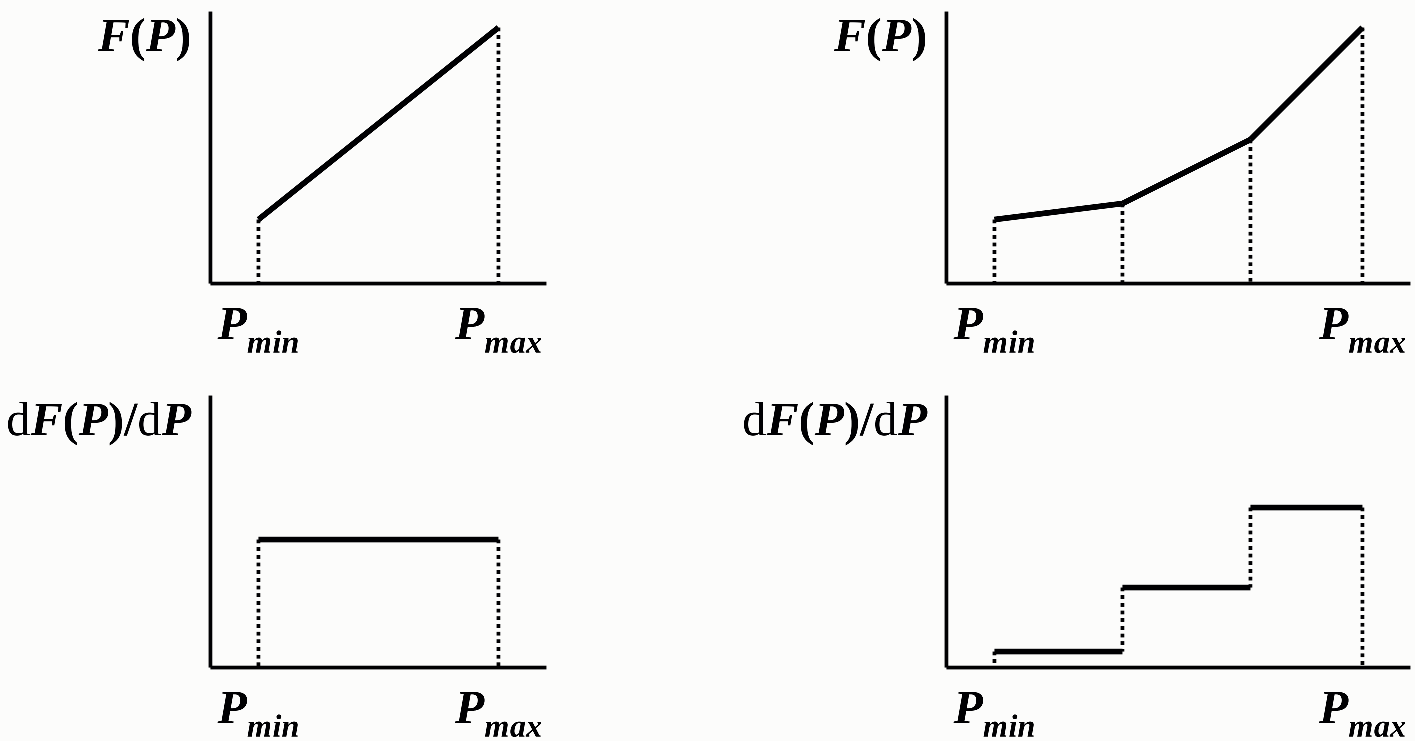
EEL 6266
Power System Operation and Control

Chapter 3

Economic Dispatch
Using Dynamic Programming

Piecewise Linear Cost Functions

- Common practice
  - many utilities prefer to represent their generator cost functions as single- or multiple-segment, linear cost functions
- Typical examples:



Piecewise Linear Cost Functions

- Piecewise linear cost functions can not be used with gradient based optimization methods
  - like the lambda-iteration
  - such methods will always land on P_{min} or P_{max}
- A table-based method resolves this problem
  - technique
    - for all units running, begin to raise the output of the unit with the lowest incremental cost segment
      - if this unit hits the right-hand end of a segment or hits P_{max}, find the unit with the next lowest incremental cost segment and begin to raise its output
    - eventually, the total of all units outputs equals the total load
      - the last unit is adjusted to have a generation, which is partially loaded for one segment

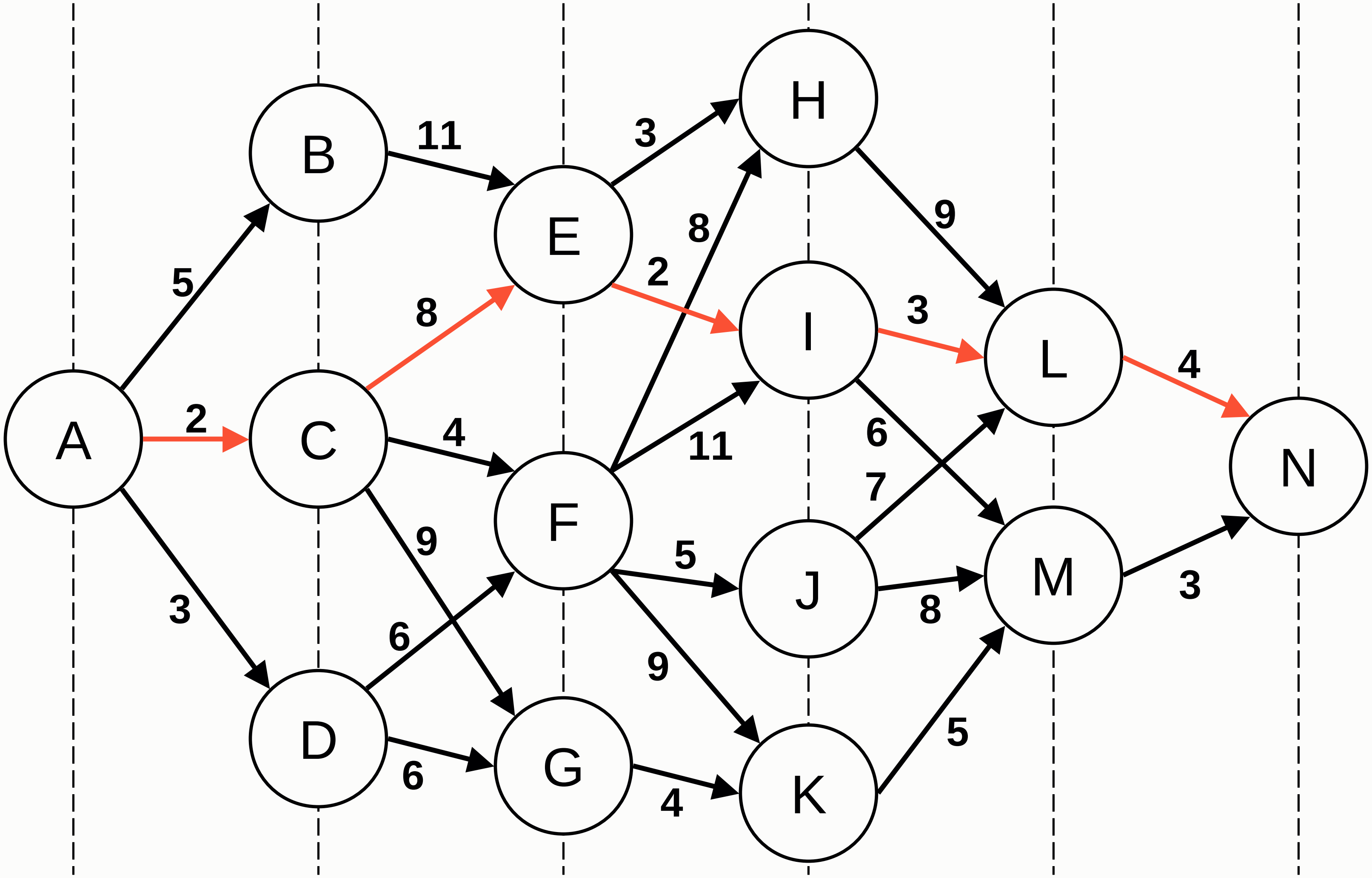
Dynamic Programming

- A wide variety of control and dynamic optimization problems use dynamic programming (DP) to find solutions
  - can greatly reduce the computation effort in finding optimal trajectories or control policies
  - DP applications have been developed for
    - economic dispatch
    - hydro-thermal economic-scheduling
    - unit commitment
  - methods are based on the calculus of variations
    - but, applications are not difficult to implement or program
  - principles are introduced by presenting examples of one-dimensional problems

Dynamic Programming

- Example
  - consider the cost of transporting a unit shipment from location A to location N
    - there are many short paths that connect many stops along the way, which offers numerous parallel routes from getting from A to N
    - each path has an associated cost
      - e.g., distance and level of difficulty results in fuel costs
    - the total cost is the sum of the path costs of the selected route from the originating location to the terminating location
    - the problem is to find the minimum cost route

Dynamic Programming



1-D Dynamic Programming Example

Dynamic Programming

- There are various stages traversed
  - starting at A, the minimum cost path to N is ACEILN
    - starting at C, the least cost path to N is CEILN
    - starting at E, the least cost path to N is EILN
    - starting at I, the least cost path to N is ILN
    - starting at L, the least cost path to N is LN
- Obtaining the optimal route
  - the choice of the route is made in sequence
  - Theory of optimality
    - the optimal sequence is called the *optimal policy*
    - any sub-sequence is called a *sub-policy*
    - the optimal policy contains only optimal sub-policies

Dynamic Programming

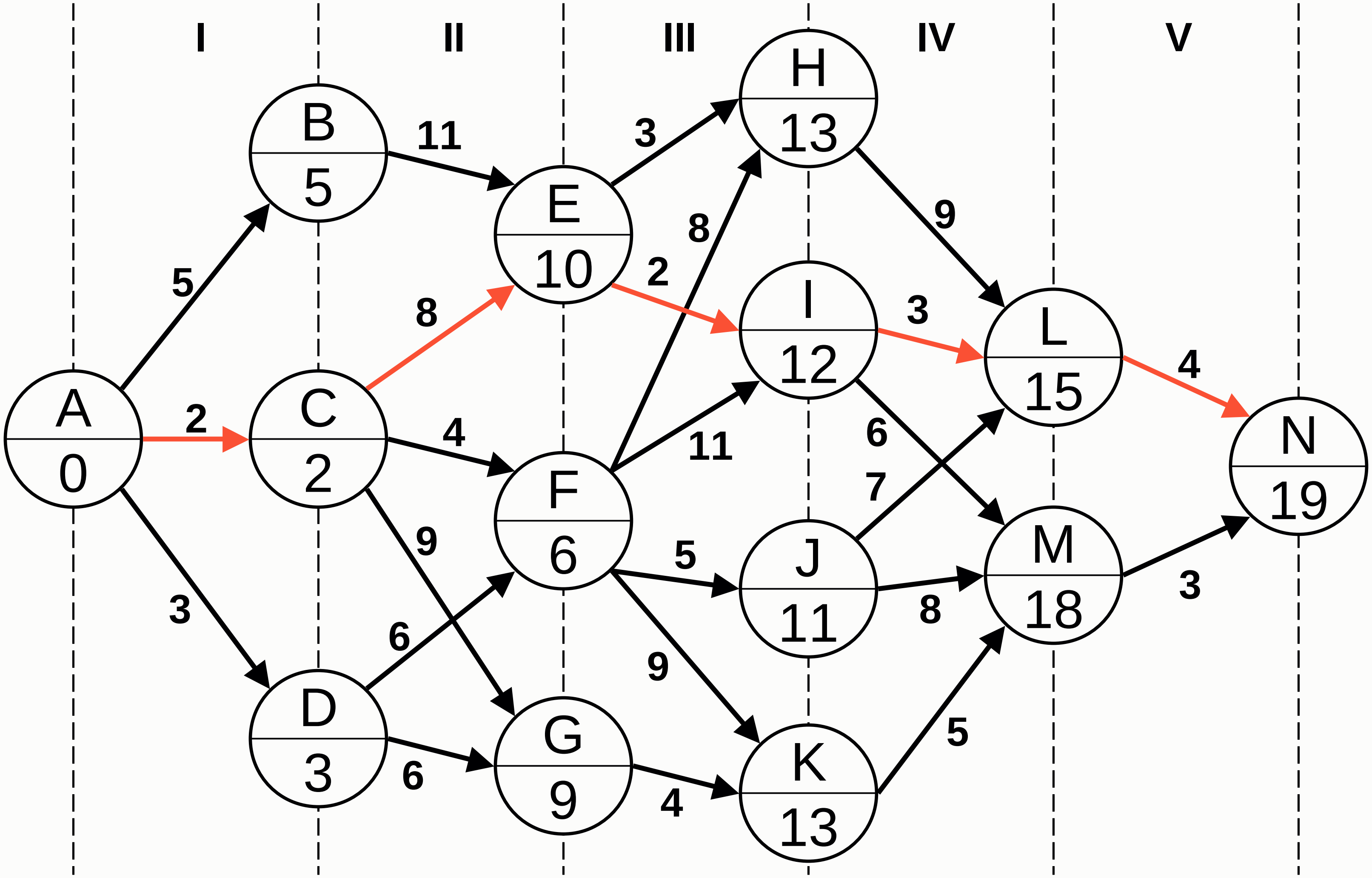
- Example (continued)
  - divide up the field of paths into stages (I, II, III, IV, V)
    - at the terminus of each stage, there is a set of nodes (stops), $\{X_i\}$
      - at stage III, the stops are $[\{X_3\} = \{H, I, J, K\}]$
    - a set of costs can be found for crossing a stage, $\{V_{III}(X_2, X_3)\}$
      - a cost is dependent on the starting and terminating nodes of a stage, $V_{III}(E, H) = 3$, $V_{III}(F, I) = 11$
    - the minimum cost for traversing from stage I to stage i and arrive at some particular node (stop), X_i, is defined as $f_I(X_i)$
      - the minimum costs from stage I to stage II for nodes {B, C, D} are:

        $f_I(B) = V_I(A, B) = 5$, $f_I(C) = V_I(A, C) = 2$, $f_I(D) = V_I(A, D) = 3$
      - the minimum cost from stage I to stage III for node {E} is:

$$f_{II}(E) = \min_{\{X_1\}} [\, f_I(X_1) + V_{II}(X_1, E)\,] = \min[\, \underset{X_1 = B}{5 + 11},\ \underset{= C}{2 + 8},\ \underset{= D}{3 + inf.}\,]$$

$f_{II}(E) = 10$ via ACE

Dynamic Programming



1-D dynamic programming example: cost at each node

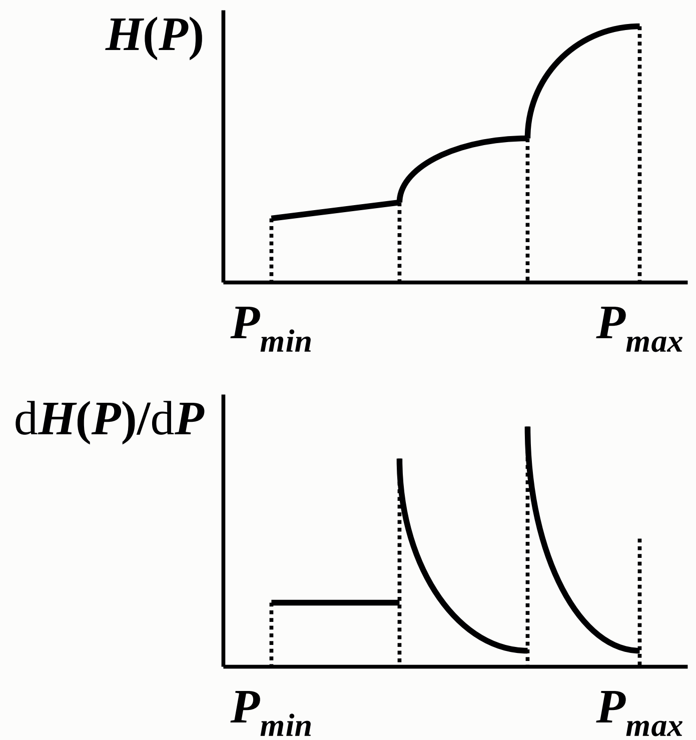
Dynamic Programming

- at each stage, the minimum cost should be recorded for all the terminus nodes (stops)
  - use the minimum cost of the terminus of the previous stage
  - identify the minimum cost path for each of the terminating nodes of the current stage

(X_1)	$f_{\mathrm{I}}(X_1)$	path	(X_2)	$f_{\mathrm{II}}(X_2)$	path	(X_3)	$f_{\mathrm{III}}(X_3)$	path	(X_4)	$f_{\mathrm{IV}}(X_4)$	path	(X_5)	$f_{\mathrm{V}}(X_5)$	path
B	5	A	E	10	AC	H	13	ACE	L	15	ACEI	N	19	ACEIL
C	2	A	F	6	AC	I	12	ACE	M	18	ADGK			
D	3	A	G	9	AD	J	11	ACF						
						K	13	ADG						

Dynamic Programming

- Economic dispatch
  - when the heat-rate curves exhibit nonconvex characteristics it is not possible to use an equal incremental cost method
    - multiple values of MW output exist for a given value of incremental cost
  - dynamic programming finds optimal dispatch under such circumstances
    - the DP solution is accomplished as an allocation problem
    - the approach generates a set of outputs for an entire set of load values



A nonconvex heat rate curve and its corresponding incremental heat rate curve

Dynamic Programming

- Example
  - consider a three-generator system serving a 310 MW demand
  - the generator I/O characteristics are not smooth nor convex

Power Levels (MW) $P_1 = P_2 = P_3$	Costs ($/hour) F_1	F_2	F_3
0	∞	∞	∞
50	810	750	806
75	1355	1155	1108.5
100	1460	1360	1411
125	1772.5	1655	1704.5
150	2085	1950	1998
175	2427.5	∞	2358
200	2760	∞	∞
225	∞	∞	∞

- the demand does not fit the data exactly, interpolate is needed between the available closest values, 300 and 325 MW

Dynamic Programming

- Example
  - the minimum cost function for scheduling units 1 and 2:

$$f_2(D) = F_1(D - P_2) + F_2(P_2)$$

- let P_2 cover its allowable range for demands of 100 to 350 MW

D (MW)	$F_1(D)$ ($/h)	P_2 = (MW) / $F_2(P_2)$= ($/h)	50 / 750	75 / 1155	100 / 1360	125 / 1655	150 / 1950	f_2 ($/h)	P_2^* (MW)
50	810		∞	∞	∞	∞	∞	∞	-
75	1355		∞	∞	∞	∞	∞	∞	-
100	1460		1560	∞	∞	∞	∞	1560	50
125	1772.5		2105	1965	∞	∞	∞	1965	75
150	2085		2210	2510	2170	∞	∞	2170	100
175	2427.5		2522.5	2615	2715	2465	∞	2465	125
200	2760		2835	2927.5	2820	3010	2760	2760	150
225	∞		3177.5	3240	3132.5	3115	3305	3115	125
250	∞		3510	2582.5	3445	3427.5	3410	3410	150
275	∞		∞	3915	3787.5	3740	3722.5	3722.5	150
300	∞		∞	∞	4120	4082.5	4035	4035	150
325	∞		∞	∞	∞	4415	4377.5	4377.5	150
350	∞		∞	∞	∞	∞	4710	4710	150

Dynamic Programming

- Example
  - the minimum cost function for scheduling units 1, 2 and 3:

$$f_3(D) = f_2(D - P_3) + F_3(P_3)$$

    - let P_3 cover its allowable range for the demand

D (MW)	$f_2(D)$ ($/h)	P_3 = 50, $F_3(P_3)$ = 806	75, 1108.5	100, 1411	125, 1704.5	150, 1998	175, 2358	f_3 ($/h)	P_3^* (MW)
100	1560	∞	∞	∞	∞	∞	∞	∞	-
125	1965	∞	∞	∞	∞	∞	∞	∞	-
150	2070	2366	∞	∞	∞	∞	∞	2366	50
175	2465	2771	2668.5	∞	∞	∞	∞	2668.5	75
200	2760	2976	3073.5	2971	∞	∞	∞	2971	100
225	3115	3271	3278.5	3376	3264.5	∞	∞	3264.5	125
250	3410	3566	3573.5	3581	3669.5	3558	∞	3558	150
275	3722.5	3921	3868.5	3876	3874.5	3963	3918	3868.5	75
300	4035	4216	4223.5	4171	4169.5	4168	4323	4168	150
325	4377.5	4528.5	4518.5	4526	4464.5	4463	4528	4463	150
350	4710	4841	4831	4821	4819.5	4758	4823	4758	150
375	∞	5183.5	5143.5	5133.5	5114.5	5113	5118	5113	150
400	∞	5516	5486	5446	5427	5408	5473	5408	150
425	∞	∞	5818.5	5788.5	5739.5	5720.5	5768	5720.5	150

Dynamic Programming

- Example
  - the results show:

D	Cost	P_1	P_2	P_3
300	4168	150	100	50
325	4463	150	125	50

- generator #2 is the marginal unit
  - it picks up all of the additional demand increase between 300 MW and 325 MW
  - $P_1 = 50$ MW, $P_2 = 110$ MW, $P_3 = 150$ MW, and $P_{total} = 310$ MW
- the cost is easily determined using interpolation

$$F_2(110) = \frac{1655-1360}{125-100} \cdot (110-100) + 1360 = 1478$$

  - $F_1 = \$\,810$, $F_2 = \$1478$, $F_3 = \$\,1998$, and $F_{total} = \$4286$

Ramp Rate Constraints

- Generators are usually under automatic generation control (AGC)
  - a small change in load and a new dispatch causes the AGC to change the outputs of appropriate units
  - generators must be able to move to the new generation value within a short period of time
  - large steam units have a prescribed "maximum rate limit", $\Delta P/\Delta t$ (MW per minute)
    - the AGC must allocate the change in generation to other units, so that the load change can be accommodated quickly enough
  - the new dispatch may be at the most economic values, but the control action may not be acceptable if the ramp rate for any of the units are violated

Ramp Rate Constraints

- To produce an acceptable dispatch to the control system, the ramp rate limits are added to the economic dispatch formulation
  - requires a short-range load forecast to determine the most likely load and load-ramping requirements of the units
    - system load is given to be supplied at time increments $t = 1 \ldots t_{\text{max}}$ with loading levels of P^{t}_{load}
    - the N generators on-line supply the load at each time increment

$$\sum_{i=1}^{N} P_i^t = P_{load}^t$$

    - each unit must obey a rate limit such that

$$P_i^{t+1} = P_i^t + \Delta P_i$$

$$-\Delta P_i^{\max} \leq \Delta P_i \leq \Delta P_i^{\max}$$

Ramp Rate Constraints

- The units are scheduled to minimize the cost to deliver the power over the time period

$$F^{total} = \sum_{t=1}^{t_{\max}} \sum_{i=1}^{N} F_i\left(P_i^t\right)$$

  - constraints

$$\sum_{i=1}^{N} P_i^t = P_{load}^t \qquad \forall t = 1 \ldots t_{\max}$$

    and

$$P_i^{t+1} = P_i^t + \Delta P_i$$

$$-\Delta P_i^{\max} \le \Delta P_i \le \Delta P_i^{\max}$$

  - the optimization problem can be solved with dynamic programming