CH # 1.2 : EXERCISES

③ PT. $(w^R)^R = w \quad \forall w \in \Sigma^*$

(Induction)

$(uv)^R = v^R u^R$

$(n+1)$ length : $v = wa$

$(uwa)^R = (wa)^R u^R$

$= a w^R u^R$

$\therefore$ true $\forall w \in \Sigma^*$

④ $L = \{ab, aa, baa\}$
which of the following strings are in L^*, L^4?

abaabaabaa	L^4, L^*
aaaabaaaa	L^4, L^*
baaaaabaaaab?	
baaaabaa	L^4, L^*

⑤ let $\Sigma = \{a, b\}$ Use set notation to describe $\bar{L}$.

$L = \{aa, bb\}$

$\bar{L} = U - \{aa, bb\}$

$\bar{L} = \{\lambda, a, b, ab, ba\} \cup \{w : |w| > 2, w \in \Sigma^*\}$

⑥ Let L be any language on a non-empty alphabet. Show that $L, \bar{L}$ cannot be both finite.

Case (i) L is finite

→ we know U is infinite

→ $\bar{L} = U - L$

= infinite lang − finite lang

= infinite lang.

Case (ii) L is infinite

U is infinite

$\bar{L} = U - L$

= finite

From above, in any case, both cannot be finite

(7) Are there any languages for which $\overline{L^*} = (\overline{L})^*$?

$\lambda \in L^*$

$\Rightarrow \lambda \notin \overline{L^*}$

but $(\overline{L})^*$ contains λ

$\therefore$ No language satisfies $\overline{L^*} = (\overline{L})^*$

(8) PT. $(L_1L_2)^R = L_2^R L_1^R \quad \forall \; L_1 L_2$

let $u \in L_1$, $v \in L_2$

$L_1L_2 \Rightarrow uv$

$(L_1L_2)^R \Rightarrow (uv)^R$

$= v^R u^R = L_2^R L_1^R \quad \forall \; u, v.$

$\therefore (L_1L_2)^R = L_2^R L_1^R \quad \forall \; L_1, L_2$

(9) Show that $(L^*)^* = L^* \; \forall \; L.$

let $\Sigma = \{a, b\}$

$L^* \Rightarrow \Sigma^* \Rightarrow \{a,b\}^*$

$(L^*)^* \Rightarrow (\{a,b\}^*)^* = \{a,b\}^*$

$= L^* \quad \forall \; \Sigma$

(10) (a) $(L_1 \cup L_2)^R = L_1^R \cup L_2^R \quad \forall \; L_1, L_2$

$(L_1 \cup L_2)^R \Rightarrow$ if $u \in L_1$ $v \in L_2$

$uv \in L_1 \cup L_2$,

$(L_1 \cup L_2)^R = (uv)^R = v^R u^R$

$\neq L_1^R \cup L_1^R$

(EXERCISES)

10. (b)

$(L^R)^* = (L^*)^R \quad \forall L$

let $uv \in L$

$L^R = (uv)^R$

$= v^R u^R$

$(L^R)^* = (v^R u^R)^*$

$L^* = (uv)^*$

$(L^*)^R = [(uv)^*]^R$

$\therefore \quad (L^R)^* \neq (L^*)^R$

(11) Find the Grammars that generate the sets of following for $\Sigma = \{a, b\}$

(a) all strings with exactly one a.

P:

$S \to AaA$

$A \to bA / \lambda$

$G = (\{A, S\}, \{a, b\}, S, P)$

(b) all strings with atleast one a.

P:

$S \to AaA$

$A \to aA \mid bA / \lambda$

$G = (\{A, S\}, \Sigma, S, P)$

(c) all strings with no more than 3 a's.

P: ~~$S \to AaAaAaA$~~ 0,1,2,3 a's

~~$A \to bA/\lambda$~~

$G = (\{A, S\}, \Sigma, S, P)$

$S \to AaA / AaAaA / AaAaAaA$ / [illegible]

$A \to bA / \lambda$

Δ

$S \to AaA / AaAaA / AaAaAaA / A$

$A \to bA / \lambda$

(d) All strings with atleast 3 a's.

P: $S \to AaAaAaA$

$A \to aA / bA / \lambda$

$G = (\{A, S\}, \Sigma, S, P)$

test:

aaa: $S \to AaAaAaA \to aaa$

baababa: $S \to AaAaAaA \to baAaAaA \to baaAaAaA \to baabaAaA \to$ baababa ✓

(12)

$S \to aA$

$A \to bS$

$S \to \lambda$

ab, abab ...

$L = \{(ab)^n : n \geq 0\}$

(13) What language does the Grammar with these productions generate?

$S \to Aa$

$A \to B$

$B \to Aa$

$S \to Aa \to Ba \to Aaa$

$L = \emptyset$: no terminal symbol to generate strings.

(14) $\Sigma = \{a, b\}$. For each of below languages, find a grammar that generates it

(a) $L_1 = \{a^n b^m : n \geq 0, m > n\}$

P_1:

$S \to Ab$

$A \to aAb / \lambda / Ab$

$G: (\{A, S\}, \{a, b\}, S, P_1)$

$S \to aSb / B$

$B \to bB / b$

test: b : $S \to Ab \to b$ ✓

$abb \to Ab \to aAbb \to abb$ ✓

ab : $S \to Ab$ ×

$bb \to S \to Ab \to bb$

(b) $L_2 = \{a^n b^{2n} : n \geq 0\}$

P_2:

$S \to aSbb / \lambda$

$G: (\{S\}, \Sigma, S, P_2)$

tests:

λ : $S \to \lambda$

aab : $S \to aSbb$ ×

abb: $S \to aSbb \to abb$ ✓

(c) $L_3 = \{a^{n+2} b^n : n \geq 1\}$

test. $n=1$: $a^3 b^1$: $S \to aaA \to aaaAb$

$\to aaab$

$S \to aaA$

$A \to aAb / \lambda$

14.

(c) $L_4 = \{a^n b^{n-3} ; n \geq 3\}$

$n-3 = m$

$n = m+3$

$n \geq 3$

$m \geq 0$

$\Rightarrow L_3 = \{a^{m+3} b^m ; m \geq 0\}$

P_3:

$S \to aaaA$

$A \to aSb / \lambda$

$\therefore G = (\{A, S\}, \Sigma, S, P_3)$

d) $L_8 = L_1 L_2$

$L_5 = \{a^n b^m a^n b^{2n} : n \geq 0, m > n\}$

$= \{a^n b^m a^k b^{2k} : n, k \geq 0, m > n\}$

$S \to S_1 S_2$

$S \to AB$

$A \to Cb$

$C \to aCb / \lambda / Cb$

$B \to aBbb / \lambda$

$\Rightarrow$

$S \to AbB$

$A \to aAb / \lambda / Ab$

$B \to aBbb / \lambda$

test: abbabb: $S \to AbB \to aAbbB \to abbB \to$ abbabb ✓

b: $S \to AbB \to bB \to b$ ✓

bb: ✗

(e) $L_1 \cup L_2$:

$S \to S_1 / S_2$

$S \to Ab / B$

$A \to aAb / \lambda$

$B \to aBbb / \lambda$

(g) L_1^3 : $\{a^n b^m a^n b^m a^n b^m \ : \ n \geq 0, m > n\}$

$S \to AbAbAb$ $\qquad S \to S S_1 S_1$

$A \to aAb / Ab / \lambda$

Test: $n=0 : m=1$ $\quad bbb$

$S \to AbAbAb \to bbb$

$n=0 : m=2$ $\quad bbbbbb$

$S \to AbAbAb \to bbbbbb$

reject: $ababaca$

$S \to AbAbAb$

$\to aAbbAbAb$ ×

(h) L_1^* : $\qquad L_1 = \{a^n b^m : n \geq 0, m > n\}$

$S \to SA / \lambda$ $\qquad S \to S S_1 | \lambda$.

$A \to aAb / Ab / \lambda$

test: $\lambda : S \to \lambda$

$abbaabbb$: $S \to SA \to SaAb \to SaaAbb \to Saabbb \to$

$SAaabbb \to aAbaabbb \to abbaabbb$ ✓

reject: aba : $S \to SA \to SaAb$ ×

(i) $L_1 - \overline{L_4}$: $\qquad L_4 = \{a^{n+3} b^n : n \geq 0\}$

$L_1 = \{a^n b^m : n \geq 0, m > n\}$

$L_1 - \overline{L_4} = L_1 - (U - L_4)$

$= L_1 - U + L_4$

$= L_1 + L_4 - U \quad = \emptyset$

(15) Find the grammars for the following on $\Sigma = \{a\}$

(a) $L = \{w : |w| \bmod 3 = 0\}$

$S \to aaaS \,/\, \lambda$

(b) $L = \{w : |w| \bmod 3 > 0\}$

1, 2, 4, 5, 7, 8, 10, 11

$S \to A \,/\, B$

$A \to aaaA \,/\, a.$

$B \to aaaB \,/\, aa$

$S \to aaaS/a/aa$

Test:

aaaaaaa : $S \to A \to aaaA \to aaaaaaA \to aaaaaaa$

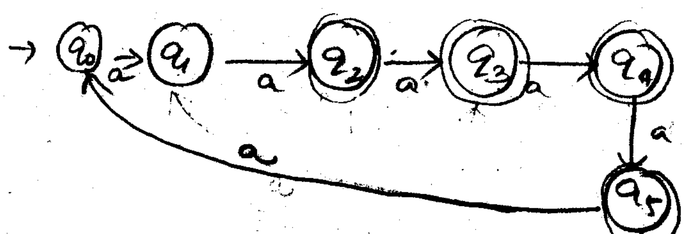
aaaaaaaaa : ×

(c) $\boxed{L = \{w : |w| \bmod 3 \neq |w| \bmod 2\}}$

mod 3 ↓ {0, 1, 2}

mod 2 ↓ {0, 1}

→ Skip every 6th & 7th



#	mod2	mod3
0	0	0
1	1	1
2	0	2
3	1	0
4	0	1
5	1	2
6	0	0
7	1	1
8	0	2
9	1	0
10	0	1
11	1	2
12	0	0
13	1	1
14	0	2

$S \to aaA$

$A \to \lambda/a/aa/aaa/\,aaaaS/aaaaaaA$

$S \to aa/aaa/aaaa/aaaaa/aaaaaaS$

(2) (3) (4) (5) (6)+S

test

λ ×

a ×

aa ✓

9a's ✓

8a's ✓

13a's ×

$S \to aaA$

$A \to \lambda/a/aa/aaa/aaaaS$

(d) $\boxed{L = \{w : |w| \bmod 3 \geq |w| \bmod 2\}}$

$\lvert w \rvert$	mod 2	mod 3	$\geq$
0	0	0	✓
1	1	1	✓
2	0	2	✓
3	1	0	×
4	0	1	✓
5	1	2	✓
6	0	0	✓
7	1	1	✓
8	0	2	✓
9	1	0	×
10	0	1	✓
11	1	2	✓
12	0	0	✓
13	1	1	✓
14	0	2	✓
15	1	0	×

$S \to \lambda | a | aa / aaaA$

$A \to a / aa / aaa / aaaa / aaaaa / aaaaaaA$

Test: 3a's : $S \to aaaA$ ×

5a's : $S \to aaaA \to aaaaa$ ✓

(16) Find a grammar that generates the language $L = \{ww^R : w \in \{a,b\}^+\}$

$\boxed{S \to aSa / bSb / aa | bb | ba}$ $\{a,b\}^*$

abba : $S \to aSa \to abSba \to abba$

bbbb $S \to bSb \to bbSbb \to bbbb$

$S \to aSa | bSb | aa | bb$

(17) Give verbal description of

$S \to aSb / bSa / a$

In no order of a and b, no. of a's are one more in any string.

$S \to aSb \to aaSbb \to aaabb$

$\to bSa \to baa$

$bSa \to bbSaa \to bbaaa$

b's : n

a's : n+1

(a) $L = \{w : n_a(w) = n_b(w) + 1\}$

we know for $L_1 = \{w : n_a(w) = n_b(w)\}$ equal a's & b's

$G \Rightarrow S \to SS$ $\qquad A \to AA / aAb / bAa / \lambda$

$S \to aSb / bSa / \lambda$ $\qquad S \to \boxed{AaA}$ additional A

$\therefore$ $S \to SSa / aSS / aSb / bSa / \lambda / SaS$

Test:

$S \to SSa \to aSbSa \to a\lambda b\lambda a \to aba \to \boxed{aba}$ ✓

$S \to \boxed{a}$ ✓

$S \to SSa \to aSbbSaa \to a\lambda bb\lambda aa \to \boxed{abbaa}$ ✓

$S \to SSa \to bSaa \to bbSaaa \to bbbSaaaa \to \boxed{bbbaaaa}$ ✓

$S \to SaS \to bSaaaSb \to \boxed{baaab}$ ✓

(b) $L = \{w : n_a(w) > n_b(w)\}$ $\qquad S \to SS / aSb / bSa / aS / a$ $\qquad$ add any no. of a's

$S \to SS / \cancel{aS} / aSb / bSa / \lambda$

(c) $L = \{w : n_a(w) = 2 n_b(w)\}$

$S \to SS / aSba / aaSb / bSaa / abSa / aSab / baSa / \lambda$

test: reject aabb : $aaSb \to aab$ ✗

aaab : $aaSb \to \cancel{aa}$ ✗

aab : $S \to aab$ ✓

ababbaaaa : $S \to SS \to abSa \to ababSaa \to ababbaSaaa \to ababbaaaa$ ✓

aaaaaabbb : $SS \to aaSb \to aaaaSbb \to aaaaaaSbbb \to aaaaaabbb$ ✓

(d) $L = \{w \in \{a,b\}^* : |n_a(w) - n_b(w)| = 1$ — Equal a's & b's

$A \to AA / aAb / bAa / \lambda$

$S \to AaA / AbA$

$\Rightarrow n_a(w) = n_b(w) + 1$ ✓

$n_b(w) = n_a(w) + 1$ ✓

~~$S \to A / B$~~

~~$A \to AAa / aAA / AaA / aAb / bAa / \lambda$~~

~~$B \to BBb / bBB / BbB / aBb / bBa / \lambda$~~

(19) $\Sigma = \{a,b,c\}$

(a) $L = \{w : n_a(w) = n_b(w) + 1\}$

$a=b$: c varying

$S \to SS / aSb / bSa / cS / \lambda$

$a = b+1$: c varying

$S \to AaA$

$A \to AA / aAb / bAa / cA / \lambda$

we know for $\Sigma = \{a,b\}$ $\quad n_a(w) = n_b(w)$

$S \to SS / aSb / bSa / \lambda$

$\therefore \Sigma = \{a,b,c\}$: $\quad S \to SS / aSb / bSa / C$

$C \to cC / \lambda$

~~$S \to AaA$~~

~~$A \to aAb / bAa / C$~~

~~$C \to cC / \lambda$~~

$\therefore n_a(w) = n_b(w) + 1 \Rightarrow$

$S \to aSS / SSa / SaS / aSb / bSa / C$

$C \to cC / \lambda$

(or) $S \to SSa / aSS / SaS / aSb / bSa / cS / \lambda$

(b) $L = \{w : n_a(w) > n_b(w)\}$

$S \to SS / aSb / bSa / cS / aS / a$

$S \to SS / aS / aSb / bSa / cS / \lambda$

(c) $L = \{w : n_a(w) = 2n_b(w)\}$

$S \to SS / cS / aaSb / aSba / aSab / abSa / baSa / bSaa / \lambda$

(d) $L = \{w : |n_a(w) - n_b(w)| = 1\}$ ~~$S_1 \to S_1 / S_2$~~ add one a / add one b

$S \to AaA / AbA$

$A \to aAb / bAa / AA / cA / \lambda$

~~$S_2 \to S_2S_2 / cS_2 / baSb / abSb / bbSa / bSab / bSba / \lambda$~~

CH #1-2

EXERCISES

(20) PT.

$S \to aAb / \lambda$
$A \to aAb / \lambda$ generates $\{a^n b^n : n \geq 0\}$

$S \to \lambda$
$S \to aAb \to ab$
$S \to aAb \to aaAbb \to aabb$

$\therefore L = \{\lambda, ab, aabb \dots\}$

$L = \{a^n b^n : n \geq 0\}$ is true

(21) $S \to aSb / ab / \lambda \equiv S \to aAb / ab$
$A \to aAb / \lambda$

$S \to \lambda$
$S \to ab$
$S \to aSb \to aabb$
$L = \{a^n b^n : n \geq 0\}$

$S \to ab$
$S \to aAb \to ab$
$S \to aaAbb \to aabb$
$L = \{a^n b^n : n > 0\}$

as both grammars represent different languages, they are not equivalent.

(22) ST. $S \to SS / SSS / aSb / bSa / \lambda$ is equivalent to $S \to SS / aSb / bSa / \lambda$.

If we rewrite SS as SSS $\{\because S \to SS\}$

both are representing same Grammars.

where $n_a(w) = n_b(w)$

(23) ST. $S \to aSb / bSa / SS / a \neq S \to aSb / bSa / a$

$S \to a$
$S \to SS \to aa$
$aa \in L_1$

$S \to aSb \to aab$
$aa \notin L_1$

Example 1.15

① id is a sequence of letters, digits, underscores

② id must start with a letter or underscore

③ id allow upper & lower case letters.

<id> → <letter> <rest> / <undscr><rest>

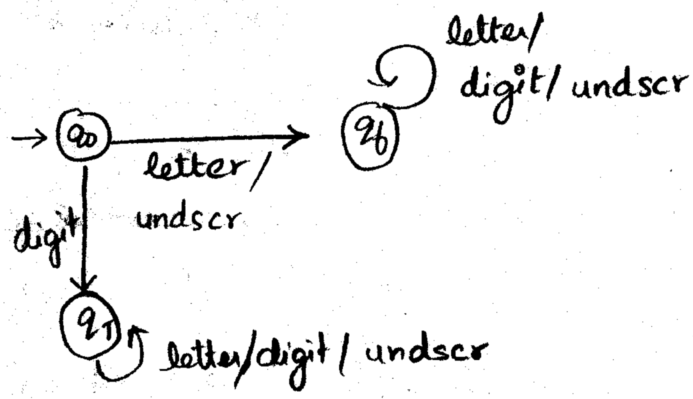
<rest> → <letter><rest> / <digit><rest> / <undscr><rest> / λ

<letter> → a|b|c| - - - z | A|B|C - - Z

<digit> → 0|1|2 - - - |9

<undscr> → _

1.16



Example 1.17

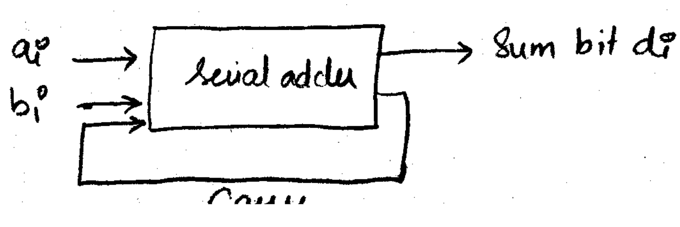
Binary Adder

		b_i: 0	1
a_i	0	0 No Carry	1 No Carry
	1	1 No Carry	0 Carry

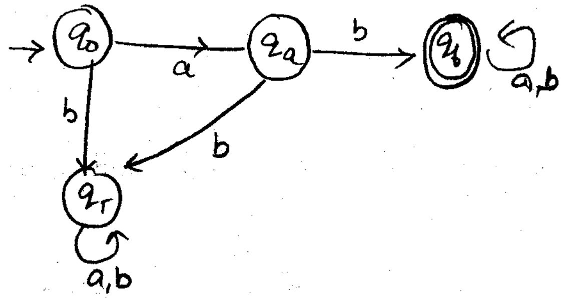
$$v(x) = \sum_{i=0}^{n} a_i 2^i$$

i/p : (a_i, b_i)

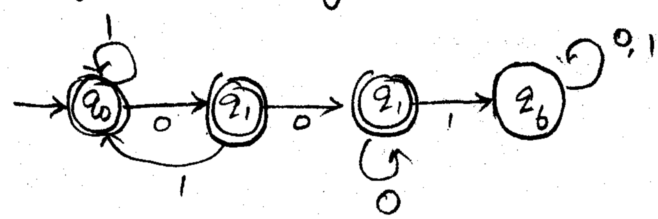
o/p = sum bit d_i



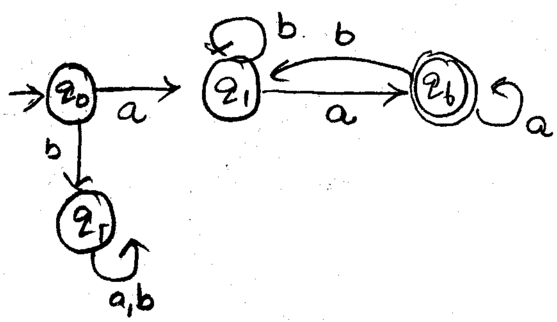
Example 2.3 Find Dfa that recognises all strings on $\Sigma = \{a, b\}$ with prefix ab.



Example 2.4 Find a dfa that accepts all the strings on $\{0,1\}$ except those containing the substring 001.



Example 2.5 Show that $L = \{awa : w \in \{a,b\}^*\}$ is regular.

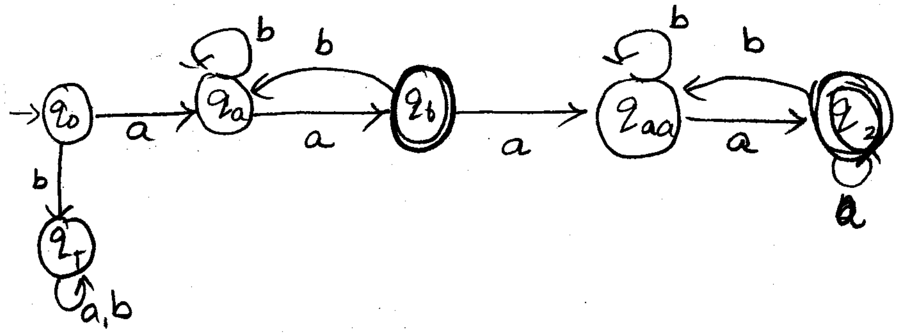


Show that Regular ⇒ Draw Dfa

Example 2.6

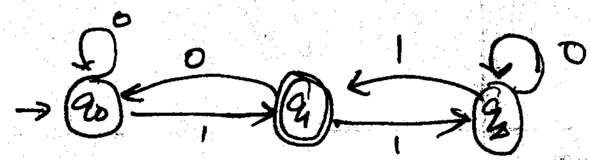
$L = \{a w_1 a a w_2 a : w_1, w_2 \in \{a,b\}^*\}$ is regular.

L regular ⇒ $L^1, L^2, L^3 \ldots$ are also regular.



EXERCISES

① Which of following are accepted by



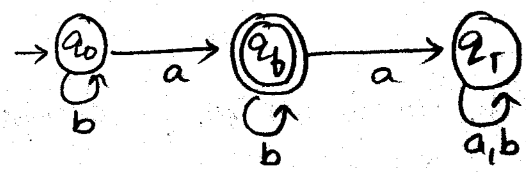
0001 ✓

01001 ✓

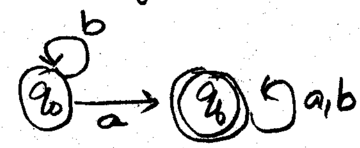
0000110. ✗

② For $\Sigma = \{a,b\}$ construct dfa's

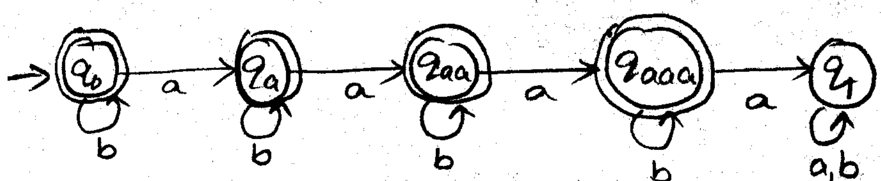
(a) all strings with exactly one a.



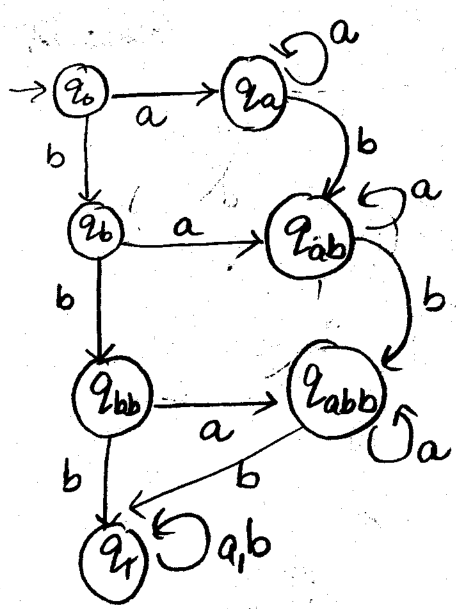
(b) all strings with @least one a



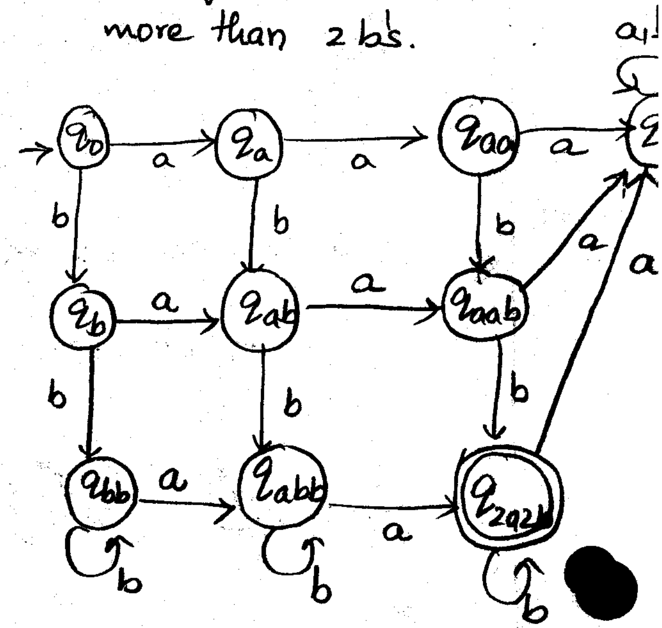
(c) all strings with no more than 3 a's.



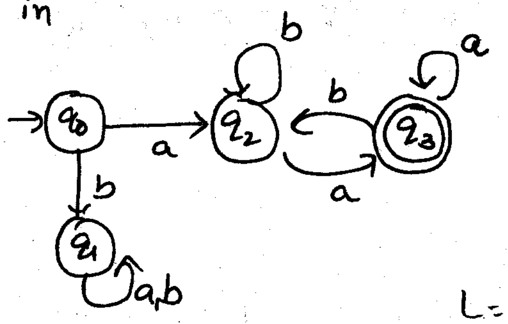
(d) @least one a & exactly 2 b's.



(e) all strings with 2 a's & more than 2 b's.



(3) ST in



If: $q_3 \notin F$
$q_0, q_1, q_2 \in F$, resulting dfa accepts $\overline{L}$.

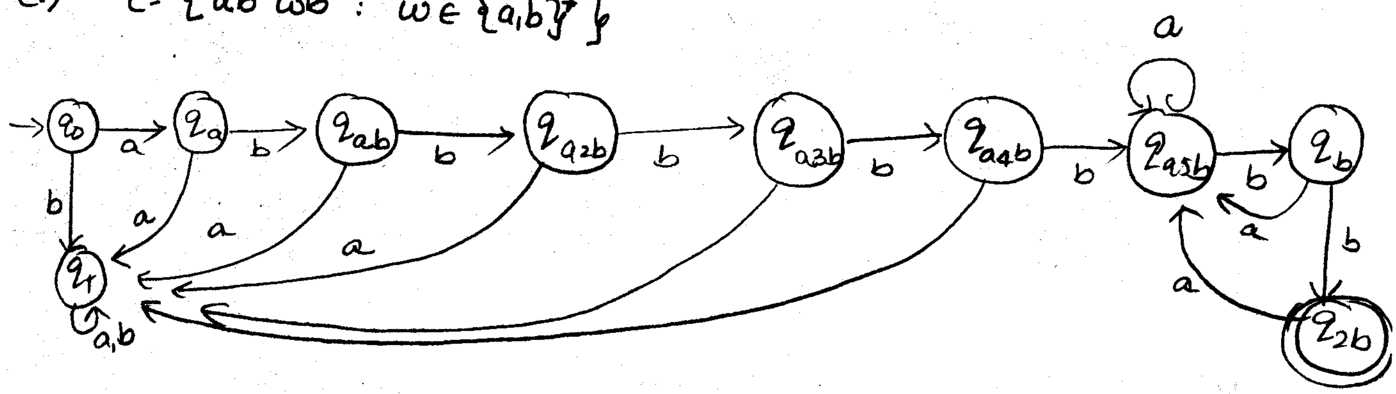
$L = \{awa : w \in \Sigma^*\}$

$\overline{L}$: is accepted by the changes to L.

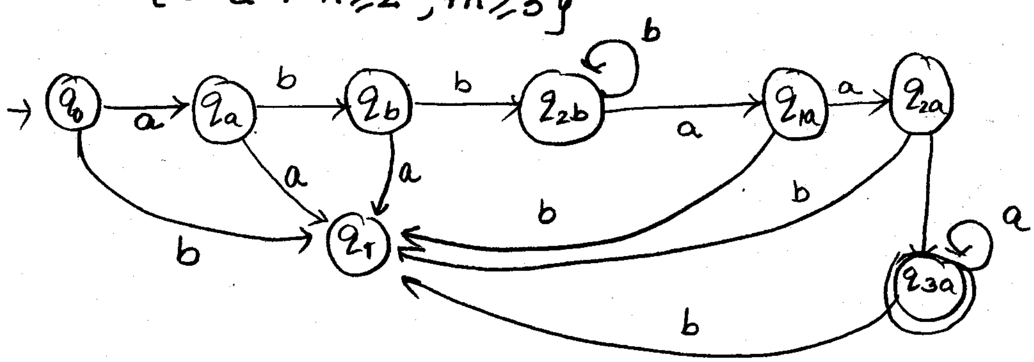
(4) $M = (Q, \Sigma, \delta, q_0, F)$
$\hat{M} = (Q, \Sigma, \delta, q_0, Q-F)$

then $\overline{L(M)} = L(\hat{M})$

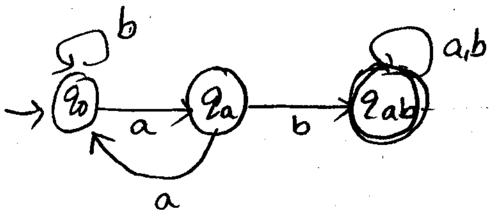
(5) (a) $L = \{ab^5wb^2 : w \in \{a,b\}^*\}$



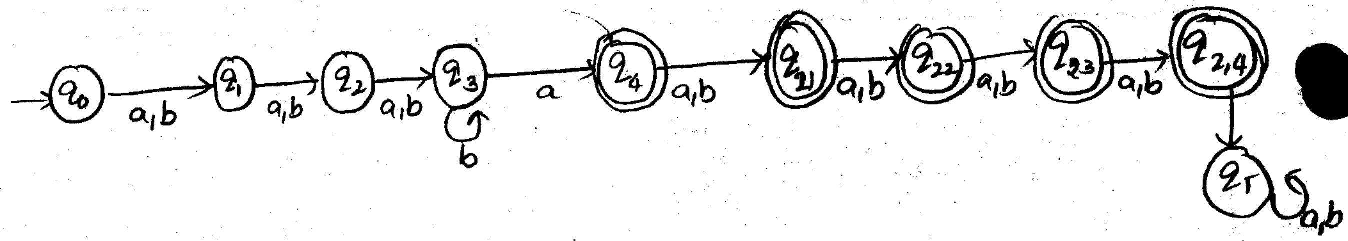
(b) $L = \{ab^n a^m : n \geq 2, m \geq 3\}$



(c) $L = \{w_1 ab w_2 : w_1 \in \{a,b\}^*, w_2 \in \{a,b\}^*\}$

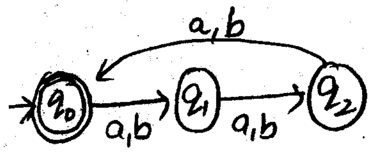


(6) $\Sigma = \{a,b\}$ give dfa for $L = \{w_1 a w_2 : |w_1| \geq 3, |w_2| \leq 5\}$

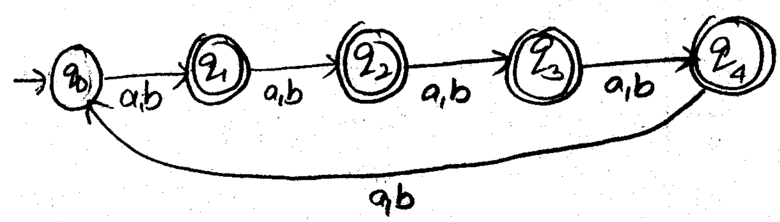


(7) on $\Sigma = \{a,b\}$

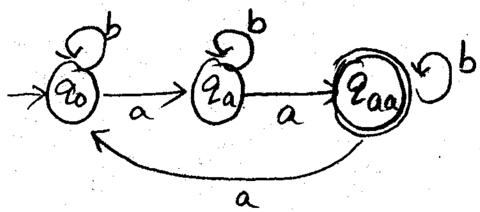
(a) $L = \{w : |w| \bmod 3 = 0\}$



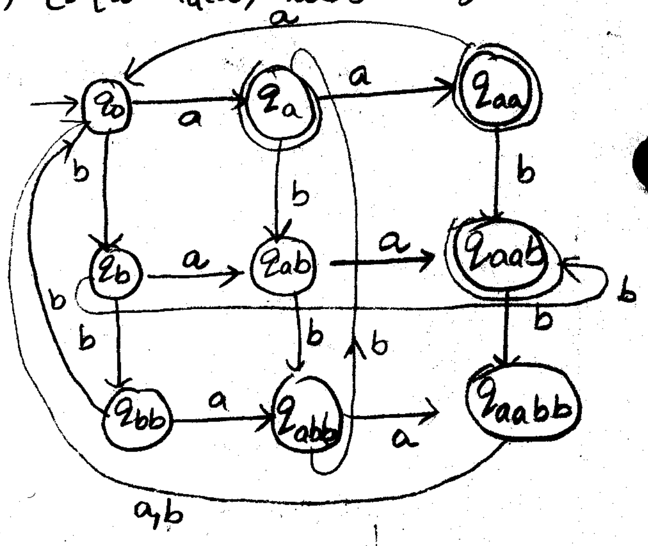
(b) $L = \{w : |w| \bmod 5 \neq 0\}$



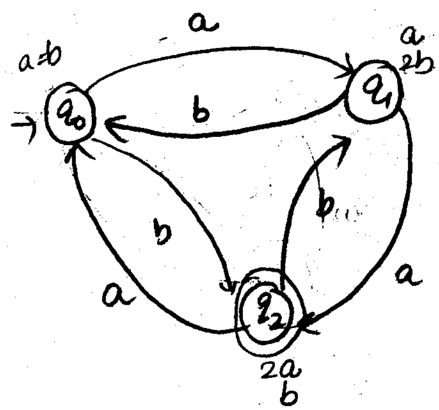
(c) $L = \{w : n_a(w) \bmod 3 > 1\}$



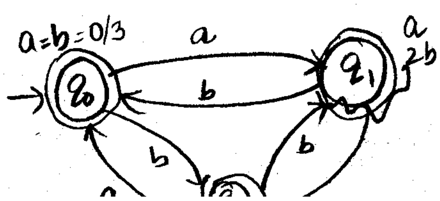
(d) $L = \{w : n_a(w) \bmod 3 > n_b(w) \bmod 3\}$



(e) $L = \{w : (n_a(w) - n_b(w)) \bmod 3 > 0\}$



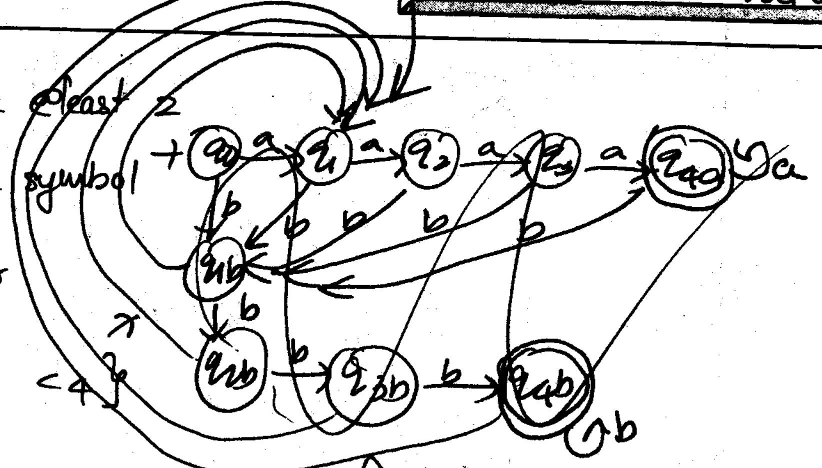
(f) $L = \{w : (n_a(w) + 2n_b(w)) \bmod 3 < 2\}$.



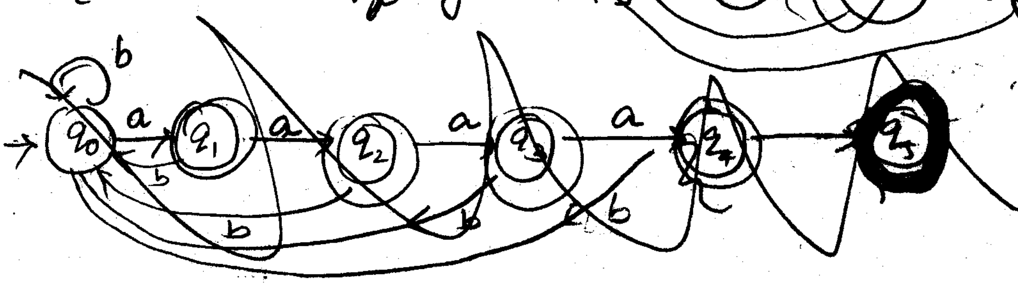
(8) run = substring of length ~~at least~~ 2

& entirely of same symbol

on $\Sigma = \{a, b\}$ find dfa for



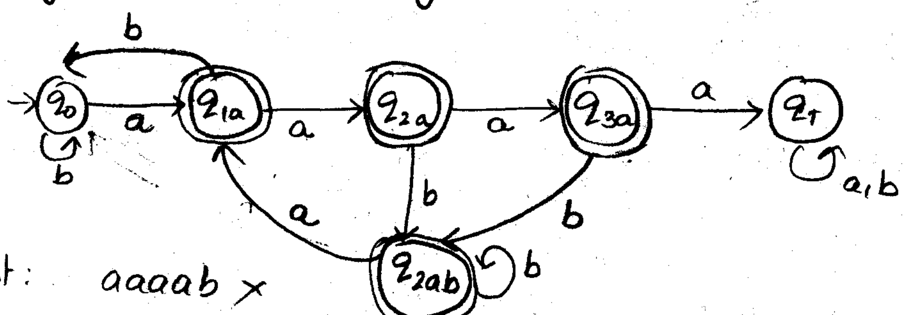
(a) $L = \{w :$ no runs of ~~length~~ exactly length $< 4\}$



zero, one a should be accepted also.

(b) $L = \{w :$ every run of a's has length 2 or 3.

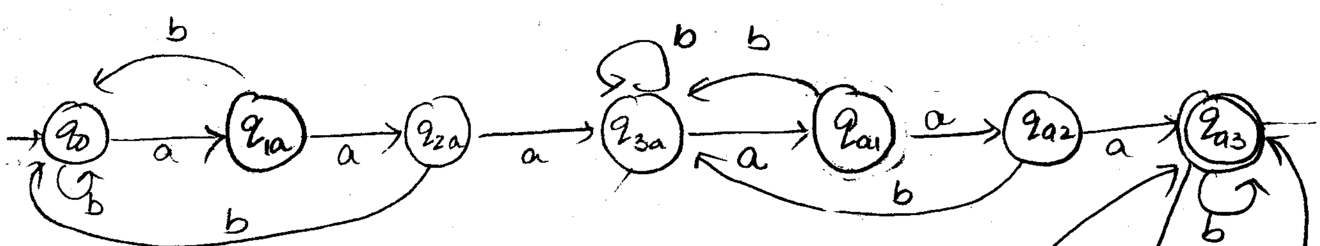
{ implies one a accepted }

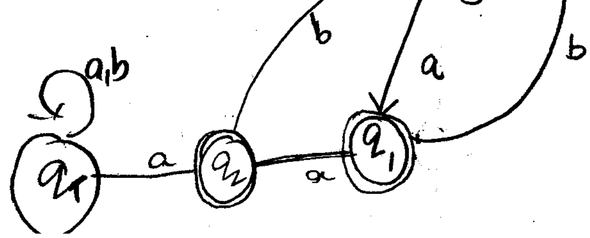


Test: aaaab ×
aabba ✓
baaab ✓
baba ✓
baaaa ×

(d) $L = \{w :$ ~~at least~~ exactly 2 runs of a's of length 3.$\}$

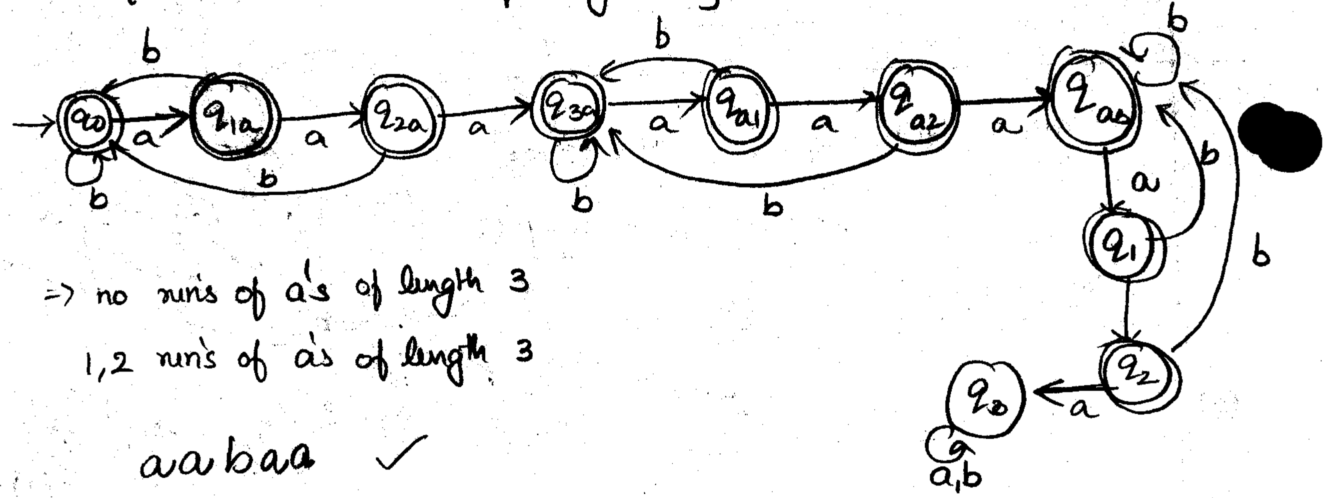
waaaw / waaawaaaw





Test: aaabaaa ✓ aabaaabaaa
aaabaaabaa

(d) $L = \{w:$ @most 2 runs of a's of length 3$\}$



=> no runs of a's of length 3

1,2 run's of a's of length 3

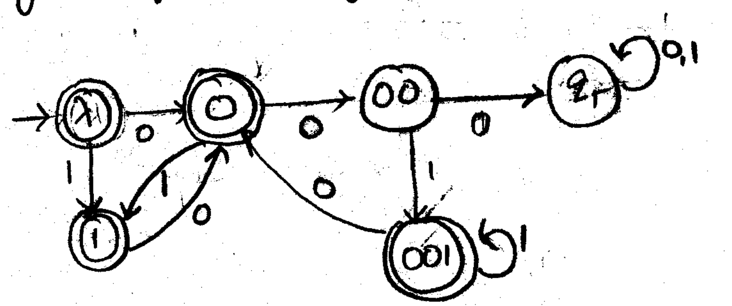
aabaa ✓

aaab

aaabaaa

(9) $\Sigma = \{0,1\}$

(a) Every 00 followed by 1



Test:

00100111 ✓

010001 ✗

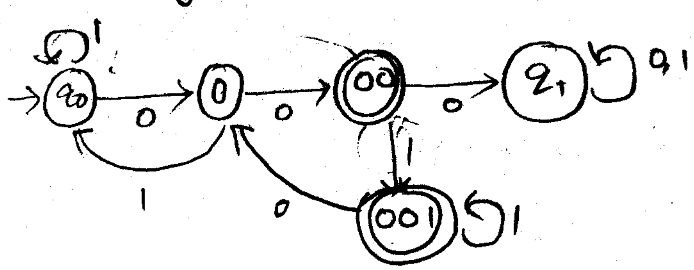
01 ✓

10 ✓

1 ✓

0 ✓

(b) All strings containing 00 but not 000.

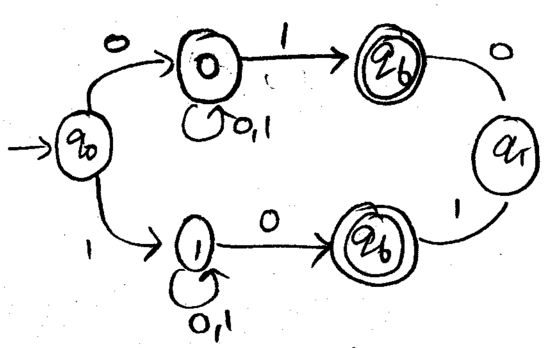


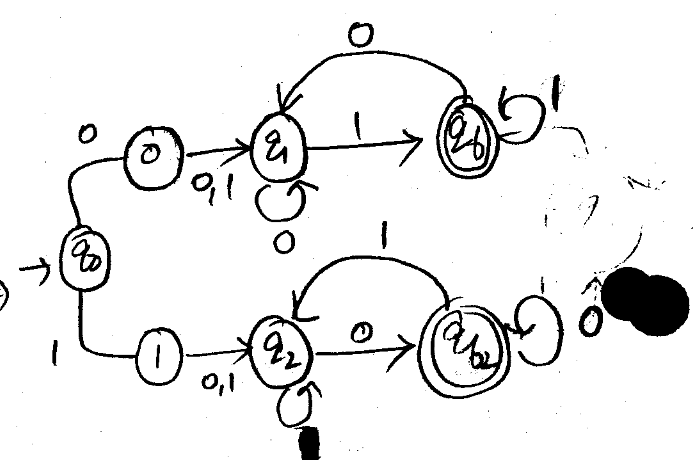
Test: 0010011 ✓

00011 ✓

1001000 ✗

(c) leftmost differs from rightmost.



=> 

nfa

1 w 0

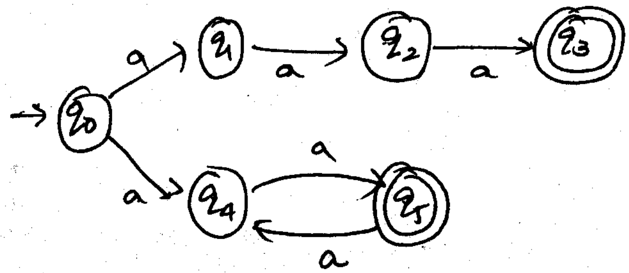
Nfa:

$$M = (Q, \Sigma, \delta, q_0, F)$$

$$\delta : Q \times (\Sigma \cup \{\lambda\}) \rightarrow 2^Q$$

Example 2.7

fig 2.8



Example 2.9

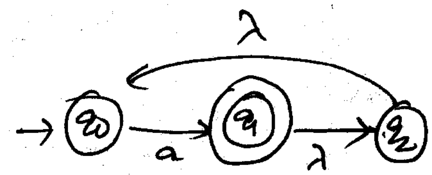
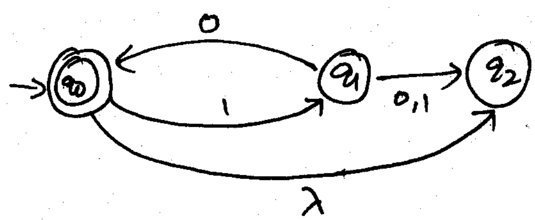


fig 2.10

Example 2.8

fig 2.9

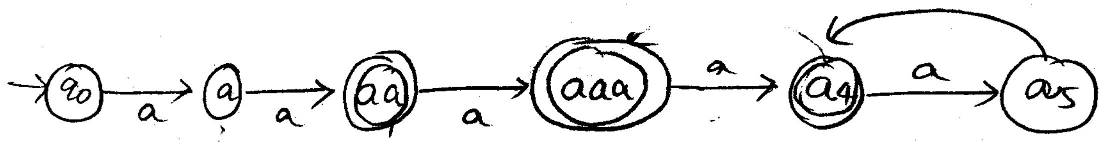


$$L(M) = \{ w \in \Sigma^* : \delta^*(q_0, w) \cap F \neq \emptyset \}$$

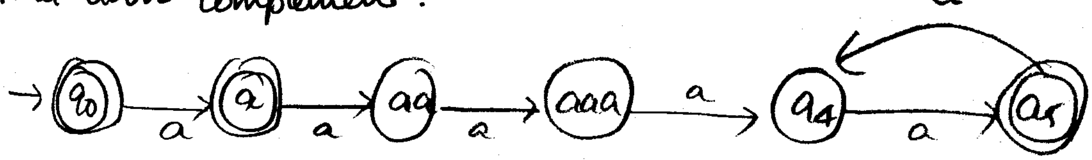
EXERCISES

(2) find dfa defined by: fig 2.8

L: $\{aaa\} \cup \{a^{2n} : n \geq 1\}$



(3) find above complement.



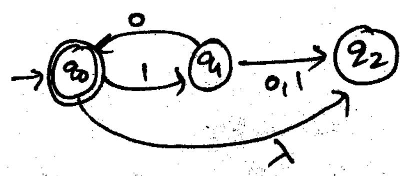
④ Fig 2.9 $\delta^*(q_0, 1011)$: $\delta^*(q_1, 011) \rightarrow \delta^*(q_0, 11) \rightarrow$ q_2

$\delta^*(q_1, 01)$: q_2

⑤ Fig 2.10 : $\delta^*(q_0, a)$ $\{q_0, q_1, q_2\}$

$\delta^*(q_1, \lambda)$ $\{q_0, q_2\}$

⑥ for:



$\delta^*(q_0, 1010) \rightarrow \delta^*(q_1, 010) \rightarrow \delta^*(q_0, 10) \rightarrow \delta^*(q_1, 0) \rightarrow q_2$ $\{\cancel{q_0}\ q_2\}$

$\rightarrow \delta^*(q_2, 1010)$ × $\rightarrow \delta^*(q_2, 10)$ × $\delta^*(q_2, 10)$ ×

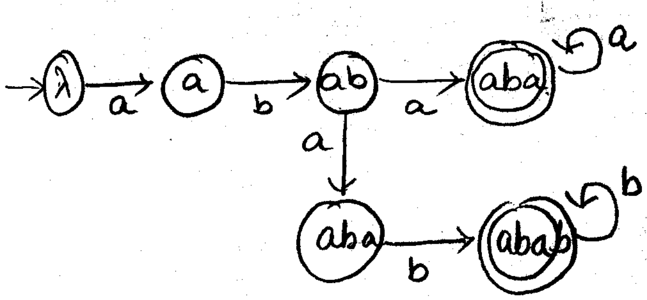
$\delta^*(q_1, 00)$

$\downarrow$ $\delta^*(q_1, 00) \rightarrow \delta^*(q_0, 0) \rightarrow q_2$

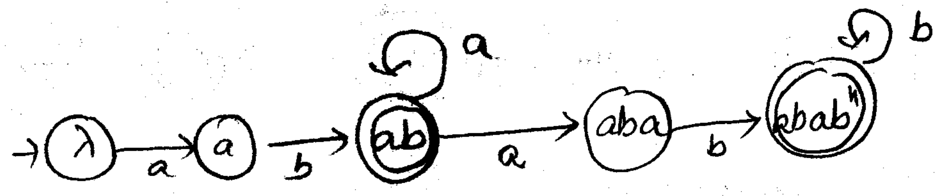
$\delta^*(q_2, 0)$ ×

⑦ no more than 5 states, design nfa for

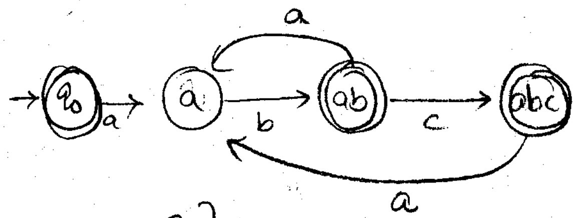
$\{abab^n : n \geq 0\} \cup \{aba^n : n \geq 0\}$

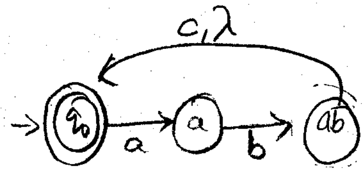


$\Rightarrow$



(8) Construct nfa with 3 states for $\{ab, abc\}^*$





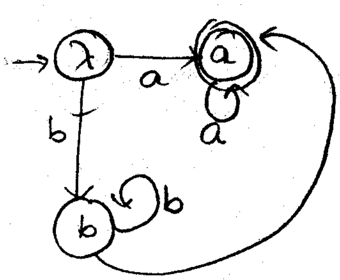
(9) Can it be done in fewer states? than 3?

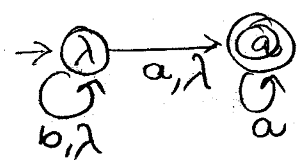
No as $|abab^n|$ least $= 3$ for $n=0$.

(10) (a) find nfa with 3 states that accepts

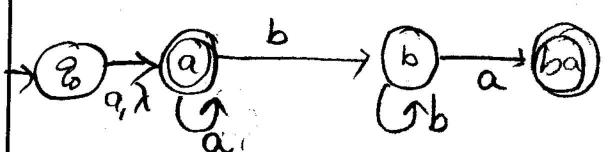
$L = \{a^n : n \geq 1\} \cup \{b^m a^k : m \geq 0, k \geq 0\}$

(b) can fewer than 3 states be possible?

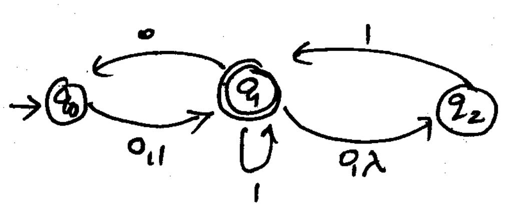




(11) Nfa - 4 states for $L = \{a^n : n \geq 0\} \cup \{b^n a : n \geq 1\}$



(12)



00 : $\delta^*(q_0, 00) \rightarrow \{q_0\ q_2\} \cap F = \emptyset$ reject

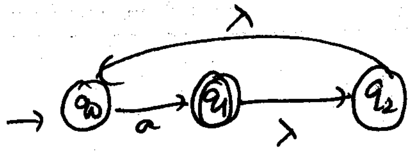
01001 : $\{q_1\} \cap F \neq \emptyset$ accept

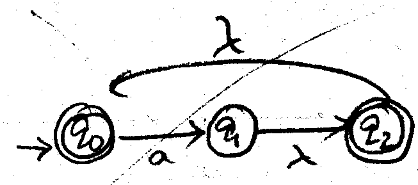
10010 : $\{q_0, q_2\} \cap F = \emptyset$ reject

000 : $\{q_1, q_2\} \cap F \neq 0$ accept

0000 : $\{q_0\ q_2\}$ reject

(13) What is complement of

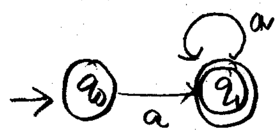




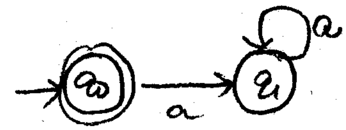
$\{\lambda, a, aa, \ldots\}$:

cant Take complement of nfa

dfa



complement $\longrightarrow$

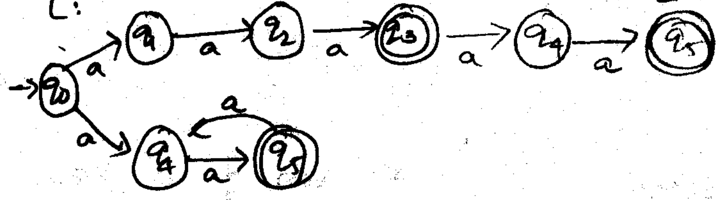


all strings $n_a(w) > 1$

all strings $n_a(w) < 1$

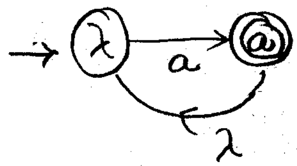
(14) $L \cup \{a^5\}$?

L:



(15) 13 ? $\{a\}^* - \{a\}^+$

(16) Find nfa for $\{a\}^*$ such that removing one edge will accept $\{a\}$



(17) Can above be done with dfa ?

No : need two paths to accept one a / more a's

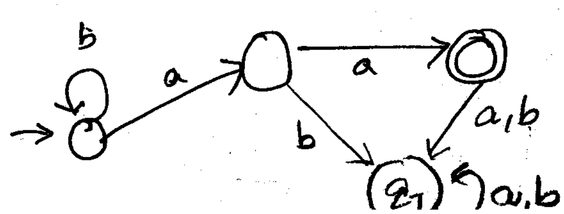
(18) let $p_0 \in Q_0$.

$\delta(p_0, \lambda) \to Q_0$

if Q_0 not initial, equivalent to M.

(19) Yes.

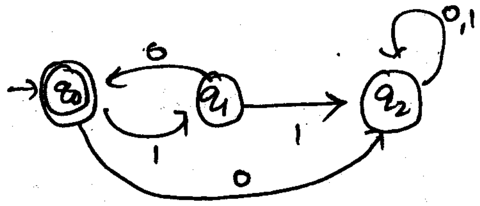
(20)



Equivalence of Nfa & Dfa:

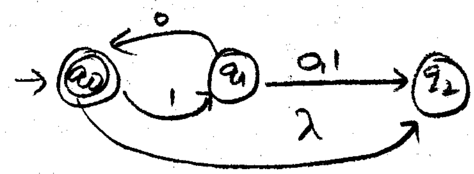
fig 2.11

dfa



$\simeq$

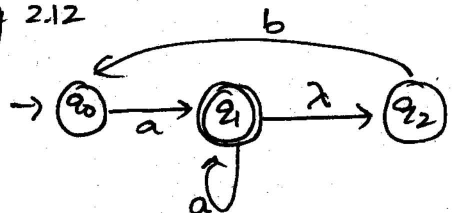
nfa



$\{\lambda, 10, 1010 \ldots\}$ $\Rightarrow$ $\{(10)^n : n \geq 0\}$

$\{(10)^n : n \geq 0\}$

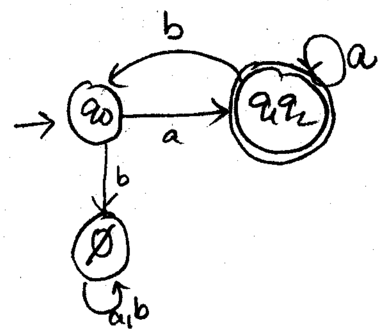
fig 2.12



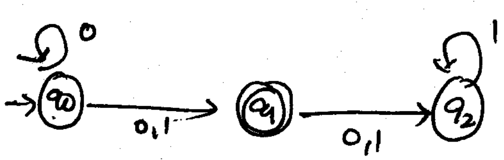
nfa

	a	b
$\{q_0\}$	$\{q_1, q_2\}$	$\emptyset$
F $\{q_1 q_2\}$	$\{q_1, q_2\}$	$\{q_0\}$
$\emptyset$	$\emptyset$	$\emptyset$

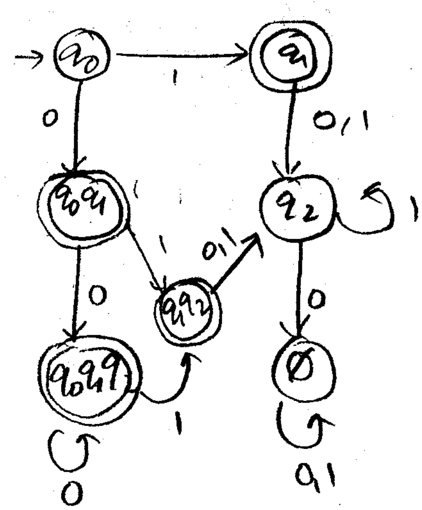
dfa



Example 2.13



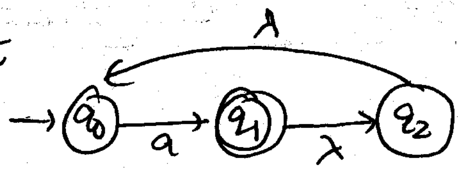
	0	1
$\{q_0\}$	$\{q_0\ q_1\}$	$\{q_1\}$
$\{q_0 q_1\}$	$\{q_0\ q_1 q_2\}$	$\{q_1,\ q_2\}$
$\{q_1\}$	$\{q_2\}$	$\{q_2\}$
$\{q_2\}$	$\emptyset$	$\{q_2\}$
$\{q_1 q_2\}$	$\{q_2\}$	$\{q_2\}$
$\{q_0 q_1 q_2\}$	[illegible]	[illegible]



If too many states getting combined, dont end, go on & enumerate all

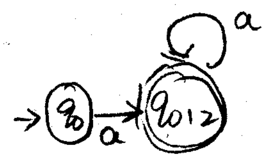
2-3 EXERCISES

① Convert

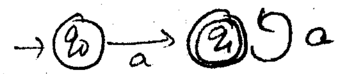


to dfa. Is there a simpler way?

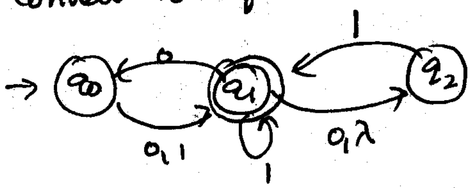
	a
$\{q_0\}$	$\{q_0 q_1 q_2\}$
$\{q_0 q_1 q_2\}$	$\{q_0 q_1 q_2\}$



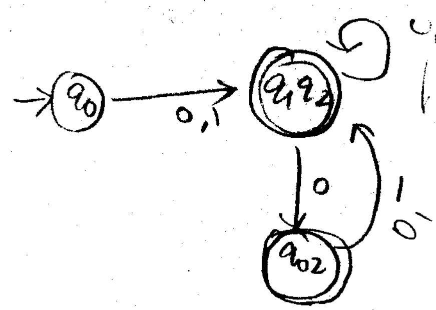
Yes: directly as $L = \{a^n : n > 0\}$



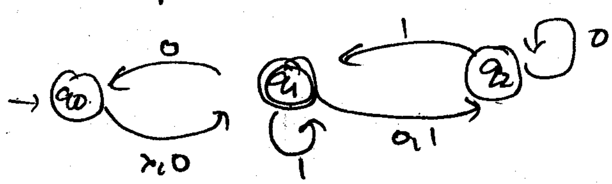
② Convert to dfa



	0	1
$\{q_0\}$	$\{q_1, q_2\}$	$\{q_1, q_2\}$
+ $\{q_1 q_2\}$	$\{q_0 q_2\}$	$\{q_1, q_2\}$
$\{q_0 q_2\}$	$\{q_1 q_2\}$	$\{q_1 q_2\}$

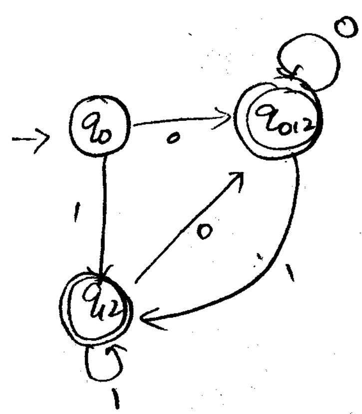


③ Convert nfa → dfa



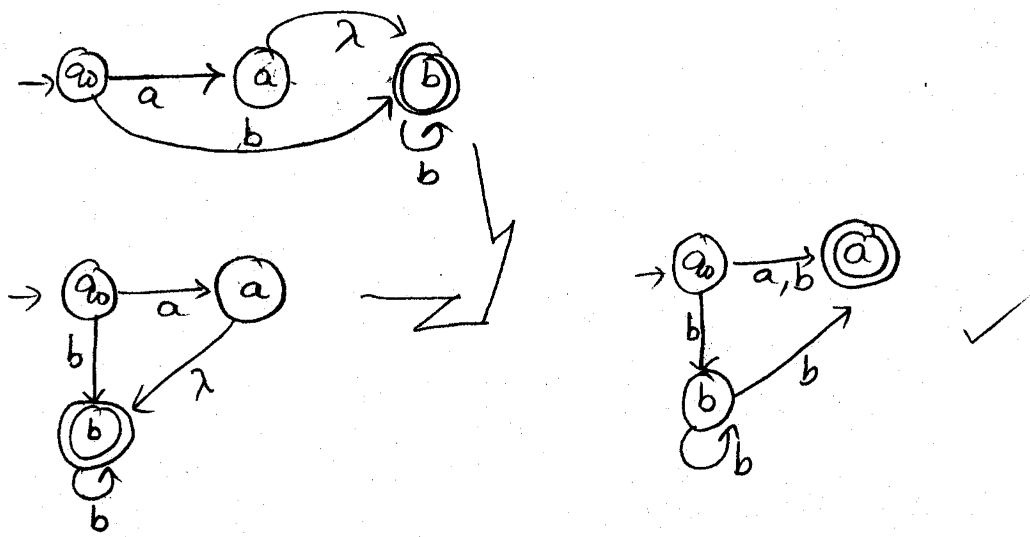
$q_0 = q_1$

	0	1
$\{q_0\}$	$\{q_0 q_1 q_2\}$	$\{q_1 q_2\}$
$\{q_1 q_2\}$	$\{q_0 q_1 q_2\}$	$\{q_1 q_2\}$
$\{q_0 q_1 q_2\}$	$\{q_0 q_1 q_2\}$	$\{q_1 q_2\}$



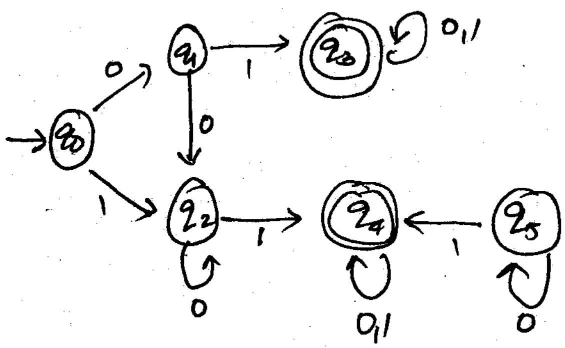
⑧ find nfa without λ-transitions, single final state for

$\{a\} \cup \{b^n : n \geq 1\}$



CH # 2.4 (Reduction of states in dfa)

Example 2.17



	q_0	q_1	q_2	q_3	q_4
q_0		D	D	D	D
q_1			ID	D	D
q_2				D	D
q_3					ID
q_4					

I states: q_5

D: $\{q_0, q_1\}$
$\{q_0, q_2\}$

ID: $\{q_3\ q_4\}$
$\{q_1\ q_2\}$

$\{q_0\}, \{q_1 q_2\}, \{q_3\ q_4\}$

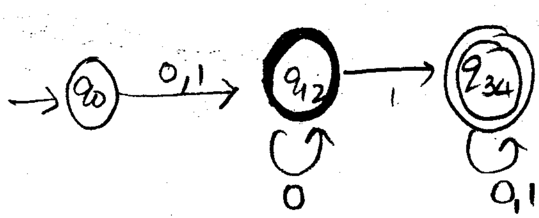
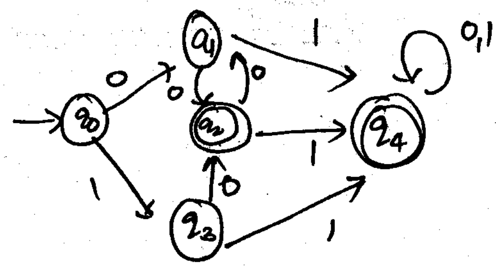


fig: 2.18



D: $\{q_1, q_4\}$
$\{q_2, q_4\}$
$\{q_3, q_4\}$
$\{q_0, q_4\}$

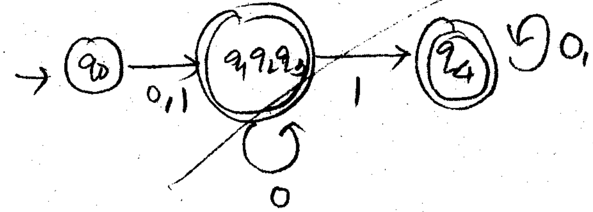
ID: $\{q_1, q_3\}$

$\downarrow 0 \quad \downarrow$

$\{q_2\} \quad \{q_4\}$

	q_0	q_1	q_2	q_3	q_4
q_0		D	D	D	D
q_1			ID	ID	D
q_2				ID	D
q_3					D
q_4					

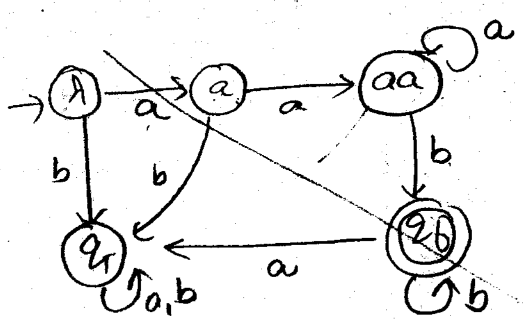
$\{q_0\}, \{q_1 q_2 q_3\} \; \{q_4\}$



EXERCISES

(2) (a) find minimal dfa for

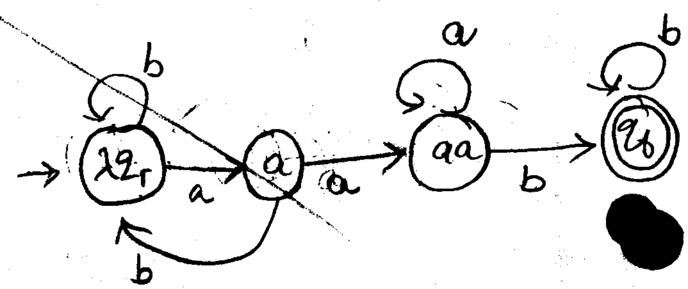
$L = \{a^n b^m : n \geq 2, m \geq 1\}$



	λ	a	aa	q_f	q_T
λ		D	D	D	ID
a			D	D	D
aa				ID	D
q_f					D
q_T					

$\{\lambda, q_T\}, \{a\}, \{a\}, \{aa\} \; \{q_f\}$

ID: (aa, q_f)
(λ, q_T)



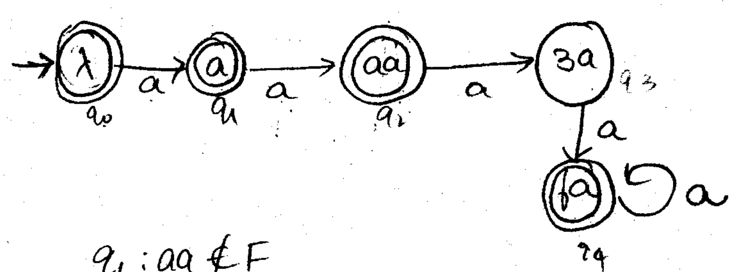
D: (a, q_f) (λ, q_f) (q_T, aa)
(q_T, q_f) (λ, aa) (q_f, aa)

② (a) $L = \{a^n b^m : n \geq 2, m \geq 1\}$

minimal

(b) $L = \{a^n b : n \geq 0\} \cup \{b^n a : n \geq 1\}$

(c) $L = \{a^n : n \geq 0, n \neq 3\}$



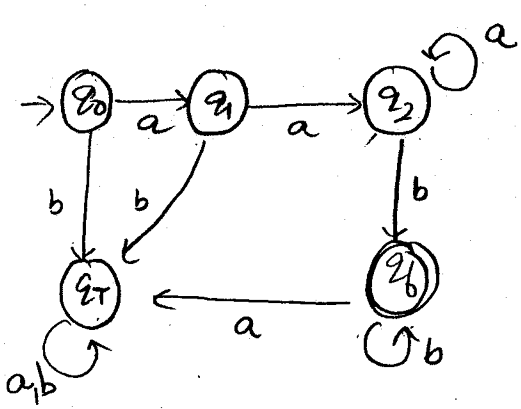
	λ	a	aa	3a	fa
λ		D	D	D	D
a			D	D	D
aa				D	D
3a					D
fa					

$q_1 : aa \notin F$
$(q_4, aa) \in F$ $\therefore (q_1 q_4)$ D

$(\lambda, aaa) \notin F$
$(3a, aaa) \in F$ $\therefore (\lambda, 3a)$, D

already in minimal //

(a)



	q_0	q_1	q_2	q_f	q_T
q_0		D	D	D	D
q_1			D	D	D
q_2				D	D
q_f					D
q_T					

$q_f \in F$
Rest $\notin F$ $\therefore$ D-states

$\therefore$ minimal

CHAPTER : 3

$\boxed{\text{RL \& RG}}$

→ $L(r_1 + r_2) = L(r_1) \cup L(r_2)$

$L(r_1 . r_2) = L(r_1) . L(r_2)$

$L(r_1^*) = (L(r_1))^*$

Example 3.2 $L(a^* . (a+b))$?

$L(a^* . (a+b)) = L(a^*) . L(a+b)$

$= L(a^*) . \{L(a) \cup L(b)\}$

$= L(a^*) . \{a, b\}$

$= \{\lambda, a, aa \ldots\} . \{a, b\}$

$= \{a\}^+ \cup \{a^n b^{n+1} : n \geq 0\}$

Example 3.3 $\Sigma = \{a, b\}$

$r = (a+b)^* . (a+bb)$

$\{a, b\}^* . \{a, bb\}$: $= \{a, bb, aa, abb, ba, bbb \ldots\}$

Example 3.4 $r = (aa)^* (bb)^* b$

$L = \{(aa)^n (bb)^n b : n \geq 0\}$

$L = \{a^{2n} b^{2m+1} : n, m \geq 0\}$

Example 3.5 $\Sigma = \{0, 1\}$: w has atleast one pair of consecutive zeroes.

$(0+1)^* 00 (0+1)^*$

3.6. no consecutive zeroes : $(1+01)^* (0+\lambda)$

RG, RL's

EXERCISES

① $L[(a+b)^* b (a+ab)^*]$ find strings $|w| < 4$.

$\{\lambda, a, b, ab, ba, aa, bb, aba, baa, aaa, bba, bab, abb, aab, \dots\}. b.$

$\{\lambda, a, ab, aab, aba, aaa\} \dots$

$|w| < 4$: $\{b, ab, bb, ba, bab \dots\}$

② $((0+1)(0+1)^*)^* 00 (0+1)^*$ denote @least one pair of consecutive 0's.

yes.

③ $r = (1+01)^* (0+1^*)$ also denotes no consecutive zeroes.

$\downarrow$

$(1+01)^* (0+\lambda+\{1\}^+)$

$((1+01)^* (0+\lambda)) + (1+01)^* \{1\}^+$

$\downarrow$

$(1+01)^*$

$= (1+01)^* (0+\lambda) \Rightarrow$ no consecutive zeroe's

④ RE? $\{a^n b^m : n \geq 3, m \text{ is even}\}$

$aaa(a^*)(bb)^*$

⑤ RE = ? $\{a^n b^m : (n+m) \text{ is even}\}$

$[(aa)^*(bb)^* + (aa)^* a (bb)^* b]$

RE = ?

(6) (a)

$$L_1 = \{a^n b^m : n \geq 4, m \leq 3\}$$

$$aaaa \cdot a^* (\lambda + b + bb + bbb)$$

(b) $L_2 = \{a^n b^m : n < 4; m \leq 3\}$

$$(\lambda + a + aa + aaa)(\lambda + b + bb + bbb)$$

(c) $\overline{L_1}$: $\{a^n b^m : n < 4, m > 3\}$ (d)

$\cancel{X}$ $(\lambda + a + aa + aaa)\, bbbb\, b^* +$

RE($\overline{L}$)= all possible rule breakers in RE(L) →

either $n < 4$ or $m \geq 4$ (or) $\overset{n}{a}\overset{k}{b}\overset{l}{a}$

$$(\lambda + a + aa + aaa)\, b^* + a^* bbbbb^* +$$

$$(a+b)^* ba (a+b)^*$$

(d) $\overline{L_2}$: $\{a^n b^m : n < 4, m \leq 3\}$

$n \geq 4 \,/\, m > 3$

$$\overline{L_2}: \quad aaaaa^* b^* + a^* bbbbb^* + (a+b)^* ba (a+b)^*$$

(8) $L\,[(aa)^* b (aa)^* + a(aa)^* ba(aa)^*]$

$w b w$: $w : a^{2n} \; : n \geq 0$

$w : a^{2n+1} \; n \geq 0$

b having even a's on both ends or
b having odd a's on both ends.

3-1
Exercises :

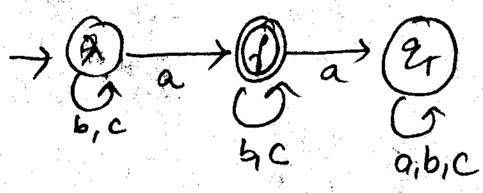
(16)

(a) $\Sigma = \{a, b, c\}$

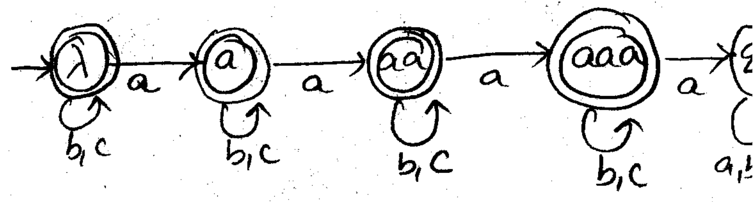
Exactly one a.

$(b+c)^* a (b+c)^*$

a ✓ bcbca ✓
ab ✓ bcaa ✗



(b) no more than 3 a's.



$[(b+c)^* a (b+c)^* a (b+c)^* a (b+c)^*$
$+ (b+c)^* + (b+c)^* a (b+c)^* +$
$(b+c)^* a (b+c)^* a (b+c)^*]$

Test: aa ✓ aaaa ✗
bca ✓

(c) @least one occurance of each symbol in Σ

$(a+b+c)^* a (a+b+c)^* b (a+b+c)^* c (a+b+c)^*$

(d) no run of a's $|w|_a > 2$

0, 1, 2

$\cancel{(b+c)^* (\lambda+a)(b+c)^* (\lambda+a)(b+c)^*}$

$(\lambda + a + aa + b + c)^*$

(e) run's of a's are multiples of 3.

$\cancel{(b+c)^* aaaa^* (b+c)^*} aaaa^* (b+c)^*$

$(\lambda + aaaa^* + b + c)^*$

17.

(a) Ending in 01

$(0+1)^* 01$

(17)

(b) Not Ending in 01:

$1^* (0 + 01 + 11)^* 11^* (0 + \lambda)$ ✓

(c) Even no. of zeros

$[1^* 0 1^* 0 1^* + 1]^*$

CHAPTER 3-2

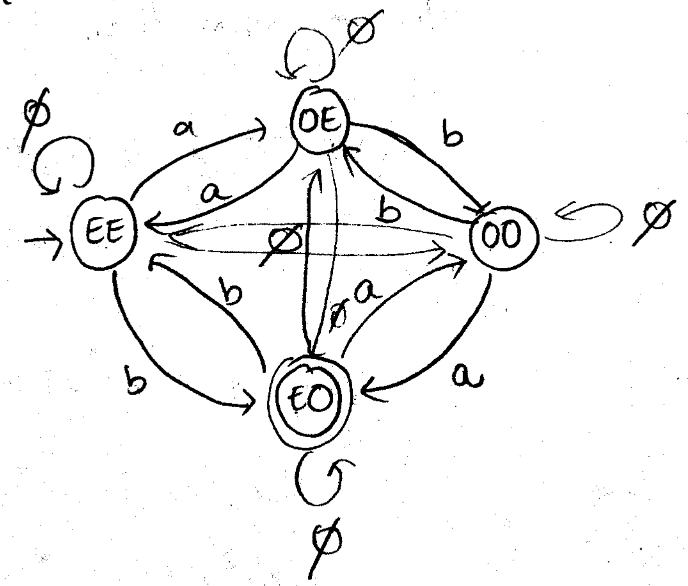
Example 3.11

RE = ?

$L = \{ w \in \{a,b\}^* : n_a(w) \text{ is even}, n_b(w) \text{ is odd} \}$

FIND RE for a nfa?

$\boxed{\emptyset \text{ to all unknown moves!}}$



$r + \emptyset = r$

$r\emptyset = \emptyset$

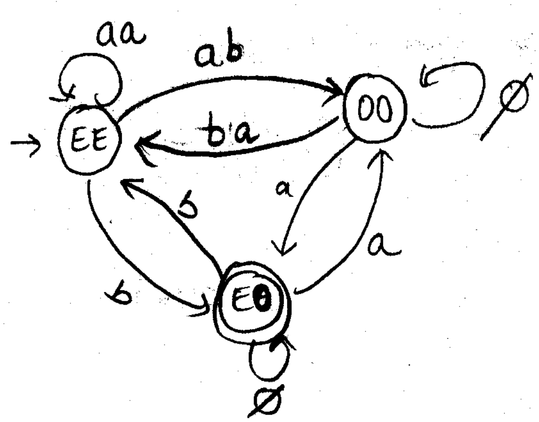
$\emptyset^* = \lambda$

step ①

$$r_{EE} = \emptyset + a\emptyset^* a$$
$$= aa$$

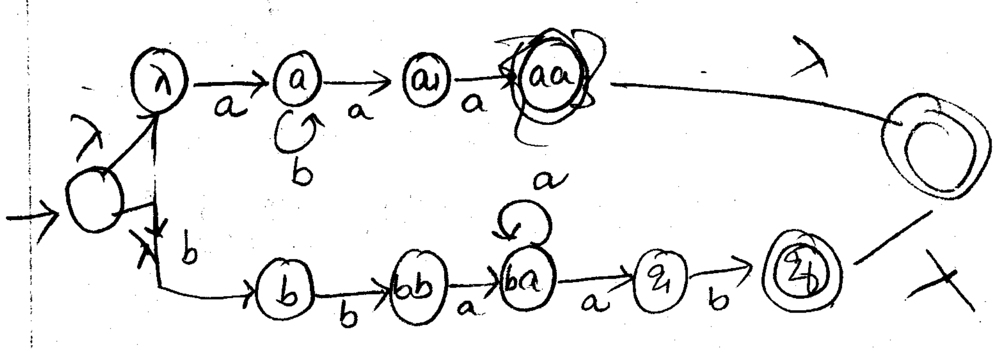
step 2

now 3-state Rule



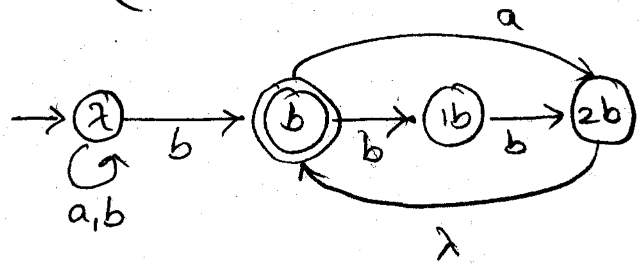
EXERCISE

$L(ab^*aa + bba^*ab)$:



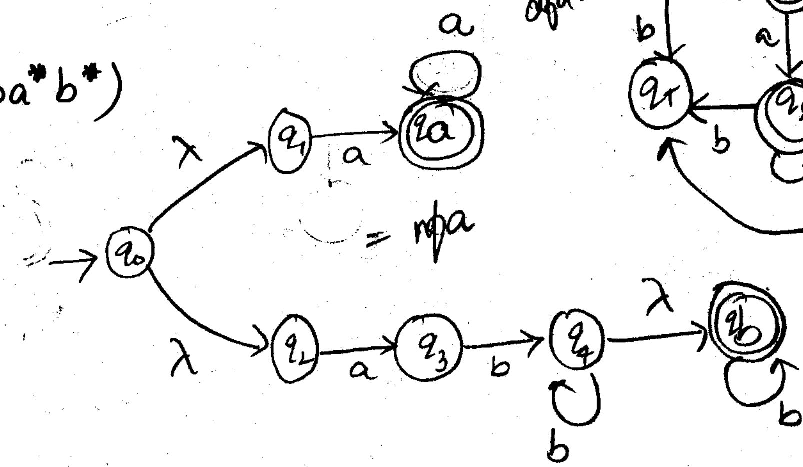
Prathima Bhima
CLASS: AT : 3-2
PAGE#: 23
DATE: 11 OCT 06

(3) nfa? $((a+b)^* b (a+bb)^*)$



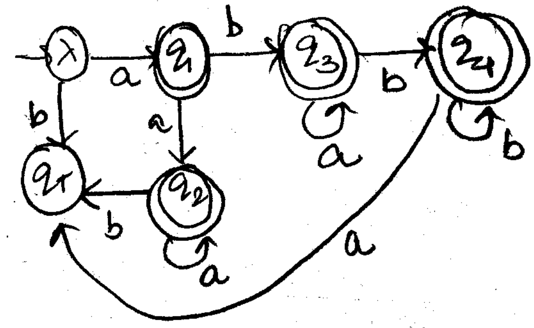
(4) (a) dfa?

$L(aa^* + aba^*b^*)$



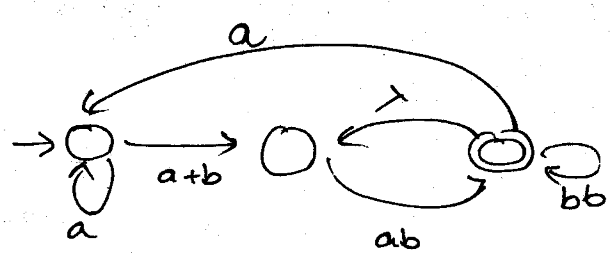
= nfa

dfa:

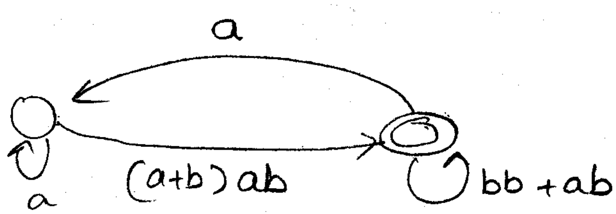


Removing an edge in a transition graph ($q_0 \notin F$)

(8)

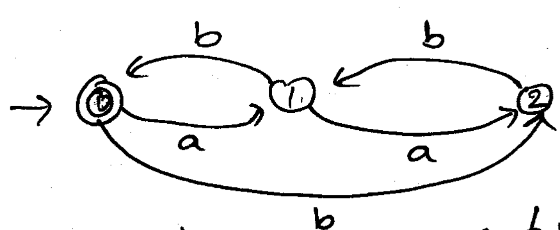


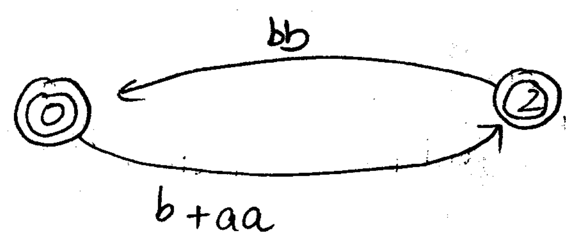
$r = a^*(a+b)ab(ab + bb + aa^*(a+b)ab)^*$



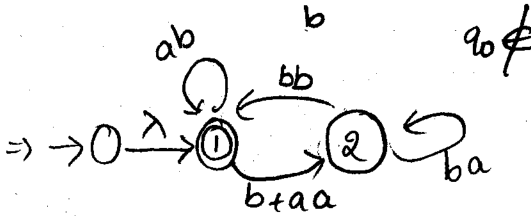
aabbbbb

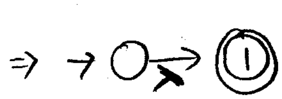
(10) (b)





$q_0 \notin F$.

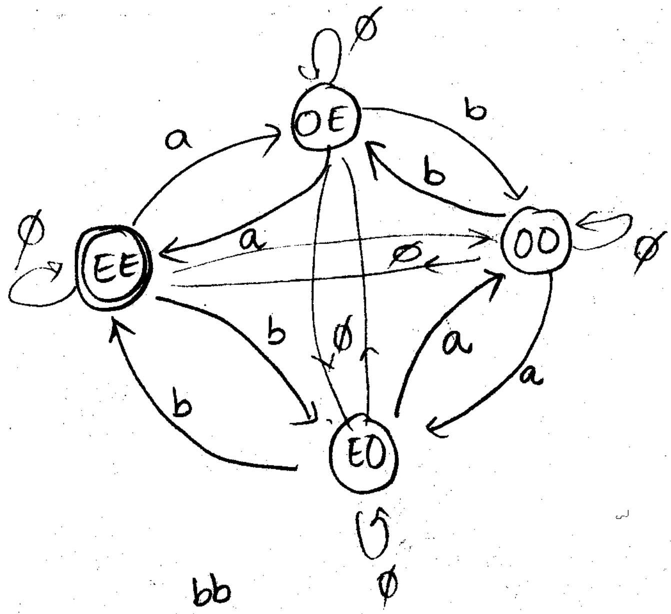


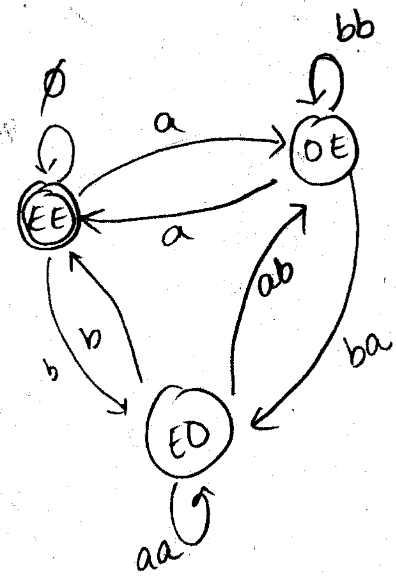


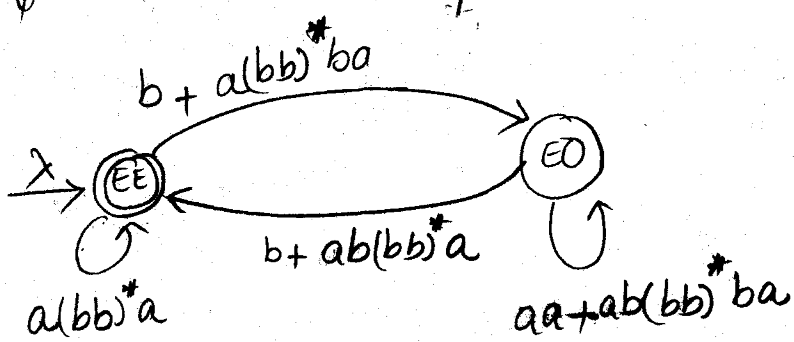
$ab + (b+aa)(ba)^* bb$

RE = ? on $\Sigma = \{a, b\}$

(13) (a) $L = \{w : n_a(w), n_b(w) \text{ are even}\}$







RE :

$$\lambda + [a(bb)^*a]\,[b + a(bb)^*ba]\,[aa + ab(bb)^*ba]^*\,[b + ab(bb)^*($$

λ ✓

aa

CH # 3.3

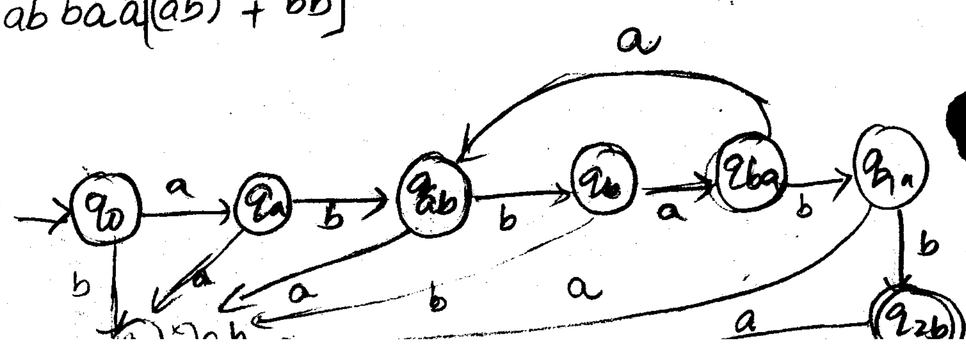
(1) dfa = ?

$S \to abA$

$A \to baB$

$B \to aA / bb$

$ab\,baa[(ab)^* + bb]$



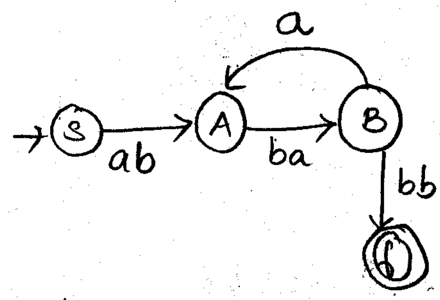
(3) LLG =? for 1

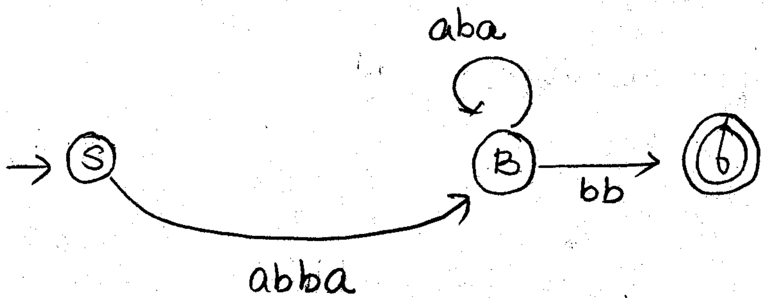
$S \to abA$

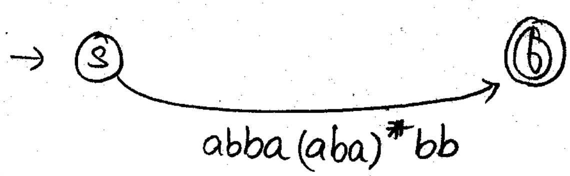
$A \to baB$

$B \to aA / bb$

RE =







Test: abbabb ✓

abbaabaababb ✓

$$\boxed{\begin{aligned} &S \to ABbb \\ &A \to abba \\ &B \to Baba \,/\, \lambda \end{aligned}}$$

$S \to ABbb \to abbaBbb \to$

$abba\,Baba\,bb \to$

$abba(aba)^*\,bb$ ✓

(4) RLG, LLG = ?

$\{a^n b^m : n \geq 2, m \geq 3\}$

LG

$S \to aaA\,bbbB$

$A \to aA / \lambda$

$B \to bB / \lambda$

RLG:

$S \to aaAB$

$A \to aA / \lambda$

$B \to bbbC$

$C \to bC / \lambda$

LLG:

$S \to ABbbb$

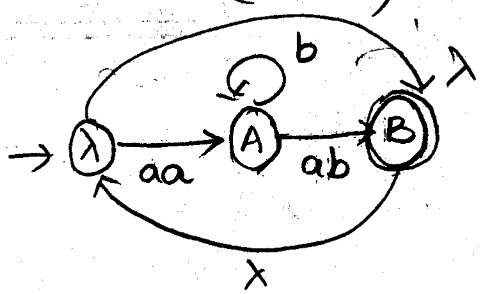
$A \to Caa$

$C \to Ca / \lambda$

$B \to Bb / \lambda$

(6) RLG = ?

$(aab^*ab)^*$



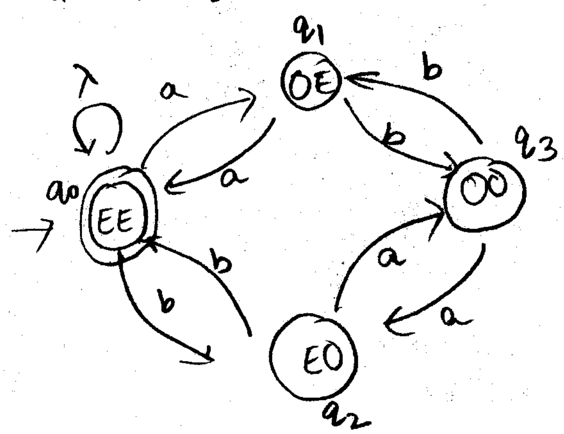
$S \to aaA / \lambda$

$A \to bA / abS$

(13)

(a) ~~REG~~ RG = ? $\Sigma = \{a,b\}$

(a) $n_a(w)$, $n_b(w)$ are even.



$q_0 \to aq_1 / \lambda / bq_2$

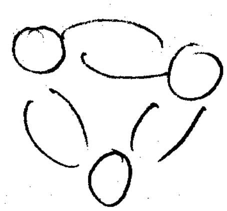
$q_1 \to bq_3 / aq_0$

$q_2 \to aq_3 / bq_0$

$q_3 \to aq_2 / bq_1$

$(n_a - n_b) \bmod 3 = 1$

draw

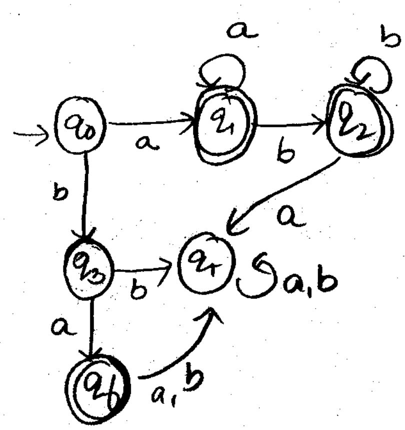


Right Quotient : L_1/L_2

RIGHT QUOTIENT
L_1/L_2
→ $\boxed{\text{draw dfa}}$ for L_1
→ ∀ nodes $\boxed{\text{apply } L_2}$ to check final state

$L_1 : \{a^n b^m : n \geq 1, m \geq 0\} \cup \{ba\}$

$L_2 : \{b^m : m \geq 1\}$



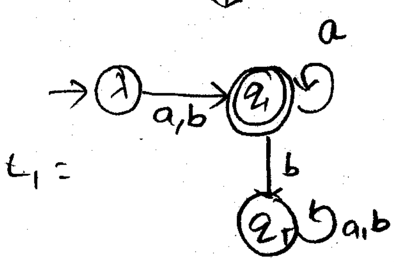
for L_1/L_2

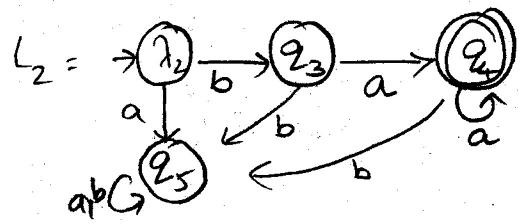
final states are:

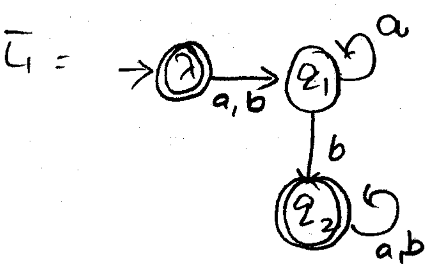
q_1, q_2

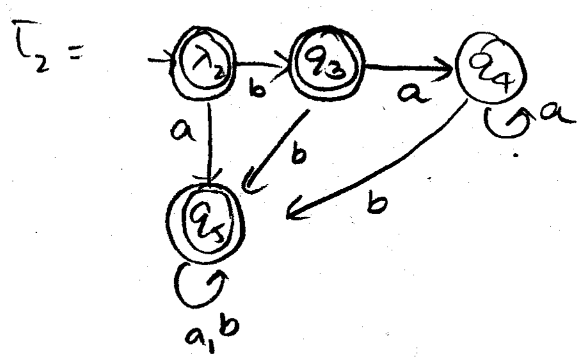
Exercises

(2) (a) $(a+b)a^* \cap (baa^*)$ nfa = ?

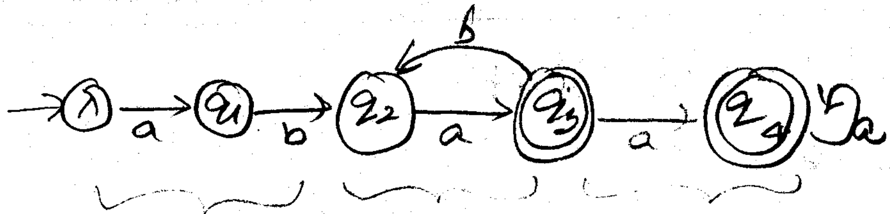








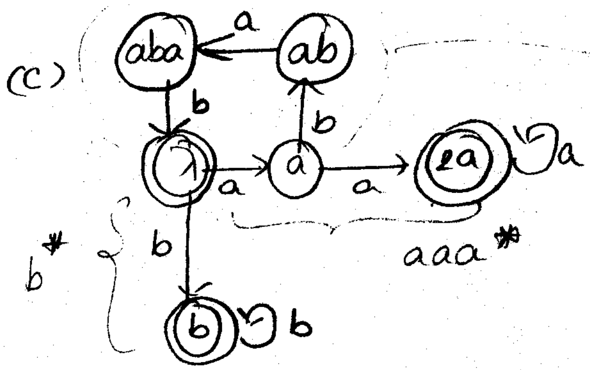
(4) (b) $ab(a+ab)^*(a+aa)$



ab $\quad ab^*$ $\quad a^*$

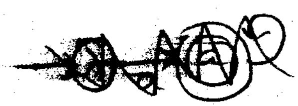
a/aa

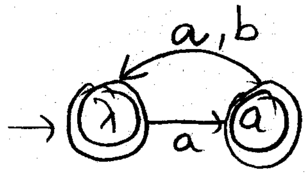
(c)



$(abab)^*$

(d)



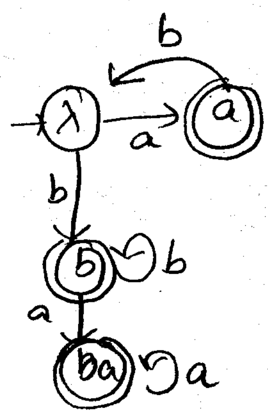


$\lambda, a, aa, ab, abab$

$(aa^*b$

(b) $\quad \emptyset$

(5) (a)



ab^*a^*

a ✓

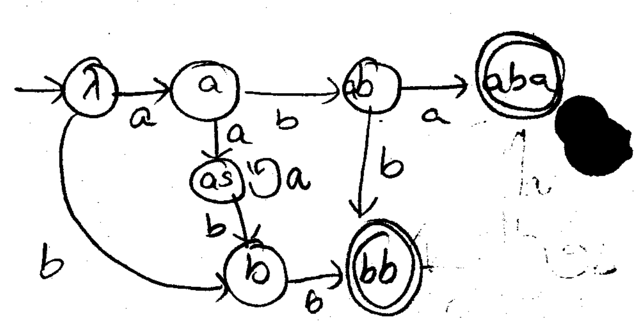
aba ✓

abba ✓

ab^*a

abb

(7)



bb ✓ $\quad ab^*ba$

abb ✓ $\quad$ aba ✓

a^*bb $\quad$ abba

abbba ✓ $\quad a^*bba$

CONTEXT-FREE GRAMMARS

A Grammar $G = (V, T, S, P)$ is CF if all productions in P are of the form

$$\boxed{A \rightarrow x}$$

$$\left(x \in (V \cup T)^*, A \in V \right)$$

Example 5.1

$G = (\{S\}, \{a, b\}, S, P)$

P:
$S \rightarrow aSa$
$S \rightarrow bSb$
$S \rightarrow \lambda$

$S \rightarrow aSa \rightarrow abba$

$S \rightarrow aSa \rightarrow aaSaa \rightarrow aabbaa$

$L(G) = \{ww^R : w \in \Sigma^*\}$

Example 5.2

G: P:
$S \rightarrow abB$
$A \rightarrow aaBb$
$B \rightarrow bbAa$
$A \rightarrow \lambda$

$S \rightarrow abbbAa \rightarrow abbba(ba)^0$

$\rightarrow abbbaaBba \rightarrow abbbaabbaba$

$\rightarrow abbbaabbaaBbaba \rightarrow abbbaabbaabbababa$

$\rightarrow abbbaabbaabbAababa \rightarrow abbbaabbaabbababa$

$L(G) = \{ab(bbaa)^n bba(ba)^m : n, m \geq 0\}$

Example 5.3

ST $L = \{a^n b^m : n \neq m\}$ is context free.

$\left(\begin{array}{l} L : \{a^n b^n : n \geq 0\} \\ S \rightarrow aSb / \lambda \end{array} \right)$

$S \rightarrow aSb / aA / bB$
$A \rightarrow aA / \lambda$
$B \rightarrow bB / \lambda$

$S \rightarrow AS_1 / BS_1$
$A \rightarrow aA / \lambda$
$B \rightarrow bB / \lambda$
$S_1 \rightarrow aS_1b / \lambda$

[redacted]

Example 5.4

$S \to aSb \,/\, SS \,/\, \lambda$

$L(G) = \{ w : w \in \{a,b\}^*,\ n_a(w) = n_b(w)\ \&\ n_a(v) \geq n_b(v),\ v \text{ is any prefix of } w \}$

$S \to aSb \to aaSbb \to aabb$

$S \to SS \to aSbaSb \to abab \ldots$

Leftmost & Right Most derivations:

$S \to AB$

$A \to aaA$

$A \to \lambda$

$B \to Bb / \lambda$

$S \to aaABb$

$A \to aaA / \lambda$

$B \to Bb / \lambda$

$L(G) = \{a^{2n} b^m : n, m \geq 0\}$

Example 5.5

$S \to aAB$

$A \to bBb$

$B \to A / \lambda$

$S \to aAB \to abBb \to abb$

$S \to aAB \to abBbbBb \to abbbb$

$S \to aAB \to abBbbBb \to abbbbBbb \to abbbbbb$

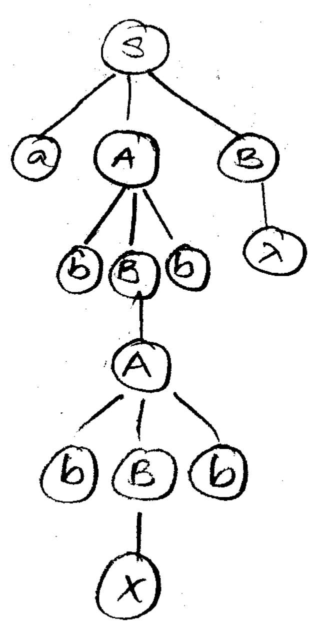
$L(G) = \{ab^{2n} : n > 0\}$

Example 5.6.

$S \to aAB$

$A \to bBb$

$B \to A / \lambda$



CHAPTER : 5-1
(CONTEXT FREE GRAMMARS)

Def: $G = (V, T, S, P)$

$\boxed{A \to x}$

$x \in (V \cup T)^*$

Eg: 5.1

$G = (\{S\}, \{a, b\}, S, P)$

$S \to aSa$
$S \to bSb$
$S \to \lambda$

$L(G) = \{ww^R : w \in \{a,b\}^*\}$

G is CFG, but not regular.

Eg: 5.2

$S \to abB$
$A \to aaBb$
$B \to bbAa$
$A \to \lambda$

$S \Rightarrow abbbAa \Rightarrow ab\boxed{bba}$
$\Rightarrow abbbaa\boxed{bba}ba$
$\Rightarrow abbbaabbAaba \Rightarrow ab\boxed{bbaa}bbaa\boxed{bba}baba$

$L(G) = \{ab(bbaa)^n bba(ba)^n : n \geq 0\}$

Eg: 5.3

$L = \{a^n b^m : n \neq m\}$

$$\left(\begin{array}{c} n = m \\ S \to aSb / \lambda \end{array} \right)$$

$n > m$

$a^{n+x} b^n$

$S_1 \to AB$
$A \to aA / a$
$B \to aBb / \lambda$

$n < m$, $m > n$

$a^n b^{n+x}$

$S_2 \to BC$
$C \to Cb / b$

$\therefore$

$S \to AB / BC$
$B \to aBb / \lambda$
$A \to aA / a$
$C \to bC / b$

$G = (V, T, S, P)$

Eg. 5.4 $S \to aSb \,/\, SS \,/\, \lambda$

$\Rightarrow L(G) = \{ w \in \{a,b\}^* : n_a(w) = n_b(w),\ n_a(v) \geq n_b(v)$ where v is prefix of $w \}$

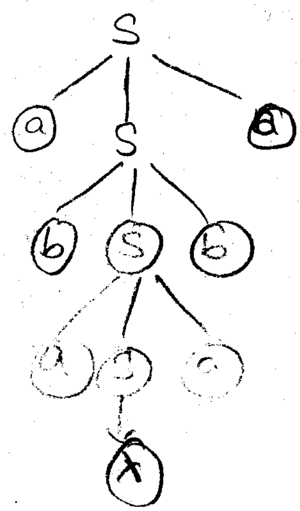
$$\left.\begin{array}{l} S \to AB \\ A \to aaA \,/\, \lambda \\ B \to Bb \,/\, \lambda \end{array}\right\} \quad L = \{a^{2n} b^m : m, n \geq 0\}$$

(EXERCISES)

(2) $S \to aSa$

$S \to bSb$

$S \to \lambda$



(7) Find CFG for $n \geq 0, m \geq 0$

(a) $L = \{a^n b^m : n \leq m+3\}$

$n = m$

$\{ S \to aSb \,/\, \lambda \}$

(1) $n = m+3 \longrightarrow$

(2) $n < m+3 \Rightarrow$ add any no. of b's

$\downarrow$

$S \to aaaA$

$A \to aAb \,/\, B$

$B \to bB \,/\, \lambda$

$n \leq m+3 \Rightarrow n = 0,1,2$

$S \to aA \,/\, aaA \,/\, aaaA \,/\, \lambda$

$A \to aAb \,/\, B$

$B \to bB \,/\, \lambda$

$S \to aSb \,/\, aaa$

$n = m+3$

$a^n b^m \Rightarrow a^{m+3} b^m$

$S \to AB$

$A \to aaa$

$B \to aBb \,/\, \lambda$

$\to \boxed{\begin{array}{l} \cancel{S \to aaaA} \\ \cancel{A \to aAb \,/} \end{array}}$

$m=0$: $n=0$ λ ✓ $n=1$ a ✓ $n=2$ aa ✓

$m=1$: $n=1$ ab ✓ $n=2$ aab ✓ $n=3$ $aaab$ ✓ $n=4$ $aaaab$

$m=1$ $n=5$ ✗

$aaaaab$

$aaaA \to aaa$

CH:5·1

EXERCISES.

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[redacted]

7 (b) HW $L = \{a^n b^m : n \neq m-1\}$

$n=0\ m=1$; b ✓
$n=1\ m=2$: abb ✓
$n=1\ m=1$; ab X

$n = m-1$
$m = n+1$

$S \to Ab$
$A \to aAb / \lambda$

$n < m-1$
add b's

$S \to Ab$
$A \to aAb / B$
$B \to bB / b$

$n > m-1$
add a's

$S \to Ab$
$A \to aAb / C$
$C \to aC / a$

Test: $n: 0,1,2$; $m=4$

✓ bbbb : $S \to Ab \to Bb \to bbbb$ ✓
abbbb : $S \to Ab \to aAbb \to aBbb \to abbbb$ ✓
aabbbb : $S \to Ab \to aaAbbb \to aabbbb$ ✓
✓ aaabbbb : $S \to Ab \to aaaAbbbb \to aaabbbb$ X
aaaabbbb : $S \to aaaaAbbbbb$ X

Test $m=3$ $n: 3,4,5,6 \ldots$ ✓
aaabbb : $S \to aaAbbb \to aaabbb$
aaaabbb: $aaAbbb \to aaaCbb$
$\to aaaabbb$ ✓

$\therefore$ $n \neq m-1$

$\Rightarrow$

$S \to Ab$
$A \to aAb / B / C$
$B \to bB / b$
$C \to aC / a$

CFG = (   )

Test

HW

7. ⓒ

$L = \{a^n b^m : n \neq 2m\}$

aaabb

$\boxed{n = 2m}$

$S \to aaSb / \lambda$

$\boxed{n: \text{even}}$

$S \to aaSb / \lambda$

add a's / b's

$S \to aaSb / A / B$

$A \to aA / a$

$B \to bB / b$

$\boxed{n: \text{odd}}$

$S \to aaS / aB$

$B \to bB / \lambda$

$S \to S_1 / S_2$

$S_1 \to aaS_1b / A / B$

$A \to aA / a$

$B \to bB / b$

$S_2 \to aaS_2 / aC$

$C \to bC / \lambda$

$S \to E / O$

$E \to aaEb / \lambda$

and with more a's or more b's.

$O \to aaO / aC$

$C \to bC / \lambda$

$\Rightarrow$

$E \to aaEb / A / B$

$A \to aA / a$

$B \to bB / b$

$\{\lambda \notin L(G)\}$

$\therefore$ CFG = (. . . .)

Test: . . . -

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[redacted]

CH # 5.1
(EXERCISES)

(7) HW
d. $L = \{a^n b^m : 2n \le m \le 3n\}$

$\Rightarrow \quad m = 2n \,/\, m = 3n$

$\therefore \quad S \rightarrow aSbb \,/\, aSbbb \,/\, \lambda$

(e) HW $L = \{w \in \{a,b\}^* : n_a(w) \ne n_b(w)\}$

$\left(n_a(w) = n_b(w) \quad S \rightarrow \cancel{SS}\, aSb \,/\, bSa \,/\, \lambda \right)$

add a's or add b's $\Rightarrow$ $S \rightarrow SS \,/\, aSb \,/\, bSa \,/\, aS \,/\, bS \,/\, a \,/\, b$

(f) HW $L = \{w \in \{a,b\}^* : n_a(v) \ge n_b(v) : v \text{ is prefix of } w\}$

$\downarrow$

$S \rightarrow SS \,/\, aSb \,/\, \lambda$

(g) HW $L = \{w \in \{ab\}^*, n_a(w) = 2n_b(w) + 1\}$

$n_a(w) = 2n_b(w)$

$\downarrow$

$S \rightarrow SS \,/\, aaSb \,/\, bSaa \,/\, aSba \,/\, aSab \,/\, abSa \,/\, baSa \,/\, \lambda$

$n_a(w) = n_b(w) + 1$

$S \rightarrow SS \,/\, aaSb \,/\, aSba \,/\, aSab \,/\, bSaa \,/\, baSa \,/\, abSa \,/\, a$

Test:
aaab ; $S \rightarrow aaSb \rightarrow aaab$ ✓
aab : $S \rightarrow aSab \rightarrow$ ✗

$n \geq 0, m \geq 0, k \geq 0.$

8 HW

(a) $L = \{a^n b^m c^k : n = m \text{ or } m \leq k\}$

$n = m$, (k)

$S_1 \to AB$

$A \to aAb / \lambda$

$B \to cB / \lambda$

$m \leq k$, (n)

$S \to AB$

$A \to aA / \lambda$

$B \to bBC$

$C \to cC / \lambda$

$S_2 \to DE$

$D \to aD / \lambda$

$E \to bEF$

$F \to cF / \lambda$

$\therefore$ $\boxed{S \to S_1 / S_2}$

$n = m$ (k)

$S_1 \to AB$

$A \to aAb / \lambda$

$B \to cB / \lambda$

$m \leq k$ (n)

$S_2 \to CD$

$C \to aC / \lambda$

$D \to bDc / E$

$E \to cE / \lambda$

$m \leq k$

$m = k$

$X \to bXc$

add c's

$X \to bXc / C$

$C \to cC / \lambda$

HW

(b) $L = \{a^n b^m c^k : n = m \text{ or } m \neq k\}$

$S \to S_1 / S_2$

$S_1 \to AB$

$A \to aAb / \lambda$

$B \to cB / \lambda$

$m = k$

$X \to bXc / \lambda$

add b's / c's

$X \to bXc / Y / Z$

$Y \to bY / b$

$Z \to cZ / c$

$m \neq k$

$S_2 \to CD$

$C \to aC / \lambda$

$D \to bDc / E / F$

$E \to bE / b$

$F \to cF / c$

HW

(c) $L = \{a^n b^m c^k : k = n + m\}$

$\underbrace{aa\ldots a}_{n}\underbrace{bb\ldots b}_{m}\underbrace{cc\ldots c}_{n+m}$

for every 'a' add a 'c'

for every 'b' add a 'c'

$S \to aSc / B$

$B \to bSc / \lambda$

$G: (\{S, B\}, \{a, b, c\}, S, P)$

(8) (d) $L = \{a^n b^m c^k : n + 2m = k\}$

HW

$\underbrace{aaa \ldots aa}_{n}\underbrace{bb \ldots bb}_{m}\underbrace{cc \ldots c}_{n+2m}$

$\boxed{\text{Every } a \text{ add one } C \quad \text{Every } b \text{ add } 2\ C\text{'s}}$

$S \to aSc \,/\, B$

$B \to bBcc \,/\, \lambda$

$n:0$, $m:0$, $k:0$ — λ ✓

ac

$abccc$ ✓

(e) $L = \{a^n b^m c^k : k = |n-m|\}$

HW

$\underbrace{a \ldots a}_{n}\underbrace{b \ldots b}_{m}\underbrace{c \ldots c}_{\text{excess } a\text{'s or } b\text{'s in the string so far.}}$

$k = n - m$ / $k = m - n$

$\boxed{n = m + k}$ $\quad$ $\boxed{m = n + k}$

$S_1 \to aS_1c \,/$ $\qquad$ $S_2 \to aS_2b \,/\, B$

$\to a\ b \,/\, \lambda$ $\qquad$ $B \to bBc \,/\, \lambda$

$S \to S_1 / S_2$

HW

(f) $L = \{w \in \Sigma^* : n_a(w) + n_b(w) \neq n_c(w)\}$

$n + m < k$ $\qquad$ $n + m > k$

$\Rightarrow$ add any no. of c's $\qquad$ add any no. of a's or b's or both

$S \to S_1 / S_2$

⑧ (g) $L = \{a^n b^m c^k , k \neq n+m\}$

HW

$k = n+m$

$S \to aSc / B$

$B \to bBc / \lambda$

$(S \to S_1 / S_2)$

$n+m < k$

add any no. of c's

$n = m = k$

$S \to aSc / B$

$B \to bBc / \lambda$

$$\boxed{\begin{array}{l} S \to aSc / cS / c / B \\ B \to bSc / \lambda \end{array}}$$

Test abcc : $aSc \to abScc$ ✗

abccc ✓

acc ✓

$n+m > k$

$\cancel{\{a, b, aa, ab, ba, bb, abc \ldots\}}$

$n = m = k$

$S \to aSc / B$

$B \to bBc / \lambda$

add atleast one a or more &

add atleast one b or more

$$\Rightarrow \boxed{\begin{array}{l} S_2 \to aS_2c / aS_2 / bS_2 / a / b / B \\ B \to bBc / \lambda \end{array}}$$

Test a : $S \to a$ ✓

aabbcc : $aSc \to aaScc \to aabccc$ ✗

aabcc : $aSc \to aaScc \to aabcc$ ✓

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(8) (h) HW $L = \{a^n b^n c^k : k \geq 3\}$

$k \geq 3$

$a^n b^n c^k \;: n,k \geq 0$

$S \to AB$

$A \to aAb / \lambda$

$B \to cB / \lambda$

$B \to cB / ccc$

↓

minimum 3 c's or more

$\therefore$ $S \to AB$

$A \to aAb / \lambda$

$B \to cB / ccc$

Test

$ccc : S \to AB \to B \to ccc$ ✓

$abccc : S \to abB \to abccc$ ✓

9. ST $L = \{w \in \{a,b,c\}^* : |w| = 3n_a(w)\}$ is a CFG.

a^n

b^m $\quad n+m+k = 3n.$

c^k $\quad \boxed{m+k = 2n}$

$\exists$ for every b an a

$\exists$ for every c an a

(no order $\because \Sigma^*$)

$\therefore$ $S \to SS / aSc / B$

$B \to bBc / cBb / \lambda$

$$\boxed{\begin{array}{l} S \to aSX / bSY / cSZ \\ X \to bc/cb/bb/cc \\ Y \to ac/ca/ab/ba \\ Z \to ab/ba/ac/ca \end{array}}$$

Test

subcase

$\boxed{m+k = 2n}$

$$\underbrace{aaa}_{n}\ \underbrace{\underbrace{bb..b}_{m}\ \underbrace{ccc}_{k}}_{2n}$$

$S \to ABC$

$A \to$

$m+k = 2n$

$$2\left[n_a(w)\right] = (n_b(w) + n_c(w))$$

Every a has 2 more symbols → Either [b/c]

$\Rightarrow$ $S \to aSX \mid bSY \mid cSZ \mid SS \mid \lambda$

$X \to bb \mid cc \mid bc \mid cb$ ⟶ already 1 a

$Y \to ab \mid ba \mid ac \mid ca$

$Z \to ac \mid ca \mid ab \mid ba$

Test: $n_a(w) = 1$, $3n_a(w) = 3$

$w = abc$: $S \to aSX \to abc$ ✓

$w = bac$: $S \to bSY \to bac$ ✓

$n_a(w) = 2$, $3n_a(w) = 6$

w: aabbbb

$S \to SS \to aSX \to aaSXX \to aabbbb$ ✓

5.1

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[redacted]

11. HW

CFG? $L = \{a^n w w^R b^n : w \in \Sigma^* \; n \geq 1\}$ $\Sigma = \{a, b\}$

$\left(\begin{array}{l} ww^R \text{ on } \Sigma = \{a,b\} \quad : n \geq 1 \\ S \to aSa \,/\, bSb \,/\, a \,/\, b \end{array} \right)$

a ✓ aa ✓
b ✓ aba ✓
abba ✓

$S \to aSb \,/\, W$

$W \to aWa \,/\, bWb \,/\, a \,/\, b$

Test

aabab : $S \to aSb \to aaWab \to aabab$ ✓

abab : $S \to aSb \to$ ✗

13. $L = \{a^n b^n : n \geq 0\}$

a) ST L^2 is CFG

b) ST L^k is CFG $\forall \; k \geq 1$

c) ST $\bar{L}$ & L^* are CFG.

L^2: $a^n b^n a^m b^m$

$S \to AA$

$A \to aAb \,/\, \lambda$

L^k : $\underset{1}{\underline{a^n b^n}} \; \underset{2}{\underline{a^m b^m}} \; \cdots \cdots \underset{k}{\underline{a^i b^i}}$

$S \to A_1 A_2 \cdots A_{k+1}$

$A \to aAb \,/\, \lambda$

$\bar{L}$: $\Sigma^* - a^n b^n \searrow$ CFG
$\downarrow$
$S \to SS \,/\, aSb \,/\, bSa \,/\, \lambda$
$= CFG$

$= CFG$

L^* : $\lambda \in L$, L^0 CFG
$L^k \in CFG$

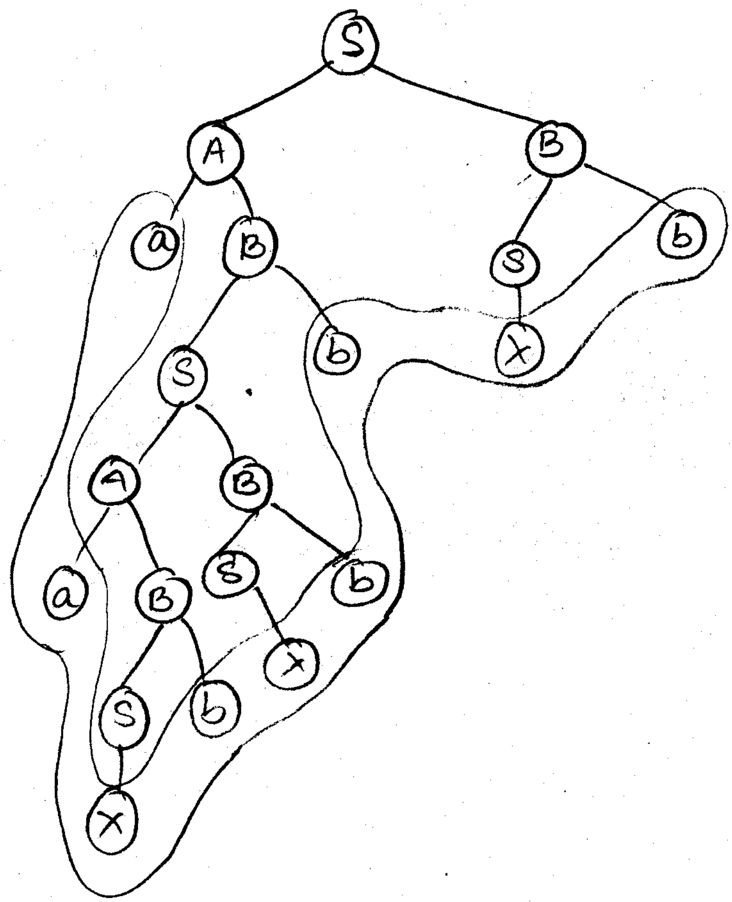
$\therefore L^*$ is CFG

(19) Show derivation Tree for aabbbb

$S \to AB / \lambda$

$A \to aB$

$B \to Sb$



CH # 5.2
(PARSING & AMBIGUITY)

* **Parsing:** finding a sequence of productions by which $w \in L(G)$ is derived.

Exhaustive seach has flaws.

① Tedious
② it is possible that it never terminates for a $w \notin L(G)$

* **SIMPLE GRAMMAR:**

A context free Grammar $G = (V, T, S, P)$ is said to be a simple Grammar or s-grammar if all productions are of the form

→ $\boxed{A \to ax.}$

– $A \in V$, $a \in T$, $x \in V^*$

→ Any pair $\boxed{(A, a)}$ occurs at most once in P.

* A CFG is said to be **ambiguous** if there exists some $w \in L(G)$ that has atleast two distinct derivation trees

*1) $S \to aS / bSS / c$ ✓ S-Grammar

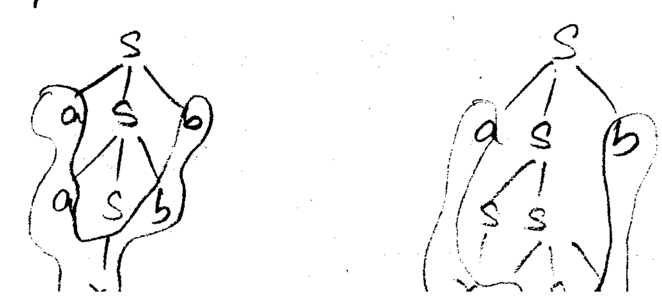
∵ $A \to ax$, (A, a) never repeats

$S \to aS / bSS / aSS / c$ ✗ not S-Grammar

∵ (A, a) repeat though $A \to ax$

* $S \to aSb / SS / \lambda$

w: aabb



∴ ambiguous.

→ One way to resolve ambiguity is

① Associate precedence rules ⇒ change semantics

→ Another way is to rewrite the Grammar.

→ If every Grammar that generates L is ambiguous, then L is called $\boxed{\text{Inherently ambiguous.}}$

→

EXERCISES

① HW find an S-Grammar for $L(aaa^*b+b)$

aaa^*b.

$A \to aX$

(A, a) ×

$S \to aaAb / b$

$A \to aA / \lambda$

$$\boxed{\begin{aligned} &S \to aaAb / b \\ &A \to aA / \lambda \end{aligned}}$$

~~$S \to aA / b$~~

~~$A \to a$~~

$B \to aB$

$(aaa^*b+b) = (aab+b) + aaa^*b$

$aab \to$ 0 of a^* ; $aaa^*b \to$ min one a

aab

$S \to aX$

$X \to aY$

$Y \to b$

(aaa^*b+b)

$S \to aX / b$

$X \to aY$

$Y \to aY / b$

Test

aab: $S \to aX \to aaY \to aab$ ✓

aaab $S \to aX \to aaY \to aaaY \to aaab$ ✓

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CH# 5.2

2. find an s-Grammar for $L = \{a^n b^n : n \geq 1\}$

HW

$\{a^n b^n \; n \geq 1\}$ $\lambda \notin L(G)$

$S \to aSb / ab$

$B \to b$

$S \to aSB / aB$ (S, a) ×

~~$S \to aB / b$~~

~~$B \to S /$~~

$S \to aA$

$A \to b / aAB$

$B \to b$

③ find an s-Grammar for $L = \{a^n b^{n+1} : n \geq 2\}$

$a^n b^{n+1} : n \geq 2$

$S \to aSb / aabbb$

$a^n b^{n+1} : n \geq 0$

$S \to aSb / b$

$n \geq 2$

Substitute $n = 2$.

aabbb :

$$\boxed{\begin{array}{l} X \to aY \\ Y \to aZ \\ Z \to bW \\ W \to bV \\ V \to b \end{array}}$$

$\underbrace{----}_{a*n}$. aabbb $\underbrace{----}_{b*n}$

aabbb

$$\boxed{\begin{array}{l} S \to aA \\ A \to aB \\ B \to bX \\ X \to bY \\ Y \to b \end{array}}$$

$/ aBY$

$a^n b^n$

Test

aabbb: $S \to aA \to aaB \to aabX \to aabbY \to aabbb$ ✓

aaabbbb: $S \to aA \to aaB \to aaaBY \to aaabXY \to aaabbYY \to aaabbbb$ ✓

⑥ HW

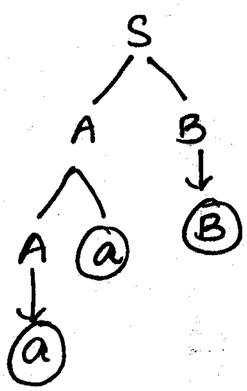
Show that the following Grammar is ambiguous:

$S \to AB / aaB$

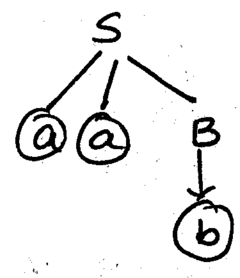
$A \to a / Aa$

$B \to b$

$w = aab$



$w = aab$



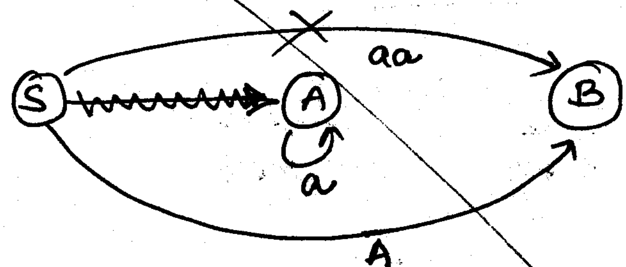
$\exists$ $w = aab$ ST $\exists$ two distinct derivation Trees as above.

$\therefore$ The Grammar is AMBIGUOUS.

⑦ HW

Construct unambiguous grammar for above Grammar.

$S \to aaB$ is repetitive.



$\therefore$ $S \to AB$

$A \to a / Aa$

$B \to b$

✓ ab

✓ aab

✓ aaab

a^*b

$S \to aA$

$A \to b / aX$

$X \to aX / b$

$\Rightarrow$ $a^*b : \{a^n b : n \geq 1\}$

$\therefore$

$S \to aA$

$A \to aA / b$

$S \to aS / b$

CH# 5.2

Prathima Bhima

HW (8) Give Derivation Tree for (((a+b) * c)) + a+b using

$E \to T$

$T \to F$

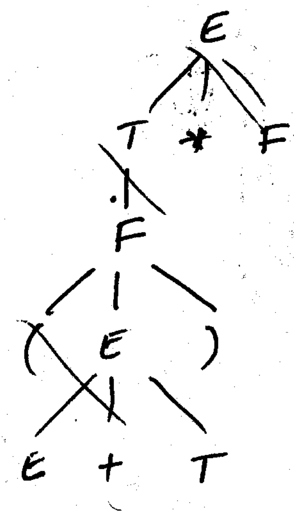
$F \to I$

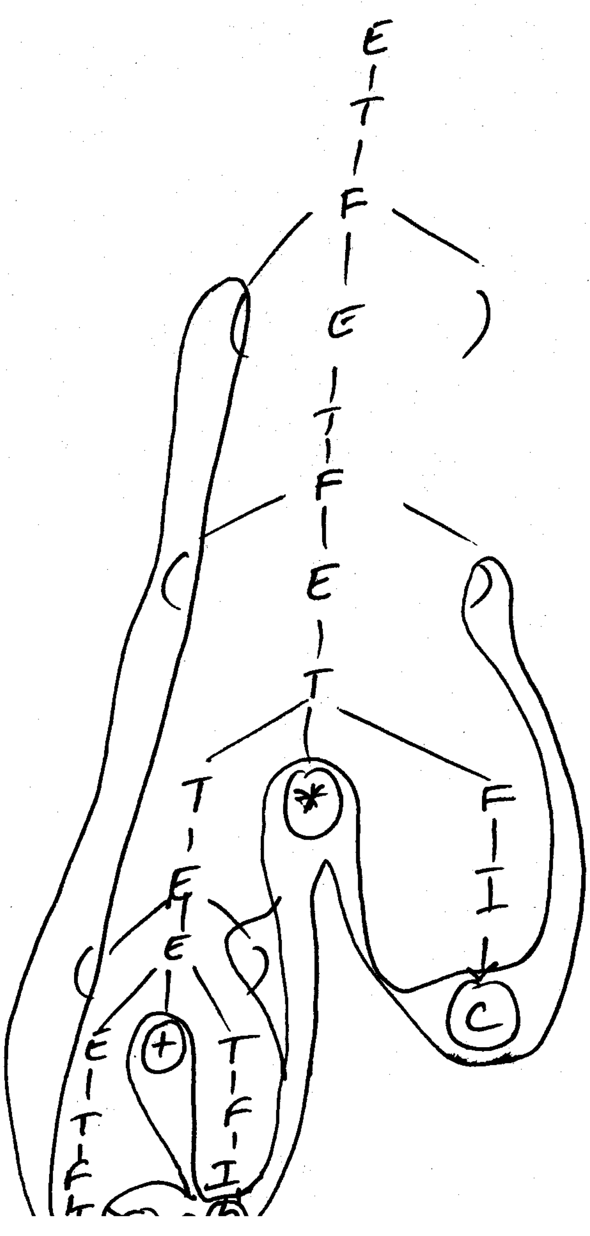
$E \to E+T$

$T \to T*F$

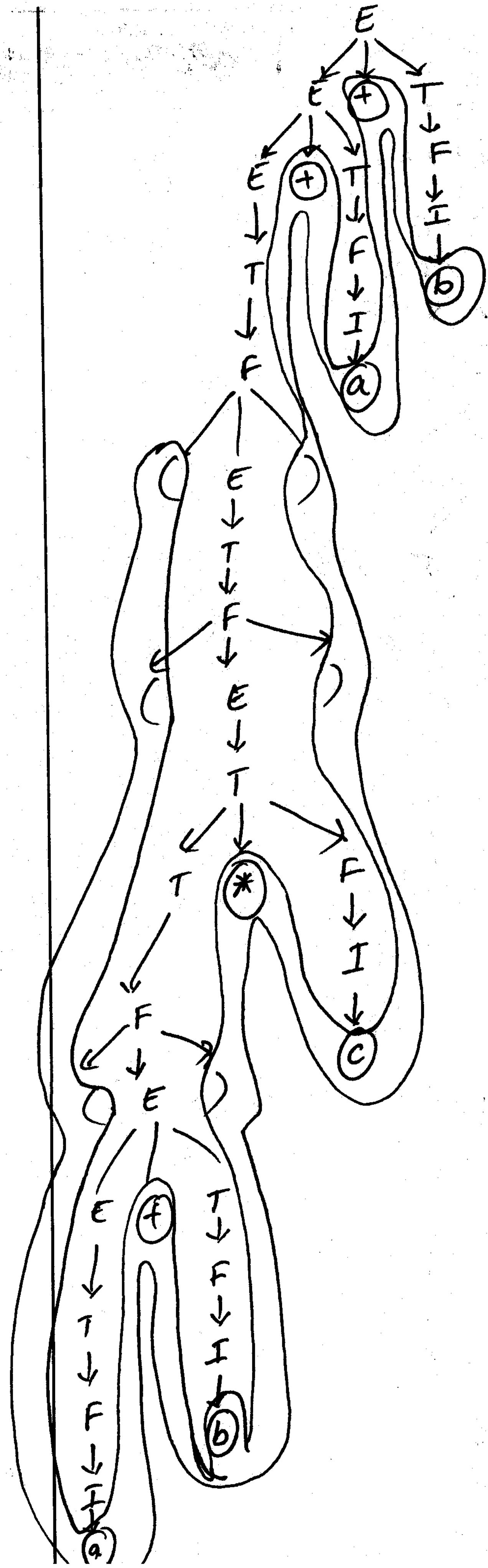
$F \to (E)$

$I \to a/b/c$





(((a+b) * c))



$(((a+b)*c))+a+b$

=

CH# 5.2

Prathima Bhima

(10) Give unambiguous grammar equivalent to . set of all regular expressions on $\Sigma = \{a,b\}$

$\{\lambda, a, b, ab, ba, abb \ldots\ldots$

$\downarrow \downarrow \downarrow \downarrow$

. SS

$(S \to aSb / bSa / SS / a / b / \lambda$
ambiguous, strings $\in \{a,b\}^*)$

RE:

✓ $S \to SS / aSb / bSa / a / b$

?

$(a+b)^*$

$S \to aS / bS / \lambda$

$S \to aS / bS / a / b$

$S \to aSX / bSX$
$X \to a / b$

ab ✓
abab ✓

(12) S.T the language $L = \{ww^R : w \in \{a,b\}^*\}$ is not $\underbrace{\text{inherently ambiguous}}_{\text{all Grammars are ambiguous}}$.

ww^R:

$S \to aSa / bSb / a / b / \lambda$

Test ab ✗ abbaabba ✓ aa ✓

aba ✓ $\lambda \notin L(G)$

ww^R

$S \to aSa / bSb / a / b$.

$S \to aSX / bSY$
$X \to a$
$Y \to b$

(1) Eliminate λ

$S \to aSa / bSb / a / b / \lambda$

$S \to aSa / bSb / a / b / aa / bb$

(2) eliminate UNIT-Ps

$S \to aSa / bSb / aa / bb / aaa / bab / aba / bbb$

$S \to aSa / bSb / aaa / aba / bbb / bab / aa / bb$?

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[redacted]

CH# 6.1

Simplification of CFG & Normal forms

Eg: 6.1 $G = (\{A, B\}, \{a, b, c\}, A, P)$

$A \to a \mid aaA \mid abBc$

$B \to abbA \mid b$

$A \to a \mid aaA \mid ababbAc \mid abbc$

Eg: 6.2

$S \to A$
$A \to aA \mid \lambda$
$\cancel{B \to bA}$

$S \to A$
$A \to aA \mid \lambda$

$S \to A$
$A \to aA \mid a$

$S \to A$
$A \to aA \mid a$

6.3

$S \to aS \mid A \mid C$
$A \to a$
$B \to aa$
$C \to aCb$

$S \to aS \mid a \mid C$
$\cancel{C \to aCb}$

$S \to aS \mid a$

6.4.

$S \to aS_1b$
$S_1 \to aS_1b \mid \lambda$

$S \to aS_1b \mid ab$
$S_1 \to aS_1b \mid ab$

6.5 find CFG without λ-productions

$S \to ABaC$
$\cancel{A \to BC}$
$B \to b \mid \lambda$
$C \to D \mid \lambda$
$D \to d$

① $\lambda \notin L(G)$
② $V_N : \{A, B, C\}$

$S \to ABaC \mid BaC \mid AaC \mid ABa \mid aC \mid Ba \mid Aa \mid a$

$A \to BC \mid B \mid C$

$B \to b$

$C \to D$

$D \to d$

Rules to Eliminate λ-Productions

① Check that $\lambda \notin L(G)$

② $V_N = \{ - - - \}$

③ Eliminate all λ-productions

④ make all combinations of nullable variables.

Rules to eliminate UNIT-Productions

STEP #1: find dependancy Graph for unit-Productions.

nodes $\longrightarrow$ variable

connections $\exists$ where Unit Production $\exists$.

STEP #2:

$S \overset{*}{\Rightarrow} A$ $\quad$ $A \overset{*}{\Rightarrow} B$

$S \overset{*}{\Rightarrow} B$

$B \overset{*}{\Rightarrow} A$

STEP #3: Grammar without UNIT-Productions + make Extensions

Eg: 6.6

$S \to Aa \mid B$

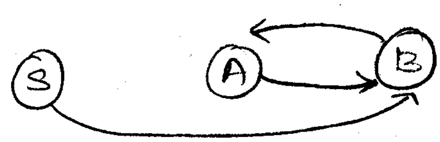
$B \to A \mid bb$

$A \to a \mid bc \mid B$

$S \to B$

$B \to A$

$A \to B$



$S \overset{*}{\Rightarrow} A$

$A \overset{*}{\Rightarrow} B$

$B \overset{*}{\Rightarrow} A$

$S \overset{*}{\Rightarrow} B$

$S \to Aa \;/ a / bc / bb$

$B \to bb \;/ a / bc$

$A \to a / bc \;/ bb$

CH#6.1

[EXERCISES]

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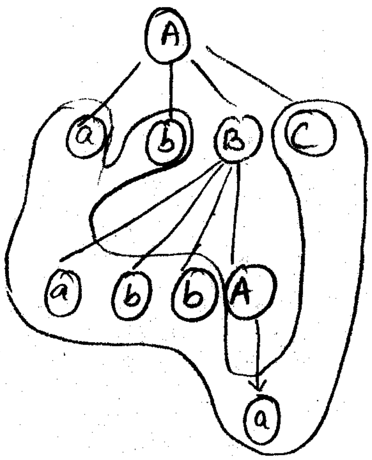
(2) HW

$A \to a \mid aaA \mid abBc$

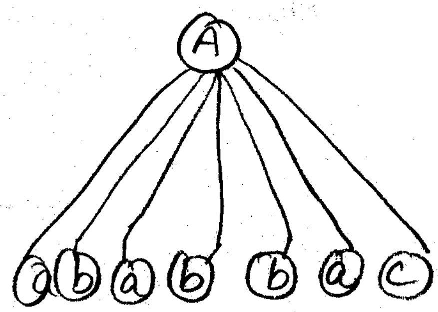
$B \to abbA \mid b$

$\Rightarrow$ $A \to a \mid aaA \mid$ $ababbac \mid ababbaaAc \mid abbc$.

Derivation tree for $w = ababbac$?



$w = ababbac$



$w = ababbac$

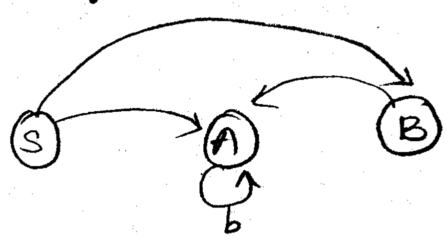
(5) HW

Eliminate all useless Productions for the Grammar.

$S \to aS \mid AB$

$A \to bA$

$B \to AA$



Substitution:

$S \to aS \mid AAA$

$A \to bA$

$\cancel{B \to AA}$

$S \to aS \mid AAA$

$\cancel{A \to bA}$ never ends

$\therefore S \to aS$ never Ends

$\therefore \lambda.$

$L = \{w = a^{\infty} b^{\infty} : w \in \{a,b\}^*\}$

⑥ HW

Eliminate Useless Productions from

$S \to a / aA / B / C$

$A \to aB / \lambda$

$B \to Aa$

$C \to cCD$

$\cancel{D \to ddd}$

Substitution:

$S \to a / aA / B / \cancel{cCddd}$

$A \to aB / \lambda$

$\cancel{B \to Aa}$

$\cancel{C \to cCddd}$

$S \to a / aA / Aa$

$A \to aAa / \lambda$

⑦

Eliminate λ-Productions from

$S \to AaB / aaB$

$A \to \lambda$

$B \to bbA / \lambda$

① $\lambda \notin L(G)$

② $V_N = \{A, B\}$

$S \to AaB / aaB$

$B \to bbA / \lambda$

$A \to \lambda$

$S \to aB / aaB / a / aa$

$B \to bb$

$\therefore S \to aB / aaB / a / aa$

$B \to bb$

Simplified:-

$S \to abb / aabb / a / aa$

(8) Remove all UNIT-Productions, useless Productions & λ-Productions

$S \to aA / aBB$

$A \to aaA / \lambda$

$B \to bB / bbC$

$C \to B$

λ-production elimination

① $\lambda \notin L(G)$

② $V_N = \{A\}$

$S \to aA / aBB / a$

$A \to aaA / aa$

$B \to bB / bbC$

$C \to B$.

Unit Production Removal

$S \quad A \quad B \leftarrow C$

$C \stackrel{*}{\Rightarrow} B$

$\boxed{\begin{array}{l} S \to aA / aBB / a \\ A \to aaA / aa \\ \cancel{B \to bB / bbC /} \end{array}}$ | B

$S \to aA / a\cancel{BB} / a$

$A \to aaA / aa$

$S \to aA / a$

$A \to aaA / aa$

What does the language generate?

$\{a^n\} \cup \{a^{2n+1}\}$ $\qquad (aa)^* a$

(9) Eliminate UNIT-Productions from (6)

$S \to a / aA / B / C$

$A \to aB / \lambda$

$B \to Aa$

$C \to cCD$

$D \to ddd$

$S \quad A \quad B \quad C \quad D$

$S \stackrel{*}{\Rightarrow} B$

$S \stackrel{*}{\Rightarrow} C$

$S \to a / aA / Aa / cCD$

$A \to aB / \lambda$

$B \to Aa$

$C \to cCD$

$D \to ddd$

12

Remove λ-Productions

$\cancel{S \to aS / AB\!C}$
$\cancel{A \to a}$
$\cancel{B \to aa}$
$\cancel{C \to aCb}$

$S \to aSb / SS / \lambda$

① $\lambda \in L(G)$

$sa + \{\lambda\}$ sa $\longrightarrow$ $S \to aSb / SS / ab$

CHAPTER 6-2

CHOMSKY NORMAL FORM:

$A \to BC$

$A \to a$

$\lambda \notin L(G)$

$\to$ restrictions on length of Production.

$\{A, B, C\} \in V$

$a \in T$

$S \to AS / a$	$S \to AS / AAS$
$A \to SA / b$	$A \to SA / aa$
$\in CNF$	$\notin CNF$

Eg 6.8 Convert the Grammar to CNF

$S \to ABa$

$A \to aab$

$B \to AC$

$X \to a$ $Z \to c$

$Y \to b$

$S \to ABX$	$X \to a$
$A \to XXY$	$Y \to b$
$B \to AZ$	$Z \to c$

$S \to AC$	$D \to XY$	$Y \to b$
$C \to BX$	$B \to AZ$	$Z \to c$
$A \to XD$	$X \to a$	

CH # 6.2

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[redacted]

GRIEBACH NORMAL FORM:

→ restriction NOT on length of Production
→ but on POSITIONS in which terminals & variables can appea

$$\boxed{A \to ax}$$

$a \in T \quad x \in V^*$

→ looks similar to s-Grammar
→ But no-restriction on (A,a) at Productions.

Eg:6.9

$S \to AB$
$A \to aA \mid bB \mid b$
$B \to b$

not GNF
$A \to aX$

$S \to aAB \mid bBB \mid bB$
$A \to aA \mid bB \mid b$
$B \to b$

$\in$ GNF.

Eg:6.10 Convert the Grammar $S \to abSb \mid aa$ into GNF.

$X \to a$
$Y \to b$

$S \to aYSY \mid aX$
$X \to a$
$Y \to b$

for every CFG G, $\lambda \notin L(G)$

$\exists$ Equivalent $\hat{G}$, in GNF.

- EXERCISES -

CH#6.2

(2) Convert to CNF

CNF

$\boxed{\begin{array}{l} A \to BC \\ A \to a \end{array}}$

$S \to aSb / ab$

$S \to XSY / XY$
$X \to a$
$Y \to b$

$S \to XA / XY$
$A \to SY$
$X \to a$
$Y \to b$

$\in$ CNF

HW (3) Convert to CNF:

$\boxed{\begin{array}{l} \text{CNF:} \\ A \to BC \\ A \to a \end{array}}$

$S \to aSaA / A$
$A \to abA / b$

Substitution:

$S \to aSaA / abA / b$
~~$A \to abA$~~ $/ b$

$S \to aSXA / aYA / b$
$A \to aYA / b$
$X \to a$
$Y \to b$

$S \to XB / XC / b$
$A \to XC / b$
$B \to SD$
$C \to YA$
$D \to XA$
~~$X \to a$~~
$Y \to b$

HW (4) Convert to CNF

$\lambda \notin L(G)$

$S \to abAB$
$A \to bAB / \lambda$
$B \to BAa / A / \lambda$

λ Elimination

$V_N : \{A, B\}$

$S \to abAB / abA / abB$
$A \to bAB / bA / bB$
$B \to BAa / A / Ba / Aa$

$\boxed{B \to BAa / Ba / Aa / bAB / bA / bB}$

$S \to \overline{XY}AB / \overline{XY}A / \overline{XY}B$
$A \to \underline{YA}B / \underline{YA} / \underline{YB}$
$B \to BAX / BX / AX / \underline{YA}B / \underline{YA} / \underline{YB}$
$X \to a$
$Y \to b$

$S \to XCB / XC / XD$
$A \to CB / YA / YB$
$B \to EX / BX / AX / CB / YA / YB$
$C \to YA \quad E \to BA \quad Y \to b$

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$S \to FB / XC / XD$

$A \to CB / YA / YB$

$B \to EX / BX / AX / CB / YA / YB$

$C \to YA$

$D \to YB$

$E \to BA$

$F \to XC$

$X \to a$

$Y \to b$

CNF

5 Convert to CNF:

$\lambda \notin L(G)$

$S \to AB / aB$

$A \to aab / \lambda$

$B \to bbA$

λ-Elimination

$V_N : \{A\}$

$S \to AB / aB / B$

$A \to aab$

$B \to bbA / bb$

Substitution

$S \to AB / aB / bbA / bb$

$A \to aab$

$B \to bbA / bb$

$S \to AB / XB / \underline{YY}B / YY$

$A \to \underline{XX}Y$

$B \to \underline{YY}A / YY$

$X \to a \quad Y \to b$

$S \to AB / XB / CB / YY$

$A \to DY$

$B \to CA / YY$

$C \to YY$

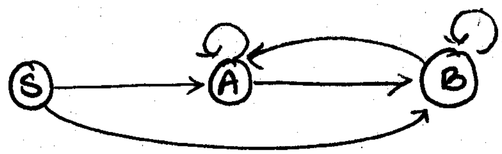
$D \to XX$

HW (9) Draw dependency Graph for

$S \to abAB$

$A \to bAB \mid \lambda$

$B \to BAa \mid A \mid \lambda$



(10) Convert to GNF

$S \to aSb \mid bSa \mid a \mid b$

$\boxed{S \to aX}$ GNF

$S \to aSY \mid bSX \mid a \mid b$

$X \to a$

$Y \to b$

(11) Convert to GNF

$S \to aSb \mid ab$

$S \to aSY \mid aY$

$Y \to b$

HW (12) Convert to GNF

$S \to ab \mid aS \mid aaS$

$S \to aY \mid aS \mid aXS$

$X \to a$

$Y \to b$

(13) Convert to GNF

<u>Substitution</u>

$S \to ABb \mid a$

$A \to aaA \mid B$

$B \to bAb$

$S \to aaAbb \mid BBb \mid a$

$A \to aaA \mid bAb$

$B \to bAb$

$S \to aaABb \mid bAbBb \mid a$

$A \to aaA \mid bAb$

$B \to bAb$

$S \to aXABY \mid bAYBY \mid a$

$A \to aXA \mid bAY$

$B \to bAY$

$X \to a$

*) Palindrome: CNF = ?

$\lambda \notin L(G)$

$\downarrow$

$ww^R \rightarrow$ $\cancel{aba}$ $\cancel{bab}$: add a/b

$\downarrow$

$S \rightarrow aSa \,/\, bSb \,/\, \lambda \,/\, a \,/\, b$

$S \rightarrow aSa \,/\, bSb \,/\, aa \,/\, bb$

$S \rightarrow XSX \,|\, YSY \,/\, XX \,/\, YY \,/\, a \,/\, b$

$X \rightarrow a$

$Y \rightarrow b$

$S \rightarrow XA \,|\, YB \,|\, XX \,|\, YY \,|\, a \,|\, b$

$A \rightarrow SX$

$X \rightarrow a$

$Y \rightarrow b$

$B \rightarrow SY$

* $L = \{a^{4n} : n \geq 1\}$ CNF = ?

$S \rightarrow aaaaS \,/\, aaaa$

$S \rightarrow AAAAS \,/\, AAAA$

$A \rightarrow a$

$S \rightarrow XXS \,/\, XX$

$X \rightarrow AA$

$A \rightarrow a$

$S \rightarrow YS \,/\, XX$

$X \rightarrow AA$

$Y \rightarrow XX$

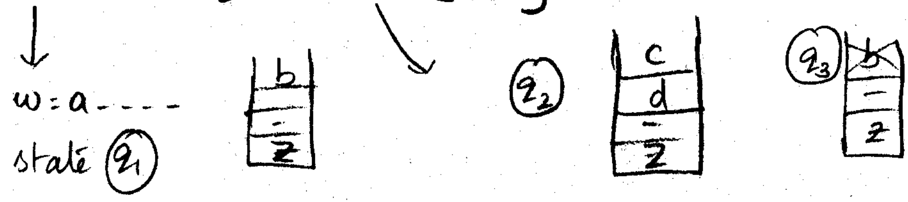
$A \rightarrow a$

$\boxed{\text{Npda:}}$ Nondeterministic Pushdown Automata:

$$M = (Q, \Sigma, \Gamma, \delta, q_0, z, F)$$

Eg: 7.1

$$\delta(q_1, a, b) \rightarrow \{(q_2, cd), (q_3, \lambda)\}$$



Eg: 7.2

$Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{a, b\}$

$\Gamma = \{0, 1\}$

$z = 0$

$F = \{q_3\}$

$\delta(q_0, a, 0) = \{(q_1, 10), (q_3, \lambda)\}$

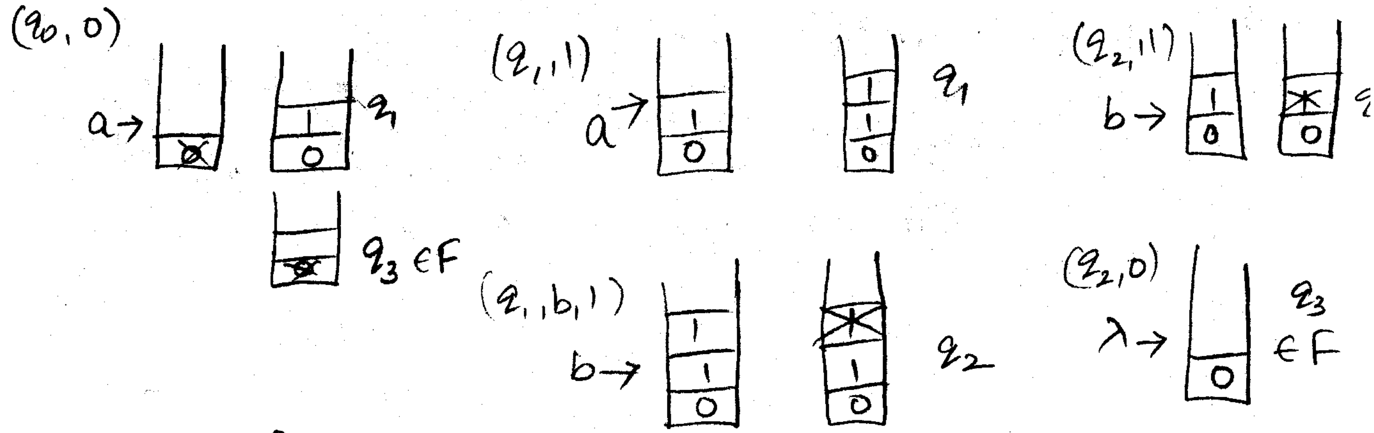
$\delta(q_0, \lambda, 0) = \{(q_3, \lambda)\}$,

$\delta(q_1, a, 1) = \{(q_1, 11)\}$,

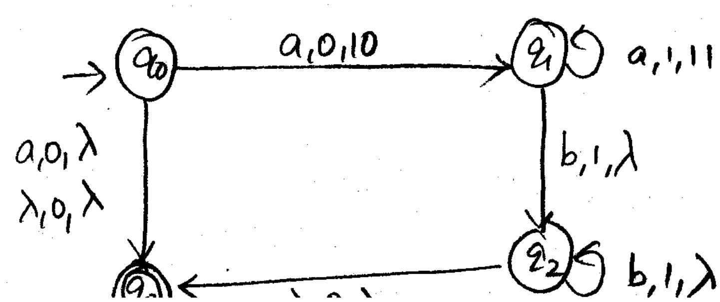
$\delta(q_1, b, 1) = \{(q_2, \lambda)\}$,

$\delta(q_2, b, 1) = \{(q_2, \lambda)\}$

$\delta(q_2, \lambda, 0) = \{(q_3, \lambda)\}$



$$a \cup \{a^n b^n : n \geq 0\}$$



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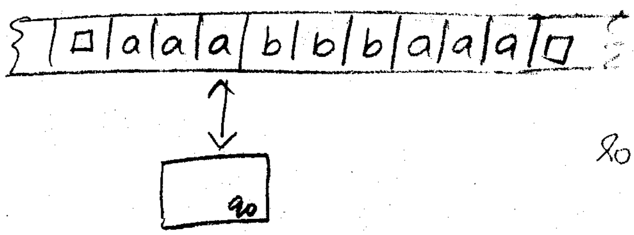
PAGE#:

(Class)

A)

$L = \{a^n b^n a^n : n \geq 0\}$



Sol^n: $a \to x$
$b \to y$
$a \to z$

$\cancel{a}aa\ bb\cancel{b}^{a}a\cancel{a}\cancel{a}$

$\cancel{a}a\ b\cancel{b}\cancel{a}\cancel{a}$ (a)

$\cancel{a}\cancel{b}\cancel{a}$ (a)

$\delta(q_0, a) = (q_1, \square, R)$

$\delta(q_1, a) = (q_1, a, R)$

$\delta(q_1, b) = (q_2, b, R)$

$\delta(q_2, b) = (q_2, b, R)$

$\delta(q_2, a) = (q_3, a, L)$

$\delta(q_3, b) = (q_3, a, R)$

$\delta(q_3, a) = (q_3, a, R)$

$\delta(q_3, \square) = (q_4, \square, L)$

$\delta(q_4, a) = (q_5, \square, L)$

$\delta(q_5, a) = (q_6, \square, L)$

$\delta(q_6, a) = (q_7, a, L)$

$\delta(q_7, a) = (q_7, a, L)$

$\delta(q_7, b) = (q_7, b, L)$

$\delta(q_7, \square) = (q_0, \square, R)$

$\delta(q_6, \square) = (q_f, \square, R)$

$\delta(q_0, \square) = (q_f, \square, R)$

not λ. $M = (\quad)$

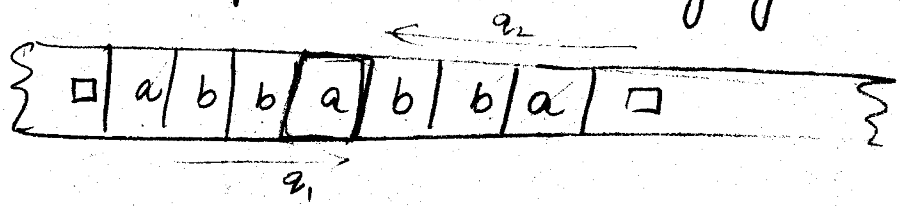
Test $w = aba : \in L$

$$q_0 aba \vdash \square q_1 ba \vdash b q_2 a \vdash q_3 ba \vdash a q_3 a \vdash$$

$$aa q_3 \square \vdash a q_4 a \vdash q_5 a \vdash q_6 \square \vdash q_f \square$$

accepted

*) Design TM that accepts PALINDROME language.



LIRYL

$$\delta(q_0, a) = (q_1, \square, R)$$
$$\delta(q_0, b) = (q_2, \square, R)$$

$$\delta(q_1, a) = (q_1, a, R) \qquad \delta(q_2, b) = (q_2, b, R)$$
$$\delta(q_1, b) = (q_1, b, R) \qquad \delta(q_2, a) = (q_2, a, R)$$
$$\delta(q_1, \square) = (q_3, \square, L) \qquad \delta(q_2, \square) = (q_5, \square, L)$$
$$\delta(q_3, a) = (q_4, \square, L) \qquad \delta(q_5, b) = (q_4, \square, L)$$
$$\delta(q_4, a) = (q_4, a, L)$$
$$\delta(q_4, b) = (q_4, b, L)$$
$$\delta(q_4, \square) = (q_0, \square, R)$$

$$\delta(q_0, \square) = (q_f, \square, R)$$ even string

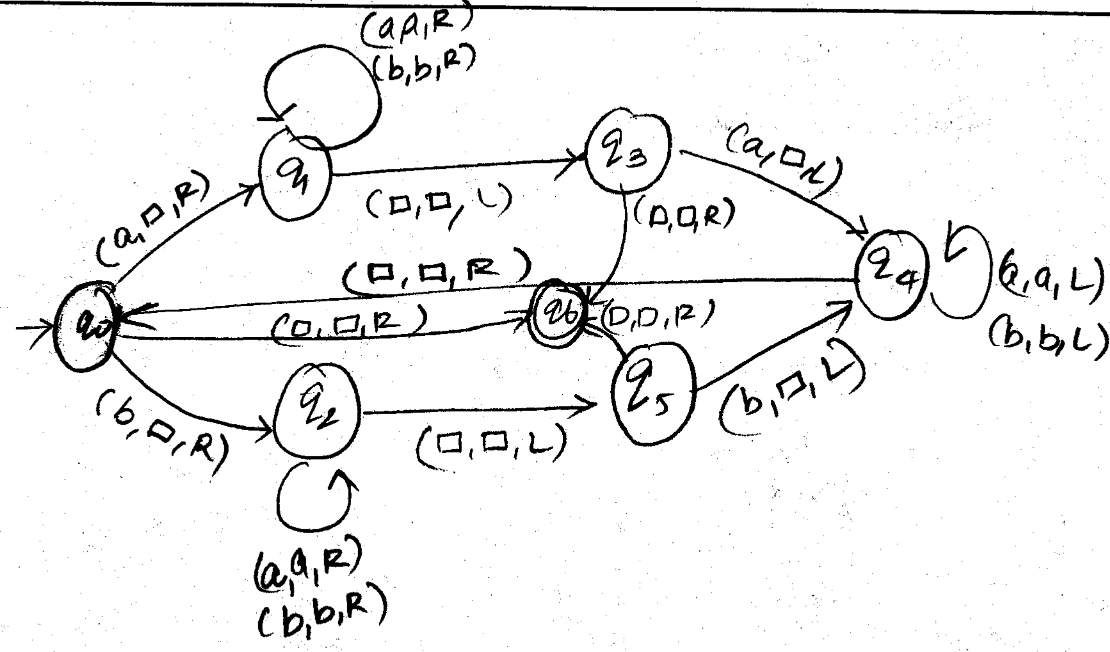
$$\delta(q_5, \square) = (q_f, \square, R)$$
$$\delta(q_3, \square) = (q_f, \square, R)$$

} odd string
middle = a/b

$M : (\ \_\ )$

Test : ababa

instead of expecting another a/b to delete, if no symbol ⇒ accepted.



Turing Machine as Transducer:

rejected strings of acceptor $= \overline{L}$

$$\hat{w} = f(w)$$

$$\boxed{q_0 w \vdash^{*}_{TM} q_f \hat{w}} \quad (q_f \in F)$$

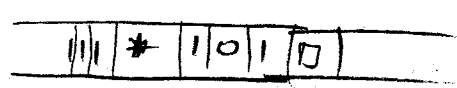
→ Computable function: $\Leftrightarrow$ has TM.

⇒ fn ends @ finite no. of steps

→ whatever the complexity.

≃ Algorithm, whatever the complexity.

*) Addition with TM



111
101

use unary NS:

Addition

1	1	1	0	1	1	1	1	1

- skip till spl. char
- replace it with 1 & move to L
- till □ move to L & del last 1.

$$\delta(q_0, 1) = (q_0, 1, R)$$
$$\delta(q_0, 0) = (q_1, 1, R)$$
$$\delta(q_1, 1) = (q_1, 1, R)$$
$$\delta(q_1, \square) = (q_2, \square, L)$$
$$\delta(q_2, 1) = (q_3, 0, L)$$

$$\delta(q_3, 1) = (q_3, 1, L)$$
$$\delta(q_3, \square) = (q_f, \square, R)$$

* Every TM has to have R/W @ begning.

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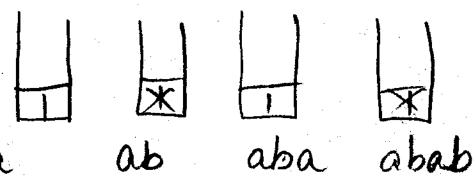
Eg: 7.4 Construct an npda for the language

$$L = \{ w \in \{a, b\}^* : n_a(w) = n_b(w) \}$$

abab 

$$\delta(q_0, \lambda, z) = \{(q_f, z)\}$$

$M = (- - - - -)$

$$\delta(q_0, a, z) = \{(q_0, 1z)\}$$

$$\delta(q_0, b, z) = \{(q_0, 0z)\}$$

$$\delta(q_0, a, 1) = \{(q_0, 11)\}$$

$$\delta(q_0, b, 0) = \{(q_0, 00)\}$$

$$\delta(q_0, a, 0) = \{(q_0, \lambda)\}$$

$$\delta(q_0, b, 1) = \{(q_0, \lambda)\}$$

Test: $w = abab$

$$(q_0, abab, z) \vdash (q_0, bab, 1z) \vdash (q_0, ab, z) \vdash (q_0, b, 1z) \vdash$$

$$(q_0, \lambda, z) \vdash (q_f, \lambda, z)$$

$\downarrow \in F$ accepted

$w = bbaa$

$$(q_0, bbaa, z) \vdash (q_0, baa, 0z) \vdash (q_0, aa, 00z) \vdash (q_0, a, 0z) \vdash$$

$$(q_0, \lambda, z) \vdash (q_f, \lambda, z)$$

$\downarrow \in F$ accepted.

Eg: 7.5 Construct an npda for $L = \{ww^R : w \in \{a,b\}^+\}$

$\delta(q_0, a, z) = \{(q_0, az)\}$

$\delta(q_0, b, z) = \{(q_0, bz)\}$

$\delta(q_0, a, a) = \{(q_0, aa)\}$

$\delta(q_0, b, b) = \{(q_0, bb)\}$

$\delta(q_0, a, b) = \{(q_0, ab)\}$

$\delta(q_0, b, a) = \{(q_0, ba)\}$

$\delta(q_1, a, a) = (q_1, \lambda)$

$\delta(q_1, b, b) = (q_1, \lambda)$

$\delta(q_1, \lambda, z) = (q_f, z)$

$\delta(q_0, a, a) = \{(q_1, a)\}$

$\delta(q_0, b, b) = \{(q_1, b)\}$

$\boxed{\text{Test:}}$ $\boxed{w = abba}$

$(q_0, abba, z) \vdash (q_0, bba, az) \vdash (q_0, ba, baz) \vdash$

$(q_1, ba, baz) \vdash (q_1, a, az) \vdash (q_1, \lambda, z) \vdash (q_f, \lambda, z)$

$q_f \in F$

accepted

$\boxed{w = abab}$

$(q_0, abab, z) \vdash (q_0, bab, az) \vdash (q_0, ab, baz) \vdash (q_0, b, abaz) \vdash (q_0, \lambda, babaz)$

$\notin F$

$\Rightarrow$ rejected

$M = (\{q_0, q_1, q_f\}, \{a, b\}, \{a, b, z\}, \delta, z, q_f)$

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(3) Construct npda's that accept the following Regular Languages

(a) $L_1 = L(aaa^*b)$

$\delta(q_0, a, z) = \{(q_1, az)\}$

$\delta(q_1, a, a) = \{(q_2, aa)\}$

$\delta(q_2, a, a) = \{(q_2, aa)\}$

$\delta(q_2, b, a) = \{(q_f, ba)\}$

$w = aab$

$(q_0, aab, z) \vdash (q_1, ab, az) \vdash (q_2, b, aaz)$

$\vdash (q_f, baaz)$

$\downarrow$

$\in F$

accepted

(b) $L_2 = L(aab^*aba^*)$

$\delta(q_0, a, z) = \{(q_1, az)\}$

$\delta(q_1, a, a) = \{(q_2, aa)\}$

$\delta(q_2, b, a) = \{(q_2, ba)\}$

$\delta(q_2, b, b) = \{(q_2, bb)\}$

$\delta(q_2, a, a) = \{(q_3, aa)\}$

$\delta(q_2, a, b) = \{(q_3, ab)\}$

$\delta(q_3, b, a) = \{(q_f, ba)\}$

$\delta(q_f, a, b) = \{(q_f, ab)\}$

$\delta(q_f, a, a) = \{(q_f, aa)\}$

$\delta(q_f, \lambda, b) = \{(q_f, b)\}$

$\delta(q_f, \lambda, a) = \{(q_f, a)\}$

$M(\ldots)$

$\boxed{w = aaab}$

$(q_0, aaab, z) \vdash (q_1, aab, az) \vdash (q_2, ab, aa$

$\vdash (q_3, b, aaa) \vdash (q_f, \lambda, baaa) \vdash$

$(q_f, \lambda, baaa)$

$\downarrow$

$\in F$ accepted

(c) $L_1 \cup L_2$: $(aaa^*b) \cup (aab^*aba^*)$

$\delta(q_0, a, z) = \{(q_1, az)\}$

$\delta(q_1, a, a) = \{(q_2, aa)\}$

$\delta(q_2, a, a) = \{(q_3, aa)\}$ a^+ aaa^*

$\delta(q_2, b, a) = \{(q_4, ba), (q_f, ba)\}$ $\boxed{aab \text{ final}}$, $\boxed{aaa^*b \text{ final}}$

$\delta(q_3, b, a) = \{(q_4, ba)\}$

$\delta(q_3, b, b) = \{(q_3, bb)\}$

$\delta(q_3, a, b) = \{(q_4, ab)\}$

$\delta(q_4, b, a) = \{(q_5, ba)\}$ $\boxed{aab^*ab \text{ final}}$

$\delta(q_5, a, a) = \{(q_f, aa)\}$ $\boxed{aab^*aba^+ \text{ final}}$

$\delta(q_f, \lambda, z) = \{(q_f, \lambda)\}$

Test: aabab:

store

$\delta(\underline{q_0}, \underline{a}abab, \underline{z}) \vdash \delta(\underline{q_1}, \underline{a}bab, \underline{a}z) \vdash \delta(q_2, \underline{b}ab, \underline{a}az) \vdash \boxed{\cancel{\delta(q_f, ab, baaz)}}$

$\vdash \delta(\underline{q_4}, \underline{a}b, \underline{b}aaaz) \vdash$

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npda $\Leftrightarrow$ CFG

CFG $\rightarrow$ npda

$\boxed{\text{CFG} \rightarrow \text{GNF} \rightarrow \text{npda}}$

$\downarrow$

$\boxed{A \rightarrow aX}$

$\delta(q_0, \lambda, z) = (q, Sz)$

$\boxed{\delta(q_1, a, A) = (q_1, X)}$

$\delta(q_1, \lambda, z) = (q_f, \lambda)$

76. Eg: $S \rightarrow aSbb \,/\, a$

$$\left.\begin{array}{l} S \rightarrow aSYY \,/\, a \\ Y \rightarrow b \end{array}\right\} \text{GNF}$$

$\delta(q_0, \lambda, z) = \{(q_1, Sz)\}$

$\delta(q_1, a, S) = \{(q_1, SYY), (q_1, \lambda)\}$

$\delta(q_1, \lambda, z) = \{(q_f, \lambda)\}$

$\delta(q_1, b, Y) = \{(q_1, \lambda)\}$

$M = (. \; . \; .)$

Test

Q: 7.7. $S \to aA$ npda = ?

$A \to aABC \mid bB \mid a$

$B \to b$

$C \to c$

$\to \delta(q_0, \lambda, z) = \{(q_1, Sz)\}$

$\delta(q_1, a, S) = \{(q_1, A)\}$

$\delta(q_1, a, A) = \{(q_1, ABC), (q_1, \lambda)\}$

$\delta(q_1, b, A) = \{(q_1, B)\}$

$\delta(q_1, b, B) = \{(q_1, \lambda)\}$

$\delta(q_1, c, C) = \{(q_1, \lambda)\}$

$\to \delta(q_1, \lambda, z) = \{(q_f, \lambda)\}$

EXERCISES

1

2

5 $S \to aABB \mid aAA$ npda = ?

$A \to aBB \mid a$

$B \to bBB \mid A.$

$\delta(q_0, \lambda, z) = \{(q_1, Sz)\}$

$\delta(q_1, a, S) = \{(q_1, ABB), (q_1, AA)\}$

$\delta(q_1, a, A) = \{(q_1, BB), (q_1, \lambda)\}$

$\delta(q_1, b, B) = \{(q_1, BB), (q_1, A)\}$

$\delta(q_1, \lambda, z) = \{(q_f, \lambda)\}$

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CH: 7 #2

(a) find npda with 2 states for $L = \{a^n b^{n+1} : n \geq 0\}$

$$S \to aSb \mid b$$

GNF: $S \to aSY \mid b$

$Y \to b$

$\delta(q_0, \lambda, z) = \{(q_1, Sz)\}$

$\delta(q_1, a, S) = \{(q_1, SY)\}$

$\delta(q_1, b, Y) = \{(q_1, \lambda)\}$

$\delta(q_1, \lambda, z) = \{(q_f, \lambda)\}$

$\delta(q_1, b, S) = \{(q_1, \lambda)\}$

$\delta(q_0, \lambda, z) = \{(q_0, Sz_1)\}$

$\delta(q_0, a, S) = \{(q_0, SY)\}$

$\delta(q_0, b, S) = \{(q_0, \lambda)\}$

$\delta(q_0, b, Y) = \{(q_0, \lambda)\}$

$\delta(q_0, \lambda, z_1) = \{(q_f, \lambda)\}$

(b) find npda with 2 states that accepts $L = \{a^n b^{2n} : n \geq 1\}$

$$S \to aSbb \mid \lambda$$

GNF: $S \to aSbb \mid abb$

$S \to aSBB \mid aBB$

$B \to b$

$\delta(q_0, \lambda, z) = \{(q_0, Sz_1)\}$

$\delta(q_0, a, S) = \{(q_0, SBB), (q_0, BB)\}$

$\delta(q_0, b, B) = \{(q_0, \lambda)\}$

$\delta(q_0, \lambda, z_1) = \{(q_f, \lambda)\}$

CH # 7.3

$\boxed{\text{Dpda : DCFL}}$

Dpda

① $\delta(q, a, b)$ contains atmost one element

② if $\delta(q, \lambda, b) \neq \phi$

then $\delta(q, c, b) = \phi \quad \forall\ c \in \Sigma$

Eg: 7.10 $\quad L = \{a^n b^n : n \geq 0\}$ is DCFL.

Dpda = ?

$$\delta(q_0, a, 0) = \{(q_1, 10)\}$$
$$\delta(q_1, a, 1) = \{(q_1, 11)\}$$
$$\delta(q_1, b, 1) = \{(q_2, \lambda)\}$$
$$\delta(q_2, b, 1) = \{(q_2, \lambda)\}$$
$$\delta(q_2, \lambda, 0) = \{(q_0, \lambda)\}$$

$q_0 \in F$.

*) $\quad L = \{a^n b^n : n \geq 0\} \cup \{a\}$ $\quad$ dpda = ?

$$\delta(q_0, a, z) = (q_f, az)$$
$$\delta(q_f, a, a) = (q_1, aa)$$
$$\delta(q_f, b, a) = (q_f, \lambda)$$
$$\delta(q_f, \lambda, z) = (q_f, \lambda)$$
$$\delta(q_1, a, a) = (q_1, aa)$$
$$\delta(q_1, b, a) = (q_2, \lambda)$$
$$\delta(q_2, b, a) = (q_2, \lambda)$$

$\{q_0, q_f, q_2\} \in F$.

CH # 7.3
Dpda EXERCISES

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① ST. $L = \{a^n b^{2n} : n \geq 0\}$ is a DCFL. abb.

$\delta(q_0, \lambda, z) = (q_f, \lambda)$
$\delta(q_0, a, z) = (q_1, 11z)$
$\delta(q_1, a, 1) = (q_1, 111)$ dpda
$\delta(q_1, b, 1) = (q_1, \lambda)$ ∴ DCFL
$\delta(q_1, \lambda, z) = (q_f, \lambda)$

Test abb: accepted

$\delta(q_0, abb, z) \vdash \delta(q_1, bb, 11z) \vdash \delta(q_1, b, 1z) \vdash \delta(q_1, \lambda, z) \vdash (q_f, \lambda)$

aabbbb: accepted

$\delta(q_0, aabb^{bb}, z) \vdash \delta(q_1, abb^{bb}, 11z) \vdash \delta(q_1, bb^{bb}, 1111z) \vdash \delta(q_1, bbb, \ldots$

③ b $L = \{a^n b^n : n \geq 1\} \cup \{b\}$ DCFL?

$\delta(q_0, a, z) = (q_1, 1z)$
$\delta(q_0, b, z) = (q_f, \lambda)$ ∴ dpda
$\delta(q_1, a, 1) = (q_1, 11)$
$\delta(q_1, b, 1) = (q_1, \lambda)$ ∴ DCFL ✓
$\delta(q_1, \lambda, z) = (q_f, \lambda)$

⑤
⑧ $L = \{ a^n b^m : n = m \text{ or } n = m+2 \}$ is DCFL ?

$\{a^n b^n\} \cup \{a^{n+2} b^n\}$

⑨ wcw^R

$\downarrow$

start matching .

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[redacted]

CH # 8.1
Properties of Cf

Eg 8.1 ST $L = \{a^n b^n c^n : n \geq 0\}$ is not context free.

1. $w = a^m b^m c^m \in L$

a . . . ab . . . bc c

(1) (2) (3) (4) (5)

2.1 $v = a$
$y = a$

$w_i = a^{m+2i-2} b^m c^m$

$i > 1 \Rightarrow m+2i-2 > m$

$w_i \notin L$

$\because n_a(w_i) \neq n_b(w_i)$
$\neq n_c(w_i)$

$\boxed{w_i \notin L}$

2.2 $v = b$, $y = b$ | 2.3 $v = c$, $y = c$ — similar cases

2.4. $v = a$
$y = b$

$w_i = a^{m+i-1} b^{m+i-1} c^m$

$i > 1 \Rightarrow m+i-1 > m$

$\Rightarrow n_a(w_i) \neq n_c(w_i)$
$n_b(w_i) \neq n_c(w_i)$

$\boxed{w_i \notin L}$

2.5. $v = b$, $y = c$ — Similar case

$\therefore$ as Pumping Lemma fails, L is not a CFL.

Eg # 8.2 $L = \{ww : w \in \{a,b\}^*\}$ is L CFL?

a . . . ab . . . ba . . . ab . . . b

(1) (2) (3) (4) (5) (6) (7)

1. $w = a^n b^n a^m b^m$

2.1 $w_i = a^{n+2i-2} b^n a^m b^m$

$i > 1 \Rightarrow n+2i-2 > m$

$w_i \notin L$

$v = a$, $y = a$ {first w}

2.2. $v = b, y = b$
2.3. $v = a, y = a$
2.4 $v = b, y = b$

similar cases

2.5 $v = a, y = b$

$w_i = a^{n+i-1} b^{n+i-1} a^m b^m$

$i > 1 \Rightarrow n_a(\text{first } w) > n_a(\text{second } w)$

$\therefore w_i \notin L$

2.6 $v = b, y = a$
2.7 $v = a, y = b$

similar case

L is not CFL as PL fails.

Eg: 8.3 ST. $L = \{a^{n!} : n \geq 0\}$ is not context free.

1. $w = a^{m!} \in L$

2. $v = a^k$

$y = a^l$

$w_i = a^{[m-(k+l)+2]!}$

$k+l < m$: $m-(k+l) > 0$

$\therefore m-(k+l) > m!$

$\therefore$ not CFL.

Eg: 8.4 ST $L = \{a^n b^j : n = j^2\}$ is not CFL.

$$\overset{③}{aa \underset{①}{-} - - aab \underset{②}{-} - - b}$$

1. $w = a^{m^2} b^m \in L$

2.1 $v = a$

$y = a$

$w_i = a^{m^2+2i-1} b^m$

$i > 1 \Rightarrow m^2+2i-1 > m^2$

$\therefore w_i \notin L$

$(\simeq$ 2.2 $v = b, y = b)$

2.3 $v = a$

$y = b$

$w_i = a^{m^2+i-1} b^{m+i-1}$

$i = 0$: $m^2-1 \neq (m-1)^2$

$w_i \notin L$

$\therefore L$ is not CFL

4) $L = \{a^n b^j a^j b^n : n, j > 0\}$ is CFG or not?

$\delta(q_0, a, z) = \{(q_0, az)\}$

$\delta(q_0, a, a) = \{(q_0, aa)\}$

$\delta(q_0, b, a) = \{(q_1, ba)\}$

$\delta(q_1, b, b) = \{(q_1, bb)\}$

$\delta(q_1, a, b) = \{(q_2, \lambda)\}$

$\delta(q_2, a, b) = \{(q_2, \lambda)\}$

$\delta(q_2, b, a) = \{(q_3, \lambda)\}$

$\delta(q_3, b, a) = \{(q_3, \lambda)\}$

$\delta(q_3, \lambda, z) = \{(q_f, \lambda)\}$

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CH#8.1

$L = \{a^n b^n c^j : n \le j\}$

1. $w = a^m b^m c^{m+1} \in L$

$(a \ldots a b \ldots b c \ldots c)$

① ② ③ ④ ⑤

2.1 $v = a$, $y = a$

$w_i = a^{m+2i-2} b^m c^{m+1}$

$i > 1$: $m+2i-2 > m$

$\Rightarrow n_a(w) \neq n_b(w)$

$w_i \notin L$

2.2 Similar cases

2.3 $v = c$, $y = c$

$w_i = a^m b^m c^{m+2i-2+1}$

$i = 0$: $m-1 < m$

$\therefore w_i \notin L$

2.4 $v = a$, $y = b$

$w_i = a^{m+i-1} b^{m+i-1} c^{m+1}$

$i > 2 \Rightarrow m+i-1 > m+1$

$\therefore w_i \notin L$

$n_c(w) \ngtr n_a(w)$

$\ngtr n_b(w)$

$\cong$ 2.5

$\therefore L \notin CFL$

$\boxed{\text{EXERCISES}}$

(2) ST $L = \{a^n : n \text{ is a prime no.}\}$ is not CFL.

1. $w = a^m$ m is prime : $\in L$

2. $a \ldots\ldots\ldots a$

$v = a$, $y = a$

$w_i = a^{m+2i-2}$

$i = 0$: $m+2i-2 = m-2$

$m-2$ is not prime

$i > 0$: $m+2i-2$
$i < 0$: $m+2i-2$
} not necessarily prime

$\therefore$ PL fails $\Rightarrow$ NOT CFL

(5) Is $L = \{a^n b^m : n = 2^m\}$ CFL?

1. $w = a^{2^m} b^m \in L$

2.

$$\overbrace{aaa \ldots abb}^{3} \ldots b$$

1 (under aa) 2 (under bb)

2.1 $v = a$, $y = a$

$$w_i = a^{2^m + 2i - 2} b^m$$

$\boxed{2^m + 2i - 2}$

$i = 0$: $2^m - 2 \neq 2^m$

ie. $Pow(a) \neq 2^m$

$\therefore w_i \notin L$

2.2 Similar for $v = b$, $y = b$

2.3 $v = a$, $y = b$

$$w_i = a^{2^m + i - 1} b^{m + i - 1}$$

$i = 0$: $a^{2^m - 1} b^{m-1}$.

$$2^{m-1} = 2^m - 2 = 2^m - 1 - 1$$

$\boxed{2^{m-1} > 2^m - 1}$

ie. $Pow(a) \neq 2^m$

$\therefore w \notin L$

ST not CFL:

(7) a) $L = \{a^n b^j : n \leq j^2\}$

$w = a^{j^2} b^j$

not CFL

b) $L = \{a^n b^j : n \geq (j-1)^3\}$

$w = a^{(j-1)^3} b^j$.

not CFG

(f) $n_a(w) < n_b(w) < n_c(w)$

$$a^n b^{n+1} c^{n+2}$$

(8) CFL or not?

(a) $L = \{a^n w w^R a^n : n \geq 0;\ w \in \{ab\}^*\}$

ww^R : CFL
a^n : CFL
} CFL closed under concatenation $\Rightarrow \boxed{CFL}$

$\delta(q_2, a, b) = \{(q_3, \lambda)\}$
$\delta(q_3, a, a) = \{(q_3, \lambda)\}$
$\delta(q_3, \lambda, z) = \{(q_4, \lambda)\}$

$\delta(q_0, a, z) = \{(q_0, az), (q_1, az)\}$
$\delta(q_0, a, a) = \{(q_0, aa), (q_1, aa)\}$
$\delta(q_1, a, a) = \{(q_1, aa)\}$
$\delta(q_1, b, a) = \{(q_1, ba)\}$
$\delta(q_1, a, b) = \{(q_1, ab), (q_2, \lambda)\}$
$\delta(q_1, b, b) = \{(q_1, bb), (q_2, \lambda)\}$
$\delta(q_2, a, a) = \{(q_2, \lambda)\}$
$\delta(q_2, b, b) = \{(q_2, \lambda)\}$

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CH # 8.1

(8) (b).

$L = \{a^n b^j a^n b^j : n \geq 0, j \geq 0\}$

not CFG

1. $w = a^m b^k a^m b^k \in L$

$$\overset{\textcircled{5}}{\;}\quad \overset{\textcircled{6}}{\;}\quad \overset{\textcircled{7}}{\;}$$

$aa \ldots ab \ldots ba \ldots ab \ldots b$

① ② ③ ④

2.1 $v = a$

$\boxed{2.2}$ $\boxed{2.3}$ $\boxed{2.4}$ $y = a$

$w_i = a^{m+2i-2} b^k a^m b^k$

$i > 0$: $m + 2i - 2 > m$

$w_i \notin L$

2.5 $v = a$

$\boxed{2.6}$ $\boxed{2.7}$ $y = b$

$w_i = a^{m+i-1} b^{m+i-1} a^m b^m$

$i = 0$ $m - 1 \neq m$

$w_i \notin L$

PL fails $\Rightarrow$ NOT CFL

(c) $L = \{a^n b^j a^j b^n : n \geq 0, j \geq 0\}$

$\boxed{a^n b^j}$

$\delta(q_0, a, z) = \{(q_0, az)\}$

$\delta(q_0, a, a) = \{(q_0, aa)\}$

$\delta(q_0, b, a) = \{(q_1, ba)\}$

$\delta(q_0, b, z) = \delta(q_1, bz)$ $\boxed{a^0 b^j}$

$\delta(q_1, b, b) = \{(q_2, bb)\ (q_3, \lambda)\}$

$\delta(q_2, a, b) = \{(q_1, \lambda)\}$

$\delta(q_2, b, b) = \{(q_3, \lambda)\}$

$\boxed{b^j a^j}$ $\boxed{b^0 a^0}$

$\delta(q_3, b, b) = \{(q_3, \lambda)\}$

$\delta(q_3, \lambda, z) = \{(q_f, \lambda)\}$

$\boxed{a^n \times b^n}$

$\delta(q_0, \lambda, z) = \{(q_f, \lambda)\} \longrightarrow \lambda \in L(G)$

$\therefore$ CFL

(d) $L = \{a^n b^j a^k b^l : n+j < k+l\}$

1\. $w = a^n b^n a^m b^m \qquad n < m$

$$a..\underbrace{ab..}_{5}ba..\underbrace{.ab.}_{6}.b$$

$$\uparrow\uparrow_{1} \quad \uparrow\uparrow_{2} \quad \uparrow\uparrow_{3} \quad \uparrow\uparrow_{4}$$

2.1 $v = a$

$y = a$

$w_i = a^{n+2i-2} b^n a^m b^m$

$i > 0 \quad n + 2i > m$

but $n < m$

$\therefore w_i \notin L$

2.2

2.3

2.4

2.5 $v = a$

$y = b$

2.6

$w_i = a^{n+i-1} b^{n+i-1} a^m b^m$

ROW(LHS) COW(RHS)

$2n+2i-2 \quad : \quad 2m$

$\boxed{n+i-1} \quad : \quad \boxed{m}$

$n < m$

$i > 0 \quad n + i > m$

$w_i \notin L$

As PL fails, NOT CFL

(e) $L = \{a^n b^j a^k b^l : n \le k,\ j \le l\}$

NOT CFL

(f) $L = \{a^n b^n c^j : n \le j\}$

NOT CFL

(g) $L = \{w \in \{a,b,c\}^* : n_a(w) = n_b(w) = 2n_c(w)\}$

NOT CFL.

Prathima Bhima
[redacted]

CH # 8.2

CFL closed under $\cup$ union

$\cdot$ concatenation

$\rightarrow$ $*$ Star Closure.

$\rightarrow$

$\boxed{RL \cap CFL = CFL}$

NOT closed under $\left.\right\}$ $\rightarrow$ Intersection $\cap$

$\rightarrow$ Complement $\bar{A}$:

Eg: 8.7 ST $L = \{a^n b^n : n \geq 0, n \neq 100\}$ is CFL.

$L_2 = \{a^n b^n : n \geq 0\}$

$L_1 = \{a^n b^n : n = 100\}$

$L_1 = \{a^{100} b^{100}\}$ $\longrightarrow$ Regular

Regular languages are closed under complement

$\therefore$ $\bar{L_1}$ is also regular.

$L = \underset{\text{CFL}}{L_2} \cap \underset{\text{RL}}{\bar{L_1}} = \{a^n b^n \cdot n \neq 100, n \geq 0\}$

$\therefore$ L is a CFL.

Eg: 8.8 ST $L = \{\{a,b,c\}^* : n_a(w) = n_b(w) = n_c(w)\}$ is not CFL.

PL fails:

also: $L_1 = (a^* b^* c^*)$ $\rightarrow$ Regular

we know $L_2 \{a^n b^n c^n\}$ is NOT CFL.

$L \cap L_1 = L_2$

? Regular NOT CFG

Case (i) L is CFL $\Rightarrow$ L_2 should be CFL, but is not $\Rightarrow$ $\boxed{L \text{ is NOT CFL}}$

Case (ii) $\boxed{L \text{ is not CFL}} \Rightarrow L_2$ is not CFL, true.

Is Empty / Is NOT Empty

$\boxed{\begin{array}{l} N \to t \\ N \to X \end{array}}$

Eg:

$S \to XY$	$S \to XY$	$S \to aabb$
$X \to AX$	$\cancel{X \to aX}$	
$X \to AA$	$X \to aa$	$L(G)$ NOT empty
$A \to a$	$\cancel{Y \to bY}$	
$Y \to BY$	$Y \to bb$	
$Y \to BB$		
$B \to b$		

Eg:

$S \to XY$	$S \to XY$	$S \to Xbb$
$X \to AX$	$X \to aX$	$X \to aX$
$A \to a$	$\cancel{Y \to bY}$	
$Y \to BY$	$Y \to bb$	$L(G)$ Is Empty.
$Y \to BB$		
$B \to b$		

Eg*)

$S \to XS \,/\, YZ$

$X \to YX$

$Y \to YY$

$Y \to XX$

$X \to A$

$Z \to SX$

Is Empty

Eg*)

$S \to AB$	$S \to aB \,/\, bB$	$S \to abb \,/\, abbb \,/\, abbbb \,/\, bbb \,/\, bbbb \,/\, bbbbb$
$\cancel{A \to BSB}$	$\cancel{B \to aaS \,/\, bbS \,/\, abS \,/\, baS}$	
$B \to AAS$	$B \to bb \,/\, bbb \,/\, bbbb$	
$\cancel{A \to CC}$		$L(G)$ Not Empty
$B \to CC$		
$\cancel{C \to SS}$		
$A \to a/b$		

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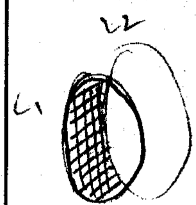
(7)

~~CFL - CFL not closed~~
~~RL - RL is closed~~

ST $L_1 - L_2 = CFL$
if $L_1 : CFL$ &
$L_2 : RL$.

$L_1 - L_2$: assume closed under difference. — (1)

$L_1 - L_2 = L_1 \cap \overline{L_2}$



(1) => LHS is CFL

RHS is not CFL as CF languages are not closed under concatenation

∴ assumption of (1) is wrong

So CFL not closed under '−'

$L_1 = CFL$

$L_2 = RL$

$L_1 - L_2 = L_1 \cap \overline{L_2}$

$L_1 \cap \overline{L_2}$: contex free language.
↓ ↓
CFL RL

⇒ If L_1:CFL, L_2:RL then closed under difference.

(11) ST DCFL not closed under ∪ & ∩

DCFL ⇒ DPDA

$L_1, L_2 \in DCFL$

$L = L_1 \cup L_2 \Rightarrow \boxed{S \to S_1 / S_2}$ → non-deterministic

$S \to SS$

... not DCFL

$L_1 \cap L_2 \Rightarrow \overline{\overline{L_1} \cup \overline{L_2}} = \overline{L}$

not DCFL

(18) S1 $L = \{ w \in \{a,b\}^* : n_a(w) = n_b(w) : w$ dont contain substring $aab \}$

$$L_2 = (a+b)^* aab (a+b)^*$$

↓ Regular language ⇒ $\bar{L}_2$ also RL

$$L_1 = \{ \{a,b\}^* : n_a(w) = n_b(w) \}$$

we know NOT CFL (PL fails)

$a^m b^k a^l b^j$
$(m+l = k+j)$

$L_1 \cap \bar{L}_2 = L$

↓ RL

Case (i) L is CFL ⇒ L_1 is CFL / not true

Case (ii) L is NOT CFL ⇒ L_1 is NOT CFL / true

∴ L is not CFL