Arthur E. Fitzgerald, Charles Kingsley, Jr., and Stephen D. Umans, *Electric Machinery*, Sixth Edition, McGraw-Hill, 2003.

1. 三相電路 (Three-Phase Circuits)
2. 磁路與磁性材料 (Magnetic Circuits and Magnetic Materials)
3. 變壓器 (Transformers)
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Bibliography

1. Yamayee and Bala, *Electromechanical Energy Devices and Power Systems*, Wiley, 1994.
2. Ryff, *Electric Machinery*, 2/e, Prentice Hall, 1994.
3. Sen, *Principles of Electric Machines and Power Electronics*, 2/e, Wiley, 1997.
4. Cathey, *Electric Machines*, McGraw-Hill, 2001.
5. Sarma, *Electric Machines*, 2/e, West, 1994.
6. Wildi, *Electrical Machines, Drives, and Power Systems*, 5/e, Prentice Hall, 2002.
7. McGeorge, *Marine Electrical Equipment and Practice*, Newnes, 1993.
8. Pallett, *Aircraft Electrical Systems*, Wiley, 1987.
9. 劉昌煥（主編），電機機械，東華書局，1999。
10. 李永忠，船舶電機及設備，第三版，中央圖書，1990。

Chapter 1 Magnetic Circuits and Magnetic Materials

- The objective of this course is to study the devices used in the interconversion of electric and mechanical energy, with emphasis placed on electromagnetic rotating machinery.
- The transformer, although not an electromechanical-energy-conversion device, is an important component of the overall energy-conversion process.
- Practically all transformers and electric machinery use ferro-magnetic material for shaping and directing the magnetic fields that acts as the medium for transferring and converting energy. Permanent-magnet materials are also widely used.
- The ability to analyze and describe systems containing magnetic materials is essential for designing and understanding electromechanical-energy-conversion devices.
- The techniques of magnetic-circuit analysis, which represent algebraic approximations to exact field-theory solutions, are widely used in the study of electromechanical-energy-conversion devices.

§1.1 Introduction to Magnetic Circuits

- Assume the frequencies and sizes involved are such that the displacement-current term in Maxwell's equations, which accounts for magnetic fields being produced in space by time-varying electric fields and is associated with electromagnetic radiations, can be neglected.
  - H : magnetic field intensity, amperes/m, A/m, A-turn/m, A-t/m
  - B : magnetic flux density, webers/m^2, Wb/m^2, tesla (T)
  - 1 Wb $=10^8$ lines (maxwells); 1 T $=10^4$ gauss
  - From (1.1), we see that the source of H is the current density J. The line integral of the tangential component of the magnetic field intensity H around a closed contour C is equal to the total current passing through any surface S linking that contour.

$$\oint_c H dl = \int_s J \cdot da \tag{1.1}$$

  - Equation (1.2) states that the magnetic flux density B is conserved. No net flux enters or leaves a closed surface. There exists no monopole charge sources of magnetic fields.

$$\oint_s B \cdot da = 0 \tag{1.2}$$

- A magnetic circuit consists of a structure composed for the most part of high-permeability magnetic material. The presence of high-permeability material tends to cause magnetic flux to be confined to the paths defined by the structure.

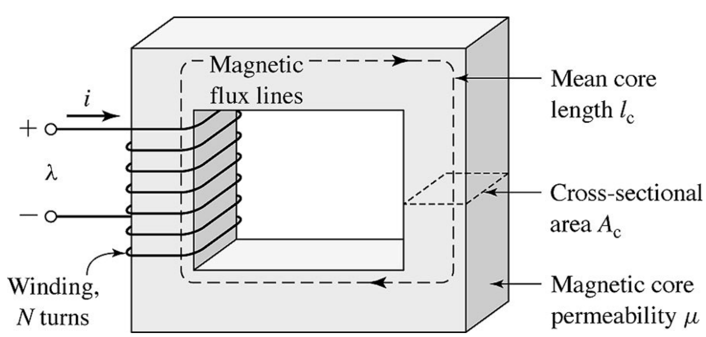


Figure 1.1 Simple magnetic circuit.

- In Fig. 1.1, the source of the magnetic field in the core is the ampere-turn product Ni, the magnetomotive force (mmf) F acting on the magnetic circuit.
- The magnetic flux ϕ (in weber, Wb) crossing a surface S is the surface integral of the normal component B:

$$\phi = \oint_s B \cdot da \tag{1.3}$$

- ϕ_c: flux in core, B_c: flux density in core

$$\phi_c = B_c A_c \tag{1.4}$$

- H_c: average magnitude H in the core. The direction of H_c can be found from the RHR.

$$F = Ni = \oint Hdl \tag{1.5}$$

$$F = Ni = H_c l_c \tag{1.6}$$

- The relationship between the magnetic field intensity H and the magnetic flux density B:

$$B = \mu H \tag{1.7}$$

  - Linear relationship?
  - $\mu = \mu_r \mu_0$, μ: magnetic permeability, Wb/A-t-m = H/m
  - $\mu_0 = 4\pi \times 10^{-7}$: the permeability of free space
  - μ_r: relative permeability, typical values: 2000-80,000

- A magnetic circuit with an air gap is shown in Fig. 1.2. Air gaps are present for moving elements. The air gap length is sufficiently small. ϕ: the flux in the magnetic circuit.

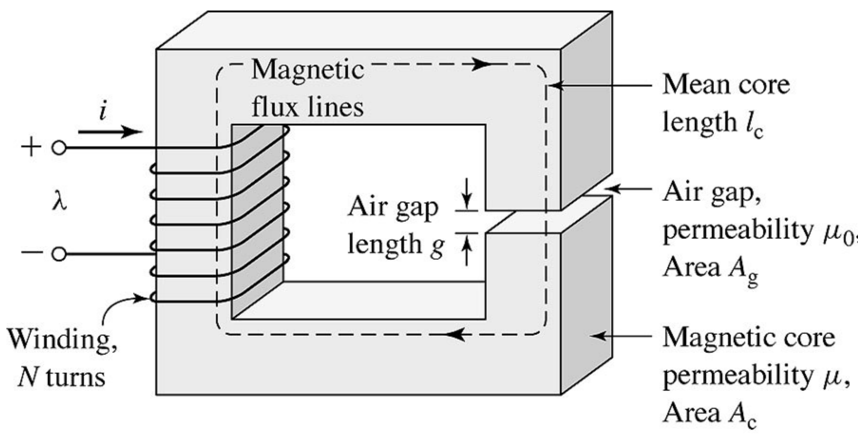


Figure 1.2 Magnetic circuit with air gap.

$$B_c = \frac{\phi}{A_c} \tag{1.8}$$

$$B_g = \frac{\phi}{A_g} \tag{1.9}$$

$$F = H_c l_c + H_g l_g \tag{1.10}$$

$$F = \frac{B_c}{\mu} l_c + \frac{B_g}{\mu_0} g \tag{1.11}$$

$$F = \phi \left(\frac{l_c}{\mu A_c} + \frac{g}{\mu_0 A_g} \right) \tag{1.12}$$

- R_c, R_g : the reluctance of the core and the air gap, respectively,

$$R_c = \frac{l_c}{\mu A_c}, \quad R_g = \frac{g}{\mu_0 A_g} \qquad (1.13),\ (1.14)$$

$$F = \phi(R_c + R_g) \qquad (1.15)$$

$$\phi = \frac{F}{R_c + R_g} \qquad (1.16)$$

$$\phi = \frac{F}{\frac{l_c}{\mu A_c} + \frac{g}{\mu_0 A_g}} \qquad (1.17)$$

- In general, for any magnetic circuit of total reluctance R_{tot}, the flux can be found as

$$\phi = \frac{F}{R_{tot}} \qquad (1.18)$$

- The permeance P is the inverse of the reluctance

$$P_{tot} = \frac{1}{R_{tot}} \qquad (1.19)$$

- Fig. 1.3: Analogy between electric and magnetic circuits:

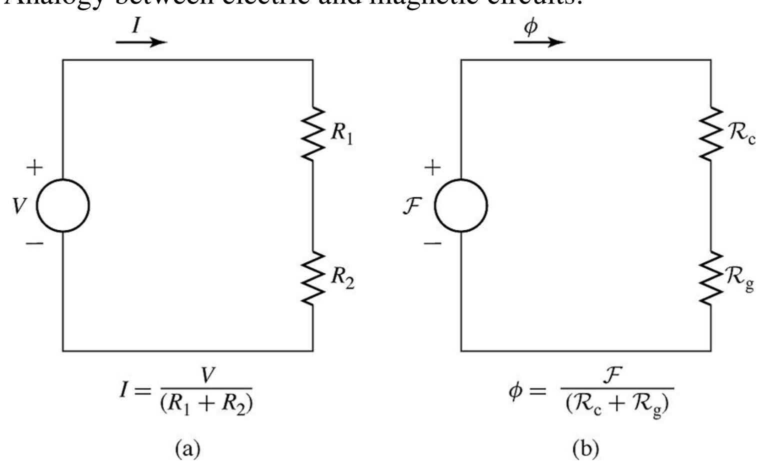


Figure 1.3 Analogy between electric and magnetic circuits: (a) electric ckt, (b) magnetic ckt.

- Note that with high material permeability: $R_c << R_g$ and thus $R_{tot} << R_g$,

$$\phi \approx \frac{F}{R_g} = \frac{F\mu_0 A_g}{g} = Ni\frac{\mu_0 A_g}{g} \qquad (1.20)$$

- Fig. 1.4: Fringing effect, effective A_g increased.

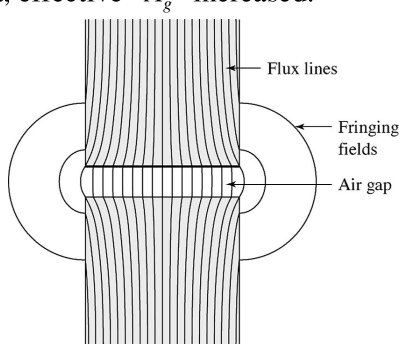


Figure 1.4 Air-gap fringing fields.

- In general, magnetic circuits can consist of multiple elements in series and parallel.

$$F = \oint Hdl = \sum_k F_k = \sum_k H_k l_k \tag{1.21}$$

$$F = \int_s J \cdot da \tag{1.22}$$

$$V = \sum_k R_k i_k \tag{1.23}$$

$$\sum_n i_n = 0 \tag{1.24}$$

$$\sum_n \phi_n = 0 \tag{1.25}$$

EXAMPLE 1.1

The magnetic circuit shown in Fig. 1.2 has dimensions $A_c = A_g = 9\ \text{cm}^2$, $g = 0.050$ cm, $l_c = 30$ cm, and $N = 500$ turns. Assume the value $\mu_r = 70{,}000$ for core material. (*a*) Find the reluctances $\mathcal{R}_c$ and $\mathcal{R}_g$. For the condition that the magnetic circuit is operating with $B_c = 1.0$ T, find (*b*) the flux ϕ and (*c*) the current i.

■ Solution

a. The reluctances can be found from Eqs. 1.13 and 1.14:

$$\mathcal{R}_c = \frac{l_c}{\mu_r \mu_0 A_c} = \frac{0.3}{70,000\,(4\pi \times 10^{-7})(9 \times 10^{-4})} = 3.79 \times 10^3\ \frac{\text{A} \cdot \text{turns}}{\text{Wb}}$$

$$\mathcal{R}_g = \frac{g}{\mu_0 A_g} = \frac{5 \times 10^{-4}}{(4\pi \times 10^{-7})(9 \times 10^{-4})} = 4.42 \times 10^5\ \frac{\text{A} \cdot \text{turns}}{\text{Wb}}$$

b. From Eq. 1.4,

$$\phi = B_c A_c = 1.0(9 \times 10^{-4}) = 9 \times 10^{-4}\ \text{Wb}$$

c. From Eqs. 1.6 and 1.15,

$$i = \frac{\mathcal{F}}{N} = \frac{\phi(\mathcal{R}_c + \mathcal{R}_g)}{N} = \frac{9 \times 10^{-4}(4.46 \times 10^5)}{500} = 0.80\ \text{A}$$

EXAMPLE 1.2

The magnetic structure of a synchronous machine is shown schematically in Fig. 1.5. Assuming that rotor and stator iron have infinite permeability ($\mu \to \infty$), find the air-gap flux ϕ and flux density B_g. For this example $I = 10$ A, $N = 1000$ turns, $g = 1$ cm, and $A_g = 2000\ \text{cm}^2$.

■ Solution

Notice that there are two air gaps in series, of total length $2g$, and that by symmetry the flux density in each is equal. Since the iron permeability here is assumed to be infinite, its reluctance is negligible and Eq. 1.20 (with g replaced by the total gap length $2g$) can be used to find the flux

$$\phi = \frac{NI\mu_0 A_g}{2g} = \frac{1000(10)(4\pi \times 10^{-7})(0.2)}{0.02} = 0.13\ \text{Wb}$$

and

$$B_g = \frac{\phi}{A_g} = \frac{0.13}{0.2} = 0.65\ \text{T}$$

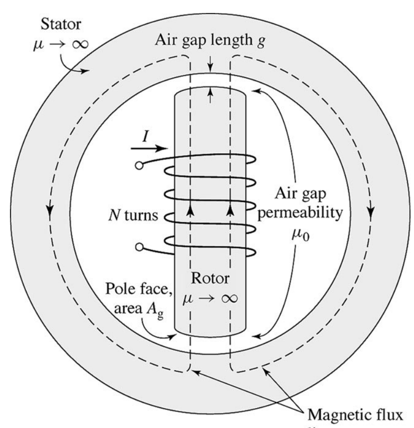


Figure 1.5 Simple synchronous machine.

§1.2 Flux Linkage, Inductance, and Energy

- Faraday's Law:

$$\oint_c E \cdot ds = -\frac{d}{dt}\int_s B \cdot da \tag{1.26}$$

  - λ: the flux linkage of the winding, φ: the instantaneous value of a time-varying flux,
  - e: the induced voltage at the winding terminals

$$e = N\frac{d\varphi}{dt} = \frac{d\lambda}{dt} \tag{1.27}$$

$$\lambda = N\varphi \tag{1.28}$$

- L: the inductance (with material of constant permeability), H = Wb-t/A

$$L = \frac{\lambda}{i} \tag{1.29}$$

$$L = \frac{N^2}{R_{tot}} \tag{1.30}$$

  - The inductance of the winding in Fig. 1.2:

$$L = \frac{N^2}{(g/\mu_0 A_g)} = \frac{N^2 \mu_0 A_g}{g} \tag{1.31}$$

EXAMPLE 1.3

The magnetic circuit of Fig. 1.6a consists of an N-turn winding on a magnetic core of infinite permeability with two parallel air gaps of lengths g_1 and g_2 and areas A_1 and A_2, respectively.

Find (*a*) the inductance of the winding and (*b*) the flux density B_1 in gap 1 when the winding is carrying a current i. Neglect fringing effects at the air gap.

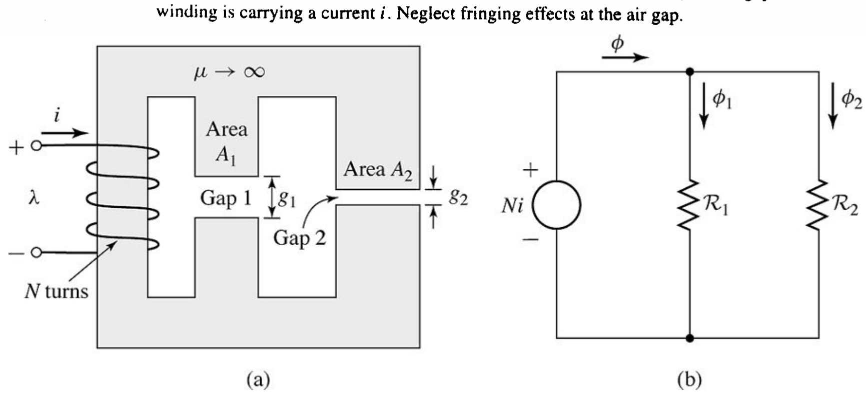


Figure 1.6 (a) Magnetic circuit and (b) equivalent circuit for Example 1.3.

■ **Solution**

a. The equivalent circuit of Fig. 1.6b shows that the total reluctance is equal to the parallel combination of the two gap reluctances. Thus

$$\phi = \frac{Ni}{\frac{\mathcal{R}_1\mathcal{R}_2}{\mathcal{R}_1+\mathcal{R}_2}}$$

where

$$\mathcal{R}_1 = \frac{g_1}{\mu_0 A_1} \qquad \mathcal{R}_2 = \frac{g_2}{\mu_0 A_2}$$

From Eq. 1.29,

$$L = \frac{\lambda}{i} = \frac{N\phi}{i} = \frac{N^2(\mathcal{R}_1+\mathcal{R}_2)}{\mathcal{R}_1\mathcal{R}_2}$$

$$= \mu_0 N^2\left(\frac{A_1}{g_1}+\frac{A_2}{g_2}\right)$$

b. From the equivalent circuit, one can see that

$$\phi_1 = \frac{Ni}{\mathcal{R}_1} = \frac{\mu_0 A_1 Ni}{g_1}$$

and thus

$$B_1 = \frac{\phi_1}{A_1} = \frac{\mu_0 Ni}{g_1}$$

EXAMPLE 1.4

In Example 1.1, the relative permeability of the core material for the magnetic circuit of Fig. 1.2 is assumed to be $\mu_r = 70{,}000$ at a flux density of 1.0 T.

a. For this value of μ_r calculate the inductance of the winding.
b. In a practical device, the core would be constructed from electrical steel such as M-5

electrical steel which is discussed in Section 1.3. This material is highly nonlinear and its relative permeability (defined for the purposes of this example as the ratio B/H) varies from a value of approximately $\mu_r = 72{,}300$ at a flux density of $B = 1.0$ T to a value of on the order of $\mu_r = 2900$ as the flux density is raised to 1.8 T. (a) Calculate the inductance under the assumption that the relative permeability of the core steel is 72,300. (b) Calculate the inductance under the assumption that the relative permeability is equal to 2900.

■ **Solution**

a. From Eqs. 1.13 and 1.14 and based upon the dimensions given in Example 1.1,

$$\mathcal{R}_c = \frac{l_c}{\mu_r \mu_0 A_c} = \frac{0.3}{72{,}300\,(4\pi \times 10^{-7})(9 \times 10^{-4})} = 3.67 \times 10^3\ \frac{\text{A}\cdot\text{turns}}{\text{Wb}}$$

while $\mathcal{R}_g$ remains unchanged from the value calculated in Example 1.1 as $\mathcal{R}_g = 4.42 \times 10^5$ A · turns / Wb.

Thus the total reluctance of the core and gap is

$$\mathcal{R}_{tot} = \mathcal{R}_c + \mathcal{R}_g = 4.46 \times 10^5\ \frac{\text{A}\cdot\text{turns}}{\text{Wb}}$$

and hence from Eq. 1.30

$$L = \frac{N^2}{\mathcal{R}_{tot}} = \frac{500^2}{4.46 \times 10^5} = 0.561\ \text{H}$$

b. For $\mu_r = 2900$, the reluctance of the core increases from a value of 3.79×10^3 A · turns / Wb to a value of

$$\mathcal{R}_c = \frac{l_c}{\mu_r \mu_0 A_c} = \frac{0.3}{2900\,(4\pi \times 10^{-7})(9 \times 10^{-4})} = 9.15 \times 10^4\ \frac{\text{A}\cdot\text{turns}}{\text{Wb}}$$

and hence the total reluctance increases from 4.46×10^5 A · turns / Wb to 5.34×10^5 A · turns / Wb. Thus from Eq. 1.30 the inductance decreases from 0.561 H to

$$L = \frac{N^2}{\mathcal{R}_{tot}} = \frac{500^2}{5.34 \times 10^5} = 0.468\ \text{H}$$

This example illustrates the linearizing effect of a dominating air gap in a magnetic circuit. In spite of a reduction in the permeablity of the iron by a factor of 72,300/2900 = 25, the inductance decreases only by a factor of 0.468/0.561 = 0.83 simply because the reluctance of the air gap is significantly larger than that of the core. In many situations, it is common to assume the inductance to be constant at a value corresponding to a finite, constant value of core permeability (or in many cases it is assumed simply that $\mu_r \rightarrow \infty$). Analyses based upon such a representation for the inductor will often lead to results which are well within the range of acceptable engineering accuracy and which avoid the immense complication associated with modeling the nonlinearity of the core material.

- Magnetic circuit with more than one windings, Fig. 1.8:

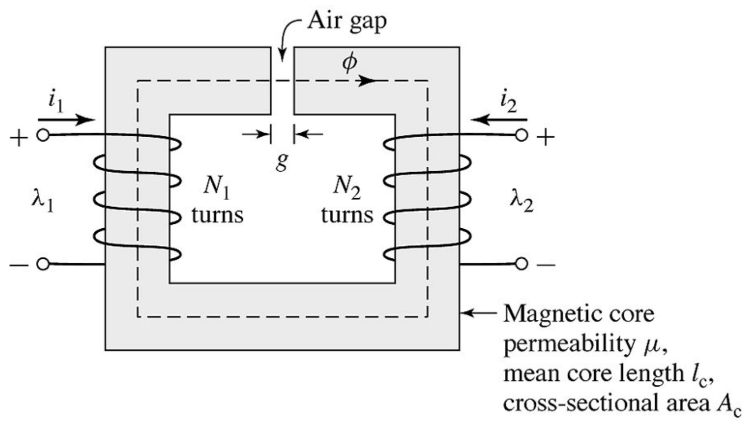


Figure 1.8 Magnetic circuit with two windings.

$$F = N_1 i_1 + N_2 i_2 \tag{1.32}$$

$$\phi = (N_1 i_1 + N_2 i_2)\frac{\mu_0 A_c}{g} \tag{1.33}$$

$$\lambda_1 = N_1 \phi = N_1^2 \left(\frac{\mu_0 A_c}{g}\right) i_1 + N_1 N_2 \left(\frac{\mu_0 A_c}{g}\right) i_2 \tag{1.34}$$

$$\lambda_1 = L_{11} i_1 + L_{12} i_2 \tag{1.35}$$

$$L_{11} = N_1^2 \frac{\mu_0 A_c}{g} \tag{1.36}$$

$$L_{12} = N_1 N_2 \frac{\mu_0 A_c}{g} = L_{21} \tag{1.37}$$

$$\lambda_2 = N_2 \phi = N_1 N_2 \left(\frac{\mu_0 A_c}{g}\right) i_1 + N_2^2 \left(\frac{\mu_0 A_c}{g}\right) i_2 \tag{1.38}$$

$$\lambda_2 = L_{21} i_1 + L_{22} i_2 \tag{1.39}$$

$$L_{22} = N_2^2 \frac{\mu_0 A_c}{g} \tag{1.40}$$

- Induced voltage, power (W = J/s), and stored energy:

$$e = \frac{d}{dt}(Li) \tag{1.41}$$

$$e = L\frac{d}{dt}(Li) \tag{1.42}$$

$$e = L\frac{di}{dt} + i\frac{dL}{dt} \tag{1.43}$$

$$p = ie = i\frac{d\lambda}{dt} \tag{1.44}$$

$$\Delta W = \int_{t_1}^{t_2} p\, dt = \int_{\lambda_1}^{\lambda_2} i\, d\lambda \tag{1.45}$$

$$\Delta W = \int_{\lambda_1}^{\lambda_2} i\, d\lambda = \int_{\lambda_1}^{\lambda_2} \frac{\lambda}{L} d\lambda = \frac{1}{2L}\left(\lambda_2^2 - \lambda_1^2\right) \tag{1.46}$$

$$W = \frac{1}{2L}\lambda^2 = \frac{L}{2} i^2 \tag{1.47}$$

EXAMPLE 1.6

For the magnetic circuit of Example 1.1 (Fig. 1.2), find (*a*) the inductance L, (*b*) the magnetic stored energy W for $B_c = 1.0$ T, and (*c*) the induced voltage e for a 60-Hz time-varying core flux of the form $B_c = 1.0 \sin \omega t$ T where $\omega = (2\pi)(60) = 377$.

Solution

a. From Eqs. 1.16 and 1.29 and Example 1.1,

$$L = \frac{\lambda}{i} = \frac{N\phi}{i} = \frac{N^2}{\mathcal{R}_c + \mathcal{R}_g}$$
$$= \frac{500^2}{4.46 \times 10^5} = 0.56 \text{ H}$$

Note that the core reluctance is much smaller than that of the gap ($\mathcal{R}_c \ll \mathcal{R}_g$). Thus to a good approximation the inductance is dominated by the gap reluctance, i.e.,

$$L \approx \frac{N^2}{\mathcal{R}_g} = 0.57 \text{ H}$$

b. In Example 1.1 we found that when $B_c = 1.0$ T, $i = 0.80$A. Thus from Eq. 1.47,

$$W = \frac{1}{2}Li^2 = \frac{1}{2}(0.56)(0.80)^2 = 0.18 \text{ J}$$

c. From Eq. 1.27 and Example 1.1,

$$e = \frac{d\lambda}{dt} = N\frac{d\varphi}{dt} = NA_c\frac{dB_c}{dt}$$
$$= 500 \times (9 \times 10^{-4}) \times (377 \times 1.0 \cos(377t))$$
$$= 170 \cos(377t) \quad V$$

§1.3 Properties of Magnetic Materials

- The importance of magnetic materials is twofold:
  - Magnetic materials are used to obtain large magnetic flux densities with relatively low levels of magnetizing force.
  - Magnetic materials can be used to constrain and direct magnetic fields in well-defined paths.
- Ferromagnetic materials, typically composed of iron and alloys of iron with cobalt, tungsten, nickel, aluminum, and other metals, are by far the most common magnetic materials.
  - They are found to be composed of a large number of domains.
  - When unmagnetized, the domain magnetic moments are randomly oriented.
  - When an external magnetizing force is applied, the domain magnetic moments tend to align with the applied magnetic field until all the magnetic moments are aligned with the applied field, and the material is said to be fully saturated.
  - When the applied field is reduced to zero, the magnetic dipole moments will no longer be totally random in their orientation and will retain a net magnetization component along the applied field direction.
- The relationship between B and H for a ferromagnetic material is both nonlinear and multivalued.
  - In general, the characteristics of the material cannot be described analytically but are commonly presented in graphical form.
  - The most common used curve is the $B-H$ curve.
  - Dc or normal *magnetization curve*:
  - *Hysteresis loop* (Note the remanance):

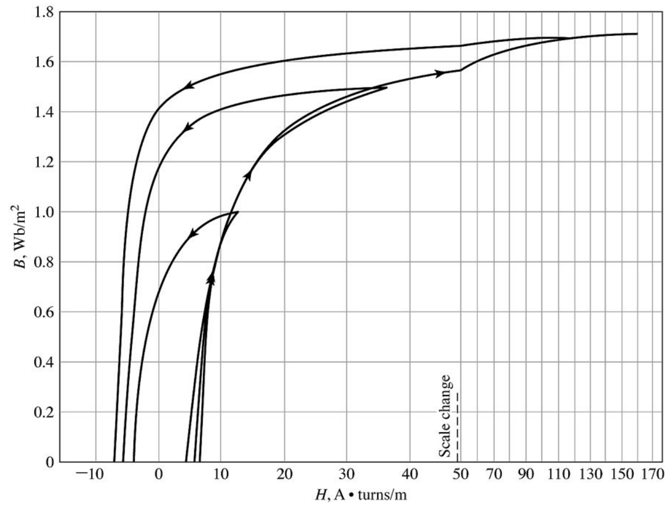


Figure 1.9 *B-H* loops for M-5 grain-oriented electrical steel 0.012 in thick. Only the top halves of the loops are shown here. *(Armco Inc.)*

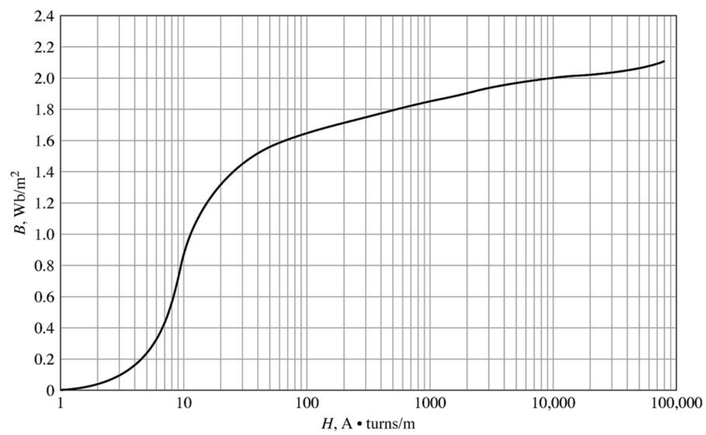


Figure 1.10 Dc magnetization curve for M-5 grain-oriented electrical steel 0.012 in thick. *(Armco Inc.)*

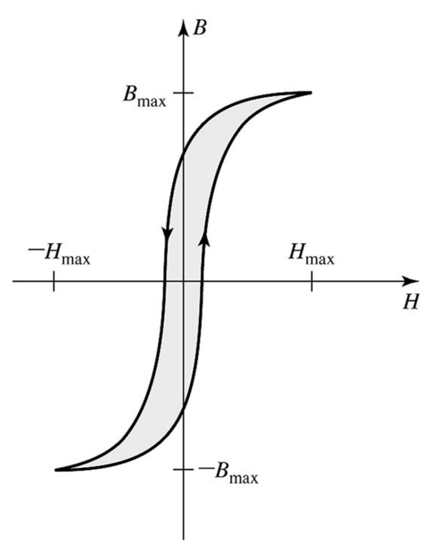


Figure 1.13 Hysteresis loop.

EXAMPLE 1.7

Assume that the core material in Example 1.1 is M-5 electrical steel, which has the dc magnetization curve of Fig. 1.10. Find the current i required to produce $B_c = 1$ T.

■ Solution

The value of H_c for $B_c = 1$ T is read from Fig. 1.10 as

$$H_c = 11 \text{ A} \cdot \text{turns/m}$$

The mmf drop for the core path is

$$\mathcal{F}_c = H_c l_c = 11(0.3) = 3.3 \text{ A} \cdot \text{turns}$$

The mmf drop across the air gap is

$$\mathcal{F}_g = H_g g = \frac{B_g g}{\mu_0} = \frac{5 \times 10^{-4}}{4\pi \times 10^{-7}} = 396 \text{ A} \cdot \text{turns}$$

The required current is

$$i = \frac{\mathcal{F}_c + \mathcal{F}_g}{N} = \frac{399}{500} = 0.80 \text{ A}$$

§1.4 AC Excitation

- In ac power systems, the waveforms of voltage and flux closely approximate sinusoidal functions of time. We are to study the excitation characteristics and losses associated with magnetic materials under steady-state ac operating conditions.

  - Assume a sinusoidal variation of the core flux $\varphi(t)$:

$$\varphi(t) = \phi_{\max} \sin \omega t = A_c B_{\max} \sin \omega t \tag{1.48}$$

where $\phi_{\max}$ = amplitude of core flux φ in webers

$B_{\max}$ = amplitude of flux density B_c in teslas

ω = angular frequency = $2\pi f$

f = frequency in Hz

  - The voltage induced in the N-turn winding is

$$e(t) = \omega N \phi_{\max} \cos(\omega t) = E_{\max} \cos \omega t \tag{1.49}$$

$$E_{\max} = \omega N \phi_{\max} = 2\pi f N A_c B_{\max} \tag{1.50}$$

  - The Root-Mean-Squared (rms) value:

$$F_{\text{rms}} = \sqrt{\left(\frac{1}{T}\int_o^T f^2(t)dt\right)} \tag{1.51}$$

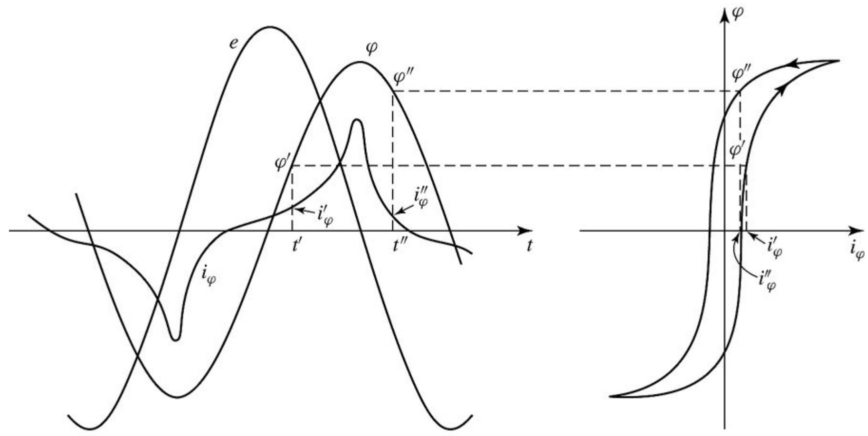
$$E_{\text{rms}} = \frac{2\pi}{\sqrt{2}} f N A_c B_{\max} = \sqrt{2}\pi f N A_c B_{\max} \tag{1.52}$$

Note that the rums value of a sinusoidal wave is $1/\sqrt{2}$ times its peak value.

- Excitation phenomena, Fig. 1.11:

  - φ vs $i_\varphi \Leftrightarrow B_c$ vs H_c, i_φ: exciting current.

  - Note that $\varphi = B_c A_c$ and that $i_\varphi = H_c \lambda_c / N$.



(a) (b)

Figure 1.11 Excitation phenomena. (a) Voltage, flux, and exciting current; (b) corresponding hysteresis loop.

$$I_{\varphi,rms} = \frac{l_c H_{c,rms}}{N} \tag{1.53}$$

$$\begin{aligned} E_{rms} I_{\varphi,rms} &= \sqrt{2}\pi f N A_c B_{\max} \frac{l_c H_{rms}}{N} \\ &= \sqrt{2}\pi f N B_{\max} H_{rms} \left(A_c l_c\right) \end{aligned} \tag{1.54}$$

$$P_a = \frac{E_{rms} I_{\varphi,rms}}{\text{mass}} = \frac{\sqrt{2}\pi f}{\rho_c} B_{\max} H_{rms} \tag{1.55}$$

P_a: the exciting rms voltamperes per unit mass, $\text{mass} = \rho_c A_c \lambda_c$

- The rms exciting voltampere can be seen to be a property of the material alone. It depends only on $B_{\max}$ because H_{rms} is a unique function of $B_{\max}$.

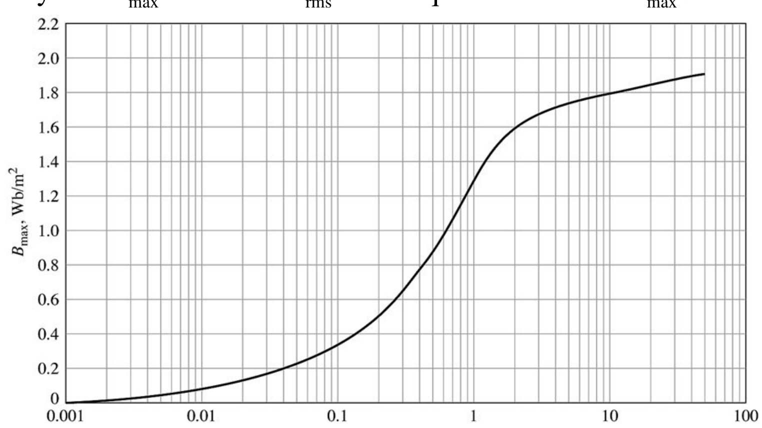


Figure 1.12 Exciting rms voltamperes per kilogram at 60 Hz for M-5 grain-oriented electrical steel 0.012 in thick. *(Armco Inc.)*

- The exciting current supplies the mmf required to produce the core flux and the power input associated with the energy in the magnetic field in the core.
  - Part of this energy is dissipated as losses and results in heating of the core.
  - The rest appears as reactive power associated with energy storage in the magnetic field. This reactive power is not dissipated in the core; it is cyclically supplied and absorbed by the excitation source.

- Two loss mechanisms are associated with time-varying fluxes in magnetic materials.
  - The first is ohmic I^2R heating, associated with induced currents in the core material.
    - Eddy currents circulate and oppose changes in flux density in the material.
    - To reduce the effects, magnetic structures are usually built of thin sheets of laminations of the magnetic material.
    - Eddy-current loss $\propto$ f^2, $B_{\max}^2$
  - The second loss mechanic is due to the hysteretic nature of magnetic material.
    - The energy input W to the core as the material undergoes a single cycle

$$W = \oint i_\varphi d\lambda = \oint \left(\frac{H_c l_c}{N} \right) (A_c N dB_c) = A_c l_c \oint H_c dB_c \tag{1.56}$$

    - For a given flux level, the corresponding hysteresis losses are proportional to the area of the hysteresis loop and to the total volume of material.
    - Hysteresis power loss $\propto$ f
  - Information on core loss is typically presented in graphical form. It is plotted in terms of watts per unit weight as a function of flux density; often a family of curves for different frequencies are given. See Fig. 1.14.

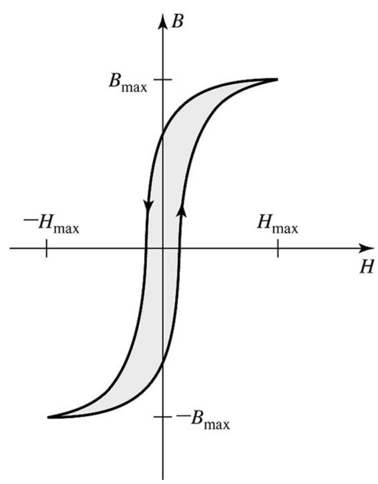


Figure 1.13 Hysteresis loop; hysteresis loss is proportional to the loop area (shaded).

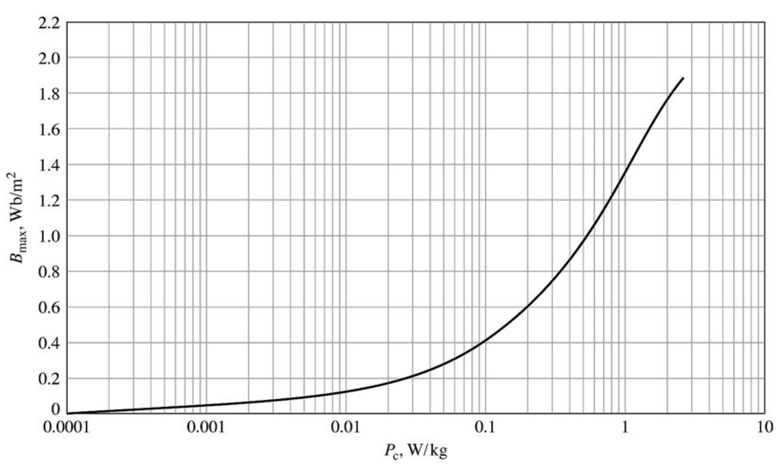


Figure 1.14 Core loss at 60 Hz in watts per kilogram for M-5 grain-oriented electrical steel 0.012 in thick. *(Armco Inc).*

EXAMPLE 1.8

The magnetic core in Fig. 1.15 is made from laminations of M-5 grain-oriented electrical steel. The winding is excited with a 60-Hz voltage to produce a flux density in the steel of $B = 1.5 \sin \omega t$ T, where $\omega = 2\pi 60 \approx 377$ rad/sec. The steel occupies 0.94 of the core cross-sectional area. The mass-density of the steel is 7.65 g/cm³. Find (*a*) the applied voltage, (*b*) the peak current, (*c*) the rms exciting current, and (*d*) the core loss.

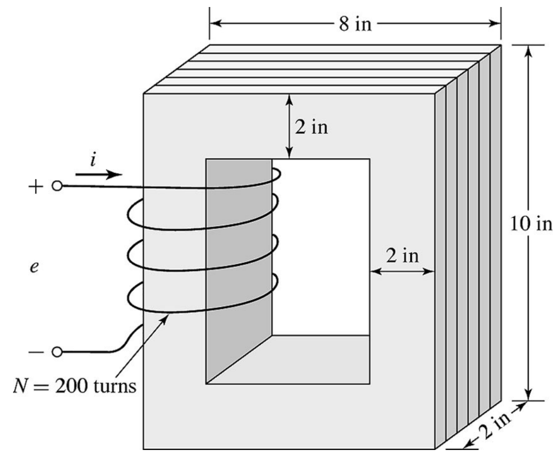


Figure 1.15 Laminated steel core with winding for Example 1.8.

■ **Solution**

a. From Eq. 1.27 the voltage is

$$e = N\frac{d\varphi}{dt} = NA_c\frac{dB}{dt}$$

$$= 200 \times 4\text{ in}^2 \times 0.94 \times \left(\frac{1.0\text{ m}^2}{39.4^2\text{ in}^2}\right) \times 1.5 \times (377\ \cos(377t))$$

$$= 274 \cos(377t)\text{ V}$$

b. The magnetic field intensity corresponding to $B_{\max} = 1.5$ T is given in Fig. 1.10 as $H_{\max} = 36$ A turns/m. Notice that, as expected, the relative permeability $\mu_r = B_{\max}/(\mu_0 H_{\max}) = 33{,}000$ at the flux level of 1.5 T is lower than the value of $\mu_r = 72{,}300$ found in Example 1.4 corresponding to a flux level of 1.0 T, yet significantly larger than the value of 2900 corresponding to a flux level of 1.8 T.

$$l_c = (6 + 6 + 8 + 8)\text{ in}\left(\frac{1.0\text{ m}}{39.4\text{ in}}\right) = 0.71\text{ m}$$

The peak current is

$$I = \frac{H_{\max} l_c}{N} = \frac{36(0.71)}{200} = 0.13\text{ A}$$

c. The rms current is obtained from the value of P_a of Fig. 1.12 for $B_{\max} = 1.5$ T.

$$P_a = 1.5\text{ VA/kg}$$

The core volume and weight are

$$V_c = (4\text{ in}^2)(0.94)(28\text{ in}) = 105.5\text{ in}^3$$

$$W_c = (105.5\text{ in}^3)\left(\frac{2.54\text{ cm}}{1.0\text{ in}}\right)^3\left(\frac{7.65\text{ g}}{1.0\text{ cm}^3}\right) = 13.2\text{ kg}$$

The total rms voltamperes and current are

$$P_a = (1.5\text{ VA/kg})(13.2\text{ kg}) = 20\text{ VA}$$

$$I_{\varphi,\text{rms}} = \frac{P_a}{E_{\text{rms}}} = \frac{20}{275(0.707)} = 0.10\text{ A}$$

d. The core-loss density is obtained from Fig. 1.14 as $P_c = 1.2$ W/kg. The total core loss is

$$P_c = (1.2\text{ W/kg})(13.2\text{ kg}) = 16\text{ W}$$

§1.5 Permanent Magnets

- Certain magnetic materials, commonly known as permanent-magnet materials, are characterized by large values of remanent magnetization and coercivity. These materials produce significant magnetic flux even in magnetic circuits with air gaps.
- The second quadrant of a hysteresis loop (the magnetization curve) is usually employed for analyzing a permanent-magnet material.
  - B_r : residual flux density or remanent magnetization,
  - H_c : coercivity, (1) a measure of the magnitude of the mmf required to demagnetize the material, and (2) a measure of the capability of the material to produce flux in a magnetic circuit which includes an air gap.
  - Large value (> 1 kA/m): hard magnetic material, o.w.: soft magnetic material
  - Fig. 1.16(a): Alnico 5, $B_r \cong 1.22\,\mathrm{T}$, $H_c \cong -49\,\mathrm{kA/m}$
  - Fig. 1.16(b): M-5 steel, $B_r \cong 1.4\,\mathrm{T}$, $H_c \cong -6\,\mathrm{kA/m}$
  - Both Alnico 5 and M-5 electrical steel would be useful in producing flux in unexcited magnetic circuits since they both have large values of remanent magnetization.
- The significant of remanent magnetization is that it can produce magnetic flux in a magnetic circuit in the absence of external excitation (such as winding currents).

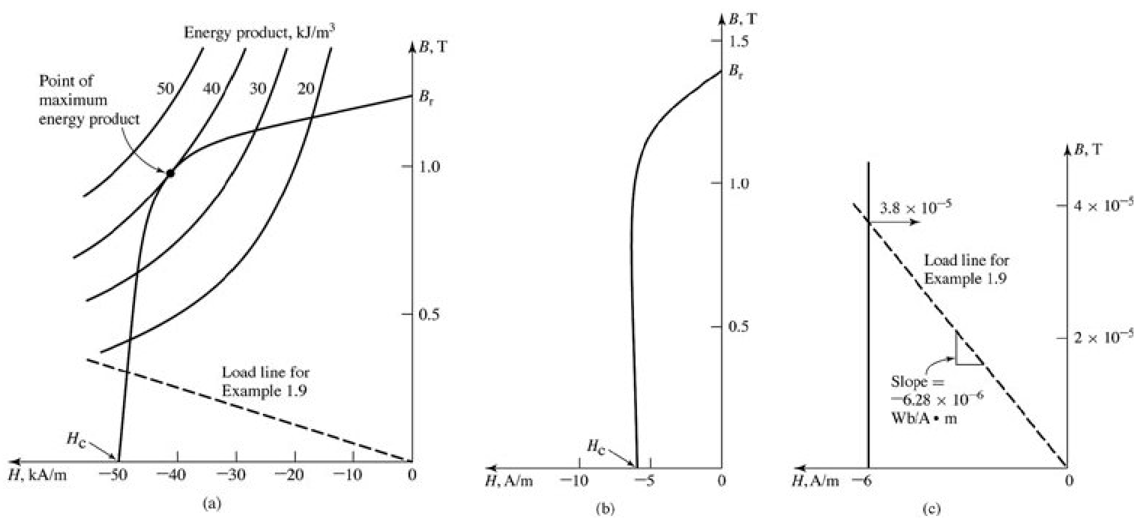


Figure 1.16 (a) Second quadrant of hysteresis loop for Alnico 5; (b) second quadrant of hysteresis loop for M-5 electrical steel; (c) hysteresis loop for M-5 electrical steel expanded for small B. *(Armco Inc.)*

EXAMPLE 1.9

As shown in Fig. 1.17, a magnetic circuit consists of a core of high permeability ($\mu \to \infty$), an air gap of length $g = 0.2$ cm, and a section of magnetic material of length $l_m = 1.0$ cm. The cross-sectional area of the core and gap is equal to $A_m = A_g = 4\ \mathrm{cm}^2$. Calculate the flux density B_g in the air gap if the magnetic material is (*a*) Alnico 5 and (*b*) M-5 electrical steel.

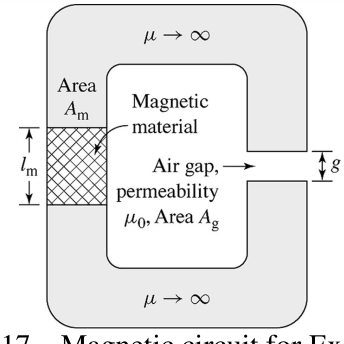


Figure 1.17 Magnetic circuit for Example 1.9.

■ **Solution**

a. Since the core permeability is assumed infinite, H in the core is negligible. Recognizing that the mmf acting on the magnetic circuit of Fig. 1.17 is zero, we can write

$$\mathcal{F} = 0 = H_g g + H_m l_m$$

or

$$H_g = -\left(\frac{l_m}{g}\right) H_m$$

where H_g and H_m are the magnetic field intensities in the air gap and the magnetic material, respectively.

Since the flux must be continuous through the magnetic circuit,

$$\phi = A_g B_g = A_m B_m$$

or

$$B_g = \left(\frac{A_m}{A_g}\right) B_m$$

where B_g and B_m are the magnetic flux densities in the air gap and the magnetic material, respectively.

These equations can be solved to yield a linear relationship for B_m in terms of H_m

$$B_m = -\mu_0 \left(\frac{A_g}{A_m}\right)\left(\frac{l_m}{g}\right) H_m = -5\,\mu_0\, H_m = -6{,}28 \times 10^{-6} H_m$$

To solve for B_m we recognize that for Alnico 5, B_m and H_m are also related by the curve of Fig. 1.16a. Thus this linear relationship, also known as a *load line,* can be plotted on Fig. 1.16a and the solution obtained graphically, resulting in

$$B_g = B_m = 0.30\ \text{T}$$

b. The solution for M-5 electrical steel proceeds exactly as in part (a). The load line is the same as that of part (a) because it is determined only by the permeability of the air gap and the geometries of the magnet and the air gap. Hence from Fig. 1.16c

$$B_g = 3.8 \times 10^{-5}\ \text{T} = 0.38\ \text{gauss}$$

which is much less than the value obtained with Alnico 5.

- Maximum Energy Product: a useful measure of the capability of permanent-magnet material.
  - The product of B and H has the dimension of energy density (J/m^3)
  - Choosing a material with the largest available maximum energy product can result in the smallest required magnet volume.
  - Consider Example 1.9. From (1.58) and (1.57), (1.59) can be obtained.

$$B_g = \frac{A_m}{A_g} B_m, \quad \frac{H_m l_m}{H_g g} = -1 \qquad (1.57)\ (1.58)$$

$$\begin{aligned} B_g^2 &= \mu_0 \left(\frac{l_m A_m}{g A_g}\right)(-H_m B_m) \\ &= \mu_0 \left(\frac{\text{Vol}_{\text{mag}}}{\text{Vol}_{\text{air gap}}}\right)(-H_m B_m) \end{aligned} \qquad (1.59)$$

$$\text{Vol}_{\text{mag}} = \frac{\text{Vol}_{\text{air gap}} B_g^2}{\mu_0(-H_m B_m)} \qquad (1.60)$$

  - Equation (1.60) indicates that to achieve a desired flux density in the air gap the required volume of the magnet can be minimized by operating the magnet at the point of maximum energy product.
  - A curve of constant B-H product is a hyperbola.
  - In Fig. 1.16a, the maximum energy product for Alnico 5 is 40 kJ/m^3, occurring at the point $B = 1.0\ \text{T}$ and $H = -40\ \text{kA/m}$.

EXAMPLE 1.10

The magnetic circuit of Fig. 1.17 is modified so that the air-gap area is reduced to $A_g = 2.0\ \text{cm}^2$, as shown in Fig. 1.18. Find the minimum magnet volume required to achieve an air-gap flux density of 0.8 T.

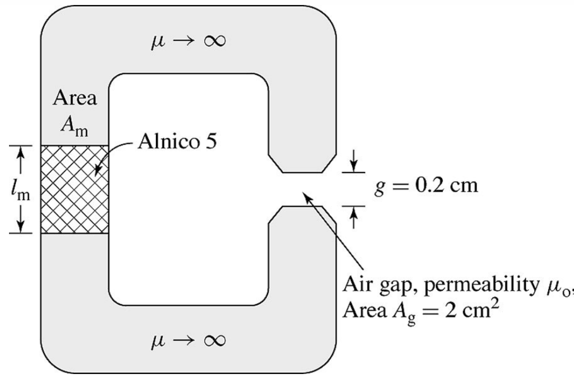


Figure 1.18 Magnetic circuit for Example 1.10.

■ **Solution**

The smallest magnet volume will be achieved with the magnet operating at its point of maximum energy product, as shown in Fig. 1.16a. At this operating point, $B_m = 1.0$ T and $H_m = -40$ kA/m.

Thus from Eq. 1.57,

$$A_m = A_g\left(\frac{B_g}{B_m}\right)$$

$$= 2\ \text{cm}^2\left(\frac{0.8}{1.0}\right) = 1.6\ \text{cm}^2$$

and from Eq. 1.58

$$l_m = -g\left(\frac{H_g}{H_m}\right) = -g\left(\frac{B_g}{\mu_0 H_m}\right)$$

$$= -0.2\ \text{cm}\left(\frac{0.8}{(4\pi \times 10^{-7})(-40 \times 10^3)}\right)$$

$$= 3.18\ \text{cm}$$

Thus the minimum magnet volume is equal to $1.6\ \text{cm}^2 \times 3.18\ \text{cm} = 5.09\ \text{cm}^3$.

§1.6 Application of Permanent Magnet Materials

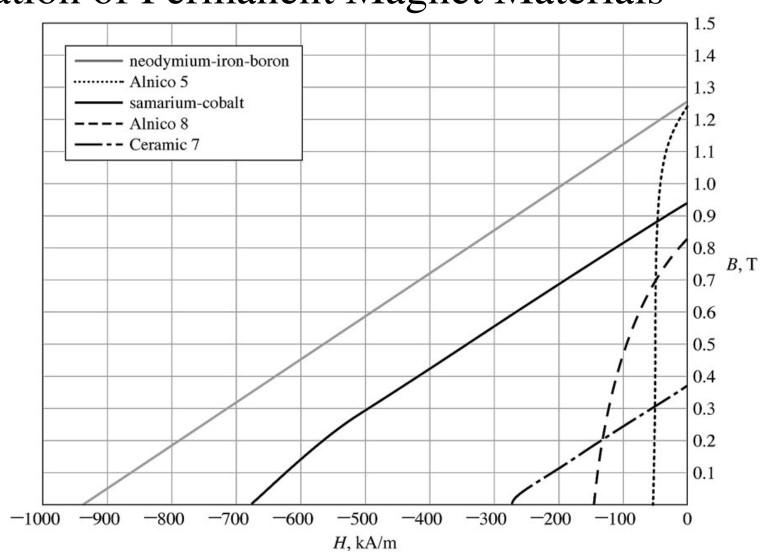


Figure 1.19 Magnetization curves for common permanent-magnet materials.

Chapter 2 Transformers

- This chapter is to discuss certain aspects of the theory of magnetically-coupled circuits, with emphasis on transformer action.
- The static transformer is not an energy conversion device, but an indispensable component in many energy conversion systems.
  - ➢ It is a significant component in ac power systems:
    - → Electric generation at the most economical generator voltage
    - → Power transfer at the most economical transmission voltage
    - → Power utilization at the most voltage for the particular utilization device
  - ➢ It is widely used in low-power, low-current electronic and control circuits:
    - → Matching the impedances of a source and its load for maximum power transfer
    - → Isolating one circuit from another
    - → Isolating direct current while maintaining ac continuity between two circuits
- The transformer is one of the simpler devices comprising two or more electric circuits coupled by a common magnetic circuit.
  - ➢ Its analysis involves many of the principles essential to the study of electric machinery.

§2.1 Introduction to Transformers

- Essentially, a transformer consists of two or more windings coupled by mutual magnetic flux.
  - ➢ One of these windings, the primary, is connected to an alternating-voltage.
  - ➢ An alternating flux will be produced whose magnitude will depend on the primary voltage, the frequency of the applied voltage, and the number of turns.
  - ➢ The mutual flux will link the other winding, the secondary, and will induce a voltage in it whose value will depend on the number of secondary turns as well as the magnitude of the mutual flux and the frequency.
  - ➢ By properly proportioning the number of primary and secondary turns, almost any desired voltage ratio, or ratio of transformation, can be obtained.
- The essence of transformer action requires only the existence of time-varying mutual flux linking two windings.
  - ➢ Iron-core transformer: coupling between the windings can be made much more effectively using a core of iron or other ferromagnetic material.
  - ➢ The magnetic circuit usually consists of a stack of thin laminations.
  - ➢ Silicon steel has the desirable properties of low cost, low core loss, and high permeability at high flux densities (1.0 to 1.5 T).
    - → Silicon-steel laminations 0.014 in thick are generally used for transformers operating at frequencies below a few hundred hertz.
  - ➢ Two common types of construction: core type and shell type (Fig. 2.1).

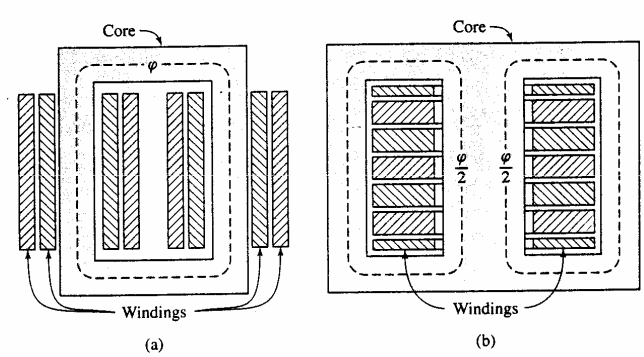


Figure 2.1 Schematic views of (a) core-type and (b) shell-type transformers.

- Most of the flux is confined to the core and therefore links both windings.
  - → Leakage flux links one winding without linking the other.
  - → Leakage flux is a small fraction of the total flux.
  - → Leakage flux is reduced by subdividing the windings into sections and by placing them as close together as possible.

§2.2 No-Load Conditions

- Figure 2.4 shows in schematic form a transformer with its secondary circuit open and an alternating voltage v_1 applied to its primary terminals.

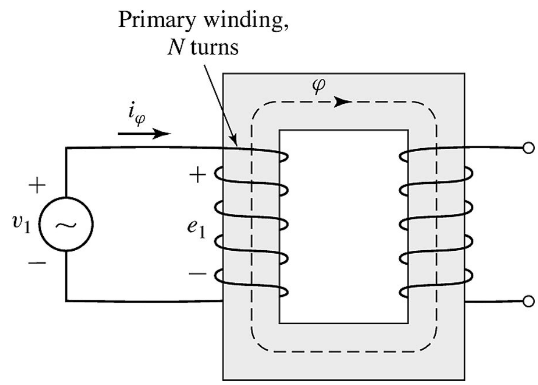


Figure 2.4 Transformer with open secondary.

- The primary and secondary windings are actually interleaved in practice.
- A small steady-state current i_φ (the exciting current) flows in the primary and establishes an alternating flux in the magnetic current.
- e_1 = emf induced in the primary (counter emf)
  λ_1 = flux linkage of the primary winding
  φ = flux in the core linking both windings
  N_1 = number of turns in the primary winding
  - → The induced emf (counter emf) leads the flux by 90°.

$$e_1 = \frac{d\lambda_1}{dt} = N_1 \frac{d\varphi}{dt} \tag{2.1}$$

$$v_1 = R_1 i_\varphi + e_1 \tag{2.2}$$

- $e_1 \approx v_1$ if the no-load resistance drop is very small and the waveforms of voltage and flux are very nearly sinusoidal.

$$\varphi = \phi_{max} \sin \omega t \tag{2.3}$$

$$e_1 = N_1 \frac{d\varphi}{dt} = \omega \phi_{max} \cos \omega t \tag{2.4}$$

$$E_1 = \frac{2\pi}{\sqrt{2}} f N_1 \phi_{max} = \sqrt{2}\pi f N_1 \phi_{max} \tag{2.5}$$

$$\boxed{\phi_{max} = \frac{V_1}{\sqrt{2}\pi f N_1}} \tag{2.6}$$

- The core flux is determined by the applied voltage, its frequency, and the number of turns in the winding. The core flux is fixed by the applied voltage, and the required exciting current is determined by the magnetic properties of the core; the exciting current must adjust itself so as to produce the mmf required to create the flux demanded by (2.6).

- A curve of the exciting current as a function of time can be found graphically from the ac hysteresis loop as shown in Fig. 1.11.

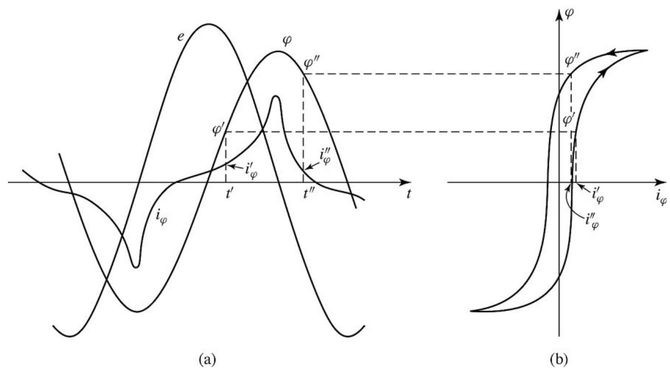


Figure 1.11 Excitation phenomena. (a) Voltage, flux, and exciting current; (b) corresponding hysteresis loop.

- If the exciting current is analyzed by Fourier-series methods, its is found to consist of a fundamental component and a series of odd harmonics.
  - The fundamental component can, in turn, be resolved into two components, one in phase with the counter emf and the other lagging the counter emf by 90^{o}.
    → Core-loss component: the in-phase component supplies the power absorbed by hysteresis and eddy-current losses in the core.
    → Magnetizing current: It comprises a fundamental component lagging the counter emf by 90^{o}, together with all the harmonics, of which the principal is the third (typically 40%).
  - The peculiarities of the exciting-current waveform usually need not by taken into account, because the exciting current itself is small, especially in large transformers. (typically about 1 to 2 percent of full-load current)
  - Phasor diagram in Fig. 2.5.
    $\hat{E}_1$ = the rms value of the induced emf
    $\hat{\Phi}$ = the rms value of the flux
    $\hat{I}_\varphi$ = the rms value of the equivalent sinusoidal exciting current
    → $\hat{I}_\varphi$ lags $\hat{E}_1$ by a phase angle θ_c.

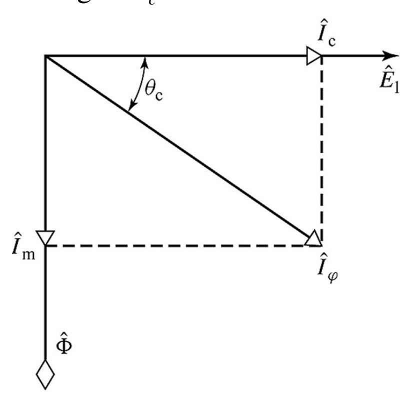


Figure 2.5 No-load phasor diagram.

→ The core loss P_c equals the product of the in-phase components of the $\hat{E}_1$ and $\hat{I}_\varphi$:

$$P_c = E_1 I_\varphi \cos\theta_c \tag{2.7}$$

→ $\hat{I}_c$ = core-loss current, $\hat{I}_m$ = magnetizing current

EXAMPLE 2.1

In Example 1.8 the core loss and exciting voltamperes for the core of Fig. 1.15 at $B_{max} = 1.5$ T and 60 Hz were found to be

$$P_c = 16 \text{ W} \qquad (VI)_{rms} = 20 \text{ VA}$$

and the induced voltage was $274/\sqrt{2} = 194$ V rms when the winding had 200 turns.

Find the power factor, the core-loss current I_c, and the magnetizing current I_m.

■ Solution

Power factor $\cos\theta_c = \frac{16}{20} = 0.80$ (lag) thus $\theta_c = -36.9°$

Note that we know that the power factor is lagging because the system is inductive.

Exciting current $I_\varphi = \frac{20}{194} = 0.10$ A rms

Core-loss component $I_c = \frac{16}{194} = 0.082$ A rms

Magnetizing component $I_m = I_\varphi|\sin\theta_c| = 0.060$ A rms

§2.3 Effect of Secondary Current; Ideal Transformer

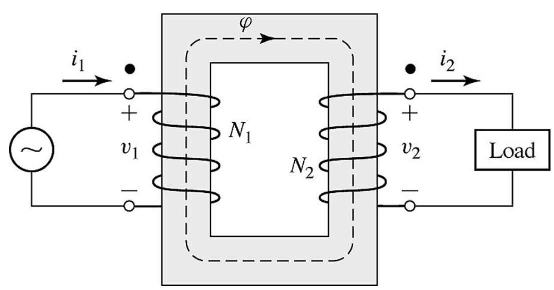


Figure 2.6 Ideal transformer and load.

- Ideal Transformer (Fig. 2.6)
  - Assumptions:
    1. Winding resistances are negligible.
    2. Leakage flux is assumed negligible.
    3. There are no losses in the core.
    4. Only a negligible mmf is required to establish the flux in the core.
  - The impressed voltage, the counter emf, the induced emf, and the terminal voltage:

$$v_1 = e_1 = N_1 \frac{d\varphi}{dt}, \quad v_2 = e_2 = N_2 \frac{d\varphi}{dt} \tag{2.8)(2.9}$$

$$\boxed{\frac{v_1}{v_2} = \frac{N_1}{N_2}} \tag{2.10}$$

→ An ideal transformer transforms voltages in the direct ratio of the turns in its windings.

  - Let a load be connected to the secondary.

$$N_1 i_1 - N_2 i_2 = 0, \quad N_1 i_1 = N_2 i_2 \tag{2.11)(2.12}$$

$$\boxed{\frac{i_1}{i_2} = \frac{N_2}{N_1}} \tag{2.13}$$

→ An ideal transformer transforms currents in the inverse ratio of the turns in its windings.

- From (2.10) and (2.13),

$$\boxed{v_1 i_1 = v_2 i_2} \tag{2.14}$$

→ Instantaneous power input to the primary equals the instantaneous power output from the secondary.

- Impedance transformation properties: Fig. 2.7.

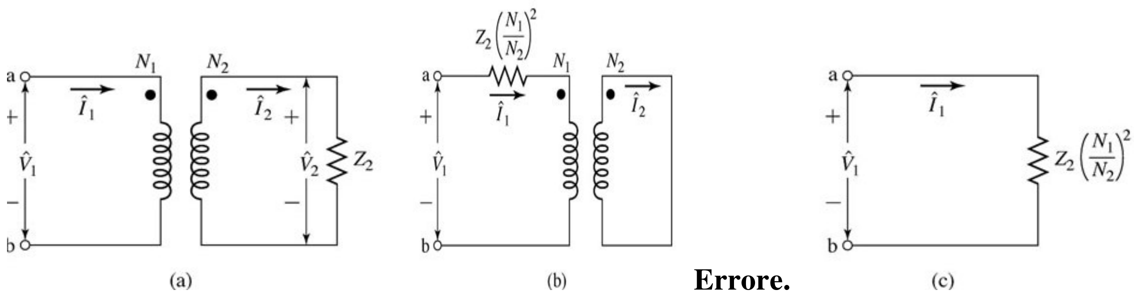


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Figure 2.7 Three circuits which are identical at terminals **ab** when the transformer is ideal.

$$\hat{v}_1 = \frac{N_1}{N_2}\hat{v}_2 \text{ and } \hat{v}_2 = \frac{N_2}{N_1}\hat{v}_1 \tag{2.15}$$

$$\hat{I}_1 = \frac{N_1}{N_2}\hat{I}_2 \text{ and } \hat{I}_2 = \frac{N_2}{N_1}\hat{I}_1 \tag{2.16}$$

$$\frac{\hat{V}_1}{\hat{I}_1} = \left(\frac{N_1}{N_2}\right)^2 \frac{\hat{V}_2}{\hat{I}_2} \tag{2.17}$$

$$Z_2 = \frac{\hat{V}_2}{\hat{I}_2} \tag{2.18}$$

$$\boxed{Z_1 = \left(\frac{N_1}{N_2}\right)^2 Z_2} \tag{2.19}$$

→ Transferring an impedance from one side to the other is called "referring the impedance to the other side." Impedances transform as the square of the turns ratio.

- Summary for the ideal transformer:
  → Voltages are transformed in the direct ratio of turns.
  → Currents are transformed in the inverse ratio of turns.
  → Impedances are transformed in the direct ratio squared.
  → Power and voltamperes are unchanged.

EXAMPLE 2.2

The equivalent circuit of Fig. 2.8a shows an ideal transformer with an impedance $R_2 + jX_2 = 1 + j4\ \Omega$ connected in series with the secondary. The turns ratio $N_1/N_2 = 5{:}1$. (*a*) Draw an equivalent circuit with the series impedance referred to the primary side. (*b*) For a primary voltage of 120 V rms and a short connected across the terminals A-B, calculate the primary current and the current flowing in the short.

■ **Solution**

a. The new equivalent is shown in Fig. 2.8b. The secondary impedance is referred to the primary by the turns ratio squared. Thus

$$R_2' + jX_2' = \left(\frac{N_1}{N_2}\right)^2 (R_2 + jX_2)$$
$$= 25 + j100\ \Omega$$

b. From Eq. 2.19, a short at terminals A-B will appear as a short at the primary of the ideal transformer in Fig. 2.8b since the zero voltage of the short is reflected by the turns ratio N_1/N_2 to the primary. Hence the primary current will be given by

$$\hat{I}_1 = \frac{\hat{V}_1}{R_2' + jX_2'} = \frac{120}{25 + j100} = 0.28 - j1.13 \text{ A rms}$$

corresponding to a magnitude of 1.16 A rms. From Eq. 2.13, the secondary current will equal $N_1/N_2 = 5$ times that of the current in the primary. Thus the current in the short will have a magnitude of $5(1.16) = 5.8$ A rms.

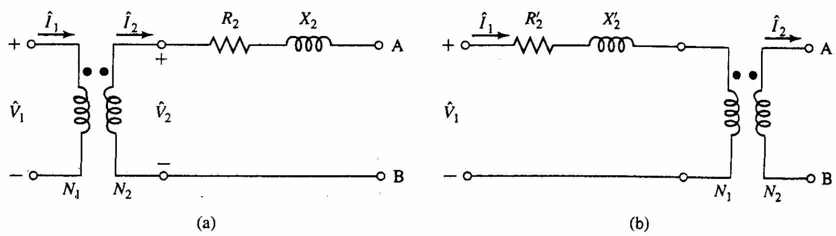

Figure 2.8 Equivalent circuits for Example 2.2. (a) Impedance in series with the secondary. (b) Impedance referred to the primary.

§2.4 Transformer Reactances and Equivalent Circuits

- A more complete model must take into account the effects of winding resistances, leakage fluxes, and finite exciting current due to the finite and nonlinear permeability of the core.
  - ➢ Note that the capacitances of the windings will be neglected.
  - ➢ Method of the equivalent circuit technique is adopted for analysis.
- Development of the transformer equivalent circuit
  - ➢ Leakage flux: Fig. 2.9.

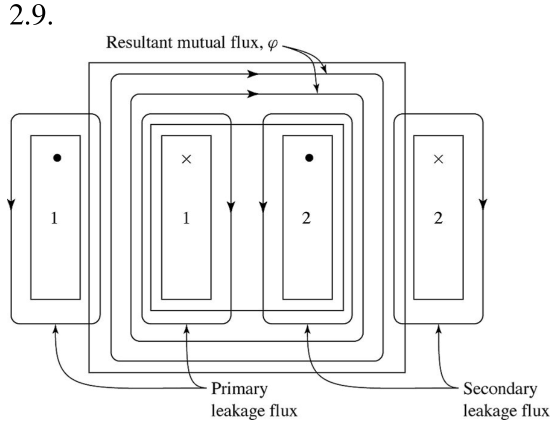

Figure 2.9 Schematic view of mutual and leakage fluxes in a transformer.

→ L_{l_1} = primary leakage inductance, X_{l_1} = primary leakage reactance

$$X_{l_1} = 2\pi f L_{l_1} \tag{2.20}$$

➢ Effect of the primary winding resistance: R_1

➢ Effect of the exciting current:

$$\begin{aligned} N_1\hat{I}_\varphi &= N_1\hat{I}_1 - N_2\hat{I}_2 \\ &= N_1\left(\hat{I}_\varphi + \hat{I}_2'\right) - N_2\hat{I}_2 \end{aligned} \tag{2.21}$$

$$\hat{I}_2{}' = \frac{N_2}{N_1}\hat{I}_2 \tag{2.22}$$

➢ L_m = magnetizing inductance, X_m = magnetizing reactance

$$X_m = 2\pi f L_m \tag{2.23}$$

➢ Ideal transformer:

$$\frac{\hat{E}_1}{\hat{E}_2} = \frac{N_1}{N_2} \tag{2.24}$$

➢ Secondary resistance, secondary leakage reactance

➢ Equivalent-T circuit for a transformer:

$$\hat{X}_{1_2} = \left(\frac{N_1}{N_2}\right)^2 X_{1_2}, \quad R_2' = \left(\frac{N_1}{N_2}\right)^2 R_2, \quad V_2' = \left(\frac{N_1}{N_2}\right)^2 V_2 \qquad \text{(2.25)-(2.27)}$$

- Steps in the development of the transformer equivalent circuit: Fig. 2.10.
  ➢ The actual transformer can be seen to be equivalent to an ideal transformer plus external impedances
  ➢ Refer to the assumptions for an ideal transformer to understand the definitions and meanings of these resistances and reactances.

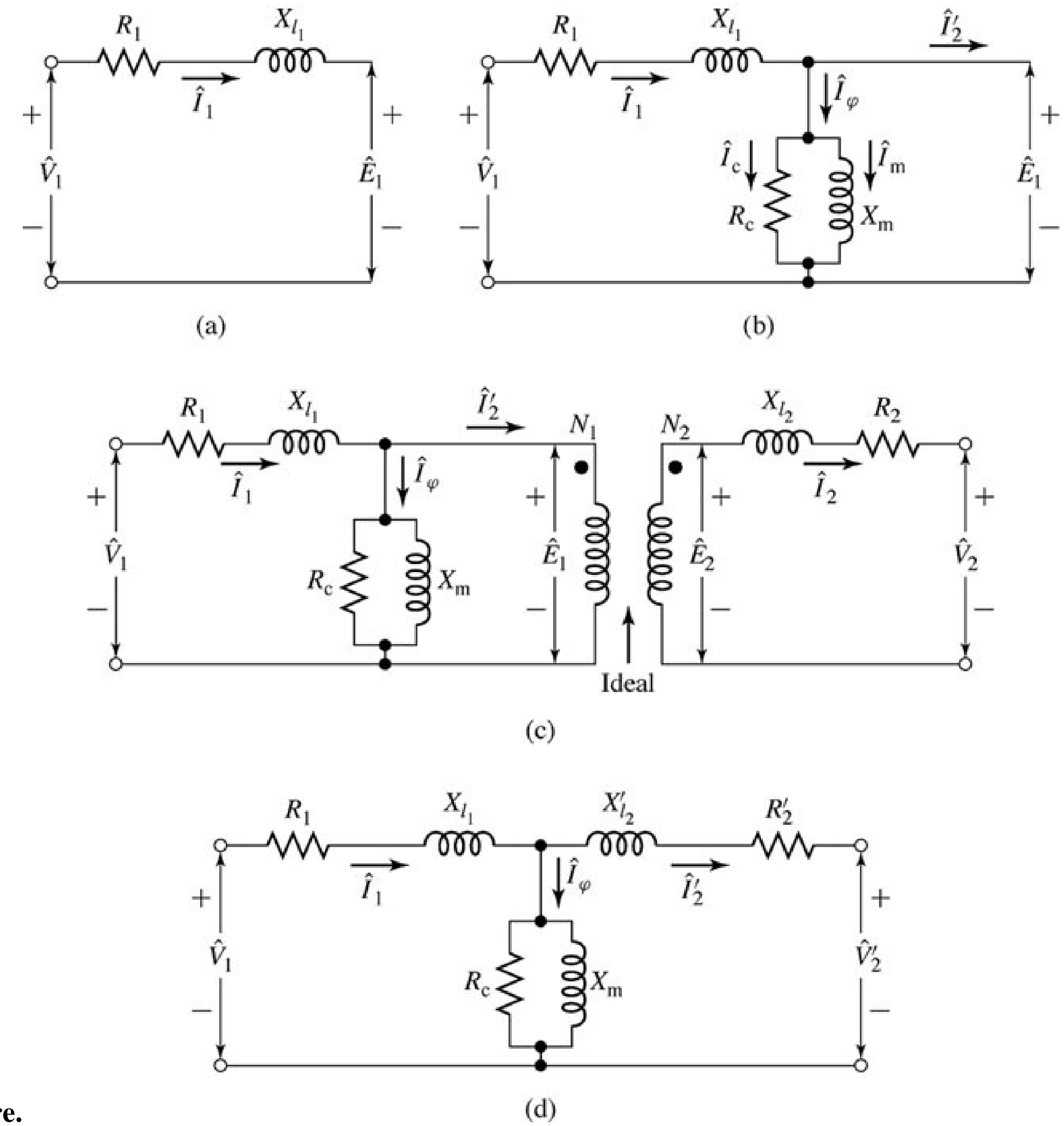

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Figure 2.10 Steps in the development of the transformer equivalent circuit.

EXAMPLE 2.3

A 50-kVA 2400:240-V 60-Hz distribution transformer has a leakage impedance of $0.72 + j0.92\ \Omega$ in the high-voltage winding and $0.0070 + j0.0090\ \Omega$ in the low-voltage winding. At rated voltage and frequency, the impedance Z_φ of the shunt branch (equal to the impedance of R_c and jX_m in parallel) accounting for the exciting current is $6.32 + j43.7\ \Omega$ when viewed from the low-voltage side. Draw the equivalent circuit referred to (*a*) the high-voltage side and (*b*) the low-voltage side, and label the impedances numerically.

■ Solution

The circuits are given in Fig. 2.11a and b, respectively, with the high-voltage side numbered 1 and the low-voltage side numbered 2. The voltages given on the nameplate of a power system transformer are based on the turns ratio and neglect the small leakage-impedance voltage drops under load. Since this is a 10-to-1 transformer, impedances are referred by multiplying or dividing by 100; for example, the value of an impedance referred to the high-voltage side is greater by a factor of 100 than its value referred to the low-voltage side.

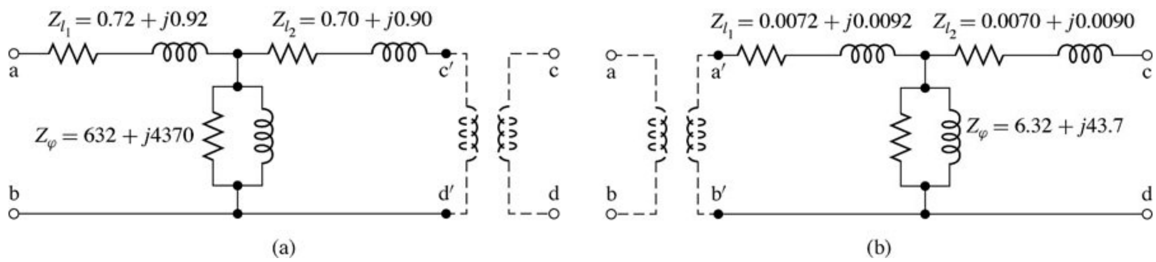

Figure 2.11 Equivalent circuits for transformer of Example 2.3 referred to (a) the high-voltage side and (b) the low-voltage side.

The ideal transformer may be explicitly drawn, as shown dotted in Fig. 2.11, or it may be omitted in the diagram and remembered mentally, making the unprimed letters the terminals. If this is done, one must of course remember to refer all connected impedances and sources to be consistent with the omission of the ideal transformer.

§2.5 Engineering Aspects of Transformer Analysis

- Approximate forms of the equivalent circuit:

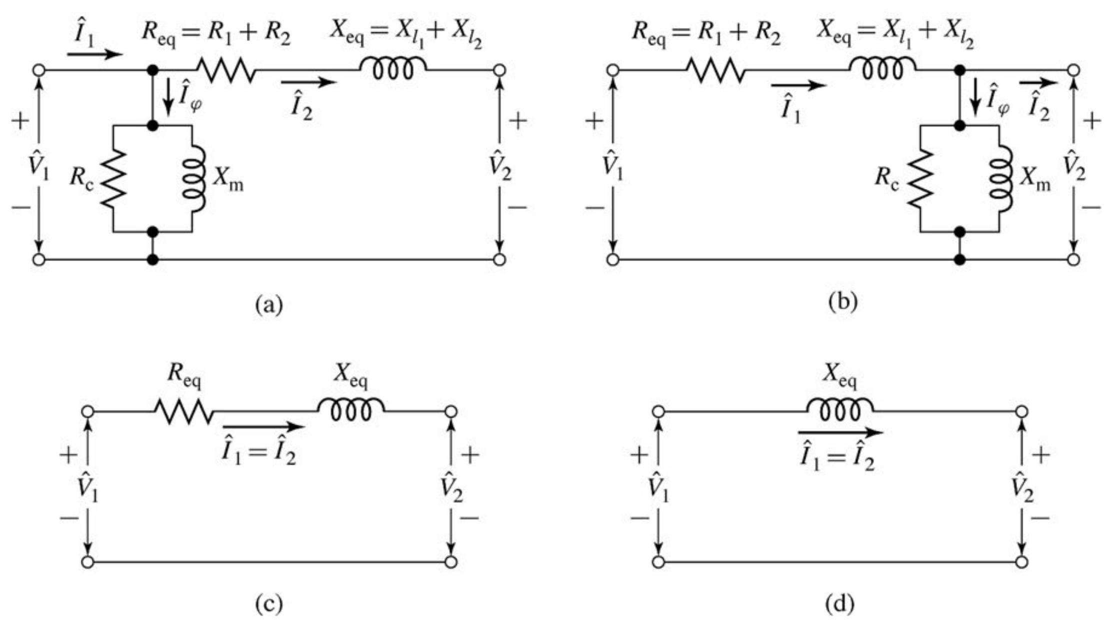

Figure 2.12 Approximate transformer equivalent circuits.

EXAMPLE 2.4

Consider the equivalent-T circuit of Fig. 2.11a of the 50-kVA 2400:240 V distribution transformer of Example 2.3 in which the impedances are referred to the high-voltage side. (*a*) Draw the cantilever equivalent circuit with the shunt branch at the high-voltage terminal. Calculate and label R_{eq} and X_{eq}. (*b*) With the low-voltage terminal open-circuit and 2400 V applied to the high-voltage terminal, calculate the voltage at the low-voltage terminal as predicted by each equivalent circuit.

■ Solution

a. The cantilever equivalent circuit is shown in Fig. 2.13. R_{eq} and X_{eq} are found simply as the sum of the high- and low-voltage winding series impedances of Fig. 2.11a

$$R_{eq} = 0.72 + 0.70 = 1.42\ \Omega$$

$$X_{eq} = 0.92 + 0.90 = 1.82\ \Omega$$

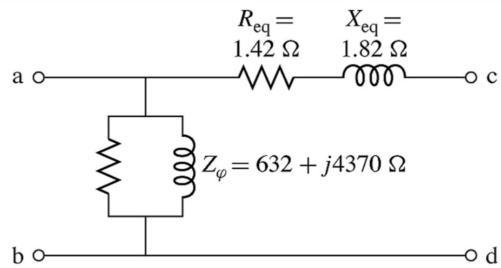

Figure 2.13 Cantilever equivalent circuit for Example 2.4.

b. For the equivalent-T circuit of Fig. 2.11a, the voltage at the terminal labeled c′-d′ will be given by

$$\hat{V}_{c'\text{-}d'} = 2400\left(\frac{Z_{\varphi}}{Z_{\varphi} + Z_{l_1}}\right) = 2399.4 + j0.315\ \text{V}$$

This corresponds to an rms magnitude of 2399.4 V. Reflected to the low-voltage terminals by the low- to high-voltage turns ratio, this in turn corresponds to a voltage of 239.94 V.

Because the exciting impedance is connected directly across the high-voltage terminals in the cantilever equivalent circuit of Fig. 2.13, there will be no voltage drop across any series leakage impedance and the predicted secondary voltage will be 240 V. These two solutions differ by 0.025 percent, well within reasonable engineering accuracy and clearly justifying the use of the cantilever equivalent circuit for analysis of this transformer.

EXAMPLE 2.5

The 50-kVA 2400:240-V transformer whose parameters are given in Example 2.3 is used to step down the voltage at the load end of a feeder whose impedance is $0.30 + j1.60\ \Omega$. The voltage V_s at the sending end of the feeder is 2400 V.

Find the voltage at the secondary terminals of the transformer when the load connected to its secondary draws rated current from the transformer and the power factor of the load is 0.80 lagging. Neglect the voltage drops in the transformer and feeder caused by the exciting current.

■ Solution

The circuit with all quantities referred to the high-voltage (primary) side of the transformer is shown in Fig. 2.14a, where the transformer is represented by its equivalent impedance, as in Fig. 2.12c. From Fig. 2.11a, the value of the equivalent impedance is $Z_{eq} = 1.42 + j1.82\ \Omega$ and the combined impedance of the feeder and transformer in series is $Z = 1.72 + j3.42\ \Omega$. From the transformer rating, the load current referred to the high-voltage side is $I = 50{,}000/2400 = 20.8$ A.

This solution is most easily obtained with the aid of the phasor diagram referred to the high-voltage side as shown in Fig. 2.14b. Note that the power factor is defined at the load side of the transformer and hence defines the phase angle θ between the load current $\hat{I}$ and the voltage $\hat{V}_2$

$$\theta = -\cos^{-1}(0.80) = -36.87^{\circ}$$

From the phasor diagram

$$Ob = \sqrt{V_s^2 - (bc)^2} \quad \text{and} \quad V_2 = Ob - ab$$

Note that

$$bc = IX\cos\theta - IR\sin\theta \quad ab = IR\cos\theta + IX\sin\theta$$

where R and X are the combined resistance and reactance, respectively. Thus

$$bc = 20.8(3.42)(0.80) - 20.8(1.72)(0.60) = 35.5 \text{ V}$$

$$ab = 20.8(1.72)(0.80) + 20.8(3.42)(0.60) = 71.4 \text{ V}$$

Substitution of numerical values shows that $V_2 = 2329$ V, referred to the high-voltage side. The actual voltage at the secondary terminals is 2329/10, or

$$V_2 = 233 \text{ V}$$

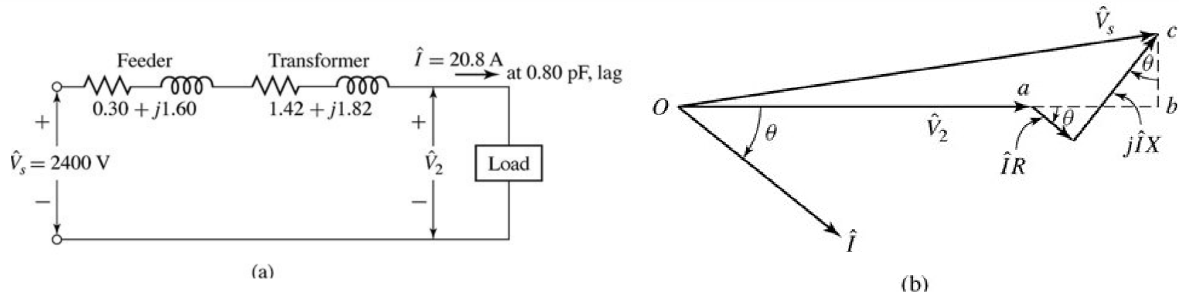

Figure 2.14 (a) Equivalent circuit and (b) phasor diagram for Example 2.5.

- Two tests serve to determine the parameters of the equivalent circuits of Figs. 2.10 and 2.12.
  - Short-circuit test and open-circuit test

- Short-Circuit Test
  - The test is used to find the equivalent series impedance $R_{eq} + jX_{eq}$.
  - The high voltage side is usually taken as the primary to which voltage is applied.
  - The short circuit is applied to the secondary
  - Typically an applied voltage on the order of 10 to 15 % or less of the rated value will result in rated current.
  - See Fig. 2.15. Note that $Z_\varphi = R_c // jX_m$.

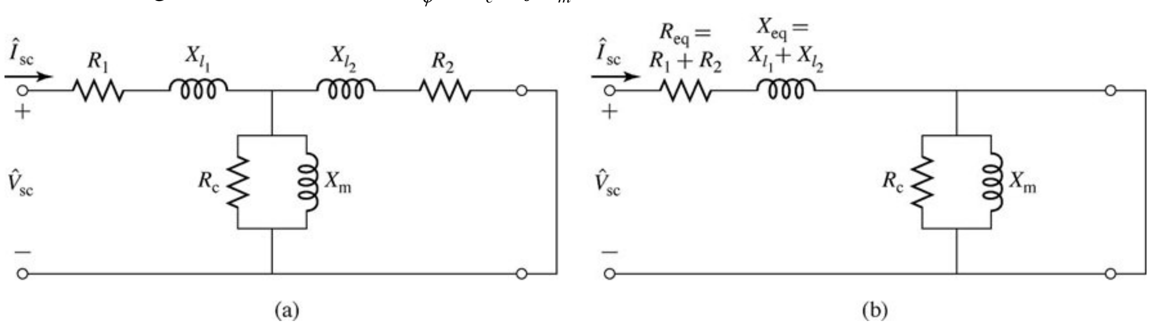

Figure 2.15 Equivalent circuit with short-circuited secondary. (a) Complete equivalent circuit. (b) Cantilever equivalent circuit with the exciting branch at the transformer secondary.

$$Z_{sc} = R_1 + jX_{1_1} + \frac{Z_\varphi\left(R_2 + jX_{1_2}\right)}{Z_\varphi + R_2 + jX_{1_2}} \tag{2.28}$$

$$Z_{sc} \approx R_1 + jX_{1_1} + R_2 + jX_{1_2} = R_{eq} + jX_{eq} \tag{2.29}$$

- Typically the instrumentation will measure the rms magnitude of the applied voltage V_{sc}, the short-circuit current I_{sc}, and the power P_{sc}. The circuit parameters (referred to the primary) can be found as (2.30)-(2.32).

$$|Z_{eq}| = |Z_{sc}| = \frac{V_{sc}}{I_{sc}} \tag{2.30}$$

$$R_{eq} = R_{sc} = \frac{P_{sc}}{I_{sc}^2} \tag{2.31}$$

$$X_{eq} = X_{sc} = \sqrt{|Z_{sc}|^2 - R_{sc}^2} \tag{2.32}$$

- The equivalent impedance can be referred from one side to the other.
- Approximate values of the individual primary and secondary resistances and leakage reactances can be obtained by assuming that $R_1 = R_2 = 0.5R_{eq}$ and $X_{l_1} = X_{l_2} = 0.5X_{eq}$ when all impedances are referred to the same side.
- Note that it is possible to measure R_1 and R_2 directly by a dc resistance measurement on each winding. However, no such simple test exists for X_{l_1} and X_{l_2}.

- Open-Circuit Test
  - The test is used to find the equivalent shunt impedance $R_c \,//\, jX_m$.
  - The test is performed with the secondary open-circuited and rated voltage impressed on the primary. If the transformer is to be used at other than its rated voltage, the test should be done at that voltage.
  - An exciting current of a few percent of full-load current is obtained.
  - See Fig. 2.16. Note that $Z_\varphi = R_c \,//\, jX_m$.

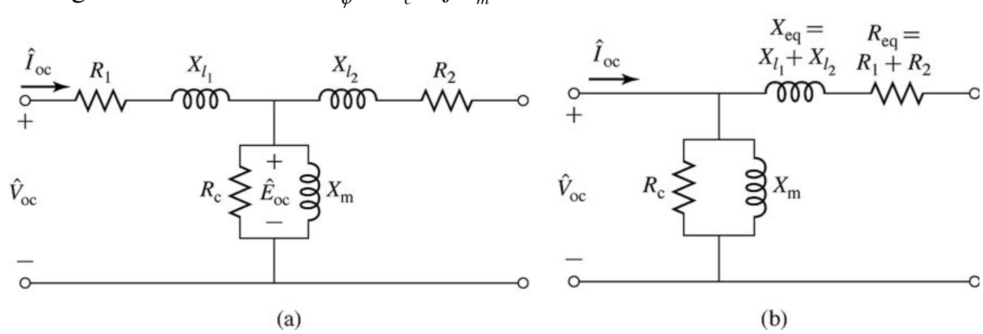

Figure 2.16 Equivalent circuit with open-circuited secondary. (a) Complete equivalent circuit. (b) Cantilever equivalent circuit with the exciting branch at the transformer primary.

$$Z_{oc} = R_1 + jX_{1_1} + Z_\varphi = R_1 + jX_{1_1} + \frac{R_c(jX_m)}{R_c + jX_m} \tag{2.33}$$

$$Z_{oc} \approx Z_\varphi = \frac{R_c(jX_m)}{R_c + jX_m} \tag{2.34}$$

- Typically the instrumentation will measure the rms magnitude of the applied voltage V_{oc}, the open-circuit current I_{oc}, and the power P_{oc}. The circuit parameters (referred to the primary) can be found as (2.35)-(2.37).

$$R_c = \frac{V_{oc}^2}{P_{oc}} \tag{2.35}$$

$$|Z_\varphi| = \frac{V_{oc}}{P_{oc}} \tag{2.36}$$

$$X_m = \frac{1}{\sqrt{(1/|Z_\varphi|)^2 - (1/R_c)^2}} \tag{2.37}$$

- The open-circuit test can be used to obtain the core loss for efficiency computations and to check the magnitude of the exciting current.

- Note the term "Voltage Regulation" which is to be discussed in Example 2.6.

EXAMPLE 2.6

With the instruments located on the high-voltage side and the low-voltage side short-circuited, the short-circuit test readings for the 50-kVA 2400:240-V transformer of Example 2.3 are 48 V, 20.8 A, and 617 W. An open-circuit test with the low-voltage side energized gives instrument readings on that side of 240 V, 5.41 A, and 186 W. Determine the efficiency and the voltage regulation at full load, 0.80 power factor lagging.

■ Solution

From the short-circuit test, the magnitude of the equivalent impedance, the equivalent resistance, and the equivalent reactance of the transformer (referred to the high-voltage side as denoted by the subscript H) are

$$|Z_{\mathrm{eq,H}}| = \frac{48}{20.8} = 2.31\ \Omega \qquad R_{\mathrm{eq,H}} = \frac{617}{20.8^2} = 1.42\ \Omega$$

$$X_{\mathrm{eq,H}} = \sqrt{2.31^2 - 1.42^2} = 1.82\ \Omega$$

Operation at full-load, 0.80 power factor lagging corresponds to a current of

$$I_{\mathrm{H}} = \frac{50{,}000}{2400} = 20.8\ \mathrm{A}$$

and an output power

$$P_{\mathrm{output}} = P_{\mathrm{load}} = (0.8)50{,}000 = 40{,}000\ \mathrm{W}$$

The total loss under this operating condition is equal to the sum of the winding loss

$$P_{\mathrm{winding}} = I_{\mathrm{H}}^2 R_{\mathrm{eq,H}} = 20.8^2(1.42) = 617\ \mathrm{W}$$

and the core loss determined from the open-circuit test

$$P_{\mathrm{core}} = 186\ \mathrm{W}$$

Thus

$$P_{\mathrm{loss}} = P_{\mathrm{winding}} + P_{\mathrm{core}} = 803\ \mathrm{W}$$

and the power input to the transformer is

$$P_{\mathrm{input}} = P_{\mathrm{output}} + P_{\mathrm{loss}} = 40{,}803\ \mathrm{W}$$

The *efficiency* of a power conversion device is defined as

$$\text{efficiency} = \frac{P_{\mathrm{output}}}{P_{\mathrm{input}}} = \frac{P_{\mathrm{input}} - P_{\mathrm{loss}}}{P_{\mathrm{input}}} = 1 - \frac{P_{\mathrm{loss}}}{P_{\mathrm{input}}}$$

which can be expressed in percent by multiplying by 100 percent. Hence, for this operating condition

$$\text{efficiency} = 100\% \left(\frac{P_{\mathrm{output}}}{P_{\mathrm{input}}}\right) = 100\% \left(\frac{40{,}000}{40{,}000 + 803}\right) = 98.0\%$$

The *voltage regulation* of a transformer is defined as the change in secondary terminal voltage from no load to full load and is usually expressed as a percentage of the full-load value. In power systems applications, regulation is one figure of merit for a transformer; a low value indicates that load variations on the secondary of that transformer will not significantly affect the magnitude of the voltage being supplied to the load. It is calculated under the assumption that the primary voltage remains constant as the load is removed from the transformer secondary.

The equivalent circuit of Fig. 2.12c will be used with all quantities referred to the high-voltage side. The primary voltage is assumed to be adjusted so that the secondary terminal voltage has its rated value at full load, or $V_{2\mathrm{H}} = 2400$ V. For a load of rated value and 0.8 power factor lagging (corresponding to a power factor angle $\theta = -\cos^{-1}(0.8) = -36.9°$), the load current will be

$$\hat{I}_{\mathrm{H}} = \left(\frac{50 \times 10^3}{2400}\right) e^{-j36.9°} = 20.8(0.8 - j0.6)\ \mathrm{A}$$

The required value of the primary voltage $V_{1\mathrm{H}}$ can be calculated as

$$\begin{aligned} \hat{V}_{1\mathrm{H}} &= \hat{V}_{2\mathrm{H}} + \hat{I}_{\mathrm{H}}(R_{\mathrm{eq,H}} + jX_{\mathrm{eq,H}}) \\ &= 2400 + 20.8(0.80 - j0.60)(1.42 + j1.82) \\ &= 2446 + j13 \end{aligned}$$

The magnitude of $\hat{V}_{\mathrm{lH}}$ is 2446 V. If this voltage were held constant and the load removed, the secondary voltage on open circuit would rise to 2446 V referred to the high-voltage side. Then

$$\text{Regulation} = \frac{2446 - 2400}{2400}(100\%) = 1.92\%$$

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§2.6 Autotransformers; Multiwinding Transformers

- Two-winding $\Rightarrow$ Other winding configurations.

§2.6.1 Autotransformers

- Autotransformer connection: Fig. 2.17.

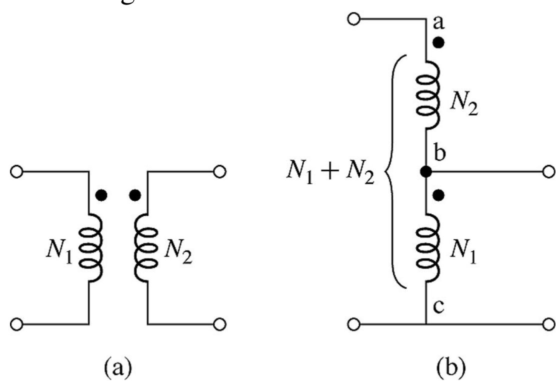

Figure 2.17 (a) Two-winding transformer. (b) Connection as an autotransformer.

- The windings of the two-winding transformer are electrically isolated whereas those of the autotransformer are connected directly together.
- In the transformer connection, winding **ab** must be provided with extra insulation.
- Autotransformer have lower leakage reactances, lower losses, and smaller exciting current and cost less than two-winding transformers when the voltage ration does not differ too greatly from 1:1.
- The rated voltages of the transformer can be expressed in terms of those of the two-winding transformer as

$$V_{L_{rated}} = V_{1_{rated}} \tag{2.38}$$

$$V_{H_{rated}} = V_{1_{rated}} + V_{2_{rated}} = \left(\frac{N_1 + N_2}{N_1}\right) V_{L_{rated}} \tag{2.39}$$

→ The effective turns ratio of the autotransformer is thus $(N_1 + N_2)/N_1$.

→ The power rating of the autotransformer is equal to $(N_1 + N_2)/N_2$ times that of the two-winding transformer.

EXAMPLE 2.7

The 2400:240-V 50-kVA transformer of Example 2.6 is connected as an autotransformer, as shown in Fig. 2.18a, in which ab is the 240-V winding and bc is the 2400-V winding. (It is assumed that the 240-V winding has enough insulation to withstand a voltage of 2640 V to ground.)

a. Compute the voltage ratings V_H and V_X of the high- and low-voltage sides, respectively, for this autotransformer connection.
b. Compute the kVA rating as an autotransformer.
c. Data with respect to the losses are given in Example 2.6. Compute the full-load efficiency as an autotransformer operating with a rated load of 0.80 power factor lagging.

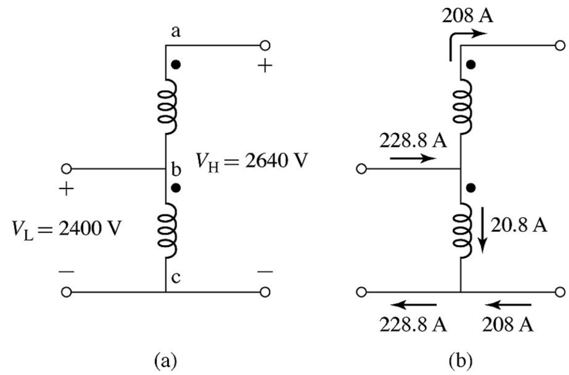

Figure 2.18 (a) Autotransformer connection for Example 2.7.
(b) Currents under rated load.

■ **Solution**

a. Since the 2400-V winding bc is connected to the low-voltage circuit, $V_L = 2400$ V. When $V_{bc} = 2400$ V, a voltage $V_{ab} = 240$ V in phase with V_{bc} will be induced in winding ab (leakage-impedance voltage drops being neglected). The voltage of the high-voltage side therefore is

$$V_H = V_{ab} + V_{bc} = 2640 \text{ V}$$

b. From the rating of 50 kVA as a normal two-winding transformer, the rated current of the 240-V winding is 50,000/240 = 208 A. Since the high-voltage lead of the autotransformer is connected to the 240-V winding, the rated current I_H at the high-voltage side of the autotransformer is equal to the rated current of the 240-V winding or 208 A. The kVA rating as an autotransformer therefore is

$$\frac{V_H I_H}{1000} = \frac{2640(208)}{1000} = 550 \text{ kVA}$$

Note that, in this connection, the autotransformer has an equivalent turns ratio of 2640/2400. Thus the rated current at the low-voltage winding (the 2400-V winding in this connection) must be

$$I_L = \left(\frac{2640}{2400}\right) 208 \text{ A} = 229 \text{ A}$$

At first, this seems rather unsettling since the 2400-V winding of the transformer has a rated current of 50 kVA/2400 V = 20.8 A. Further puzzling is that fact that this transformer, whose rating as a normal two-winding transformer is 50 kVA, is capable of handling 550 kVA as an autotransformer.

The higher rating as an autotransformer is a consequence of the fact that not all the 550 kVA has to be transformed by electromagnetic induction. In fact, all that the transformer has to do is to boost a current of 208 A through a potential rise of 240 V, corresponding to a power transformation capacity of 50 kVA. This fact is perhaps best illustrated by Fig. 2.18b which shows the currents in the autotransformer under rated conditions. Note that the windings carry only their rated currents in spite of higher rating of the transformer.

c. When it is connected as an autotransformer with the currents and voltages shown in Fig. 2.18, the losses are the same as in Example 2.6, namely, 803 W. But the output as an autotransformer at full load, 0.80 power factor is 0.80(550,000) = 440,000 W. The efficiency therefore is

$$\left(1 - \frac{803}{440{,}803}\right) 100\% = 99.82\%$$

The efficiency is so high because the losses are those corresponding to transforming only 50 kVA.

§2.6.2 Multiwinding Transformers

- Transformers having three or more windings, known as multiwinding or multicircuit transformers, are often used to interconnect three or more circuits which may have different voltages.
  - Trsansformers having a primary and multiple secondaries are frequently found in multiple-output dc power supplies.
  - Distribution transformers used to supply power for domestic purposes usually have two 120-V secondaries connected in series.
  - The three-phase transformer banks used to interconnect two transmission system of different voltages often have a third, or tertiary, set of windings to provide voltage for auxiliary power purposes in substation or to supply a local distribution system.
    - → Static capacitors or synchronous condensers may be connected to the tertiary windings for power factor correction or voltage regulation.
    - → Sometimes Δ-connected tertiary windings are put on three-phase banks to provide a low-impedance path for third harmonic components of the exciting current to reduce third-harmonic components of the neutral voltage.

§2.7 Transformers in Three-Phase Circuits

- Three single-phase transformers can be connected to form a three-phase transformer bank in any of the four ways shown in Fig. 2.19. Note that $a = N_1 / N_2$.

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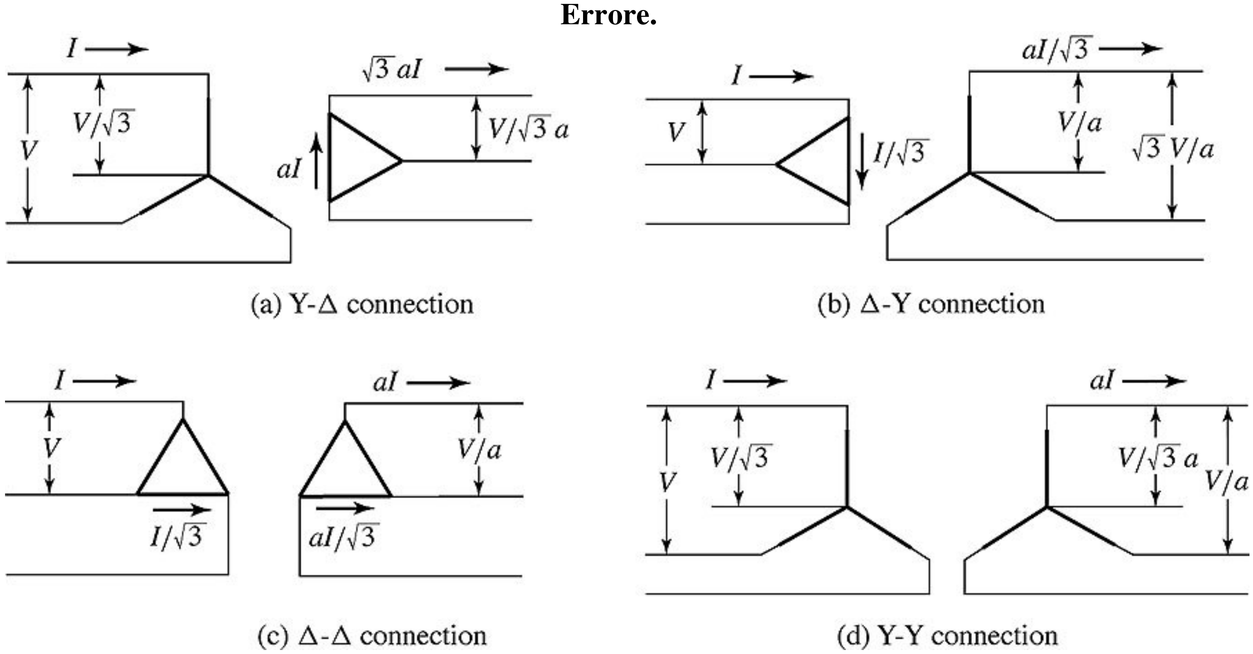

Figure 2.19 Common three-phase transformer connections; the transformer windings are indicated by the heavy lines.

- The Y-Δ connection is commonly used in stepping down from a high voltage to a medium or low voltage.
- The Δ-Y connection is commonly used for stepping up to a high voltage.
- The Δ-Δ connection has the advantage that one transformer can be removed for repair or maintenance while the remaining two continue to function as a three-phase bank with the

rating reduced to 58 percent of that of the original bank. (Open-delta, or V, connection)

- The Y-Y connection is seldom used because of difficulties with exciting-current phenomenon.
  - → Because there is no neutral connection to carry harmonics of the exciting current and harmonic voltages are produced which significantly distort the transformer voltages.

- A three-phase bank may consist of one three-phase transformer having all six windings on a common multi-legged core and contained in a single tank.
  - They cost less, weigh less, require less floor space, and have somewhat higher efficiency.

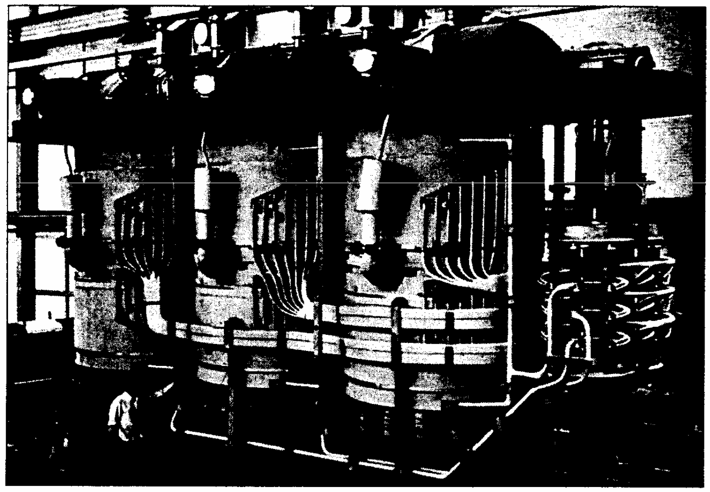

Figure 2.20 A 200-MVA, three-phase, 50-Hz, three-winding, 210/80/10.2-kV transformer removed from its tank. The 210-kV winding has an on-load tap changer for adjustment of the voltage. (*Brown Boveri Corporation.*)

- It is usually convenient to carry out circuit computations involving three-phase transformer banks under balanced conditions on a single-phase (per-phase-Y, line-to-neutral) basis.
  - Y-Δ, Δ-Y, and Δ-Δ connections $\Rightarrow$ equivalent Y-Y connections
  - A balanced Δ-connected circuit of Z_Δ Ω/phase is equivalent to a balanced Y-connected circuit of Z_Y Ω/phase if

$$Z_Y = \frac{1}{3} Z_\Delta \tag{2.40}$$

EXAMPLE 2.8

Three single-phase, 50-kVA 2400:240-V transformers, each identical with that of Example 2.6, are connected Y-Δ in a three-phase 150-kVA bank to step down the voltage at the load end of a feeder whose impedance is $0.15 + j1.00$ Ω/phase. The voltage at the sending end of the feeder is 4160 V line-to-line. On their secondary sides, the transformers supply a balanced three-phase load through a feeder whose impedance is $0.0005 + j0.0020$ Ω/phase. Find the line-to-line voltage at the load when the load draws rated current from the transformers at a power factor of 0.80 lagging.

■ Solution

The computations can be made on a single-phase basis by referring everything to the high-voltage, Y-connected side of the transformer bank. The voltage at the sending end of the feeder is equivalent to a source voltage V_s of

$$V_s = \frac{4160}{\sqrt{3}} = 2400 \text{ V line-to-neutral}$$

From the transformer rating, the rated current on the high-voltage side is 20.8 A/phase Y. The low-voltage feeder impedance referred to the high voltage side by means of the square of the rated line-to-line voltage ratio of the transformer bank is

$$Z_{\text{lv,H}} = \left(\frac{4160}{240}\right)^2 (0.0005 + j0.0020) = 0.15 + j0.60\ \Omega$$

and the combined series impedance of the high- and low-voltage feeders referred to the high-voltage side is thus

$$Z_{\text{feeder,H}} = 0.30 + j1.60\ \Omega/\text{phase Y}$$

Because the transformer bank is Y-connected on its high-voltage side, its equivalent single-phase series impedance is equal to the single-phase series impedance of each single-phase transformer as referred to its high-voltage side. This impedance was originally calculated in Example 2.4 as

$$Z_{\text{eq,H}} = 1.42 + j1.82\ \Omega/\text{phase Y}$$

Due to the choice of values selected for this example, the single-phase equivalent circuit for the complete system is identical to that of Example 2.5, as can been seen with specific reference to Fig. 2.14a. In fact, the solution on a per-phase basis is exactly the same as the solution to Example 2.5, whence the load voltage referred to the high-voltage side is 2329 V to neutral. The actual line-neutral load voltage can then be calculated by referring this value to the low-voltage side of the transformer bank as

$$V_{\text{load}} = 2329\left(\frac{240}{4160}\right) = 134 \text{ V line-to-neutral}$$

which can be expressed as a line-to-line voltage by multiplying by $\sqrt{3}$

$$V_{\text{load}} = 134\sqrt{3} = 233 \text{ V line-to-line}$$

Note that this line-line voltage is equal to the line-neutral load voltage calculated in Example 2.5 because in this case the transformers are delta connected on their low-voltage side and hence the line-line voltage on the low-voltage side is equal to the low-voltage terminal voltage of the transformers.

EXAMPLE 2.9

The three transformers of Example 2.8 are reconnected Δ-Δ and supplied with power through a 2400-V (line-to-line) three-phase feeder whose reactance is 0.80 Ω/phase. At its sending end, the feeder is connected to the secondary terminals of a three-phase Y-Δ-connected transformer whose rating is 500 kVA, 24 kV:2400 V (line-to-line). The equivalent series impedance of the sending-end transformer is $0.17 + j0.92\ \Omega$/phase referred to the 2400-V side. The voltage applied to the primary terminals of the sending-end transformer is 24.0 kV line-to-line.

A three-phase short circuit occurs at the 240-V terminals of the receiving-end transformers. Compute the steady-state short-circuit current in the 2400-V feeder phase wires, in the primary and secondary windings of the receiving-end transformers, and at the 240-V terminals.

■ Solution

The computations will be made on an equivalent line-to-neutral basis with all quantities referred to the 2400-V feeder. The source voltage then is

$$\frac{2400}{\sqrt{3}} = 1385 \text{ V line-to-neutral}$$

From Eq. 2.40, the single-phase-equivalent series impedance of the Δ-Δ transformer seen at its 2400-V side is

$$Z_{eq} = R_{eq} + jX_{eq} = \frac{1.42 + j1.82}{3} = 0.47 + j0.61\ \Omega/\text{phase}$$

The total series impedance to the short circuit is then the sum of this impedance, that of sending-end transformer and the reactance of the feeder

$$Z_{tot} = (0.47 + j0.61) + (0.17 + j0.92) + j0.80 = 0.64 + j2.33\ \Omega/\text{phase}$$

which has a magnitude of

$$|Z_{tot}| = 2.42\ \Omega/\text{phase}$$

The magnitude of the phase current in the 2400-V feeder can now simply be calculated as the line-neutral voltage divided by the series impedance

$$\text{Current in 2400-V feeder} = \frac{1385}{2.42} = 572\ \text{A}$$

and, as is shown in Fig. 2.19c, the winding current in the 2400-V winding of the receiving-end transformer is equal to the phase current divided by $\sqrt{3}$ or

$$\text{Current in 2400-V windings} = \frac{572}{\sqrt{3}} = 330\ \text{A}$$

while the current in the 240-V windings is 10 times this value

$$\text{Current in 240-V windings} = 10 \times 330 = 3300\ \text{A}$$

Finally, again with reference to Fig. 2.19c, the phase current at the 240-V terminals into the short circuit is given by

$$\text{Current at the 240-V terminals} = 3300\sqrt{3} = 5720\ \text{A}$$

Note of course that this same result could have been computed simply by recognizing that the turns ratio of the Δ-Δ transformer bank is equal to 10:1 and hence, under balanced-three-phase conditions, the phase current on the low voltage side will be 10 times that on the high-voltage side.

§2.8 Voltage and Current Transformers

- Transformers are often used in instrumentation applications to match the magnitude of a voltage or current to the range of a meter or other instrumentation.
  - ➢ Most 60-Hz power-systems' instrumentation is based upon voltages in the range of 0-120 V rms and currents in the range of 0-5 A rms.
  - ➢ Power system voltages range up to 765-kV line-to-line and currents can be 10's of kA.
    - → Some method of supplying an accurate, low-level representation of these signals to the instrumentation is required.

- Potential Transformer (PT) and Current Transformer (CT), also referred to as Instrumentation Transformer, are designed to approximate the ideal transformer as closely as is practically possible.
  - ➢ The load on an instrumentation transformer is frequently referred to as the burden on that transformer.
  - ➢ A potential transformer should ideally accurately measure voltage while appearing as an open circuit to the system under measurement, i.e. drawing negligible current and power.
    - → Its load impedance should be "large" in some sense.
  - ➢ An ideal current transformer would accurately measure current while appearing as a short circuit to the system under measurement, i.e. developing negligible voltage drop and drawing negligible power.

→ Its load impedance should be "small" in some sense.

§2.9 The Per-Unit System

- Computations relating to machines, transformers, and systems of machines are often carried out in per-unit system.
  - ➢ All pertinent quantities are expressed as decimal fractions of appropriately chose base values.
  - ➢ All the usual computations are then carried out in these per unit values instead of the familiar volts, amperes, ohms, and so on.
  - ➢ Advantages:
    - → The parameter values typically fall in a reasonably narrow numerical range when expressed in a per-unit system based upon their rating.
    - → When transformer equivalent-circuit parameters are converted to their per-unit values, the ideal transformer turns ratio becomes 1:1 and hence the ideal transformer can be eliminated.
  - ➢ Actual quantities: $V, I, P, Q, VA, R, X, Z, G, B, Y$

$$\text{Quantity in per unit} = \frac{\text{Actual quantity}}{\text{Base value of quantity}} \tag{2.47}$$

  - ➢ To a certain extent, base values can be chosen arbitrarily, but certain relations between them must be observed. For a single-phase system:

$$P_{\text{base}}, Q_{\text{base}}, VA_{\text{base}} = V_{\text{base}} I_{\text{base}} \tag{2.48}$$

$$R_{\text{base}}, X_{\text{base}}, Z_{\text{base}} = \frac{V_{\text{base}}}{I_{\text{base}}} \tag{2.49}$$

    - → Only two independent base quantities can be chose arbitrarily; the remaining quantities are determined by (2.48) and (2.49).
    - → In typical usage, values of VA_{base} and V_{base} are chosen first; values of I_{base} and all other quantities in (2.48) and (2.49) are then uniquely established.
    - → The value of VA_{base} must be the same over the entire system under analysis.
    - → When a transformer is encountered, the values of V_{base} differ on each side and should be chosen in the same ratio as the turns ratio of the transformer.
      - ✧ The per-unit ideal transformer will have a unity turns ratio and hence can be eliminated.
      - ✧ Usually the rated or nominal voltages of the respective sides are chosen.

- The procedure for performing system analyses in per-unit is summarized as follows:

1. Select a *VA* base and a base voltage at some point in the system.
2. Convert all quantities to per unit on the chosen *VA* base and with a voltage base that transforms as the turns ratio of any transformer which is encountered as one moves through the system.
3. Perform a standard electrical analysis with all quantities in per unit.
4. When the analysis is completed, all quantities can be converted back to real units (e.g., volts, amperes, watts, etc.) by multiplying their per-unit values by their corresponding base values.

- Machine Ratings as Bases
  - ➢ When expressed in per-unit form on their rating as a base, the per-unit values of machine

parameters fall within a relatively narrow range.

→ The physics behind each type of device is the same and, in a crude sense, they can each be considered to be simply scaled versions of the same basic device.

→ When normalized to their own rating, the effect of the scaling is eliminated and the result is a set of per-unit parameter values which is quite similar over the whole size range of that device. For power and distribution transformers, $I_\varphi = 0.02 \sim 0.06\text{pu}$, $R = 0.005 \sim 0.02\text{pu}$, and $X = 0.015 \sim 0.10\text{pu}$.

➢ Manufacturers often supply device parameters in per unit on the device base.

→ When performing a system analysis, it may be necessary to convert the supplied per-unit parameter values to per-unit values on the base chosen for the analysis.

$$(P,Q,VA)_{\text{pu on base 2}} = (P,Q,VA)_{\text{pu on base 1}}\left[\frac{VA_{\text{base 1}}}{VA_{\text{base 2}}}\right] \tag{2.50}$$

$$(P,X,Z)_{\text{pu on base 2}} = (P,X,Z)_{\text{pu on base 1}}\left[\frac{(V_{\text{base 1}})^2 VA_{\text{base 2}}}{(V_{\text{base 2}})^2 VA_{\text{base 1}}}\right] \tag{2.51}$$

$$V_{\text{pu on base 2}} = V_{\text{pu on base 1}}\left[\frac{V_{\text{base 1}}}{V_{\text{base 2}}}\right] \tag{2.52}$$

$$I_{\text{pu on base 2}} = I_{\text{pu on base 1}}\left[\frac{V_{\text{base 2}} VA_{\text{base 1}}}{V_{\text{base 1}} VA_{\text{base 2}}}\right] \tag{2.53}$$

EXAMPLE 2.12

The equivalent circuit for a 100-MVA, 7.97-kV:79.7-kV transformer is shown in Fig. 2.22a. The equivalent-circuit parameters are:

$$X_L = 0.040\ \Omega \quad X_H = 3.75\ \Omega \quad X_m = 114\ \Omega$$
$$R_L = 0.76\ \text{m}\Omega \quad R_H = 0.085\ \Omega$$

Note that the magnetizing inductance has been referred to the low-voltage side of the equivalent circuit. Convert the equivalent circuit parameters to per unit using the transformer rating as base.

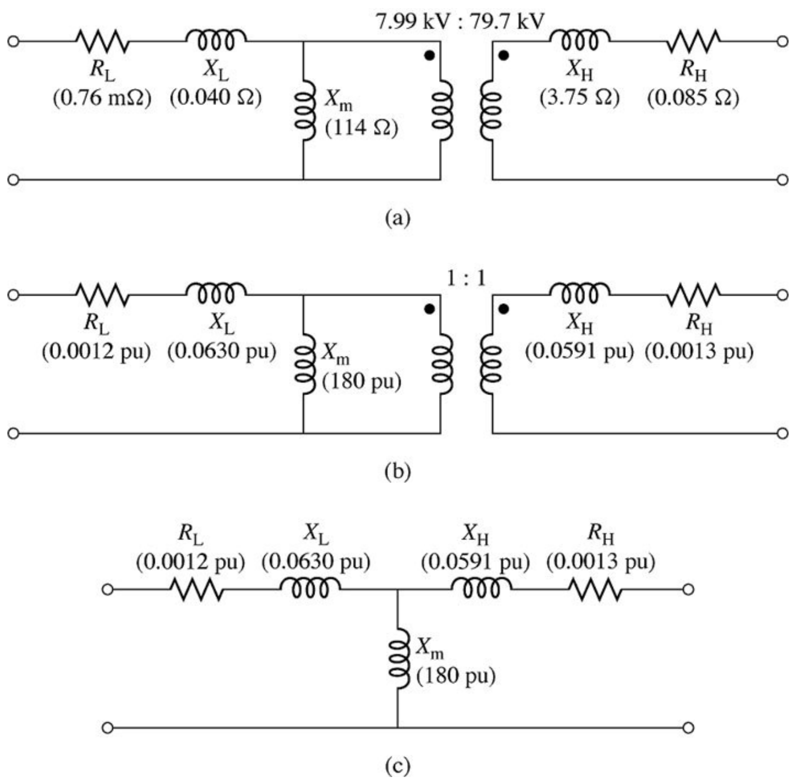

Figure 2.22 Transformer equivalent circuits for Example 2.12.
(a) Equivalent circuit in actual units. (b) Per-unit equivalent circuit with 1:1 ideal transformer.
(c) Per-unit equivalent circuit following elimination of the ideal transformer.

■ Solution

The base quantities for the transformer are:

Low-voltage side:

$$VA_{\text{base}} = 100 \text{ MVA} \quad V_{\text{base}} = 7.97 \text{ kV}$$

and from Eqs. 2.48 and 2.49

$$R_{\text{base}} = X_{\text{base}} = \frac{V_{\text{base}}^2}{VA_{\text{base}}} = 0.635 \ \Omega$$

High-voltage side:

$$VA_{\text{base}} = 100 \text{ MVA} \quad V_{\text{base}} = 79.7 \text{ kV}$$

and from Eqs. 2.48 and 2.49

$$R_{\text{base}} = X_{\text{base}} = \frac{V_{\text{base}}^2}{VA_{\text{base}}} = 63.5 \ \Omega$$

The per-unit values of the transformer parameters can now be calculated by division by their corresponding base quantities.

$$X_L = \frac{0.040}{0.635} = 0.0630 \text{ per unit}$$

$$X_H = \frac{3.75}{63.5} = 0.0591 \text{ per unit}$$

$$X_m = \frac{114}{0.635} = 180 \text{ per unit}$$

$$R_L = \frac{7.6 \times 10^{-4}}{0.635} = 0.0012 \text{ per unit}$$

$$R_H = \frac{0.085}{63.5} = 0.0013 \text{ per unit}$$

Finally, the voltages representing the turns ratio of the ideal transformer must each be divided by the base voltage on that side of the transformer. Thus the turns ratio of 7.97-kV:79.7-kV becomes in per unit

$$\text{Per-unit turns ratio} = \left(\frac{7.97\text{ kV}}{7.97\text{ kV}}\right) : \left(\frac{79.7\text{ kV}}{79.7\text{ kV}}\right) = 1 : 1$$

The resultant per-unit equivalent circuit is shown in Fig. 2.22b. Because it has unity turns ratio, there is no need to keep the ideal transformer and hence this equivalent circuit can be reduced to the form of Fig. 2.22c.

EXAMPLE 2.13

The exciting current measured on the low-voltage side of a 50-kVA, 2400:240-V transformer is 5.41 A. Its equivalent impedance referred to the high-voltage side is $1.42 + j1.82\ \Omega$. Using the transformer rating as the base, express in per unit on the low- and high-voltage sides (*a*) the exciting current and (*b*) the equivalent impedance.

■ Solution

The base values of voltages and currents are

$$V_{\text{base,H}} = 2400\text{ V} \quad V_{\text{base,L}} = 240\text{ V} \quad I_{\text{base,H}} = 20.8\text{ A} \quad I_{\text{base,L}} = 208\text{ A}$$

where subscripts H and L indicate the high- and low-voltage sides, respectively.

From Eq. 2.49

$$Z_{\text{base,H}} = \frac{2400}{20.8} = 115.2\ \Omega \quad Z_{\text{base,L}} = \frac{240}{208} = 1.152\ \Omega$$

a. From Eq. 2.47, the exciting current in per unit referred to the low-voltage side can be calculated as:

$$I_{\varphi,\text{L}} = \frac{5.41}{208} = 0.0260\text{ per unit}$$

The exciting current referred to the high-voltage side is 0.541 A. Its per-unit value is

$$I_{\varphi,\text{H}} = \frac{0.541}{20.8} = 0.0260\text{ per unit}$$

Note that, as expected, the per-unit values are the same referred to either side, corresponding to a unity turns ratio for the ideal transformer in the per-unit transformer. This is a direct consequence of the choice of base voltages in the ratio of the transformer turns ratio and the choice of a constant volt-ampere base.

b. From Eq. 2.47 and the value for Z_{base}

$$Z_{\text{eq,H}} = \frac{1.42 + j1.82}{115.2} = 0.0123 + j0.0158\text{ per unit}$$

The equivalent impedance referred to the low-voltage side is $0.0142 + j0.0182\Omega$. Its per-unit value is

$$Z_{\text{eq,L}} = \frac{0.142 + 0.0182}{1.152} = 0.0123 + j0.0158\text{ per unit}$$

The per-unit values referred to the high- and low-voltage sides are the same, the transformer turns ratio being accounted for in per unit by the base values. Note again that this is consistent with a unity turns ratio of the ideal transformer in the per-unit transformer equivalent circuit.

- Balanced Three-Phase System:
  - Relations for base values:

$$(P_{\text{base}}, Q_{\text{base}}, VA_{\text{base}})_{3-\text{phase}} = 3VA_{\text{base, per phase}} \tag{2.54}$$

  - The three-phase volt-ampere base ($VA_{\text{base, 3-phase}}$) and the line-to-line voltage base ($V_{\text{base, 3-phase}} = V_{\text{base, l-l}}$) are usually chosen first.
  - The base values for the phase (line-to-neutral) voltage then is

$$V_{\text{base, l-n}} = \frac{1}{\sqrt{3}} V_{\text{base, l-l}} \tag{2.55}$$

- The base current for three-phase system is equal to the phase current, which is the same as the base current for a single-phase (per-phase) analysis.

$$I_{\text{base, 3-phase}} = I_{\text{base, per phase}} = \frac{VA_{\text{base, 3-phase}}}{\sqrt{3} V_{\text{base, 3-phase}}} \tag{2.56}$$

- The three-phase base impedance is chosen to be the single-phase base impedance.

$$\begin{aligned} Z_{\text{base, 3-phase}} &= Z_{\text{base, per phase}} \\ &= \frac{V_{\text{base, l-n}}}{I_{\text{base, per phase}}} \\ &= \frac{V_{\text{base, 3-phase}}}{\sqrt{3} I_{\text{base, 3-phase}}} \\ &= \frac{\left(V_{\text{base, 3-phase}}\right)^2}{VA_{\text{base, 3-phase}}} \end{aligned} \tag{2.57}$$

- Note that the factors of $\sqrt{3}$ and 3 are automatically taken care of in per unit by the base values. Three-phase problems can thus be solved in per unit as if they were single-phase problems.

EXAMPLE 2.14

Rework Example 2.9 in per unit, specifically calculating the short-circuit phase currents which will flow in the feeder and at the 240-V terminals of the receiving-end transformer bank. Perform the calculations in per unit on the three-phase, 150-kVA, rated-voltage base of the receiving-end transformer.

■ Solution

We start by converting all the impedances to per unit. The impedance of the 500-kVA, 24 kV:2400 V sending end transformer is $0.17 + j0.92$ Ω/phase as referred to the 2400-V side. From Eq. 2.57, the base impedance corresponding to a 2400-V, 150-kVA base is

$$Z_{\text{base}} = \frac{2400^2}{150 \times 10^3} = 38.4\ \Omega$$

From Example 2.9, the total series impedance is equal to $Z_{\text{tot}} = 0.64 + j2.33$ Ω/phase and thus in per unit it is equal to

$$Z_{\text{tot}} = \frac{0.64 + j2.33}{38.4} = 0.0167 + j0.0607 \text{ per unit}$$

which is of magnitude

$$|Z_{\text{tot}}| = 0.0629 \text{ per unit}$$

The voltage applied to the high-voltage side of the sending-end transformer is V_s = 24.0 kV = 1.0 per unit on a rated-voltage base and hence the short-circuit current will equal

$$I_{\text{sc}} = \frac{V_s}{|Z_{\text{tot}}|} = \frac{1.0}{0.0629} = 15.9 \text{ per unit}$$

To calculate the phase currents in amperes, it is simply necessary to multiply the per-unit short-circuit current by the appropriate base current. Thus, at the 2400-V feeder the base current is

$$I_{\text{base, 2400-V}} = \frac{150 \times 10^3}{\sqrt{3}\, 2400} = 36.1 \text{ A}$$

and hence the feeder current will be

$$I_{\text{feeder}} = 15.9 \times 36.1 = 574 \text{ A}$$

The base current at the 240-V secondary of the receiving-end transformers is

$$I_{\text{base, 240-V}} = \frac{150 \times 10^3}{\sqrt{3}\,240} = 361 \text{ A}$$

and hence the short-circuit current is

$$I_{\text{240-V secondary}} = 15.9 \times 361 = 5.74 \text{ kA}$$

As expected, these values are equivalent within numerical accuracy to those calculated in Example 2.9.

EXAMPLE 2.15

A three-phase load is supplied from a 2.4-kV:460-V, 250-kVA transformer whose equivalent series impedance is $0.026 + j0.12$ per unit on its own base. The load voltage is observed to be 438-V line-line, and it is drawing 95 kW at unity power factor. Calculate the voltage at the high-voltage side of the transformer. Perform the calculations on a 460-V, 100-kVA base.

■ Solution

The 460-V side base impedance for the transformer is

$$Z_{\text{base, transformer}} = \frac{460^2}{250 \times 10^3} = 0.846 \ \Omega$$

while that based upon a 100-kVA base is

$$Z_{\text{base, 100-kVA}} = \frac{460^2}{100 \times 10^3} = 2.12 \ \Omega$$

Thus, from Eq. 2.51 the per-unit transformer impedance on a 100-kVA base is

$$Z_{\text{transformer}} = (0.026 + j0.12)\left(\frac{0.864}{2.12}\right) = 0.0106 + j.0489 \text{ per unit}$$

The per unit load voltage is

$$\hat{V}_{\text{load}} = \frac{438}{460} = 0.952 \angle 0^\circ \text{ per unit}$$

where the load voltage has been chosen as the reference for phase-angle calculations.

The per-unit load power is

$$P_{\text{load}} = \frac{95}{100} = 0.95 \text{ per unit}$$

and hence the per-unit load current, which is in phase with the load voltage because the load is operating at unity power factor, is

$$\hat{I}_{\text{load}} = \frac{P_{\text{load}}}{V_{\text{load}}} = \frac{0.95}{0.952} = 0.998 \angle 0^\circ \text{ per unit}$$

Thus we can now calculate the high-side voltage of the transformer

$$\begin{aligned}\hat{V}_{\text{H}} &= \hat{V}_{\text{load}} + \hat{I}_{\text{load}} Z_{\text{transformer}} \\ &= 0.952 + 0.998(0.0106 + j0.0489) \\ &= 0.963 + j0.0488 = 0.964 \angle 29.0^\circ \text{ per unit}\end{aligned}$$

Thus the high-side voltage is equal to 0.964×2400 V = 2313 V (line-line).

Chapter 3 Electromechanical-Energy-Conversion Principles

- The electromechanical-energy-conversion process takes place through the medium of the electric or magnetic field of the conversion device of which the structures depend on their respective functions.
  - Transducers: microphone, pickup, sensor, loudspeaker
  - Force producing devices: solenoid, relay, electromagnet
  - Continuous energy conversion equipment: motor, generator

- This chapter is devoted to the principles of electromechanical energy conversion and the analysis of the devices accomplishing this function. Emphasis is placed on the analysis of systems that use magnetic fields as the conversion medium.
  - The concepts and techniques can be applied to a wide range of engineering situations involving electromechanical energy conversion.
  - Based on the energy method, we are to develop expressions for forces and torques in magnetic-field-based electromechanical systems.

§3.1 Forces and Torques in Magnetic Field Systems

- The Lorentz Force Law gives the force F on a particle of charge q in the presence of electric and magnetic fields.

$$F = q(E + v \times B) \tag{3.1}$$

F : newtons, q : coulombs, E : volts/meter, B : telsas, v : meters/second

  - In a pure electric-field system,

$$F = qE \tag{3.2}$$

  - In pure magnetic-field systems,

$$F = q(v \times B) \tag{3.3}$$

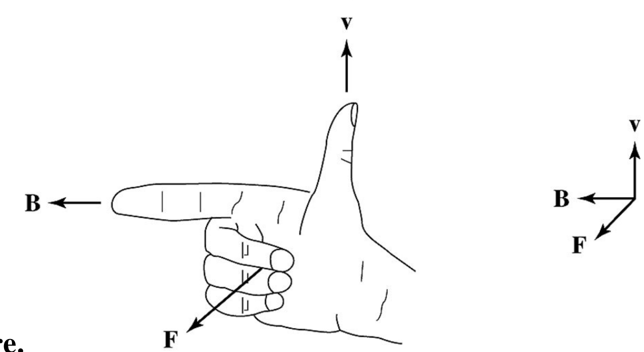

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Figure 3.1 Right-hand rule for $F = q(v \times B)$.

  - For situations where large numbers of charged particles are in motion,

$$F_v = \rho(E + v \times B) \tag{3.4}$$

$$J = \rho v \tag{3.5}$$

$$\boxed{F_v = J \times B} \tag{3.6}$$

ρ (charge density): coulombs/m^3, F_v (force density): newtons/m^3,
$J = \rho v$ (current density): amperes/m^2.

EXAMPLE 3.1

A nonmagnetic rotor containing a single-turn coil is placed in a uniform magnetic field of magnitude B_0, as shown in Fig. 3.2. The coil sides are at radius R and the wire carries current I

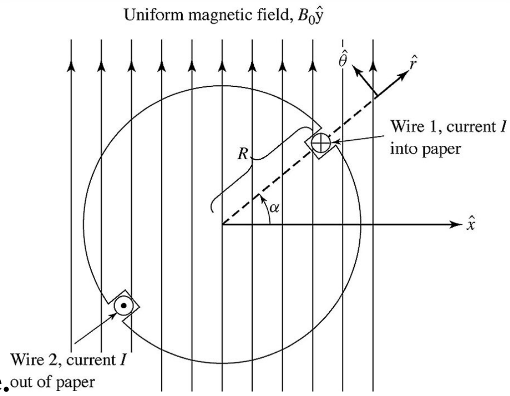

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Figure 3.2 Single-coil rotor for Example 3.1.

as indicated. Find the θ-directed torque as a function of rotor position α when $I = 10$ A, $B_0 = 0.02$ T and $R = 0.05$ m. Assume that the rotor is of length $l = 0.3$ m.

■ Solution

The force per unit length on a wire carrying current I can be found by multiplying Eq. 3.6 by the cross-sectional area of the wire. When we recognize that the product of the cross-sectional area and the current density is simply the current **I**, the force per unit length acting on the wire is given by

$$\mathbf{F} = \mathbf{I} \times \mathbf{B}$$

Thus, for wire 1 carrying current I into the paper, the θ-directed force is given by

$$F_{1_\theta} = -I B_0 l \sin\alpha$$

and for wire 2 (which carries current in the opposite direction and is located 180° away from wire 1)

$$F_{2_\theta} = -I B_0 l \sin\alpha$$

where l is the length of the rotor. The torque T acting on the rotor is given by the sum of the force-moment-arm products for each wire

$$T = -2I B_0 R l \sin\alpha = -2(10)(0.02)(0.05)(0.3)\sin\alpha = -0.006\sin\alpha \quad \text{N}\cdot\text{m}$$

- Unlike the case in Example 3.1, most electromechanical-energy-conversion devices contain magnetic material.
  - ➢ Forces act directly on the magnetic material of these devices which are constructed of rigid, nondeforming structures.
  - ➢ The performance of these devices is typically determined by the net force, or torque, acting on the moving component. It is rarely necessary to calculate the details of the internal force distribution.
  - ➢ Just as a compass needle tries to align with the earth's magnetic field, the two sets of fields associated with the rotor and the stator of rotating machinery attempt to align, and torque is associated with their displacement from alignment.
    - → In a motor, the stator magnetic field rotates ahead of that of the rotor, pulling on it and performing work.
    - → For a generator, the rotor does the work on the stator.

- The Energy Method
  - Based on the principle of conservation of energy: energy is neither created nor destroyed; it is merely changed in form.
  - Fig. 3.3(a): a magnetic-field-based electromechanical-energy-conversion device.
    - → A lossless magnetic-energy-storage system with two terminals
    - → The electric terminal has two terminal variables: e (voltage), i (current).
    - → The mechanical terminal has two terminal variables: f_{fld} (force), x (position)
    - → The loss mechanism is separated from the energy-storage mechanism.
      - – Electrical losses: ohmic losses…
      - – Mechanical losses: friction, windage…
  - Fig. 3.3(b): a simple force-producing device with a single coil forming the electric terminal, and a movable plunger serving as the mechanical terminal.
    - → The interaction between the electric and mechanical terminals, i.e. the electromechanical energy conversion, occurs through the medium of the magnetic stored energy.

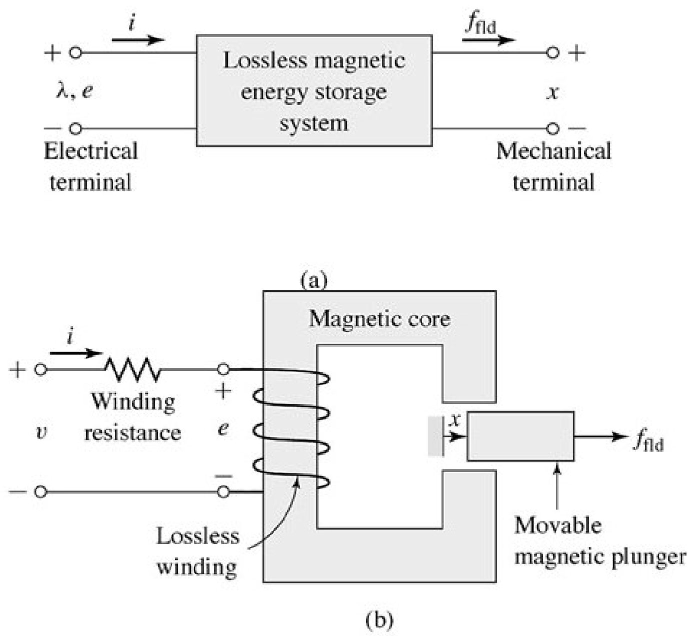


Figure 3.3 (a) Schematic magnetic-field electromechanical-energy-conversion device; (b) simple force-producing device.

→ W_{fld} : the stored energy in the magnetic field

$$\boxed{\frac{d\,W_{\text{fld}}}{dt} = ei - f_{\text{fld}}\frac{dx}{dt}} \tag{3.7}$$

$$e = \frac{d\lambda}{dt} \tag{3.8}$$

$$\boxed{d\,W_{\text{fld}} = i d\lambda - f_{\text{fld}} dx} \tag{3.9}$$

→ Equation (3.9) permits us to solve for the force simply as a function of the flux λ and the mechanical terminal position x.

→ Equations (3.7) and (3.9) form the basis for the energy method.

§3.2 Energy Balance

- Consider the electromechanical systems whose predominant energy-storage mechanism is in magnetic fields. For motor action, we can account for the energy transfer as

$$\begin{pmatrix}\text{Energy input}\\ \text{form electric}\\ \text{sources}\end{pmatrix} = \begin{pmatrix}\text{Mechanical}\\ \text{energy}\\ \text{output}\end{pmatrix} + \begin{pmatrix}\text{Increase in energy}\\ \text{stored in magnetic}\\ \text{field}\end{pmatrix} + \begin{pmatrix}\text{Energy}\\ \text{converted}\\ \text{into heat}\end{pmatrix} \tag{3.10}$$

  - Note the generator action.

- The ability to identify a lossless-energy-storage system is the essence of the energy method.
  - This is done mathematically as part of the modeling process.
  - For the lossless magnetic-energy-storage system of Fig. 3.3(a), rearranging (3.9) in form of (3.10) gives

$$\boxed{dW_{\text{elec}} = dW_{\text{mech}} + dW_{\text{fld}}} \tag{3.11}$$

    where

    $dW_{\text{elec}} = i d\lambda$ = differential electric energy input

    $dW_{\text{mech}} = f_{\text{fld}} dx$ = differential mechanical energy output

    dW_{fld} = differential change in magnetic stored energy

  - Here e is the voltage induced in the electric terminals by the changing magnetic stored energy. It is through this reaction voltage that the external electric circuit supplies power to the coupling magnetic field and hence to the mechanical output terminals.

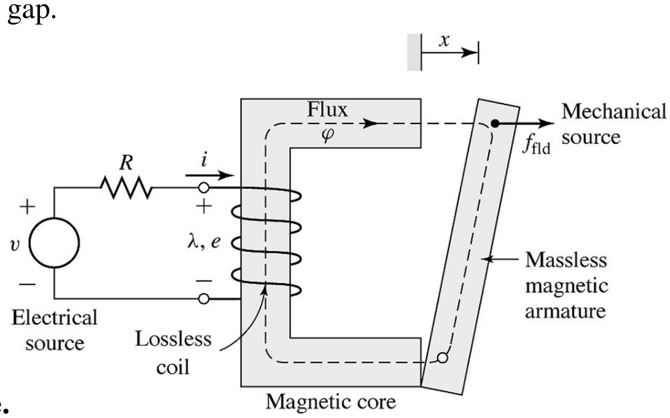
$$dW_{\text{elec}} = ei\ dt \tag{3.12}$$

  - The basic energy-conversion process is one involving the coupling field and its action and reaction on the electric and mechanical systems.
  - Combining (3.11) and (3.12) results in

$$\boxed{dW_{\text{elec}} = ei\ dt = dW_{\text{mech}} + dW_{\text{fld}}} \tag{3.13}$$

§3.3 Energy in Singly-Excited Magnetic Field Systems

- We are to deal energy-conversion systems: the magnetic circuits have air gaps between the stationary and moving members in which considerable energy is stored in the magnetic field.
  - This field acts as the energy-conversion medium, and its energy is the reservoir between the electric and mechanical system.
- Fig. 3.4 shows an electromagnetic relay schematically. The predominant energy storage occurs in the air gap, and the properties of the magnetic circuit are determined by the dimensions of the air gap.



Errore.

Figure 3.4 Schematic of an electromagnetic relay.

$$\lambda = L(x)i \tag{3.14}$$

$$dW_{\text{mech}} = f_{\text{fld}}dx \tag{3.15}$$

$$\boxed{dW_{\text{fld}} = id\lambda - f_{\text{fld}}dx} \tag{3.16}$$

- W_{fld} is uniquely specified by the values of λ and x. Therefore, λ and x are referred to as state variables.
- Since the magnetic energy storage system is lossless, it is a conservative system. W_{fld} is the same regardless of how λ and x are brought to their final values. See Fig. 3.5 where tow separate paths are shown.

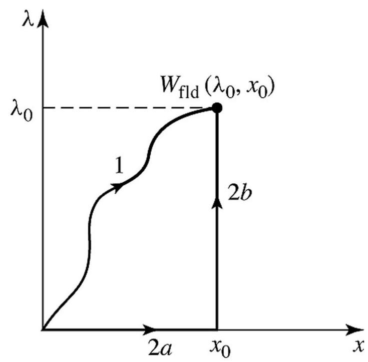


Figure 3.5 Integration paths for W_{fld}.

$$W_{\text{fld}}(\lambda_0, x_0) = \int_{\text{path 2a}} dW_{\text{fld}} + \int_{\text{path 2b}} dW_{\text{fld}} \tag{3.17}$$

On path 2a, $d\lambda = 0$ and $f_{\text{fld}} = 0$. Thus, $dW_{\text{fld}} = 0$ on path 2a.
On path 2b, $dx = 0$.
Therefore, (3.17) reduces to the integral of $id\lambda$ over path 2b.

$$W_{\text{fld}}(\lambda_0, x_0) = \int_0^{\lambda_0} i(\lambda, x_0)d\lambda \tag{3.18}$$

For a linear system in which λ is proportional to i, (3.18) gives

$$W_{\text{fld}}(\lambda, x) = \int_0^{\lambda} i(\lambda', x)d\lambda' = \int_0^{\lambda} \frac{\lambda'}{L(x)}d\lambda' = \frac{1}{2}\frac{\lambda^2}{L(x)} \tag{3.19}$$

- V : the volume of the magnetic field

$$W_{\text{fld}} = \int_V \left(\int_0^B H \cdot dB' \right) dV \tag{3.20}$$

If $B = \mu H$,

$$W_{\text{fld}} = \int_V \left(\frac{B^2}{2\mu} \right) dV \tag{3.21}$$

EXAMPLE 3.2

The relay shown in Fig. 3.6a is made from infinitely-permeable magnetic material with a movable plunger, also of infinitely-permeable material. The height of the plunger is much greater than the air-gap length ($h \gg g$). Calculate the magnetic stored energy W_{fld} as a function of plunger position ($0 < x < d$) for $N = 1000$ turns, $g = 2.0$ mm, $d = 0.15$ m, $l = 0.1$ m, and $i = 10$ A.

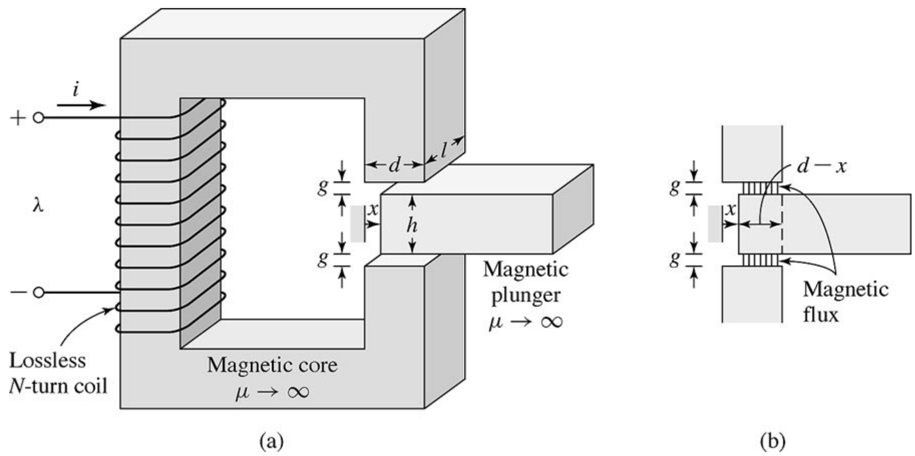


Figure 3.6 (a) Relay with movable plunger for Example 3.2.
(b) Detail showing air-gap configuration with the plunger partially removed.

■ Solution

Equation 3.19 can be used to solve for W_{fld} when λ is known. For this situation, i is held constant, and thus it would be useful to have an expression for W_{fld} as a function of i and x. This can be obtained quite simply by substituting Eq. 3.14 into Eq. 3.19, with the result

$$W_{fld} = \frac{1}{2}L(x)i^2$$

The inductance is given by

$$L(x) = \frac{\mu_0 N^2 A_{gap}}{2g}$$

where A_{gap} is the gap cross-sectional area. From Fig. 3.6b, A_{gap} can be seen to be

$$A_{gap} = l(d-x) = ld\left(1 - \frac{x}{d}\right)$$

Thus

$$L(x) = \frac{\mu_0 N^2 ld(1 - x/d)}{2g}$$

and

$$\begin{aligned} W_{fld} &= \frac{1}{2}\frac{N^2\mu_0 ld(1-x/d)}{2g}\,i^2 \\ &= \frac{1}{2}\frac{(1000^2)(4\pi \times 10^{-7})(0.1)(0.15)}{2(0.002)} \times 10^2\left(1 - \frac{x}{d}\right) \\ &= 236\left(1 - \frac{x}{d}\right)\text{ J} \end{aligned}$$

Errore.

§3.4 Determination of Magnetic Force and Torque form Energy

- The magnetic stored energy W_{fld} is a state function, determined uniquely by the values of the independent state variables λ and x.

$$dW_{\text{fld}}(\lambda, x) = i d\lambda - f_{\text{fld}} dx \tag{3.22}$$

$$dF(x_1, x_2) = \left.\frac{\partial F}{\partial x_1}\right|_{x_2} dx_1 + \left.\frac{\partial F}{\partial x_2}\right|_{x_1} dx_2 \tag{3.23}$$

$$dW_{\text{fld}}(\lambda, x) = \left.\frac{\partial W_{\text{fld}}}{\partial \lambda}\right|_{x} d\lambda + \left.\frac{\partial W_{\text{fld}}}{\partial x}\right|_{\lambda} dx \tag{3.24}$$

Comparing (3.22) with (3.24) gives (3.25) and (3.26):

$$i = \left.\frac{\partial W_{\text{fld}}(\lambda, x)}{\partial \lambda}\right|_{x} \tag{3.25}$$

$$\boxed{f_{\text{fld}} = -\left.\frac{\partial W_{\text{fld}}(\lambda, x)}{\partial x}\right|_{\lambda}} \tag{3.26}$$

- Once we know W_{fld} as a function of λ and x, (3.25) can be used to solve for $i(\lambda, x)$.
- Equation (3.26) can be used to solve for the mechanical force $f_{\text{fld}}(\lambda, x)$. The partial derivative is taken while holding the flux linkages λ constant.
- For linear magnetic systems for which $\lambda = L(x)i$, the force can be found as

$$f_{\text{fld}} = -\left.\frac{\partial}{\partial x}\left(\frac{1}{2}\frac{\lambda^2}{L(x)}\right)\right|_{\lambda} = \frac{\lambda^2}{2L(x)^2}\frac{dL(x)}{dx} \tag{3.27}$$

$$\boxed{f_{\text{fld}} = \frac{i^2}{2}\frac{dL(x)}{dx}} \tag{3.28}$$

EXAMPLE 3.3

Table 3.1 contains data from an experiment in which the inductance of a solenoid was measured as a function of position x, where $x = 0$ corresponds to the solenoid being fully retracted.

Table 3.1 Data for Example 3.3.

x [cm]	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
L [mH]	2.8	2.26	1.78	1.52	1.34	1.26	1.20	1.16	1.13	1.11	1.10

Plot the solenoid force as a function of position for a current of 0.75 A over the range $0.2 \le x \le 1.8$ cm.

■ Solution

The solution is most easily obtained using MATLAB.† First, a fourth-order polynomial fit of the inductance as a function of x is obtained using the MATLAB function *polyfit*. The result is of the form

$$L(x) = a(1)x^4 + a(2)x^3 + a(3)x^2 + a(4)x + a(5)$$

Figure 3.7a shows a plot of the data points along with the results of the polynomial fit.

Once this fit has been obtained, it is a straight forward matter to calculate the force from Eq. 3.28.

$$f_{\text{fld}} = \frac{i^2}{2}\frac{dL(x)}{dx} = \frac{i^2}{2}(4a(1)x^3 + 3a(2)x^2 + 2a(3)x + a(4))$$

This force is plotted in Figure 3.7b. Note that the force is negative, which means that it is acting in such a direction as to pull the solenoid inwards towards $x = 0$.

† MATLAB is a registered trademark of The MathWorks, Inc.

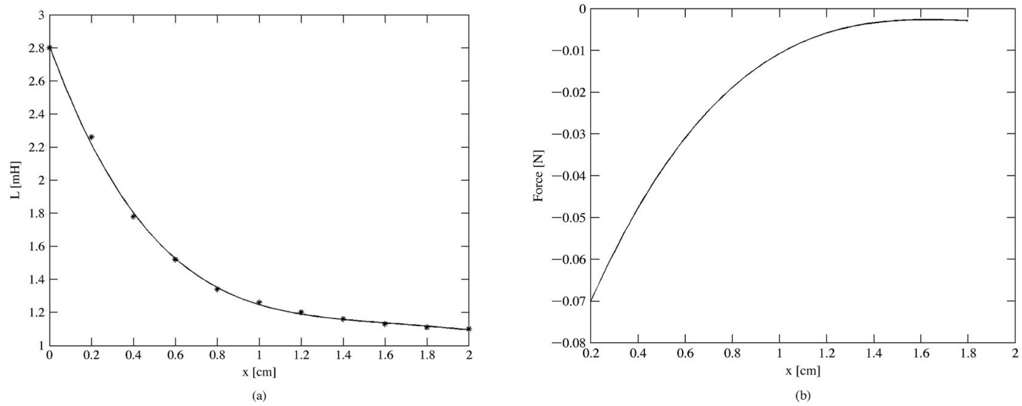


Figure 3.7 Example 3.3. (a) Polynomial curve fit of inductance.
(b) Force as a function of position x for $i = 0.75$ A.

- For a system with a rotating mechanical terminal, the mechanical terminal variables become the angular displacement θ and the torque T_{fld}.

$$dW_{\text{fld}}(\lambda,\theta) = i\,d\lambda - T_{\text{fld}}\,d\theta \tag{3.29}$$

$$T_{\text{fld}} = -\left.\frac{\partial W_{\text{fld}}(\lambda,\theta)}{\partial\theta}\right|_{\lambda} \tag{3.30}$$

  - For linear magnetic systems for which $\lambda = L(\theta)i$:

$$W_{\text{fld}}(\lambda,\theta) = \frac{1}{2}\frac{\lambda^2}{L(\theta)} \tag{3.31}$$

$$T_{\text{fld}} = -\left.\frac{\partial}{\partial\theta}\left(\frac{1}{2}\frac{\lambda^2}{L(\theta)}\right)\right|_{\lambda} = \frac{1}{2}\frac{\lambda^2}{L(\theta)^2}\frac{dL(\theta)}{d\theta} \tag{3.32}$$

(3.33)

$$T_{\text{fld}} = \frac{i^2}{2}\frac{dL(\theta)}{d\theta} \tag{3.34}$$

EXAMPLE 3.4

The magnetic circuit of Fig. 3.9 consists of a single-coil stator and an oval rotor. Because the air-gap is nonuniform, the coil inductance varies with rotor angular position, measured between the magnetic axis of the stator coil and the major axis of the rotor, as

$$L(\theta) = L_0 + L_2\cos(2\theta)$$

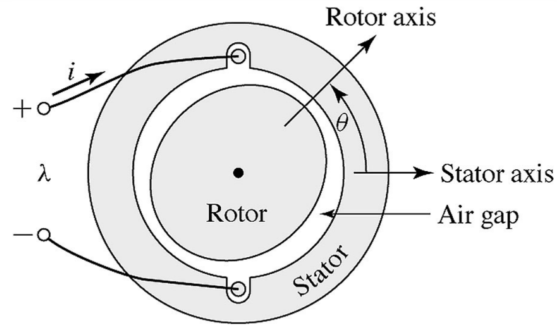


Figure 3.9 Magnetic circuit for Example 3.4.

where $L_0 = 10.6$ mH and $L_2 = 2.7$ mH. Note the second-harmonic variation of inductance with rotor angle θ. This is consistent with the fact that the inductance is unchanged if the rotor is rotated through an angle of 180°.

Find the torque as a function of θ for a coil current of 2 A.

■ Solution

From Eq. 3.33

$$T_{\text{fld}}(\theta) = \frac{i^2}{2}\frac{dL(\theta)}{d\theta} = \frac{i^2}{2}\left(-2L_2 \sin(2\theta)\right)$$

Numerical substitution gives

$$T_{\text{fld}}(\theta) = -1.08 \times 10^{-2} \sin(2\theta) \quad \text{N} \cdot \text{m}$$

Note that in this case the torque acts in such a direction as to pull the rotor axis in alignment with the coil axis and hence to maximize the coil inductance.

---

§3.5 Determination of Magnetic Force and Torque from Coenergy

- Recall that in §3.4, the magnetic stored energy W_{fld} is a state function, determined uniquely by the values of the independent state variables λ and x.

$$dW_{\text{fld}}(\lambda, x) = i d\lambda - f_{\text{fld}} dx \tag{3.22}$$

$$dW_{\text{fld}}(\lambda, x) = \left.\frac{\partial W_{\text{fld}}}{\partial \lambda}\right|_x d\lambda + \left.\frac{\partial W_{\text{fld}}}{\partial x}\right|_\lambda dx \tag{3.24}$$

$$i = \left.\frac{\partial W_{\text{fld}}(\lambda, x)}{\partial \lambda}\right|_x \tag{3.25}$$

$$\boxed{f_{\text{fld}} = -\left.\frac{\partial W_{\text{fld}}(\lambda, x)}{\partial x}\right|_\lambda} \tag{3.26}$$

- Coenergy: from which the force can be obtained directly as a function of the current. The selection of energy or coenergy as the state function is purely a matter of convenience.
  - ➢ The coenergy $W'_{fld}(i, x)$ is defined as a function of i and x such that

$$W'_{\text{fld}}(i, x) = i\lambda - W_{\text{fld}}(\lambda, x) \tag{3.34}$$

$$d(i\lambda) = i d\lambda + \lambda di \tag{3.35}$$

$$dW'_{\text{fld}}(i, x) = d(i\lambda) - dW_{fld}(\lambda, x) \tag{3.36}$$

$$\boxed{dW'_{\text{fld}}(i, x) = \lambda\, di + f_{\text{fld}} dx} \tag{3.37}$$

  - ➢ From (3.37), the coenergy $W'_{fld}(i, x)$ can be seen to be a state function of the two independent variables i and x.

$$dW'_{\text{fld}}(i, x) = \left.\frac{\partial W'_{fld}}{\partial i}\right|_x di + \left.\frac{\partial W'_{fld}}{\partial x}\right|_i dx \tag{3.38}$$

$$\lambda = \left.\frac{\partial W'_{\text{fld}}(i, x)}{\partial i}\right|_x \tag{3.39}$$

$$\boxed{f_{\text{fld}} = \left.\frac{\partial W'_{\text{fld}}(i, x)}{\partial x}\right|_i} \tag{3.40}$$

  - ➢ For any given system, (3.26) and (3.40) will give the same result.

- By analogy to (3.18) in §3.3, the coenergy can be found as (3.41)

$$W_{\text{fld}}(\lambda_0, x_0) = \int_0^{\lambda_0} i(\lambda, x_0)\, d\lambda \tag{3.18}$$

$$W'_{\text{fld}}(i, x) = \int_0^{i} \lambda(i', x)\, di' \tag{3.41}$$

For linear magnetic systems for which $\lambda = L(x)i$,

$$W'_{\text{fld}}(i, x) = \frac{1}{2} L(x)\, i^2 \tag{3.42}$$

$$\boxed{f_{\text{fld}} = \frac{i^2}{2}\frac{dL(x)}{dx}} \tag{3.43}$$

→ (3.43) is identical to the expression given by (3.28).

- For a system with a rotating mechanical displacement,

$$W'_{\text{fld}}(i, \theta) = \int_0^{i} \lambda(i', \theta)\, di' \tag{3.44}$$

$$T_{\text{fld}} = \left.\frac{\partial W'_{\text{fld}}(i, \theta)}{\partial \theta}\right|_i \tag{3.45}$$

If the system is magnetically linear,

$$W'_{\text{fld}}(i, \theta) = \frac{1}{2} L(\theta) i^2 \tag{3.46}$$

$$T_{\text{fld}} = \frac{i^2}{2}\frac{dL(\theta)}{d\theta} \tag{3.47}$$

→ (3.47) is identical to the expression given by (3.33).

- In field-theory terms, for soft magnetic materials

$$W'_{\text{fld}} = \int_V \left(\int_0^{H_0} B \cdot dH \right) dV \tag{3.48}$$

$$W'_{\text{fld}} = \int_v \frac{\mu H^2}{2}\, dV \tag{3.49}$$

For permanent-magnet (hard) materials

$$W'_{\text{fld}} = \int_V \left(\int_{H_c}^{H_0} B \cdot dH \right) dV \tag{3.50}$$

EXAMPLE 3.5

For the relay of Example 3.2, find the force on the plunger as a function of x when the coil is driven by a controller which produces a current as a function of x of the form

$$i(x) = I_0 \left(\frac{x}{d}\right) \text{ A}$$

■ **Solution**

From Example 3.2

$$L(x) = \frac{\mu_0 N^2 l d(1 - x/d)}{2g}$$

This is a magnetically-linear system for which the force can be calculated as

$$f_{\text{fld}} = \frac{i^2}{2}\frac{dL(x)}{dx} = -\frac{i^2}{2}\left(\frac{\mu_0 N^2 l}{2g}\right)$$

Substituting for $i(x)$, the expression for the force as a function of x can be determined as

$$f_{\text{fld}} = -\frac{I_0^2 \mu_0 N^2 l}{4g}\left(\frac{x}{d}\right)^2$$

Note that from Eq. 3.46, the coenergy for this system is equal to

$$W'_{\text{fld}}(i, x) = \frac{i^2}{2} L(x) = \frac{i^2}{2}\frac{N^2 \mu_0 l d(1 - x/d)}{2g}$$

Substituting for $i(x)$, this can be written as

$$W'_{\text{fld}}(i, x) = \frac{I_0^2 N^2 \mu_0 l d(1 - x/d)}{4g}\left(\frac{x}{d}\right)^2$$

Note that, although this is a perfectly correct expression for the coenergy as a function of x under the specified operating conditions, if one were to attempt to calculate the force from taking the partial derivative of this expression for W'_{fld} with respect to x, the resultant expression would not give the correct expression for the force. The reason for this is quite simple: As seen from Eq. 3.40, the partial derivative must be taken holding the current constant. Having substituted the expression for $i(x)$ to obtain the expression, the current is no longer a constant, and this requirement cannot be met. This illustrates the problems that can arise if the various force and torque expressions developed here are misapplied.

- For a magnetically-linear system, the energy and coenergy (densities) are numerically equal: $\frac{1}{2}\lambda^2/L = \frac{1}{2}Li^2$, $\frac{1}{2}B^2/\mu = \frac{1}{2}\mu H^2$. For a nonlinear system in which λ and i or B and H are not linearly proportional, the two functions are not even numerically equal.

$$W_{\text{fld}} + W'_{\text{fld}} = \lambda i \tag{3.51}$$

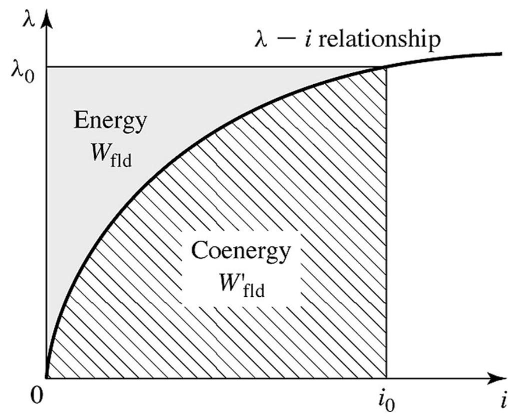


Figure 3.10 Graphical interpretation of energy and coenergy in a singly-excited system.

> Consider the relay in Fig. 3.4. Assume the relay armature is at position x so that the device operating at point a in Fig. 3.11. Note that

$$f_{\text{fld}} = -\frac{\partial W_{\text{fld}}(\lambda, x)}{\partial x}\bigg|_{\lambda} \cong \lim_{\Delta x \to 0}\frac{-\Delta W_{\text{fld}}}{\Delta x}\bigg|_{\lambda} \quad \text{and} \quad f_{\text{fld}} = \frac{\partial W'_{\text{fld}}(i, x)}{\partial x}\bigg|_{i} \cong \lim_{\Delta x \to 0}\frac{\Delta W'_{\text{fld}}}{\Delta x}\bigg|_{i}$$

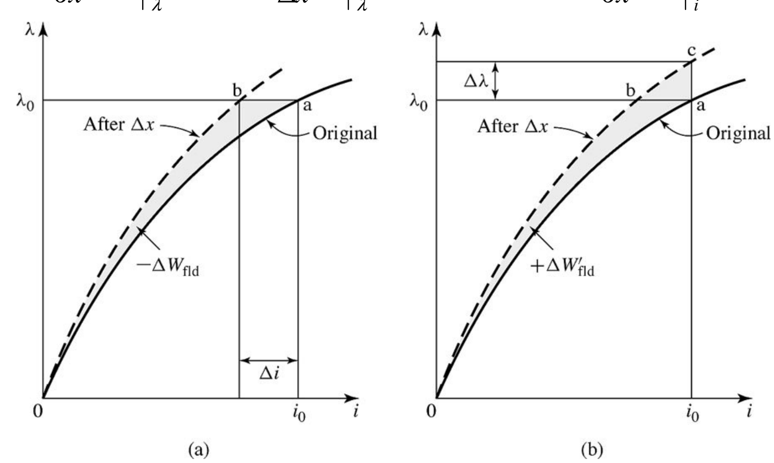


Figure 3.11 Effect of Δx on the energy and coenergy of a singly-excited device: (a) change of energy with λ held constant; (b) change of coenergy with i held constant.

- The force acts in a direction to decrease the magnetic field stored energy at constant flux or to increase the coenergy at constant current.
  → In a singly-excited device, the force acts to increase the inductance by pulling on members so as to reduce the reluctance of the magnetic path linking the winding.

EXAMPLE 3.6

The magnetic circuit shown in Fig. 3.12 is made of high-permeability electrical steel. The rotor is free to turn about a vertical axis. The dimensions are shown in the figure.

a. Derive an expression for the torque acting on the rotor in terms of the dimensions and the magnetic field in the two air gaps. Assume the reluctance of the steel to be negligible (i.e., $\mu \rightarrow \infty$) and neglect the effects of fringing.
b. The maximum flux density in the overlapping portions of the air gaps is to be limited to approximately 1.65 T to avoid excessive saturation of the steel. Compute the maximum torque for $r_1 = 2.5$ cm, $h = 1.8$ cm, and $g = 3$ mm.

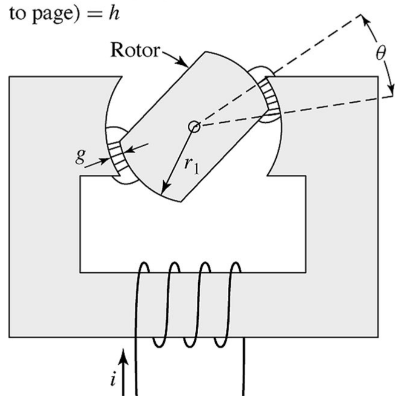


Figure 3.12 Magnetic system of Example 3.6.

Solution

a. There are two air gaps in series, each of length g, and hence the air-gap field intensity H_{ag} is equal to

$$H_{ag} = \frac{Ni}{2g}$$

Because the permeability of the steel is assumed infinite and B_{steel} must remain finite, $H_{steel} = B_{steel}/\mu$ is zero and the coenergy density (Eq. 3.49) in the steel is zero ($\mu H_{steel}^2/2 = B_{steel}^2/2\mu = 0$). Hence the system coenergy is equal to that of the air gaps, in which the coenergy density in the air gap is $\mu_0 H_{ag}^2/2$. The volume of the two overlapping air gaps is $2gh(r_1 + 0.5g)\theta$. Consequently, the coenergy is equal to the product of the air-gap coenergy density and the air-gap volume

$$W'_{ag} = \left(\frac{\mu_0 H_{ag}^2}{2}\right)(2gh(r_1 + 0.5g)\theta) = \frac{\mu_0 (Ni)^2 h(r_1 + 0.5g)\theta}{4g}$$

and thus, from Eq. 3.40

$$T_{fld} = \left.\frac{\partial W'_{ag}(i, \theta)}{\partial \theta}\right|_i = \frac{\mu_0 (Ni)^2 h(r_1 + 0.5g)}{4g}$$

The sign of the torque is positive, hence acting in the direction to increase the overlap angle θ and thus to align the rotor with the stator pole faces.

b. For $B_{ag} = 1.65$ T,

$$H_{ag} = \frac{B_{ag}}{\mu_0} = \frac{1.65}{4\pi \times 10^{-7}} = 1.31 \times 10^6 \text{ A/m}$$

and thus

$$Ni = 2gH_{ag} = 2(3 \times 10^{-3})1.31 \times 10^6 = 7860 \text{ A-turns}$$

T_{fld} can now be calculated as

$$T_{\text{fld}} = \frac{4\pi \times 10^{-7}(7860)^2(1.8 \times 10^{-2})(2.5 \times 10^{-2} + 0.5(3 \times 10^{-3}))}{4(3 \times 10^{-3})}$$

$$= 3.09 \text{ N} \cdot \text{m}$$

§3.6 Multiply-Excited Magnetic Field Systems

- Many electromechanical devices have multiple electrical terminals.
  - Measurement systems: torque proportional to two electric signals; power as the product of voltage and current.
  - Energy conversion devices: multiply-excited magnetic field system.
  - A simple system with two electrical terminals and one mechanical terminal: Fig. 3.13.
    → Three independent variables: $\{\theta, \lambda_1, \lambda_2\}$, $\{\theta, i_1, i_2\}$, $\{\theta, \lambda_1, i_2\}$, or $\{\theta, i_1, \lambda_2\}$.

$$dW_{\text{fld}}(\lambda_1, \lambda_2, \theta) = i_1 d\lambda_1 + i_2 d\lambda_2 - T_{\text{fld}} d\theta \tag{3.52}$$

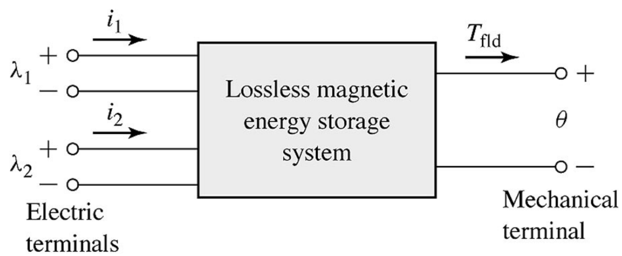


Figure 3.13 Multiply-excited magnetic energy storage system.

$$i_1 = \left.\frac{\partial W_{\text{fld}}(\lambda_1, \lambda_2, \theta)}{\partial \lambda_1}\right|_{\lambda_2, \theta} \tag{3.53}$$

$$i_2 = \left.\frac{\partial W_{\text{fld}}(\lambda_1, \lambda_2, \theta)}{\partial \lambda_2}\right|_{\lambda_1, \theta} \tag{3.54}$$

$$\boxed{T_{\text{fld}} = -\left.\frac{\partial W_{\text{fld}}(\lambda_1, \lambda_2, \theta)}{\partial \theta}\right|_{\lambda_1, \lambda_2}} \tag{3.55}$$

To find W_{fld}, use the path of integration in Fig. 3.14.

$$W_{\text{fld}}(\lambda_{1_0}, \lambda_{2_0}, \theta_0) = \int_0^{\lambda_{2_0}} i_2(\lambda_1 = 0, \lambda_2, \theta = \theta_0) d\lambda_2 + \int_0^{\lambda_{1_0}} i_1(\lambda_1, \lambda_2 = \lambda_{2_0}, \theta = \theta_0) d\lambda_1 \tag{3.56}$$

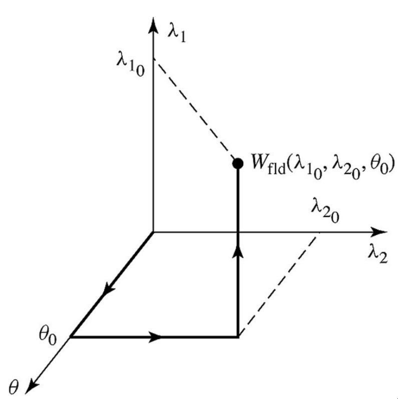


Figure 3.14 Integration path to obtain $W_{\text{fld}}(\lambda_{1_0}, \lambda_{2_0}, \theta_0)$.

➢ In a magnetically-linear system,

$$\lambda_1 = L_{11} i_1 + L_{12} i_2 \tag{3.57}$$

$$\lambda_2 = L_{21} i_1 + L_{22} i_2 \tag{3.58}$$

$$L_{12} = L_{21} \tag{3.59}$$

Note that $L_{ij} = L_{ij}(\theta)$.

$$i_1 = \frac{L_{22}\lambda_1 - L_{12}\lambda_2}{D} \tag{3.60}$$

$$i_2 = \frac{-L_{21}\lambda_1 + L_{11}\lambda_2}{D} \tag{3.61}$$

$$D = L_{11}L_{22} - L_{12}L_{21} \tag{3.62}$$

The energy for this linear system is

$$\begin{aligned} W_{\mathrm{fld}}\left(\lambda_{1_0}, \lambda_{2_0}, \theta_0\right) &= \int_0^{\lambda_{2_0}} \frac{L_{11}(\theta_0)\lambda_2}{D(\theta_0)} d\lambda_2 + \int_0^{\lambda_{1_0}} \frac{\left(L_{22}(\theta_0)\lambda_1 - L_{12}(\theta_0)\lambda_{2_0}\right)}{D(\theta_0)} d\lambda_1 \\ &= \frac{1}{2D(\theta_0)} L_{11}(\theta_0)\lambda_{2_0}^2 + \frac{1}{2D(\theta_0)} L_{22}(\theta_0)\lambda_{1_0}^2 - \frac{L_{12}(\theta_0)}{D(\theta_0)}\lambda_{1_0}\lambda_{2_0} \end{aligned} \tag{3.63}$$

➢ Coenergy function for a system with two windings can be defined as (3.46)

$$W'_{\mathrm{fld}}\left(i_1, i_2, \theta\right) = \lambda_1 i_1 + \lambda_2 i_2 - W_{\mathrm{fld}} \tag{3.64}$$

$$dW'_{\mathrm{fld}}\left(i_1, i_2, \theta\right) = \lambda_1 di_1 + \lambda_2 di_2 + T_{\mathrm{fld}} d\theta \tag{3.65}$$

$$\lambda_1 = \left.\frac{\partial W_{\mathrm{fld}}\left(i_1, i_2, \theta\right)}{\partial i_1}\right|_{i_2, \theta} \tag{3.66}$$

$$\lambda_2 = \left.\frac{\partial W_{\mathrm{fld}}\left(i_1, i_2, \theta\right)}{\partial i_2}\right|_{i_1, \theta} \tag{3.67}$$

$$\boxed{T_{\mathrm{fld}} = \left.\frac{\partial W'_{\mathrm{fld}}\left(i_1, i_2, \theta\right)}{\partial \theta}\right|_{i_1, i_2}} \tag{3.68}$$

$$W'_{\mathrm{fld}}\left(i_1, i_2, \theta_0\right) = \int_0^{i_{2_0}} \lambda_2\left(i_1 = 0, i_2, \theta = \theta_0\right) di_2 + \int_0^{\lambda_{1_0}} \lambda_1\left(i_1, i_2 = i_{2_0}, \theta = \theta_0\right) di_1 \tag{3.69}$$

➢ For the linear system described as (3.57) to (3.59)

$$W'_{\mathrm{fld}}\left(i_1, i_2, \theta_0\right) = \frac{1}{2} L_{11}(\theta) i_1^2 + \frac{1}{2} L_{22}(\theta) i_2^2 + L_{12}(\theta) i_1 i_2 \tag{3.70}$$

$$T_{\mathrm{fld}} = \left.\frac{\partial W'_{\mathrm{fld}}\left(i_1, i_2, \theta_0\right)}{\partial \theta}\right|_{i_1, i_2} = \frac{i_1^2}{2}\frac{dL_{11}(\theta)}{d\theta} + \frac{i_2^2}{2}\frac{dL_{22}(\theta)}{d\theta} + i_1 i_2 \frac{dL_{12}(\theta)}{d\theta} \tag{3.71}$$

→ Note that (3.70) is simpler than (3.63). That is, the coenergy function is a relatively simple function of displacement.

→ The use of a coenergy function of the terminal currents simplifies the determination of torque or force.

→ Systems with more than two electrical terminals are handled in analogous fashion.

EXAMPLE 3.7

In the system shown in Fig. 3.15, the inductances in henrys are given as $L_{11} = (3 + \cos 2\theta) \times 10^{-3}$; $L_{12} = 0.3 \cos\theta$; $L_{22} = 30 + 10 \cos 2\theta$. Find and plot the torque $T_{\mathrm{fld}}(\theta)$ for current $i_1 = 0.8$ A and $i_2 = 0.01$ A.

■ Solution

The torque can be determined from Eq. 3.71.

$$T_{\text{fld}} = \frac{i_1^2}{2}\frac{dL_{11}(\theta)}{d\theta} + \frac{i_2^2}{2}\frac{dL_{22}(\theta)}{d\theta} + i_1 i_2 \frac{dL_{12}(\theta)}{d\theta}$$
$$= \frac{i_1^2}{2}(-2 \times 10^{-3}) \sin 2\theta + \frac{i_2^2}{2}(-20 \sin 2\theta) - i_1 i_2 (0.3) \sin\theta$$

For $i_1 = 0.8$ A and $i_2 = 0.01$ A, the torque is

$$T_{\text{fld}} = -1.64 \times 10^{-3} \sin 2\theta - 2.4 \times 10^{-3} \sin\theta$$

Notice that the torque expression consists of terms of two types. One term, proportional to $i_1 i_2 \sin\theta$, is due to the mutual interaction between the rotor and stator currents; it acts in a direction to align the rotor and stator so as to maximize their mutual inductance. Alternately, it can be thought of as being due to the tendency of two magnetic fields (in this case those of the rotor and stator) to align.

The torque expression also has two terms each proportional to $\sin 2\theta$ and to the square of one of the coil currents. These terms are due to the action of the individual winding currents alone and correspond to the torques one sees in singly-excited systems. Here the torque is due to the fact that the self inductances are a function of rotor position and the corresponding torque acts in a direction to maximize each inductance so as to maximize the coenergy. The 2θ variation is due to the corresponding variation in the self inductances (exactly as was seen previously in Example 3.4), which in turn is due to the variation of the air-gap reluctance; notice that rotating the rotor by 180° from any given position gives the same air-gap reluctance (hence the twice-angle variation). This torque component is known as the ***reluctance torque***. The two torque components (mutual and reluctance), along with the total torque, are plotted with MATLAB in Fig. 3.16.

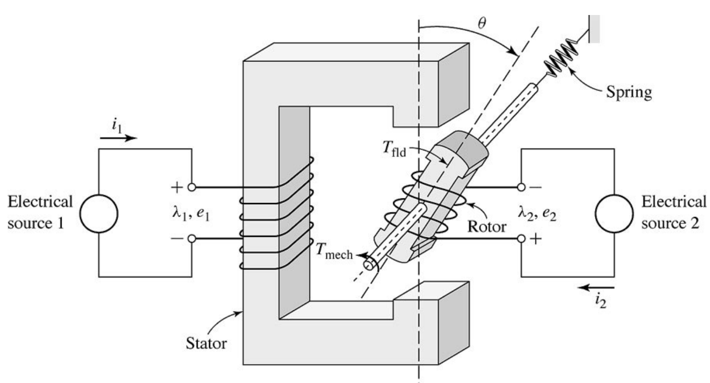


Figure 3.15 Multiply-excited magnetic system for Example 3.7.

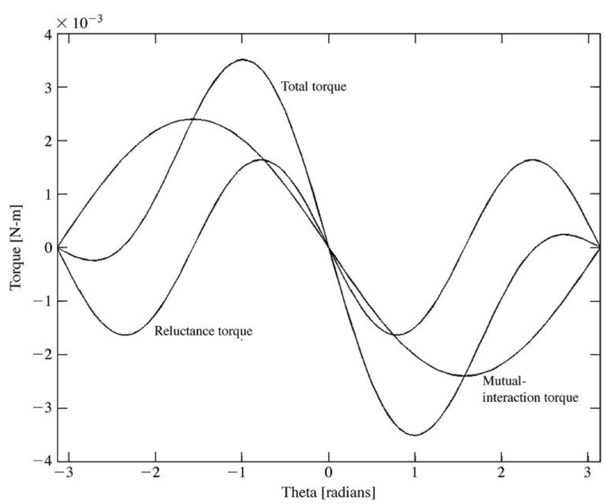


Figure 3.16 Plot of torque components for the multiply-excited system of Example 3.7.

Practice Problem 3.7

Find an expression for the torque of a symmetrical two-winding system whose inductances vary as

$$L_{11} = L_{22} = 0.8 + 0.27\cos 4\theta$$
$$L_{12} = 0.65\cos 2\theta$$

for the condition that $i_1 = -i_2 = 0.37$ A .

Solution: $T_{\text{fld}} = -0.296\sin 4\theta + 0.178\sin 2\theta$

- System with linear displacement:

$$W_{\text{fld}}\left(\lambda_{1_0}, \lambda_{2_0}, x_0\right) = \int_0^{\lambda_{2_0}} i_2\left(\lambda_1 = 0, \lambda_2, x = x_0\right) d\lambda_2 + \int_0^{\lambda_{1_0}} i_1\left(\lambda_1, \lambda_2 = \lambda_{2_0}, x = x_0\right) d\lambda_1 \tag{3.72}$$

$$W'_{\text{fld}}\left(i_{1_0}, i_{2_0}, x_0\right) = \int_0^{\lambda_{2_0}} \lambda_2\left(i_1 = 0, i_2, x = x_0\right) di_2 + \int_0^{\lambda_{1_0}} \lambda_1\left(i_1, i_2 = i_{2_0}, x = x_0\right) di_1 \tag{3.73}$$

$$f_{\text{fld}} = -\left.\frac{\partial W_{\text{fld}}\left(\lambda_1, \lambda_2, x\right)}{\partial x}\right|_{\lambda_1, \lambda_2} \tag{3.74}$$

$$f_{\text{fld}} = -\left.\frac{\partial W'_{\text{fld}}\left(i_1, i_2, x\right)}{\partial x}\right|_{i_1, i_2} \tag{3.75}$$

For a magnetically-linear system,

$$W'_{\text{fld}}\left(i_1, i_2, x\right) = \frac{1}{2} L_{11}(x) i_1^2 + \frac{1}{2} L_{22}(x) i_2^2 + L_{12}(x) i_1 i_2 \tag{3.76}$$

$$f_{\text{fld}} = \frac{i_1^2}{2}\frac{dL_{11}(x)}{dx} + \frac{i_2^2}{2}\frac{dL_{22}(x)}{dx} + i_1 i_2 \frac{dL_{12}(x)}{dx} \tag{3.77}$$

Chapter 4 Introduction to Rotating Machines

- The objective of this chapter is to introduce and discuss some of the principles underlying the performance of electric machinery, both ac and dc machines.

§4.1 Elementary Concepts

- Voltages can be induced by time-varying magnetic fields. In rotating machines, voltages are generated in windings or groups of coils by rotating these windings mechanically through a magnetic field, by mechanically rotating a magnetic field past the winding, or by designing the magnetic circuit so that the reluctance varies with rotation of the rotor.
  - ➢ The flux linking a specific coil is changed cyclically, and a time-varying voltage is generated.
  - ➢ Electromagnetic energy conversion occurs when changes in the flux linkage result from mechanical motion.
  - ➢ A set of such coils connected together is typically referred to as an armature winding, a winding or a set of windings carrying ac currents.
    - → In ac machines such as synchronous or induction machines, the armature winding is typically on the stator. (the stator winding)
    - → In dc machines, the armature winding is found on the rotor. (the rotor winding)
  - ➢ Synchronous and dc machines typically include a second winding (or set of windings), referred to as the field winding, which carrys dc current and which are used to produce the main operating flux in the machine.
    - → In dc machines, the field winding is found on the stator.
    - → In synchronous machines, the field winding is found on the rotor.
    - → Permanent magnets can be used in the place of field windings.
  - ➢ In most rotating machines, the stator and rotor are made of electrical steel, and the windings are installed in slots on these structures. The stator and rotor structures are typically built from thin laminations of electrical steel, insulated from each other, to reduce eddy-current losses.

§4.2 Introduction to AC And DC Machines

§4.2.1 AC Machines

- Traditional ac machines fall into one of two categories: synchronous and induction.
  - ➢ In synchronous machines, rotor-winding currents are supplied directly from the stationary frame through a rotating contact.
  - ➢ In induction machines, rotor currents are induced in the rotor windings by a combination of the time-variation of the stator currents and the motion of the rotor relative to the stator.
- Synchronous Machines
  - ➢ Fig. 4.4: a simplified salient-pole ac synchronous generator with two poles.
    - → The armature winding is on the stator, and the field winding is on the rotor.
    - → The field winding is excited by direct current conducted to it by means of stationary carbon brushes that contact rotating slip rings or collector rings.
    - → It is advantages to have the single, low-power field winding on the rotor while having the high-power, typically multiple-phase, armature winding on the stator.
    - → Armature winding $(a,-a)$ consists of a single coil of N turns.
    - → Conductors forming these coil sides are connected in series by end connections.
    - → The rotor is turned at a constant speed by a source of mechanical power connected to its shaft. Flux paths are shown schematically by dashed lines.

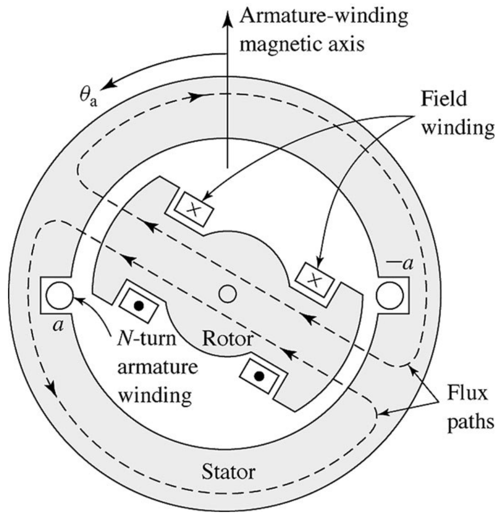


Figure 4.4 Schematic view of a simple, two-pole, single-phase synchronous generator.

- Assume a sinusoidal distribution of magnetic flux in the air gap of the machine in Fig. 4.4.
  - → The radial distribution of air-gap flux density B is shown in Fig. 4.5(a) as a function of the spatial angle θ around the rotor periphery.
  - → As the rotor rotates, the flux –linkages of the armature winding change with time and the resulting coil voltage will be sinusoidal in time as shown in Fig 4.5(b). The frequency in cycles per second (Hz) is the same as the speed of the rotor in revolutions in second (rps).
  - → A two-pole synchronous machine must revolve at 3600 rpm to produce a 60-Hz voltage.
  - → Note the terms “rpm” and “rps”.

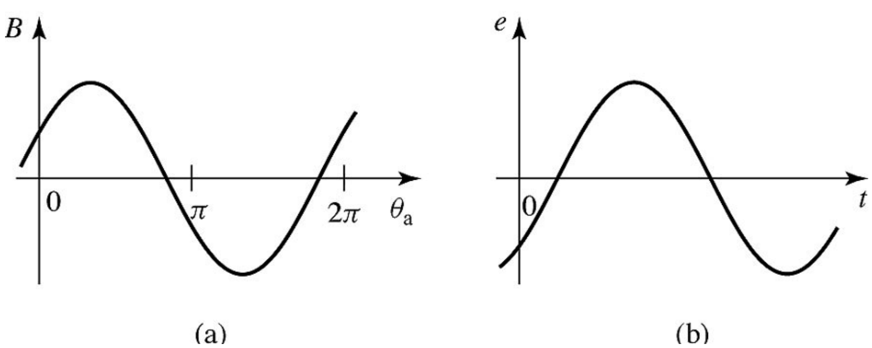


Figure 4.5 (a) Space distribution of flux density and (b) corresponding waveform of the generated voltage for the single-phase generator of Fig. 4.4.

- A great many synchronous machines have more than two poles. Fig 4.6 shows in schematic form a four-pole single-phase generator.
  - → The field coils are connected so that the poles are of alternate polarity.
  - → The armature winding consists of two coils $(a_1, -a_1)$ and $(a_2, -a_2)$ connected in series by their end connections.
  - → There are two complete wavelengths, or cycles, in the flux distribution around the periphery, as shown in Fig. 4.7.
  - → The generated voltage goes through two complete cycles per revolution of the rotor.
  - → The frequency in Hz is thus twice the speed in rps.

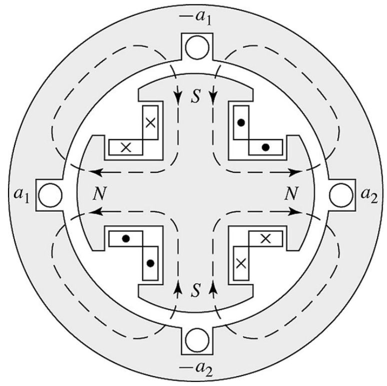


Figure 4.6 Schematic view of a simple, four-pole, single-phase synchronous generator.

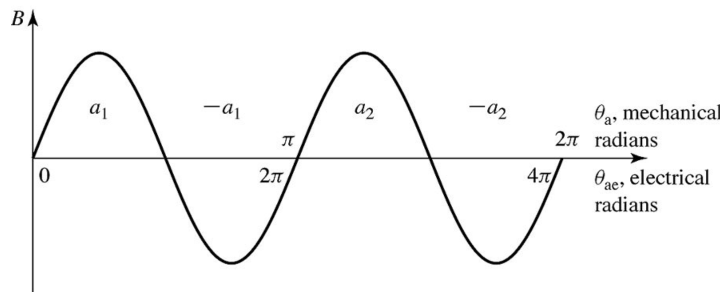


Figure 4.7 Space distribution of the air-gap flux density in an idealized, four-pole synchronous generator.

- When a machine has more than two poles, it is convenient to concentrate on a single pair of poles and to express angles in electrical degrees or electrical radians rather than in physical units.
  - → One pair of poles equals 360 electrical degrees or 2π electrical radians.
  - → Since there are poles/2 wavelengths, or cycles, in one revolution, it follows that

$$\theta_{\mathrm{ae}} = \left(\frac{\text{poles}}{2}\right)\theta_{\mathrm{a}} \tag{4.1}$$

    where θ_{ae} is the angle in electrical units and θ_{a} is the spatial angle.
  - → The coil voltage of a multipole machine passes through a complete cycle every time a pair of poles sweeps by, or (poles/2) times each revolution. The electrical frequency f_{e} of the voltage generated is therefore

$$f_{\mathrm{e}} = \left(\frac{\text{poles}}{2}\right)\frac{n}{60} \quad \text{Hz} \tag{4.2}$$

    where n is the mechanical speed in rpm. Note that $\boxed{\omega_{\mathrm{e}} = (\text{poles}/2)\omega_{\mathrm{m}}}$.
- The rotors shown in Figs. 4.4 and 4.6 have salient, or projecting, poles with concentrated windings. Fig. 4.8 shows diagrammatically a nonsalient-pole, or cylindrical, rotor.
  - → The field winding is a two-pole distributed winding; the coil sides are distributed in multiple slots around the rotor periphery and arranged to produce an approximately sinusoidal distribution of radial air-gap flux.
  - → Most power systems in the world operate at frequencies of either 50 or 60 Hz.
  - → A salient-pole construction is characteristic of hydroelectric generators because hydraulic turbines operate at relatively low speeds, and hence a relatively large number of poles is required to produce the desired frequency.
  - → Steam turbines and gas turbines operate best at relatively high speeds, and turbine-driven alternators or turbine generators are commonly two- or four-pole cylindrical-rotor machines.

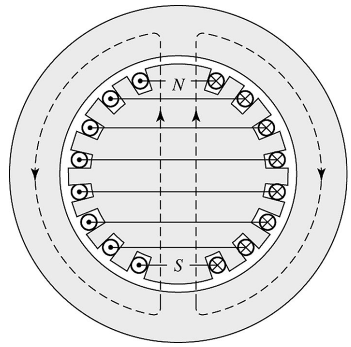
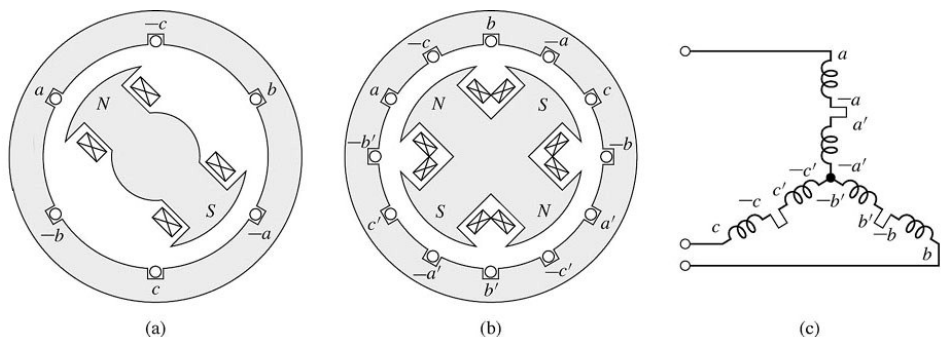


Figure 4.8 Elementary two-pole cylindrical-rotor field winding.

- Most of the world's power systems are three-phase systems. With very few exceptions, synchronous generators are three-phase machines.
  - → A simplified schematic view of a three-phase, two-pole machine with one coil per phase is shown in Fig. 4.12(a)
  - → Fig. 4.12(b) depicts a simplified three-phase, four-pole machine. Note that a minimum of two sets of coils must be used. In an elementary multipole machine, the minimum number of coils sets is given by one half the number of poles.
  - → Note that coils $(a,-a)$ and $(a',-a')$ can be connected in series or in parallel. Then the coils of the three phases may then be either Y- or Δ-connected. See Fig. 4.12(c).



Errore.

Figure 4.12 Schematic views of three-phase generators: (a) two-pole, (b) four-pole, and (c) Y connection of the windings.

- The electromechanical torque is the mechanism through which a synchronous generator converts mechanical to electric energy.
  - → When a synchronous generator supplies electric power to a load, the armature current creates a magnetic flux wave in the air gap that rotates at synchronous speed.
  - → This flux reacts with the flux created by the field current, and an electromechanical torque results from the tendency of these two magnetic fields to align.
  - → In a generator this torque opposes rotation, and mechanical torque must be applied from the prime mover to sustain rotation.
- The counterpart of the synchronous generator is the synchronous motor.
  - → Ac current supplied to the armature winding on the stator, and dc excitation is supplied to the field winding on the rotor. The magnetic field produced by the armature currents rotates at synchronous speed. (Why?)
  - → To produce a steady electromechanical torque, the magnetic fields of the stator and rotor must be constant in amplitude and stationary with respect to each other.
  - → In a motor the electromechanical torque is in the direction of rotation and balances the opposing torque required to drive the mechanical load.
  - → In both generators and motors, an electromechanical torque and a rotational voltage are

produced which are the essential phenomena for electromechanical energy conversion.
→ Note that the flux produced by currents in the armature of a synchronous motor rotates ahead of that produced by the field, thus pulling on the field (and hence on the rotor) and doing work. This is the opposite of the situation in a synchronous generator, where the field does work as its flux pulls on that of the armature, which is lagging behind.

- Induction Machines
  - Alternating currents are applied directly to the stator windings. Rotors currents are then produced by induction, i.e., transformer action.
    - → Alternating currents flow in the rotor windings of an induction machine, in contrast to a synchronous machine in which a field winding on the rotor is excited with dc current.
    - → The induction machine may be regarded as a generalized transformer in which electric power is transformed between rotor and stator together with a change of frequency and a flow of mechanical power.
  - The induction motor is the most common of all motors.
    - → The induction machine is seldom used as a generator.
    - → In recent years it has been found to be well suited for wind-power applications.
    - → It may also be used as a frequency changer.
  - In the induction motor, the stator windings are essentially the same as those of a synchronous machine. The rotor windings are electrically short-circuited.
    - → The rotor windings frequently have no external connections.
    - → Currents are induced by transformer action from the stator winding.
    - → Squirrel-cage induction motor: relatively expensive and highly reliable.
  - The armature flux in the induction motor leads that of the rotor and produces an electromechanical torque.
    - → The rotor does not rotate synchronously.
    - → It is the slipping of the rotor with respect to the synchronous armature flux that gives rise to the induced rotor currents and hence the torque.
    - → Induction motors operate at speeds less than the synchronous mechanical speed.
    - → A typical speed-torque characteristic for an induction motor is shown in Fig. 4.15.

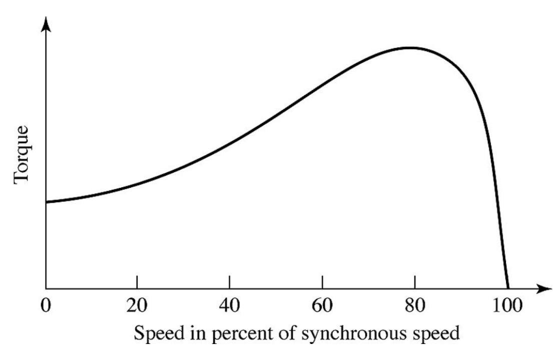


Figure 4.15 Typical induction-motor speed-torque characteristic.

§4.2.2 DC Machines

- DC Machines
  - There are two sets of windings in a dc machine.
    - → The armature winding is on the rotor with current conducted from it by means of carbon brushes.
    - → The field winding is on the stator and is excited by direct current.
  - An elementary two-pole dc generator is shown in Fig. 4.17.

→ Armature winding: $(a, -a)$, pitch $= 180^{\circ}$

→ The rotor is normally turned at a constant speed by a source of mechanical power connected the shaft.

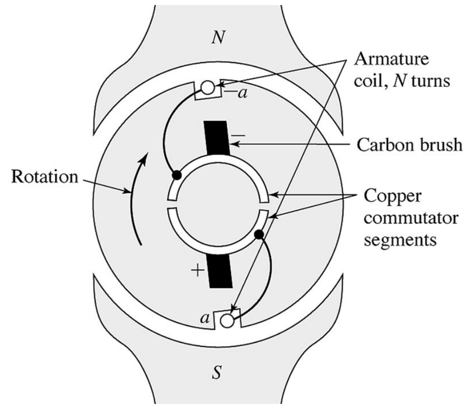


Figure 4.17 Elementary dc machine with commutator.

→ The air-gap flux distribution usually approximates a flat-topped wave, rather than the sine wave found in ac machines, and is shown in Fig. 4.18(a).

→ Rotation of the coil generates a coil voltage which is a time function having the same waveform as the spatial flux-density distribution.

→ The voltage induced in an individual armature coil is an alternating voltage and rectification is produced mechanically by means of a commutator. Stationary carbon brushes held against the commutator surface connect the winding to the external armature terminal.

→ The need for commutation is the reason why the armature windings are placed on the rotor.

→ The commutator provides full-wave rectification, and the voltage waveform between brushes is shown in Fig. 4.18(b).

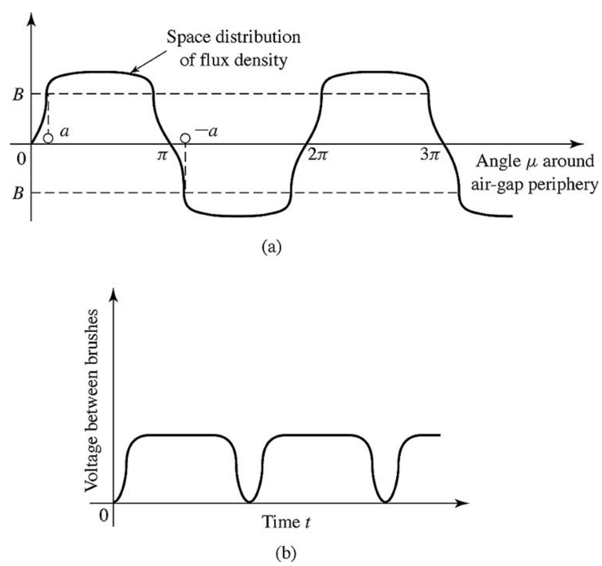


Figure 4.18 (a) Space distribution of air-gap flux density in an elementary dc machine; (b) waveform of voltage between brushes.

➢ It is the interaction of the two flux distributions created by the direct currents in the field and the armature windings that creates an electromechanical torque.

→ If the machine is acting as a generator, the torque opposes rotation.
→ If the machine is acting as a motor, the torque acts in the direction of the rotation.

§4.3 MMF of Distributed Windings

- Most armatures have distributed windings, i.e. windings which are spread over a number of slots around the air-gap periphery.
  - The individual coils are interconnected so that the result is a magnetic field having the same number of poles as the field winding.
  - Consider Fig. 4.19(a).
    - → Full-pitch coil: a coil which spans 180 electrical degrees.
    - → In Fig. 4.19(b), the air gap and winding are in developed form (laid out flat) and the air-gap mmf distribution is shown by the steplike distribution of amplitude $Ni/2$.

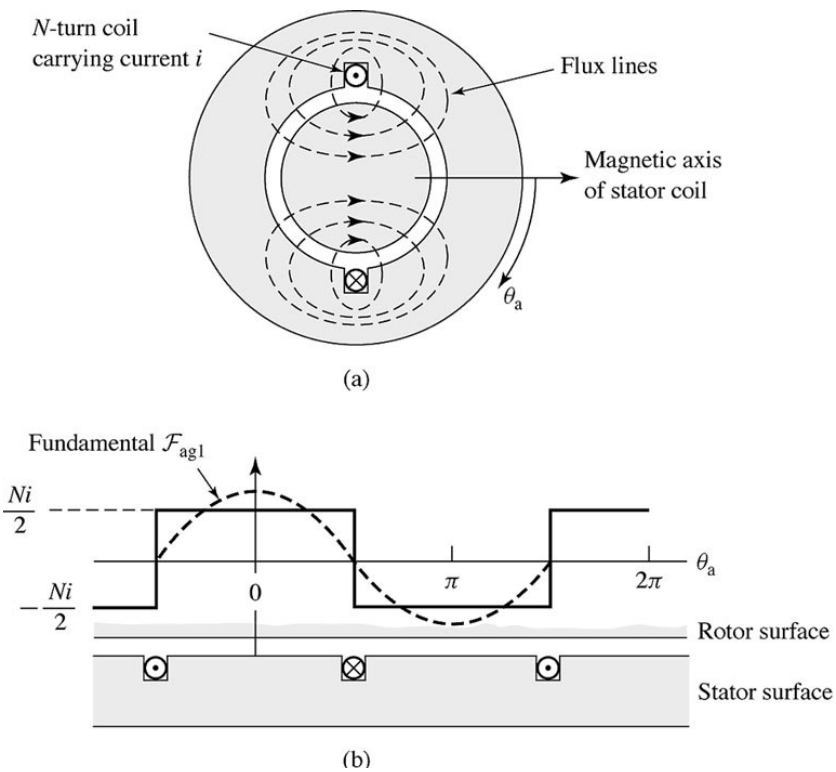


Figure 4.19 (a) Schematic view of flux produced by a concentrated, full-pitch winding in a machine with a uniform air gap. (b) The air-gap mmf produced by current in this winding.

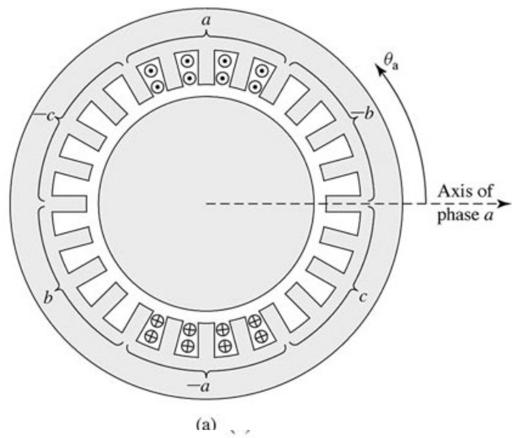
§4.3.1 AC Machines

- It is appropriate to focus our attention on the space-fundamental sinusoidal component of the air-gap mmf.
  - In the design of ac machines, serious efforts are made to distribute the coils making up the windings so as to minimize the higher-order harmonic components.
  - The rectangular air-gap mmf wave of the concentrated two-pole, full-pitch coil of Fig. 4.19(b) can be resolved to a Fourier series comprising a fundamental component and a series of odd harmonics.
    - → The fundamental component F_{ag1} and its amplitude $(F_{ag1})_{peak}$ are

$$F_{ag1} = \frac{4}{\pi}\left(\frac{Ni}{2}\right)\cos\theta_a \tag{4.3}$$

$$\left(F_{\text{agl}}\right)_{\text{peak}} = \frac{4}{\pi}\left(\frac{Ni}{2}\right) \tag{4.4}$$

- Consider a distributed winding, consisting of coils distributed in several slots.
  - → Fig. 4.20(a) shows phase a of the armature winding of a simplified two-pole, three-phase ac machine and phases b and c occupy the empty slots.
  - → The windings of the three phases are identical and are located with their magnetic axes 120 degrees apart. The winding is arranged in two layers, each full-pitch coil of N_{c} turns having one side in the top of a slot and the other coil side in the bottom of a slot a pole pitch away.
  - → Fig. 4.20(b) shows that the mmf wave is a series of steps each of height $2N_{\text{c}}i_{\text{a}}$. It can be seen that the distributed winding produces a closer approximation to a sinusoidal mmf wave than the concentrated coil of Fig. 4.19 does.



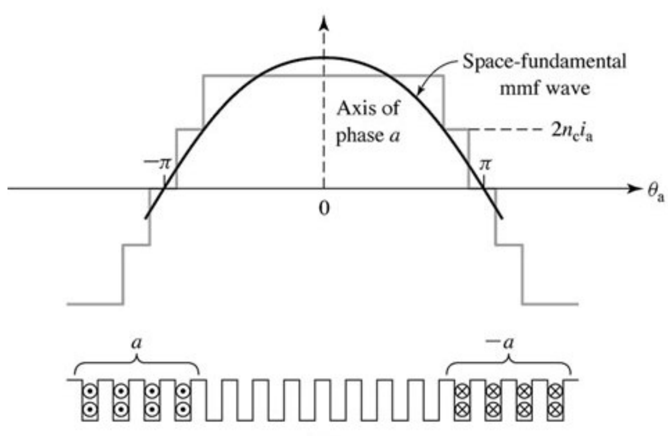


Figure 4.20 The mmf of one phase of a distributed two-pole, three-phase winding with full-pitch coils.

- The modified form of (4.3) for a distributed multipole winding is

$$F_{\text{agl}} = \frac{4}{\pi}\left(\frac{k_{\text{w}}N_{\text{ph}}}{\text{poles}}\right) i_{\text{a}} \cos\left(\frac{\text{poles}}{2}\theta_{\text{a}}\right) \tag{4.5}$$

N_{ph} : number of series turns per phase,

k_{w} : winding factor, a reduction factor taking into account the distribution of the winding, typically in the range of 0.85 to 0.95, $k_{\text{w}} = k_{\text{b}}k_{\text{p}}$ (or $k_{\text{d}}k_{\text{p}}$).

  - → The peak amplitude of this mmf wave is

$$(F_{\text{ag1}})_{\text{peak}} = \frac{4}{\pi}\left(\frac{k_{\text{w}} N_{\text{ph}}}{\text{poles}}\right) i_{\text{a}} \tag{4.6}$$

- Eq. (4.5) describes the space-fundamental component of the mmf wave produced by current in phase a of a distributed winding.
  - → If $i_{\text{a}} = I_{\text{m}} \cos \omega t$ the result will be an mmf wave which is stationary in space and varies sinusoidally both with respect to θ_{a} and in time.
  - → The application of three-phase currents will produce a rotating mmf wave.
- Rotor windings are often distributed in slots to reduce the effects of space harmonics.
  - → Fig. 4.21(a) shows the rotor of a typical two-pole round-rotor generator.
  - → As shown in Fig. 4.21(b), there are fewer turns in the slots nearest the pole face.
  - → The fundamental air-gap mmf wave of a multipole rotor winding is

$$F_{\text{agl}} = \frac{4}{\pi}\left(\frac{k_{\text{r}} N_{\text{r}}}{\text{poles}}\right) I_{\text{r}} \cos\left(\frac{\text{poles}}{2}\theta_{\text{r}}\right) \tag{4.7}$$

$$(F_{\text{ag1}})_{\text{peak}} = \frac{4}{\pi}\left(\frac{k_{\text{r}} N_{\text{r}}}{\text{poles}}\right) I_{\text{r}} \tag{4.8}$$

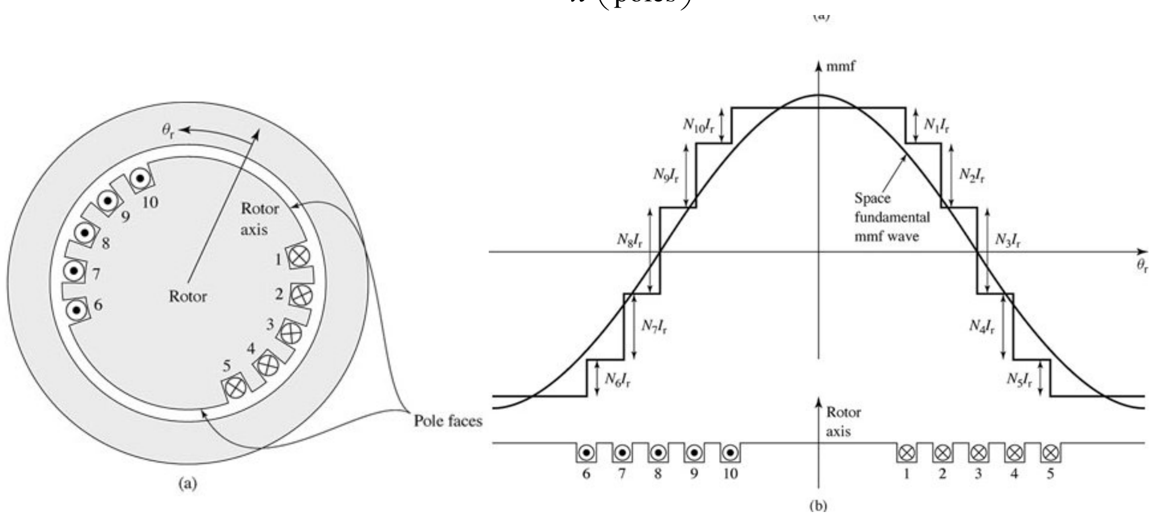


Figure 4.21 The air-gap mmf of a distributed winding on the rotor of a round-rotor generator.

EXAMPLE 4.1

The phase-a two-pole armature winding of Fig. 4.20a can be considered to consist of 8 N_c-turn, full-pitch coils connected in series, with each slot containing two coils. There are a total of 24 armature slots, and thus each slot is separated by $360°/24 = 15°$. Assume angle θ_a is measured from the magnetic axis of phase a such that the four slots containing the coil sides labeled a are at $\theta_a = 67.5°, 82.5°, 97.5°$, and $112.5°$. The opposite sides of each coil are thus found in the slots found at $-112.5°, -97.5°, -82.5°$ and $-67.5°$, respectively. Assume this winding to be carrying current i_a.

(*a*) Write an expression for the space-fundamental mmf produced by the two coils whose sides are in the slots at $\theta_a = 112.5°$ and $-67.5°$. (*b*) Write an expression for the space-fundamental mmf produced by the two coils whose sides are in the slots at $\theta_a = 67.5°$ and $-112.5°$. (*c*) Write an expression for the space-fundamental mmf of the complete armature winding. (*d*) Determine the winding factor k_w for this distributed winding.

■ **Solution**

a. Noting that the magnetic axis of this pair of coils is at $\theta_a = (112.5° - 67.5°)/2 = 22.5°$ and that the total ampere-turns in the slot is equal to $2N_c i_a$, the mmf produced by this pair of coils can be found from analogy with Eq. 4.3 to be

$$(\mathcal{F}_{ag1})_{22.5°} = \frac{4}{\pi}\left(\frac{2N_c i_a}{2}\right)\cos(\theta_a - 22.5°)$$

b. This pair of coils produces the same space-fundamental mmf as the pair of part (a) with the exception that this mmf is centered at $\theta_a = -22.5°$. Thus

$$(\mathcal{F}_{ag1})_{-22.5°} = \frac{4}{\pi}\left(\frac{2N_c i_a}{2}\right)\cos(\theta_a + 22.5°)$$

c. By analogy with parts (a) and (b), the total space-fundamental mmf can be written as

$$\begin{aligned}(\mathcal{F}_{ag1})_{total} &= (\mathcal{F}_{ag1})_{-22.5°} + (\mathcal{F}_{ag1})_{-7.5°} + (\mathcal{F}_{ag1})_{7.5°} + (\mathcal{F}_{ag1})_{22.5°} \\ &= \frac{4}{\pi}\left(\frac{2N_c}{2}\right) i_a [\cos(\theta_a + 22.5°) + \cos(\theta_a + 7.5°) \\ &\qquad + \cos(\theta_a - 7.5°) + \cos(\theta_a - 22.5°)] \\ &= \frac{4}{\pi}\left(\frac{7.66N_c}{2}\right) i_a \cos\theta_a \\ &= 4.88 N_c i_a \cos\theta_a\end{aligned}$$

d. Recognizing that, for this winding $N_{ph} = 8N_c$, the total mmf of part (c) can be rewritten as

$$(\mathcal{F}_{ag1})_{total} = \frac{4}{\pi}\left(\frac{0.958N_{ph}}{2}\right) i_a \cos\theta_a$$

Comparison with Eq. 4.5 shows that for this winding, the winding factor is $k_w = 0.958$.

---

§4.3.2 DC Machines

- Because of the restrictions imposed on the winding arrangement by the commutator, the mmf wave of a dc machine armature approximates a sawtooth waveform more nearly than the sine wave of ac machines.
  - ➢ Fig. 4.22 shows diagrammatically in cross section the armature of a two-pole dc machine.
    - → The armature coil connections are such that the armature winding produces a magnetic field whose axis is vertical and thus is perpendicular to the axis of the field winding.
    - → As the armature rotates, the magnetic field of the armature remains vertical due to commutator action and a continuous unidirectional torque results.
    - → The mmf wave is illustrated and analyzed in Fig. 4.23.

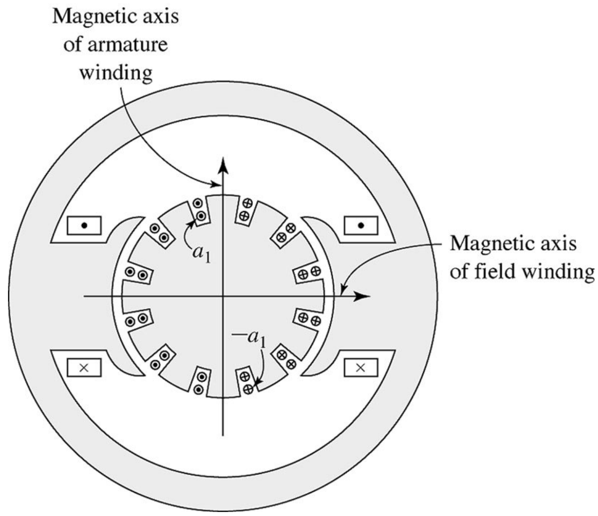


Figure 4.22 Cross section of a two-pole dc machine.

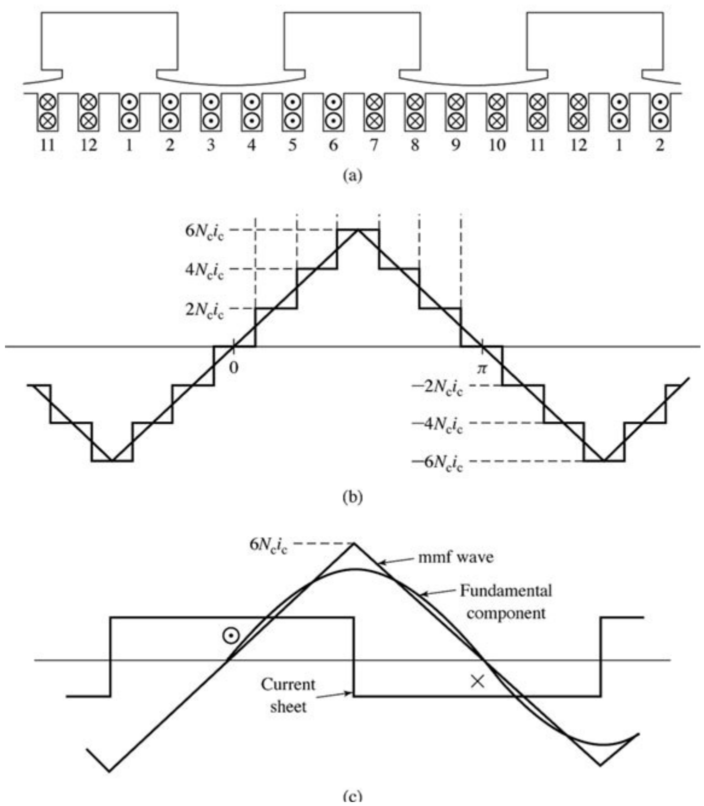


Figure 4.23 (a) Developed sketch of the dc machine of Fig. 4.22; (b) mmf wave; (c) equivalent sawtooth mmf wave, its fundamental component, and equivalent rectangular current sheet.

- DC machines often have a magnetic structure with more than two poles.
  - → Fig. 4.24(a) shows schematically a four-pole dc machine.
  - → The machine is shown in laid-out form in Fig. 4.24(b).

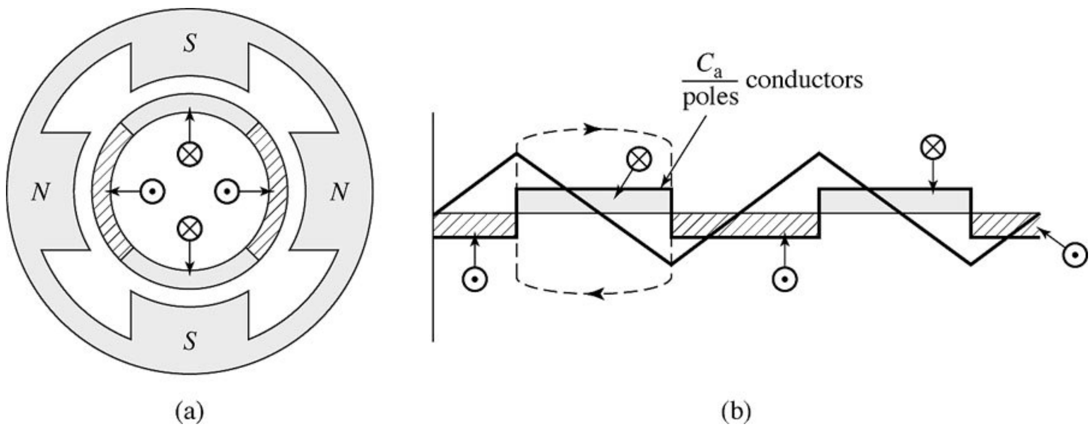


Figure 4.24 (a) Cross section of a four-pole dc machine; (b) development of current sheet and mmf wave.

- The peak value of the sawtooth armature mmf wave can be written as

$$\left(F_{\text{ag}}\right)_{\text{peak}} = \left(\frac{C_{\text{a}}}{2m \cdot \text{poles}}\right) i_{\text{a}} \qquad \text{A} \cdot \text{turns/pole} \tag{4.9}$$

C_a = total number of conductors in armature winding
m = number of parallel paths through armature winding
i_a = armature current, A

$$\left(F_{\text{ag}}\right)_{\text{peak}} = \left(\frac{N_{\text{a}}}{\text{poles}}\right) i_{\text{a}}, \quad N_{\text{a}} = C_{\text{a}}/(2m)\text{: no. of series armature turns} \tag{4.10}$$

$$\left(F_{\text{ag}}\right)_{\text{peak}} = \frac{8}{\pi^2}\left(\frac{N_{\text{a}}}{\text{poles}}\right) i_{\text{a}} \tag{4.11}$$

(4.12)

§4.4 Magnetic Fields In Rotating Machinery

- The behavior of electric machinery is determined by the magnetic fields created by currents in the various windings of the machine.
  - The investigations of both ac and dc machines are based on the assumption of sinusoidal spatial distribution of mmf.
  - Results from examining a two-pole machine can immediately be extrapolated to a multipole machine.

§4.4.1 Magnetic with Uniform Air Gaps

- Consider machines with uniform air gaps.
  - Fig. 4.25(a) shows a single full-pitch, *N*-turn coil in a high-permeability magnetic structure $(\mu \to \infty)$, with a concentric, cylindrical rotor.
    - → In Fig. 4.25(b) the air-gap mmf F_{ag} is plotted versus angle θ_{a}.
    - → Fig. 4.25(c) demonstrates the air-gap constant radial magnetic field H_{ag}.

$$H_{\text{ag}} = \frac{F_{\text{ag}}}{g} \tag{4.12}$$

$$\left(H_{\text{ag1}}\right) = \frac{F_{\text{ag1}}}{g} = \frac{4}{\pi}\left(\frac{Ni}{2g}\right)\cos\theta_{\text{a}} \tag{4.13}$$

$$\left(H_{agl}\right)_{peak} = \frac{4}{\pi}\left(\frac{Ni}{2g}\right) \tag{4.14}$$

- For a distributed winding such as that of Fig. 4.20, the air-gap magnetic field intensity is

$$H_{agl} = \frac{4}{\pi}\left(\frac{k_w N_{ph}}{g \cdot \text{poles}}\right) i_a \cos\left(\frac{\text{poles}}{2}\theta_a\right) \tag{4.15}$$

EXAMPLE 4.2

A four-pole synchronous ac generator with a smooth air gap has a distributed rotor winding with 263 series turns, a winding factor of 0.935, and an air gap of length 0.7 mm. Assuming the mmf drop in the electrical steel to be negligible, find the rotor-winding current required to produce a peak, space-fundamental magnetic flux density of 1.6 T in the machine air gap.

■ Solution

The space-fundamental air-gap magnetic flux density can be found by multiplying the air-gap magnetic field by the permeability of free space μ_0, which in turn can be found from the space-fundamental component of the air-gap mmf by dividing by the air-gap length g. Thus, from Eq. 4.8

$$(B_{agl})_{peak} = \frac{\mu_0(\mathcal{F}_{agl})_{peak}}{g} = \frac{4\mu_0}{\pi g}\left(\frac{k_r N_r}{\text{poles}}\right) I_r$$

and I_r can be found from

$$\begin{aligned} I_r &= \left(\frac{\pi g \cdot \text{poles}}{4\mu_0 k_r N_r}\right)(B_{agl})_{peak} \\ &= \left(\frac{\pi \times 0.0007 \times 4}{4 \times 4\pi \times 10^{-7} \times 0.935 \times 263}\right) 1.6 \\ &= 11.4\ \text{A} \end{aligned}$$

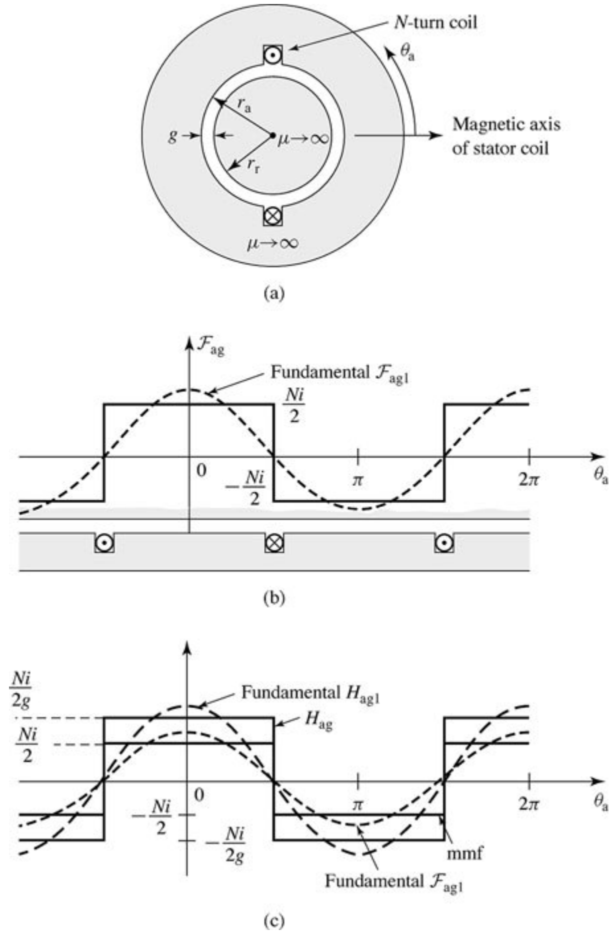


Figure 4.25 The air-gap mmf and radial component of H_{ag} for a concentrated full-pitch winding.

§4.4.2 Machines with Nonuniform Air Gaps

- The air-gap magnetic-field distribution of machines with nonuniform air gaps is more complex than that of uniform-air-gap machines.
  - Fig. 4.26(a) shows the structure of a typical dc machine and Fig. 4.26(b) shows the structure of a typical salient-pole synchronous machine.

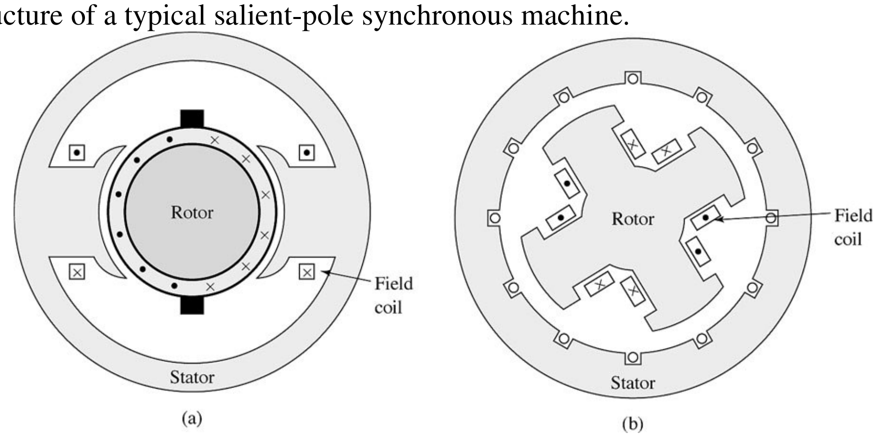


Figure 4.26 Structure of typical salient-pole machines: (a) dc machine and (b) salient-pole synchronous machine.

  - Detailed analysis of the magnetic field distributions requires complete solutions of the field problem.
    - → Fig. 4.27 shows the magnetic field distribution in a salient-pole dc generator (obtained by finite-element solution).

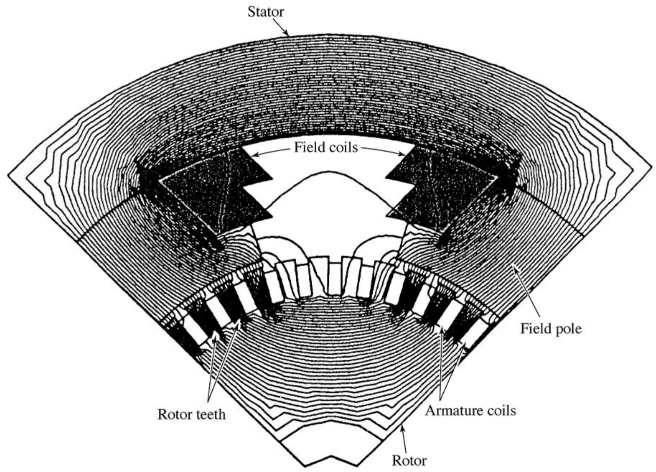


Figure 4.27 Finite-element solution of the magnetic field distribution in a salient-pole dc generator. Field coils excited; no current in armature coils. (*General Electric Company*.)

§4.5 Rotating MMF Waves in AC Machines

- To understand the theory and operation of polyphase ac machines, it is necessary to study the nature of the mmf wave produced by a polyphase winding.

§4.5.1 MMF Wave of a Single-Phase Winding

- Fig. 4.28(a) shows the space-fundamental mmf distribution of a single-phase winding.
  - Note that from Eq. (4.5), F_{ag1} is

$$F_{\mathrm{ag1}} = \frac{4}{\pi}\left(\frac{k_{\mathrm{w}} N_{\mathrm{ph}}}{\mathrm{poles}}\right) i_{\mathrm{a}} \cos\left(\frac{\mathrm{poles}}{2}\theta_{\mathrm{a}}\right) \tag{4.16}$$

When the winding is exicted by a current

$$i_{\mathrm{a}} = I_{\mathrm{a}} \cos \omega_{\mathrm{e}} t \tag{4.17}$$

the mmf distribution is given by

$$\begin{aligned} F_{\mathrm{ag1}} &= F_{\max} \cos\left(\frac{\mathrm{poles}}{2}\theta_{\mathrm{a}}\right) \cos \omega_{\mathrm{e}} t \\ &= F_{\max} \cos\left(\theta_{\mathrm{ae}}\right) \cos \omega_{\mathrm{e}} t \end{aligned} \tag{4.18}$$

$$F_{\max} = \frac{4}{\pi}\left(\frac{k_{\mathrm{w}} N_{\mathrm{ph}}}{\mathrm{poles}}\right) I_{\mathrm{a}} \tag{4.19}$$

→ This mmf distribution remains fixed in space with an amplitude that varies sinusoidally in time at frequency ω_{e}, as shown in Fig. 4.28(a).

  - The air-gap mmf of a single-phase winding exicted by a source of ac current can be resolved into rotating traveling waves.

→ By the identity $\cos\alpha\cos\beta = \frac{1}{2}\cos(\alpha-\beta) + \frac{1}{2}\cos(\alpha+\beta)$,

$$F_{\mathrm{ag1}} = F_{\max}\left[\frac{1}{2}\cos\left(\theta_{\mathrm{ae}} - \omega_{\mathrm{e}} t\right) + \frac{1}{2}\cos\left(\theta_{\mathrm{ae}} + \omega_{\mathrm{e}} t\right)\right] \tag{4.20}$$

$$F_{\mathrm{ag1}}^{+} = \frac{1}{2} F_{\max} \cos\left(\theta_{\mathrm{ae}} - \omega_{\mathrm{e}} t\right) \tag{4.21}$$

$$F_{\mathrm{ag1}}^{-} = \frac{1}{2} F_{\max} \cos\left(\theta_{\mathrm{ae}} + \omega_{\mathrm{e}} t\right) \tag{4.22}$$

→ F_{ag1}^{+} travels in the $+\theta_{\text{a}}$ direction and F_{ag1}^{-} travels in the $-\theta_{\text{a}}$ direction.

→ This decomposition is shown graphically in Fig. 4.28(b) and in a phasor representation in Fig. 4.28(c).

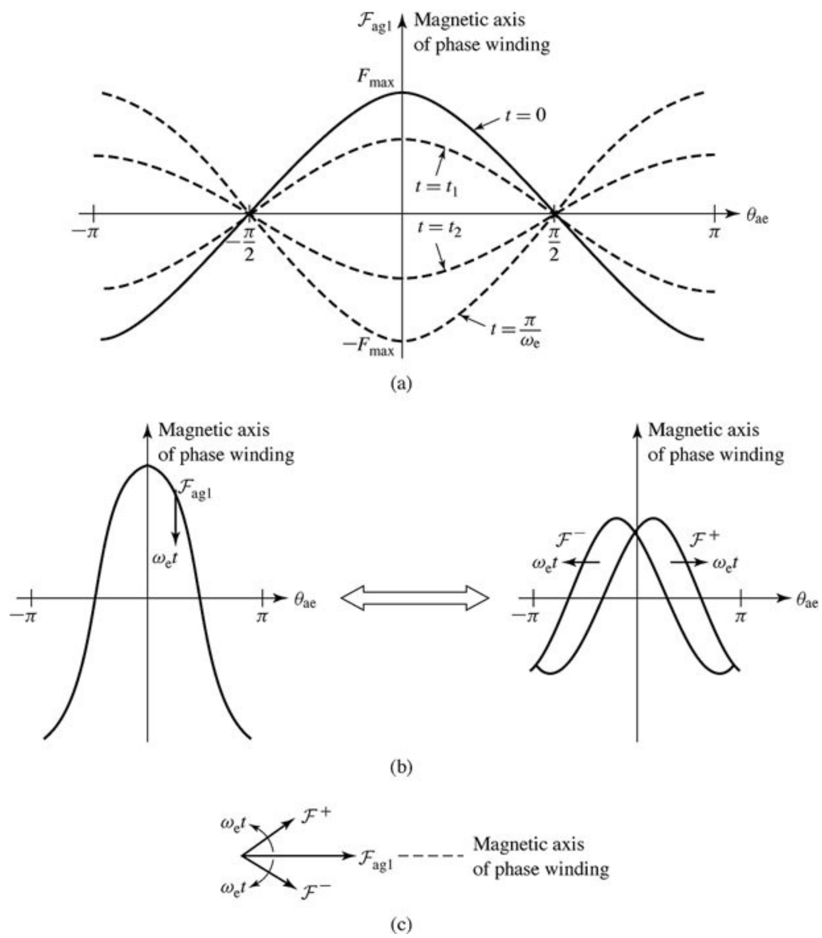


Figure 4.28 Single-phase-winding space-fundamental air-gap mmf: (a) mmf distribution of a single-phase winding at various times; (b) total mmf F_{agl} decomposed into two traveling waves F^{-} and F^{+}; (c) phasor decomposition of F_{agl}.

§4.5.2 MMF Wave of a Polyphase Winding

- We are to study the mmf distribution of three-phase windings such as those found on the stator of three-phase induction and synchronous machines.
  - In a three-phase machine, the windings of the individual phases are displaced from each other by 120 electrical degrees in space around the air-gap circumference as shown in Fig. 4.29 in which the concentrated full-pitch coils may be considered to represent distributed windings.
    → Under balanced three-phase conditions, the excitation currents (Fig. 4.30) are

$$i_{\text{a}} = I_{\text{m}} \cos \omega_{\text{e}} t \tag{4.23}$$

$$i_{\text{b}} = I_{\text{m}} \cos\left(\omega_{\text{e}} t - 120^{\circ}\right) \tag{4.24}$$

$$i_{\text{c}} = I_{\text{m}} \cos\left(\omega_{\text{e}} t + 120^{\circ}\right) \tag{4.25}$$

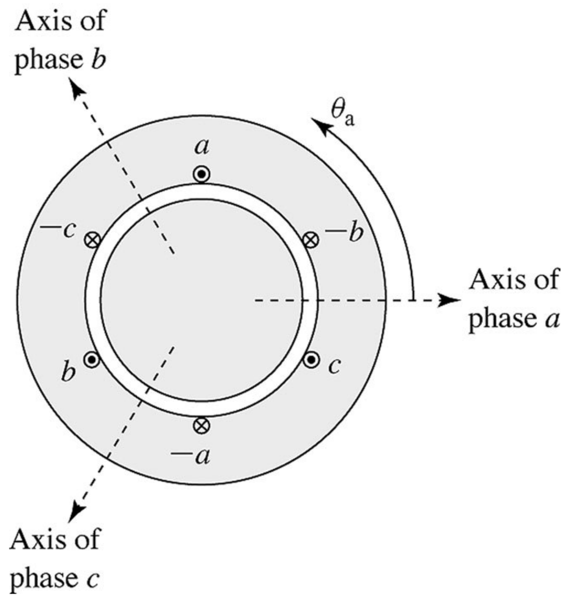


Figure 4.29 Simplified two-pole three-phase stator winding.

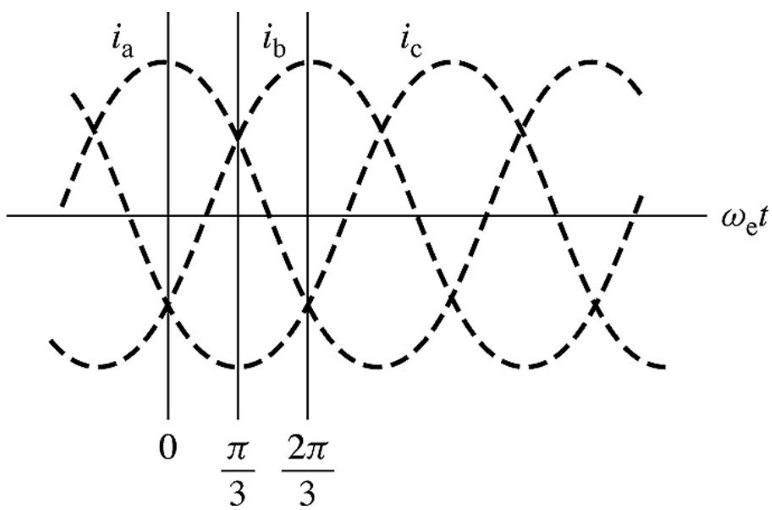


Figure 4.30 Instantaneous phase currents under balanced three-phase conditions.

→ The mmf of phase a has been shown to be

$$F_{a1} = F_{a1}^{+} + F_{a1}^{-} \tag{4.26}$$

$$F_{a1}^{+} = \frac{1}{2} F_{max} \cos\left(\theta_{ae} - \omega_e t\right) \tag{4.27}$$

$$F_{a1}^{-} = \frac{1}{2} F_{max} \cos\left(\theta_{ae} + \omega_e t\right) \tag{4.28}$$

$$F_{max} = \frac{4}{\pi}\left(\frac{k_w N_{ph}}{\text{poles}}\right) I_m \tag{4.29}$$

→ Similarly, for phases b and c

$$F_{b1} = F_{b1}^{+} + F_{b1}^{-} \tag{4.30}$$

$$F_{b1}^{+} = \frac{1}{2} F_{max} \cos\left(\theta_{ae} - \omega_e t\right) \tag{4.31}$$

$$F_{b1}^{-} = \frac{1}{2} F_{max} \cos\left(\theta_{ae} + \omega_e t + 120^o\right) \tag{4.32}$$

$$F_{c1} = F_{c1}^{+} + F_{c1}^{-} \tag{4.33}$$

$$F_{c1}^{+} = \frac{1}{2} F_{max} \cos\left(\theta_{ae} - \omega_e t\right) \tag{4.34}$$

$$F_{c1}^{-} = \frac{1}{2} F_{max} \cos\left(\theta_{ae} + \omega_e t - 120^o\right) \tag{4.35}$$

→ The total mmf is the sum

$$F\left(\theta_{ae}, t\right) = F_{a1} + F_{b1} + F_{c1} \tag{4.36}$$

It can be performed in terms of the positive- and negative- traveling waves.

$$\begin{aligned} F^{-}\left(\theta_{ae}, t\right) &= F_{a1}^{-} + F_{b1}^{-} + F_{c1}^{-} \\ &= \frac{1}{2} F_{max} \left[\cos\left(\theta_{ae} + \omega_e t\right) + \cos\left(\theta_{ae} + \omega_e t - 120^o\right) + \cos\left(\theta_{ae} + \omega_e t + 120^o\right)\right] \\ &= 0 \end{aligned} \tag{4.37}$$

$$\begin{aligned} F^{+}\left(\theta_{ae}, t\right) &= F_{a1}^{+} + F_{b1}^{+} + F_{c1}^{+} \\ &= \frac{3}{2} F_{max} \cos(\theta_{ae} - \omega_e t) \end{aligned} \tag{4.38}$$

→ The result of displacing the three windings by 120^o in space phase and displacing the winding currents by 120^o in time phase is a single positive-traveling mmf wave

$$\begin{aligned} F\left(\theta_{ae}, t\right) &= \frac{3}{2} F_{max} \cos\left(\theta_{ae} - \omega_e t\right) \\ &= \frac{3}{2} F_{max} \cos\left(\left(\frac{\text{poles}}{2}\right)\theta_a - \omega_e t\right) \end{aligned} \tag{4.39}$$

→ Under balanced three-phase conditions, the three-phase winding produces an air-gap mmf wave which rotates at synchronous angular velocity ω_s (rad/sec)

$$\omega_s = \left(\frac{2}{\text{poles}}\right) \omega_e \tag{4.40}$$

ω_e : angular velocity of the applied electrical excitation (rad/sec)

→ n_s : synchronous speed

$f_e = \omega_e / (2\pi)$: applied electrical frequency

$$n_s = \left(\frac{120}{\text{poles}}\right) f_e \quad \text{r/min} \tag{4.41}$$

- A polyphase winding exicted by balanced polyphase currents produces a rotating mmf wave.
  - → It is the interaction of this magnetic flux wave with that of the rotor which produces torque.
  - → Constant torque is produced when rotor-produced magnetic flux rotates in synchronism with that of the stator.

§4.5.3 Graphical Analysis of Polyphase MMF

- For balanced three-phase currents, the production of a rotating mmf can also be shown graphically.

- Refer to Fig. 4.30 and Fig. 4.31.
  - → As time passes, the resultant mmf wave retains its sinusoidal form and amplitude but rotates progressively around the air gap.
  - → The net result is an mmf wave of constant amplitude rotating at uniform angular velocity.

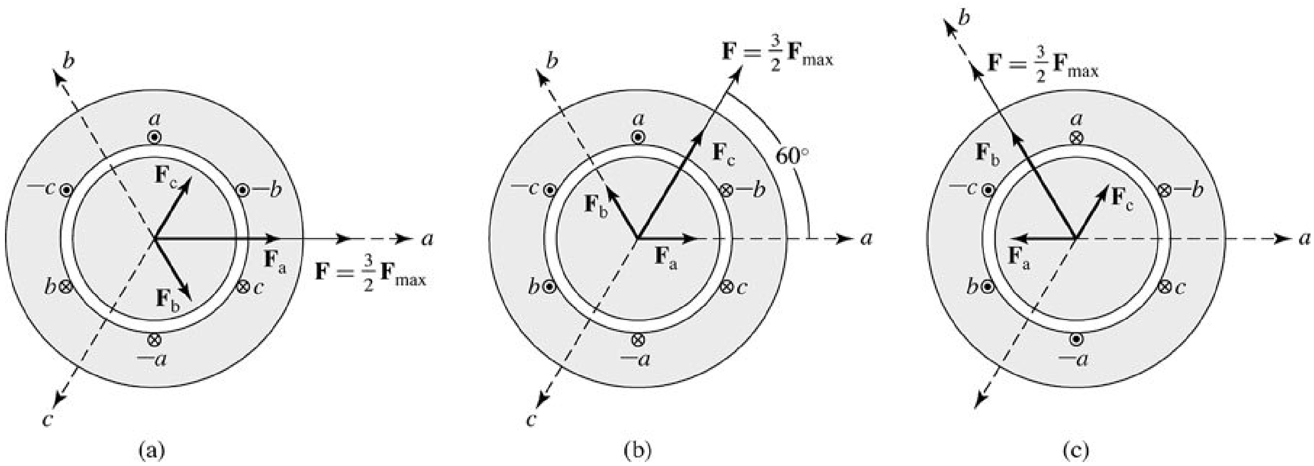


Figure 4.31 The production of a rotating magnetic field by means of three-phase currents.

EXAMPLE 4.3

Consider a three-phase stator excited with balanced, 60-Hz currents. Find the synchronous angular velocity in rad/sec and speed in r/min for stators with two, four, and six poles.

■ Solution

For a frequency of $f_e = 60$ Hz, the electrical angular frequency is equal to

$$\omega_e = 2\pi f_e = 120\pi \approx 377 \text{ rad/sec}$$

Using Eqs. 4.40 and 4.41, the following table can be constructed:

Poles	n_s (r/min)	ω_s (rad/sec)
2	3600	$120\pi \approx 377$
4	1800	60π
6	1200	40π

- Practice Problem 4.3
  Repeat Example 4.3 for a three-phase stator excited by balanced 50-Hz currents.

§4.6 Generated Voltage

§4.7 Torque in Nonsalient-Pole Machines

§4.6 Generated Voltage

§4.6.1 AC Machines

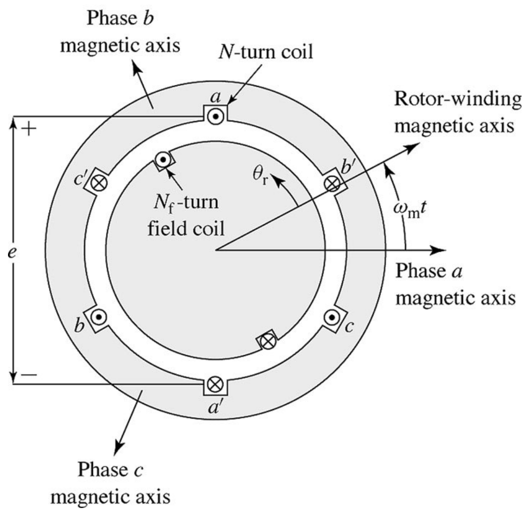


Figure 4.32 Cross-sectional view of an elementary three-phase ac machine.

$$B_{\text{peak}} = \frac{4\mu_0}{\pi g}\left(\frac{k_f N_f}{\text{poles}}\right) I_f \tag{4.42}$$

$$B = B_{\text{peak}} \cos\left(\frac{\text{poles}}{2}\theta_r\right) \tag{4.43}$$

$$\begin{aligned} \Phi &= l\int_{-\pi/\text{poles}}^{+\pi/\text{poles}} B_{\text{peak}} \cos\left(\frac{\text{poles}}{2}\theta_r\right) r d\theta_r \\ &= \left(\frac{2}{\text{poles}}\right) 2B_{\text{peak}} lr \end{aligned} \tag{4.44}$$

$$\begin{aligned} \lambda_a &= k_w N_{ph} \Phi_p \cos\left(\left(\frac{\text{poles}}{2}\right)\omega_m t\right) \\ &= k_w N_{ph} \Phi_p \cos \omega_{me} t \end{aligned} \tag{4.45}$$

$$\omega_{me} = \left(\frac{\text{poles}}{2}\right)\omega_m \tag{4.46}$$

$$e_a = \frac{d\lambda_a}{dt} = k_w N_{ph} \frac{d\Phi_p}{dt} \cos \omega_{me} t - \omega_{me} k_w N_{ph} \Phi_p \sin \omega_{me} t \tag{4.47}$$

$$e_a = -\omega_{me} k_w N_{ph} \Phi_p \sin \omega_{me} t \tag{4.48}$$

EXAMPLE 4.4

The so-called *cutting-of-flux* equation states that the voltage v induced in a wire of length l (in the frame of the wire) moving with respect to a constant magnetic field with flux density of magnitude B is given by

$$v = l\mathrm{v}_\perp B$$

where $\mathrm{v}_\perp$ is the component of the wire velocity perpendicular to the direction of the magnetic flux density.

Consider the two-pole elementary three-phase machine of Fig. 4.32. Assume the rotor-produced air-gap flux density to be of the form

$$B_{\mathrm{ag}}(\theta_\mathrm{r}) = B_{\mathrm{peak}} \sin \theta_\mathrm{r}$$

and the rotor to rotate at constant angular velocity ω_e. (Note that since this is a two-pole machine, $\omega_\mathrm{m} = \omega_\mathrm{e}$). Show that if one assumes that the armature-winding coil sides are in the air gap and not in the slots, the voltage induced in a full-pitch, N-turn concentrated armature phase coil can be calculated from the cutting-of-flux equation and that it is identical to that calculated using Eq. 4.48. Let the average air-gap radius be r and the air-gap length be g ($g \ll r$).

■ Solution

We begin by noting that the cutting-of-flux equation requires that the conductor be moving and the magnetic field to be nontime varying. Thus in order to apply it to calculating the stator magnetic field, we must translate our reference frame to the rotor.

In the rotor frame, the magnetic field is constant and the stator coil sides, when moved to the center of the air gap at radius r, appear to be moving with velocity $\omega_{\mathrm{me}} r$ which is perpendicular to the radially-directed air-gap flux. If the rotor and phase-coil magnetic axes are assumed to be aligned at time $t = 0$, the location of a coil side as a function of time will be given by $\theta_\mathrm{r} = -\omega_{\mathrm{me}} t$. The voltage induced in one side of one turn can therefore be calculated as

$$e_1 = l\mathrm{v}_\perp B_{\mathrm{ag}}(\theta_\mathrm{r}) = l\omega_{\mathrm{me}} r B_{\mathrm{peak}} \sin(-\omega_{\mathrm{me}} t)$$

There are N turns per coil and two sides per turn. Thus the total coil voltage is given by

$$e = 2Ne_1 = -2Nl\omega_{\mathrm{me}} r B_{\mathrm{peak}} \sin \omega_{\mathrm{me}} t$$

From Eq. 4.48, the voltage induced in the full-pitched, 2-pole stator coil is given by

$$e = -\omega_{\mathrm{me}} N \Phi_\mathrm{p} \sin \omega_{\mathrm{me}} t$$

Substituting $\Phi_\mathrm{p} = 2B_{\mathrm{peak}} l r$ from Eq. 4.44 gives

$$e = -\omega_{\mathrm{me}} N (2B_{\mathrm{peak}} l r) \sin \omega_{\mathrm{me}} t$$

which is identical to the voltage determined using the cutting-of-flux equation.

$$E_{\max} = \omega_{me} k_w N_{ph} \Phi_p = 2\pi f_{me} k_w N_{ph} \Phi_p \tag{4.49}$$

$$E_{rms} = \frac{2\pi}{\sqrt{2}} f_{me} k_w N_{ph} \Phi_p = \sqrt{2} f_{me} k_w N_{ph} \Phi_p \tag{4.50}$$

EXAMPLE 4.5

A two-pole, three-phase, Y-connected 60-Hz round-rotor synchronous generator has a field winding with N_f distributed turns and winding factor k_f. The armature winding has N_a turns per phase and winding factor k_a. The air-gap length is g, and the mean air-gap radius is r. The armature-winding active length is l. The dimensions and winding data are

$N_f = 68$ series turns $\quad k_f = 0.945$

$N_a = 18$ series turns/phase $\quad k_a = 0.933$

$r = 0.53$ m $\quad g = 4.5$ cm

$l = 3.8$ m

The rotor is driven by a steam turbine at a speed of 3600 r/min. For a field current of $I_f = 720$ A dc, compute (*a*) the peak fundamental mmf $(F_{ag1})_{peak}$ produced by the field winding, (*b*) the peak fundamental flux density $(B_{ag1})_{peak}$ in the air gap, (*c*) the fundamental flux per pole Φ_p, and (*d*) the rms value of the open-circuit voltage generated in the armature.

Solution

a. From Eq. 4.8

$$(F_{ag1})_{peak} = \frac{4}{\pi}\left(\frac{k_f N_f}{\text{poles}}\right) I_f = \frac{4}{\pi}\left(\frac{0.945 \times 68}{2}\right) 720$$

$$= \frac{4}{\pi}(32.1)720 = 2.94 \times 10^4 \text{ A} \cdot \text{turns/pole}$$

b. Using Eq. 4.12, we get

$$(B_{ag1})_{peak} = \frac{\mu_0 (F_{ag1})_{peak}}{g} = \frac{4\pi \times 10^{-7} \times 2.94 \times 10^4}{4.5 \times 10^{-2}} = 0.821 \text{ T}$$

Because of the effect of the slots containing the armature winding, most of the air-gap flux is confined to the stator teeth. The flux density in the teeth at a pole center is higher than the value calculated in part (*b*), probably by a factor of about 2. In a detailed design this flux density must be calculated to determine whether the teeth are excessively saturated.

c. From Eq. 4.44

$$\Phi_p = 2(B_{ag1})_{peak} l r = 2(0.821)(3.8)(0.53) = 3.31 \text{ Wb}$$

d. From Eq. 4.50 with $f_{me} = 60$ Hz

$$E_{rms,line-neutral} = \sqrt{2}\,\pi f_{me} k_a N_a \Phi_p = \sqrt{2}\,\pi(60)(0.933)(18)(3.31)$$

$$= 14.8 \text{ kV rms}$$

The line-line voltage is thus

$$E_{rms,line-line} = \sqrt{3}\,(14.8 \text{ kV}) = 25.7 \text{ kV rms}$$

$$E_a = \frac{1}{\pi}\int_0^{\pi} \omega_{me} N \Phi_p \sin(\omega_{me} t) d(\omega_{me} t) = \frac{2}{\pi}\omega_{me} N \Phi_p \tag{4.51}$$

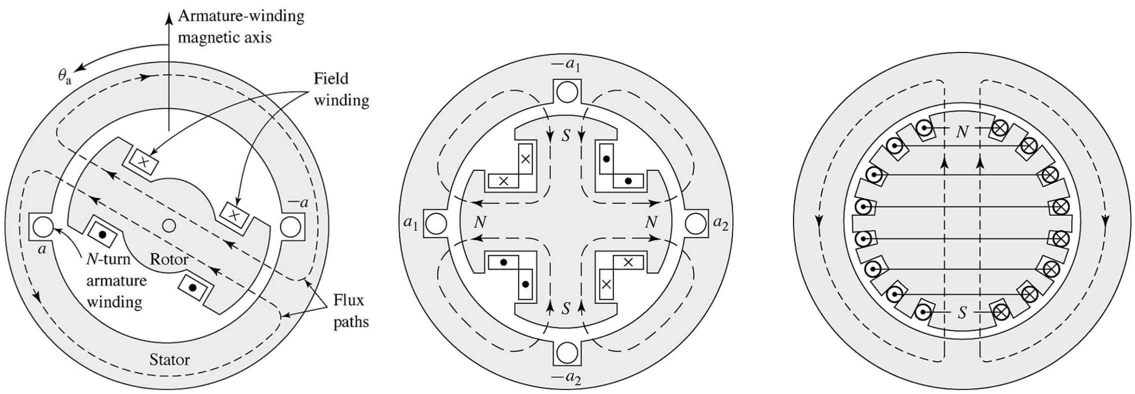
$$E_a = \left(\frac{\text{poles}}{\pi}\right) N \Phi_p \omega_m = \text{poles}\, N \Phi_p \left(\frac{n}{30}\right) \tag{4.52}$$

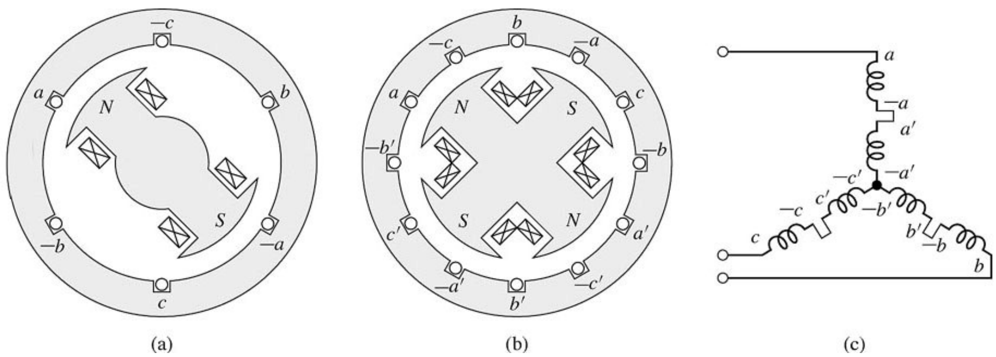
$$E_a = \left(\frac{\text{poles}}{2\pi}\right)\left(\frac{C_a}{m}\right)\Phi_p \omega_m = \left(\frac{\text{poles}}{60}\right)\left(\frac{C_a}{m}\right)\Phi_p n \tag{4.53}$$

§4.7 Torque in Nonsalient-pole Machines

Chapter 5 Synchronous Machines

- Main features of synchronous machines:
  - A synchronous machine is an ac machine whose speed under steady-state conditions is proportional to the frequency of the current in its armature.
  - The rotor, along with the magnetic field created by the dc field current on the rotor, rotates at the same speed as, or in synchronism with, the rotating magnetic field produced by the armature currents, and a steady torque results.





Errore.
Figure 4.12 Schematic views of three-phase generators: (a) two-pole, (b) four-pole, and (c) *Y* connection of the windings.

§5.1 Introduction to Polyphase Synchronous Machines

- Synchronous machines:
  - Armature winding: on the stator, alternating current.
  - Field winding: on the rotor, dc power supplied by the excitation system.
    - → Cylindrical rotor: for two- and four-pole turbine generators.
    - → Salient-pole rotor: for multipolar, slow-speed, hydroelectric generators and for most synchronous motors.
  - Acting as a voltage source:
    - → Frequency determined by the speed of its mechanical drive (or prime mover).
    - → The amplitude of the generated voltage is proportional to the frequency and the field current.

$$\begin{aligned}\lambda_a &= k_w N_{ph} \Phi_p \cos\left(\left(\frac{\text{poles}}{2}\right)\omega_m t\right) \\ &= k_w N_{ph} \Phi_p \cos \omega_{me} t\end{aligned} \tag{4.45}$$

$$\omega_{\text{me}} = \left(\frac{\text{poles}}{2}\right)\omega_{\text{m}} \tag{4.46}$$

$$e_a = \frac{d\lambda_a}{dt} = k_w N_{ph} \frac{d\Phi_p}{dt} \cos\omega_{me} t - \omega_{me} k_w N_{ph} \Phi_p \sin\omega_{me} t \tag{4.47}$$

$$e_a = -\omega_{me} k_w N_{ph} \Phi_p \sin\omega_{me} t \tag{4.48}$$

$$E_{\max} = \omega_{\text{me}} k_{\text{w}} N_{\text{ph}} \Phi_{\text{p}} = 2\pi f_{\text{me}} k_{\text{w}} N_{\text{ph}} \Phi_{\text{p}} \tag{4.49}$$

$$E_{\text{rms}} = \frac{2\pi}{\sqrt{2}} f_{\text{me}} k_{\text{w}} N_{\text{ph}} \Phi_{\text{p}} = \sqrt{2}\pi f_{\text{me}} k_{\text{w}} N_{\text{ph}} \Phi_{\text{p}} \tag{4.50}$$

- Synchronous generators can be readily operated in parallel: interconnected power systems.
- When a synchronous generator is connected to a large interconnected system containing many other synchronous generators, the voltage and frequency at its armature terminals are substantially fixed by the system.
  - → It is often useful, when studying the behavior of an individual generator or group of generators, to represent the remainder of the system as a constant-frequency, constant-voltage source, commonly referred to as an infinite bus.
  - → Analysis of a synchronous machine connected to an infinite bus.
- Torque equation:

$$T = -\frac{\pi}{2}\left(\frac{\text{poles}}{2}\right)^2 \Phi_{\text{R}} F_{\text{f}} \sin\delta_{\text{RF}} \tag{5.1}$$

where

Φ_{R} = resultant air-gap flux per pole

F_{f} = mmf of the dc field winding

δ_{RF} = electric phase angle between magnetic axes of Φ_{R} and F_{f}

- → The minus sign indicates that the electromechanical torque acts in the direction to bring the interacting fields into alignment.
- → In a generator, the prime-mover torque acts in the direction of rotation of the rotor, and the electromechanical torque opposes rotation. The rotor mmf wave leads the resultant air-gap flux.
- → In a motor, the electromechanical torque is in the direction of rotation, in opposition to the retarding torque of the mechanical load on the shaft.
- → Torque-angle curve: Fig. 5.1.

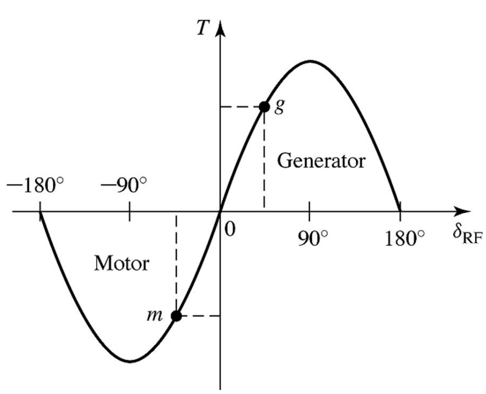


Figure 5.1 Torque-angle characteristics.

→ An increase in prime-mover torque will result in a corresponding increase in the torque angle.

→ $T = T_{\max}$: pull-out torque at $\delta_{\mathrm{RF}} = 90^{\circ}$. Any further increase in prime-mover torque cannot be balanced by a corresponding increase in synchronous electromechanical torque, with the result that synchronism will no longer be maintained and the rotor will speed up. $\Rightarrow$ loss of synchronism, pulling out of step.

§5.2 Synchronous-Machine Inductances; Equivalent Circuits

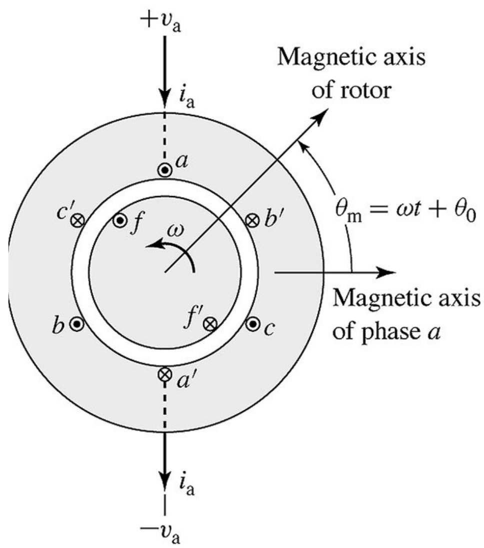


Figure 5.2 Schematic diagram of a two-pole, three-phase cylindrical-rotor synchronous machine.

§5.2.1 Rotor Self-Inductance
§5.2.2 Stator-to-Rotor Mutual Inductances
§5.2.3 Stator Inductances; Synchronous Inductance
§5.2.4 Equivalent Circuit

- Equivalent circuit for the synchronous machine:
  - Single-phase, line-to-neutral equivalent circuits for a three-phase machine operating under balanced, three-phase conditions.

L_s = effective inductance seen by phase a under steady-state, balanced three-phase machine operating conditions.

$X_s = \omega_e L_s$: synchronous reactance

R_a = armature winding resistance

e_{af} = voltage induced by the field winding flux (generated voltage, internal voltage)

I_a = armature current

v_a = terminal voltage

Motor reference direction:

$$\hat{V}_a = R_a \hat{I}_a + jX_s \hat{I}_a + \hat{E}_{af} \tag{5.23}$$

Generator reference direction:

$$\hat{V}_a = -R_a \hat{I}_a - jX_s \hat{I}_a + \hat{E}_{af} \tag{5.24}$$

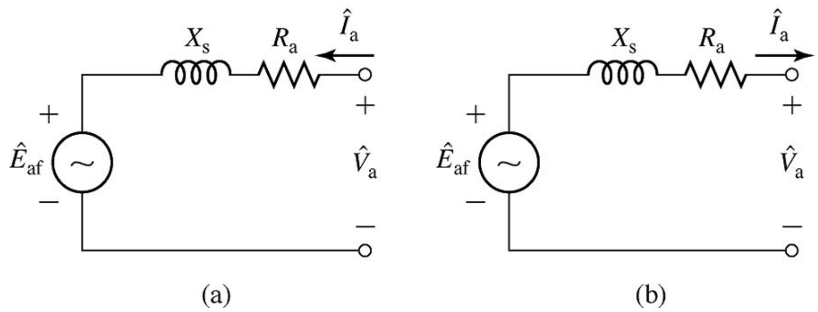


Figure 5.3 Synchronous-machine equivalent circuits:
(a) motor reference direction and (b) generator reference direction.

$$X_s = X_{al} + X_{\varphi} \tag{5.25}$$

X_{al} = armature leakage reactance

X_{φ} = magnetizing reactance of the armature winding

$\hat{E}_R$ = air-gap voltage or the voltage behind leakage reactance

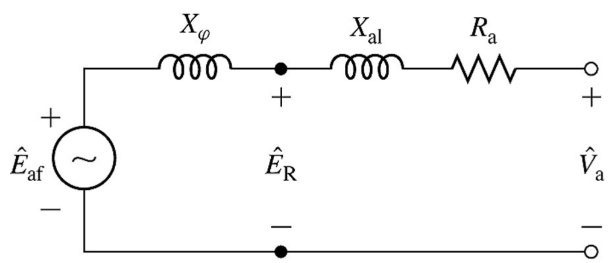


Figure 5.4 Synchronous-machine equivalent circuit showing air-gap and leakage components of synchronous reactance and air-gap voltage.

EXAMPLE 5.1

A 60-Hz, three-phase synchronous motor is observed to have a terminal voltage of 460 V (line-line) and a terminal current of 120 A at a power factor of 0.95 lagging. The field-current under this operating condition is 47 A. The machine synchronous reactance is equal to 1.68 Ω (0.794 per unit on a 460-V, 100-kVA, 3-phase base). Assume the armature resistance to be negligible.

Calculate (*a*) the generated voltage E_{af} in volts, (*b*) the magnitude of the field-to-armature mutual inductance L_{af}, and (*c*) the electrical power input to the motor in kW and in horsepower.

■ Solution

a. Using the motor reference direction for the current and neglecting the armature resistance, the generated voltage can be found from the equivalent circuit of Fig. 5.3a or Eq. 5.23 as

$$\hat{E}_{af} = \hat{V}_a - jX_s\hat{I}_a$$

We will choose the terminal voltage as our phase reference. Because this is a line-to-neutral equivalent, the terminal voltage V_a must be expressed as a line-to-neutral voltage

$$\hat{V}_a = \frac{460}{\sqrt{3}} = 265.6 \text{ V, line-to-neutral}$$

A lagging power factor of 0.95 corresponds to a power factor angle $\theta = -\cos^{-1}(0.95) = -18.2°$. Thus, the phase-*a* current is

$$\hat{I}_a = 120\, e^{-j18.2°} \text{ A}$$

Thus

$$\hat{E}_{af} = 265.6 - j1.68(120\, e^{-j18.2^\circ})$$
$$= 278.8\, e^{-j43.4^\circ} \text{ V, line-to-neutral}$$

and hence, the generated voltage E_{af} is equal to 278.8 V rms, line-to-neutral.

b. The field-to-armature mutual inductance can be found from Eq. 5.21. With $\omega_e = 120\pi$,

$$L_{af} = \frac{\sqrt{2}\, E_{af}}{\omega_e I_f} = \frac{\sqrt{2} \times 279}{120\pi \times 47} = 22.3 \text{ mH}$$

c. The three-phase power input to the motor P_{in} can be found as three times the power input to phase a. Hence,

$$P_{in} = 3V_a I_a(\text{power factor}) = 3 \times 265.6 \times 120 \times 0.95$$
$$= 90.8 \text{ kW} = 122 \text{ hp}$$

EXAMPLE 5.2

Assuming the input power and terminal voltage for the motor of Example 5.1 remain constant, calculate (a) the phase angle δ of the generated voltage and (b) the field current required to achieve unity power factor at the motor terminals.

■ Solution

a. For unity power factor at the motor terminals, the phase-a terminal current will be in phase with the phase-a line-to-neutral voltage $\hat{V}_a$. Thus

$$\hat{I}_a = \frac{P_{in}}{3V_a} = \frac{90.6 \text{ kW}}{3 \times 265.6 \text{ V}} = 114 \text{ A}$$

From Eq. 5.23,

$$\hat{E}_{af} = \hat{V}_a - jX_s\hat{I}_a$$
$$= 265.6 - j1.68 \times 114 = 328\, e^{-j35.8^\circ} \text{ V, line-to-neutral}$$

Thus, $E_{af} = 328$ V line-to-neutral and $\delta = -35.8^\circ$.

b. Having found L_{af} in Example 5.1, we can find the required field current from Eq. 5.21.

$$I_f = \frac{\sqrt{2}\, E_{af}}{\omega_e L_{af}} = \frac{\sqrt{2} \times 328}{377 \times 0.0223} = 55.2 \text{ A}$$

---

§5.4 Steady-State Power-Angle Characteristics

- The maximum power a synchronous machine can deliver is determined by the maximum torque that can be applied without loss of synchronism with the external system to which it is connected.
  - Both the external system and the machine itself can be represented as an impedance in series with a voltage source.

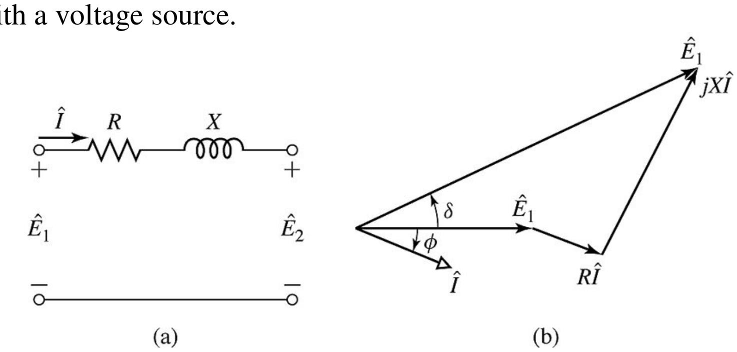


Figure 5.11 (a) Impedance interconnecting two voltages; (b) phasor diagram.

$$P_2 = E_2 I \cos\phi \tag{5.34}$$

$$\hat{I} = \frac{\hat{E}_1 - \hat{E}_2}{Z} \tag{5.35}$$

$$\hat{E}_1 = E_1 e^{j\delta} \tag{5.36}$$

$$\hat{E}_2 = E_2 \tag{5.37}$$

$$Z = R + jX = |Z| e^{j\phi_Z} \tag{5.38}$$

$$\hat{I} = Ie^{j\phi} = \frac{E_1 e^{j\delta} - E_2}{|Z| e^{j\phi_Z}} = \frac{E_1}{|Z|} e^{j(\delta - \phi_Z)} - \frac{E_2}{|Z|} e^{-j\phi_Z} \tag{5.39}$$

$$I \cos\phi = \frac{E_1}{|Z|} \cos(\delta - \phi_Z) - \frac{E_2}{|Z|} \cos(-\phi_Z) \tag{5.40}$$

$$P_2 = \frac{E_1 E_2}{|Z|} \cos(\delta - \phi_Z) - \frac{E_2^2 R}{|Z|^2} \tag{5.41}$$

$$P_2 = \frac{E_1 E_2}{|Z|} \sin(\delta + \alpha_Z) - \frac{E_2^2 R}{|Z|^2} \tag{5.42}$$

where

$$\alpha_Z = 90^\circ - \phi_Z = \tan^{-1}\left(\frac{R}{X}\right) \tag{5.43}$$

$$P_1 = \frac{E_1 E_2}{|Z|} \sin(\delta - \phi_Z) - \frac{E_1^2 R}{|Z|^2} \tag{5.44}$$

Frequently, $R << |Z|, |Z| \approx X$ and $\alpha_Z \approx 0$,

$$\boxed{P_1 = P_2 = \frac{E_1 E_2}{X} \sin\delta} \tag{5.45}$$

- Equation (5.45) is commonly referred to as the power-angle characteristic for a synchronous machine.
  - → The angle δ is known as the power angle.
  - → Note that E_1 and E_2 are the line-to-neutral voltages.
  - → For three-phase systems, a factor "3" shall be placed in front of the equation.
  - → The maximum power transfer is

$$P_{1,\max} = P_{2,\max} = \frac{E_1 E_2}{X} \tag{5.46}$$

    occurring when $\delta = \pm 90^\circ$.
  - → If $\delta > 0$, $\hat{E}_1$ leads $\hat{E}_2$ and power flows from source $\hat{E}_1$ to $\hat{E}_2$.
  - → When $\delta < 0$, $\hat{E}_1$ lags $\hat{E}_2$ and power flows from source $\hat{E}_2$ to $\hat{E}_1$.
  - → Consider Fig. 5.12 in which a synchronous machine with generated voltage $\hat{E}_{\mathrm{af}}$ and synchronous X_{s} is connected to a system whose Thevenin equivalent is a voltage source $\hat{V}_{\mathrm{EQ}}$ in series with a reactive impedance jX_{EQ}. The power-angle characteristic can be written

$$P = \frac{E_{\mathrm{af}} V_{\mathrm{EQ}}}{X_{\mathrm{s}} + X_{\mathrm{EQ}}} \sin\delta \tag{5.47}$$

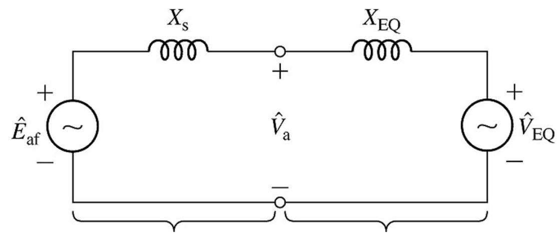


Figure 5.12 Equivalent-circuit representation of a synchronous machine connected to an external system.

→ Note that $P \propto E_1 E_2$, $P \propto X^{-1}$, $P_{\max} \propto E_1 E_2$, and $P_{\max} \propto X^{-1}$.

→ In general, stability considerations dictate that a synchronous machine achieve steady-state operation for a power angle considerably less than 90°.

EXAMPLE 5.6

A three-phase, 75-MVA, 13.8-kV synchronous generator with saturated synchronous reactance $X_s = 1.35$ per unit and unsaturated synchronous reactance $X_{s,u} = 1.56$ per unit is connected to an external system with equivalent reactance $X_{EQ} = 0.23$ per unit and voltage $V_{EQ} = 1.0$ per unit, both on the generator base. It achieves rated open-circuit voltage at a field current of 297 amperes.

a. Find the maximum power $P_{\max}$ (in MW and per unit) that can be supplied to the external system if the internal voltage of the generator is held equal to 1.0 per unit.
b. Using MATLAB,† plot the terminal voltage of the generator as the generator output is varied from zero to $P_{\max}$ under the conditions of part (a).
c. Now assume that the generator is equipped with an *automatic voltage regulator* which controls the field current to maintain constant terminal voltage. If the generator is loaded to its rated value, calculate the corresponding power angle, per-unit internal voltage, and field current. Using MATLAB, plot per-unit E_{af} as a function of per-unit power.

■ Solution

a. From Eq. 5.47

$$P_{\max} = \frac{E_{af} V_{EQ}}{X_s + X_{EQ}}$$

Note that although this is a three-phase generator, no factor of 3 is required because we are working in per unit.

Because the machine is operating with a terminal voltage near its rated value, we should express $P_{\max}$ in terms of the saturated synchronous reactance. Thus

$$P_{\max} = \frac{1}{1.35 + 0.23} = 0.633 \text{ per unit} = 47.5 \text{ MW}$$

b. From Fig 5.12, the generator terminal current is given by

$$\hat{I}_a = \frac{\hat{E}_{af} - \hat{V}_{EQ}}{j(X_s + X_{EQ})} = \frac{E_{af}\, e^{j\delta} - V_{EQ}}{j(X_s + X_{EQ})} = \frac{e^{j\delta} - 1.0}{j1.58}$$

The generator terminal voltage is then given by

$$\hat{V}_a = \hat{V}_{EQ} + jX_{EQ}\hat{I}_a = 1.0 + \frac{.23}{1.58}(e^{j\delta} - 1.0)$$

Figure 5.13a is the desired MATLAB plot. The terminal voltage can be seen to vary from 1.0 at $\delta = 0°$ to approximately 0.87 at $\delta = 90°$.

c. With the terminal voltage held constant at $V_a = 1.0$ per unit, the power can be expressed as

$$P = \frac{V_a V_{BQ}}{X_{BQ}} \sin \delta_t = \frac{1}{0.23} \sin \delta_t = 4.35 \sin \delta_t$$

where δ_t is the angle of the terminal voltage with respect to $\hat{V}_{BQ}$.

For $P = 1.0$ per unit, $\delta_t = 13.3°$ and hence $\hat{I}$ is equal to

$$\hat{I}_a = \frac{V_a e^{j\delta_t} - V_{BQ}}{jX_{BQ}} = 1.007\, e^{j6.65°}$$

and

$$\hat{E}_{af} = \hat{V}_{BQ} + j(X_{BQ} + X_s)\hat{I}_a = 1.78\, e^{j62.7°}$$

or $E_{af} = 1.78$ per unit, corresponding to a field current of $I_f = 1.78 \times 297 =$ 529 amperes. The corresponding power angle is 62.7°.

Figure 5.13b is the desired MATLAB plot. E_{af} can be seen to vary from 1.0 at $P = 0$ to 1.78 at $P = 1.0$.

EXAMPLE 5.7

A 2000-hp, 2300-V, unity-power-factor, three-phase, Y-connected, 30-pole, 60-Hz synchronous motor has a synchronous reactance of 1.95 Ω/phase. For this problem all losses may be neglected.

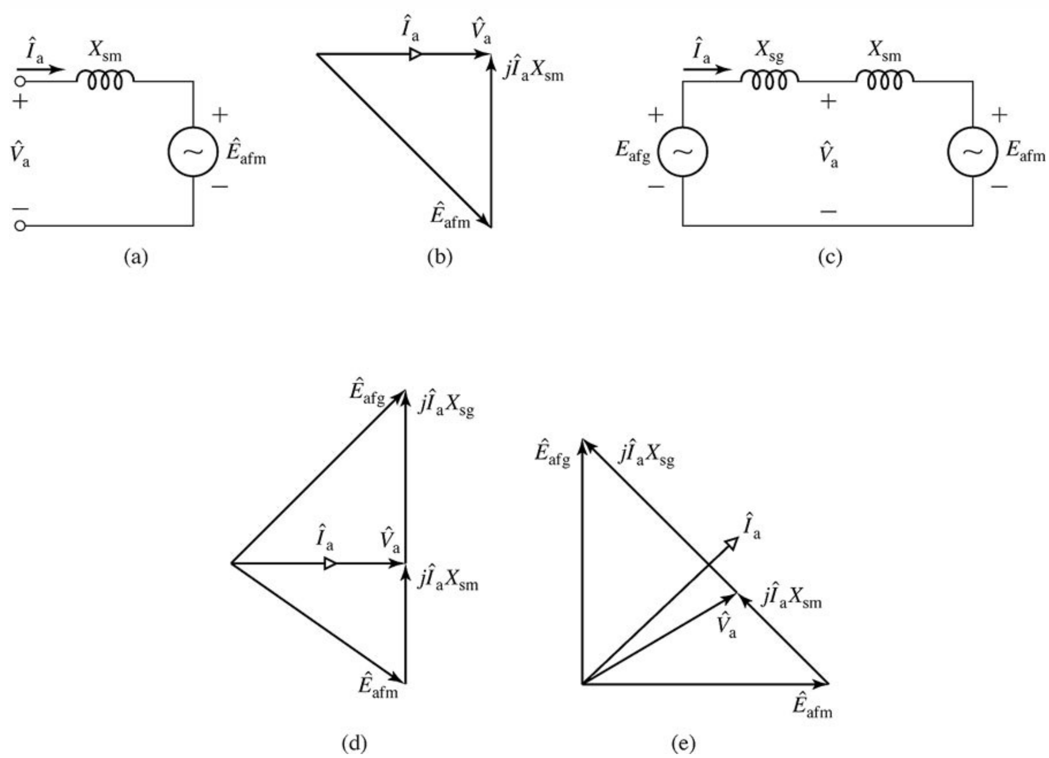


Figure 5.14 Equivalent circuits and phasor diagrams for Example 5.7.

a. Compute the maximum power and torque which this motor can deliver if it is supplied with power directly from a 60-Hz, 2300-V infinite bus. Assume its field excitation is maintained constant at the value which would result in unity power factor at rated load.

b. Instead of the infinite bus of part (a), suppose that the motor is supplied with power from a three-phase, Y-connected, 2300-V, 1500-kVA, two-pole, 3600 r/min turbine generator whose synchronous reactance is 2.65 Ω/phase. The generator is driven at rated speed, and the field excitations of generator and motor are adjusted so that the motor runs at unity power factor and rated terminal voltage at full load. Calculate the maximum power and torque which could be supplied corresponding to these values of field excitation.

■ Solution

Although this machine is undoubtedly of the salient-pole type, we will solve the problem by simple cylindrical-rotor theory. The solution accordingly neglects reluctance torque. The machine actually would develop a maximum torque somewhat greater than our computed value, as discussed in Section 5.7.

a. The equivalent circuit is shown in Fig. 5.14a and the phasor diagram at full load in Fig. 5.14b, where $\hat{E}_{afm}$ is the generated voltage of the motor and X_{sm} is its synchronous reactance. From the motor rating with losses neglected,

$$\text{Rated kVA} = 2000 \times 0.746 = 1492 \text{ kVA, three-phase}$$

$$= 497 \text{ kVA/phase}$$

$$\text{Rated voltage} = \frac{2300}{\sqrt{3}} = 1328 \text{ V line-to-neutral}$$

$$\text{Rated current} = \frac{497{,}000}{1328} = 374 \text{ A/phase-Y}$$

From the phasor diagram at full load

$$E_{afm} = \sqrt{V_a^2 + (I_a X_{sm})^2} = 1515 \text{ V}$$

When the power source is an infinite bus and the field excitation is constant, V_a and E_{afm} are constant. Substitution of V_a for E_1, E_{afm} for E_2, and X_{sm} for X in Eq. 5.46 then gives

$$P_{max} = \frac{V_a E_{afm}}{X_{sm}} = \frac{1328 \times 1515}{1.95} = 1032 \text{ kW/phase}$$

$$= 3096 \text{ kW, three-phase}$$

In per unit, $P_{max} = 3096/1492 = 2.07$ per unit. Because this power exceeds the motor rating, the motor cannot deliver this power for any extended period of time.

With 30 poles at 60 Hz, the synchronous angular velocity is found from Eq. 4.40

$$\omega_s = \left(\frac{2}{\text{poles}}\right)\omega_e = \left(\frac{2}{30}\right)(2\pi 60) = 8\pi \text{ rad/sec}$$

and hence

$$T_{max} = \frac{P_{max}}{\omega_s} = \frac{3096 \times 10^3}{8\pi} = 123.2 \text{ kN} \cdot \text{m}$$

b. When the power source is the turbine generator, the equivalent circuit becomes that shown in Fig. 5.14c, where $\hat{E}_{afg}$ is the generated voltage of the generator and X_{sg} is its synchronous reactance. Here the synchronous generator is equivalent to an external voltage $\hat{V}_{EQ}$ and reactance X_{EQ} as in Fig. 5.12. The phasor diagram at full motor load, unity power factor, is shown in Fig. 5.14d. As before, $V_a = 1330$ V/phase at full load and $E_{afm} = 1515$ V/phase.

From the phasor diagram

$$E_{afg} = \sqrt{V_{ta}^2 + (I_a X_{sg})^2} = 1657 \text{ V}$$

Since the field excitations and speeds of both machines are constant, E_{afg} and E_{afm} are constant. Substitution of E_{afg} for E_1, E_{afm} for E_2, and $X_{sg} + X_{sm}$ for X in Eq. 5.46 then gives

$$P_{max} = \frac{E_{afg} E_{afm}}{X_{sg} + X_{sm}} = \frac{1657 \times 1515}{4.60} = 546 \text{ kW/phase}$$

$$= 1638 \text{ kW, three-phase}$$

In per unit, $P_{max} = 1638/1492 = 1.10$ per unit.

$$T_{max} = \frac{P_{max}}{\omega_s} = \frac{1635 \times 10^3}{8\pi} = 65.2 \text{ kN} \cdot \text{m}$$

Synchronism would be lost if a load torque greater than this value were applied to the motor shaft. Of course, as in part (a), this loading exceeds the rating of the motor and could not be sustained under steady-state operating conditions.

§5.3 Open- and Short-Circuit Characteristics

§5.3.1 Open-Circuit Saturation Characteristic and No-Load Rotational Losses

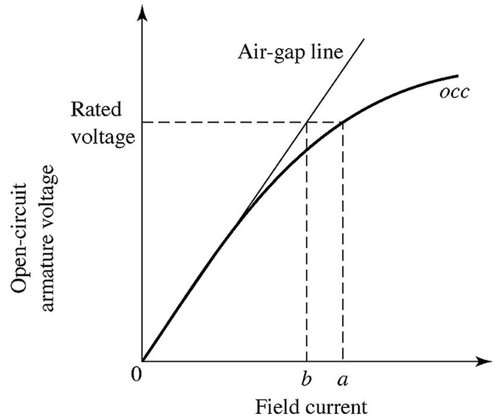


Figure 5.5 Open-circuit characteristic of a synchronous machine.

EXAMPLE 5.3

An open-circuit test performed on a three-phase, 60-Hz synchronous generator shows that the rated open-circuit voltage of 13.8 kV is produced by a field current of 318 A. Extrapolation of the air-gap line from a complete set of measurements on the machine shows that the field-current corresponding to 13.8 kV on the air-gap line is 263 A. Calculate the saturated and unsaturated values of L_{af}.

■ **Solution**

From Eq. 5.21, L_{af} is found from

$$L_{af} = \frac{\sqrt{2}\,E_{af}}{\omega_e I_f}$$

Here, $E_{af} = 13.8\ \text{kV}/\sqrt{3} = 7.97\ \text{kV}$. Hence the saturated value of L_{af} is given by

$$(L_{af})_{sat} = \frac{\sqrt{2}(7.97 \times 10^3)}{377 \times 318} = 94\ \text{mH}$$

and the unsaturated value is

$$(L_{af})_{unsat} = \frac{\sqrt{2}(7.97 \times 10^3)}{377 \times 263} = 114\ \text{mH}$$

In this case, we see that saturation reduces the magnetic coupling between the field and armature windings by approximately 18 percent.

§5.3.2 Short-Circuit Characteristic and Load Loss

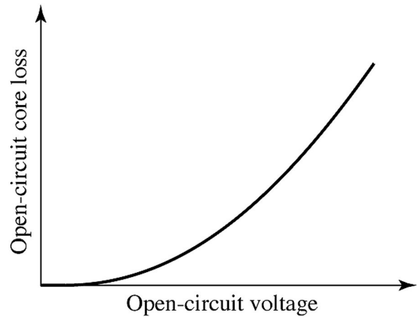


Figure 5.6 Typical form of an open-circuit core-loss curve.

$$\hat{E}_{af} = \hat{I}_a\left(R_a + jX_s\right) \tag{5.26}$$

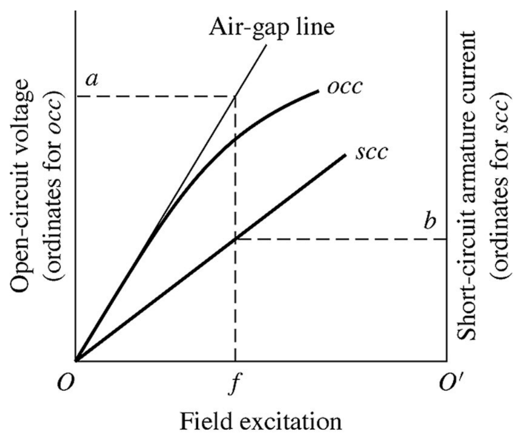


Figure 5.7 Open- and short-circuit characteristics of a synchronous machine.

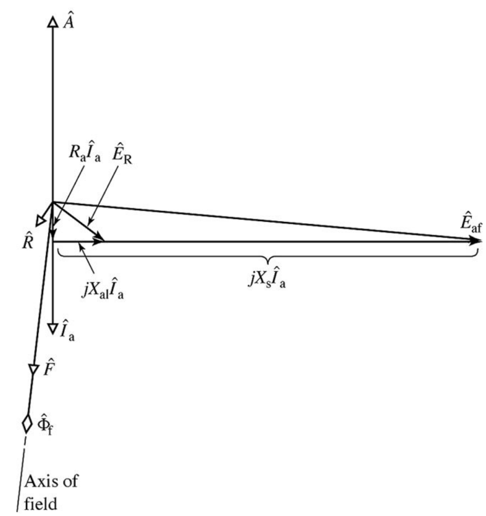


Figure 5.8 Phasor diagram for short-circuit conditions.

$$\hat{E}_R = \hat{I}_a\left(R_a + jX_{al}\right) \tag{5.27}$$

$$X_{s,u} = \frac{V_{a,ag}}{I_{a,ac}} \tag{5.28}$$

$$X_s = \frac{V_{a,rated}}{I'_a} \tag{5.29}$$

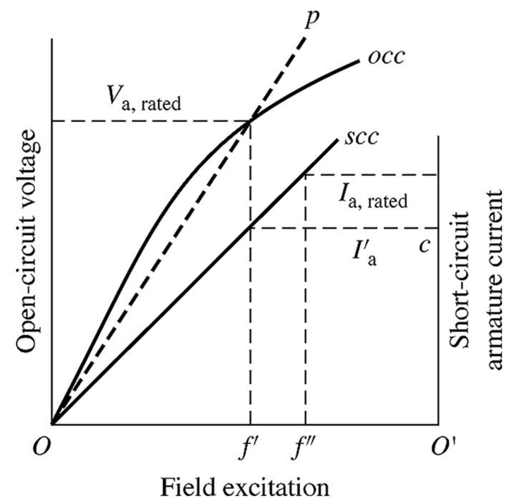


Figure 5.9 Open- and short-circuit characteristics showing equivalent magnetization line for saturated operating conditions.

$$\text{SCR} = \frac{Of'}{Of''} \tag{5.30}$$

$$\text{SCR} = \frac{\text{AFNL}}{\text{AFSC}} \tag{5.31}$$

EXAMPLE 5.4

The following data are taken from the open- and short-circuit characteristics of a 45-kVA, three-phase, Y-connected, 220-V (line-to-line), six-pole, 60-Hz synchronous machine. From the open-circuit characteristic:

Line-to-line voltage = 220 V   Field current = 2.84 A

From the short-circuit characteristic:

Armature current, A	118	152
Field current, A	2.20	2.84

From the air-gap line:

Field current = 2.20 A   Line-to-line voltage = 202 V

Compute the unsaturated value of the synchronous reactance, its saturated value at rated voltage in accordance with Eq. 5.29, and the short-circuit ratio. Express the synchronous reactance in ohms per phase and in per unit on the machine rating as a base.

■ Solution

At a field current of 2.20 A the line-to-neutral voltage on the air-gap line is

$$V_{\text{a,ag}} = \frac{202}{\sqrt{3}} = 116.7\ \text{V}$$

and for the same field current the armature current on short circuit is

$$I_{\text{a,sc}} = 118\ \text{A}$$

From Eq. 5.28

$$X_{\text{s,u}} = \frac{116.7}{118} = 0.987\ \Omega/\text{phase}$$

Note that rated armature current is

$$I_{a,\text{rated}} = \frac{45{,}000}{\sqrt{3} \times 220} = 118 \text{ A}$$

Therefore, $I_{a,sc} = 1.00$ per unit. The corresponding air-gap-line voltage is

$$V_{a,ag} = \frac{202}{220} = 0.92 \text{ per unit}$$

From Eq. 5.28 in per unit

$$X_{s,u} = \frac{0.92}{1.00} = 0.92 \text{ per unit}$$

The saturated synchronous reactance can be found from the open- and short-circuit characteristics and Eq. 5.29

$$X_s = \frac{V_{a,\text{rated}}}{I'_a} = \frac{(220/\sqrt{3})}{152} = 0.836 \ \Omega/\text{phase}$$

In per unit $I'_a = \frac{152}{118} = 1.29$, and from Eq. 5.29

$$X_s = \frac{1.00}{1.29} = 0.775 \text{ per unit}$$

Finally, from the open- and short-circuit characteristics and Eq. 5.30, the short-circuit ratio is given by

$$\text{SCR} = \frac{2.84}{2.20} = 1.29$$

Note that as was indicated following Eq. 5.30, the inverse of the short-circuit ratio is equal to the per-unit saturated synchronous reactance

$$X_s = \frac{1}{\text{SCR}} = \frac{1}{1.29} = 0.775 \text{ per unit}$$

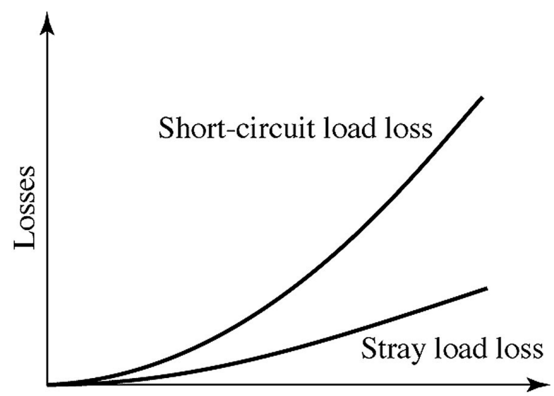


Figure 5.10 Typical form of short-circuit load loss and stray load-loss curves.

$$\frac{R_T}{R_t} = \frac{234.5 + T}{234.5 + t} \tag{5.32}$$

$$R_{a,\text{eff}} = \frac{\text{short} - \text{circuit load loss}}{(\text{short} - \text{circuitarmaturecurrent})^2} \tag{5.33}$$

EXAMPLE 5.5

For the 45-kVA, three-phase, Y-connected synchronous machine of Example 5.4, at rated armature current (118 A) the short-circuit load loss (total for three phases) is 1.80 kW at a temperature of 25°C. The dc resistance of the armature at this temperature is 0.0335 Ω/phase. Compute the effective armature resistance in per unit and in ohms per phase at 25°C.

Solution

The short-circuit load loss is 1.80/45 = 0.040 per unit at I_a = 1.00 per unit. Therefore,

$$R_{a,\text{eff}} = \frac{0.040}{(1.00)^2} = 0.040 \text{ per unit}$$

On a per-phase basis the short-circuit load loss is 1800/3 = 600 W/phase and consequently the effective resistance is

$$R_{a,\text{eff}} = \frac{600}{(118)^2} = 0.043\ \Omega/\text{phase}$$

The ratio of ac-to-dc resistance is

$$\frac{R_{a,\text{eff}}}{R_{a,\text{dc}}} = \frac{0.043}{0.0335} = 1.28$$

Because this is a small machine, its per-unit resistance is relatively high. The effective armature resistance of machines with ratings above a few hundred kilovoltamperes usually is less than 0.01 per unit.

§5.5 Steady-State Operating Characteristics

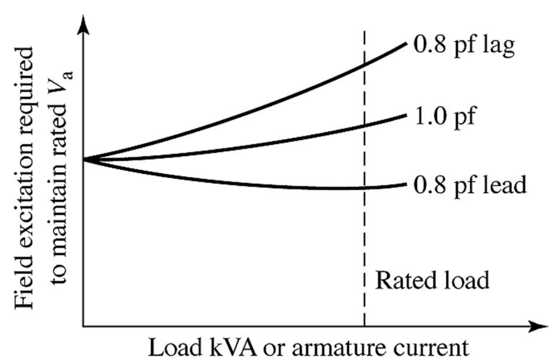


Figure 5.15 Characteristic form of synchronous-generator compounding curves.

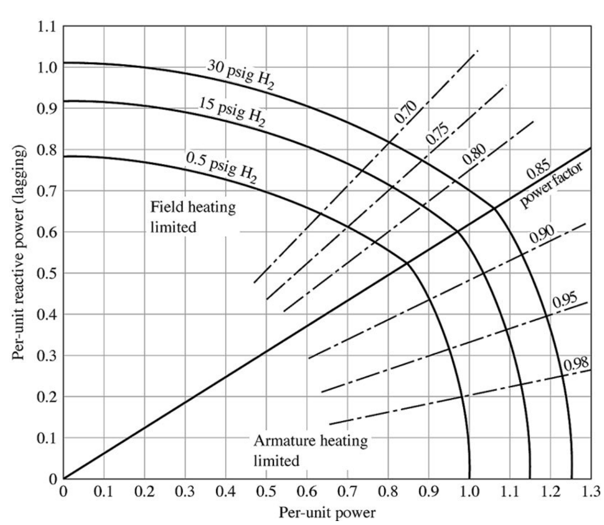


Figure 5.16 Capability curves of an 0.85 power factor, 0.80 short-circuit ratio, hydrogen-cooled turbine generator. Base MVA is rated MVA at 0.5 psig hydrogen.

$$\text{Apparent power} = \sqrt{P^2 + Q^2} = V_a I_a \tag{5.48}$$

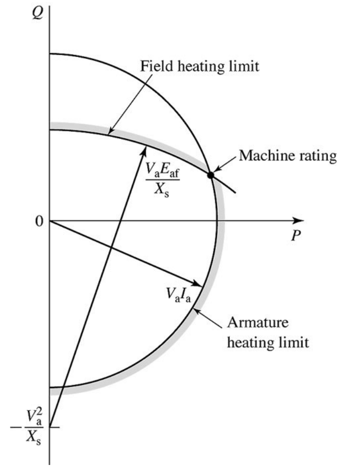


Figure 5.17 Construction used for the derivation of a synchronous generator capability curve.

$$P - jQ = \hat{V}_a + jX_s\hat{I}_a \tag{5.49}$$

$$\hat{E}_{af} = \hat{V}_a + jX_s\hat{I}_a \tag{5.50}$$

$$P^2 + \left(Q + \frac{V_a^2}{X_s}\right)^2 = \left(\frac{V_a E_{af}}{X_s}\right)^2 \tag{5.51}$$

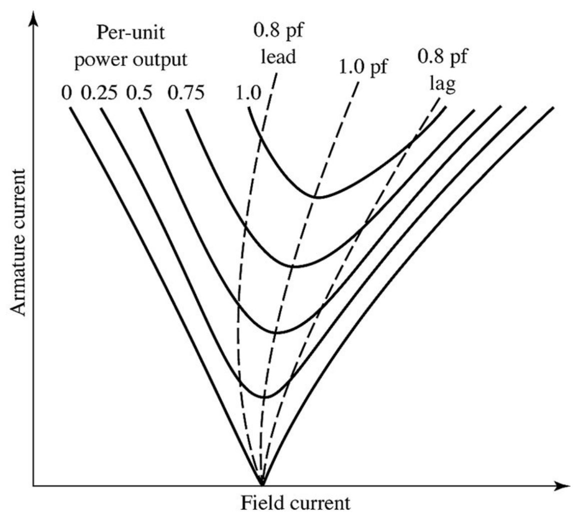


Figure 5.18 Typical form of synchronous-generator V curves.

EXAMPLE 5.8

Data are given in Fig. 5.19 with respect to the losses of the 45-kVA synchronous machine of Examples 5.4 and 5.5. Compute its efficiency when it is running as a synchronous motor at a terminal voltage of 220 V and with a power input to its armature of 45 kVA at 0.80 lagging power factor. The field current measured in a load test taken under these conditions is I_f (test) = 5.50 A. Assume the armature and field windings to be at a temperature of 75°C.

■ **Solution**

For the specified operating conditions, the armature current is

$$I_a = \frac{45 \times 10^3}{\sqrt{3} \times 230} = 113 \text{ A}$$

The I^2R losses must be computed on the basis of the dc resistances of the windings at 75°C. Correcting the winding resistances by means of Eq. 5.32 gives

$$\text{Field-winding resistance } R_f \text{ at } 75°\text{C} = 35.5\ \Omega$$

$$\text{Armature dc resistance } R_a \text{ at } 75°\text{C} = 0.0399\ \Omega/\text{phase}$$

The field I^2R loss is therefore

$$I_f^2 R_f = 5.50^2 \times 35.5 = 1.07 \text{ kW}$$

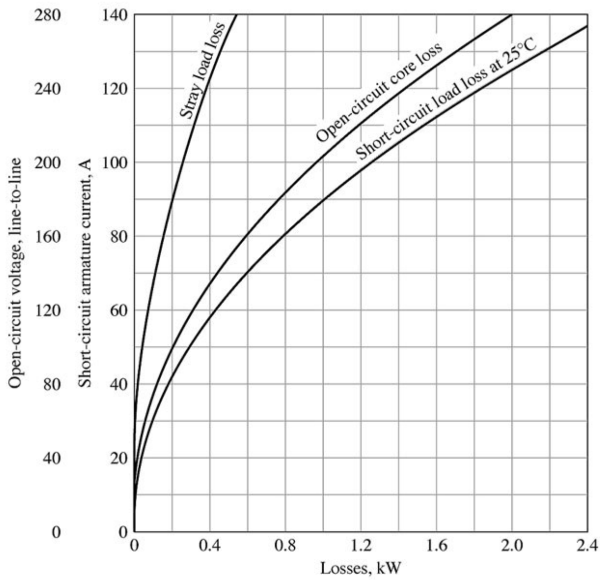
According to ANSI standards, losses in the excitation system, including those in any field-rheostat, are not charged against the machine.

The armature I^2R loss is

$$3I_a^2 R_a = 3 \times 113^2 \times 0.0399 = 1.53 \text{ kW}$$

and from Fig. 5.19 at $I_a = 113$ A the stray-load loss = 0.37 kW. The stray-load loss is considered to account for the losses caused by the armature leakage flux. According to ANSI standards, no temperature correction is to be applied to the stray load loss.

[2] See, for example, IEEE Std. 115-1995, "IEEE Guide: Test Procedures for Synchronous Machines," Institute of Electrical and Electronic Engineers, Inc., 345 East 47th Street, New York, New York, 10017 and NEMA Standards Publication No. MG-1-1998, "Motors and Generators," National Electrical Manufacturers Association, 1300 North 17th Street, Suite 1847, Rosslyn, Virginia, 22209.



Friction and windage loss = 0.91 kW
Armature dc resistance at 25°C = 0.0335 Ω per phase
Field-winding resistance at 25°C = 29.8 Ω

Figure 5.19 Losses in a three-phase, 45-kVA, Y-connected, 220-V, 60-Hz, six-pole synchronous machine.

Core loss under load is primarily a function of the main core flux in the motor. As is discussed in Chapter 2, the voltage across the magnetizing branch in a transformer (corresponding to the transformer core flux) is calculated by subtracting the leakage impedance drop from the terminal voltage. In a directly analogous fashion, the main core flux in a synchronous machine (i.e., the air-gap flux) can be calculated as the voltage behind the leakage impedance of the machine. Typically the armature resistance is small, and hence it is common to ignore the resistance and to calculate the voltage behind the leakage reactance. The core loss can then be estimated from the open-circuit core-loss curve at the voltage behind leakage reactance.

In this case, we do not know the machine leakage reactance. Thus, one approach would be simply to assume that the air-gap voltage is equal to the terminal voltage and to determine the core-loss under load from the core-loss curve at the value equal to terminal voltage.[3] In this case, the motor terminal voltage is 230 V line-to-line and thus from Fig. 5.19, the open-circuit core loss is 1.30 kW.

[3] Although not rigorously correct, it has become common practice to ignore the leakage impedance drop when determining the under-load core loss.

To estimate the effect of ignoring the leakage reactance drop, let us assume that the leakage reactance of this motor is 0.20 per unit or

$$X_{al} = 0.2\left(\frac{220^2}{45 \times 10^3}\right) = 0.215\ \Omega$$

Under this assumption, the air-gap voltage is equal to

$$\hat{V}_a - jX_{al}\hat{I}_a = \frac{230}{\sqrt{3}} - j0.215 \times 141(0.8 + j0.6)$$

$$= 151 - j24.2 = 153\, e^{-j9.1^\circ}\ \text{V, line-to-neutral}$$

which corresponds to a line-to-line voltage of $\sqrt{3}\,(153) = 265$ V. From Fig. 5.19, the corresponding core-loss is 1.8 kW, 500 W higher than the value determined using the terminal voltage. We will use this value for the purposes of this example.

Including the friction and windage loss of 0.91 kW, all losses have now been found:

$$\text{Total losses} = 1.07 + 1.53 + 0.37 + 1.80 + 0.91 = 5.68\ \text{kW}$$

The total motor input power is the input power to the armature plus that to the field.

$$\text{Input power} = 0.8 \times 45 + 1.07 = 37.1\ \text{kW}$$

and the output power is equal to the total input power minus the total losses

$$\text{Output power} = 37.1 - 5.68 = 31.4\ \text{kW}$$

Therefore

$$\text{Efficiency} = \frac{\text{Output power}}{\text{Input power}} = 1 - \frac{31.4}{37.1} = 0.846 = 84.6\%$$

§5.6 Effects of Salient Poles; Introduction to Direct-And Quadrature-Axis Theory

§5.6.1 Flux and MMF Waves

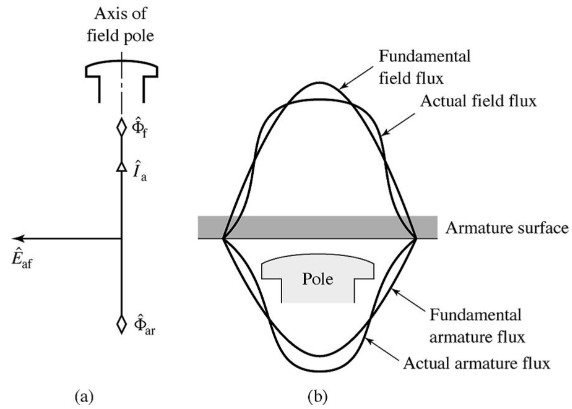


Figure 5.20 Direct-axis air-gap fluxes in a salient-pole synchronous machine.

$$E_{3,a} = \sqrt{2}V_3 \cos\left(3\omega_e t + \phi_3\right) \tag{5.52}$$

$$E_{3,b} = \sqrt{2}V_3 \cos\left(3\left(\omega_e t - 120^{\circ}\right) + \phi_3\right) = \sqrt{2}V_3 \cos\left(3\omega_e t + \phi_3\right) \tag{5.53}$$

$$E_{3,c} = \sqrt{2}V_3 \cos\left(3\left(\omega_e t - 120^{\circ}\right) + \phi_3\right) = \sqrt{2}V_3 \cos\left(3\omega_e t + \phi_3\right) \tag{5.54}$$

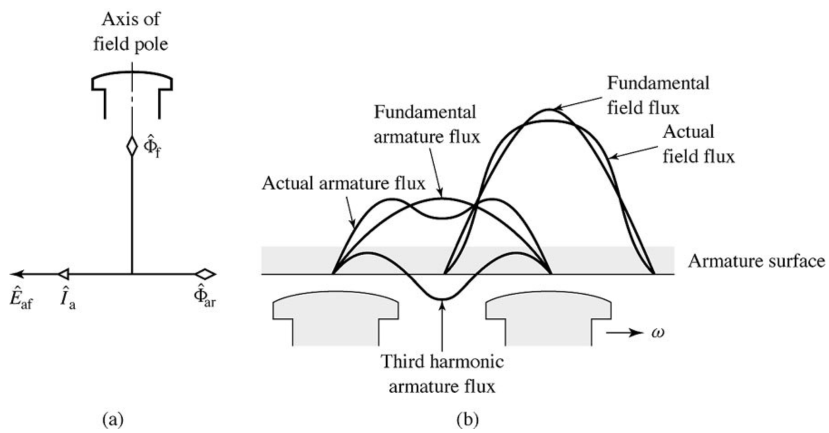


Figure 5.21 Quadrature-axis air-gap fluxes in a salient-pole synchronous machine.

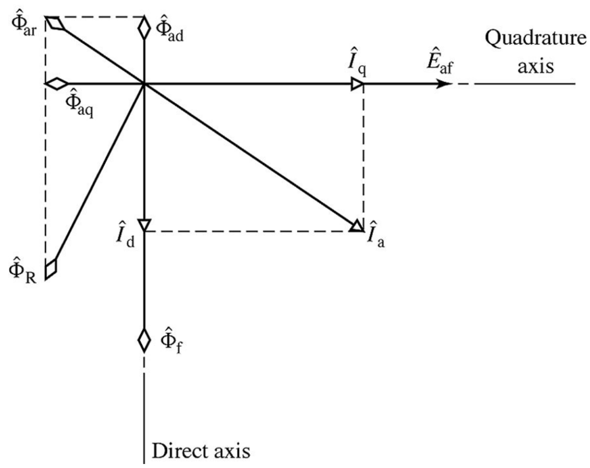


Figure 5.22 Phasor diagram of a salient-pole synchronous generator.

§5.3.2 Phasor Diagrams for Salient-Pole Machines

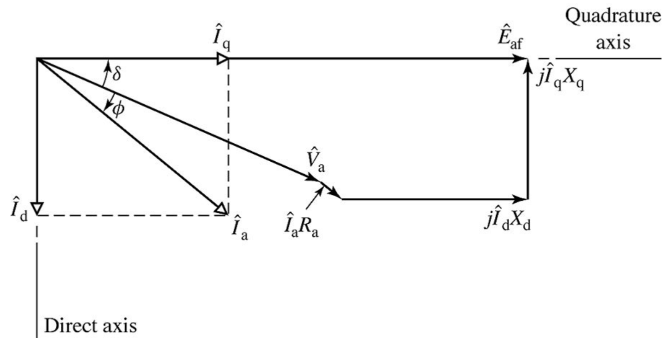


Figure 5.23 Phasor diagram for a synchronous generator showing the relationship between the voltages and the currents.

$$X_d = X_{al} + X_{\varphi d} \tag{5.55}$$

$$X_q = X_{al} + X_{\varphi q} \tag{5.56}$$

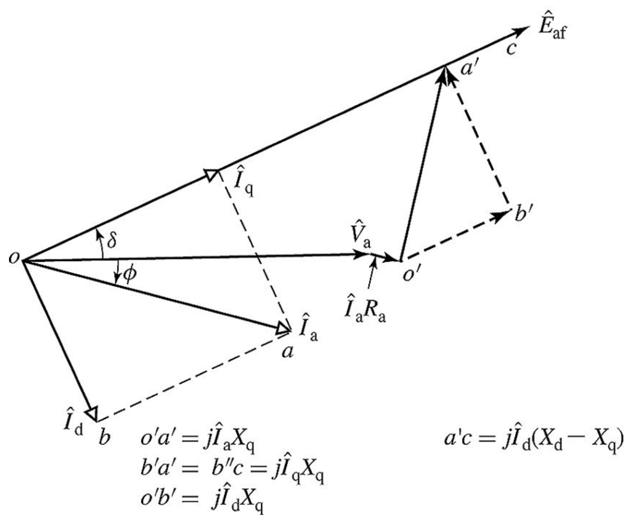


Figure 5.24 Relationships between component voltages in a phasor diagram.

$$\frac{\text{o'a'}}{\text{oa}} = \frac{\text{b'a'}}{\text{ba}} \tag{5.57}$$

$$\text{o'a'} = \left(\frac{\text{b'a'}}{\text{ba}}\right)\text{oa} = \frac{\left|\hat{I}_q\right|X_q}{\left|\hat{I}_q\right|}\left|\hat{I}_a\right| = X_q\left|\hat{I}_a\right| \tag{5.58}$$

$$\hat{E}_{af} = \hat{V}_a + R_a\hat{I}_a + jX_d\hat{I}_d + jX_q\hat{I}_q \tag{5.59}$$

EXAMPLE 5.9

The reactances X_d and X_q of a salient-pole synchronous generator are 1.00 and 0.60 per unit, respectively. The armature resistance may be considered to be negligible. Compute the generated voltage when the generator delivers its rated kVA at 0.80 lagging power factor and rated terminal voltage.

■ Solution

First, the phase of $\hat{E}_{af}$ must be found so that $\hat{I}_a$ can be resolved into its direct- and quadrature-axis components. The phasor diagram is shown in Fig. 5.25. As is commonly done for such problems, the terminal voltage $\hat{V}_a$ will be used as the reference phasor, i.e., $\hat{V}_a = V_a e^{j0.0°} = V_a$. In per unit

$$\hat{I}_a = I_a e^{j\phi} = 0.80 - j0.60 = 1.0\, e^{-j36.9°}$$

The quadrature axis is located by the phasor

$$\hat{E}' = \hat{V}_a + jX_q\hat{I}_a = 1.0 + j0.60(1.0\,e^{-j36.9^\circ}) = 1.44\,e^{j19.4^\circ}$$

Thus, $\delta = 19.4^\circ$, and the phase angle between $\hat{E}_{af}$ and $\hat{I}_a$ is $\delta - \phi = 19.4^\circ - (-36.9^\circ) = 56.3^\circ$. Note, that although it would appear from Fig. 5.25 that the appropriate angle is $\delta + \phi$, this is not correct because the angle ϕ as drawn in Fig. 5.25 is a negative angle. In general, the desired angle is equal to the difference between the power angle and the phase angle of the terminal current.

The armature current can now be resolved into its direct- and quadrature-axis components. Their magnitudes are

$$I_d = |\hat{I}_a| \sin(\delta - \phi) = 1.00 \sin(56.3^\circ) = 0.832$$

and

$$I_q = |\hat{I}_a| \cos(\delta - \phi) = 1.00 \cos(56.3^\circ) = 0.555$$

As phasors,

$$\hat{I}_d = 0.832\,e^{j(-90^\circ+19.4^\circ)} = 0.832\,e^{j70.6^\circ}$$

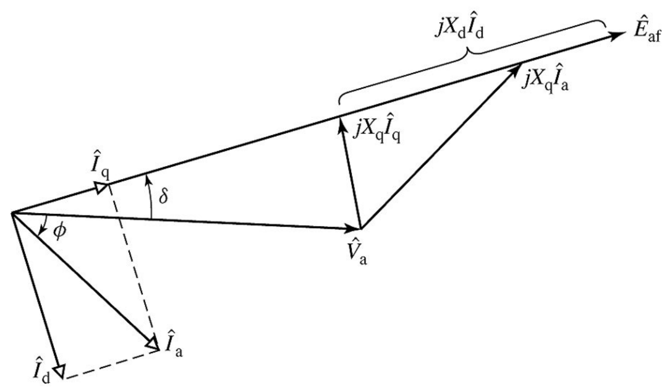


Figure 5.25 Generator phasor diagram for Example 5.9.

and

$$\hat{I}_q = 0.555\,e^{j19.4^\circ}$$

We can now find E_{af} from Eq. 5.59

$$\begin{aligned}\hat{E}_{af} &= \hat{V}_a + jX_d\hat{I}_d + jX_qj\hat{I}_q \\ &= 1.0 + j1.0(0.832\,e^{j70.6^\circ}) + j0.6(0.555\,e^{j19.4^\circ}) \\ &= 1.77\,e^{j19.4^\circ}\end{aligned}$$

and we see that $E_{af} = 1.77$ per unit. Note that, as expected, $\angle\hat{E}_{af} = 19.4^\circ = \delta$, thus confirming that $\hat{E}_{af}$ lies along the quadrature axis.

EXAMPLE 5.10

In the simplified theory of Section 5.2, the synchronous machine is assumed to be representable by a single reactance, the synchronous reactance X_s of Eq. 5.25. The question naturally arises: How serious an approximation is involved if a salient-pole machine is treated in this simple fashion? Suppose that a salient-pole machine were treated by cylindrical-rotor theory as if it had a single synchronous reactance equal to its direct-axis value X_d? To investigate this question, we will repeat Example 5.9 under this assumption.

■ **Solution**

In this case, under the assumption that

$$X_q = X_d = X_s = 1.0 \text{ per unit}$$

the generated voltage can be found simply as

$$\begin{aligned}\hat{E}_{af} &= V_a + jX_s\hat{I}_a \\ &= 1.0 + j1.0(1.0\,e^{-j36.9^\circ}) = 1.79\,e^{j26.6^\circ} \text{ per unit}\end{aligned}$$

Comparing this result with that of Example 5.9 (in which we found that $E_{af} = 1.77\,e^{j19.4^\circ}$), we see that the magnitude of the predicted generated voltage is relatively close to the correct value. As a result, we see that the calculation of the field current required for this operating condition will be relatively accurate under the simplifying assumption that the effects of saliency can be neglected.

However, the calculation of the power angle δ (19.4° versus a value of 26.6° if saliency is neglected) shows a considerably larger error. In general, such errors in the calculation of generator steady-state power angles may be of significance when studying the transient behavior of a system including a number of synchronous machines. Thus, although saliency can perhaps be ignored when doing "back-of-the-envelope" system calculations, it is rarely ignored in large-scale, computer-based system studies.

Chapter 6 Polyphase Induction Machines

- Study on the behavior of polyphase induction machines:
  - The analysis begins with the development of single-phase equivalent circuits.
    - → The general form is suggested by the similarity of an induction machine to a transformer.
    - → The equivalent circuits can be used to study the electromechanical characteristics of an induction machine as well as the loading presented by the machine on its supply source.

§6.1 Introduction to Polyphase Induction Machines

- An induction machine is one in which alternating current is supplied to the stator directly and to the rotor by induction or transformer action from the stator.
  - The stator winding is excited from a balanced polyphase source and produces a magnetic field in the air gap rotating at synchronous speed.
  - The rotor winding may one of two types.
    - → A wound rotor is built with a polyphase winding similar to, and wound with the same number of poles as, the stator. The rotor terminals are available external to the motor.
    - → A squirrel-cage rotor has a winding consisting of conductor bars embedded in slots in the rotor iron and short-circuited at each end buy conducting end rings. It is the most commonly used type of motor in sizes ranging from fractional horsepower on up.
  - The difference between synchronous speed and the rotor speed is commonly referred to as the slip of the rotor. The fractional slip s is

$$s = \frac{n_\mathrm{s} - n}{n_\mathrm{s}} \tag{6.1}$$

    - → The slip is often expressed in percent.
    - → n : rotor speed in rpm

$$n = (1 - s) n_s \tag{6.2}$$

    - → ω_m : mechanical angular velocity

$$\omega_m = (1 - s)\omega_s \tag{6.3}$$

    - → f_r : the frequency of induced voltages, the slip frequency

$$f_\mathrm{r} = s\, f_\mathrm{e} \tag{6.4}$$

      – A wound-rotor induction machine can be used as a frequency changer.
  - The rotor currents produce an air-gap flux wave that rotates at synchronous speed and in synchronism with that produced by the stator currents.
    - → With the rotor revolving in the same direction of rotation as the stator field, the rotor currents produce a rotating flux wave rotating at sn_s with respect to the rotor in the forward direction.
    - → With respect to the stator, the speed of the flux wave produced by the rotor currents (with frequency sf_e) equals

$$sn_\mathrm{s} + n = sn_\mathrm{s} + n_\mathrm{s}(1 - s) = n_\mathrm{s} \tag{6.5}$$

    - → Because the stator and rotor fields each rotate synchronously, they are stationary with respect to each other and produce a steady torque, thus maintaining rotation of the rotor. Such torque is called an asynchronous torque.

→ Equation (4.81) $T = -\frac{\pi}{2}\left(\frac{\text{poles}}{2}\right)^2 \Phi_{sr} F_r \sin\delta_r$ can be expressed in the form

$$T = -\ KI_r \sin\delta_r \tag{6.6}$$

I_r : the rotor current

δ_r : the angle by which the rotor mmf wave leads the resultant air-gap mmf wave

→ Fig. 6.4 shows a typical polyphase squirrel-cage induction motor torque-speed curve. The factors influencing the shape of this curve can be appreciated in terms of the torque equation.

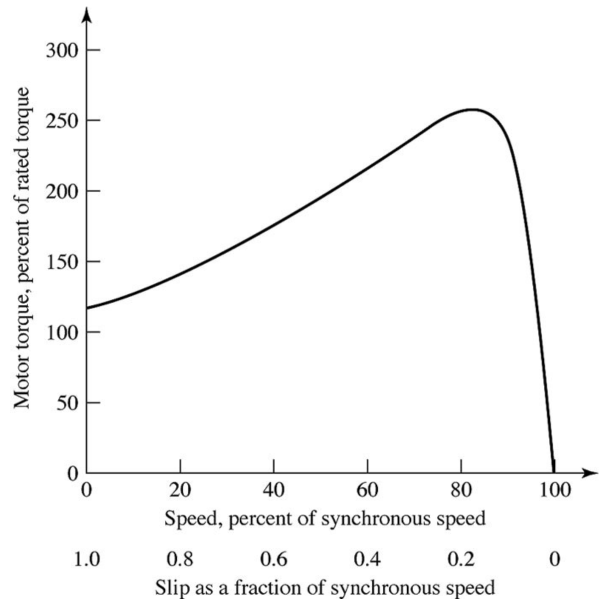


Figure 6.4 Typical induction-motor torque-speed curve for constant-voltage, constant-frequency operation.

→ Under normal running conditions the slip is small: 2 to 10 percent at full load.
→ The maximum torque is referred to as the breakdown torque.
→ The slip at which the peak torque occurs is proportional to the rotor resistance.

§6.2 Currents and Fluxes in Polyphase Induction Machines

§6.3 Induction-Motor Equivalent Circuit

- Only machines with symmetric polyphase windings exited by balanced polyphase voltages are considered. It is helpful to think of three-phase machines as being Y-connected.

  - Stator equivalent circuit:

$$\hat{V}_1 = \hat{E}_2 + \hat{I}_1(R_1 + jX_1) \tag{6.8}$$

$\hat{V}_1 =$ Stator line-to-neutral terminal voltage

$\hat{E}_2 =$ Counter emf (line-to-neutral) generated by the resultant air-gap flux

$\hat{I}_1 =$ Stator current

$R_1 =$ Stator effective resistance

$X_1 =$ Stator leakage reactance

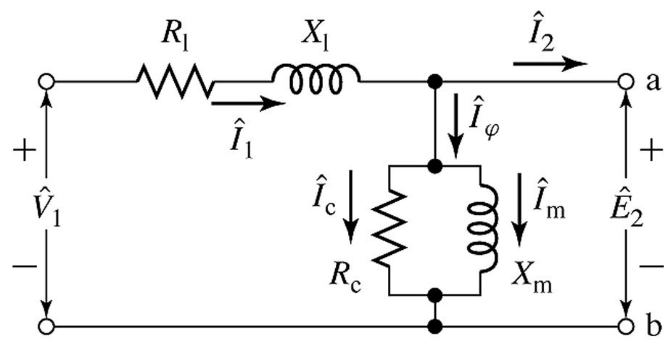


Figure 6.7 Stator equivalent circuit for a polyphase induction motor.

- Rotor equivalent circuit:

$$Z_2 = \frac{\hat{E}_2}{\hat{I}_2} \tag{6.9}$$

$$Z_{2s} = \frac{\hat{E}_{2s}}{\hat{I}_{2s}} = N_{\text{eff}}^2 \left(\frac{\hat{E}_{\text{rotor}}}{\hat{I}_{\text{rotor}}} \right) = N_{\text{eff}}^2 Z_{\text{rotor}} \tag{6.10}$$

Z_{2s} : the slip-frequency leakage impedance of the equivalent rotor

Z_{rotor} : the slip-frequency leakage impedance

$$Z_{2s} = \frac{\hat{E}_{2s}}{\hat{I}_{2s}} = R_2 + jsX_2 \tag{6.11}$$

R_2 = Referred rotor resistance

sR_2 = Referred rotor leakage reactance at slip frequency

X_2 = Referred rotor leakage reactance at stator frequency f_e

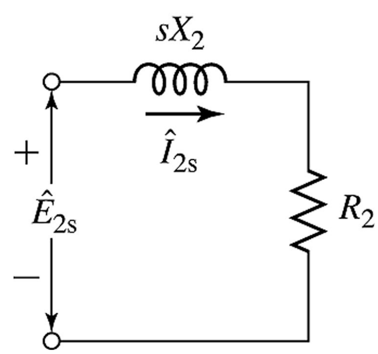


Figure 6.8 Rotor equivalent circuit for a polyphase induction motor at slip frequency.

$$\hat{I}_{2s} = \hat{I}_2 \tag{6.12}$$

$$E_{2s} = sE_2 \tag{6.13}$$

$$\hat{E}_{2s} = s\hat{E}_2 \tag{6.14}$$

$$\frac{\hat{E}_{2s}}{\hat{I}_{2s}} = \frac{s\hat{E}_2}{\hat{I}_2} = Z_{2s} = R_2 + jsX_2 \tag{6.15}$$

$$Z_2 = \frac{\hat{E}_2}{\hat{I}_2} = \frac{R_2}{s} + jX_2 \tag{6.16}$$

➢ Fig. 6.9 shows the single-phase equivalent circuit.

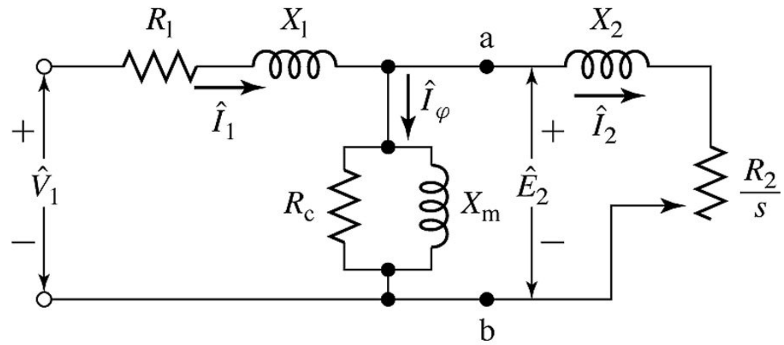


Figure 6.9 Single-phase equivalent circuit for a polyphase induction motor.

§6.4 Analysis of the Equivalent Circuit

- The single-phase equivalent circuit can be used to determine a wide variety of steady-state performance characteristics of polyphase induction machines.
  - ➢ P_{gap} : the total power transferred across the air gap from the stator

    P_{rotor} : the total rotor ohmic loss

$$P_{gap} = n_{ph} I_2^2 \left(\frac{R_2}{s} \right) \tag{6.17}$$

$$P_{rotor} = n_{ph} I_{2s}^2 R_2 \tag{6.18}$$

$$P_{rotor} = n_{ph} I_2^2 R_2 \tag{6.19}$$

$$P_{mech} = P_{gap} - P_{rotor} = n_{ph} I_2^2 \left(\frac{R_2}{s} \right) - n_{ph} I_2^2 R_2 \tag{6.20}$$

$$P_{mech} = n_{ph} I_2^2 R_2 \left(\frac{1-s}{s} \right) \tag{6.21}$$

$$P_{mech} = (1-s) P_{gap} \tag{6.22}$$

$$P_{rotor} = s P_{gap} \tag{6.23}$$

→ Of the total power delivered across the air gap to the rotor, the fraction $1-s$ is converted to mechanical power and the fraction s is dissipated as ohmic loss in the rotor conductors.

→ When power aspects are to be emphasized, the equivalent circuit can be redrawn in the manner of Fig. 6.10.

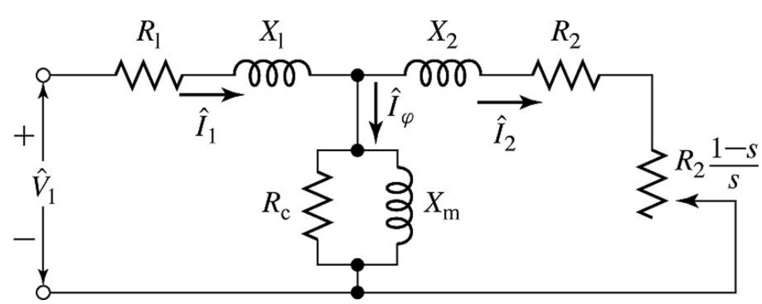


Figure 6.10 Alternative form of equivalent circuit.

EXAMPLE 6.1

A three-phase, two-pole, 60-Hz induction motor is observed to be operating at a speed of 3502 r/min with an input power of 15.7 kW and a terminal current of 22.6 A. The stator-winding resistance is 0.20 Ω/phase. Calculate the I^2R power dissipated in rotor.

■ Solution

The power dissipated in the stator winding is given by

$$P_{\text{stator}} = 3I_1^2R_1 = 3(22.6)^2 0.2 = 306 \text{ W}$$

Hence the air-gap power is

$$P_{\text{gap}} = P_{\text{input}} - P_{\text{stator}} = 15.7 - 0.3 = 15.4 \text{ kW}$$

The synchronous speed of this machine can be found from Eq. 4.41

$$n_s = \left(\frac{120}{\text{poles}}\right) f_e = \left(\frac{120}{2}\right) 60 = 3600 \text{ r/min}$$

and hence from Eq. 6.1, the slip is $s = (3600 - 3502)/3600 = 0.0272$. Thus, from Eq. 6.23,

$$P_{\text{rotor}} = sP_{\text{gap}} = 0.0272 \times 15.4 \text{ kW} = 419 \text{ W}$$

➢ Consider the electromechanical torque T_{mech}.

$$P_{\text{mech}} = \omega_m T_{\text{mech}} = (1-s)\omega_s T_{\text{mech}} \tag{6.24}$$

$$T_{\text{mech}} = \frac{P_{\text{mech}}}{\omega_{\text{m}}} = \frac{P_{\text{gap}}}{\omega_{\text{s}}} = \frac{n_{\text{ph}} I_2^2 (R_2/s)}{\omega_s} \tag{6.25}$$

$$\omega_s = \frac{4\pi f_e}{\text{poles}} = \left(\frac{2}{\text{poles}}\right)\omega_e \tag{6.26}$$

$$P_{\text{shaft}} = P_{\text{mech}} - P_{\text{rot}} \tag{6.27}$$

$$T_{\text{shaft}} = \frac{P_{\text{shaft}}}{\omega_{\text{m}}} = T_{\text{mech}} - T_{\text{rot}} \tag{6.28}$$

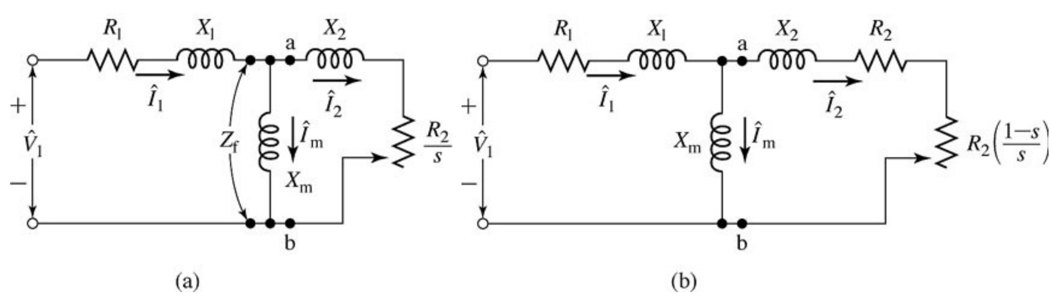


Figure 6.11 Equivalent circuits with the core-loss resistance *R*c neglected corresponding to (a) Fig. 6.9 and (b) Fig. 6.10.

EXAMPLE 6.2

A three-phase Y-connected 220-V (line-to-line) 7.5-kW 60-Hz six-pole induction motor has the following parameter values in Ω/phase referred to the stator:

$$R_1 = 0.294 \quad R_2 = 0.144$$

$$X_1 = 0.503 \quad X_2 = 0.209 \quad X_m = 13.25$$

The total friction, windage, and core losses may be assumed to be constant at 403 W, independent of load.

For a slip of 2 percent, compute the speed, output torque and power, stator current, power factor, and efficiency when the motor is operated at rated voltage and frequency.

■ Solution

Let the impedance Z_f (Fig. 6.11a) represent the per phase impedance presented to the stator by the magnetizing reactance and the rotor. Thus, from Fig. 6.11a

$$Z_f = R_f + jX_f = \left(\frac{R_2}{s} + jX_2\right) \text{ in parallel with } jX_m$$

Substitution of numerical values gives, for $s = 0.02$,

$$R_f + jX_f = 5.41 + j3.11\ \Omega$$

The stator input impedance can now be calculated as

$$Z_{in} = R_1 + jX_1 + Z_f = 5.70 + j3.61 = 6.75\ \angle 32.3°\ \Omega$$

The line-to-neutral terminal voltage is equal to

$$V_1 = \frac{220}{\sqrt{3}} = 127\ \text{V}$$

and hence the stator current can be calculated as

$$\hat{I}_1 = \frac{V_1}{Z_{in}} = \frac{127}{6.75\ \angle 32.3°} = 18.8\ \angle{-32.3°}\ \text{A}$$

The stator current is thus 18.8 A and the power factor is equal to cos (−32.3°) = 0.845 lagging.

The synchronous speed can be found from Eq. 4.41

$$n_s = \left(\frac{120}{\text{poles}}\right) f_e = \left(\frac{120}{6}\right) 60 = 1200\ \text{r/min}$$

or from Eq. 6.26

$$\omega_s = \frac{4\pi f_e}{\text{poles}} = 125.7\ \text{rad/sec}$$

The rotor speed is

$$n = (1 - s)n_s = (0.98)1200 = 1176\ \text{r/min}$$

or

$$\omega_m = (1 - s)\omega_s = (0.98)125.7 = 123.2\ \text{rad/sec}$$

From Eq. 6.17,

$$P_{gap} = n_{ph} I_2^2 \left(\frac{R_2}{s}\right)$$

Note however that because the only resistance included in Z_f is R_2/s, the power dissipated in Z_f is equal to the power dissipated in R_2/s and hence we can write

$$P_{gap} = n_{ph} I_1^2 R_f = 3(18.8)^2(5.41) = 5740\ \text{W}$$

We can now calculate P_{mech} from Eq. 6.21 and the shaft output power from Eq. 6.27. Thus

$$P_{shaft} = P_{mech} - P_{rot} = (1 - s)P_{gap} - P_{rot}$$

$$= (0.98)5740 - 403 = 5220\ \text{W}$$

and the shaft output torque can be found from Eq. 6.28 as

$$T_{shaft} = \frac{P_{shaft}}{\omega_m} = \frac{5220}{123.2} = 42.4\ \text{N}\cdot\text{m}$$

The efficiency is calculated as the ratio of shaft output power to stator input power. The input power is given by

$$P_{in} = n_{ph}\text{Re}[\hat{V}_1 \hat{I}_1^*] = 3\text{Re}[127(18.8\ \angle 32.3°)]$$

$$= 3 \times 127 \times 18.8 \cos(32.2°) = 6060\ \text{W}$$

Thus the efficiency η is equal to

$$\eta = \frac{P_{shaft}}{P_{in}} = \frac{5220}{6060} = 0.861 = 86.1\%$$

The complete performance characteristics of the motor can be determined by repeating these calculations for other assumed values of slip.

§6.5 Torque and Power by Use of Thevenin's Theorem

- Considerable simplification will be obtained from application of Thevenin's network theorem to the induction-motor equivalent circuit.

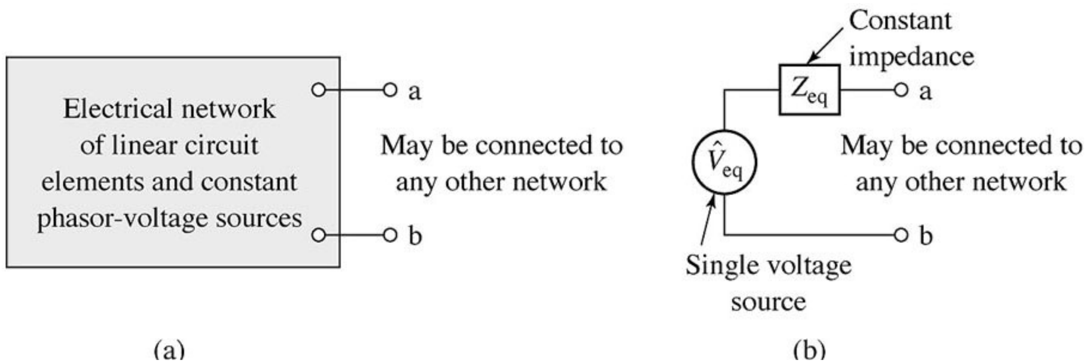


Figure 6.12 (a) General linear network and (b) its equivalent at terminals ab by Thevenin's theorem.

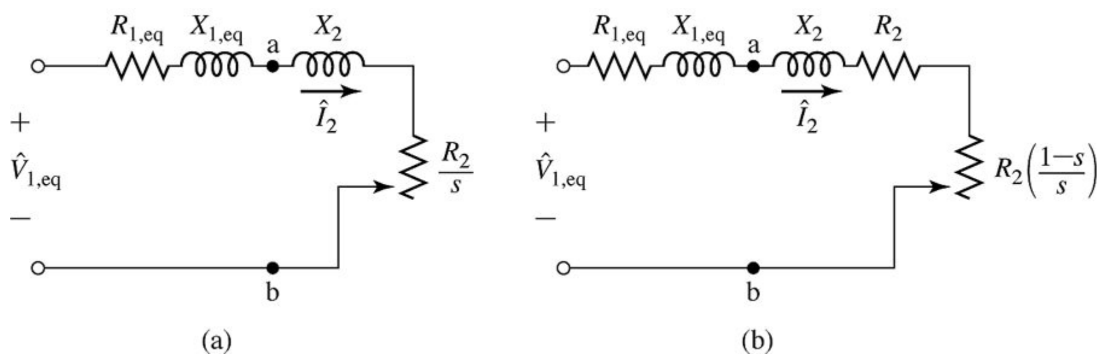


Figure 6.13 Induction-motor equivalent circuits simplified by Thevenin's theorem.

$$\hat{V}_{1,\mathrm{eq}} = \hat{V}_1\left(\frac{jX_m}{R_1 + j(X_1 + X_m)}\right) \tag{6.29}$$

$$Z_{1,\mathrm{eq}} = R_{1,\mathrm{eq}} + jX_{1,\mathrm{eq}} = (R_1 + jX_1) \text{ in parallel with } jX_m \tag{6.30}$$

$$Z_{1,\mathrm{eq}} = \hat{V}_1\,\frac{j\,X_m(R_1 + jX_1)}{R_1 + j(X_1 + X_m)} \tag{6.31}$$

$$\hat{I}_2 = \frac{\hat{V}_{1,\mathrm{eq}}}{Z_{1,\mathrm{eq}} + jX_2 + R_2/s} \tag{6.32}$$

$$T_{\mathrm{mech}} = \frac{1}{\omega_s}\left[\frac{n_{\mathrm{ph}}V_{1,\mathrm{eq}}^2(R_2/s)}{(R_{1,\mathrm{eq}} + (R_2/s))^2 + (X_{1,\mathrm{eq}} + X_2)^2}\right] \tag{6.33}$$

- The general shape of the torque-speed or torque-slip curve with motor connected to a constant-voltage, constant-frequency source is shown in Figs. 6.14 and 6.15.

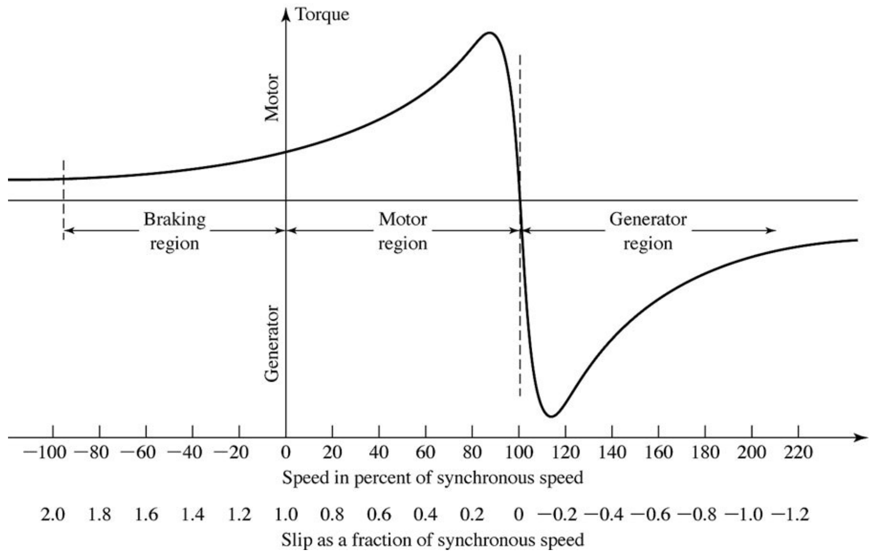


Figure 6.14 Induction-machine torque-slip curve showing braking, motor, and generator regions.

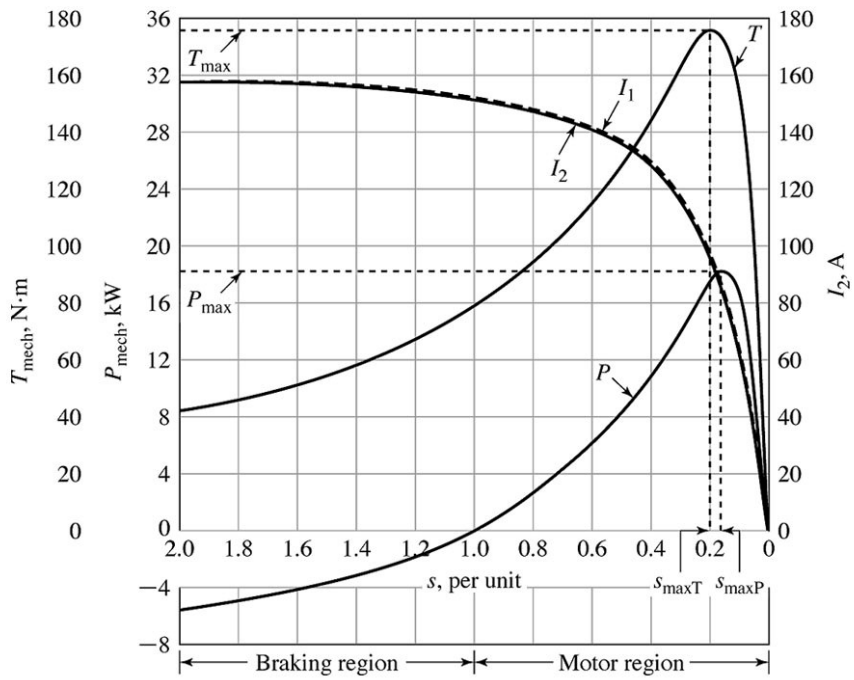


Figure 6.15 Computed torque, power, and current curves for the 7.5-kW motor in Exps 6.2 and 6.3.

- Maximum electromechanical torque will occur at a value of slip $s_{\max T}$ for which

$$\frac{R_2}{s_{\max T}} = \sqrt{R_{1,\text{eq}}^2 + \left(X_{1,\text{eq}} + X_2\right)^2} \tag{6.34}$$

$$s_{\max T} = \frac{R_2}{\sqrt{R_{1,\text{eq}}^2 + \left(X_{1,\text{eq}} + X_2\right)^2}} \tag{6.35}$$

$$T_{\max} = \frac{1}{\omega_s}\left[\frac{0.5 n_{ph} V_{1,\text{eq}}^2}{R_{1,\text{eq}} + \sqrt{R_{1,\text{eq}}^2 + \left(X_{1,\text{eq}} + X_2\right)^2}}\right] \tag{6.36}$$

EXAMPLE 6.3

For the motor of Example 6.2, determine (*a*) the load component I_2 of the stator current, the electromechanical torque T_{mech}, and the electromechanical power P_{mech} for a slip $s = 0.03$; (*b*) the maximum electromechanical torque and the corresponding speed; and (*c*) the electromechanical starting torque T_{start} and the corresponding stator load current $I_{2,start}$.

■ Solution

First reduce the circuit to its Thevenin-equivalent form. From Eq. 6.29, $V_{1,eq} = 122.3$ V and from Eq. 6.31, $R_{1,eq} + jX_{1,eq} = 0.273 + j0.490\ \Omega$.

a. At $s = 0.03$, $R_2/s = 4.80$. Then, from Fig. 6.13a,

$$I_2 = \frac{V_{1,eq}}{\sqrt{(R_{1,eq} + R_2/s)^2 + (X_{1,eq} + X_2)^2}} = \frac{122.3}{\sqrt{(5.07)^2 + (0.699)^2}} = 23.9\text{ A}$$

From Eq. 6.25

$$T_{mech} = \frac{n_{ph} I_2^2 (R_2/s)}{\omega_s} = \frac{3 \times 23.9^2 \times 4.80}{125.7} = 65.4\text{ N}\cdot\text{m}$$

and from Eq. 6.21

$$P_{mech} = n_{ph} I_2^2 (R_2/s)(1 - s) = 3 \times 23.9^2 \times 4.80 \times 0.97 = 7980\text{ W}$$

The curves of Fig. 6.15 were computed by repeating these calculations for a number of assumed values of s.

b. At the maximum-torque point, from Eq. 6.35,

$$s_{maxT} = \frac{R_2}{\sqrt{R_{1,eq}^2 + (X_{1,eq} + X_2)^2}} = \frac{0.144}{\sqrt{0.273^2 + 0.699^2}} = 0.192$$

and thus the speed at T_{max} is equal to $(1 - s_{maxT})n_s = (1 - 0.192) \times 1200 = 970$ r/min

From Eq. 6.36

$$T_{max} = \frac{1}{\omega_s}\left[\frac{0.5 n_{ph} V_{1,eq}^2}{R_{1,eq} + \sqrt{R_{1,eq}^2 + (X_{1,eq} + X_2)^2}}\right] = \frac{1}{125.7}\left[\frac{0.5 \times 3 \times 122.3^2}{0.273 + \sqrt{0.273^2 + 0.699^2}}\right] = 175\text{ N}\cdot\text{m}$$

c. At starting, $s = 1$. Therefore

$$I_{2,start} = \frac{V_{1,eq}}{\sqrt{(R_{1,eq} + R_2)^2 + (X_{1,eq} + X_2)^2}} = \frac{122.3}{\sqrt{0.417^2 + 0.699^2}} = 150\text{ A}$$

From Eq. 6.25

$$T_{start} = \frac{n_{ph} I_2^2 R_2}{\omega_s} = \frac{3 \times 150^2 \times 0.144}{125.7} = 77.3\text{ N}\cdot\text{m}$$

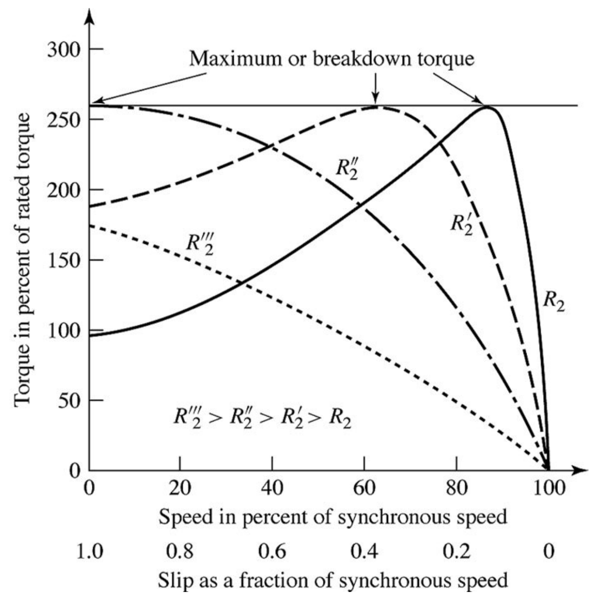


Figure 6.16 Induction-motor torque-slip curves showing effect of changing rotor-circuit resistance.

§6.5 Parameter Determination from No-Load and Blocked-Rotor Tests

- The equivalent-circuit parameters needed for computing the performance of a poly-phase induction motor under load can be obtained from the results of a no-load test, a blocked-rotor test, and measurement of the dc resistances of the stator windings.

§6.6.1 No-Load Test

- Like the open-circuit test on a transformer, the no-load test on an induction motor gives information with respect to exciting current and no-load losses.

§6.6.2 Blocked-Rotor Test

- Like the short-circuit test on a transformer, the blocked-rotor test on an induction motor give information with respect to the leakage impedances.

EXAMPLE 6.5

The following test data apply to a 7.5-hp, three-phase, 220-V, 19-A, 60-Hz, four-pole induction motor with a double-squirrel-cage rotor of design class C (high-starting-torque, low-starting-current type):

Test 1: No-load test at 60 Hz

Applied voltage $V = 219$ V line-to-line

Average phase current $I_{1,nl} = 5.70$ A

Power $P_{nl} = 380$ W

Test 2: Blocked-rotor test at 15 Hz

$$\text{Applied voltage } V = 26.5 \text{ V line-to-line}$$

$$\text{Average phase current } I_{1,\text{bl}} = 18.57 \text{ A}$$

$$\text{Power } P_{\text{bl}} = 675 \text{ W}$$

Test 3: Average dc resistance per stator phase (measured immediately after test 2)

$$R_1 = 0.262\ \Omega$$

Test 4: Blocked-rotor test at 60 Hz

$$\text{Applied voltage } V = 212 \text{ V line-to-line}$$

$$\text{Average phase current } I_{1,\text{bl}} = 83.3 \text{ A}$$

$$\text{Power } P_{\text{bl}} = 20.1 \text{ kW}$$

$$\text{Measured starting torque } T_{\text{start}} = 74.2 \text{ N}\cdot\text{m}$$

a. Compute the no-load rotational loss and the equivalent-circuit parameters applying to the normal running conditions. Assume the same temperature as in test 3. Neglect any effects of core loss, assuming that core loss can be lumped in with the rotational losses.

b. Compute the electromechanical starting torque from the input measurements of test 4. Assume the same temperature as in test 3.

■ Solution

a. From Eq. 6.37, the rotational losses can be calculated as

$$P_{\text{rot}} = P_{\text{nl}} - n_{\text{ph}} I_{1,\text{nl}}^2 R_1 = 380 - 3 \times 5.70^2 \times 0.262 = 354 \text{ W}$$

The line-to-neutral no-load voltage is equal to $V_{1,\text{nl}} = 219/\sqrt{3} = 126.4$ V and thus, from Eqs. 6.43 and 6.44,

$$Q_{\text{nl}} = \sqrt{(n_{\text{ph}} V_{1,\text{nl}} I_{1,\text{nl}})^2 - P_{\text{nl}}^2} = \sqrt{(3 \times 126.4 \times 5.7)^2 - 380^2} = 2128 \text{ W}$$

and thus from Eq. 6.45

$$X_{\text{nl}} = \frac{Q_{\text{nl}}}{n_{\text{ph}} I_{1,\text{nl}}^2} = \frac{2128}{3 \times 5.7^2} = 21.8\ \Omega$$

We can assume that the blocked-rotor test at a reduced frequency of 15 Hz and rated current reproduces approximately normal running conditions in the rotor. Thus, from test 2 and Eqs. 6.47 and 6.48 with $V_{1,\text{bl}} = 26.5/\sqrt{3} = 15.3$ V

$$Q_{\text{bl}} = \sqrt{(n_{\text{ph}} V_{1,\text{bl}} I_{1,\text{bl}})^2 - P_{\text{bl}}^2} = \sqrt{(3 \times 15.3 \times 18.57)^2 - 675^2} = 520 \text{ VA}$$

and thus from Eq. 6.49

$$X_{\text{bl}} = \left(\frac{f_r}{f_{\text{bl}}}\right)\left(\frac{Q_{\text{bl}}}{n_{\text{ph}} I_{1,\text{bl}}^2}\right) = \left(\frac{60}{15}\right)\left(\frac{520}{3 \times 18.57^2}\right) = 2.01\ \Omega$$

Since we are told that this is a Class C motor, we can refer to Table 6.1 and assume that $X_1 = 0.3(X_1 + X_2)$ or $X_1 = kX_2$, where $k = 0.429$. Substituting into Eq. 6.57 results in a quadratic in X_2

$$k^2 X_2^2 + (X_{\text{bl}}(1-k) - X_{\text{nl}}(1+k))X_2 + X_{\text{nl}} X_{\text{bl}} = 0$$

or

$$(0.429)^2 X_2 + (2.01(1-0.429) - 22.0(1+0.429))X_2 + 22.0(2.01)$$
$$= 0.184 X_2^2 - 30.29 X_2 + 44.22 = 0$$

Solving gives two roots: 1.48 and 163.1. Clearly, X_2 must be less than X_{nl} and hence it is easy to identify the proper solution as

$$X_2 = 1.48\ \Omega$$

and thus

$$X_1 = 0.633\ \Omega$$

From Eq. 6.58,

$$X_m = X_{nl} - X_1 = 21.2\ \Omega$$

R_{bl} can be found from Eq. 6.50 as

$$R_{bl} = \frac{P_{bl}}{n_{ph} I_{1,bl}^2} = \frac{675}{3 \times 18.57^2} = 0.652\ \Omega$$

and thus from Eq. 6.56

$$R_2 = (R_{bl} - R_1)\left(\frac{X_2 + X_m}{X_m}\right)^2$$

$$= (0.652 - 0.262)\left(\frac{22.68}{21.2}\right)^2 = 0.447\ \Omega$$

The parameters of the equivalent circuit for small values of slip have now been calculated.

b. Although we could calculate the electromechanical starting torque from the equivalent-circuit parameters derived in part (a), we recognize that this is a double-squirrel-cage motor and hence these parameters (most specifically the rotor parameters) will differ significantly under starting conditions from their low-slip values calculated in part (a). Hence, we will calculate the electromechanical starting torque from the rated-frequency, blocked-rotor test measurements of test 4.

From the power input and stator I^2R losses, the air-gap power P_{gap} is

$$P_{gap} = P_{bl} - n_{ph} I_{1,bl}^2 R_1 = 20{,}100 - 3 \times 83.3^2 \times 0.262 = 14{,}650\ \mathrm{W}$$

Since this is a four-pole machine, the synchronous speed can be found from Eq. 6.26 as $\omega_s = 188.5$ rad/sec. Thus, from Eq. 6.25 with $s = 1$

$$T_{start} = \frac{P_{gap}}{\omega_s} = \frac{14{,}650}{188.5} = 77.7\ \mathrm{N \cdot m}$$

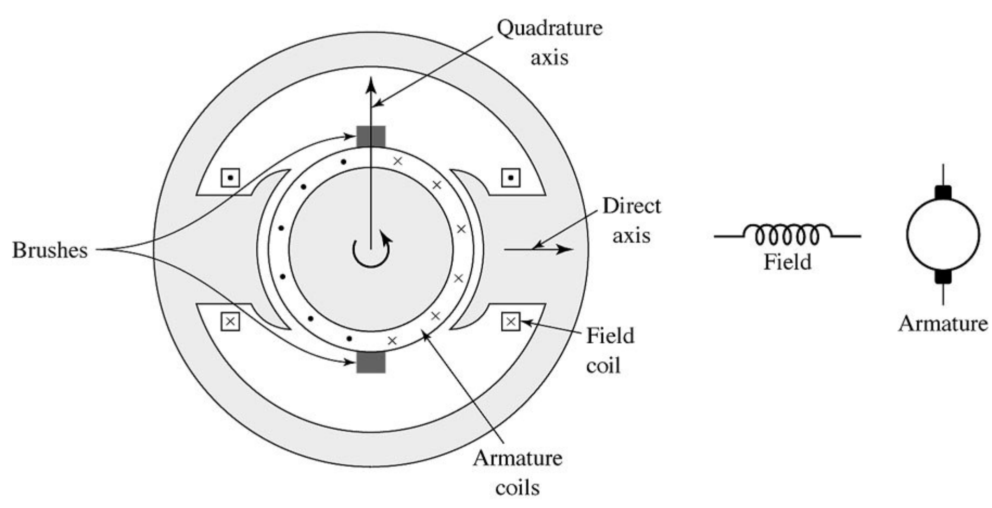
The test value, $T_{start} = 74.2\ \mathrm{N \cdot m}$ is a few percent less than the calculated value because the calculations do not account for the power absorbed in the stator core loss or in stray-load losses.

Chapter 7 DC Machines

- Dc machines are characterized by their versatility.
  - By means of various combinations of shunt-, series-, and separately-excited field windings they can be designed to display a wide variety of volt-ampere or speed-torque characteristics for both dynamic and steady-state operation.
  - Because of the ease with which they can be controlled, systems of dc machines have been frequently used in applications requiring a wide range of motor speeds or precise control of motor output.

§7.1 Introduction

- The essential features of a dc machine are shown schematically in Fig. 7.1.
  - Fig. 7.1(b) shows the circuit representation of the machine.
  - The stator has salient poles and is excited by one or more field coils.
    - → The air-gap flux distribution created by the field windings is symmetric about the center line of the field poles. This axis is called the field axis or direct axis.
  - The ac voltage generated in each rotating armature coil is converted to dc in the external armature terminals by means of a rotating commutator and stationary brushes to which the armature leads are connected.
    - → The commutator-brush combination forms a mechanical rectifier, resulting in a dc armature voltage as well as an armature-mmf wave which is fixed in space.
    - → The brushes are located so that commutation occurs when the coil sides are in the neutral zone, midway between the field poles.
    - → The axis of the armature-mmf wave is 90 electrical degrees from the axis of the field poles, i.e., in the quadrature axis.
    - → The armature-mmf wave is along the brush axis.



(a) (b)

Figure 7.1 Schematic representations of a dc machine.

  - Recall equation (4.81). Note that the torque is proportional to the product of the magnitudes of the interacting fields and to the sine of the electrical space angle between their magnetic axes. The negative sign indicates that the electromechanical torque acts in a direction to decrease the displacement angle between the fields.

$$T = -\frac{\pi}{2}\left(\frac{\text{poles}}{2}\right)^2 \Phi_{\text{sr}} F_{\text{r}} \sin\delta_{\text{r}} \tag{4.81}$$

→ For the dc machine, the electromagnetic torque T_{mech} can be expressed in terms of the interaction of the direct-axis air-gap per pole Φ_{d} and the space-fundamental component F_{a1} of the armature-mmf wave, in a form similar to (4.81). Note that $\sin\delta_{\mathrm{r}} = 1$.

$$T_{\mathrm{mech}} = \frac{\pi}{2}\left(\frac{\mathrm{pole}}{2}\right)^2 \Phi_{\mathrm{d}} F_{\mathrm{a1}} \tag{7.1}$$

$$\boxed{T_{\mathrm{mech}} = K_{\mathrm{a}} \Phi_{\mathrm{d}}\, i_{\mathrm{a}}} \tag{7.2}$$

$$K_{\mathrm{a}} = \frac{\mathrm{poles}\ C_{\mathrm{a}}}{2\pi\, m} \tag{7.3}$$

K_{a}: a constant determined by the design of the winding, the winding constant

i_{a} = current in external armature circuit

C_{a} = total number of conductors in armature winding,

m = number of parallel paths through winding

➢ The rectified voltage e_{a} between brushes, known also as the speed voltage, is

$$\boxed{e_{\mathrm{a}} = K_{\mathrm{a}} \Phi_{\mathrm{d}} \omega_{\mathrm{m}}} \tag{7.4}$$

→ The generated voltage as observed from the brushes is the sum of the rectified voltage of all the coils in series between brushes and is shown by the rippling line labeled e_{a} in Fig. 7.2.

→ With a dozen or so commutator segments per pole, the ripple becomes very small and the average generated voltage observed from the brushes equals the sum of the average values of the rectified coils voltages.

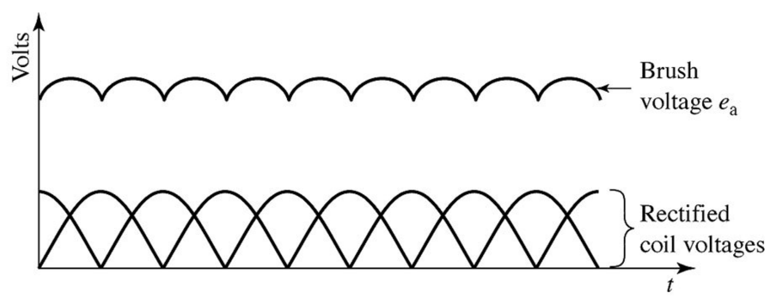


Figure 7.2 Rectified coil voltages and resultant voltage between brushes in a dc machine.

➢ Note that the electric power equals the mechanical power.

$$e_{\mathrm{a}} i_{\mathrm{a}} = T_{\mathrm{mech}} \omega_{\mathrm{m}} \tag{7.5}$$

➢ The flux-mmf characteristic is referred to as the magnetization curve.

→ The direct-axis air-gap flux is produced by the combined mmf $\sum N_{\mathrm{f}} i_{\mathrm{f}}$ of the field winding.

→ The form of a typical magnetization curve is shown in Fig. 7.3(a).

→ The dashed straight line through the origin coinciding with the straight portion of the magnetization curves is called the air-gap line.

→ It is assumed that the armature mmf has no effect on the direct-axis flux because the axis of the armature-mmf wave is along the quadrature axis and hence perpendicular to the field axis. (This assumption needs reexamining!)

→ Note the residual magnetism in the figure. The magnetic material of the field does not fully demagnetize when the net field mmf is reduced to zero.

→ It is usually more convenient to express the magnetization curve in terms of the armature emf e_{a0} at a constant speed ω_{m0} as shown in Fig. 7.3(b).

$$\frac{e_a}{\omega_m} = K_a \Phi_d = \frac{e_{a0}}{\omega_{m0}} \tag{7.6}$$

$$e_a = (\frac{\omega_m}{\omega_{m0}})\, e_{a0} \tag{7.7}$$

$$e_a = (\frac{n}{n_0})\, e_{a0} \tag{7.8}$$

→ Fig. 7.3(c) shows the magnetization curve with only one field winding excited. This curve can easily be obtained by test methods.

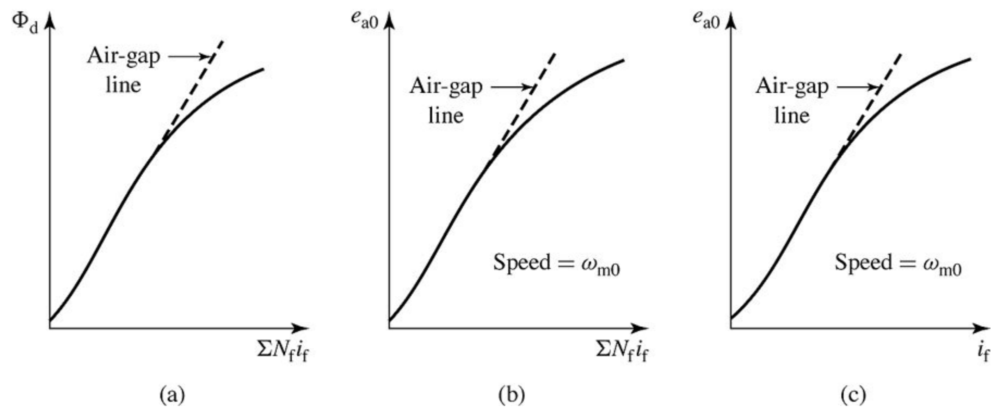


Figure 7.3 Typical form of magnetization curves of a dc machine.

- Various methods of excitation of the field windings are shown in Fig. 7.4.

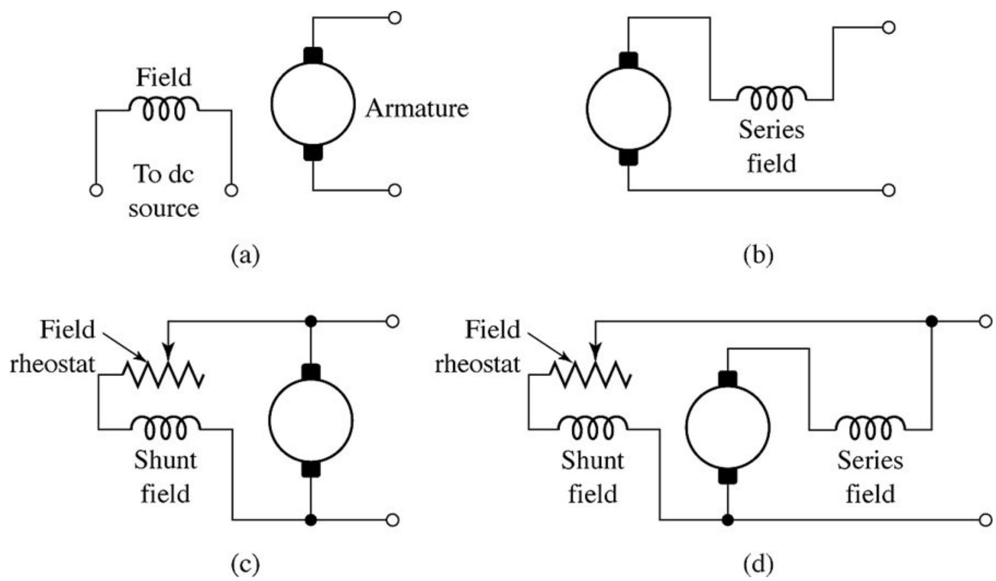


Figure 7.4 Field-circuit connections of dc machines:
(a) separate excitation, (b) series, (c) shunt, (d) compound.

- Consider first dc generators.
  - → Separately-excited generators.
  - → Self-excited generators: series generators, shunt generators, compound generators.
    - ✓ With self-excited generators, residual magnetism must be present in the machine iron to get the self-excitation process started.

- ✓ N.B.: long- and short-shunt, cumulatively and differentially compound.
- → Typical steady-state volt-ampere characteristics are shown in Fig. 7.5, constant-speed operation being assumed.
- → The relation between the steady-state generated emf E_a and the armature terminal voltage V_a is

$$V_a = E_a - I_a R_a \tag{7.10}$$

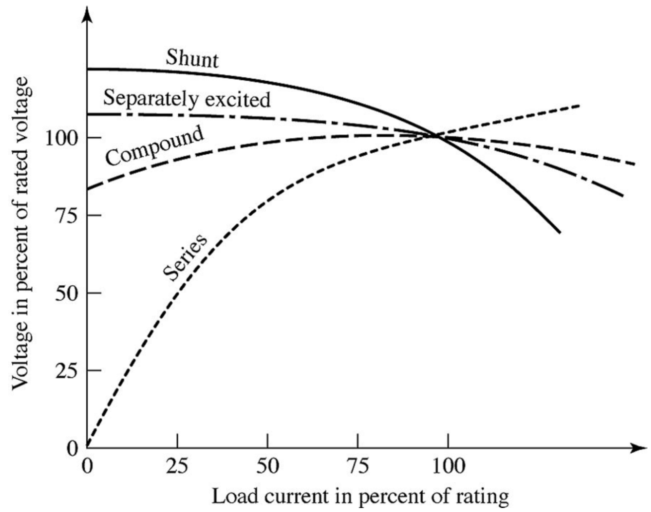


Figure 7.5 Volt-ampere characteristics of dc generators.

- ➢ Any of the methods of excitation used for generators can also be used for motors.
  - → Typical steady-state dc-motor speed-torque characteristics are shown in Fig. 7.6, in which it is assumed that the motor terminals are supplied from a constant-voltage source.
  - → In a motor the relation between the emf E_a generated in the armature and and the armature terminal voltage V_a is

$$V_a = E_a + I_a R_a \tag{7.11}$$

$$I_a = \frac{V_a - E_a}{R_a} \tag{7.12}$$

  - → The application advantages of dc machines lie in the variety of performance characteristics offered by the possibilities of shunt, series, and compound excitation.

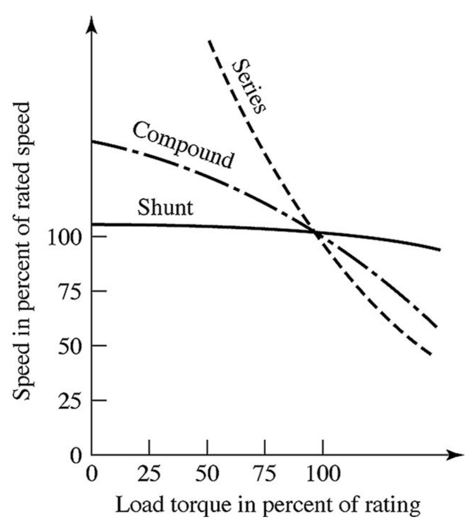


Figure 7.6 Speed-torque characteristics of dc motors.

§7.4 Analytical Fundamentals: Electric-Circuit Aspects

- Analysis of dc machines: electric-circuit and magnetic-circuit aspects

  - Torque and power:
    The electromagnetic torque T_{mech}
    $$T_{\mathrm{mech}} = K_{\mathrm{a}}\Phi_{\mathrm{d}}I_{\mathrm{a}} \tag{7.13}$$
    The generated voltage E_{a}
    $$E_{\mathrm{a}} = K_{\mathrm{a}}\Phi_{\mathrm{d}}\omega_{\mathrm{m}} \tag{7.14}$$
    $$K_{\mathrm{a}} = \frac{\mathrm{poles}C_{\mathrm{a}}}{2\pi\, m} \tag{7.15}$$
    $E_{\mathrm{a}}I_{\mathrm{a}}$: electromagnetic power
    $$T_{\mathrm{mech}} = \frac{E_{\mathrm{a}}I_{\mathrm{a}}}{\omega_{\mathrm{m}}} = K_{\mathrm{a}}\Phi_{\mathrm{d}}I_{\mathrm{a}} \tag{7.16}$$
    Note that the electromagnetic power differs from the mechanical power at the machine shaft by the rotational losses and differs from the electric power at the machine terminals by the shunt-field and armature I^2R losses.

  - Voltage and current (Refer to Fig. 7.12.):

    V_{a} : the terminal voltage of the armature winding
    V_{t} : the terminal voltage of the dc machine, including the voltage drop across the series-connected field winding
    $V_{\mathrm{a}} = V_{\mathrm{t}}$ if there is no series field winding
    R_{a} : the resistance of armature, R_{s} : the resistance of the series field

    $$V_{\mathrm{a}} = E_{\mathrm{a}} \pm I_{\mathrm{a}}R_{\mathrm{a}} \tag{7.17}$$
    $$V_{\mathrm{t}} = E_{\mathrm{a}} \pm I_{\mathrm{a}}\left(R_{\mathrm{a}} + R_{\mathrm{s}}\right) \tag{7.18}$$
    $$I_{\mathrm{L}} = I_{\mathrm{a}} \pm I_{\mathrm{f}} \tag{7.19}$$

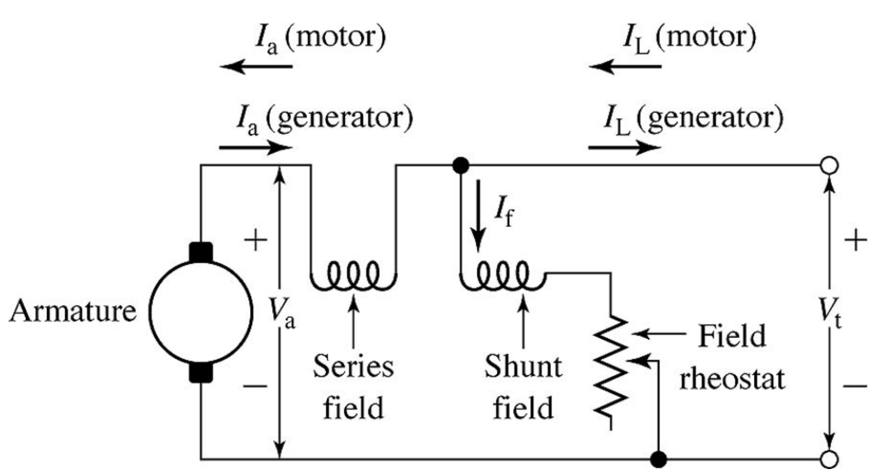


Figure 7.12 Motor or generator connection diagram with current directions.

EXAMPLE 7.1

A 25-kW 125-V separately-excited dc machine is operated at a constant speed of 3000 r/min with a constant field current such that the open-circuit armature voltage is 125 V. The armature resistance is 0.02 Ω.

Compute the armature current, terminal power, and electromagnetic power and torque when the terminal voltage is (*a*) 128 V and (*b*) 124 V.

■ **Solution**

a. From Eq. 7.17, with $V_t = 128$ V and $E_a = 125$ V, the armature current is

$$I_a = \frac{V_t - E_a}{R_a} = \frac{128 - 125}{0.02} = 150 \text{ A}$$

in the motor direction, and the power input at the motor terminal is

$$V_t I_a = 128 \times 150 = 19.20 \text{ kW}$$

The electromagnetic power is given by

$$E_a I_a = 125 \times 150 = 18.75 \text{ kW}$$

In this case, the dc machine is operating as a motor and the electromagnetic power is hence smaller than the motor input power by the power dissipated in the armature resistance.

Finally, the electromagnetic torque is given by Eq. 7.16:

$$T_{\text{mech}} = \frac{E_a I_a}{\omega_m} = \frac{18.75 \times 10^3}{100\pi} = 59.7 \text{ N} \cdot \text{m}$$

b. In this case, E_a is larger than V_t and hence armature current will flow out of the machine, and thus the machine is operating as a generator. Hence

$$I_a = \frac{E_a - V_t}{R_a} = \frac{125 - 124}{0.02} = 50 \text{ A}$$

and the terminal power is

$$V_t I_a = 124 \times 50 = 6.20 \text{ kW}$$

The electromagnetic power is

$$E_a I_a = 125 \times 50 = 6.25 \text{ kW}$$

and the electromagnetic torque is

$$T_{\text{mech}} = \frac{6.25 \times 10^3}{100\pi} = 19.9 \text{ N} \cdot \text{m}$$

EXAMPLE 7.2

Consider again the separately-excited dc machine of Example 7.1 with the field-current maintained constant at the value that would produce a terminal voltage of 125 V at a speed of 3000 r/min. The machine is observed to be operating as a motor with a terminal voltage of 123 V and with a terminal power of 21.9 kW. Calculate the speed of the motor.

■ **Solution**

The terminal current can be found from the terminal voltage and power as

$$I_a = \frac{\text{Input power}}{V_t} = \frac{21.9 \times 10^3}{123} = 178 \text{ A}$$

Thus the generated voltage is

$$E_a = V_t - I_a R_a = 119.4 \text{ V}$$

From Eq. 7.8, the rotational speed can be found as

$$n = n_0 \left(\frac{E_a}{E_{a0}} \right) = 3000 \left(\frac{119.4}{125} \right) = 2866 \text{ r/min}$$

- For compound machines, Fig. 7.12 shows a long-shunt connection and the short-shunt connection is illustrated in Fig. 7.13.

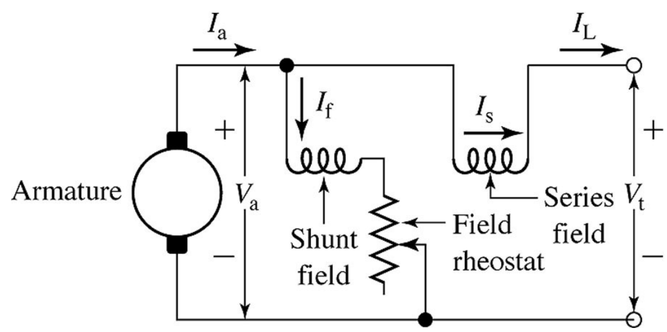


Figure 7.13 Short-shunt compound-generator connections.

§7.5 Analytical Fundamentals: Magnetic-Circuit Aspects

- The net flux per pole is that resulting from the combined mmf's of the field and armature windings.
  - First we consider the mmf intentionally placed on the stator main poles to create the working flux, i.e., the main-field mmf, and then we include armature-reaction effects.

§7.5.1 Armature Reaction Neglected

- With no load on the machine or with armature-reaction effects ignored, the resultant mmf is the algebraic sum of the mmf's acting on the main or direct axis.

$$\text{Main} - \text{field mmf} = N_f I_f \pm N_s I_s \tag{7.20}$$

$$\text{Gross mmf} = I_f + \left(\frac{N_s}{N_f} \right) I_s \text{ equivalent shunt-field amperes} \tag{7.21}$$

  - An example of a no-load magnetization characteristic is given by the curve for $I_a = 0$ in Fig. 7.14.
  - The generated voltage E_a at any speed ω_m is given by

$$E_a = \left(\frac{\omega_m}{\omega_{m0}} \right) E_{a0} \tag{7.22}$$

$$E_a = \left(\frac{n}{n_0} \right) E_{a0} \tag{7.23}$$

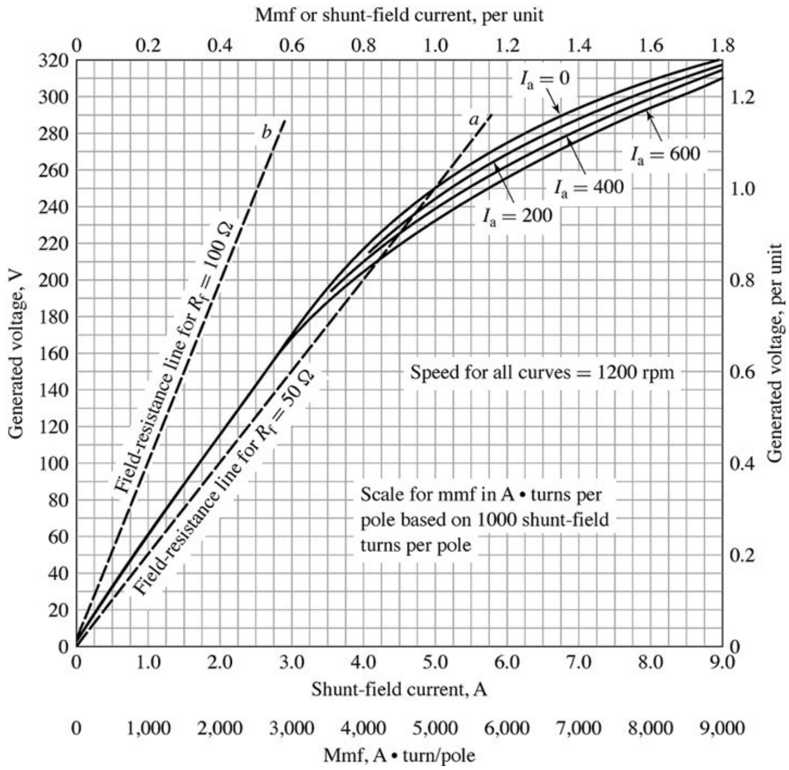


Figure 7.14 Magnetization curves for a 100-kW, 250-V, 1200-r/min dc machine. Also shown are field-resistance lines for the discussion of self-excitation in § 7.6.1.

EXAMPLE 7.3

A 100-kW, 250-V, 400-A, long-shunt compound generator has an armature resistance (including brushes) of 0.025 Ω, a series-field resistance of 0.005 Ω, and the magnetization curve of Fig. 7.14. There are 1000 shunt-field turns per pole and three series-field turns per pole. The series field is connected in such a fashion that positive armature current produces direct-axis mmf which adds to that of the shunt field.

Compute the terminal voltage at rated terminal current when the shunt-field current is 4.7 A and the speed is 1150 r/min. Neglect the effects of armature reaction.

■ Solution

As is shown in Fig. 7.12, for a long-shunt connection the armature and series field-currents are equal. Thus

$$I_s = I_a = I_L + I_f = 400 + 4.7 = 405 \text{ A}$$

From Eq. 7.21 the main-field gross mmf is

$$\text{Gross mmf} = I_f + \left(\frac{N_s}{N_f}\right) I_s$$

$$= 4.7 + \left(\frac{3}{1000}\right) 405 = 5.9 \quad \text{equivalent shunt-field amperes}$$

By examining the $I_a = 0$ curve of Fig. 7.14 at this equivalent shunt-field current, one reads a generated voltage of 274 V. Accordingly, the actual emf at a speed of 1150 r/min can be found from Eq. 7.23

$$E_a = \left(\frac{n}{n_0}\right) E_{a0} = \left(\frac{1150}{1200}\right) 274 = 263 \text{ V}$$

Then

$$V_t = E_a - I_a(R_a + R_s) = 263 - 405(0.025 + 0.005) = 251 \text{ V}$$

§7.5.2 Effects of Armature Reaction Included

- Current in the armature winding gives rise to a demagnetizing effect caused by a cross-magnetizing armature reaction.
  - One common approach is to base analyses on the measured performance of the machine.
    - → Data are taken with both the field and armature excited, and the tests are conducted so that the effects on generated emf of varying both the main-field excitation and armature mmf can be noted.
    - → Refer to Fig. 7.14. The inclusion of armature reaction becomes simply a matter of using the magnetization curve corresponding to the armature current involved.
    - → The load-saturation curves are displaced to the right of the no-load curve by an amount which is a function of I_a.
    - → The effect of armature reaction is approximately the same as a demagnetizing mmf F_{ar} acting on the main-field axis.

$$\text{Net mmf} = \text{gross mmf} - F_{ar} = N_f I_f + N_s I_s - AR \tag{7.24}$$

    - → Over the normal operating range, the demagnetizing effect of armature reaction may be assumed to be approximately proportional to the armature current.
    - → The amount of armature of armature reaction present in Fig. 7.14 is definitely more than one would expect to find in a normal, well-designed machine operating at normal currents.

EXAMPLE 7.4

Consider again the long-shunt compound dc generator of Example 7.3. As in Example 7.3, compute the terminal voltage at rated terminal current when the shunt-field current is 4.7 A and the speed is 1150 r/min. In this case however, include the effects of armature reaction.

■ Solution

As calculated in Example 7.3, $I_s = I_a = 400$ A and the gross mmf is equal to 5.9 equivalent shunt-field amperes. From the curve labeled $I_a = 400$ in Fig. 7.14 (based upon a rated terminal current of 400 A), the corresponding generated emf is found to be 261 V (as compared to 274 V with armature reaction neglected). Thus from Eq. 7.23, the actual generated voltage at a speed of 1150 r/min is equal to

$$E_a = \left(\frac{n}{n_0}\right) E_{a0} = \left(\frac{1150}{1200}\right) 261 = 250 \text{ V}$$

Then

$$V_t = E_a - I_a(R_a + R_s) = 250 - 405(0.025 + 0.005) = 238 \text{ V}$$

EXAMPLE 7.5

To counter the effects of armature reaction, a fourth turn is added to the series field winding of the dc generator of Examples 7.3 and 7.4, increasing its resistance to 0.007 Ω. Repeat the terminal-voltage calculation of Example 7.4.

■ Solution

As in Examples 7.3 and 7.4, $I_s = I_a = 405$ A. The main-field mmf can then be calculated as

$$\text{Gross mmf} = I_f + \left(\frac{N_s}{N_f}\right) I_s = 4.7 + \left(\frac{4}{1000}\right) 405$$
$$= 6.3 \quad \text{equivalent shunt-field amperes}$$

From the $I_a = 400$ curve of Fig. 7.14 with an equivalent shunt-field current of 6.3 A, one reads a generated voltage 269 V which corresponds to an emf at 1150 r/min of

$$E_a = \left(\frac{1150}{1200}\right) 269 = 258 \text{ V}$$

The terminal voltage can now be calculated as

$$V_t = E_a - I_a(R_a + R_s) = 258 - 405(0.025 + 0.007) = 245 \text{ V}$$

---

§7.6 Analysis of Steady-State Performance

- Generator operation and motor operation
  - For a generator, the speed is usually fixed by the prime mover, and problems often encountered are to determine the terminal voltage corresponding to a specified load and excitation or to find the excitation required for a specified load and terminal voltage.
  - For a motor, problems frequently encountered are to determine the speed corresponding to a specific load and excitation or to find the excitation required for specific load and speed conditions; terminal voltage is often fixed at the value of the available source.

§7.6.1 Generator Analysis

- Analysis is based on the type of field connection.
  - Separately-excited generators are the simplest to analyze.
    - → Its main-field current is independent of the generator voltage.
    - → For a given load, the equivalent main-field excitation is given by (7.21) and the associated armature-generated voltage E_a is determined by the appropriate magnetization curve.
    - → The voltage E_a, together with (7.17) or (7.18), fixes the terminal voltage.
  - Shunt-excited generators will be found to self-excite under properly chosen operating condition under which the generated voltage will build up spontaneously.
    - → The process is typically initiated by the presence of a small amount of residual magnetism in the field structure and the shunt-field excitation depends on the terminal voltage. Consider the field-resistance line, the line 0*a* in Fig. 7.14.
    - → The tendency of a shunt-connected generator to self-excite can be observed by examining the buildup of voltage for an unloaded shunt generator.
      - – Buildup continues until the volt-ampere relations represented by the magnetization curve and the field-resistance line are simultaneously satisfied.

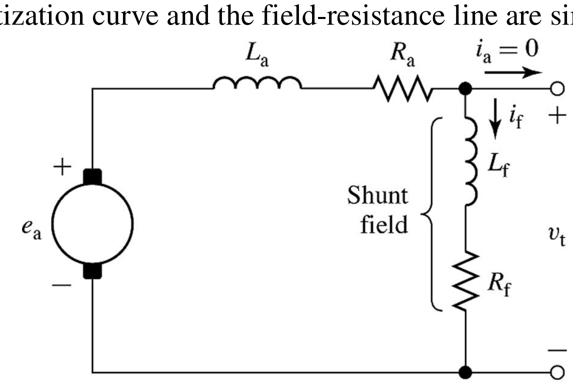


Figure 7.15 Equivalent circuit for analysis of voltage buildup in a self-excited dc generator.

Note that in Fig. 7.15,

$$\left(L_{\mathrm{a}}+L_{\mathrm{f}}\right)\frac{di_{\mathrm{f}}}{dt}=e_{\mathrm{a}}-\left(R_{\mathrm{a}}+R_{\mathrm{f}}\right)i_{\mathrm{f}} \tag{7.25}$$

→ The field resistance line should also include the armature resistance.
→ Notice that if the field resistance is too high, as shown by line 0*b* in Fig. 7.14, voltage buildup will not be achieved.
→ The critical field resistance, corresponding to the slope of the field-resistance line tangent to the magnetization curve, is the resistance above which buildup will not be obtained.

EXAMPLE 7.6

A 100-kW, 250-V, 400-A, 1200-r/min dc shunt generator has the magnetization curves (including armature-reaction effects) of Fig. 7.14. The armature-circuit resistance, including brushes, is 0.025 Ω. The generator is driven at a constant speed of 1200 r/min, and the excitation is adjusted (by varying the shunt-field rheostat) to give rated voltage at no load.

(*a*) Determine the terminal voltage at an armature current of 400 A. (*b*) A series field of four turns per pole having a resistance of 0.005 Ω is to be added. There are 1000 turns per pole in the shunt field. The generator is to be ***flat-compounded*** so that the full-load voltage is 250 V when the shunt-field rheostat is adjusted to give a no-load voltage of 250 V. Show how a resistance across the series field (referred to as a ***series-field diverter***) can be adjusted to produce the desired performance.

■ Solution

a. The 50 Ω field-resistance line 0*a* (Fig. 7.14) passes through the 250-V, 5.0-A point of the no-load magnetization curve. At $I_a = 400$ A

$$I_a R_a = 400 \times 0.025 = 10\ \text{V}$$

Thus the operating point under this condition corresponds to a condition for which the terminal voltage V_t (and hence the shunt-field voltage) is 10 V less than the generated voltage E_a.

A vertical distance of 10 V exists between the magnetization curve for $I_a = 400$ A and the field-resistance line at a field current of 4.1 A, corresponding to $V_t = 205$ V. The associated line current is

$$I_L = I_a - I_f = 400 - 4 = 396\ \text{A}$$

Note that a vertical distance of 10 V also exists at a field current of 1.2 A, corresponding to $V_t = 60$ V. The voltage-load curve is accordingly double-valued in this region. It can be shown that this operating point is unstable and that the point for which $V_t = 205$ V is the normal operating point.

b. For the no-load voltage to be 250 V, the shunt-field resistance must be 50 Ω and the field-resistance line is 0*a* (Fig. 7.14). At full load, $I_f = 5.0$ A because $V_t = 250$ V. Then

$$I_a = 400 + 5.0 = 405\ \text{A}$$

and

$$E_a = V_t + I_a(R_a + R_p) = 250 + 405(0.025 + R_p)$$

where R_p is the parallel combination of the series-field resistance $R_s = 0.005\ \Omega$ and the diverter resistance R_d

$$R_p = \frac{R_s R_d}{(R_s + R_d)}$$

The series field and the diverter resistor are in parallel, and thus the shunt-field current can be calculated as

$$I_s = 405\left(\frac{R_d}{R_s + R_d}\right) = 405\left(\frac{R_p}{R_s}\right)$$

and the equivalent shunt-field amperes can be calculated from Eq. 7.21 as

$$I_{net} = I_f + \frac{4}{1000} I_s = 5.0 + \frac{4}{1000} I_s$$
$$= 5.0 + 1.62 \left(\frac{R_p}{R_s}\right)$$

This equation can be solved for R_p which can be, in turn, substituted (along with $R_s = 0.005\ \Omega$) in the equation for E_a to yield

$$E_a = 253.9 + 1.25 I_{net}.$$

This can be plotted on Fig. 7.14 (E_a on the vertical axis and I_{net} on the horizontal axis). Its intersection with the magnetization characteristic for $I_a = 400$ A (strictly speaking, of course, a curve for $I_a = 405$ A should be used, but such a small distinction is obviously meaningless here) gives $I_{net} = 6.0$ A.

Thus

$$R_p = \frac{R_s(I_{net} - 5.0)}{1.62} = 0.0031\ \Omega$$

and

$$R_d = 0.0082\ \Omega$$

§7.6.2 Motor Analysis

- The terminal voltage of a motor is usually held substantially constant or controlled to a specific value. Motor analysis is most nearly resembles that for separately-excited generators.
  - Speed is an important variable and often the one whose value is to be found.

$$V_a = E_a \pm I_a R_a \tag{7.17}$$

$$V_t = E_a \pm I_a (R_a + R_s) \tag{7.18}$$

$$\text{Gross mmf} = I_f + \left(\frac{N_s}{N_f}\right) I_s \ \text{ equivalent shunt-field amperes} \tag{7.21}$$

$$T_{mech} = K_a \Phi_d I_a \tag{7.13}$$

$$E_a = K_a \Phi_d \omega_m \tag{7.14}$$

$$E_a = \left(\frac{\omega_m}{\omega_{m0}}\right) E_{a0} \tag{7.22}$$

$$E_a = \left(\frac{n}{n_0}\right) E_{a0} \tag{7.23}$$

EXAMPLE 7.7

A 100-hp, 250-V dc shunt motor has the magnetization curves (including armature-reaction effects) of Fig. 7.14. The armature circuit resistance, including brushes, is 0.025 Ω. No-load rotational losses are 2000 W and the stray-load losses equal 1.0% of the output. The field rheostat is adjusted for a no-load speed of 1100 r/min.

a. As an example of computing points on the speed-load characteristic, determine the speed in r/min and output in horsepower (1 hp = 746 W) corresponding to an armature current of 400 A.
b. Because the speed-load characteristic observed to in part (a) is considered undesirable, a *stabilizing winding* consisting of 1-1/2 cumulative series turns per pole is to be added. The resistance of this winding is assumed negligible. There are 1000 turns per pole in the shunt field. Compute the speed corresponding to an armature current of 400 A.

■ **Solution**

a. At no load, $E_a = 250$ V. The corresponding point on the 1200-r/min no-load saturation curve is

$$E_{a0} = 250\left(\frac{1200}{1100}\right) = 273\ \text{V}$$

for which $I_f = 5.90$ A. The field current remains constant at this value.

At $I_a = 400$ A, the actual counter emf is

$$E_a = 250 - 400 \times 0.025 = 240\ \text{V}$$

From Fig. 7.14 with $I_a = 400$ and $I_f = 5.90$, the value of E_a would be 261 V if the speed were 1200 r/min. The actual speed is then found from Eq. 7.23

$$n = 1200\left(\frac{240}{261}\right) = 1100\ \text{r/min}$$

The electromagnetic power is

$$E_a I_a = 240 \times 400 = 96\ \text{kW}$$

Deduction of the rotational losses leaves 94 kW. With stray load losses accounted for, the power output P_0 is given by

$$94\ \text{kW} - 0.01 P_0 = P_0$$

or

$$P_0 = 93.1\ \text{kW} = 124.8\ \text{hp}$$

Note that the speed at this load is the same as at no load, indicating that armature-reaction effects have caused an essentially flat speed-load curve.

b. With $I_f = 5.90$ A and $I_s = I_a = 400$ A, the main-field mmf in equivalent shunt-field amperes is

$$5.90 + \left(\frac{1.5}{1000}\right)400 = 6.50\ \text{A}$$

From Fig. 7.14 the corresponding value of E_a at 1200 r/min would be 271 V. Accordingly, the speed is now

$$n = 1200\left(\frac{240}{271}\right) = 1063\ \text{r/min}$$

The power output is the same as in part (a). The speed-load curve is now drooping, due to the effect of the stabilizing winding.