The direct compressive stress (σ_c) in the column due to the weight of the water tank is given by

$$\sigma_c = \frac{Mg}{bd} = \frac{Mg}{x_1 x_2} \tag{E$_5$}$$

and the buckling stress for a fixed-free column (σ_b) is given by [1.71]

$$\sigma_b = \left(\frac{\pi^2 EI}{4l^2}\right)\frac{1}{bd} = \frac{\pi^2 E x_2^2}{48l^2} \tag{E$_6$}$$

To avoid failure of the column, the direct stress has to be restricted to be less than σ_{max} and the buckling stress has to be constrained to be greater than the direct compressive stress induced.

Finally, the design variables have to be constrained to be positive. Thus the multiobjective optimization problem can be stated as follows:

$$\text{Find } \mathbf{X} = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} \text{ which minimizes}$$

$$f_1(\mathbf{X}) = \rho l x_1 x_2 \tag{E$_7$}$$

$$f_2(\mathbf{X}) = -\left[\frac{E x_1 x_2^3}{4l^2(M + \frac{33}{140}\rho l x_1 x_2)}\right]^{1/2} \tag{E$_8$}$$

subject to

$$g_1(\mathbf{X}) = \frac{Mg}{x_1 x_2} - \sigma_{max} \le 0 \tag{E$_9$}$$

$$g_2(\mathbf{X}) = \frac{Mg}{x_1 x_2} - \frac{\pi^2 E x_2^2}{48l^2} \le 0 \tag{E$_{10}$}$$

$$g_3(\mathbf{X}) = -x_1 \le 0 \tag{E$_{11}$}$$

$$g_4(\mathbf{X}) = -x_2 \le 0 \tag{E$_{12}$}$$

1.6 OPTIMIZATION TECHNIQUES

The various techniques available for the solution of different types of optimization problems are given under the heading of mathematical programming techniques in Table 1.1. The classical methods of differential calculus can be used to find the unconstrained maxima and minima of a function of several variables. These methods assume that the function is differentiable twice with respect to the design variables and the derivatives are continuous. For problems with equality constraints, the Lagrange multiplier method can be used. If the problem has inequality constraints, the Kuhn–Tucker conditions can be used to identify the optimum point. But these methods lead to a set of nonlinear simultaneous equations that may be difficult to solve. The classical methods of optimization are discussed in Chapter 2.

The techniques of nonlinear, linear, geometric, quadratic, or integer programming can be used for the solution of the particular class of problems indicated by the name of the technique. Most of these methods are numerical techniques wherein an approximate solution is sought by proceeding in an iterative manner by starting from an initial solution. Linear programming techniques are described in Chapters 3 and 4. The quadratic programming technique, as an extension of the linear programming approach, is discussed in Chapter 4. Since nonlinear programming is the most general method of optimization that can be used to solve any optimization problem, it is dealt with in detail in Chapters 5–7. The geometric and integer programming methods are discussed in Chapters 8 and 10, respectively. The dynamic programming technique, presented in Chapter 9, is also a numerical procedure that is useful primarily for the solution of optimal control problems. Stochastic programming deals with the solution of optimization problems in which some of the variables are described by probability distributions. This topic is discussed in Chapter 11.

In Chapter 12 we discuss some additional topics of optimization. An introduction to separable programming is presented in Section 12.2. A brief discussion of multiobjective optimization is given in Section 12.3. In Sections 12.4 to 12.6 we present the basic concepts of simulated annealing, genetic algorithms, and neural network methods, respectively. When the problem is one of minimization or maximization of an integral, the methods of the calculus of variations presented in Section 12.7 can be used to solve it. An introduction to optimal control theory, which can be used for the solution of trajectory optimization problems, is given in Seciton 12.8. Sensitivity analysis and other computational issues are discussed in the context of solution of practical optimization problems in Chapter 13.

1.7 ENGINEERING OPTIMIZATION LITERATURE

The literature on engineering optimization is large and diverse. Several textbooks are available and dozens of technical periodicals regularly publish papers related to engineering optimization. This is primarily because optimization is applicable to all areas of engineering. Researchers in many fields must be attentive to the developments in the theory and applications of optimization.

The most widely circulated journals that publish papers related to engineering optimization are *Engineering Optimization, ASME Journal of Mechanical Design, AIAA Journal, ASCE Journal of Structural Engineering, Computers* *and Structures, International Journal for Numerical Methods in Engineering, Structural Optimization, Journal of Optimization Theory and Applications, Computers and Operations Research, Operations Research*, and *Management Science.* Many of these journals are cited in the chapter references.

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REVIEW QUESTIONS

1.1 Match the following terms and descriptions.

(a) Free feasible point	$g_j(\mathbf{X}) = 0$
(b) Free infeasible point	Some $g_j(\mathbf{X}) = 0$ and other $g_j(\mathbf{X}) < 0$
(c) Bound feasible point	Some $g_j(\mathbf{X}) = 0$ and other $g_j(\mathbf{X}) \geq 0$
(d) Bound infeasible point	Some $g_j(\mathbf{X}) > 0$ and other $g_j(\mathbf{X}) < 0$
(e) Active constraints	All $g_j(\mathbf{X}) < 0$

1.2 Answer true or false.

(a) Optimization problems are also known as mathematical programming problems.

(b) The number of equality constraints can be larger than the number of design variables.

(c) Preassigned parameters are part of design data in a design optimization problem.

(d) Side constraints are not related to the functionality of the system.

(e) A bound design point can be infeasible.

(f) It is necessary that some $g_j(\mathbf{X}) = 0$ at the optimum point.

(g) An optimal control problem can be solved using dynamic programming techniques.

(h) An integer programming problem is same as a discrete programming problem.

1.3 Define the following terms.

(a) Mathematical programming problem

(b) Trajectory optimization problem

(c) Behavior constraint
(d) Quadratic programming problem
(e) Posynomial
(f) Geometric programming problem

1.4 Match the following types of problems with their descriptions.

(a) Geometric programming problem	Classical optimization problem
(b) Quadratic programming problem	Objective and constraints are quadratic
(c) Dynamic programming problem	Objective is quadratic and constraints are linear
(d) Nonlinear programming problem	Objective and constraints arise from a serial system
(e) Calculus of variations problem	Objective and constraints are polynomials with positive coefficients

1.5 How do you solve a maximization problem as a minimization problem?

1.6 State the linear programming problem in standard form.

1.7 Define an OC problem and give an engineering example.

1.8 What is the difference between linear and nonlinear programming problems?

1.9 What is the difference between design variables and preassigned parameters?

1.10 What is a design space?

1.11 What is the difference between a constraint surface and a composite constraint surface?

1.12 What is the difference between a bound point and a free point in the design space?

1.13 What is a merit function?

1.14 Suggest a simple method of handling multiple objectives in an optimization problem.

1.15 What are objective function contours?

1.16 What is operations research?

1.17 State five engineering applications of optimization.

1.18 What is an integer programming problem?

1.19 What is graphical optimization, and what are its limitations?

1.20 Under what conditions can a polynomial in n variables be called a posynomial?

1.21 Define a stochastic programming problem and give two practical examples.

1.22 What is a separable programming problem?

PROBLEMS

1.1 A fertilizer company purchases nitrates, phosphates, potash, and an inert chalk base at a cost of \$1500, \$500, \$1000, and \$100 per ton, respectively, and produces four fertilizers A, B, C, and D. The production cost, selling price, and composition of the four fertilizers are given below.

Fertilizer	Production Cost (\$/ton)	Selling Price (\$/ton)	Percentage Composition by Weight: Nitrates	Phosphates	Potash	Inert Chalk Base
A	100	350	5	10	5	80
B	150	550	5	15	10	70
C	200	450	10	20	10	60
D	250	700	15	5	15	65

During any week, no more than 1000 tons of nitrate, 2000 tons of phosphates, and 1500 tons of potash will be available. The company is required to supply a minimum of 5000 tons of fertilizer A and 4000 tons of fertilizer D per week to its customers; but it is otherwise free to produce the fertilizers in any quantities it pleases. Formulate the problem of finding the quantity of each fertilizer to be produced by the company to maximize its profit.

1.2 The two-bar truss shown in Fig. 1.14 is symmetric about the y axis. The nondimensional area of cross section of the members A/A_{ref}, and the nondimensional position of joints 1 and 2, x/h, are treated as the design variables x_1 and x_2, respectively, where A_{ref} is the reference value of the area (A) and h is the height of the truss. The coordinates of joint 3 are held constant. The weight of the truss (f_1) and the total displacement of joint 3 under the given load (f_2) are to be minimized without exceeding the permissible stress, σ_0. The weight of the truss and the displacement of joint 3 can be expressed as

$$f_1(\mathbf{X}) = 2\rho h x_2 \sqrt{1 + x_1^2}\, A_{\text{ref}}$$

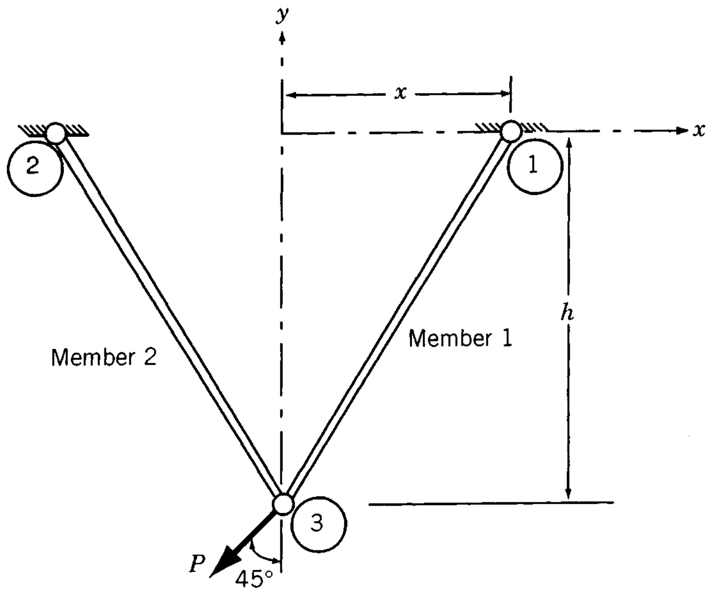

Figure 1.14 Two-bar truss.

$$f_2(\mathbf{X}) = \frac{Ph(1 + x_1^2)^{1.5}\sqrt{1 + x_1^4}}{2\sqrt{2}Ex_1^2x_2A_{\text{ref}}}$$

where ρ is the weight density, P the applied load, and E is Young's modulus. The stresses induced in members 1 and 2 (σ_1 and σ_2) are given by

$$\sigma_1(\mathbf{X}) = \frac{P(1 + x_1)\sqrt{(1 + x_1^2)}}{2\sqrt{2}x_1x_2A_{\text{ref}}}$$

$$\sigma_2(\mathbf{X}) = \frac{P(x_1 - 1)\sqrt{(1 + x_1^2)}}{2\sqrt{2}x_1x_2A_{\text{ref}}}$$

In addition, upper and lower bounds are placed on design variables x_1 and x_2 as

$$x_i^{\min} \le x_i \le x_i^{\max}; \qquad i = 1,2$$

Find the solution of the problem using a graphical method with **(a)** f_1 as the objective, **(b)** f_2 as the objective, and **(c)** $(f_1 + f_2)$ as the objective for the following data: $E = 30 \times 10^6$ psi, $\rho = 0.283$ lb/in^3, $P = 10{,}000$ lb, $\sigma_0 = 20{,}000$ psi, $h = 100$ in., $A_{\text{ref}} = 1$ in^2, $x_1^{\min} = 0.1$, $x_2^{\min} = 0.1$, $x_1^{\max} = 2.0$, and $x_2^{\max} = 2.5$.

1.3 Ten jobs are to be performed in an automobile assembly line as noted in the following table.

Job Number	Time Required to Complete the Job (min)	Jobs that Must Be Completed Before Starting This Job
1	4	None
2	8	None
3	7	None
4	6	None
5	3	1, 3
6	5	2, 3, 4
7	1	5, 6
8	9	6
9	2	7, 8
10	8	9

It is required to set up a suitable number of workstations with one worker assigned to each workstation to perform certain jobs. Formulate the problem of determining the number of workstations and the particular jobs to be assigned to eack workstation to minimize the idle time of the workers as an integer programming problem. (*Hint:* Define variables x_{ij} such that $x_{ij} = 1$ if job i is assigned to station j, and $x_{ij} = 0$ otherwise.)

1.4 A railroad track of length L is to be constructed over an uneven terrain by adding or removing dirt (Fig. 1.15). The absolute value of the slope of the track is to be restricted to a value of r_1 to avoid steep slopes. The absolute value of the rate of change of the slope is to be limited to a value r_2 to avoid rapid accelerations and decelerations. The absolute

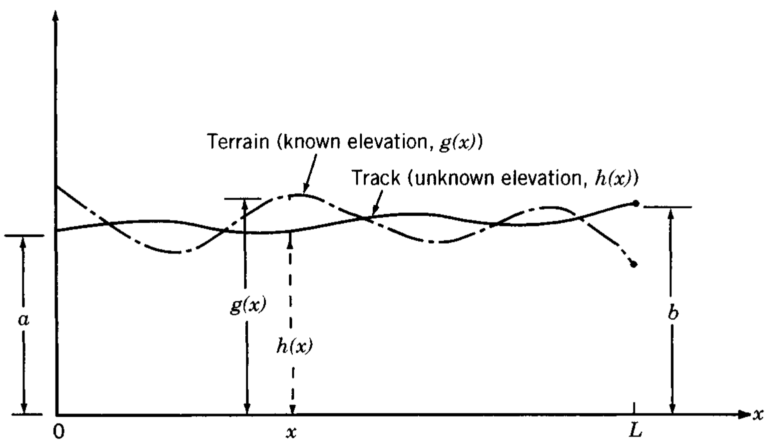

Figure 1.15 Railroad track on an uneven terrain.

value of the second derivative of the slope is to be limited to a value of r_3 to avoid severe jerks. Formulate the problem of finding the elevation of the track to minimize the construct costs as an OC problem. Assume the construction costs to be proportional to the amount of dirt added or removed. The elevation of the track is equal to a and b at $x = 0$ and $x = L$, respectively.

1.5 A manufacturer of a particular product produces x_1 units in the first week and x_2 units in the second week. The number of units produced in the first and second weeks must be at least 200 and 400, respectively, to be able to supply the regular customers. The initial inventory is zero and the manufacturer ceases to produce the product at the end of the second week. The production cost of a unit, in dollars, is given by $4x_i^2$, where x_i is the number of units produced in week i ($i = 1,2$). In addition to the production cost, there is an inventory cost of \$10 per unit for each unit produced in the first week that is not sold by the end of the first week. Formulate the problem of minimizing the total cost and find its solution using a graphical optimization method.

1.6 Consider the slider-crank mechanism shown in Fig. 1.16 with the crank rotating at a constant angular velocity ω. Use a graphical procedure to find the lengths of the crank and the connecting rod to maximize the velocity of the slider at a crank angle of $\theta = 30°$ for $\omega = 100$ rad/s. The mechanism has to satisfy Groshof's criterion $l \geq 2.5r$ to ensure 360° rotation of the crank. Additional constraints on the mechanism are given by $0.5 \leq r \leq 10$, $2.5 \leq l \leq 25$, and $10 \leq x \leq 20$.

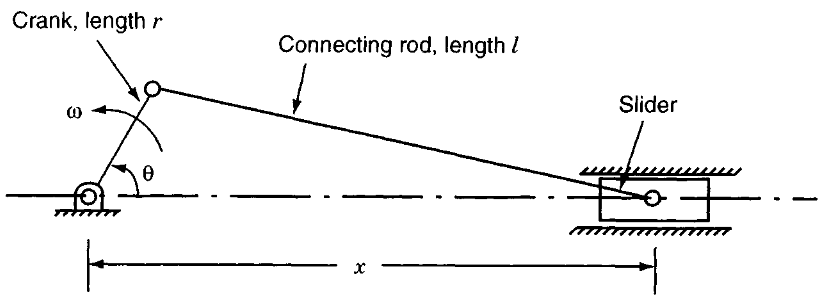

Figure 1.16 Slider-crank mechanism.

1.7 Solve Problem 1.6 to maximize the acceleration (instead of the velocity) of the slider at $\theta = 30°$ for $\omega = 100$ rad/s.

1.8 It is required to stamp four circular disks of radii R_1, R_2, R_3, and R_4 from a rectangular plate in a fabrication shop (Fig. 1.17). Formulate the problem as an optimization problem to minimize the scrap. Identify the design variables, objective function, and the constraints.

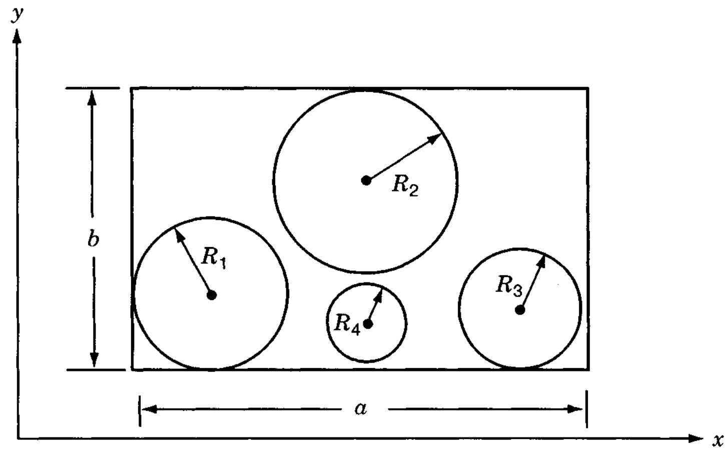

Figure 1.17 Locations of circular disks in a rectangular plate.

1.9 The torque transmitted (T) by a cone clutch, shown in Fig. 1.18, under uniform pressure condition is given by

$$T = \frac{2\pi f p}{3 \sin \alpha} (R_1^3 - R_2^3)$$

where p is the pressure between the cone and the cup, f the coefficient of friction, α the cone angle, R_1 the outer radius, and R_2 the inner radius.

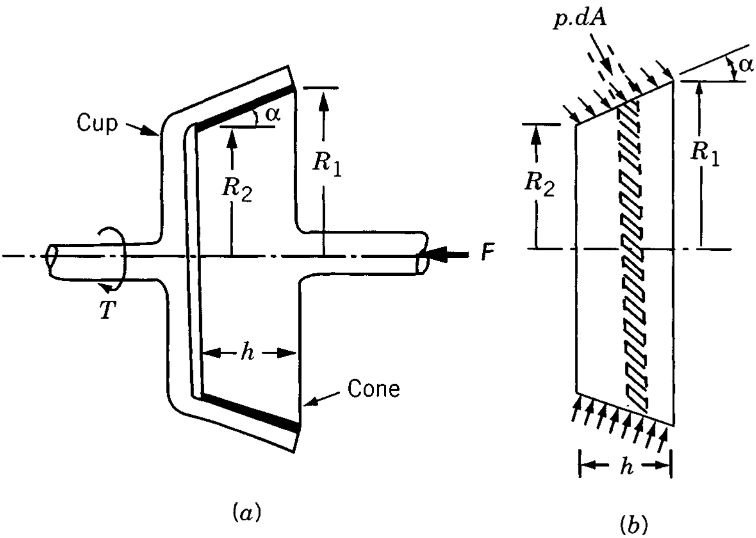

Figure 1.18 Cone clutch.

(a) Find R_1 and R_2 that minimize the volume of the cone clutch with $\alpha = 30°$, $F = 30$ lb, and $f = 0.5$ under the constraints: $T \geq 100$ lb-in., $R_1 \geq 2R_2$, $0 \leq R_1 \leq 15$ in., and $0 \leq R_2 \leq 10$ in.

(b) What is the solution if the constraint $R_1 \geq 2R_2$ is changed to $R_1 \leq 2R_2$?

(c) Find the solution of the problem stated in part (a) by assuming a uniform wear condition between the cup and the cone. The torque transmitted (T) under uniform wear condition is given by

$$T = \frac{\pi f p R_2}{\sin \alpha} (R_1^2 - R_2^2)$$

(*Note:* Use graphical optimization for the solutions.)

1.10 A hollow circular shaft is to be designed for minimum weight to achieve a minimum reliability of 0.99 when subjected to a random torque of $(\overline{T}, \sigma_T) = (10^6, 10^4)$ lb-in., where $\overline{T}$ is the mean torque and σ_T is the standard deviation of the torque, T. The permissible shear stress, τ_0, of the material is given by $(\overline{\tau}_0, \sigma_{\tau 0}) = (50{,}000,\ 5000)$ psi, where $\overline{\tau}_0$ is the mean value and $\sigma_{\tau 0}$ is the standard deviation of τ_0. The maximum induced stress (τ) in the shaft is given by

$$\tau = \frac{T r_o}{J}$$

where r_o is the outer radius and J is the polar moment of inertia of the cross section of the shaft. The manufacturing tolerances on the inner and outer radii of the shaft are specified as ± 0.06 in. The length of the shaft is given by 50 ± 1 in. and the specific weight of the material by 0.3 ± 0.03 lb/in^3. Formulate the optimization problem and solve it using a graphical procedure. Assume normal distribution for all the random variables and 3σ values for the specified tolerances.
[*Hints:* (1) The minimum reliability requirement of 0.99 can be expressed, equivalently, as [1.70]

$$z_1 = 2.326 \leq \frac{\overline{\tau} - \overline{\tau}_0}{\sqrt{\sigma_\tau^2 + \sigma_{\tau 0}^2}}$$

(2) If $f(x_1, x_2, \ldots, x_n)$ is a function of the random variables $x_1, x_2, \ldots, x_n$, the mean value of $f(\overline{f})$ and the standard deviation of $f(\sigma_f)$ are given by

$$\overline{f} = f(\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_n)$$

$$\sigma_f = \left[\sum_{i=1}^{n} \left(\left.\frac{\partial f}{\partial x_i}\right|_{\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_n} \right)^2 \sigma_{xi}^2 \right]^{1/2}$$

where $\overline{x}_i$ is the mean value of x_i and σ_{x_i} is the standard deviation of x_i.]

1.11 Certain nonseparable optimization problems can be reduced to a separable form by using suitable transformation of variables. For example, the product term $f = x_1 x_2$ can be reduced to the separable form $f = y_1^2 - y_2^2$ by introducing the transformations

$$y_1 = \tfrac{1}{2}(x_1 + x_2), \qquad y_2 = \tfrac{1}{2}(x_1 - x_2)$$

Suggest suitable transformations to reduce the following terms to separable form.

(a) $f = x_1^2 x_2^3, \; x_1 > 0, \; x_2 > 0$

(b) $f = x_1^{x_2}, \; x_1 > 0$

1.12 In the design of a shell-and-tube heat exchanger (Fig. 1.19), it is decided to have the total length of tubes equal to at least α_1 [1.8]. The cost of the tube is α_2 per unit length and the cost of the shell is given by $\alpha_3 D^{2.5} L$, where D is the diameter and L is the length of the heat exchanger shell. The floor space occupied by the heat exchanger costs α_4 per unit area and the cost of pumping cold fluid is $\alpha_5 L/d^5 N^2$ per day,

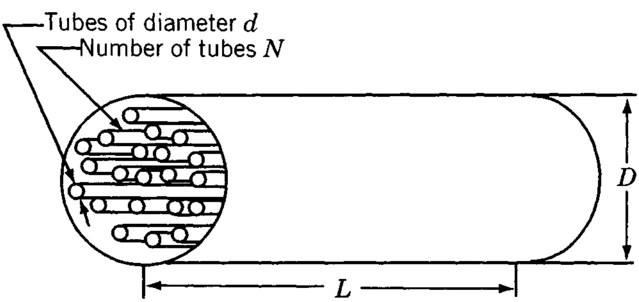

Figure 1.19 Shell-and-tube heat exchanger.

where d is the diameter of the tube and N is the number of tubes. The maintenance cost is given by $\alpha_6 NdL$. The thermal energy transferred to the cold fluid is given by $\alpha_7/N^{1.2}dL^{1.4} + \alpha_8/d^{0.2}L$. Formulate the mathematical programming problem of minimizing the overall cost of the heat exchanger with the constraint that the thermal energy transferred be greater than a specified amount α_9. The expected life of the heat exchanger is α_{10} years. Assume that α_i, $i = 1,2,\ldots,10$, are known constants, and each tube occupies a cross-sectional square of width and depth equal to d.

1.13 The bridge network shown in Fig. 1.20 consists of five resistors R_i $(i = 1,2,\ldots,5)$. If I_i is the current flowing through the resistance R_i, the problem is to find the resistances $R_1, R_2, \ldots, R_5$ so that the total

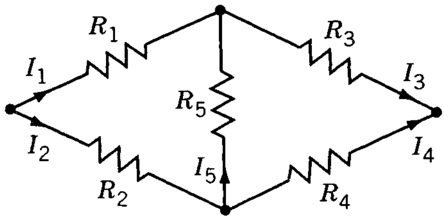

Figure 1.20 Electrical bridge network.

power dissipated by the network is a minimum. The current I_i can vary between the lower and upper limits $I_{i,\min}$ and $I_{i,\max}$, and the voltage drop, $V_i = R_i I_i$, must be equal to a constant c_i for $1 \le i \le 5$. Formulate the problem as a mathematical programming problem.

1.14 A traveling saleswoman has to cover n towns. She plans to start from a particular town numbered 1, visit each of the other $n - 1$ towns, and return to the town 1. The distance between towns i and j is given by d_{ij}. Formulate the problem of selecting the sequence in which the towns are to be visited to minimize the total distance traveled.

1.15 A farmer has a choice of planting barley, oats, rice, or wheat on his 200-acre farm. The labor, water, and fertilizer requirements, yields per acre, and selling prices are given in the following table:

Type of Crop	Labor Cost ($)	Water Required (m^3)	Fertilizer Required (lb)	Yield (lb)	Selling Price ($/lb)
Barley	300	10,000	100	1,500	0.5
Oats	200	7,000	120	3,000	0.2
Rice	250	6,000	160	2,500	0.3
Wheat	360	8,000	200	2,000	0.4

The farmer can also give part or all of the land for lease, in which case he gets $200 per acre. The cost of water is $0.02/m^3 and the cost of the fertilizer is $2/lb. Assume that the farmer has no money to start with and can get a maximum loan of $50,000 from the land mortgage bank at an interest of 8%. He can repay the loan after six months. The irrigation canal cannot supply more than 4×10^5 m^3 of water. Formulate the problem of finding the planting schedule for maximizing the expected returns of the farmer.

1.16 There are two different sites, each with four possible targets (or depths) to drill an oil well. The preparation cost for each site and the cost of drilling at site i to target j are given below.

	Drilling Cost to Target j				
Site i	1	2	3	4	Preparation Cost
1	4	1	9	7	11
2	7	9	5	2	13

Formulate the problem of determining the best site for each target so that the total cost is minimized.

1.17 A four-pole dc motor, whose cross section is shown in Fig. 1.21, is to be designed with the length of the stator and rotor x_1, the overall diameter of the motor x_2, the unnotched radius x_3, the depth of the notches x_4, and the ampere turns x_5 as design variables. The air gap is to be less

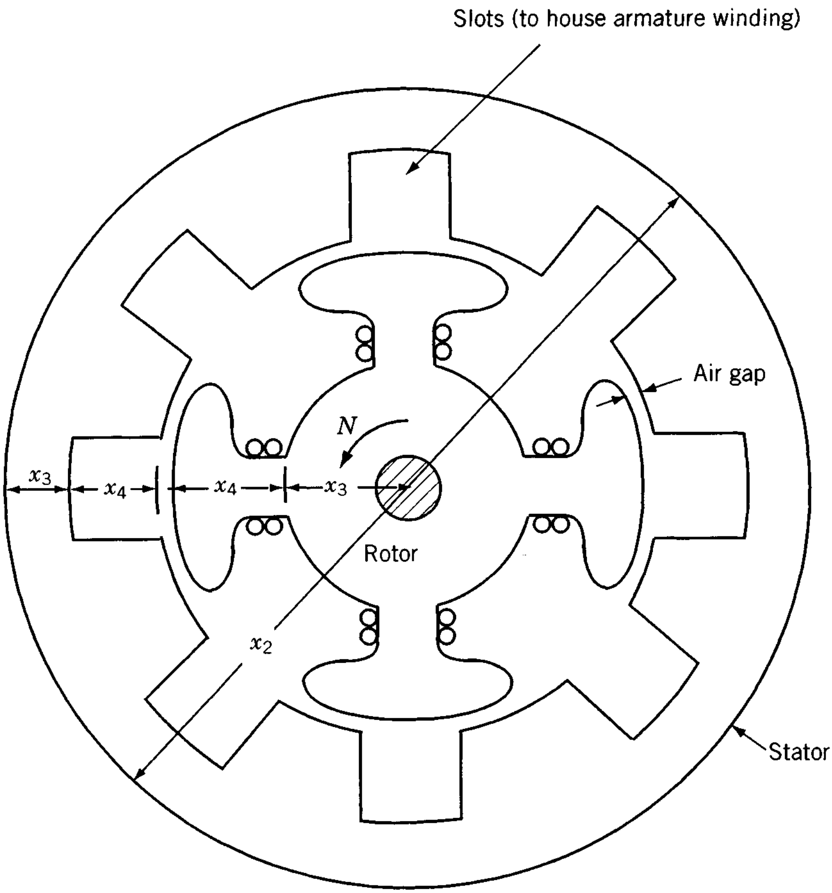

Figure 1.21 Cross section of an idealized motor.

than $k_1 \sqrt{x_2 + 7.5}$ where k_1 is a constant. The temperature of the external surface of the motor cannot exceed ΔT above the ambient temperature. Assuming that the heat can be dissipated only by radiation, formulate the problem for maximizing the power of the motor [1.44]. [*Hints:*

1. The heat generated due to current flow is given by $k_2 x_1 x_2^{-1} x_4^{-1} x_5^2$ where k_2 is a constant. The heat radiated from the external surface for a temperature difference of ΔT is given by $k_3 x_1 x_2 \Delta T$ where k_3 is a constant.
2. The expression for power is given by $k_4 N B x_1 x_3 x_5$ where k_4 is a constant, N is the rotational speed of the rotor, and B is the average flux density in the air gap.
3. The units of the various quantities are as follows. Lengths: centimeter, heat generated, heat dissipated; and power: watt; temperature: °C; rotational speed: rpm; flux density: gauss.]

1.18 A gas pipeline is to be laid between two cities A and E, making it pass through one of the four locations in each of the intermediate towns B, C, and D (Fig. 1.22). The associated costs are indicated in the following tables.

Costs for A to B and D to E:

	Station i			
	1	2	3	4
From A to point i of B	30	35	25	40
From point i of D to E	50	40	35	25

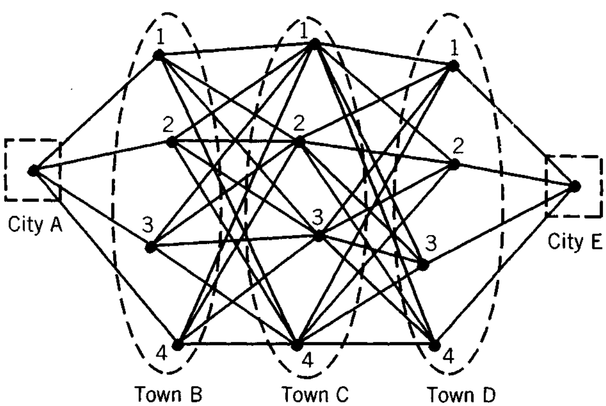

Figure 1.22 Possible paths of the pipeline between A and E.

Costs for B to C and C to D:

	To:			
From:	1	2	3	4
1	22	18	24	18
2	35	25	15	21
3	24	20	26	20
4	22	21	23	22

Formulate the problem of minimizing the cost of the pipeline.

1.19 A beam-column of rectangular cross section is required to carry an axial load of 25 lb and a transverse load of 10 lb, as shown in Fig. 1.23. It is to be designed to avoid the possibility of yielding and buckling and for minimum weight. Formulate the optimization problem by assuming

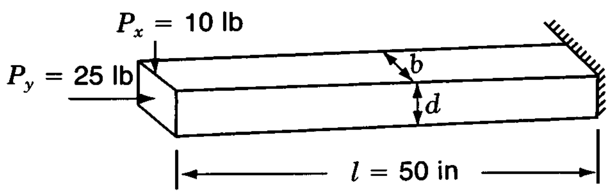

Figure 1.23 Beam-column.

that the beam-column can bend only in the vertical (xy) plane. Assume the material to be steel with a specific weight of 0.3 lb/in^3, Young's modulus of 30×10^6 psi, and a yield stress of 30,000 psi. The width of the beam is required to be at least 0.5 in. and not greater than twice the depth. Also, find the solution of the problem graphically. [*Hint:* The compressive stress in the beam-column due to P_y is P_y/bd and that due to P_x is

$$\frac{P_x l d}{2I_{zz}} = \frac{6P_x l}{bd^2}$$

The axial buckling load is given by

$$(P_y)_{\text{cri}} = \frac{\pi^2 E I_{zz}}{4l^2} = \frac{\pi^2 E b d^3}{48 l^2}.]$$

1.20 A two-bar truss is to be designed to carry a load of $2W$ as shown in Fig. 1.24. Both bars have a tubular section with mean diameter d and wall

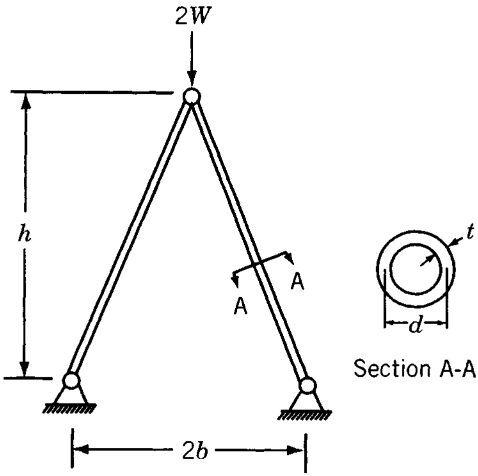

Figure 1.24 Two-bar truss.

thickness t. The material of the bars has Young's modulus E and yield stress σ_y. The design problem involves the determination of the values of d and t so that the weight of the truss is a minimum and neither yielding nor buckling occurs in any of the bars. Formulate the problem as a nonlinear programming problem.

1.21 Consider the problem of determining the economic lot sizes for four different items. Assume that the demand occurs at a constant rate over time. The stock for the ith item is replenished instantaneously upon request in lots of sizes Q_i. The total storage space available is A, whereas each unit of item i occupies an area d_i. The objective is to find the values of Q_i that optimize the per unit cost of holding the inventory and of ordering subject to the storage area constraint. The cost function is given by

$$C = \sum_{i=1}^{4} \left(\frac{a_i}{Q_i} + b_i Q_i \right), \qquad Q_i > 0$$

where a_i and b_i are fixed constants. Formulate the problem as a dynamic programming (optimal control) model. Assume that Q_i is discrete.

1.22 The layout of a processing plant, consisting of a pump (P), a water tank (T), a compressor (C), and a fan (F), is shown in Fig. 1.25. The locations of the various units, in terms of their (x,y) coordinates, are also indicated in this figure. It is decided to add a new unit, a heat exchanger (H), to the plant. To avoid congestion, it is decided to locate H within a rectangular area defined by $\{-15 \le x \le 15,\ -10 \le y \le 10\}$.

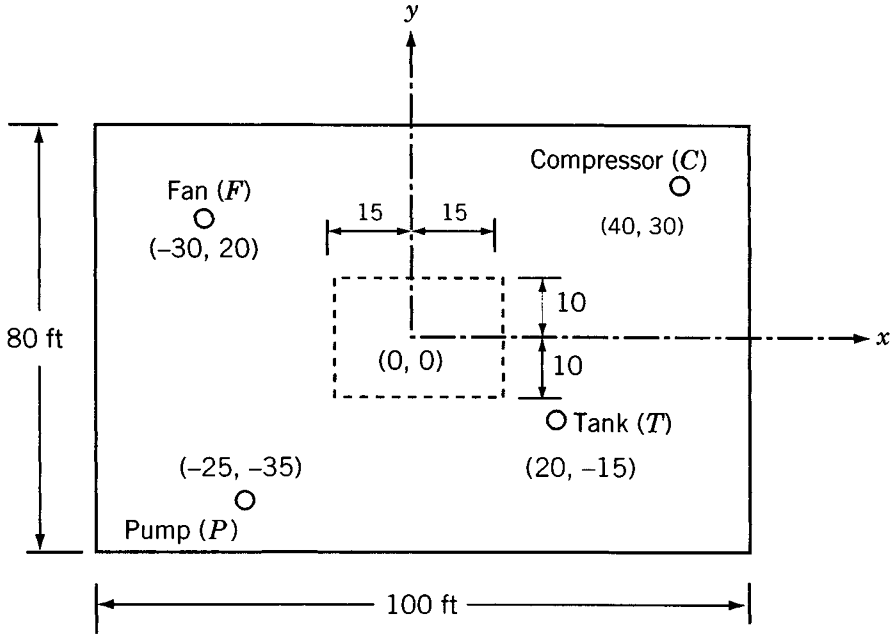

Figure 1.25 Processing plant layout (coordinates in ft.).

Formulate the problem of finding the location of H to minimize the sum of its x and y distances from the existing units, P, T, C, and F.

1.23 Two copper-based alloys (brasses), A and B, are mixed to produce a new alloy, C. The composition of alloys A and B and the requirements of alloy C are given in the following table.

	Composition by Weight			
Alloy	Copper	Zinc	Lead	Tin
A	80	10	6	4
B	60	20	18	2
C	≥ 65	≥ 15	≥ 16	≥ 3

If alloy B costs twice as much as alloy A, formulate the problem of determining the amounts of A and B to be mixed to produce alloy C at a minimum cost.

1.24 An oil refinery produces four grades of motor oil in three process plants. The refinery incurs a penalty for not meeting the demand of any particular grade of motor oil. The capacities of the plants, the production costs, the demands of the various grades of motor oil, and the penalties are given in the following table.

Process Plant	Capacity of the Plant (kgal/day)	Production Cost ($/day) to Manufacture Motor Oil of Grade: 1	2	3	4
1	100	750	900	1000	1200
2	150	800	950	1100	1400
3	200	900	1000	1200	1600
Demand (kgal/day)		50	150	100	75
Penalty (per each kilogallon shortage)		$10	$12	$16	$20

Formulate the problem of minimizing the overall cost as an LP problem.

1.25 A part-time graduate student in engineering is enrolled in a four-unit mathematics course and a three-unit design course. Since the student has to work for 20 hours a week at a local software company, he can spend a maximum of 40 hours a week to study outside the class. It is known from students who took the courses previously that the numerical grade (g) in each course is related to the study time spent outside the class as $g_m = t_m/6$ and $g_d = t_d/5$, where g indicates the numerical grade ($g = 4$ for A, 3 for B, 2 for C, 1 for D, and 0 for F), t represents the time spent in hours per week to study outside the class, and the subscripts m and d denote the courses, mathematics and design, respectively. The student enjoys design more than mathematics and hence would like to spend at least 75 minutes to study for design for every 60 minutes he spends to study mathematics. Also, as far as possible, the student does not want to spend more time on any course beyond the time required to earn a grade of A. The student wishes to maximize his grade point P, given by $P = 4g_m + 3g_d$, by suitably distributing his study time. Formulate the problem as an LP problem.

1.26 The scaffolding system, shown in Fig. 1.26, is used to carry a load of 10,000 lb. Assuming that the weights of the beams and the ropes are negligible, formulate the problem of determining the values of x_1, x_2, x_3, and x_4 to minimize the tension in ropes A and B while maintaining positive tensions in ropes C, D, E, and F.

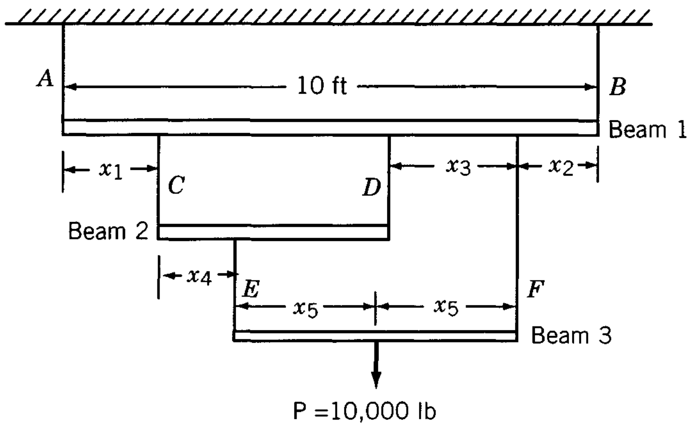

Figure 1.26 Scaffolding system.

1.27 Formulate the problem of minimum weight design of a power screw subjected to an axial load, F, as shown in Fig. 1.27 using the pitch (p), major diameter (d), nut height (h), and screw length (s) as design variables. Consider the following constraints in the formulation:

1. The screw should be self-locking [1.67].
2. The shear stress in the screw should not exceed the yield strength of the material in shear. Assume the shear strength in shear (according to distortion energy theory), to be $0.577\sigma_y$ where σ_y is the yield strength of the material.

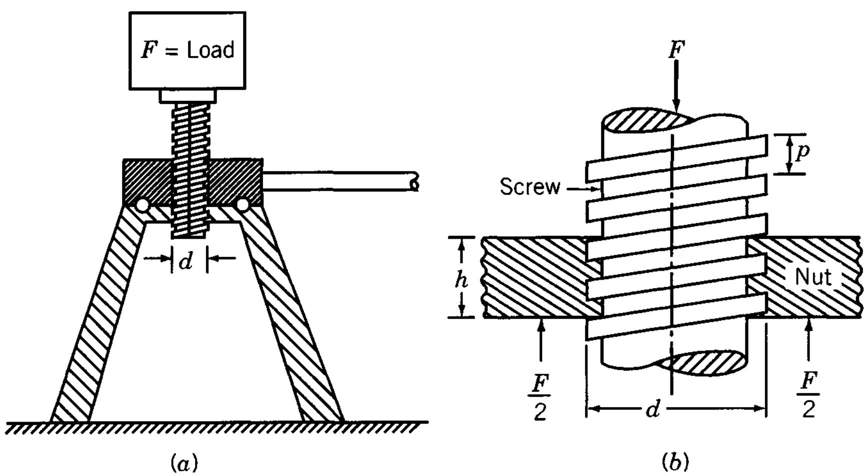

Figure 1.27 Power screw.

3. The bearing stress in the threads should not exceed the yield strength of the material, σ_y.
4. The critical buckling load of the screw should be less than the applied load, F.

1.28 **(a)** A simply supported beam of hollow rectangular section is to be designed for minimum weight to carry a vertical load F_y and an axial load P as shown in Fig. 1.28. The deflection of the beam in the y direction under the self-weight and F_y should not exceed 0.5 in. The beam should not buckle either in the yz or the xz plane under the axial load. Assuming the ends of the beam to be pin ended, formulate the optimization problem using x_i, $i = 1,2,3,4$ as design variables for the following data: $F_y = 300$ lb, $P = 40{,}000$ lb, $l = 120$ in., $E = 30 \times 10^6$ psi, $\rho = 0.284$ lb/in^3, lower bound on x_1 and $x_2 = 0.125$ in, upper bound on x_1, and $x_2 = 4$ in.

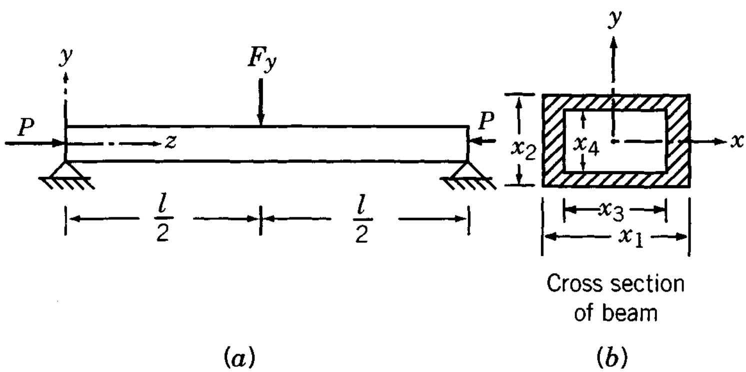

Figure 1.28 Simply supported beam under loads.

(b) Formulate the problem stated in part (a) using x_1 and x_2 as design variables, assuming the beam to have a solid rectangular cross section. Also find the solution of the problem using a graphical technique.

1.29 A cylindrical pressure vessel with hemispherical ends (Fig. 1.29) is required to hold at least 20,000 gallons of a fluid under a pressure of 2500 psia. The thicknesses of the cylindrical and hemispherical parts of the shell should be equal to at least those recommended by Section VIII of the ASME pressure vessel code, which are given by

$$t_c = \frac{pR}{Se + 0.4p}$$

$$t_h = \frac{pR}{Se + 0.8p}$$

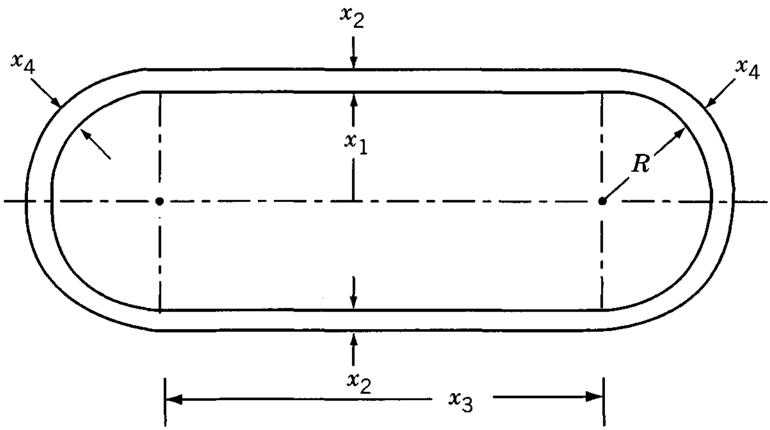

Figure 1.29 Pressure vessel.

where S is the yield strength, e the joint efficiency, p the pressure, and R the radius. Formulate the design problem for minimum structural volume using x_i, $i = 1,2,3,4$, as design variables. Assume the following data: $S = 30{,}000$ psi and $e = 1.0$.

1.30 A crane hook is to be designed to carry a load F as shown in Fig. 1.30. The hook can be modeled as a three-quarter circular ring with a rect-

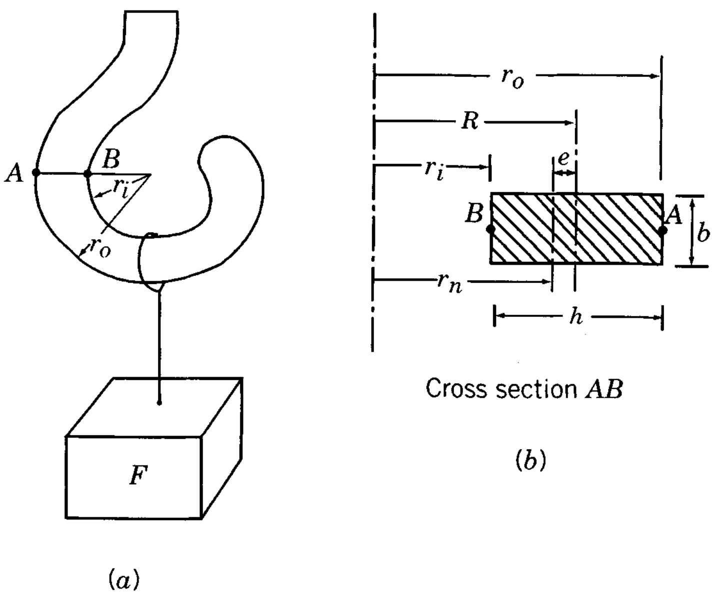

Figure 1.30 Crane hook carrying a load.

angular cross section. The stresses induced at the inner and outer fibers at section AB should not exceed the yield strength of the material. Formulate the problem of minimum volume design of the hook using r_o, r_i, b, and h as design variables. [*Note:* The stresses induced at points A and B are given by [1.67]

$$\sigma_A = \frac{Mc_o}{Aer_o}$$

$$\sigma_B = \frac{Mc_i}{Aer_i}$$

where M is the bending moment due to the load ($=FR$), R the radius of the centroid, r_o the radius of the outer fiber, r_i the radius of the inner fiber, c_o the distance of the outer fiber from the neutral axis $= R_o - r_n$, c_i the distance of inner fiber from neutral axis $= r_n - r_i$, r_n the radius of neutral axis, given by

$$r_n = \frac{h}{\ln(r_o/r_i)}$$

A the cross-sectional area of the hook $= bh$, and e the distance between the centroidal and neutral axes $= R - r_n$.]

1.31 Consider the four-bar truss shown in Fig. 1.31, in which members 1, 2, and 3 have the same cross-sectional area x_1 and the same length l, while member 4 has an area of cross section x_2 and length $\sqrt{3}\ l$. The trus is made of a lightweight material for which Young's modulus and the weight density are given by 30×10^6 psi and 0.03333 lb/in^3, respectively. The truss is subject to the loads $P_1 = 10{,}000$ lb and $P_2 = 20{,}000$ lb. The weight of the truss per unit value of l can be expressed

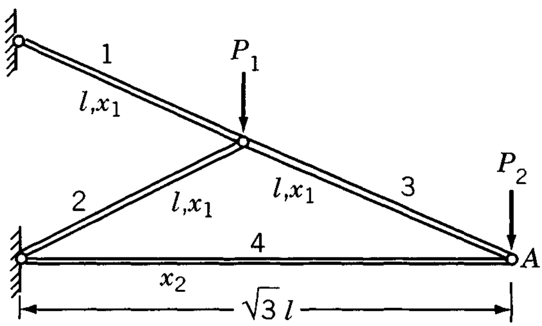

Figure 1.31 Four-bar truss.

as

$$f = 3x_1(1)(0.03333) + x_2\sqrt{3}\,(0.03333) = 0.1x_1 + 0.05773x_2$$

The vertical deflection of joint A can be expressed as

$$\delta_A = \frac{0.6}{x_1} + \frac{0.3464}{x_2}$$

and the stresses in members 1 and 4 can be written as

$$\sigma_1 = \frac{5(10{,}000)}{x_1} = \frac{50{,}000}{x_1}, \qquad \sigma_4 = \frac{-2\sqrt{3}\,(10{,}000)}{x_2} = -\frac{34{,}640}{x_2}$$

The weight of the truss is to be minimized with constraints on the vertical deflection of the joint A and the stresses in members 1 and 4. The maximum permissible deflection of joint A is 0.1 in. and the permissible stresses in members are $\sigma_{\text{max}} = 8333.3333$ psi (tension) and $\sigma_{\text{min}} = -4948.5714$ psi (compression). The optimization problem can be stated as a separable programming problem as follows:

$$\text{Minimize } f(x_1,x_2) = 0.1x_1 + 0.05773x_2$$

subject to

$$\frac{0.6}{x_1} + \frac{0.3464}{x_2} - 0.1 \le 0, \qquad 6 - x_1 \le 0, \quad 7 - x_2 \le 0$$

Determine the solution of the problem using a graphical procedure.