مولفه های تنش در این دودکش چه هستند؟

برای ساخت آن، کدام تنش ها مهم هستند؟

قبلا درمورد تنش های چندگانه در یک نقطه صحبت کرده ایم:

- تیرها – در یک نقطه هم تنش برشی و هم نرمال میگیرند
- مخازن فشار – دو تنش نرمال دارند.

اکنون به مبحث تنش های ناشی از بارهای مرکب در یک نقطه می پردازیم. روش انجام:

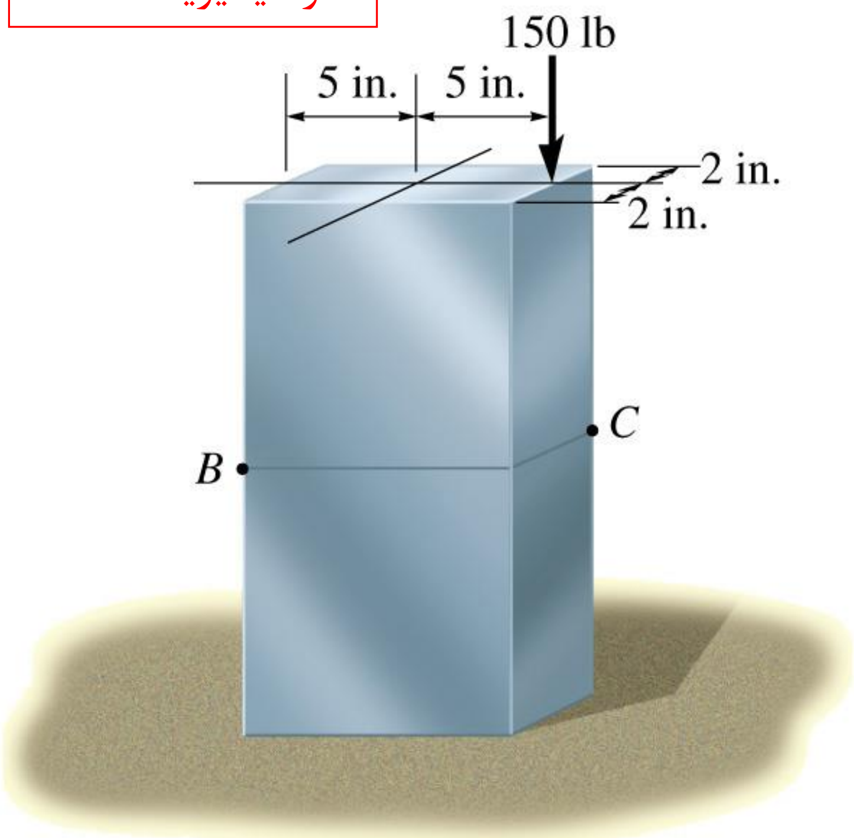
- باید مشخص کنید نقطه بحرانی کجا قرار دارد (یعنی کجا تنش ماکزیمم رخ می دهد)
- عضو را در آن نقطه برش بزنید و نیروهای داخلی را تعیین کنید.
- تنش ها را با استفاده از نیروهای داخلی تعیین کنید
- تنش های همسان را جمع کنید. یعنی

➢ : $\sigma_{total} = \sigma_1 + \sigma_2 + \sigma_3 + \ldots\ldots\ldots\ldots\ldots$

➢ $\tau_{total} = \tau_1 + \tau_2 + \tau_3 + \ldots\ldots\ldots\ldots\ldots$

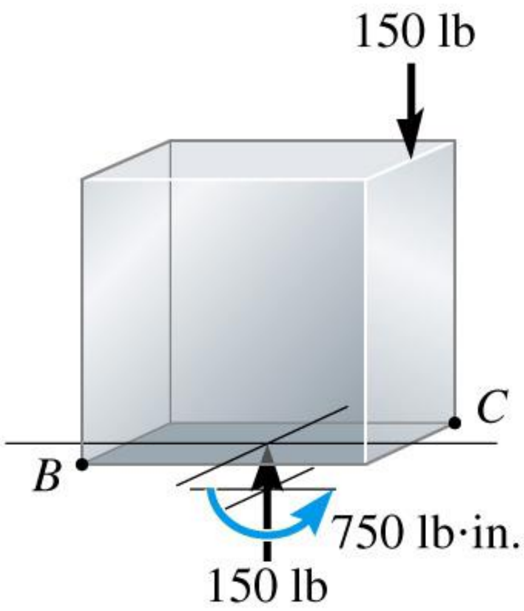
این تنش ها را روی یک المان به صورت خلاصه نشان دهید که به آن المان تنش اولیه یا المان تنش تراز شده میگویند.

۱) تصمیم بگیرید
چه نقطه ای را در
نظر میگیرید.



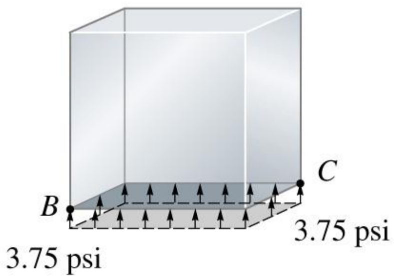
(a)

۲) مقطع بزنید و
نیروهای داخلی را
بدست آورید.



(b)

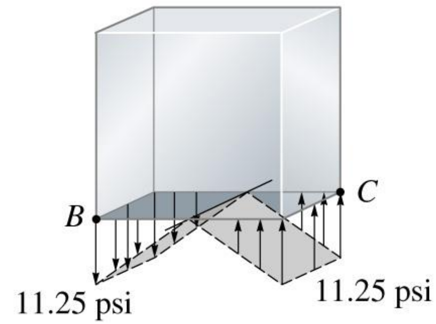
۳) تنش ها را بیابید. تنش های همسان را جمع کنید



Normal Force

(c)

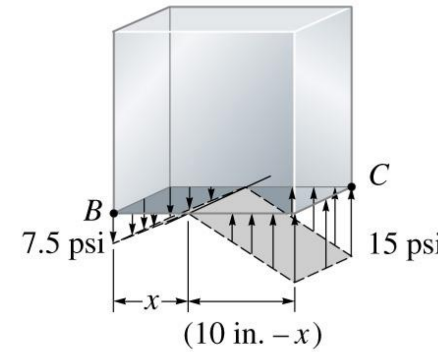
+



Bending Moment

(d)

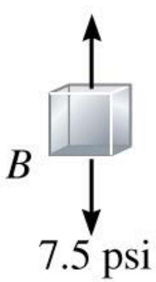
=



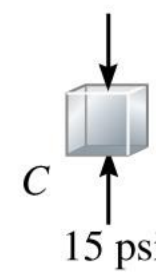
Combined Loading

(e)

۴) همه تنش ها را روی المان تنش اولیه به صورت خلاصه نشان دهید

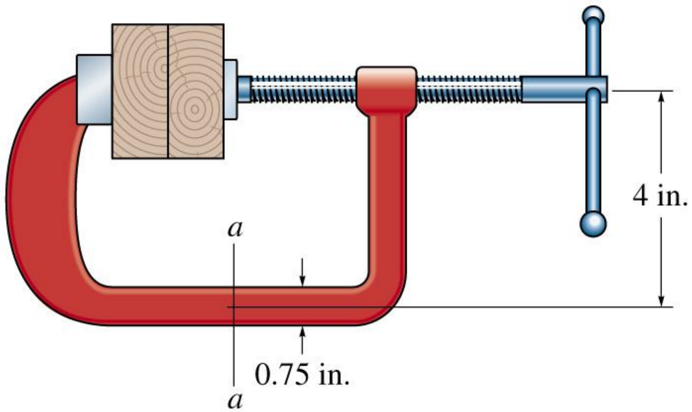


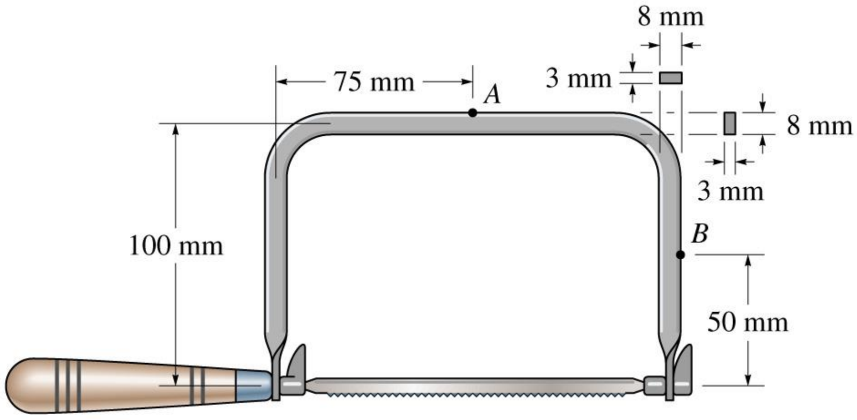
(f)

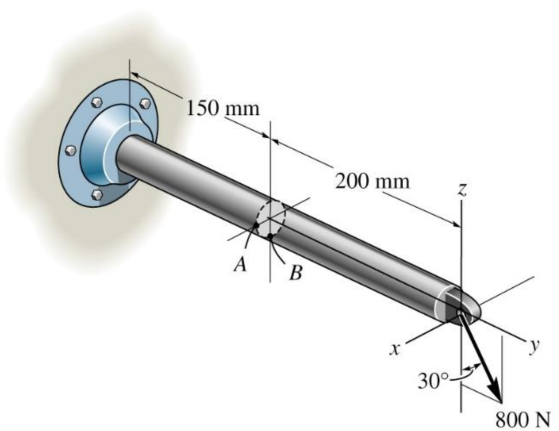


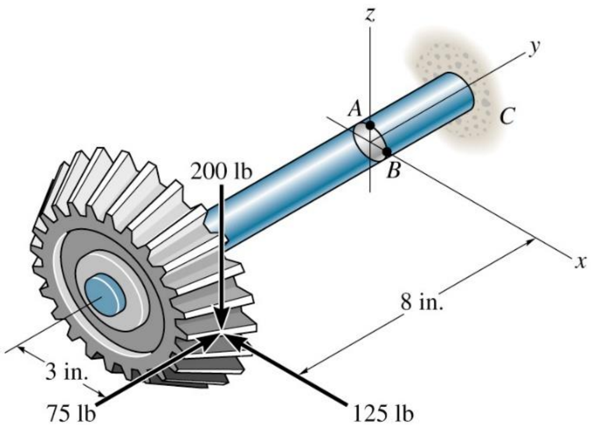
(g)

اگر به دنبال تنش ماکزیمم باشید این سازه ها را در چه نقطه ای تحلیل میکنید؟



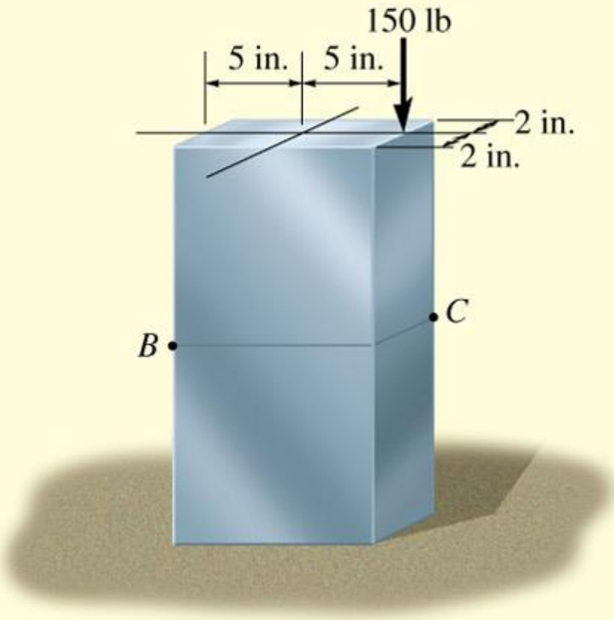






EXAMPLE 8.2

A force of 150 lb is applied to the edge of the member shown in Fig. 8–3*a*. Neglect the weight of the member and determine the state of stress at points *B* and *C*.



(a)

Solution

Internal Loadings. The member is sectioned through *B* and *C*. For equilibrium at the section there must be an axial force of 150 lb acting through the centroid and a bending moment of 750 lb · in. about the centroidal or principal axis, Fig. 8–3*b*.

Stress Components.

Normal Force. The uniform normal-stress distribution due to the normal force is shown in Fig. 8–3*c*. Here

$$\sigma = \frac{P}{A} = \frac{150 \text{ lb}}{(10 \text{ in.})(4 \text{ in.})} = 3.75 \text{ psi}$$

Bending Moment. The normal-stress distribution due to the bending moment is shown in Fig. 8–3*d*. The maximum stress is

$$\sigma_{\max} = \frac{Mc}{I} = \frac{750 \text{ lb} \cdot \text{in.}(5 \text{ in.})}{\left[\frac{1}{12}(4 \text{ in.})(10 \text{ in.})^3\right]} = 11.25 \text{ psi}$$

Superposition. If the above normal-stress distributions are added algebraically, the resultant stress distribution is shown in Fig. 8–3*e*. Although it is not needed here, the location of the line of zero stress can be determined by proportional triangles; i.e.,

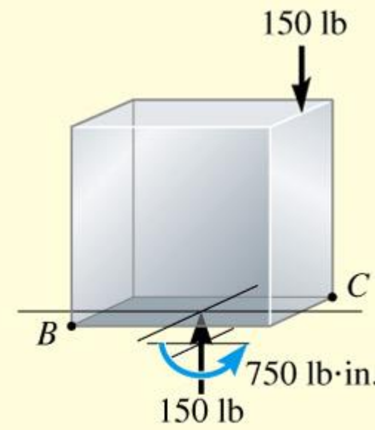
$$\frac{7.5 \text{ psi}}{x} = \frac{15 \text{ psi}}{(10 \text{ in.} - x)}$$

$$x = 3.33 \text{ in.}$$

Elements of material at B and C are subjected only to normal or *uniaxial stress* as shown in Fig. 8–3*f* and 8–3*g*. Hence,

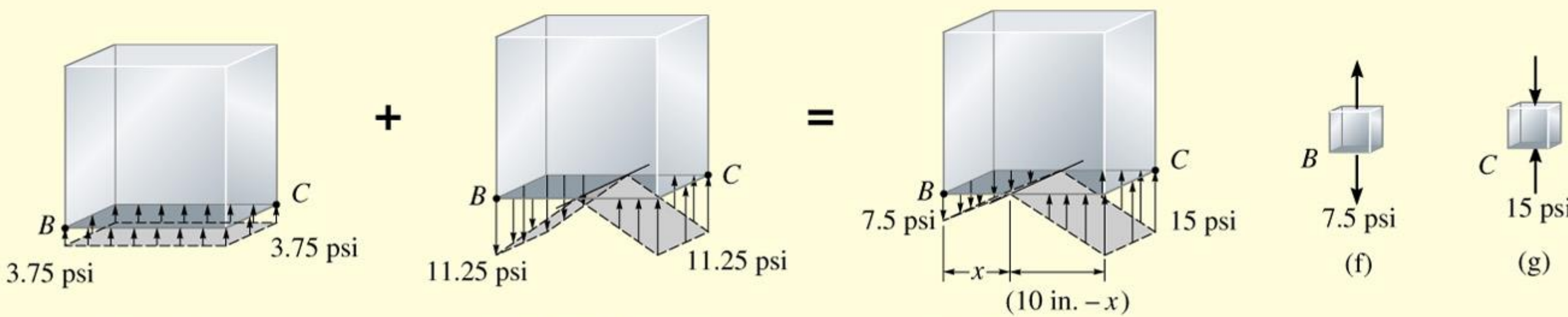
$$\sigma_B = 7.5 \text{ psi} \quad \text{(tension)} \qquad \textit{Ans.}$$

$$\sigma_C = 15 \text{ psi} \quad \text{(compression)} \qquad \textit{Ans.}$$



(b)

Fig. 8–3

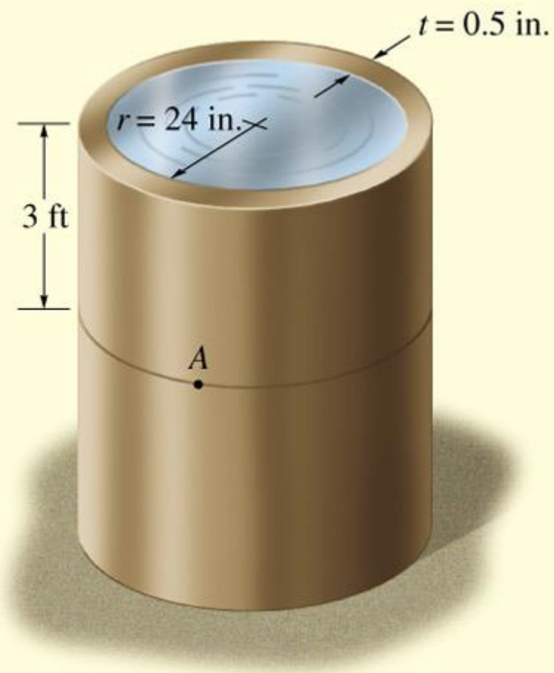


Normal Force (c)

Bending Moment (d)

Combined Loading (e)

EXAMPLE 8.3



(a)

The tank in Fig. 8–4*a* has an inner radius of 24 in. and a thickness of 0.5 in. It is filled to the top with water having a specific weight of $\gamma_w = 62.4\ \text{lb/ft}^3$. If it is made of steel having a specific weight of $\gamma_{st} = 490\ \text{lb/ft}^3$, determine the state of stress at point A. The tank is open at the top.

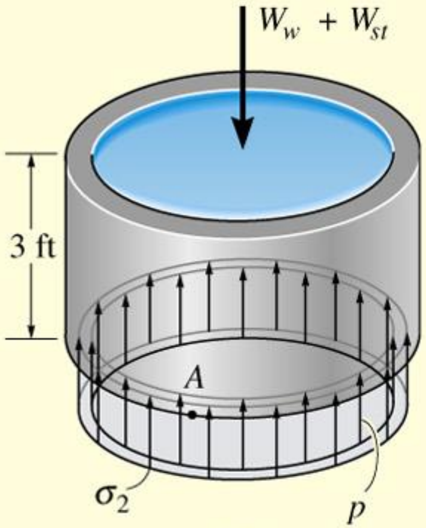
Solution

Internal Loadings. The free-body diagram of the section of both the tank and the water above point A is shown in Fig. 8–4*b*. Notice that the weight of the water is supported by the water surface just *below* the section, *not* by the walls of the tank. In the vertical direction, the walls simply hold up the weight of the tank. This weight is

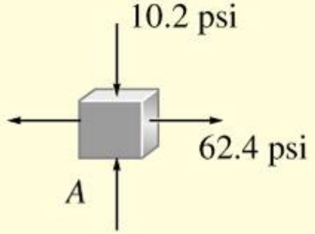
$$W_{st} = \gamma_{st}V_{st} = (490\ \text{lb/ft}^3)\left[\pi\left(\frac{24.5}{12}\text{ft}\right)^2 - \pi\left(\frac{24}{12}\text{ft}\right)^2\right](3\ \text{ft}) = 777.7\ \text{lb}$$

The stress in the circumferential direction is developed by the water pressure at level A. To obtain this pressure we must use *Pascal's law*, which states that the pressure at a point located a depth z in the water is $p = \gamma_w z$. Consequently, the pressure on the tank at level A is

$$p = \gamma_w z = (62.4\ \text{lb/ft}^3)(3\ \text{ft}) = 187.2\ \text{lb/ft}^2 = 1.30\ \text{psi}$$



(b)



(c)

Fig. 8–4

Stress Components.

Circumferential Stress. Applying Eq. 8–1, using the inner radius $r = 24$ in., we have

$$\sigma_1 = \frac{pr}{t} = \frac{1.30 \text{ lb/in}^2 \text{ (24 in.)}}{\text{(0.5 in.)}} = 62.4 \text{ psi} \qquad \textit{Ans.}$$

Longitudinal Stress. Since the weight of the tank is supported uniformly by the walls, we have

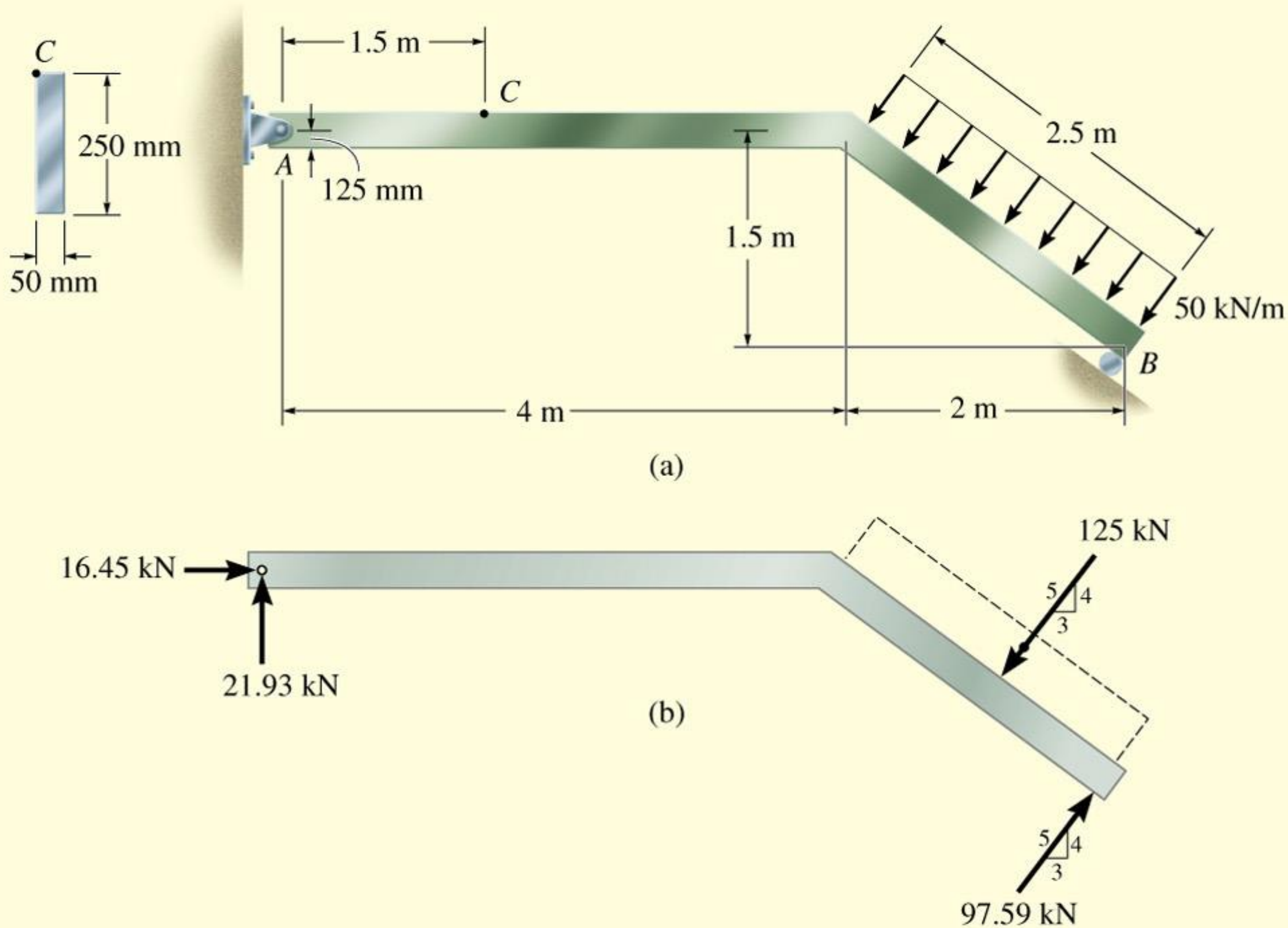
$$\sigma_2 = \frac{W_{st}}{A_{st}} = \frac{777.7 \text{ lb}}{\pi[(24.5 \text{ in.})^2 - (24 \text{ in.})^2]} = 10.2 \text{ psi} \qquad \textit{Ans.}$$

Note that Eq. 8–2, $\sigma_2 = pr/2t$, does *not apply* here, since the tank is open at the top and therefore, as stated previously, the water cannot develop a loading on the walls in the longitudinal direction.

Point A is therefore subjected to the biaxial stress shown in Fig. 8–4c.

EXAMPLE 8.4

The member shown in Fig. 8–5*a* has a rectangular cross section. Determine the state of stress that the loading produces at point *C*.



(a)

(b)

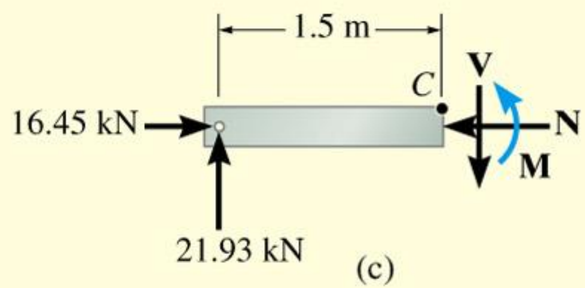


Fig. 8–5

Solution

Internal Loadings. The support reactions on the member have been determined and are shown in Fig. 8–5*b*. If the left segment *AC* of the member is considered, Fig. 8–5*c*, the resultant internal loadings at the section consist of a normal force, a shear force, and a bending moment. Solving,

$$N = 16.45 \text{ kN} \qquad V = 21.93 \text{ kN} \qquad M = 32.89 \text{ kN} \cdot \text{m}$$

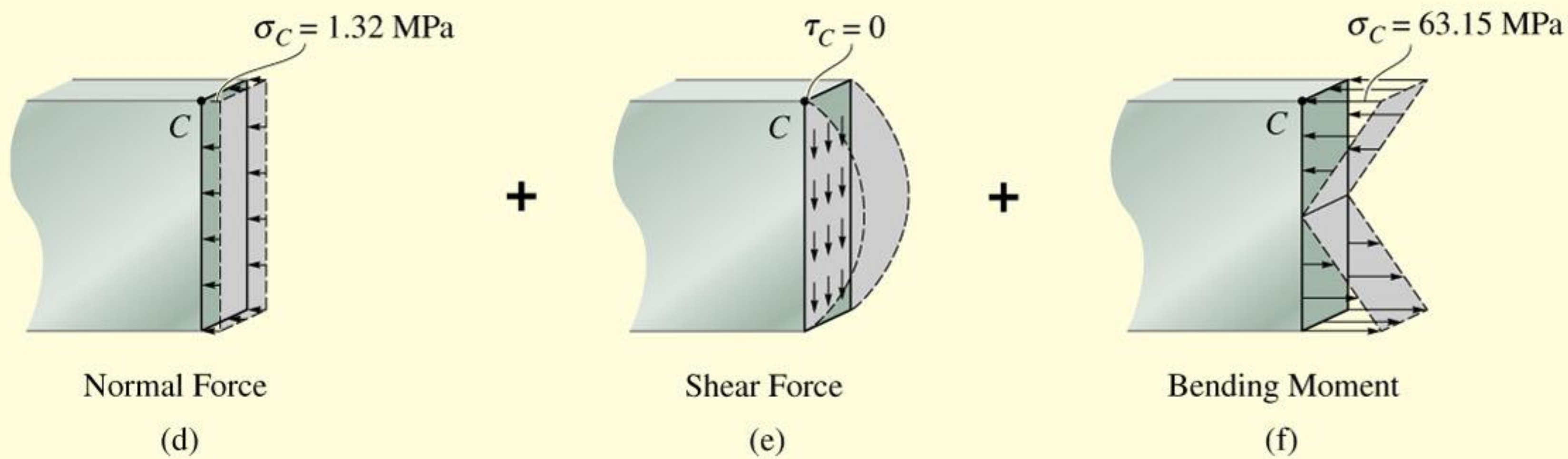


Fig. 8–5 (cont.)

Stress Components.

Normal force. The uniform normal-stress distribution acting over the cross section is produced by the normal force, Fig. 8–5*d*. At point *C*,

$$\sigma_C = \frac{P}{A} = \frac{16.45 \text{ kN}}{(0.050 \text{ m})(0.250 \text{ m})} = 1.32 \text{ MPa}$$

Shear force. Here the area $A' = 0$, since point C is located at the top of the member. Thus $Q = \bar{y}'A' = 0$ and for C, Fig. 8–5*e*, the shear stress

$$\tau_C = 0$$

Bending moment. Point C is located at $y = c = 125$ mm from the neutral axis, so the normal stress at C, Fig. 8–5*f*, is

$$\sigma_C = \frac{Mc}{I} = \frac{(32.89\text{ kN}\cdot\text{m})(0.125\text{ m})}{\left[\frac{1}{12}(0.050\text{ m})(0.250)^3\right]} = 63.15\text{ MPa}$$

(g)

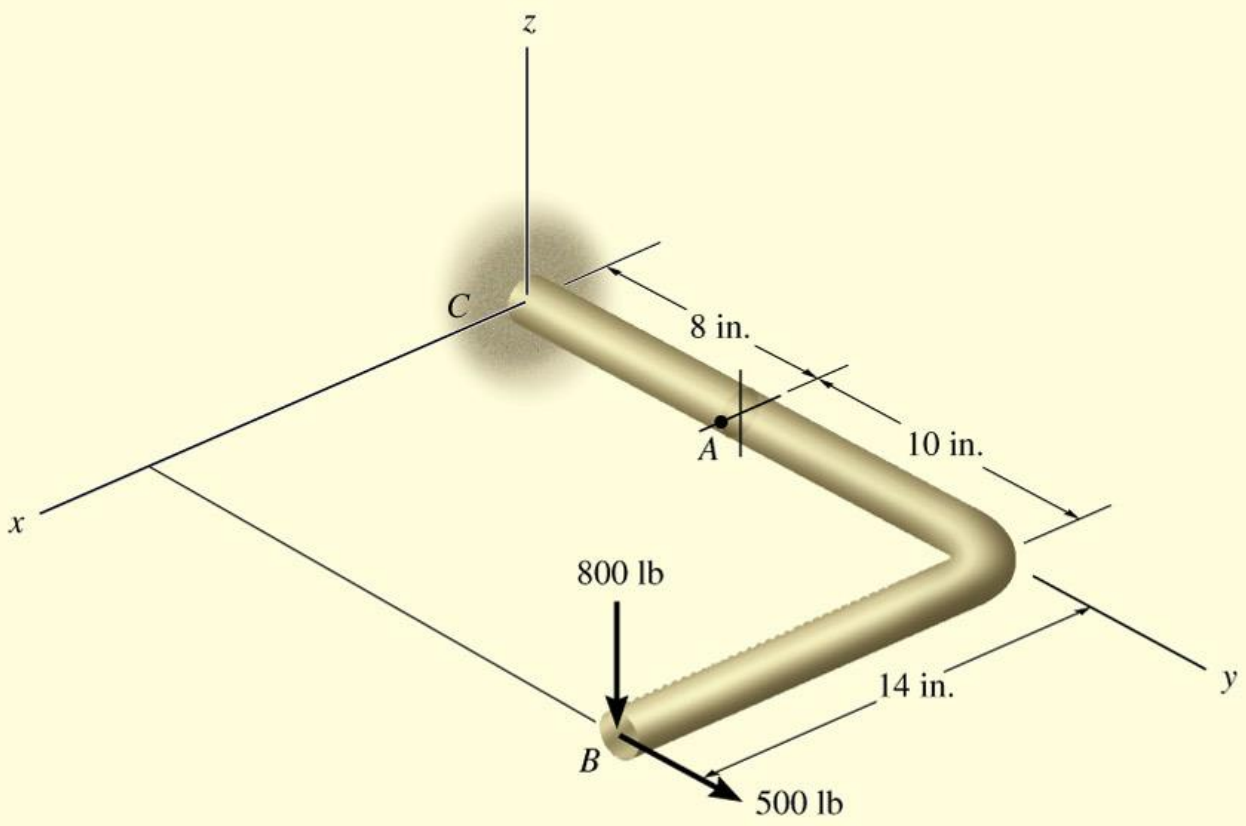
Superposition. The shear stress is zero. Adding the normal stresses determined above gives a compressive stress at C having a value of

$$\sigma_C = 1.32\text{ MPa} + 63.15\text{ MPa} = 64.5\text{ MPa} \qquad \textit{Ans.}$$

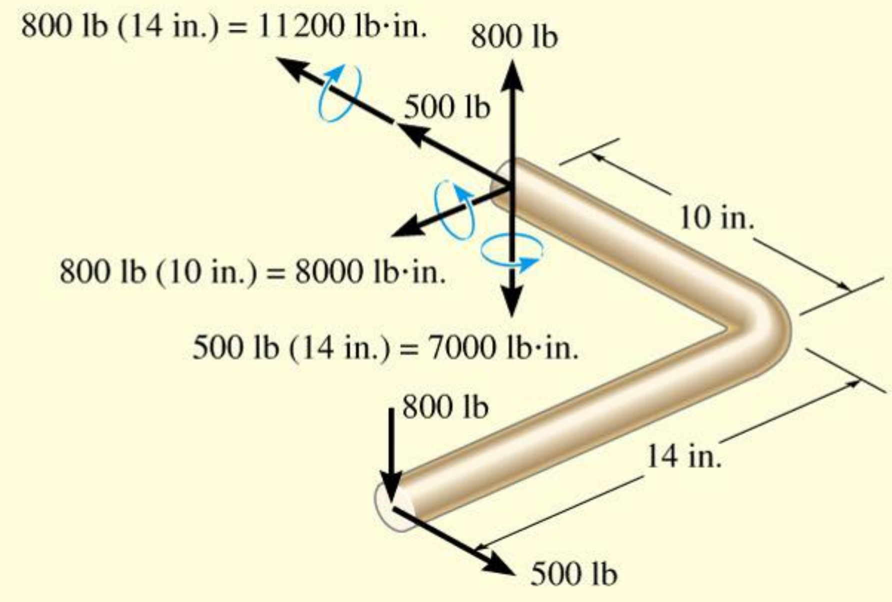
This result, acting on an element at C, is shown in Fig. 8–5*g*.

EXAMPLE 8.5

The solid rod shown in Fig. 8–6*a* has a radius of 0.75 in. If it is subjected to the loading shown, determine the state of stress at point *A*.



(a)

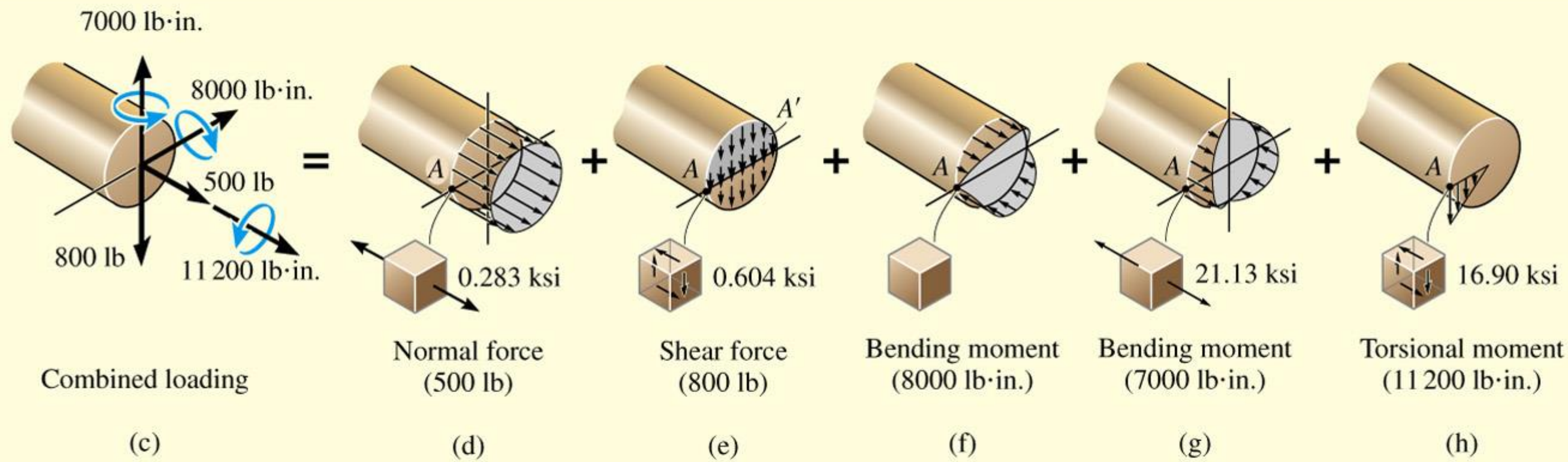


(b)

Fig. 8–6

Solution

Internal Loadings. The rod is sectioned through point A. Using the free-body diagram of segment AB, Fig. 8–6b, the resultant internal loadings can be determined from the six equations of equilibrium. Verify these results. The normal force (500 lb) and shear force (800 lb) must act through the centroid of the cross section and the bending-moment components (8000 lb · in. and 7000 lb · in.) are applied about centroidal (principal) axes. In order to better "visualize" the stress distributions due to each of these loadings, we will consider the *equal but opposite resultants* acting on AC, Fig. 8–6c.



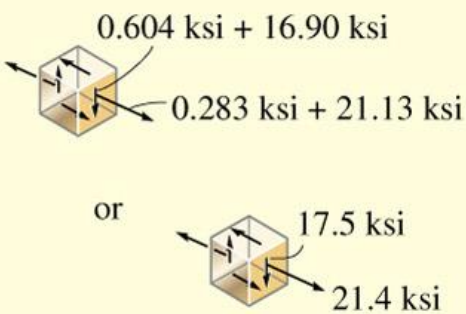
Stress Components.

Normal force. The normal-stress distribution is shown in Fig. 8–6*d*. For point A, we have

$$\sigma_A = \frac{P}{A} = \frac{500 \text{ lb}}{\pi(0.75 \text{ in.})^2} = 283 \text{ psi} = 0.283 \text{ ksi}$$

Shear force. The shear-stress distribution is shown in Fig. 8–6*e*. For point A, Q is determined from the shaded *semicircular* area. Using the table on the inside front cover, we have

$$Q = \bar{y}'A' = \frac{4(0.75 \text{ in.})}{3\pi}\left[\frac{1}{2}\pi(0.75 \text{ in.})^2\right] = 0.2813 \text{ in}^3$$



(i)

Fig. 8–6 (cont.)

so that

$$\tau_A = \frac{VQ}{It} = \frac{800 \text{ lb}(0.2813 \text{ in}^3)}{\left[\frac{1}{4}\pi(0.75 \text{ in.})^4\right]2(0.75 \text{ in.})} = 604 \text{ psi} = 0.604 \text{ ksi}$$

Bending moments. For the 8000-lb · in. component, point A lies on the neutral axis, Fig. 8–6*f*, so the normal stress is

$$\sigma_A = 0$$

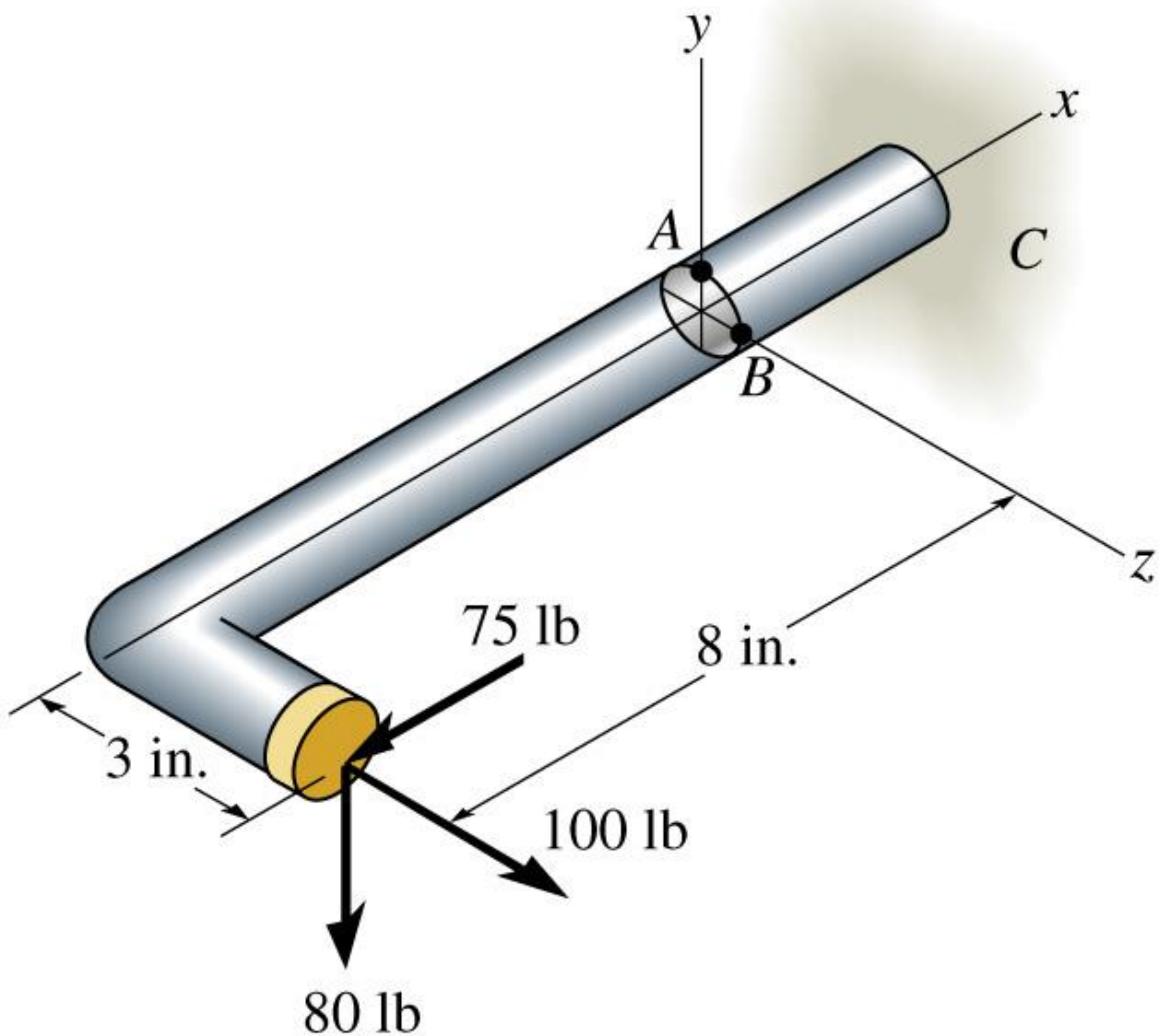
For the 7000-lb · in. moment, $c = 0.75$ in., so the normal stress at point A, Fig. 8–6*g*, is

$$\sigma_A = \frac{Mc}{I} = \frac{7000 \text{ lb} \cdot \text{in.}(0.75 \text{ in.})}{\left[\frac{1}{4}\pi(0.75 \text{ in.})^4\right]} = 21\,126 \text{ psi} = 21.13 \text{ ksi}$$

Torsional Moment. At point A, $\rho_A = c = 0.75$ in., Fig. 8–6*h*. Thus the shear stress is

$$\tau_A = \frac{Tc}{J} = \frac{11\,200 \text{ lb} \cdot \text{in.}(0.75 \text{ in.})}{\left[\frac{1}{2}\pi(0.75 \text{ in.})^4\right]} = 16\,901 \text{ psi} = 16.90 \text{ ksi}$$

Superposition. When the above results are superimposed, it is seen that an element of material at A is subjected to both normal and shear stress components, Fig. 8–6*i*.


y
x
A
C
B
z
75 lb
8 in.
3 in.
100 lb
80 lb