)^{3/2}} + \frac{kq_2Q}{b^2} + \frac{kq_2^2}{4b^2} = mb\omega^2$$

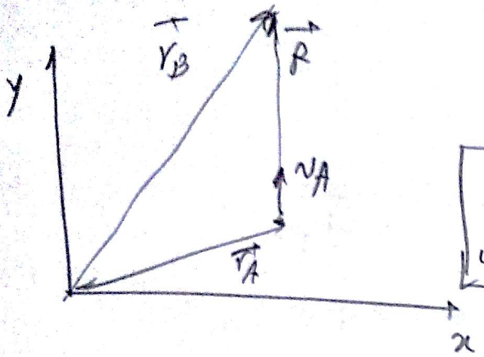
$$\rightarrow \boxed{\frac{2kq_1q_2}{(a^2+b^2)^{3/2}} + \frac{kq_1Q}{a^3} + \frac{kq_1^2}{4a^3} = m\omega^2}$$

$$= \frac{2kq_1q_2}{(a^2+b^2)^{3/2}} + \frac{kq_2Q}{b^3} + \frac{kq_2^2}{4b^3} \rightarrow \boxed{\frac{q_1Q}{a^3} + \frac{q_1^2}{4a^3} = \frac{q_2Q}{b^3} + \frac{q_2^2}{4b^3}}$$

(b2 اگر تغییری دهیم تا $\frac{qQ}{mL^3}$ تغییر نکند ω ثابت خواهد ماند $\omega^2 = \frac{kq^2}{ma^3} f(\dots)$

مسئله ۲

آ)



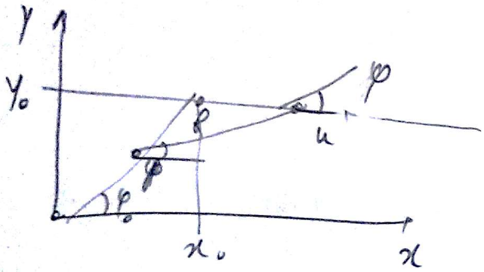
$$\dot{\vec R} = \dot{\vec r}_B - \vec v_A = \boxed{\dot{\vec r}_B - \frac{(\vec r_B - \vec r_A)}{|\vec r_B - \vec r_A|} v_A = \dot{\vec R}}$$

$$\boxed{\begin{aligned} \dot R &= u\cos\varphi - v \\ u\sin\varphi &= -R\dot\varphi \end{aligned}}$$

ب)

پ)

$$-R\frac{d\varphi}{dt} = u\sin\varphi \to dt = \frac{-R\,d\varphi}{u\sin\varphi} \to -\frac{dR\,u\sin\varphi}{R\,d\varphi} = u\cos\varphi - v$$



$$-\frac{dR}{R} = \frac{u\cos\varphi - v}{u\sin\varphi}\,d\varphi = \left(\cot\varphi - \frac{v}{u\sin\varphi}\right)d\varphi$$

$$-\ln\left(\frac{R}{R_0}\right) = \ln\left(\frac{\sin\varphi}{\sin\varphi_0}\right) + \frac{v}{u}\ln\left|\frac{\cot\varphi + \frac{1}{\sin\varphi}}{\cot\varphi_0 + \frac{1}{\sin\varphi_0}}\right|$$

$$\left(\frac{R}{R_0}\right)^{-1} = \frac{\sin\varphi}{\sin\varphi_0} \times \left(\frac{\cot\varphi + \frac{1}{\sin\varphi}}{\cot\varphi_0 + \frac{1}{\sin\varphi_0}}\right) \to \boxed{R = R_0\frac{\sin\varphi_0}{\sin\varphi}\left(\frac{\cot\varphi_0 + \frac{1}{\sin\varphi_0}}{\cot\varphi + \frac{1}{\sin\varphi}}\right)^{\frac{v}{u}}}$$

$$R = R_0\frac{\sin\varphi_0}{\sin\varphi}\left(\frac{\frac{\cos\varphi_0 + 1}{\sin\varphi_0}}{\frac{\cos\varphi + 1}{\sin\varphi}}\right)^K = R_0\frac{\sin\varphi_0}{\sin\varphi}\left(\frac{\cot\frac{\varphi_0}{2}}{\cot\varphi}\right)^K = R$$

//

$$\sin\varphi\left(\frac{\cdots\varphi_0}{\frac{\cos\varphi+1}{\sin\varphi}}\right)=R_0\frac{\sin\varphi_0}{\sin\varphi}\left(\frac{\operatorname{ct}\frac{\varphi_0}{2}}{\operatorname{ct}\frac{\varphi}{2}}\right)^K=R$$

ت)

اگر $R\to 0$: $\operatorname{ct}\frac{\varphi}{2}\to\infty \quad \frac{\varphi}{2}=0\to\varphi\ll 1\to R=\frac{R_0\sin\varphi_0}{\varphi}\left(\operatorname{ct}\frac{\varphi_0}{2}\right)^K\left(\frac{\varphi}{2}\right)^K$

$$\to R\propto\varphi^{K-1}\qquad K-1\geqslant 0\quad\boxed{K\geqslant 1}$$

ث)

$$\dot{\varphi}=\frac{-u\sin\varphi}{R_0\frac{\sin\varphi_0}{\sin\varphi}\left(\frac{\operatorname{ct}\frac{\varphi_0}{2}}{\operatorname{ct}\frac{\varphi}{2}}\right)^K}=\frac{-u\sin^2\varphi}{R_0\sin\varphi_0}\left(\frac{\tan\frac{\varphi_0}{2}}{\tan\frac{\varphi}{2}}\right)^K$$

$$\tan^2\frac{\varphi}{2}=\frac{\sin^2\frac{\varphi}{2}}{\cos^2\frac{\varphi}{2}}=\frac{1-\cos\varphi}{1+\cos\varphi}=\frac{\sin^2\varphi}{2-\sin^2\varphi}\qquad 2\tan\frac{\varphi}{2}-\sin^2\varphi\tan\frac{\varphi}{2}=\sin^2\varphi$$

$$\sin^2\varphi=\frac{2\tan\frac{\varphi}{2}}{1+\tan\frac{\varphi}{2}}\to\dot{\varphi}=\frac{-u}{R_0\sin\varphi_0}\times\left(\frac{2\tan\frac{\varphi}{2}}{1+\tan\frac{\varphi}{2}}\right)^2\left(\frac{1+\cos\varphi_0}{\sin\varphi_0\tan\frac{\varphi}{2}}\right)^K$$

$$=\boxed{\frac{-u\sqrt{x_0^2+y_0^2}}{R_0y_0}\times\left(\frac{2\tan\frac{\varphi}{2}}{1+\tan\frac{\varphi}{2}}\right)^2\left(\frac{\sqrt{x_0^2+y_0^2}+x_0}{y_0\tan\frac{\varphi}{2}}\right)^K=\dot{\varphi}}$$

ج ۱

$$\frac{d\varphi}{dt} = \frac{-4u \sin^2\frac{\varphi}{2}\cos^2\frac{\varphi}{2}}{R_0 \sin\varphi_0}\left(\frac{\tan\frac{\varphi_0}{2}}{\tan\frac{\varphi}{2}}\right)^K \qquad \frac{-4u}{R_0\sin\varphi_0}\left(\tan\frac{\varphi_0}{2}\right)^K t = \frac{d\varphi/2}{\sin^2\varphi/2\,\cos^2\varphi/2}\left(\tan\frac{\varphi}{2}\right)^K$$

$$= \frac{d\frac{\varphi}{2}}{\cos^2\varphi/2}\left(1+\frac{1}{\tan^2\varphi/2}\right)\tan^K\frac{\varphi}{2} = \frac{d\varphi/2}{\cos^2\varphi/2}\left(\tan^K\frac{\varphi}{2}+\tan^{K-2}\frac{\varphi}{2}\right)$$

$$= \frac{\tan^{K+1}\frac{\varphi}{2} - \tan^{K+1}\frac{\varphi_0}{2}}{K+1} + \frac{\tan^{K-1}\frac{\varphi}{2} - \tan^{K-1}\frac{\varphi_0}{2}}{K-1} = \frac{-2ut\left(\tan\frac{\varphi_0}{2}\right)^K}{R_0\sin\varphi_0}$$

$$\rightarrow t = \left(\frac{\tan^{K+1}\frac{\varphi_0}{2}}{K+1} + \frac{\tan^{K-1}\frac{\varphi_0}{2}}{K-1}\right)\frac{R_0\sin\varphi_0}{2u\left(\tan\frac{\varphi_0}{2}\right)^K} \qquad \tan^2\frac{\varphi}{2} = \frac{1-\cos\varphi}{1+\cos\varphi}$$

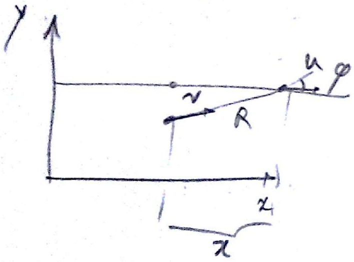
$$t = \left(\frac{1}{K+1}\left(\frac{\sqrt{x_0^2+y_0^2}-x_0}{\sqrt{x_0^2+y_0^2}+x_0}\right)^{1/2} + \frac{1}{K-1}\left(\frac{\sqrt{x_0^2+y_0^2}-x_0}{\sqrt{x_0^2+y_0^2}+x_0}\right)^{-1/2}\right)\frac{y_0}{2u}$$

$$\left(\left(\sqrt{x_0^2+y_0^2} + x_0 \right) \quad k-1 \left(\frac{\quad}{\sqrt{x_0^2+y_0^2} + x_0} \right)^2 \right) \frac{\quad}{2u}$$

$$t = \frac{y_0}{2u} \left(\frac{\sqrt{x_0^2+y_0^2} - x_0}{\sqrt{x_0^2+y_0^2} + x_0} \right)^{-\frac{1}{2}} \left(\frac{u}{v+u} \times \frac{\sqrt{x_0^2+y_0^2} - x_0}{\sqrt{x_0^2+y_0^2} + x_0} + \frac{u}{v-u} \right)$$

$$= \frac{y_0}{2} \left(\frac{\sqrt{x_0^2+y_0^2} + x_0}{\sqrt{x_0^2+y_0^2} - x_0} \right)^{\frac{1}{2}} \frac{(v-u)\left(\sqrt{x_0^2+y_0^2} - x_0\right) + (v+u)\left(\sqrt{x_0^2+y_0^2} + x_0\right)}{(v^2-u^2)\left(\sqrt{x_0^2+y_0^2} - x_0\right)}$$

$$= \frac{\cancel{y_0}}{\cancel{2}} \left(\frac{\sqrt{x_0^2+y_0^2} + x_0}{\sqrt{x_0^2+y_0^2} - x_0} \right)^{\frac{1}{2}} \times \frac{\cancel{2} v\sqrt{x_0^2+y_0^2} + \cancel{2} u x_0}{(v^2-u^2)\left(\cancel{\sqrt{x_0^2+y_0^2} + x_0}\right)^{\frac{1}{2}} \left(\cancel{y_0} \right.} = \boxed{\frac{v\sqrt{x_0^2+y_0^2} + u x_0}{v^2-u^2} = t}$$



$$\dot{x} = u - v\cos\varphi \rightarrow x - x_0 = ut - v\int \cos\varphi \, dt$$

$$\dot{R} = u\cos\varphi - v \qquad R - R_0 = u\int \cos\varphi \, dt - vt$$

$$\rightarrow \frac{R - R_0 + vt}{u} = \int \cos\varphi \, dt$$

$$\overset{0}{\cancel{x}} - x_0 = ut - \frac{v}{u}\left(\overset{0}{\cancel{R}} - R_0 + vt\right)$$

$$\rightarrow -x_0 = ut + \frac{v}{u}(R_0 - vt) \rightarrow x_0 + R_0\frac{v}{u} = \frac{v^2}{u}t - ut \rightarrow t = \frac{x_0 + R_0\frac{v}{u}}{\frac{v^2}{u} - u} = \frac{\sqrt{x_0^2+y_0^2}\, v + x_0 u}{v^2-u^2} = t$$

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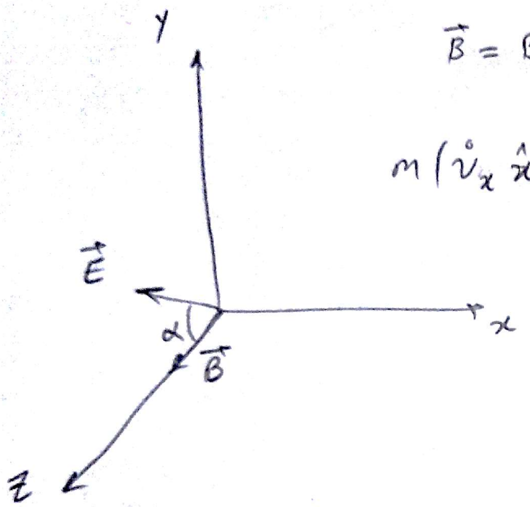
مسئله 3



$$\vec{B} = B\hat{z} \qquad \vec{E} = E\cos\alpha\,\hat{z} + \sin\alpha\,\hat{y}$$

آ) $\quad -v_x\hat{y} \quad v_y\hat{x}$

$$m(\dot{v}_x\hat{x} + \dot{v}_y\hat{y} + \dot{v}_z\hat{z}) = (E\cos\alpha\,\hat{z} + E\sin\alpha\,\hat{y})q + q(v_x\hat{x} + v_y\hat{y} + v_z\hat{z}) \times B\hat{z}$$

$$\boxed{m(\dot{v}_x\hat{x} + \dot{v}_y\hat{y} + \dot{v}_z\hat{z}) = +q v_y B\hat{x} + (-qBv_x + qE\sin\alpha)\hat{y} + qE\cos\alpha\,\hat{z}}$$

ب)

$$m\dot{v}_z = qE\cos\alpha \rightarrow \boxed{v_z = \frac{qE}{m}\cos\alpha\, t + v_{0z}}$$

$$m\dot{v}_x = q v_y B \rightarrow m(v_x - v_{0x}) = qyB$$

$$m\dot{v}_y = -qBv_x + qE\sin\alpha = -qB\times\left(\frac{qyB}{m} + v_{0x}\right) + qE\sin\alpha = \frac{-q^2B^2y}{m} - qBv_{0x} + qE\sin\alpha = m\ddot{y}$$

$$\ddot{y} = -\omega^2 y + \frac{qE}{m}\sin\alpha - \frac{qBv_{0x}}{m} \rightarrow y = \left(\frac{qE}{m\omega^2}\sin\alpha - \frac{qBv_{0x}}{m\omega^2}\right)(1-\cos\omega t) + \frac{v_{0y}}{\omega}\sin\omega t$$

$$q(E\sin\alpha - v_{0x}B)(1-\cos\omega t) + \frac{v_{0y}}{\omega}\sin\omega t$$

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$$m\ddot{x} = q v_y B \rightarrow m \dot{v}_x = q \dot{y} B$$

$$m\dot{v}_y = -qBv_x + qE\sin\alpha = -qB\times\left(\frac{qyB}{m} + v_{0x}\right) + qE\sin\alpha = \frac{-q^2B^2y}{m} - qBv_{0x} + qE\sin\alpha = m\ddot{y}$$

$$\ddot{y} = -\omega^2 y + \frac{qE}{m}\sin\alpha - \frac{qBv_{0x}}{m} \rightarrow y = \left(\frac{qE}{m\omega^2}\sin\alpha - \frac{qBv_{0x}}{m\omega^2}\right)(1-\cos\omega t) + \frac{v_{0y}}{\omega}\sin\omega t$$

$$y = \frac{q}{m\omega^2}\left(E\sin\alpha - v_{0x}B\right)(1-\cos\omega t) + \frac{v_{0y}}{\omega}\sin\omega t$$

$$\boxed{v_y = \frac{q}{m\omega}\left(E\sin\alpha - v_{0x}B\right)\sin\omega t + v_{0y}\cos\omega t}$$

$$v_x = \frac{q\cancel{B}^{\,\omega}}{\cancel{m}} y + v_{0x} = \boxed{\frac{q}{m\omega}\left(E\sin\alpha - v_{0x}B\right)(1-\cos\omega t) + v_{0y}\sin\omega t + v_{0x} = v_x}$$

$$x = \frac{q}{m\omega^2}\left(E\sin\alpha - Bv_{0x}\right)\left(\omega t - \frac{\sin\omega t}{\cancel{\omega}}\right) + \frac{v_{0y}}{\omega}(1-\cos\omega t) + v_{0x}t$$

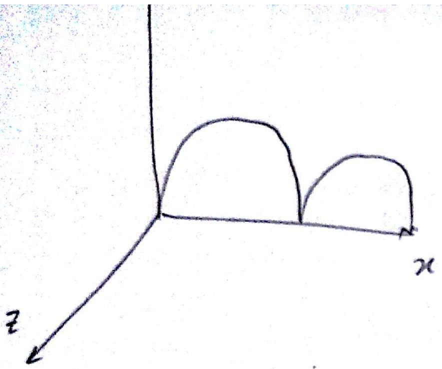
(ب)

$$\boxed{x = \frac{q}{m\omega^2}\left(E\sin\alpha - Bv_{0x}\right)(\omega t - \sin\omega t) + \frac{v_{0y}}{\omega}(1-\cos\omega t) + v_{0x}t}$$

$$\boxed{y = \frac{q}{m\omega^2}\left(E\sin\alpha - v_{0x}B\right)(1-\cos\omega t) + \frac{v_{0y}}{\omega}\sin\omega t}$$

$$\boxed{z = \frac{qE}{m}\cos\alpha\,\frac{t^2}{2} + v_{0z}t}$$

(ب)

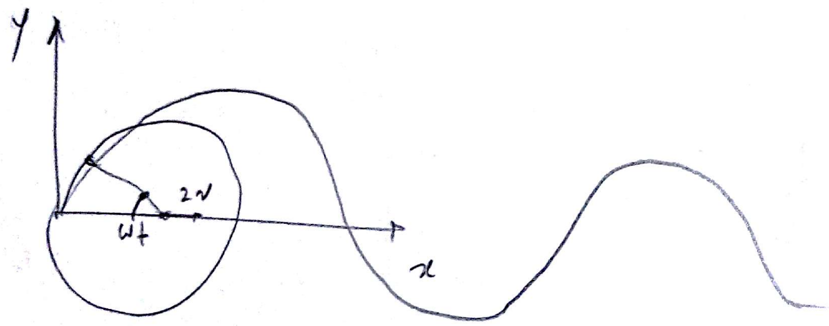


(1)

$$x = R_0(1 - \cos\omega t + 2\omega t) \qquad R_0 = \frac{E}{2B\omega}$$

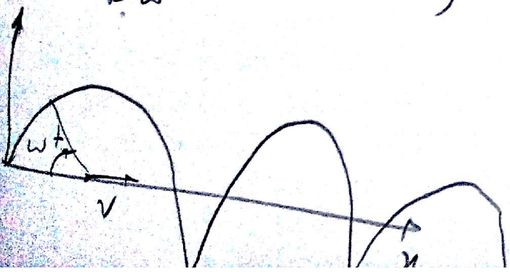
$$y = R_0 \sin\omega t$$

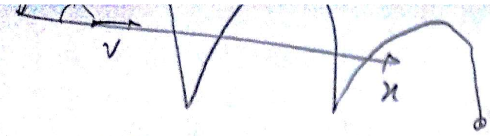
(2)



$$x = \frac{E}{B\omega}(1 - \cos\omega t + \omega t) \qquad y = \frac{E}{B\omega}\sin\omega t$$

(3)

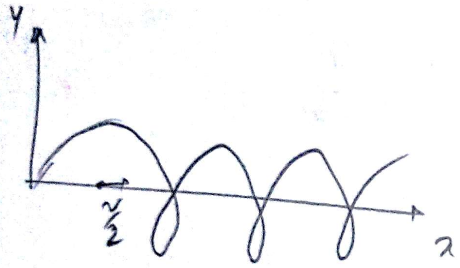




$$x = \frac{2E}{B\omega}(1-\cos\omega t) + \frac{E}{B\omega}(\omega t) = \frac{2E}{B\omega}\left(1-\cos\omega t+\frac{\omega t}{2}\right) = x$$

د (٤)

$$y = \frac{2E}{B\omega}\sin\omega t$$



(ث)

$$x = \frac{q}{m\omega^2}(E - Bv_{0x})(\omega t - \sin\omega t) + \frac{v_{0y}}{\omega}(1-\cos\omega t) + v_{0x}t$$

$$= \left(\frac{Eq}{m\omega^2} - \frac{v_{0x}}{\omega}\right)(\omega t - \sin\omega t) + \frac{v_{0y}}{\omega}(1-\cos\omega t) + v_{0x}t$$

$$\left(\frac{Eq}{m\omega\frac{qB}{m}}t - \cancel{v_{0x}t} - \frac{Eq}{m\omega^2}\sin\omega t + \frac{v_{0x}}{\omega}\sin\omega t\right) + \frac{v_{0y}}{\omega}(1-\cos\omega t)\ \cancel{+v_{0x}t}$$

$$x_{rel} = \left(\frac{v_{0x}}{\omega} - \frac{Eq}{m\omega^2}\right)\sin\omega t + \frac{v_{0y}}{\omega}(1-\cos\omega t), \quad y = \left(\frac{qE}{m\omega^2} - \frac{v_{0x}}{\omega}\right)(1-\cos\omega t) + \frac{v_{0y}}{\omega}\sin$$

دایره ی نانم

$$A := \frac{v_{0x}}{\omega} - \frac{Eq}{m\omega^2} \quad , \quad B := \frac{v_{0y}}{\omega}$$

$$x = A\sin\omega t + B(1-\cos\omega t) \rightarrow x - B = A\sin\omega t - B\cos\omega t$$

$$y = -A(1-\cos\omega t) + B\sin\omega t \rightarrow y + A = A\cos\omega t + B\sin\omega t$$

$(y+A)^2 + (x-B)^2 = A^2 + B^2 \rightarrow$ دایره‌ای به شعاع $\sqrt{\left(\frac{v_{0x}}{\omega} - \frac{Eq}{m\omega^2}\right)^2 + \left(\frac{v_{0y}}{\omega}\right)^2}$

$$\boxed{\left(y + \frac{v_{0x}}{\omega} - \frac{Eq}{m\omega^2}\right)^2 + \left(x - \frac{v_{0y}}{\omega}\right)^2 = \left(\frac{v_{0x}}{\omega} - \frac{Eq}{m\omega^2}\right)^2 + \left(\frac{v_{0y}}{\omega}\right)^2}$$

ج)

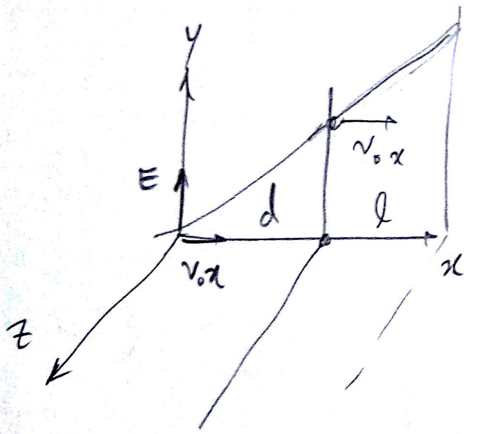
$$x = \frac{-qB_0}{m\omega^2} v_{0x}(\omega t - \sin\omega t) + v_{0x}t = -\frac{v_{0x}}{\omega}\left(\cancel{\omega t} - \sin\omega t\right) + \cancel{v_{0x}t}$$

$$x = \frac{v_{0x}}{\omega}\sin\omega t = l \rightarrow \frac{v_{0x}}{\omega} \times \omega t = l \rightarrow t = \frac{l}{\ }$$

$x = \frac{v_{0x}}{\omega}\sin\omega t = l \to \frac{v_{0x}}{\cancel{\omega}}\times\cancel{\omega} t = l \to t = \frac{l}{v_{0x}}$

$y = \frac{-qB_0}{m\omega^2}v_{0x}(1-\cos\omega t) = -\frac{v_{0x}}{\omega}(1-\cos\omega t) = -\frac{\cancel{v_{0x}}\, l}{\cancel{\omega}}\times\frac{\omega^2 t^2}{2} = -\frac{l\omega t}{2} = y$

$z = \frac{qE}{2m}t^2 \to z = \frac{qE}{2m}\times\left(\frac{2y}{l\omega}\right)^2 = \frac{\cancel{q}E\cancel{B}}{\cancel{2m}B}\times\frac{\cancel{4}^{2}y^2}{l^2\omega^{\cancel{2}}} = \boxed{\frac{E}{B}\times\frac{2y^2}{l^2\omega} = z}$



$t_1 = \frac{d}{v_{0x}}$ $\qquad v_{y_1} = \frac{Eq}{m}\times\frac{d}{v_{0x}}$

$y_1 = \frac{Eq}{2m}\times\frac{d^2}{v_{0x}^2}$ $\qquad t_2 = \frac{l-d}{v_{0x}}$

$y = \frac{Eqd}{mv_{0x}^2}\times\frac{l-d}{\cancel{v_{0x}}} + \frac{Eq}{2m}\frac{d^2}{v_{0x}^2}$

$= \frac{Eqd}{mv_{0x}^2}\left(l-d+\frac{d}{2}\right) = \boxed{\frac{Eqd}{mv_{0x}^2}\left(l-\frac{d}{2}\right) = y}$

(E

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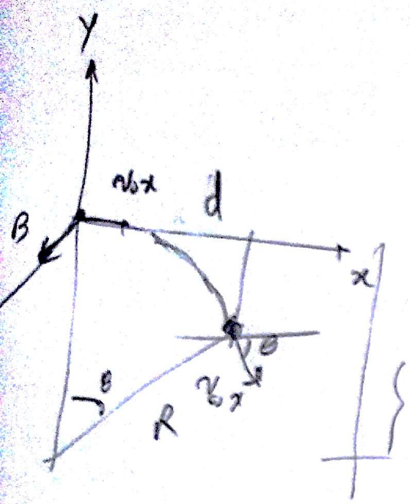


$$\frac{m v_{0x}^{2}}{R} = q\cancel{v_{0x}}B \rightarrow R = \frac{m v_{0x}}{qB}$$

$$R\sin\theta = d \rightarrow \sin\theta = \frac{qBd}{m v_{0x}} \qquad Y = -R(1-\cos\theta)$$

$$Y_1 = -R\left(1-\sqrt{1-\left(\frac{qBd}{m v_{0x}}\right)^2}\right)$$

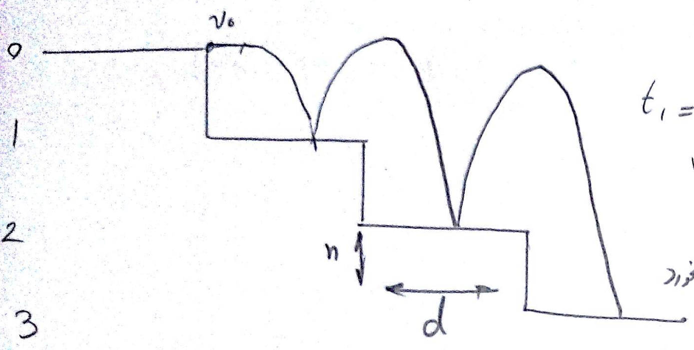
$$t_2 = \frac{l-d}{v_{0x}\cos\theta} = \frac{l-d}{v_{0x}\sqrt{1-\left(\frac{qBd}{m v_{0x}}\right)^2}}$$

$$\boxed{Y_2 = -R\left(1-\sqrt{1-\left(\frac{qBd}{m v_{0x}}\right)^2}\right) - \frac{qBd}{m}\times\frac{l-d}{v_{0x}\sqrt{1-\left(\frac{qBd}{m v_{0x}}\right)^2}}}$$

مسئله ۴

$$\frac{gt^2}{2} = h \rightarrow t = \sqrt{\frac{2h}{g}} \rightarrow \Delta x = v_0 \sqrt{\frac{2h}{g}} < d$$

الف)



$$t_1 = \sqrt{\frac{2h}{g}} \qquad t_2 = \sqrt{\frac{2h}{g}}\,(2+\sqrt{2})$$

بار دوم به پله اول نخورد : $3\sqrt{\frac{2h}{g}}\, v_0 > d \rightarrow \frac{v_0}{d}\sqrt{\frac{2h}{g}} > \frac{1}{3} \simeq 0.33$

بار دوم به پله سوم نخورد : $v_0\sqrt{\frac{2h}{g}}\,(2+\sqrt{2}) < 2d \rightarrow \frac{v_0}{d}\sqrt{\frac{2h}{g}} < \frac{2}{2+\sqrt{2}} \simeq 0.58$

بار سوم به پله دوم نخورد : $\sqrt{\frac{2h}{g}}\,(2+2\sqrt{2}+\sqrt{3}) > 2d \rightarrow \frac{v_0}{d}\sqrt{\frac{2h}{g}} > \frac{2}{2+2\sqrt{2}+\sqrt{3}} \simeq 0.348$

$$\boxed{\frac{1}{3} < \frac{v_0}{d}\sqrt{\frac{2h}{g}} < \frac{2}{2+\sqrt{2}}}$$

ب)

به پله ی n-1 نخورد : $(n-1)d < v_0\sqrt{\frac{2h}{g}}\left(2(1+\sqrt{2}+\dots+\sqrt{n-1})+\sqrt{n-1}\right)$

به پله n+1 نخورد : $v_0\sqrt{\frac{2h}{g}}\,(2(1+\sqrt{\dots}$

ب)

به پله ی $n-1$ بخورد : $(n-1)d < v_0\sqrt{\frac{2h}{g}}\left(2(1+\sqrt{2}+\dots+\sqrt{n-1})+\sqrt{n-1}\right)$

به پله $n+1$ نخورد : $v_0\sqrt{\frac{2h}{g}}\left(2(1+\sqrt{2}+\dots+\sqrt{n-1})+\sqrt{n}\right) < nd$

$$\frac{2(1+\sqrt{2}+\dots+\sqrt{n})+\sqrt{n}}{n} < \frac{2(1+\sqrt{2}+\dots+\sqrt{n+1})+\sqrt{n+1}}{n+1} = f_{(N)}$$

$$2(1+\sqrt{2}+\dots+\sqrt{n})+\sqrt{n} < 2n\sqrt{n+1}+\sqrt{n+1}\,n$$

$$\swarrow$$

$$2n\sqrt{n}+\sqrt{n} = \sqrt{n}(2n+1) < 3n\sqrt{n+1} \qquad \cancel{2n+1<3n} \rightarrow 1<n \rightarrow \text{بدیهی می‌شود}$$

$f_{(N)}$ صعودی است اگر $1\le n$

$$v_0\sqrt{\frac{2h}{g}} > \frac{(n-1)d}{2(1+\sqrt{2}+\dots+\sqrt{n-1})+\sqrt{n}} = \frac{d}{f_{(N)}} \qquad 1<n-1 \rightarrow 2<n$$

$$\rightarrow \boxed{v_0\sqrt{\frac{2h}{g}} > \frac{d}{3}}$$

$$g_{(N)} = \frac{2(1+\sqrt{2}+\dots+\sqrt{n-1})+\sqrt{n}}{n} < \frac{2(1+\sqrt{2}+\dots+\sqrt{n})+\sqrt{n+1}}{n+1} \Rightarrow$$

$$2(1+\sqrt{2}+\dots+\sqrt{n-1})+\sqrt{n} \overset{+n\sqrt{n}}{} < \cancel{2}n\sqrt{n}+n\sqrt{n+1}$$

$$2(1+\sqrt{2}+\dots+\sqrt{n-1})+\sqrt{n} < n\sqrt{n}+n\sqrt{n+1}$$

$$2(n-1)\sqrt{n-1}+\cancel{\sqrt{n}} < (n-1)\sqrt{n}+n\sqrt{n+1} \rightarrow$$

بدیهی

$g_{(n)}$ صعودی است

$$\frac{v_0}{d}\sqrt{\frac{2h}{g}} < \frac{n}{2(1+\sqrt{2}+\dots+\sqrt{n-1})+\sqrt{n}} = \frac{1}{g_{(N)}} \rightarrow$$

فقط به ازای N

کافیست

$$\boxed{\frac{v_0}{d}\sqrt{\frac{2h}{g}} < \frac{n}{2(1+\sqrt{2}+\dots+\sqrt{n-1})+\sqrt{n}}}$$

ب)

$$\frac{gt^2}{2} = 2h + x\tan\theta$$

v_0 v_0

$$v_0 t = x$$

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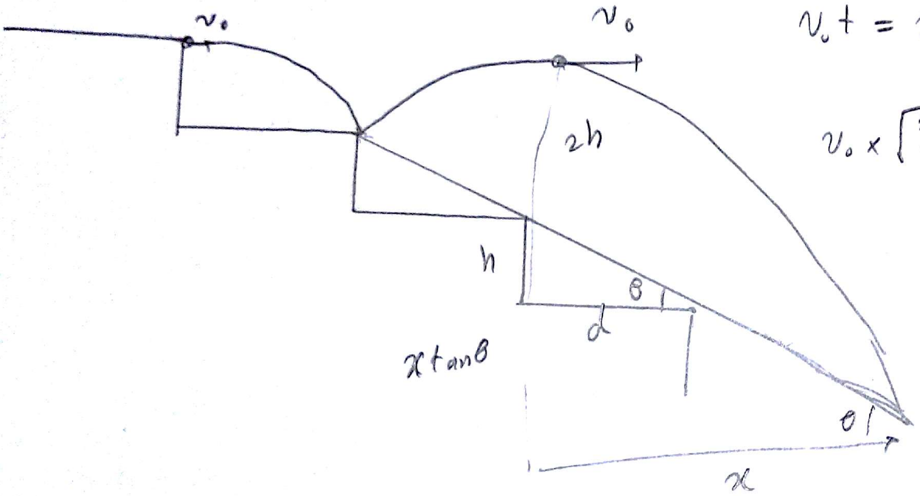
$$\boxed{\frac{v_0}{d}\sqrt{\frac{2h}{g}} < \frac{h}{2(1+\sqrt{2}+\dots+\sqrt{n-1})+\sqrt{n}}}$$

کافیست

ب)

$$\frac{gt^2}{2} = 2h + x\tan\theta$$



$$v_0 t = x$$

$$v_0 \times \sqrt{\frac{2h}{g}} = d \rightarrow v_0 = d\sqrt{\frac{g}{2h}}$$

$$t = \frac{x}{v_0} \rightarrow \frac{g}{2} \times \frac{x^2}{v_0^2} = 2h + x\tan\theta$$

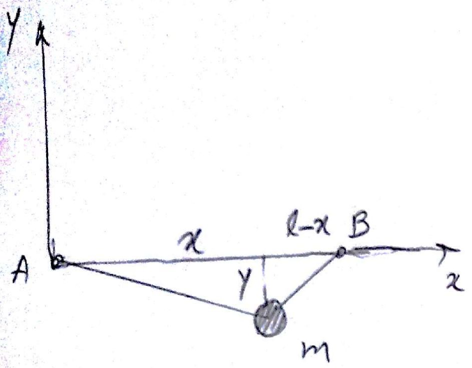
$$\frac{2v_0^2}{g} = \frac{d^2}{h}$$

$$\frac{x^2}{d^2} h = 2h + x \times \frac{h}{d} \qquad x^2 \cancel{h} = 2\cancel{h} d^2 + x \cancel{h} d \rightarrow x^2 - xd - 2d^2 = 0$$

$$x = \frac{d \pm \sqrt{d^2 + 8d^2}}{2} = \frac{d \pm 3\sqrt{d^2}}{2} = d\,\frac{1+3}{2} = 2d \rightarrow$$ به طبقه پله ی چهارم می خورد

آزمون 2 سال ۹۱

مسئله ۱



(a

$$k \times \frac{\ell}{dx} \times \frac{dx}{x} = \boxed{\frac{k\ell}{x} = k_{eq}}$$

(b

$$U = -mgy + \frac{k\ell}{2x}\left(\sqrt{x^2+y^2}-x\right)^2 + \frac{k\ell}{2(\ell-x)}\left(\sqrt{(\ell-x)^2+y^2}-(\ell-x)\right)^2$$

$$\frac{y^4}{4x^2}$$

$$x\left(1+\frac{y^2}{x^2}\right)^{\frac{1}{2}} - x = x\left(\cancel{1}+\frac{y^2}{2x^2}\right) - \cancel{x} = \frac{y^2}{2x}$$

$$U = -mgy + \frac{k\ell}{8x^3}y^4 + \frac{k\ell}{8(\ell-x)^3}y^4 = \boxed{k\ell\left(-\epsilon y + \frac{y^4}{8x^3} + \frac{y^4}{8(\ell-x)^3}\right) = U}$$

(c

$$\frac{\partial U}{\partial x} = 0 \to -\beta x^{-4} + \beta(\ell-x)^{-4} = 0 \to x = \ell - x \to \boxed{x = \frac{\ell}{2}}$$

$$= 0 \quad 4y^3$$

$\frac{\partial U}{\partial y} = 0 \rightarrow -\epsilon + \frac{4y^3}{\ell^3} = 0 \rightarrow \frac{8y^3}{\ell^3} = \epsilon \rightarrow \boxed{y = \frac{1}{2}\epsilon^{\frac{1}{3}}\ell}$ (c

$U = U_0 + \cancel{\frac{\partial U}{\partial x}\delta x} + \cancel{\frac{\partial U}{\partial y}\delta y} + \frac{\partial^2 U}{\partial x^2}\frac{\delta x^2}{2} + \frac{\partial^2 U}{\partial y^2}\frac{\delta y^2}{2} + \cancel{\frac{\partial^2 U}{\partial x \partial y}\delta x \delta y}$ (d

$\frac{\partial U}{\partial y} = k\ell\left(-\epsilon + \frac{4y^3}{2\cancel{8}x^3} + \frac{4y^3}{2\cancel{8}(\ell-x)^3}\right)$ $\frac{\partial^2 U}{\partial x \partial y} = k\ell\left(\cancel{\frac{-12y^3}{8x^4} + \frac{-3 \times 4y^3 \times -1}{8(\ell-x)^4}}\right) \rightarrow 0$

$\frac{\partial^2 U}{\partial y^2} = k\ell\left(\frac{3y^2}{2x^3} + \frac{3y^2}{2(\ell-x)^3}\right) = \frac{k\ell}{\frac{\ell^3}{8}} \times 3 \times \left(\frac{1}{2}\epsilon^{\frac{1}{3}}\ell\right)^2 = \frac{\cancel{8}k \times 3}{\ell^2} \times \frac{1}{\cancel{4}}\epsilon^{\frac{2}{3}}\ell^2$

$= \frac{6k}{\cancel{\ell^2}}\epsilon^{\frac{2}{3}}\cancel{\ell^2} = 3k\epsilon^{\frac{2}{3}}$

$\frac{\partial U}{\partial x} = \frac{k\ell y^4}{8}\left(\frac{-3}{x^4} + \frac{3}{(\ell-x)^4}\right)$ $\frac{\partial^2 U}{\partial x^2} = \frac{3k\ell y^4}{\cancel{8}_2}\left(\frac{+4}{x^5} + \frac{4}{(\ell-x)^5}\right)$

$= \frac{3k\ell y^4}{\left(\frac{\ell}{2}\right)^5} = \frac{3k\cancel{\ell}\cancel{y^4}\frac{1}{\cancel{16}}\epsilon^{\frac{4}{3}}\cancel{\ell^4}}{\frac{\ell^5}{32}} = 6k\epsilon^{\frac{4}{3}}$

$\rightarrow U = k\ell\left(-\frac{1}{2}\epsilon^{\frac{4}{3}}\ell + \frac{\left(\frac{1}{2}\right)^4\epsilon^{\frac{4}{3}}\ell^4\ell}{4 \times \left(\frac{\ell}{2}\right)^3}\right) + 3k\epsilon^{\frac{2}{3}}\frac{\delta y^2}{2} + \cancel{6}k\epsilon^{\frac{4}{3}}\frac{\delta x^2}{\cancel{2}} = -\frac{3}{8}k\ell^2\epsilon^{\frac{4}{3}} + \frac{3}{2}k\epsilon^{\frac{2}{3}}\delta y^2 + 3k\epsilon^{\frac{4}{3}}\delta x^2$

$$\boxed{U = -\frac{3}{8} k a^2 \epsilon^{4/3} + 3k\epsilon^{2/3} \delta y^2 + 3k\epsilon^{4/3}\delta x^2}$$

$$\omega_x^2 = \frac{3k\epsilon^{4/3}}{m/2} = \boxed{\frac{6k}{m}\epsilon^{4/3} = \omega_x^2}$$

(e

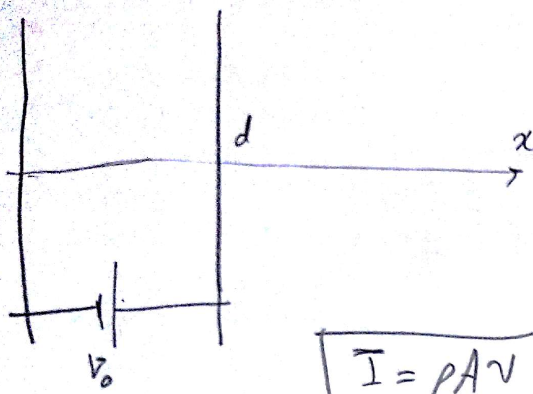
$$\omega_y^2 = \frac{3k\epsilon^{2/3}}{m/2} = \boxed{\frac{6k\epsilon^{2/3}}{m} = \omega_y^2}$$

$$\frac{\omega_x}{\omega_y} \in Q \rightarrow \left(\frac{\cancel{6}\,\epsilon^{\cancel{4/3}\ 2/3}}{\cancel{6}\,\cancel{\epsilon^{2/3}}}\right)^{1/2} = \boxed{\epsilon^{1/3} \in Q}$$

(f

مسئله 2

آ)



$$\frac{m}{2}v^2 - eV_{(x)} = 0$$

$$\rightarrow \boxed{v = \left(\frac{2eV_{(x)}}{m}\right)^{\frac{1}{2}}}$$

ب)

$$\boxed{I = \rho A v}$$

پ)

$$\frac{dE}{dx} = \frac{\rho}{\varepsilon_0} = \frac{I}{Av\varepsilon_0} = \frac{I}{A\varepsilon_0}\left(\frac{m}{2eV_{(x)}}\right)^{\frac{1}{2}} = -\frac{d^2V}{dx^2} = -\frac{dV'}{dx}$$

$$\frac{I}{A\varepsilon_0}\left(\frac{m}{2eV_{(x)}}\right)^{\frac{1}{2}} \rightarrow -V'dV' = -\frac{V'^2}{2} = \frac{I}{A\varepsilon_0}\sqrt{\frac{m}{2e}}\,V_{(x)}^{-\frac{1}{2}}\,dV = \frac{I}{A\varepsilon_0}\sqrt{\frac{m}{2e}}\,2V^{\frac{1}{2}} = -\frac{E^2}{2}$$

$$\rightarrow E = \sqrt{\frac{-4I}{A\varepsilon_0}\sqrt{\frac{mV}{2e}}} \qquad -\frac{dV}{dx} = \overbrace{\sqrt{-\frac{4I}{A\varepsilon_0}\sqrt{\frac{m}{2e}}}}^{\alpha}\,V^{\frac{1}{4}} \rightarrow -\frac{dV}{dx} = \alpha V^{\frac{1}{4}} \rightarrow -\frac{4}{3}V^{\frac{3}{4}} = \alpha x$$

$$V = \left(-\frac{3}{4}\alpha x\right)^{\frac{4}{3}} = \left(-\frac{3}{4}\times\sqrt{\frac{-4I}{A\varepsilon_0}\sqrt{\frac{m}{2e}}}\,x\right)^{\frac{4}{3}} = V$$

$$V = \left(-\frac{3}{4}\alpha x\right)^{\frac{4}{3}} = \left(-\frac{3}{4}\times\sqrt{\frac{-4I}{A\varepsilon_0\sqrt{\frac{m}{2e}}}}\,x\right)^{\frac{4}{3}} = V$$

$$E = -\frac{dV}{dx} = -\left(-\frac{3}{4}\sqrt{\frac{-4I}{A\varepsilon_0\sqrt{\frac{m}{2e}}}}\right)^{\frac{4}{3}} \times \frac{4}{3}x^{\frac{1}{3}} = -\frac{4}{3}x^{\frac{1}{3}}\left(-\frac{81}{\cancel{2}8}\times\frac{\cancel{16}I^2}{A^2\varepsilon_0^2}\times\frac{m}{\cancel{2}e}\right)^{\frac{1}{3}}$$

$$= \frac{\cancel{4}}{\cancel{3}}x^{\frac{1}{3}}\left(\frac{\overset{3\times 2}{\cancel{81}}mI^2}{\cancel{32}A^2\varepsilon_0^2 e}\right)^{\frac{1}{3}} = E = \boxed{\left(\frac{6mI^2}{A^2\varepsilon_0^2 e}x\right)^{\frac{1}{3}} = E}$$

$$V_{(x)} = \left(\frac{81}{256}\times\frac{16I^2}{A^2\varepsilon_0^2}\times\frac{m}{2e}x^{4}\right)^{\frac{1}{3}} = \boxed{\left(\frac{81}{32}\,\frac{I^2 m}{A^2\varepsilon_0^2 e}x^4\right)^{\frac{1}{3}} = V}$$

(ب

$$V_0^3 = \frac{81}{32}\,\frac{I^2 m}{A^2\varepsilon_0^2 e}d^4 \rightarrow I^2 = \frac{32}{81}\,\frac{V_0^3 A^2\varepsilon_0^2 e}{m\,d^4} \rightarrow \boxed{I = \frac{4}{9}\,\frac{A\varepsilon_0 V_0}{d^2}\sqrt{\frac{2V_0 e}{m}}}$$

(ج

$$\rho = \frac{I}{A\times\sqrt{\frac{2eV}{m}}} = \frac{4}{9}\,\frac{\varepsilon_0 V_0}{d^2}\times\sqrt{\frac{V_0}{V}} = \rho =$$

(د

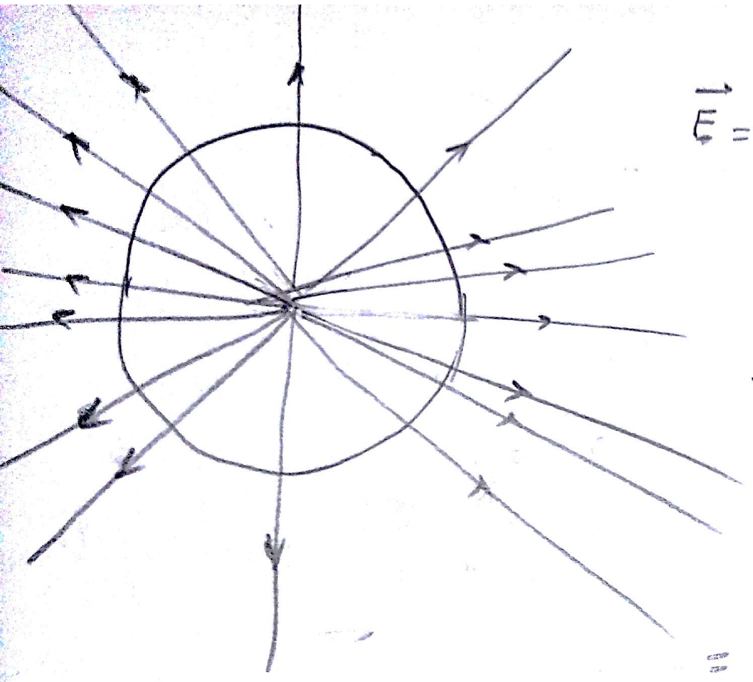
$$V = \frac{V_0}{d^{\frac{4}{3}}}\times x^{\frac{4}{3}} \rightarrow \rho = \frac{4\varepsilon_0 V_0}{9d^2}\times\left(\frac{d^{\frac{4}{3}}}{x^{\frac{4}{3}}}\right)^{\frac{1}{2}} = \boxed{\frac{4\varepsilon_0 V_0}{9d^2}\left(\frac{d}{x}\right)^{\frac{2}{3}} = \rho}$$

$$v = \left(\frac{2e}{m} \times V_0 \left(\frac{x}{d}\right)^{4/3}\right)^{1/2} = \boxed{\sqrt{\frac{2eV_0}{m}} \left(\frac{x}{d}\right)^{2/3} = v}$$ ج:

$$\sqrt{\frac{2eV_0}{m}} \, \frac{x^{2/3}}{d^{2/3}} = \frac{dx}{dt} \qquad \sqrt{\frac{2eV_0}{m}} \, \frac{t}{d^{2/3}} = 3\cancel{x^{1/3}} \nearrow d^{1/3} \quad + \quad \boxed{t = 3d\sqrt{\frac{m}{2eV_0}}}$$ (ع

مسئله 4

الف)



$$\vec{E} = \frac{q}{4\pi\varepsilon_0} \frac{\left(1-\frac{v^2}{c^2}\right)\vec{R}}{\left(1-\frac{v^2}{c^2}\sin^2\theta\right)^{3/2} R^3}$$

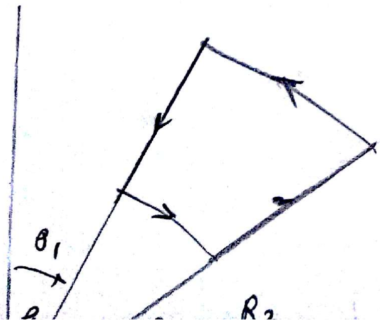
ب)

$$\oint \vec{E}\cdot d\vec{a} = \frac{q}{\frac{\cancel{4\pi}\varepsilon_0}{2}} \left(1-\frac{v^2}{c^2}\right) \int \frac{\cancel{2\pi} \cancel{R^2} \sin\theta \, d\theta}{\left(1-\frac{v^2}{c^2}\sin^2\theta\right)^{3/2} \cancel{R^2}}$$

$$= \frac{q}{2\varepsilon_0}\left(1-\frac{v^2}{c^2}\right) \int \frac{\sin\theta \, d\theta}{\left(1-\frac{v^2}{c^2}(1-\cos^2\theta)\right)^{3/2}}$$

$$= \frac{q}{2\varepsilon_0} \frac{\cancel{\left(1-\frac{v^2}{c^2}\right)}}{\left(1-\frac{v^2}{c^2}\right)^{\cancel{3/2}\,1/2}} \int \frac{\sin\theta \, d\theta}{\left(1+\frac{\frac{v^2}{c^2}\cos^2\theta}{1-\frac{v^2}{c^2}}\right)^{3/2}}$$

$$= \frac{-q}{2\varepsilon_0} \frac{c}{v} \int \frac{du}{(1+u^2)^{3/2}} = \frac{+q}{2\varepsilon_0} \frac{c}{v} \left[\frac{2v^2}{(c^2-v^2)^{1/2}}\right] = \boxed{\frac{q}{\varepsilon_0} \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \oint \vec{E}\cdot d\vec{a}}$$



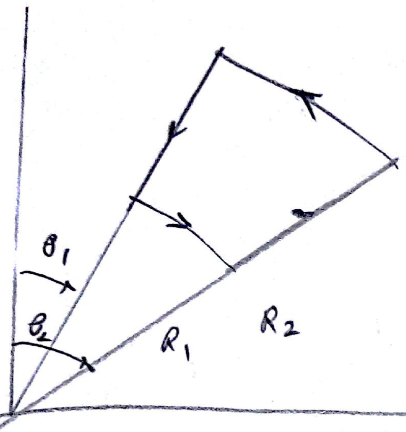
پ)

$$\int \vec{E}\cdot dR = \frac{q}{4\pi\varepsilon_0} \frac{1-\frac{v^2}{c^2}}{\left(1-\frac{v^2}{c^2}\sin^2\theta\right)^{3/2}} \int \frac{dR}{R^2}$$

$$= \frac{q}{4\pi\varepsilon_0} \frac{1-\frac{v^2}{c^2}}{\ldots} \left(\frac{1}{\ldots} - \frac{1}{\ldots}\right)$$

$$\left(1-\frac{v^2}{c^2}\right)^{\cancel{3/2}\,1/2} \qquad \left(1+\frac{\frac{v^2}{c^2}\cos^2\theta}{1-\frac{v^2}{c^2}}\right)^{3/2}$$

$$= \frac{+}{2\varepsilon_0}\,\frac{c}{v}\int\frac{du}{(1+u^2)^{3/2}} = \frac{+q}{2\varepsilon_0}\,\frac{c}{v}\left[\frac{2v^2}{(c^2-v^2)^{1/2}}\right] = \boxed{\frac{q}{\varepsilon_0}\,\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \int\vec{E}\cdot d\vec{a}}$$



$$\int\vec{E}\cdot dR = \frac{q}{4\pi\varepsilon_0}\,\frac{1-\frac{v^2}{c^2}}{\left(1-\frac{v^2}{c^2}\sin^2\theta\right)^{3/2}}\int\frac{dR}{R^2}$$ ب)

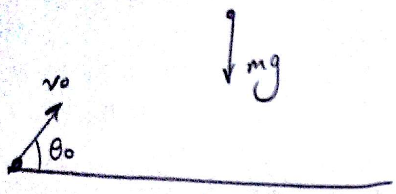
$$= \frac{q}{4\pi\varepsilon_0}\,\frac{1-\frac{v^2}{c^2}}{\left(1-\frac{v^2}{c^2}\sin^2\theta\right)^{3/2}}\left(\frac{1}{R_2}-\frac{1}{R_1}\right)$$

$$\boxed{\int\vec{E}\cdot d\vec{r} = \frac{q}{4\pi\varepsilon_0}\left(1-\frac{v^2}{c^2}\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\left(\frac{1}{\left(1-\frac{v^2}{c^2}\sin^2\theta_2\right)^{3/2}}-\frac{1}{\left(1-\frac{v^2}{c^2}\sin^2\theta_1\right)^{3/2}}\right)}$$

$$\boxed{\vec{E} = \frac{\lambda\vec{R}}{2\pi\varepsilon_0 R^2}\times\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}}$$ ج)

مسئله 5

الف)



$$m\ddot{y} = -mg \rightarrow \ddot{y} = -g \ , \ \ddot{x} = 0$$

ب)

$$R_{max} = f(v_x, v_y, g)$$

$$[x] = [v_x]^{\alpha_1}[v_y]^{\alpha_2}[g]^{\alpha_3} = \left(\frac{x}{t}\right)^{\alpha_1}\left(\frac{y}{t}\right)^{\alpha_2}\left(\frac{y}{t^2}\right)^{\cancel{\alpha_3}\,-\alpha_2}$$

$$\alpha_3 = -\alpha_2 \ , \alpha_1 = 1 \ , \alpha_2 = 1 \ , \alpha_3 = -1 \rightarrow R_{max} = \alpha_1 \frac{v_x v_y}{g} = \alpha_1 \frac{v_0 \cos\theta_0 \times v_0 \sin\theta_0}{g}$$

$$\boxed{R_{max} = \frac{v_0^2 \sin\theta_0 \cos\theta_0}{g} \times \alpha_1}$$

پ)

$$\vec{F} = -m\beta\vec{v} \rightarrow [\beta] = \frac{1}{t} \rightarrow R_{max} = \alpha_1 \frac{v_0^2 \sin\theta_0 \cos\theta_0}{g} f\left(\beta \frac{v_y}{g}\right)$$

$$\boxed{R_{max} \simeq \alpha_1 \frac{v_0^2 \sin\theta_0 \cos\theta_0}{g}\left(1 + \beta\alpha_2 \frac{v_0 \sin\theta_0}{g}\right)}$$

ت)

R° $v_0^2 \cancel{\sin\theta_0 \cos\theta_0}$

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$$R_{max} = \alpha_1 \frac{v_0 \sin\theta_0 \cos\theta_0}{g} f\left(\beta \frac{v_y}{g}\right)$$ (ب

$$\boxed{R_{max} \simeq \alpha_1 \frac{v_0^2 \sin\theta_0 \cos\theta_0}{g}\left(1+\beta\alpha_2 \frac{v_0 \sin\theta_0}{g}\right)}$$

(ج

$$R^0_{max} = \alpha_1 \frac{v_0^2 \cancel{\sin\theta_0 \cos\theta_0}}{2g} \rightarrow R_{max} = 2R^0_{max} \sin\theta_0 \cos\theta_0 \left(1+\beta\alpha_2 \frac{v_0 \sin\theta_0}{g}\right)$$

$$= 2R^{(0)}_{max}\left(\sin\theta\cos\theta + \frac{\beta\alpha_2 v_0}{g}\sin^2\theta\cos\theta\right)$$

$$\frac{dR_{max}}{d\theta} = 0 \rightarrow \cancel{\cos^2\theta - \sin^2\theta} + \frac{\beta\alpha_2 v_0}{g}\left(2\sin\theta\cos^2\theta - \sin^3\theta\right) = 0$$

$$2\cos^2\theta - 1$$

$$2\gamma - 2\cos\theta\sin\theta = -2\delta$$

$$\rightarrow -2\delta + \frac{\beta\alpha_2 v_0}{g} \times \cancel{\left(\frac{\sqrt{2}}{2}\right)^3}^{\frac{1}{2\sqrt{2}}} = 0 \rightarrow \frac{\beta\alpha_2 v_0}{g \times 2\sqrt{2}} = 2\delta \rightarrow \frac{\beta\alpha_2 v_0}{g} = 4\sqrt{2}\,\delta$$

$$\boxed{R_{max} = 2R^{(0)}_{max} \sin\theta_0 \cos\theta_0 \left(1 + 4\sqrt{2}\,\delta \sin\theta_0\right)}$$