

$$۱) -\frac{b}{pa} = ۲ \Rightarrow y_{\min} = -۳$$

$$۲) -\frac{b}{pa} = \frac{-۲}{۱} = \frac{-۱}{۲} \Rightarrow y_{\min} = ۱$$

$$۳) -\frac{b}{pa} = \frac{۲}{۲} \Rightarrow y_{\min} = \frac{-۵}{۲}$$

$$۴) -\frac{b}{pa} = \frac{-۳}{۲} \Rightarrow y_{\min} = \frac{-۱۷}{۱}$$

$$۵) y = x^p - ۲x^p + ۷ = \underbrace{(x^p - 1)^p}_{\min=0} + ۶ \Rightarrow y_{\min} = ۶$$

$$۶) y = x^p + ۲x^p + ۷ = \underbrace{(x^p + 1)^p}_{\min=1} + ۶ \Rightarrow y_{\min} = ۷$$

$$۷) y = x - \sqrt{x} = \underbrace{\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^p}_{\min=0} - \frac{1}{\sqrt{x}} \Rightarrow y_{\min} = \frac{-1}{\sqrt{x}}$$

$$۸) y = x + \sqrt{x} = \underbrace{\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^p}_{\min=\frac{1}{\sqrt{x}}} - \frac{1}{\sqrt{x}} \Rightarrow y_{\min} = ۰$$

$$۹) y = ۲x - \sqrt{x} = ۲\left(x - \frac{1}{\sqrt{x}}\right) = ۲\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) = \underbrace{۲\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^p}_{\min=0} - \frac{1}{\sqrt{x}} \Rightarrow y_{\min} = \frac{-1}{\sqrt{x}}$$

$$۱۰) y = x + \frac{1}{x} = \underbrace{\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^p}_{\min=0} + ۲ \Rightarrow y_{\min} = ۲$$

$$۱۱) y = x + \frac{۲}{x} = \underbrace{\left(\sqrt{x} - \sqrt{\frac{۲}{x}}\right)^p}_{\min=0} + ۲\sqrt{۲} \Rightarrow y_{\min} = ۲\sqrt{۲}$$

$$۱۲) y = \frac{۲x+۲}{\sqrt{x}} = \frac{۲\sqrt{x}}{\sqrt{x}} + \frac{۲}{\sqrt{x}} = \frac{۲}{\sqrt{x}}\left(A + \frac{1}{A}\right) \Rightarrow y_{\min} = \frac{۴}{\sqrt{x}}$$

$$۱۳) y = \frac{۳x^p+۳}{\sqrt{x^p}} = \frac{۳x^p}{\sqrt{x^p}} + \frac{۳}{\sqrt{x^p}} = \frac{۳}{\sqrt{x^p}}\left(A + \frac{1}{A}\right) \Rightarrow y_{\min} = \frac{۶}{\sqrt{x^p}}$$

$$۱۴) y = \frac{x^p+۶}{x^p} = x^p + \frac{۶}{x^p} \Rightarrow \frac{x^p + \frac{۶}{x^p}}{۲} \geq \sqrt{\frac{۲x^p \cdot \frac{۶}{x^p}}{۲}} \Rightarrow y \geq ۲\sqrt{۳} \Rightarrow y_{\min} = ۲\sqrt{۳}$$

$$۱۵) y = \frac{a+bx^p}{x^p} = \frac{a}{x^p} + bx^p \Rightarrow \frac{\frac{a}{x^p} + bx^p}{2} \geq \sqrt{ab} \Rightarrow y \geq \sqrt{ab} \Rightarrow y_{\min} = \sqrt{ab}$$

$$۱۶) y = \tan x + \cot x = \tan x + \frac{1}{\tan x} \quad A + \frac{1}{A} \geq 2 \Rightarrow y_{\min} = 2$$

$$۱۷) y = p \tan x + \frac{p}{\tan x} = p \tan x + \frac{p}{\tan x} \Rightarrow \frac{p \tan x + \frac{p}{\tan x}}{2} \geq \sqrt{p^2} \Rightarrow y \geq 2\sqrt{p} \Rightarrow y_{\min} = 2\sqrt{p}$$

$$۱۸) y = x^p - x + \frac{\lambda}{x^p - x + 1} = x^p - x + 1 + \frac{\lambda}{x^p - x + 1} - 1$$

بنابراین عبارت $x^p - x + 1$ همواره مثبت است. $x^p - x + 1 = 0 \Rightarrow \Delta < 0, a > 0$.

$$\frac{x^p - x + 1 + \frac{\lambda}{x^p - x + 1}}{2} \geq \sqrt{\lambda} \Rightarrow x^p - x + 1 + \frac{\lambda}{x^p - x + 1} \geq 2\sqrt{\lambda} \Rightarrow y \geq 2\sqrt{\lambda} - 1 \Rightarrow y_{\min} = 2\sqrt{\lambda} - 1$$

$$۱۹) y = \frac{1}{-x^p + px - 1} \Rightarrow y_{\min} = \frac{p}{\omega}$$

$$-\frac{b}{2a} = \frac{-p}{-2} = \frac{p}{2} \Rightarrow f\left(\frac{p}{2}\right) = \frac{\omega}{p} \Rightarrow -x^p + px - 1 \leq \frac{\omega}{p} \Rightarrow y \geq \frac{p}{\omega}$$

$$۲۰) y = \frac{px^p + \lambda x + p^2}{4x + p} = \frac{p(x+1)^p + 9}{4(x+1)} = \frac{p(x+1)}{4} + \frac{9}{4(x+1)} = A + \frac{1}{A} \geq 2 \Rightarrow y_{\min} = 2$$

$$۲۱) y = -2x^p - vx + 1 \Rightarrow -\frac{b}{2a} = \frac{-v}{-2} \Rightarrow y_{\max} = f\left(\frac{-b}{2a}\right) = \frac{\omega v}{\lambda}$$

$$۲۲) y = \frac{p}{\sqrt{px^p - vx + p}} \Rightarrow y \leq 2 \Rightarrow y_{\max} = 2$$

عبارت $px^p - vx + p$ همواره مثبت است. $px^p - vx + p = 0 \Rightarrow \Delta < 0, a > 0 \Rightarrow p > 0$.

$$-\frac{b}{2a} = \frac{v}{2p} = 1 \Rightarrow f\left(\frac{-b}{2a}\right) = 1 \Rightarrow px^p - vx + p \geq 1$$

$$x+y=12 \Rightarrow y=12-x \quad (۲۳ ج)$$

$$P=x \cdot y = x(12-x) = -x^2 + 12x \quad P_{\max} = P\left(-\frac{b}{2a}\right) = S(6) = 36 \quad (الف)$$

$$x^2 + y^2 = (x+y)^2 - 2xy = 144 - 72 = 72 \quad (ب)$$

$$x+y=4 \Rightarrow y=4-x \quad (۲۴ ج)$$

$$S = \frac{1}{p} x \cdot y = \frac{1}{p} x(4-x) = -\frac{1}{p} x^2 + \frac{4}{p} x \Rightarrow S_{\max} = S\left(-\frac{b}{2a}\right) = S(2) = \frac{4}{p} = 4,5$$

$$xy=12 \Rightarrow y=\frac{12}{x} \quad (۲۵ ج)$$

$$S = 3x+y = 3x + \frac{12}{x} = \left(\sqrt{3x} - \sqrt{\frac{12}{x}}\right)^2 + 2\sqrt{3x} \cdot \frac{12}{x} = \left(\sqrt{3x} - \sqrt{\frac{12}{x}}\right)^2 + 12$$

$$\sqrt{3x} = \sqrt{\frac{12}{x}} \Rightarrow 3x^2 = 12 \Rightarrow x^2 = 4 \Rightarrow x=2 \Rightarrow y=6$$

$$2x+y=12 \Rightarrow y=12-2x \quad (۲۶ ج)$$

$$S = x \cdot y = x(12-2x) = -2x^2 + 12x \Rightarrow S_{\max} = S\left(-\frac{b}{2a}\right) = S(3) = 18$$

$$p(x+y)=k \Rightarrow x+y=\frac{k}{p} \Rightarrow y=\frac{k}{p}-x \quad (۲۷ ج)$$

$$S = x \cdot y = x\left(\frac{k}{p}-x\right) = -x^2 + \frac{k}{p}x \Rightarrow x_{\max} = \frac{-b}{2a} = \frac{k}{2p} \Rightarrow y_{\max} = \frac{k}{p} - \frac{k}{2p} = \frac{k}{2p} \Rightarrow x_{\max} = y_{\max}$$

$$x^2 - mx + 2m - 5 = 0 \quad (۲۸ ج)$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (-m)^2 - 2(2m-5) = m^2 - 4m + 10$$

$$m_{\min} = \frac{f}{p} = 2$$

$$2x+y=12 \Rightarrow y=12-2x \quad (۲۹ ج)$$

$$S = \frac{1}{p} x \cdot y = \frac{1}{p} x(12-2x) = -\frac{2}{p} x^2 + \frac{12}{p} x \quad x_{\max} = \frac{-b}{2a} = \frac{-6}{-2} = 3 \Rightarrow y_{\max} = 6$$

$$3x + \frac{y}{p} = 24 \Rightarrow \frac{y}{p} = 24 - 3x \quad (۳۰ ج)$$

$$S' = \frac{1}{p} x \cdot y = x \cdot (24 - 3x) = -3x^2 + 24x$$

$$S'_{\max} = S\left(-\frac{b}{2a}\right) = S(4) = 4 \cdot (24 - 12) = 48$$

$$S = x \cdot y = 4F \Rightarrow y = \frac{4F}{x} \quad (31 \text{ ج})$$

$$P = P(x+y) = P\left(x + \frac{4F}{x}\right) = P \left[\underbrace{\left(\sqrt{x} - \frac{\Lambda}{\sqrt{x}}\right)^2}_{\cdot} + 4F \right]$$

$$\sqrt{x} = \frac{\Lambda}{\sqrt{x}} \Rightarrow x = \Lambda \Rightarrow y = \Lambda$$

$$3x + 2y = 100 \Rightarrow y = \frac{100 - 3x}{2} \quad (32 \text{ ج})$$

$$S = x \cdot y = x \left(\frac{100 - 3x}{2} \right) = -\frac{3}{2}x^2 + 50x$$

$$S_{\max} = S\left(\frac{-b}{2a}\right) = S\left(\frac{100}{3}\right) = \frac{100}{3} \times \left(\frac{100 - 100}{2}\right) = \frac{5000}{3}$$

$$r+h = K \Rightarrow r = K-h \quad (33 \text{ ج})$$

$$S = \pi r h = \pi h(K-h) = -\pi h^2 + \pi K h$$

$$S_{\max} = S\left(\frac{-b}{2a}\right) = S\left(\frac{K}{2}\right) = \pi \times \frac{K}{2} \times \frac{K}{2} = \frac{\pi K^2}{4}$$

$$\frac{x}{\Lambda} + \frac{y}{F} = 1 \Rightarrow x + \frac{F}{\Lambda}y = \Lambda \Rightarrow y = \frac{\Lambda - x}{\frac{F}{\Lambda}} \quad (34 \text{ ج})$$

$$S = x \cdot y = x \cdot \left(\frac{\Lambda - x}{\frac{F}{\Lambda}}\right) = \frac{\Lambda}{F}x - \frac{1}{F}x^2 \Rightarrow S_{\max} = S\left(\frac{-b}{2a}\right) = S\left(\frac{F}{2}\right) = \Lambda$$

$$y = 4 - x^2 \quad (35 \text{ ج})$$

$$P = Fx + 2y = Fx + 2(4 - x^2) = -2x^2 + Fx + 8$$

$$P_{\max} = P\left(\frac{-b}{2a}\right) = P(1) = -2 + F + 8 = F + 6$$

$$y^2 = Fx + \Lambda \quad (36 \text{ ج})$$

$$d = \sqrt{x^2 + y^2} = \sqrt{x^2 + Fx + \Lambda} \Rightarrow d_{\min} = d\left(\frac{-b}{2a}\right) = d\left(-\frac{F}{2}\right) = \sqrt{F - \Lambda + \Lambda} = \sqrt{F}$$

$$2y + F = x^2 \quad A(0, 11) \quad (37 \text{ ج})$$

$$d = \sqrt{x^2 + (y-11)^2} = \sqrt{2y + F + y^2 - 22y + 121} = \sqrt{y^2 - 20y + 121}$$

$$d_{\min} = d\left(\frac{-b}{2a}\right) = d(10) = \sqrt{100 - 200 + 121} = 5$$

$$x \cdot y = 48 \quad (38 \text{ ج})$$

$$S = (x+3)(y+2) = 2x + 3y + xy + 6 = 2x + 3y + 48 = (\sqrt{2x} - \sqrt{3y})^2 + 2\sqrt{6xy} + 48$$

$$= (\sqrt{2x} - \sqrt{3y})^2 + 48 + 24\sqrt{2} \Rightarrow S_{\max} = 48 + 24\sqrt{2}$$

ترجید: از روی شکل معلوم است که طول ضلع است است
منبت است

$$S = (x_1 + x_2)y = (1y - 3 + \frac{3-y}{3})xy = 3y - y^2 + y - \frac{y^2}{3} = -\frac{4}{3}y^2 + 4y \quad (39 \text{ ج})$$

$$S'_{\max} = S'(-\frac{b}{2a}) = S'(\frac{3}{2}) = -\frac{4}{3} \times \frac{9}{4} + 4 \times \frac{3}{2} = -3 + 6 = 3$$

$$2x + \pi y = 300 \Rightarrow y = \frac{300 - 2x}{\pi} \quad (40 \text{ ج})$$

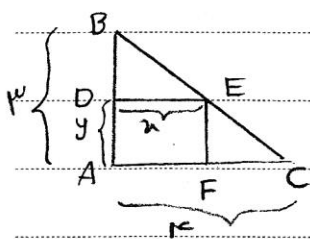
$$S = x \cdot y = x \cdot \frac{300 - 2x}{\pi} = -\frac{2}{\pi}x^2 + \frac{300}{\pi}$$

$$x_{\max} = -\frac{b}{2a} = -\frac{300}{-4} = 75 \Rightarrow y = \frac{300 - 150}{\pi} \Rightarrow P = \frac{1}{\pi} \pi y = 75$$

$$\frac{x}{5} + \frac{y}{5} = 1 \Rightarrow 2x + 5y = 10 \Rightarrow y = \frac{10 - 2x}{5} \quad (41 \text{ ج})$$

$$S = x \cdot y = x \left(\frac{10 - 2x}{5} \right) = -\frac{2}{5}x^2 + 2x$$

$$S_{\max} = S'(-\frac{b}{2a}) = S'(\frac{5}{2}) = \frac{5}{5} = 1, 5$$



$$\frac{3-y}{3} = \frac{x}{4} \Rightarrow 4x = 12 - 3y \Rightarrow 3x + 4y = 12$$

$$y = \frac{12 - 3x}{4}$$

$$S = x \cdot y = x \left(\frac{12 - 3x}{4} \right) = -\frac{3}{4}x^2 + 3x$$

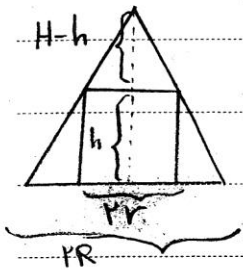
$$S_{\max} = S'(-\frac{b}{2a}) = S'(2) = 2 \times \frac{3}{4} = 3$$

(42 ج)

$$\frac{x}{4} + \frac{y}{4} = 1 \Rightarrow 2x + 4y = 8 \Rightarrow y = \frac{8 - 2x}{4} \quad (43 \text{ ج})$$

$$S = \frac{1}{4}x \cdot y = \frac{1}{4}x \cdot \left(\frac{8 - 2x}{4} \right) = -\frac{1}{8}x^2 + x$$

$$S_{\max} = S'(-\frac{b}{2a}) = S'(4) = \frac{1}{4} \times 4 \times 1 = 1$$



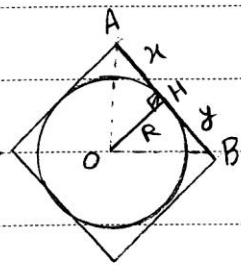
$$H = 2r \quad ; \quad \frac{r}{R} = \dots \quad (۴۴ ز)$$

$$S' = 2\pi r^2 + 2\pi r h$$

$$\frac{H-h}{H} = \frac{r}{R} \Rightarrow \frac{H-h}{r} = \frac{H}{R} \Rightarrow \frac{H-h}{r} = 2 \Rightarrow 2r = H-h \Rightarrow h = H-2r$$

$$S' = 2\pi r^2 + 2\pi r(H-2r) = -2\pi r^2 + 2\pi H r$$

$$r_{\max} = \frac{-2\pi H}{-4\pi} = \frac{H}{2} \Rightarrow \frac{r}{R} = \frac{\frac{H}{2}}{\frac{H}{2}} = \frac{1}{2}$$



$$AH = x, BH = y \quad (۴۵ ز)$$

$$P = 4(x+y) = 4\left(x + \frac{R^2}{x}\right) = 4\left(\sqrt{x} - \sqrt{\frac{R^2}{x}}\right)^2 + 4R$$

$$OH^2 = AH \cdot BH \Rightarrow x \cdot y = R^2 \Rightarrow y = \frac{R^2}{x}$$

$$P_{\min} \Rightarrow \sqrt{x} - \frac{R}{\sqrt{x}} = 0 \Rightarrow x = R \Rightarrow y = R \Rightarrow x+y = 2R$$

$$\text{معادله تقاطع: } y = ax + b \quad \begin{cases} 3 \cdot a + b = 3 \dots & a = -2 \\ 1 \cdot a + b = 1 \dots & b = 9 \dots \end{cases} \quad y = -2x + 9 \dots \quad (۴۶ ز)$$

$$\text{درآمد} = \text{قیمت کالا} \times \text{تعداد کالا} = (-2x + 9) \cdot x = -2x^2 + 9 \cdot x$$

$$-\frac{b}{2a} = \frac{-9}{-4} = 2.25$$

$$\text{عربده} = \text{درآمد} - \text{هزینه} \quad (۴۷ ز)$$

$$\text{هشود} = (9 - 2x) \left(12 + \frac{x}{5}\right) - 8 \cdot \left(12 + \frac{x}{5}\right) = \frac{-1}{5}x^2 - 12x + 24x + 12 \cdot 9 - 96 - 12x$$

$$= \frac{-1}{5}x^2 + 12x + 12 \cdot 9$$

$$x_{\max} = \frac{-b}{2a} = \frac{-12}{\frac{-2}{5}} = 30$$

حرفه‌آس

علیرضا فیضیان