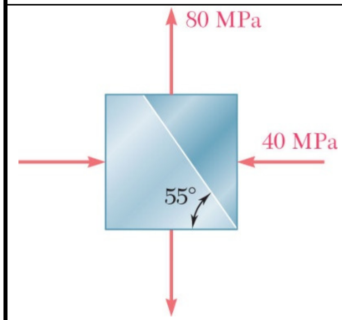
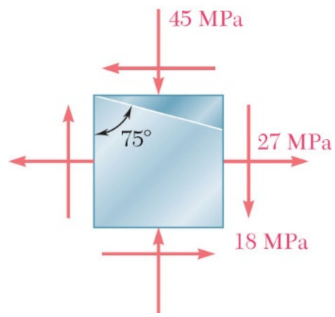
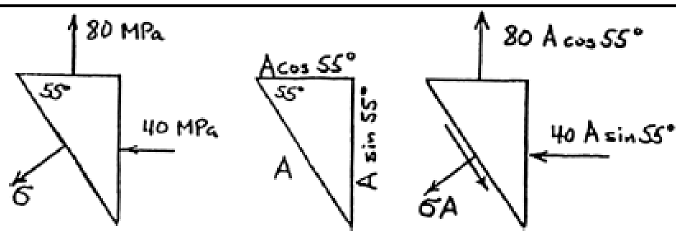
In the Name of GOD

Problem 6-1 For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown.

	**Answer:** $\sigma = -0.521$ MPa $\tau = 56.4$ MPa		**Answer:** $\sigma = -49.2$ MPa $\tau = 2.41$ MPa

SOLUTION



Stresses Areas Forces

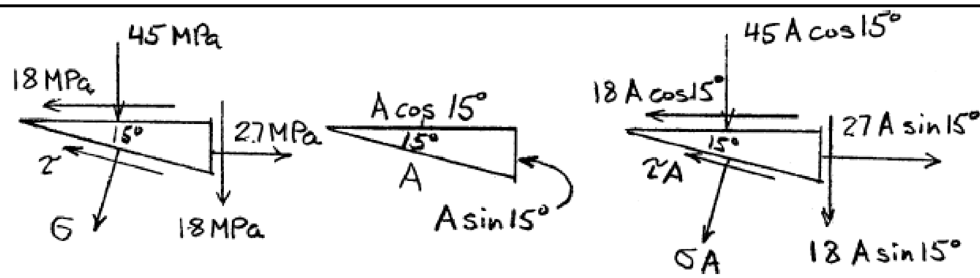
$+\swarrow \Sigma F = 0$: $\sigma A - 80A \cos 55° \cos 55° + 40A \sin 55° \sin 55° = 0$ $\quad \sigma = -0.521$ MPa ◀

$\sigma = 80 \cos^2 55° - 40 \sin^2 55°$

$+\searrow \Sigma F = 0$: $\tau A - 80A \cos 55° \sin 55° - 40A \sin 55° \cos 55°$ $\quad \tau = 56.4$ MPa ◀

$\tau = 120 \cos 55° \sin 55°$

SOLUTION



Stresses Areas Forces

$+\swarrow \Sigma F = 0$: $\sigma A + 18A \cos 15° \sin 15° + 45A \cos 15° \cos 15° - 27A \sin 15° \sin 15° + 18A \sin 15° \cos 15° = 0$

$\sigma = -18 \cos 15° \sin 15° - 45 \cos^2 15° + 27 \sin^2 15° - 18 \sin 15° \cos 15°$

$\sigma = -49.2$ MPa ◀

$+\nwarrow \Sigma F = 0$: $\tau A + 18A \cos 15° \cos 15° - 45A \cos 15° \sin 15° - 27A \sin 15° \cos 15° - 18A \sin 15° \sin 15° = 0$

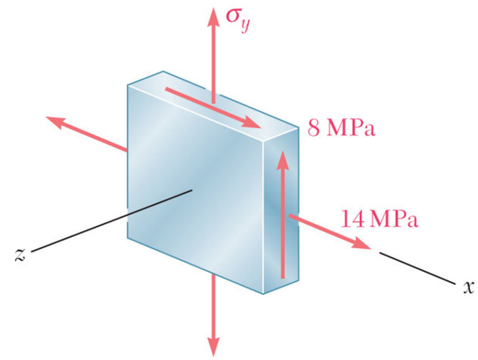
$\tau = -18(\cos^2 15° - \sin^2 15°) + (45 + 27) \cos 15° \sin 15°$

$\tau = 2.41$ MPa ◀

Problem 6-2 For the state of stress shown, determine two values of σ_y for which the maximum shearing stress is 10 MPa.

Answer: 2.00 MPa ; 9.33 MPa .

Note that the following solution is for English units, however, the procedure is the same.



SOLUTION

$\sigma_x = 14\text{ ksi}, \quad \tau_{xy} = 8\text{ ksi}, \quad \tau_{max} = 10\text{ ksi} \qquad \text{Let} \qquad u = \dfrac{\sigma_y - \sigma_x}{2} \qquad \sigma_y = 2u + \sigma_x$

$\sigma_{ave} = \dfrac{1}{2}(\sigma_x + \sigma_y) = \sigma_x + u \qquad R = \sqrt{u^2 + \tau_{xy}^2} \qquad u = \pm\sqrt{R^2 - \tau_{xy}^2}$

<u>Case (1)</u> $\underline{\tau_{max} = R} = 10\text{ ksi}, \quad u = \pm 6\text{ ksi}$

(1a) $u = +6\text{ksi} \quad \sigma_y = 2u + \sigma_x = 26\text{ ksi} \quad \text{(reject)}$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 20\text{ ksi}, \quad \sigma_a = \sigma_{ave} + R = 30\text{ ksi}, \quad \sigma_b = \sigma_{ave} - R = 10\text{ ksi}$$

$$\sigma_{max} = 30\text{ ksi}, \quad \sigma_{min} = 0, \quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 15\text{ ksi} \neq 7.5\text{ ksi}$$

(1b) $u = -6\text{ ksi} \quad \sigma_y = 2u + \sigma_x = 2\text{ ksi}$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 8\text{ ksi}, \quad \sigma_a = \sigma_{ave} + R = 18\text{ ksi}, \quad \sigma_b = \sigma_{ave} - R = -2\text{ ksi}$$

$$\sigma_{max} = 18\text{ ksi}, \quad \sigma_{min} = -2\text{ ksi}, \quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 10\text{ ksi} \quad \text{(o.k.)} \quad \sigma_y = 2.00\text{ ksi} \blacktriangleleft$$

<u>Case (2)</u> <u>Assume $\sigma_{min} = 0$.</u> $\sigma_{max} = 2\tau_{max} = 20\text{ ksi} = \sigma_a$

$$\sigma_a = \sigma_{ave} + R = \sigma_x + u + \sqrt{u^2 + \tau_{xy}^2} \qquad \sigma_a - \sigma_x - u = \sqrt{u^2 + \tau_{xy}^2}$$

$$(\sigma_a - \sigma_x - u)^2 = u^2 + \tau_{xy}^2 \qquad (\sigma_a - \sigma_x)^2 - 2(\sigma_a - \sigma_x)u + u^2 = u^2 + \tau_{xy}^2$$

$$2u = \frac{(\sigma_a - \sigma_x)^2 - \tau_{xy}^2}{\sigma_a - \sigma_x} = \frac{(20 - 14)^2 - 8^2}{20 - 14} = -4.6667\text{ ksi}$$

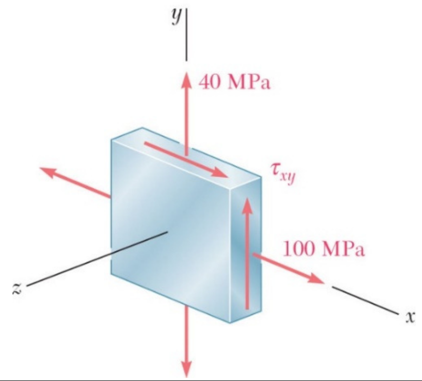
$u = -2.3333\text{ ksi} \quad \sigma_y = 2u + \sigma_x = 9.3333\text{ ksi}$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 11.6667\text{ ksi} \quad R = \sqrt{u^2 + \tau_{xy}^2} = 8.3333\text{ ksi} \qquad \sigma_a = \sigma_{ave} + R = 20\text{ ksi}$$

$$\sigma_b = \sigma_{ave} - R = 3.3334\text{ ksi} \qquad \sigma_{max} = 20\text{ ksi}, \quad \sigma_{min} = 0, \quad \tau_{max} = 10\text{ ksi} \qquad \sigma_y = 9.33\text{ ksi} \blacktriangleleft$$

Problem 6-3 For the state of stress shown, determine the value of τ_{xy} for which the maximum shearing stress is (a) 60 MPa, (b) 78 MPa.

Answer: (a) 40.0 MPa (b) 72.0 MPa



SOLUTION

$\sigma_x = 100$ MPa, $\sigma_y = 40$ MPa, $\sigma_z = 0$ $\qquad \sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 70$ MPa

(a) $\tau_{max} = 60$ MPa.

If σ_z is σ_{min}, then $\sigma_{max} = \sigma_{min} + 2\tau_{max}$.

$$\sigma_{max} = 0 + (2)(60) = 120 \text{ MPa}$$

$$\sigma_{max} = \sigma_{ave} + R$$

$$R = \sigma_{max} - \sigma_{ave} = 120 - 70 = 50 \text{ MPa}$$

$$\sigma_b = \sigma_{max} - 2R = 20 \text{ MPa} > 0$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{30^2 + \tau_{xy}^2} = 50 \text{ MPa}$$

$$\tau_{xy} = \sqrt{50^2 - 30^2}$$

$\tau_{xy} = 40$ MPa ◀

(b) $\tau_{max} = 78$ MPa.

If σ_z is σ_{min}, then $\sigma_{max} = \sigma_{min} + 2\tau_{max} = 0 + (2)(78) = 156$ MPa.

$$\sigma_{max} = \sigma_{ave} + R$$

$$R = \sigma_{max} - \sigma_{ave} = 156 - 70 = 86 \text{ MPa} > \tau_{max} = 78 \text{ MPa}$$

Set $\qquad R = \tau_{max} = 78$ MPa. $\quad \sigma_{min} = \sigma_{ave} - R = -8$ MPa < 0

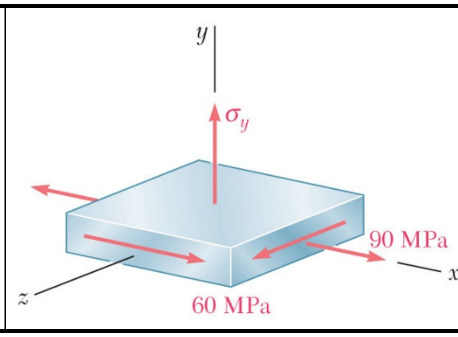
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{30^2 + \tau_{xy}^2}$$

$$\tau_{xy} = \sqrt{78^2 - 30^2}$$

$\tau_{xy} = 72$ MPa ◀

Problem 6-4 For the state of stress shown, determine two values of σ_y for which the maximum shearing stress is 80 MPa.

Answer: –40.0 MPa ; 130.0 MPa .

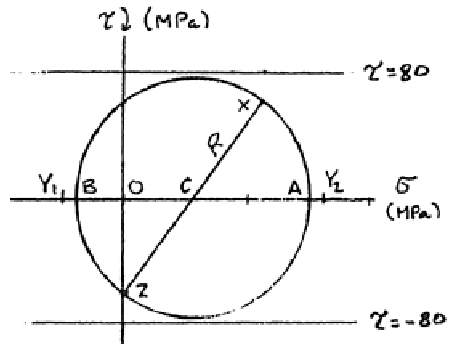


SOLUTION

$\sigma_x = 90$ MPa, $\sigma_z = 0$, $\tau_{xz} = 60$ MPa   Mohr's circle of stresses in *zx*-plane:

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_z) = 45 \text{ MPa} \qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{zx}^2} = \sqrt{45^2 + 60^2} = 75 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 120 \text{ MPa}, \quad \sigma_b = \sigma_{ave} - R = -30 \text{ MPa}$$



Assume $\sigma_{max} = \sigma_a = 120$ MPa.

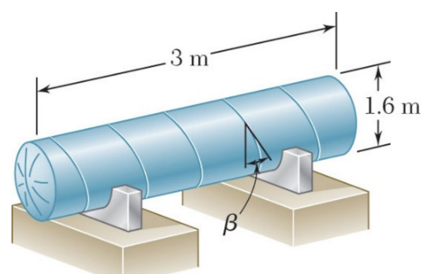
$$\sigma_y = \sigma_{min} = \sigma_{max} - 2\tau_{max}$$
$$= 120 - (2)(80) \qquad \sigma_y = -40 \text{ MPa} \blacktriangleleft$$

Assume $\sigma_{min} = \sigma_b = -30$ MPa.

$$\sigma_y = \sigma_{max} = \sigma_{min} + 2\tau_{max}$$
$$= -30 + (2)(8) \qquad \sigma_y = 130 \text{ MPa} \blacktriangleleft$$

Problem 6-5 The pressure tank shown has a 8-mm wall thickness and butt-welded seams forming an angle $\beta = 20^\circ$ with a transverse plane. For a gage pressure of 600 kPa, determine, (a) the normal stress perpendicular to the weld, (b) the shearing stress parallel to the weld.

Answer: (a) 33.2 MPa (b) 9.55 MPa

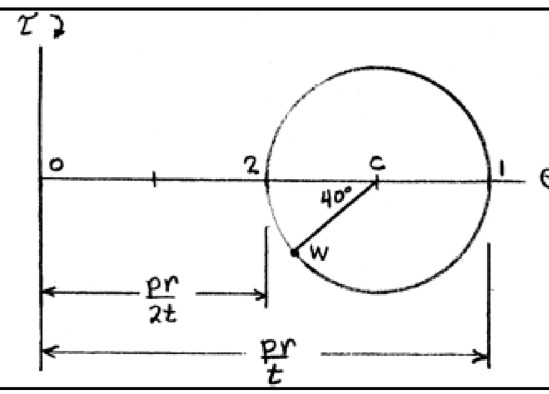


SOLUTION

$$d = 1.6 \text{ m} \quad t = 8\times10^{-3} \text{m} \quad r = \frac{1}{2}d - t = 0.792 \text{ m}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(600\times10^3)(0.792)}{8\times10^{-3}} = 59.4\times10^6 \text{Pa}$$

$$\sigma_2 = \frac{pr}{2t} = \frac{(600\times10^3)(0.792)}{(2)(8\times10^{-3})} = 29.7\times10^6 \text{Pa}$$



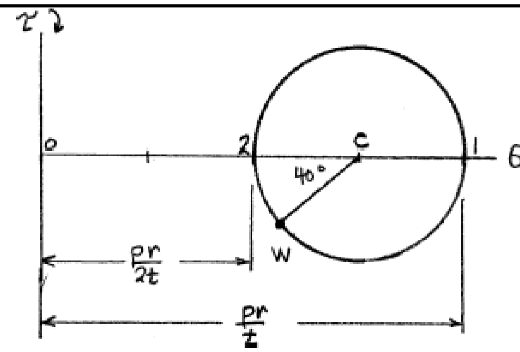
$$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = 44.56\times10^6\,\text{Pa} \qquad R = \frac{1}{2}(\sigma_1 - \sigma_2) = 14.85\times10^6\,\text{Pa}$$

(a) $\sigma_w = \sigma_{ave} - R\cos 40° = 33.17\times10^6$ Pa $\qquad \sigma_w = 33.2$ MPa ◀

(b) $\tau_w = R\sin 40° = 9.55\times10^6$ Pa $\qquad \tau_w = 9.55$ MPa ◀

Problem 6-6 For the tank of the previous problem, determine the largest allowable gage pressure, knowing that the allowable normal stress perpendicular to the weld is 120 MPa and the allowable shearing stress parallel to the weld is 80 MPa. **Answer:** 2.17 MPa .

SOLUTION



$d = 1.6$ m $\quad t = 8\times10^{-3}$ m $\quad r = \frac{1}{2}d - t = 0.792$ m

$$\sigma_1 = \frac{pr}{t} \quad \sigma_2 = \frac{pr}{2t} \quad \sigma_{ave} = \frac{1}{2}(\sigma_1+\sigma_2) = \frac{3}{4}\frac{pr}{t}$$

$$R = \frac{1}{2}(\sigma_1 - \sigma_2) = \frac{1}{4}\frac{pr}{t}$$

$$\sigma_w = \sigma_{ave} - R\cos 40° = \left(\frac{3}{4} - \frac{1}{4}\cos 40°\right)\frac{pr}{t} = 0.5585\frac{pr}{t}$$

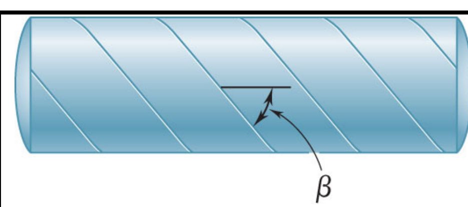
$$p = \frac{\sigma_w t}{0.5585r} = \frac{(120\times10^6)(8\times10^{-3})}{(0.5585)(0.792)} = 2.17\times10^6\,\text{Pa} = 2.17\text{ MPa}$$

$$\tau_w = R\sin 40° = \left(\frac{1}{4}\sin 40°\right)\frac{pr}{t} = 0.1607\frac{pr}{t}$$

$$p = \frac{\tau_w t}{0.1607r} = \frac{(80\times10^6)(8\times10^{-3})}{(0.1607)(0.792)} = 5.03\times10^6\,\text{Pa} = 5.03\text{ MPa}$$

The largest allowable pressure is the smaller value. $\qquad p = 2.17$ MPa ◀

Problem 6-7 The pressurized tank shown was fabricated by welding strips of plate along a helix forming an angle β with a transverse plane. Determine the largest value of β that can be used if the normal stress perpendicular to the weld is not to be larger than 85 percent of the maximum stress in the tank.



Answer: 56.8°

SOLUTION

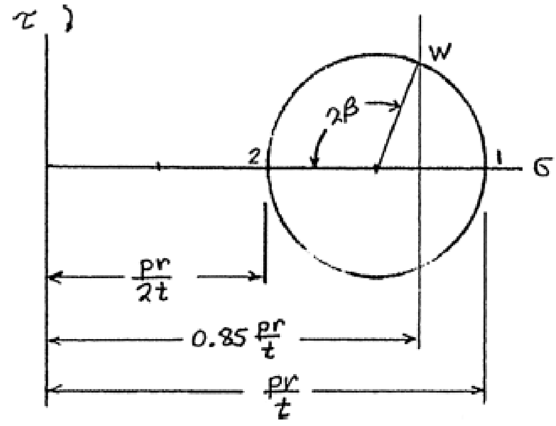
$$\sigma_1 = \frac{pr}{t} \quad \sigma_2 = \frac{pr}{2t} \qquad \sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4}\frac{pr}{t}$$

$$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4}\frac{pr}{t} \qquad \sigma_w = \sigma_{ave} - R\cos 2\beta$$

$$0.85\frac{pr}{t} = \left(\frac{3}{4} - \frac{1}{4}\cos 2\beta\right)\frac{pr}{t}$$

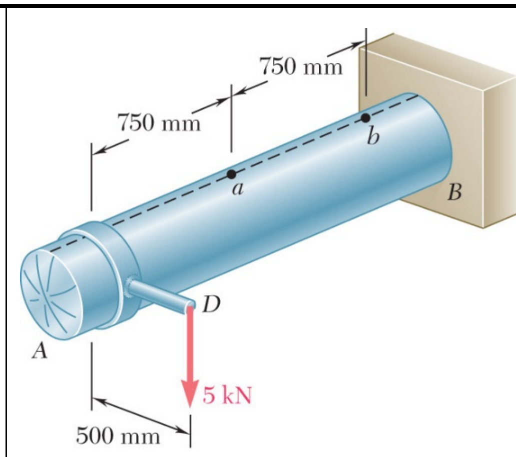
$$\cos 2\beta = -4\left(0.85 - \frac{3}{4}\right) = -0.4 \qquad 2\beta = 113.6^\circ$$

$$\beta = 56.8^\circ \blacktriangleleft$$

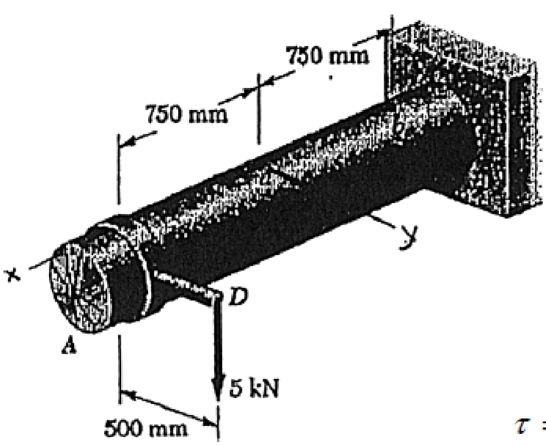


Problem 6-8 The compressed-air tank *AB* has an inner diameter of 450 mm and a uniform wall thickness of 6 mm. Knowing that the gage pressure inside the tank is 1.2 MPa, determine the maximum normal stress and the maximum in-plane shearing stress at point a on the top of the tank.

Answer: $\sigma_{max} = 45.1$ MPa ; $\tau_{max}(\text{in-plane}) = 9.40$ MPa .



SOLUTION



Internal pressure: $r = \frac{1}{2}d = 225\,\text{mm} \quad t = 6\,\text{mm}$

$$\sigma_1 = \frac{pr}{t} = \frac{(1.2)(225)}{6} = 45\,\text{MPa} \quad \sigma_2 = \frac{pr}{2t} = 22.5\,\text{MPa}$$

Torsion: $c_1 = 225$ mm, $c_2 = 225 + 6 = 231$ mm

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = 446.9\times10^6\,\text{mm}^4 = 446.9\times10^{-6}\,\text{m}^4$$

$$T = (5\times10^3)(500\times10^{-3}) = 2500\,\text{N}\cdot\text{m}$$

$$\tau = \frac{Tc}{J} = \frac{(2500)(231\times10^{-3})}{446.9\times10^{-6}} = 1.292\times10^6\,\text{Pa} = 1.292\,\text{MPa}$$

Transverse shear: $\tau = 0$ at point a.

Bending: $I = \frac{1}{2}J = 223.45\times10^{-6}\,\text{m}^4, \quad c = 231\times10^{-3}\text{m}$

At point a, $M = (5 \times 10^3)(750 \times 10^{-3}) = 3750 \text{ N} \cdot \text{m}$

$$\sigma = \frac{Mc}{I} = \frac{(3750)(231 \times 10^{-3})}{223.45 \times 10^{-6}} = 3.88 \text{ MPa}$$

Total stresses (MPa).

Longitudinal: $\sigma_x = 22.5 + 3.88 = 26.38 \text{ MPa}$

Circumferential: $\sigma_y = 45 \text{ MPa}$

Shear: $\tau_{xy} = 1.292 \text{ MPa}$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 35.69 \text{ MPa} \qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 9.40 \text{ MPa}$$

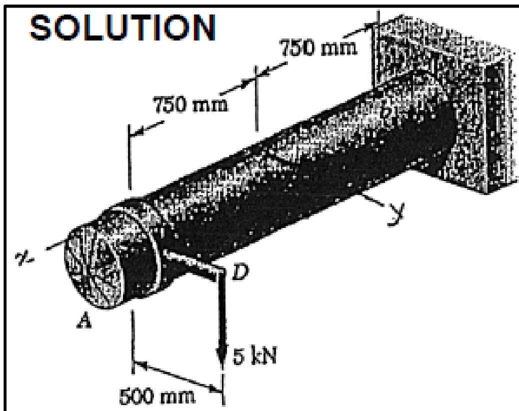
$$\sigma_{max} = \sigma_{ave} + R = 45.1 \text{ MPa} \qquad \sigma_{max} = 45.1 \text{ MPa} \blacktriangleleft$$

$$\tau_{max(\text{in-plane})} = R = 9.40 \text{ MPa} \qquad \tau_{max\,(\text{in-plane})} = 9.40 \text{ MPa} \blacktriangleleft$$

Problem 6-9 For the compressed-air tank and loading of the previous problem, determine the maximum normal stress and the maximum in-plane shearing stress at point b on the top of the tank.

Answer: $\sigma_{max} = 45.1 \text{ MPa}$; $\tau_{max}(\text{in-plane}) = 7.49 \text{ MPa}$.

SOLUTION



Internal pressure: $r = \frac{1}{2}d = 225 \text{ mm} \quad t = 6 \text{ mm}$

$$\sigma_1 = \frac{pr}{t} = \frac{(1.2)(225)}{6} = 45 \text{ MPa}$$

$$\sigma_2 = \frac{pr}{2t} = 22.5 \text{ MPa}$$

Torsion: $c_1 = 225 \text{ mm}, \quad c_2 = 225 + 6 = 231 \text{ mm}$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = 446.9 \times 10^6 \text{mm}^4 = 446.9 \times 10^{-6} \text{ m}^4$$

$$T = (5 \times 10^3)(500 \times 10^{-3}) = 2500 \text{ N} \cdot \text{m}$$

$$\tau = \frac{Tc}{J} = \frac{(2500)(231 \times 10^{-3})}{446.9 \times 10^{-6}} = 1.292 \times 10^6 \text{ Pa} = 1.292 \text{ MPa}$$

Transverse shear: $\tau = 0$ at point b.

Bending: $I = \frac{1}{2}J = 223.45 \times 10^{-6}\ \text{m}^4, \quad c = 231 \times 10^{-3}\ \text{m}$

At point b, $M = (5 \times 10^3)(2 \times 750 \times 10^{-3}) = 7500\ \text{N} \cdot \text{m}$

$$\sigma = \frac{Mc}{I} = \frac{(7500)(231 \times 10^{-3})}{223.45 \times 10^{-6}} = 7.75\ \text{MPa}$$

Total stresses (MPa).

Longitudinal: $\sigma_x = 22.5 + 7.75 = 30.25\ \text{MPa}$

Circumferential: $\sigma_y = 45\ \text{MPa}$ Shear: $\tau_{xy} = 1.292\ \text{MPa}$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 37.625\ \text{MPa} \qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 7.487\ \text{MPa}$$

$$\sigma_{\max} = \sigma_{\text{ave}} + R = 45.1\ \text{MPa} \qquad \sigma_{\max} = 45.1\ \text{MPa} \blacktriangleleft$$

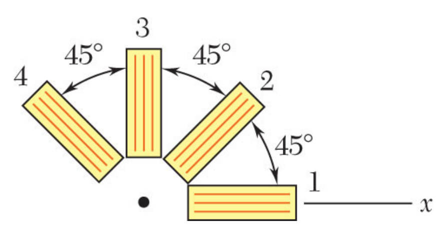
$$\tau_{\max\,(\text{in-plane})} = R = 7.49\ \text{MPa} \qquad \tau_{\max\,(\text{in-plane})} = 7.49\ \text{MPa} \blacktriangleleft$$

Problem 6-10 The rosette shown has been used to determine the following strains at a point on the surface of a crane hook:

$\varepsilon_1 = 420\,\mu \quad \varepsilon_2 = -45\,\mu \quad \varepsilon_4 = 165\,\mu$

(a) What should be the reading of gage 3? (b) Determine the principal strains and the maximum in-plane shearing strain.

Answer: (a) $-300\,\mu$ (b) $435\,\mu$, $-315\,\mu$, $750\,\mu$



SOLUTION

(*a*) Gages 2 and 4 are 90° apart.

$$\varepsilon_{\text{ave}} = \frac{1}{2}(\varepsilon_2 + \varepsilon_4) = \frac{1}{2}(-45 \times 10^{-6} + 165 \times 10^{-6}) = 60 \times 10^{-6}\,\mu$$

Gages 1 and 3 are also 90° apart.

$$\varepsilon_{\text{ave}} = \frac{1}{2}(\varepsilon_1 + \varepsilon_3) \qquad \varepsilon_3 = 2\varepsilon_{\text{ave}} - \varepsilon_1 = (2)(60 \times 10^{-6}) - 420 \times 10^{-6}$$

$$\varepsilon_3 = -300 \times 10^{-6}\,\mu \quad \blacktriangleleft$$

(*b*) $\varepsilon_x = \varepsilon_1 = 420 \times 10^{-6}\ \text{in/in} \quad \varepsilon_y = \varepsilon_3 = -300 \times 10^{-6}\,\mu$

$$\gamma_{xy} = 2\varepsilon_2 - \varepsilon_1 - \varepsilon_3 = (2)(-45\times10^{-6}) - 420\times10^{-6} - (-300\times10^{-6})$$
$$= -210\times10^{-6}\mu$$

$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = \sqrt{\left(\frac{420\times10^{-6} - (-300\times10^{-6})}{2}\right)^2 + \left(\frac{-210\times10^{-6}}{2}\right)^2}$$
$$= 375\times10^{-6}\mu$$

$\varepsilon_a = \varepsilon_{ave} + R = 60\times10^{-6} + 375\times10^{-6}$ $\qquad \varepsilon_a = 435\times10^{-6}\mu$ ◀

$\varepsilon_b = \varepsilon_{ave} - R = 60\times10^{-6} - 375\times10^{-6}$ $\qquad \varepsilon_b = -315\times10^{-6}\mu$ ◀

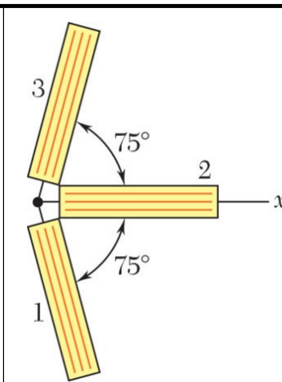
$\gamma_{\max\ (\text{in-plane})} = 2R$ $\qquad \gamma_{\max\ (\text{in-plane})} = 750\times10^{-6}\mu$ ◀

Problem 6-11 The strains determined by the use of the rosette attached as shown during the test of a machine element are

$\varepsilon_1 = -93.1\,\mu \quad \varepsilon_2 = 385\,\mu \quad \varepsilon_3 = 210\,\mu$

Determine (a) the orientation and magnitude of the principal strains in the plane of the rosette, (b) the maximum in-plane shearing strain.

Answer: (a) 30.0°, 120.0°, $560\,\mu$, $-140\,\mu$ $\;(b)$ $700\,\mu$



SOLUTION

Use $\varepsilon_{x'} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$

where $\theta = -75^\circ$ for gage 1, $\theta = 0$ for gage 2, $\theta = +75^\circ$ for gage 3.

and

$$\varepsilon_1 = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos(-150^\circ) + \frac{\gamma_{xy}}{2}\sin(-150^\circ) \qquad (1)$$

$$\varepsilon_2 = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 0 + \frac{\gamma_{xy}}{2}\sin 0 \qquad (2)$$

$$\varepsilon_3 = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos(150^\circ) + \frac{\gamma_{xy}}{2}\sin(150^\circ) \qquad (3)$$

From Eq. (2), $\qquad \varepsilon_x = \varepsilon_z = 385\times10^{-6}\mu$

Adding Eqs. (1) and (3),

$$\varepsilon_1 + \varepsilon_3 = (\varepsilon_x + \varepsilon_y) + (\varepsilon_x - \varepsilon_y)\cos 150^\circ = \varepsilon_x(1 + \cos 150^\circ) + \varepsilon_y(1 - \cos 150^\circ)$$

$$\varepsilon_y = \frac{\varepsilon_1 + \varepsilon_3 - \varepsilon_x(1+\cos 150^\circ)}{(1-\cos 150^\circ)} = \frac{-93.1\times10^{-6} + 210\times10^{-6} - 385\times10^{-6}(1+\cos 150^\circ)}{1-\cos 150^\circ}$$

$$= 35.0\times10^{-6}\mu$$

Subtracting Eq. (1) from Eq. (3), $\varepsilon_3 - \varepsilon_1 = \gamma_{xy}\sin 150^\circ$

$$\gamma_{xy} = \frac{\varepsilon_3 - \varepsilon_1}{\sin 150^\circ} = \frac{210\times10^{-6} - (-93.1\times10^{-6})}{\sin 150^\circ} = 606.2\times10^{-6}\mu$$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{606.2\times10^{-6}}{385\times10^{-6} - 35.0\times10^{-6}} = 1.732 \quad (a)\quad \theta_a = 30.0^\circ,\quad \theta_b = 120.0^\circ \blacktriangleleft$$

$$\varepsilon_{ave} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) = \frac{1}{2}(385\times10^{-6} + 35.0\times10^{-6}) = 210\times10^{-6}\mu$$

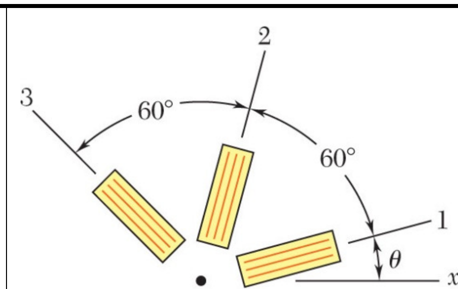
$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = \sqrt{\left(\frac{385\times10^{-6} - 35.0\times10^{-6}}{2}\right)^2 + \left(\frac{606.2}{2}\right)^2} = 350.0\times10^{-6}$$

$$\varepsilon_a = \varepsilon_{ave} + R = 210\times10^{-6} + 350.0\times10^{-6} \qquad \varepsilon_a = 560\times10^{-6}\mu \blacktriangleleft$$

$$\varepsilon_b = \varepsilon_{ave} - R = 210\times10^{-6} - 350.0\times10^{-6} \qquad \varepsilon_b = -140.0\times10^{-6}\mu \blacktriangleleft$$

(b) $$\frac{\gamma_{\max\,(\text{in-plane})}}{2} = R = 350.0\times10^{-6}\mu \qquad \gamma_{\max\,(\text{in-plane})} = 700\times10^{-6}\mu \blacktriangleleft$$

Problem 6-12 Show that the sum of the three strain measurements made with a 60° rosette is independent of the orientation of the rosette and equal to $\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 3\varepsilon_{ave}$ where ε_{ave} is the abscissa of the center of the corresponding Mohr's circle.



SOLUTION

$$\varepsilon_1 = \varepsilon_{ave} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta \qquad (1)$$

$$\varepsilon_2 = \varepsilon_{ave} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos(2\theta + 120^\circ) + \frac{\gamma_{xy}}{2}\sin(2\theta + 120^\circ)$$

$$= \varepsilon_{ave} + \frac{\varepsilon_x - \varepsilon_y}{2}(\cos 120^\circ\cos 2\theta - \sin 120^\circ\sin 2\theta) + \frac{\gamma_{xy}}{2}(\cos 120^\circ\sin 2\theta + \sin 120^\circ\cos 2\theta)$$

$$= \varepsilon_{ave} + \frac{\varepsilon_x - \varepsilon_y}{2}\left(-\frac{1}{2}\cos 2\theta - \frac{\sqrt{3}}{2}\sin 2\theta\right) + \frac{\gamma_{xy}}{2}\left(-\frac{1}{2}\sin 2\theta + \frac{\sqrt{3}}{2}\cos 2\theta\right) \qquad (2)$$

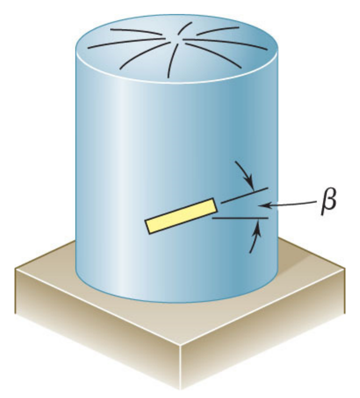
$$\varepsilon_3 = \varepsilon_{\text{ave}} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos(2\theta + 240°) + \frac{\gamma_{xy}}{2}\sin(2\theta + 240°)$$

$$= \varepsilon_{\text{ave}} + \frac{\varepsilon_x - \varepsilon_y}{2}(\cos 240° \cos 2\theta - \sin 240° \sin 2\theta) + \frac{\gamma_{xy}}{2}(\cos 240° \sin 2\theta + \sin 240° \cos 2\theta)$$

$$= \varepsilon_{\text{ave}} + \frac{\varepsilon_x - \varepsilon_y}{2}\left(-\frac{1}{2}\cos 2\theta + \frac{\sqrt{3}}{2}\sin 2\theta\right) + \frac{\gamma_{xy}}{2}\left(-\frac{1}{2}\sin 2\theta - \frac{\sqrt{3}}{2}\cos 2\theta\right) \quad (3)$$

Adding (1), (2), and (3), $\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 3\varepsilon_{\text{ave}} + 0 + 0 \qquad 3\varepsilon_{\text{ave}} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$ ◀

Problem 6-13 A single strain gage forming an angle β with a horizontal plane is used to determine the gage pressure in the cylindrical steel tank shown. The cylindrical wall of the tank is 6-mm thick, has a 600-mm inside diameter, and is made of a steel with $E = 200$ GPa and $\nu = 0.30$. Determine the pressure in the tank indicated by a strain gage reading of 280 μ (a) for $\beta = 18°$, (b) $\beta = 35°$. **Answer:** (a) 1.421 MPa (b) 1.761 MPa

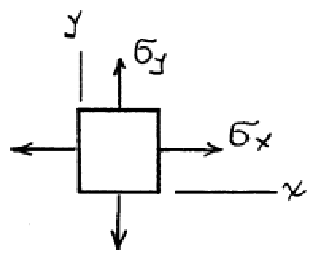


SOLUTION (a)

$$\sigma_x = \sigma_1 = \frac{pr}{t} \qquad \sigma_y = \frac{1}{2}\sigma_x, \quad \sigma_z \approx 0 \qquad \varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \left(1 - \frac{\nu}{2}\right)\frac{\sigma_x}{E} = 0.85\frac{\sigma_x}{E}$$

$$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z) = \left(\frac{1}{2} - \nu\right)\frac{\sigma_x}{E} = 0.20\frac{\sigma_x}{E}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = 0$$



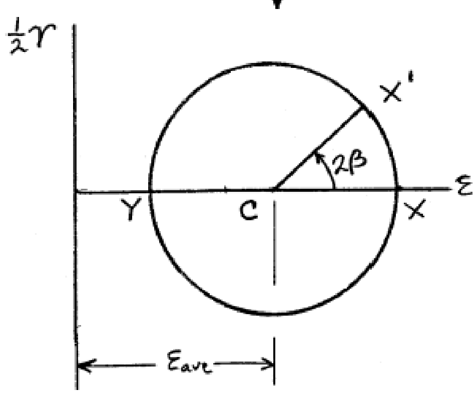
Draw Mohr's circle for strain.

$$\varepsilon_{\text{ave}} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) = 0.525\frac{\sigma_x}{E}$$

$$R = \frac{1}{2}(\varepsilon_x - \varepsilon_y) = 0.325\frac{\sigma_x}{E}$$

$$\varepsilon_{x'} = \varepsilon_{\text{ave}} + R\cos 2\beta = (0.525 + 0.325\cos 2\beta)\frac{\sigma_x}{E}$$

$$p = \frac{t\sigma_x}{r} = \frac{tE\varepsilon_{x'}}{r(0.525 + 0.325\cos 2\beta)}$$

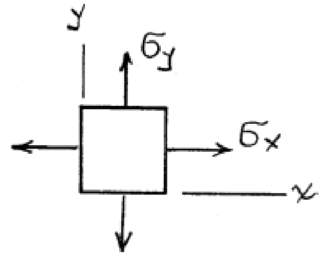


Data: $r = \frac{1}{2}d = \frac{1}{2}(600) = 300 \text{ mm} = 0.300 \text{ m}$

$t = 6\times10^{-3}$ mm $\quad E = 200\times10^{9}$ Pa, $\quad \varepsilon_{x'} = 280\times10^{-6} \quad \beta = 18°$

$$p = \frac{(6\times10^{-3})(200\times10^{9})(280\times10^{-6})}{(0.300)(0.525+0.325\cos 36°)} = 1.421\times10^{6}\text{ Pa} \qquad p = 1.421\text{ MPa} \blacktriangleleft$$

SOLUTION (b)

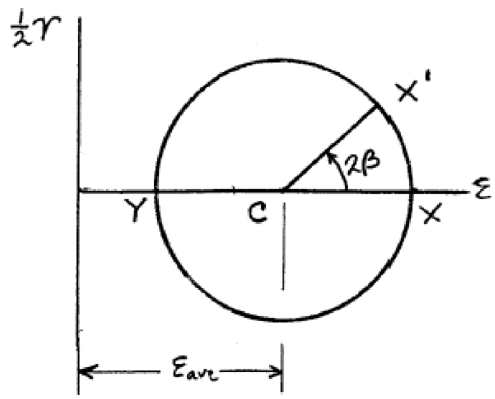


$$\sigma_x = \sigma_1 = \frac{pr}{t} \qquad \sigma_y = \frac{1}{2}\sigma_x, \quad \sigma_z \approx 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \left(1 - \frac{\nu}{2}\right)\frac{\sigma_x}{E} = 0.85\frac{\sigma_x}{E}$$

$$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z) = \left(\frac{1}{2} - \nu\right)\frac{\sigma_x}{E} = 0.20\frac{\sigma_x}{E} \qquad \gamma_{xy} = \frac{\tau_{xy}}{G} = 0$$

Draw Mohr's circle for strain.



$$\varepsilon_{\text{ave}} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) = 0.525\frac{\sigma_x}{E}$$

$$R = \frac{1}{2}(\varepsilon_x - \varepsilon_y) = 0.325\frac{\sigma_x}{E}$$

$$\varepsilon_{x'} = \varepsilon_{\text{ave}} + R\cos 2\beta$$

$$= (0.525 + 0.325\cos 2\beta)\frac{\sigma_x}{E}$$

$$p = \frac{t\sigma_x}{r} = \frac{tE\varepsilon_{x'}}{r(0.525+0.325\cos 2\beta)}$$

<u>Data</u>: $r = \frac{1}{2}d = \frac{1}{2}(600) = 300\text{ mm} = 0.300\text{ m}$

$t = 6\times10^{-3}\text{ m} \quad E = 200\times10^{9}\text{ Pa}, \quad \varepsilon_{x'} = 280\times10^{-6} \quad \beta = 35°$

$$p = \frac{(6\times10^{-3})(200\times10^{9})(280\times10^{-6})}{(0.300)(0.525+0.325\cos 70°)} = 1.761\times10^{6}\text{ Pa} \qquad p = 1.761\text{ MPa} \blacktriangleleft$$