Buckling of Columns

13

CHAPTER OBJECTIVES

In this chapter, we will discuss the behavior of columns and indicate some of the methods used for their design. The chapter begins with a general discussion of buckling, followed by a determination of the axial load needed to buckle a so-called ideal column. Afterwards, a more realistic analysis is considered, which accounts for any bending of the column. Also, inelastic buckling of a column is presented as a special topic. At the end of the chapter we will discuss some of the methods used to design both concentrically and eccentrically loaded columns made of common engineering materials.

13.1 Critical Load

Whenever a member is designed, it is necessary that it satisfy specific strength, deflection, and stability requirements. In the preceding chapters we have discussed some of the methods used to determine a member's strength and deflection, while assuming that the member was always in stable equilibrium. Some members, however, may be subjected to compressive loadings, and if these members are long and slender the loading may be large enough to cause the member to deflect laterally or sidesway. To be specific, long slender members subjected to an axial compressive force are called ***columns***, and the lateral deflection that occurs is called ***buckling***. Quite often the buckling of a column can lead to a sudden and dramatic failure of a structure or mechanism, and as a result, special attention must be given to the design of columns so that they can safely support their intended loadings without buckling.

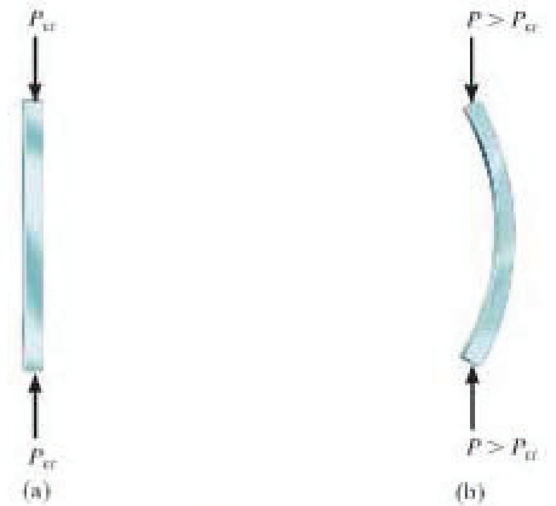

Fig. 13–1

The maximum axial load that a column can support when it is on the *verge* of buckling is called the ***critical load***, P_{cr}, Fig. 13–1*a*. Any additional loading will cause the column to buckle and therefore deflect laterally as shown in Fig. 13–1*b*. In order to better understand the nature of this instability, consider a two-bar mechanism consisting of weightless bars that are rigid and pin connected as shown in Fig. 13–2*a*. When the bars are in the vertical position, the spring, having a stiffness k, is unstretched, and a *small* vertical force **P** is applied at the top of one of the bars. We can upset this equilibrium position by displacing the pin at A by a small amount Δ, Fig. 13–2*b*. As shown on the free-body diagram of the pin when the bars are displaced, Fig. 13–2*c*, the spring will produce a restoring force $F = k\Delta$, while the applied load **P** develops two horizontal components, $P_x = P \tan\theta$, which tend to push the pin (and the bars) further out of equilibrium. Since θ is small, $\Delta \approx \theta(L/2)$ and $\tan\theta \approx \theta$. Thus the *restoring* spring force becomes $F = k\theta L/2$, and the *disturbing* force is $2P_x = 2P\theta$.

If the restoring force is greater than the disturbing force, that is, $k\theta L/2 > 2P\theta$, then, noticing that θ cancels out, we can solve for P, which gives

$$P < \frac{kL}{4} \qquad \text{stable equilibrium}$$

This is a condition for *stable equilibrium* since the force developed by the spring would be adequate to restore the bars back to their vertical position. However, if $kL\theta/2 < 2P\theta$, or

$$P > \frac{kL}{4} \qquad \text{unstable equilibrium}$$

then the mechanism would be in *unstable equilibrium*. In other words, if this load **P** is applied, and a slight displacement occurs at A, the mechanism will tend to move out of equilibrium and not be restored to its original position.

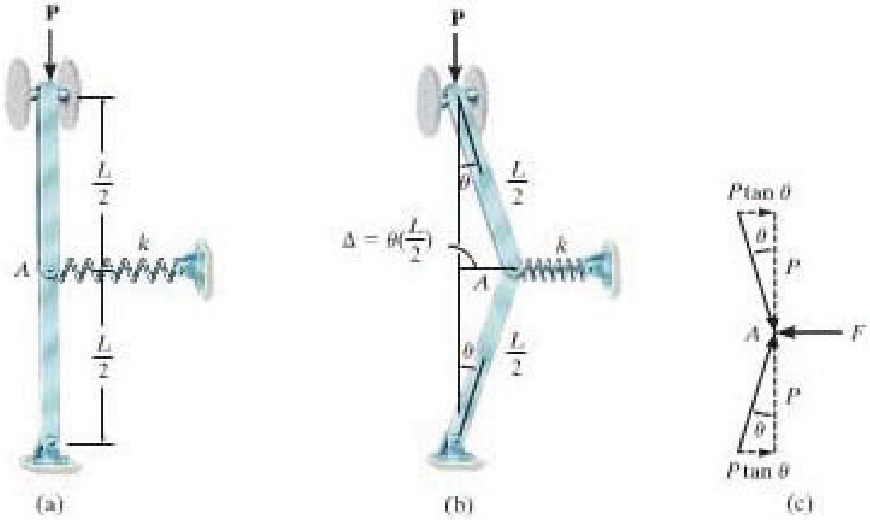

Fig. 13–2

The intermediate value of P, which requires $kL\theta/2 = 2P\theta$, is the *critical load*. Here

$$P_{cr} = \frac{kL}{4} \quad \text{neutral equilibrium}$$

This loading represents a case of the mechanism being in *neutral equilibrium*. Since P_{cr} is *independent* of the (small) displacement θ of the bars, any slight disturbance given to the mechanism will not cause it to move further out of equilibrium, nor will it be restored to its original position. Instead, the bars will *remain* in the deflected position.

These three different states of equilibrium are represented graphically in Fig. 13–3. The transition point where the load is equal to the critical value $P = P_{cr}$ is called the *bifurcation point*. At this point the mechanism will be in equilibrium for any *small value* of θ, measured either to the right or to the left of the vertical. Physically, P_{cr} represents the load for which the mechanism is on the verge of buckling. It is quite reasonable to determine this value by assuming *small displacements* as done here; however, it should be understood that P_{cr} may *not* be the largest value of P that the mechanism can support. Indeed, if a larger load is placed on the bars, then the mechanism may have to undergo a further deflection before the spring is compressed or elongated enough to hold the mechanism in equilibrium.

Like the two-bar mechanism just discussed, the critical buckling loads on columns supported in various ways can be obtained, and the method used to do this will be explained in the next section. Although in engineering design the critical load may be considered to be the largest load the column can support, realize that, like the two-bar mechanism in the deflected or buckled position, a column may actually support an

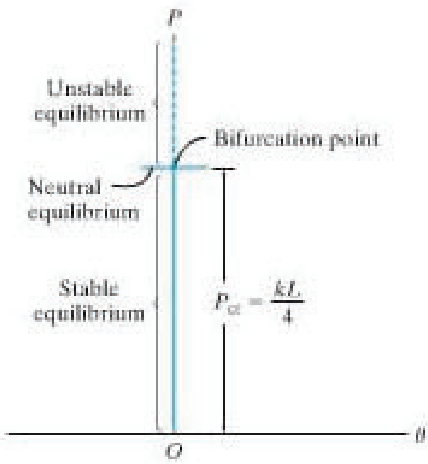

Fig. 13–3

even greater load than P_{cr}. Unfortunately, however, this loading may require the column to undergo a *large* deflection, which is generally not tolerated in engineering structures or machines. For example, it may take only a few newtons of force to buckle a meterstick, but the additional load it may support can be applied only after the stick undergoes a relatively large lateral deflection.

13.2 Ideal Column with Pin Supports

Some slender pin-connected members used in moving machinery, such as this short link, are subjected to compressive loads and thus act as columns.

In this section we will determine the critical buckling load for a column that is pin supported as shown in Fig. 13–4*a*. The column to be considered is an ***ideal column***, meaning one that is perfectly straight before loading, is made of homogeneous material, and upon which the load is applied through the centroid of the cross section. It is further assumed that the material behaves in a linear-elastic manner and that the column buckles or bends in a single plane. In reality, the conditions of column straightness and load application are never accomplished; however, the analysis to be performed on an "ideal column" is similar to that used to analyze initially crooked columns or those having an eccentric load application. These more realistic cases will be discussed later in this chapter.

Since an ideal column is straight, theoretically the axial load P could be increased until failure occurs by either fracture or yielding of the material. However, when the critical load P_{cr} is reached, the column will be on the verge of becoming unstable, so that a small lateral force F, Fig. 13–4*b*, will cause the column to remain in the deflected position when F is removed, Fig. 13–4*c*. Any slight reduction in the axial load P from P_{cr} will allow the column to straighten out, and any slight increase in P, beyond P_{cr}, will cause further increases in lateral deflection.

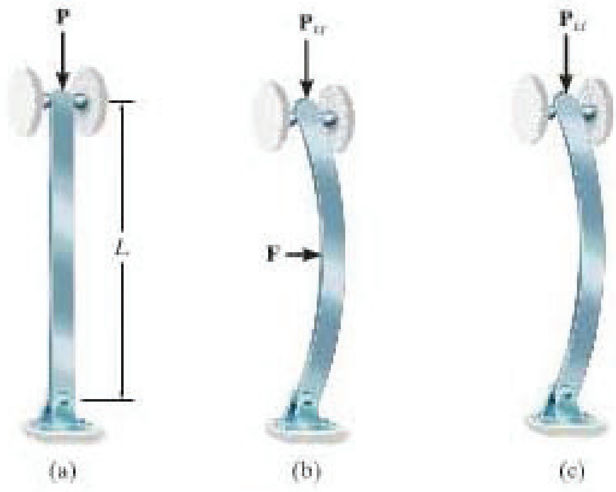

Fig. 13–4

Whether or not a column will remain stable or become unstable when subjected to an axial load will depend on its ability to restore itself, which is based on its resistance to bending. Hence, in order to determine the critical load and the buckled shape of the column, we will apply Eq. 12–10, which relates the internal moment in the column to its deflected shape, i.e.,

$$EI\frac{d^2v}{dx^2} = M \tag{13-1}$$

Recall that this equation assumes that the slope of the elastic curve is small and that deflections occur only by bending. When the column is in its deflected position, Fig. 13–5*a*, the internal bending moment can be determined by using the method of sections. The free-body diagram of a segment in the deflected position is shown in Fig. 13–5*b*. Here both the deflection v and the internal moment M are shown in the *positive direction* according to the sign convention used to establish Eq. 13–1. Moment equilibrium requires $M = -Pv$. Thus Eq. 13–1 becomes

$$EI\frac{d^2v}{dx^2} = -Pv$$

$$\frac{d^2v}{dx^2} + \left(\frac{P}{EI}\right)v = 0 \tag{13-2}$$

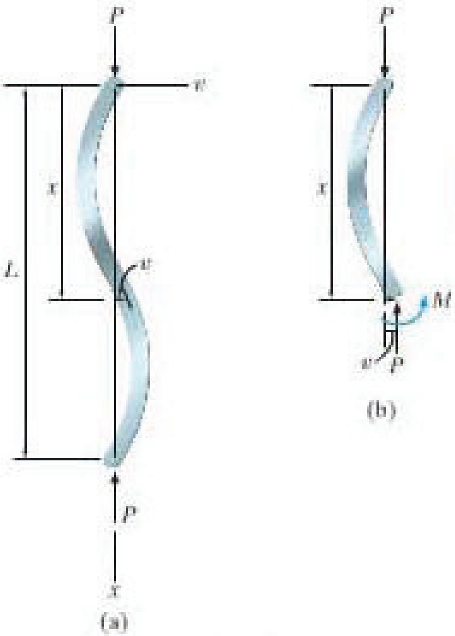

Fig. 13–5

This is a homogeneous, second-order, linear differential equation with constant coefficients. It can be shown by using the methods of differential equations, or by direct substitution into Eq. 13–2, that the general solution is

$$v = C_1 \sin\left(\sqrt{\frac{P}{EI}}x\right) + C_2 \cos\left(\sqrt{\frac{P}{EI}}x\right) \tag{13-3}$$

The two constants of integration are determined from the boundary conditions at the ends of the column. Since $v = 0$ at $x = 0$, then $C_2 = 0$. And since $v = 0$ at $x = L$, then

$$C_1 \sin\left(\sqrt{\frac{P}{EI}}L\right) = 0$$

This equation is satisfied if $C_1 = 0$; however, then $v = 0$, which is a *trivial solution* that requires the column to always remain straight, even though the load may cause the column to become unstable. The other possibility is for

$$\sin\left(\sqrt{\frac{P}{EI}}L\right) = 0$$

which is satisfied if

$$\sqrt{\frac{P}{EI}}L = n\pi$$

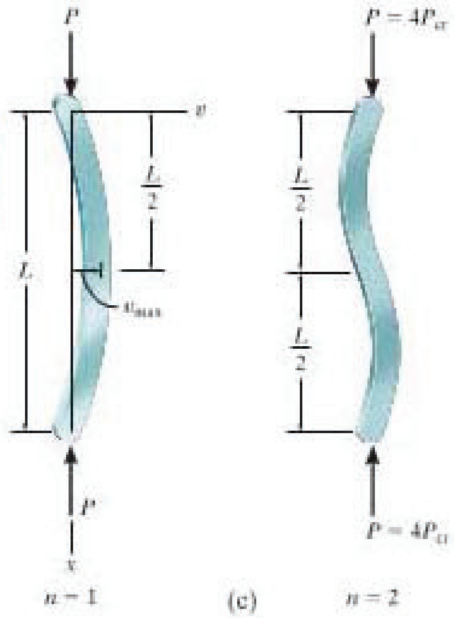

Fig. 13–5 (cont.)

or

$$P = \frac{n^2\pi^2 EI}{L^2} \quad n = 1, 2, 3, \ldots \tag{13–4}$$

The *smallest value* of P is obtained when $n = 1$, so the *critical load* for the column is therefore*

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

This load is sometimes referred to as the *Euler load*, named after the Swiss mathematician Leonhard Euler, who originally solved this problem in 1757. The corresponding buckled shape is defined by the equation

$$v = C_1 \sin\frac{\pi x}{L}$$

Here the constant C_1 represents the maximum deflection, v_{max}, which occurs at the midpoint of the column, Fig. 13–5*c*. Specific values for C_1 cannot be obtained, since the exact deflected form for the column is unknown once it has buckled. It has been assumed, however, that this deflection is small.

Note that the critical load is independent of the strength of the material; rather it only depends on the column's dimensions (I and L) and the material's stiffness or modulus of elasticity E. For this reason, as far as elastic buckling is concerned, columns made, for example, of high-strength steel offer no advantage over those made of lower-strength steel, since the modulus of elasticity for both is approximately the same. Also note that the load-carrying capacity of a column will increase as the moment of inertia of the cross section increases. Thus, efficient columns are designed so that most of the column's cross-sectional area is located as far away as possible from the principal centroidal axes for the section. This is why hollow sections such as tubes are more economical than solid sections. Furthermore, wide-flange sections, and columns that are "built up" from channels, angles, plates, etc., are better than sections that are solid and rectangular.

*n represents the number of waves in the deflected shape of the column. For example, if $n = 2$, then *two* waves will appear, Fig. 13–5*c*. Here the critical load is $4P_{cr}$ just prior to buckling, which practically speaking will not exist.

It is also important to realize that a column will buckle about the principal axis of the cross section having the ***least moment of inertia*** (the weakest axis). For example, a column having a rectangular cross section, like a meter stick, as shown in Fig. 13–6, will buckle about the a–a axis, not the b–b axis. As a result, engineers usually try to achieve a balance, keeping the moments of inertia the same in all directions. Geometrically, then, circular tubes would make excellent columns. Also, square tubes or those shapes having $I_x \approx I_y$ are often selected for columns.

Summarizing the above discussion, the buckling equation for a pin-supported long slender column can be rewritten, and the terms defined as follows:

$$P_{cr} = \frac{\pi^2 EI}{L^2} \qquad (13\text{–}5)$$

where

P_{cr} = critical or maximum axial load on the column just before it begins to buckle. This load must *not* cause the stress in the column to exceed the proportional limit

E = modulus of elasticity for the material

I = *least* moment of inertia for the column's cross-sectional area

L = unsupported length of the column, whose ends are pinned

Fig. 13–6

For purposes of design, the above equation can also be written in a more useful form by expressing $I = Ar^2$, where A is the cross-sectional area and r is the ***radius of gyration*** of the cross-sectional area. Thus,

$$P_{cr} = \frac{\pi^2 E(Ar^2)}{L^2}$$

$$\left(\frac{P}{A}\right)_{cr} = \frac{\pi^2 E}{(L/r)^2}$$

or

$$\sigma_{cr} = \frac{\pi^2 E}{(L/r)^2} \qquad (13\text{–}6)$$

Typical interior steel pipe columns used to support the roof of a single story building.

Here

σ_{cr} = critical stress, which is an average normal stress in the column just before the column buckles. This stress is an *elastic stress* and therefore $\sigma_{cr} \le \sigma_Y$

E = modulus of elasticity for the material

L = unsupported length of the column, whose ends are pinned

r = *smallest* radius of gyration of the column, determined from $r = \sqrt{I/A}$, where I is the *least* moment of inertia of the column's cross-sectional area A

The geometric ratio L/r in Eq. 13–6 is known as the ***slenderness ratio***. It is a measure of the column's flexibility, and as will be discussed later, it serves to classify columns as long, intermediate, or short.

It is possible to graph Eq. 13–6 using axes that represent the critical stress versus the slenderness ratio. Examples of this graph for columns made of a typical structural steel and aluminum alloy are shown in Fig. 13–7. Note that the curves are hyperbolic and are valid only for critical stresses below the material's yield point (proportional limit), since the material must behave elastically. For the steel the yield stress is $(\sigma_Y)_{st} = 36$ ksi $[E_{st} = 29(10^3)$ ksi$]$, and for the aluminum it is $(\sigma_Y)_{al} = 27$ ksi $[E_{al} = 10(10^3)$ ksi$]$. Substituting $\sigma_{cr} = \sigma_Y$ into Eq. 13–6, the *smallest* allowable slenderness ratios for the steel and aluminum columns are therefore $(L/r)_{st} = 89$ and $(L/r)_{al} = 60.5$, respectively. Thus, for a steel column, if $(L/r)_{st} \geq 89$, Euler's formula can be used to determine the critical load since the stress in the column remains elastic. On the other hand, if $(L/r)_{st} < 89$, the column's stress will exceed the yield point before buckling can occur, and therefore the Euler formula is not valid in this case.

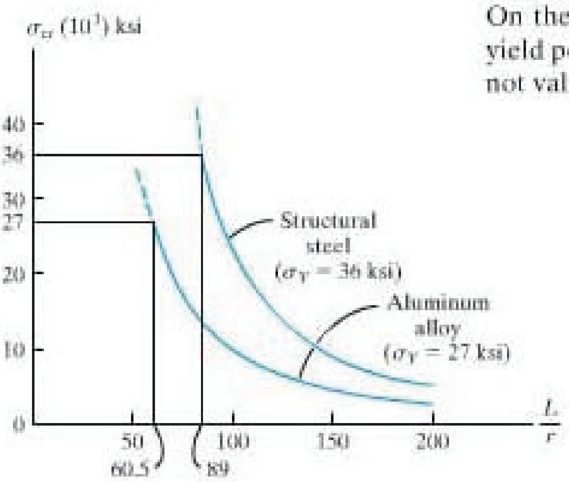

Fig. 13–7

Important Points

- *Columns* are long slender members that are subjected to axial compressive loads.
- The *critical load* is the maximum axial load that a column can support when it is on the verge of buckling. This loading represents a case of *neutral equilibrium*.
- An *ideal column* is initially perfectly straight, made of homogeneous material, and the load is applied through the centroid of the cross section.
- A pin-connected column will buckle about the principal axis of the cross section having the *least* moment of inertia.
- The *slenderness ratio* is L/r, where r is the smallest radius of gyration of the cross section. Buckling will occur about the axis where this ratio gives the greatest value.

EXAMPLE 13.1

The A-36 steel W8 × 31 member shown in Fig. 13–8 is to be used as a pin-connected column. Determine the largest axial load it can support before it either begins to buckle or the steel yields.

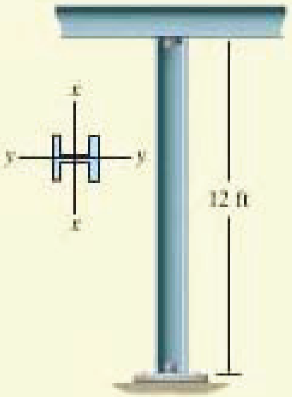

Fig. 13–8

SOLUTION

From the table in Appendix B, the column's cross-sectional area and moments of inertia are $A = 9.13 \text{ in}^2$, $I_x = 110 \text{ in}^4$, and $I_y = 37.1 \text{ in}^4$. By inspection, buckling will occur about the y–y axis. Why? Applying Eq. 13–5, we have

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2[29(10^3) \text{ kip/in}^2](37.1 \text{ in}^4)}{[12 \text{ ft}(12 \text{ in./ft})]^2} = 512 \text{ kip}$$

When fully loaded, the average compressive stress in the column is

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{512 \text{ kip}}{9.13 \text{ in}^2} = 56.1 \text{ ksi}$$

Since this stress exceeds the yield stress (36 ksi), the load P is determined from simple compression:

$$36 \text{ ksi} = \frac{P}{9.13 \text{ in}^2}; \qquad P = 329 \text{ kip} \qquad \textit{Ans.}$$

In actual practice, a factor of safety would be placed on this loading.

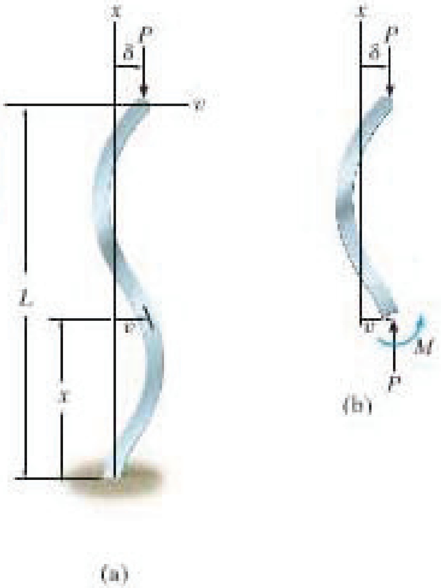

Fig. 13–9

13.3 Columns Having Various Types of Supports

The Euler load was derived for a column that is pin connected or free to rotate at its ends. Oftentimes, however, columns may be supported in some other way. For example, consider the case of a column fixed at its base and free at the top, Fig. 13–9*a*. As the column buckles the load displaces δ and at x the displacement is v. From the free-body diagram in Fig. 13–9*b*, the internal moment at the arbitrary section is $M = P(\delta - v)$. Consequently, the differential equation for the deflection curve is

$$EI\frac{d^2v}{dx^2} = P(\delta - v)$$

$$\frac{d^2v}{dx^2} + \frac{P}{EI}v = \frac{P}{EI}\delta \qquad (13\text{–}7)$$

Unlike Eq. 13–2, this equation is nonhomogeneous because of the nonzero term on the right side. The solution consists of both a complementary and a particular solution, namely,

$$v = C_1 \sin\left(\sqrt{\frac{P}{EI}}x\right) + C_2 \cos\left(\sqrt{\frac{P}{EI}}x\right) + \delta$$

The constants are determined from the boundary conditions. At $x = 0$, $v = 0$, so that $C_2 = -\delta$. Also,

$$\frac{dv}{dx} = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}x\right) - C_2\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}x\right)$$

At $x = 0$, $dv/dx = 0$, so that $C_1 = 0$. The deflection curve is therefore

$$v = \delta\left[1 - \cos\left(\sqrt{\frac{P}{EI}}x\right)\right] \qquad (13\text{–}8)$$

Since the deflection at the top of the column is δ, that is, at $x = L$, $v = \delta$, we require

$$\delta \cos\left(\sqrt{\frac{P}{EI}}L\right) = 0$$

The trivial solution $\delta = 0$ indicates that no buckling occurs, regardless of the load P. Instead,

$$\cos\left(\sqrt{\frac{P}{EI}}L\right) = 0 \quad \text{or} \quad \sqrt{\frac{P}{EI}}L = \frac{n\pi}{2},\ n = 1, 3, 5 \ldots$$

The smallest critical load occurs when $n = 1$, so that

$$P_{cr} = \frac{\pi^2 EI}{4L^2} \qquad (13\text{–}9)$$

By comparison with Eq. 13–5, it is seen that a column fixed-supported at its base and free at its top will support only one-fourth the critical load that can be applied to a column pin-supported at both ends.

The tubular columns used to support this water tank have been braced at three locations along their length to prevent them from buckling.

Other types of supported columns are analyzed in much the same way and will not be covered in detail here.* Instead, we will tabulate the results for the most common types of column support and show how to apply these results by writing Euler's formula in a general form.

Effective Length. As stated previously, the Euler formula, Eq. 13–5, was developed for the case of a column having ends that are pinned or free to rotate. In other words, L in the equation represents the unsupported distance between the points of zero moment. This formula can be used to determine the critical load on columns having other types of support provided "L" represents the distance between the zero-moment points. This distance is called the column's ***effective length***, L_e. Obviously, for a pin-ended column $L_e = L$, Fig. 13–10*a*. For the fixed- and free-ended column, the deflection curve, Eq. 13–8, was found to be one-half that of a column that is pin connected and has a length of $2L$, Fig. 13–10*b*. Thus the effective length between the points of zero moment is $L_e = 2L$. Examples for two other columns with different end supports are also shown in Fig. 13–10. The column fixed at its ends, Fig. 13–10*c*, has inflection points or points of zero moment $L/4$ from each support. The effective length is therefore represented by the middle half of its length, that is, $L_e = 0.5L$. Lastly, the pin- and fixed-ended column, Fig. 13–10*d*, has an inflection point at approximately $0.7L$ from its pinned end, so that $L_e = 0.7L$.

Rather than specifying the column's effective length, many design codes provide column formulas that employ a dimensionless coefficient K called the ***effective-length factor***. This factor is defined from

$$L_e = KL \tag{13–10}$$

Specific values of K are also given in Fig. 13–10. Based on this generality, we can therefore write Euler's formula as

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} \tag{13–11}$$

or

$$\sigma_{cr} = \frac{\pi^2 E}{(KL/r)^2} \tag{13–12}$$

Here (KL/r) is the column's ***effective-slenderness ratio***. For example, if the column is fixed at its base and free at its end, we have $K = 2$, and therefore Eq. 13–11 gives the same result as Eq. 13–9.

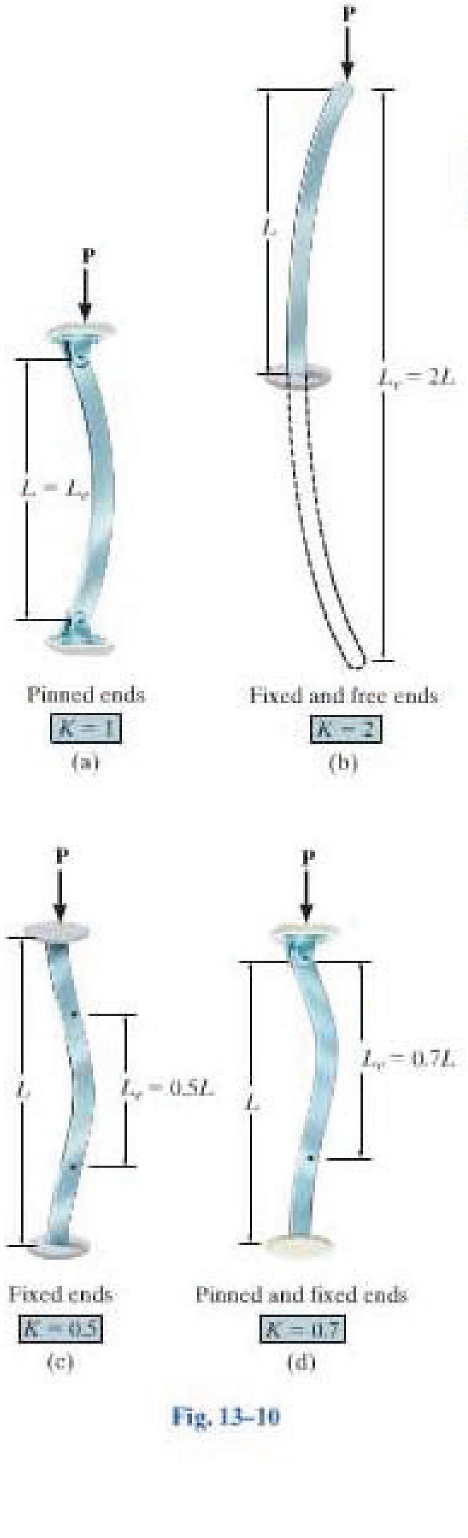

Fig. 13–10

*See Problems 13–43, 13–44, and 13–45.

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EXAMPLE 13.2

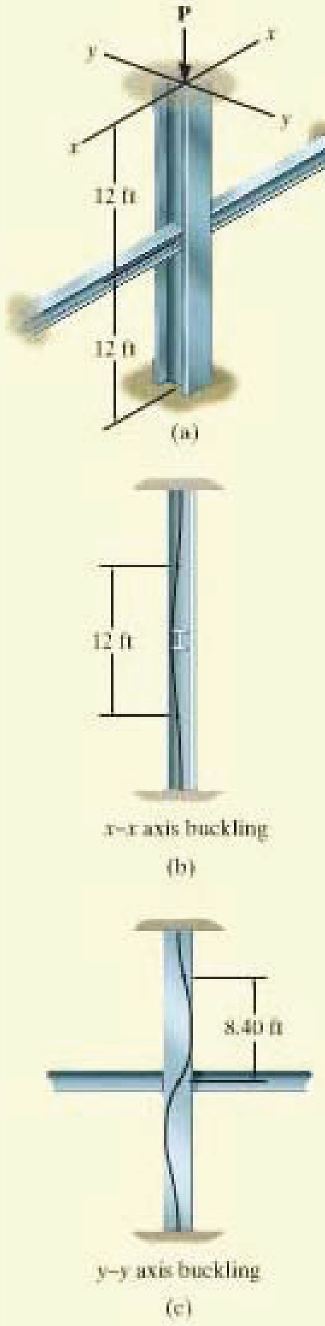

Fig. 13–11

A W6 × 15 steel column is 24 ft long and is fixed at its ends as shown in Fig. 13–11*a*. Its load-carrying capacity is increased by bracing it about the *y–y* (weak) axis using struts that are assumed to be pin connected to its midheight. Determine the load it can support so that the column does not buckle nor the material exceed the yield stress. Take $E_{st} = 29(10^3)$ ksi and $\sigma_Y = 60$ ksi.

SOLUTION

The buckling behavior of the column will be *different* about the *x–x* and *y–y* axes due to the bracing. The buckled shape for each of these cases is shown in Figs. 13–11*b* and 13–11*c*. From Fig. 13–11*b*, the effective length for buckling about the *x–x* axis is $(KL)_x = 0.5(24\text{ ft}) = 12\text{ ft} = 144$ in., and from Fig. 13–11*c*, for buckling about the *y–y* axis, $(KL)_y = 0.7(24\text{ ft}/2) = 8.40\text{ ft} = 100.8$ in. The moments of inertia for a W6 × 15 are found from the table in Appendix B. We have $I_x = 29.1\text{ in}^4$, $I_y = 9.32\text{ in}^4$.

Applying Eq. 13–11,

$$(P_{cr})_x = \frac{\pi^2 EI_x}{(KL)_x^2} = \frac{\pi^2[29(10^3)\text{ ksi}]29.1\text{ in}^4}{(144\text{ in.})^2} = 401.7\text{ kip} \quad (1)$$

$$(P_{cr})_y = \frac{\pi^2 EI_y}{(KL)_y^2} = \frac{\pi^2[29(10^3)\text{ ksi}]9.32\text{ in}^4}{(100.8\text{ in.})^2} = 262.5\text{ kip} \quad (2)$$

By comparison, buckling will occur about the *y–y* axis.

The area of the cross section is 4.43 in², so the average compressive stress in the column is

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{262.5\text{ kip}}{4.43\text{ in}^2} = 59.3\text{ ksi}$$

Since this stress is less than the yield stress, buckling will occur before the material yields. Thus,

$$P_{cr} = 263\text{ kip} \qquad \textit{Ans.}$$

NOTE: From Eq. 13–12 it can be seen that buckling will always occur about the column axis having the *largest* slenderness ratio, since a large slenderness ratio will give a small critical stress. Thus, using the data for the radius of gyration from the table in Appendix B, we have

$$\left(\frac{KL}{r}\right)_x = \frac{144\text{ in.}}{2.56\text{ in.}} = 56.2$$

$$\left(\frac{KL}{r}\right)_y = \frac{100.8\text{ in.}}{1.46\text{ in.}} = 69.0$$

Hence, *y–y* axis buckling will occur, which is the same conclusion reached by comparing Eqs. 1 and 2.

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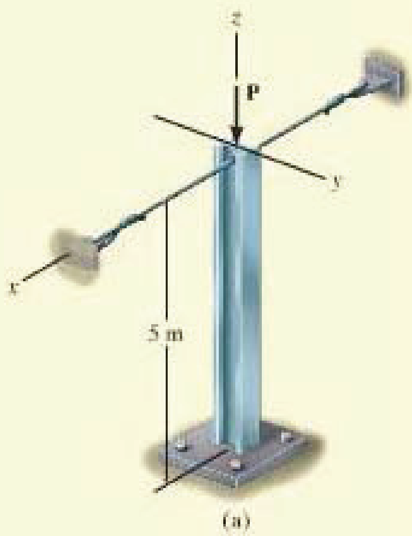
EXAMPLE 13.3

The aluminum column is fixed at its bottom and is braced at its top by cables so as to prevent movement at the top along the x axis, Fig. 13–12a. If it is assumed to be fixed at its base, determine the largest allowable load P that can be applied. Use a factor of safety for buckling of F.S. = 3.0. Take $E_{al} = 70$ GPa, $\sigma_Y = 215$ MPa, $A = 7.5(10^{-3})\text{ m}^2$, $I_x = 61.3(10^{-6})\text{ m}^4$, $I_y = 23.2(10^{-6})\text{ m}^4$.



(a)

Fig. 13–12

SOLUTION

Buckling about the x and y axes is shown in Fig. 13–12b and 13–12c, respectively. Using Fig. 13–10a, for x–x axis buckling, $K = 2$, so $(KL)_x = 2(5\text{ m}) = 10\text{ m}$. Also, for y–y axis buckling, $K = 0.7$, so $(KL)_y = 0.7(5\text{ m}) = 3.5\text{ m}$.

Applying Eq. 13–11, the critical loads for each case are

$$(P_{cr})_x = \frac{\pi^2 E I_x}{(KL)_x^2} = \frac{\pi^2[70(10^9)\text{ N/m}^2](61.3(10^{-6})\text{ m}^4)}{(10\text{ m})^2}$$
$$= 424\text{ kN}$$

$$(P_{cr})_y = \frac{\pi^2 E I_y}{(KL)_y^2} = \frac{\pi^2[70(10^9)\text{ N/m}^2](23.2(10^{-6})\text{ m}^4)}{(3.5\text{ m})^2}$$
$$= 1.31\text{ MN}$$

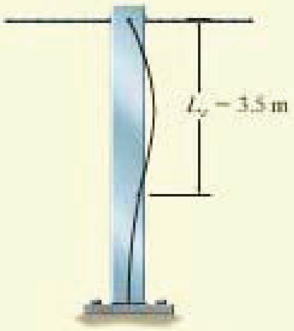
By comparison, as P is increased the column will buckle about the x–x axis. The allowable load is therefore

$$P_{\text{allow}} = \frac{P_{cr}}{\text{F.S.}} = \frac{424\text{ kN}}{3.0} = 141\text{ kN} \qquad \textit{Ans.}$$

Since

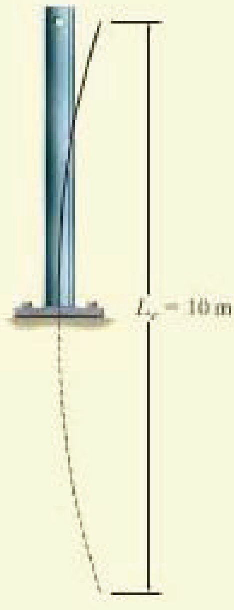
$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{424\text{ kN}}{7.5(10^{-3})\text{ m}^2} = 56.5\text{ MPa} < 215\text{ MPa}$$

Euler's equation can be applied.



y–y axis buckling

(c)



x–x axis buckling

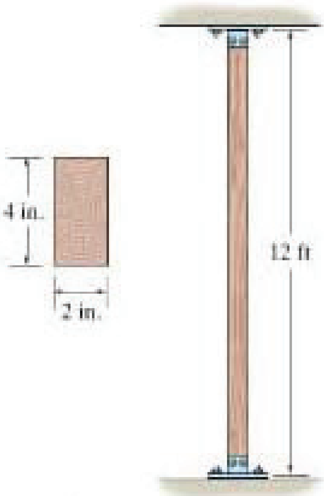
(b)

FUNDAMENTAL PROBLEMS

13

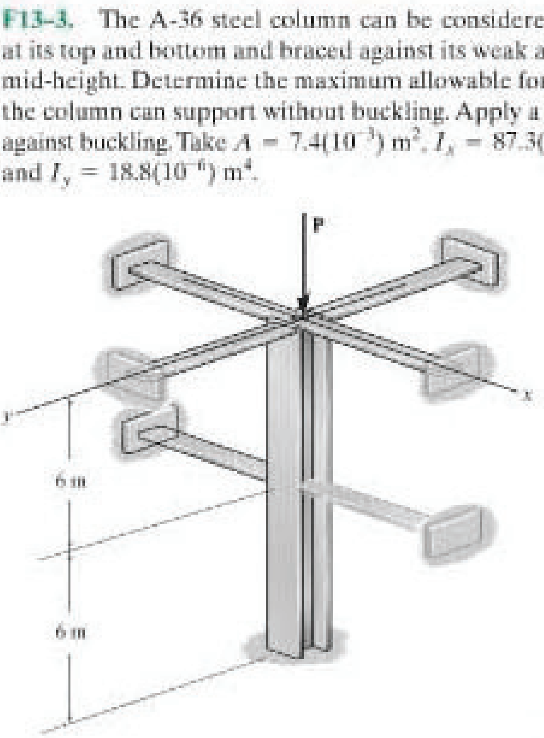
F13–1. A 50-in.-long rod is made from a 1-in.-diameter steel rod. Determine the critical buckling load if the ends are fixed supported. $E = 29(10^3)$ ksi, $\sigma_Y = 36$ ksi.

F13–2. A 12-ft wooden rectangular column has the dimensions shown. Determine the critical load if the ends are assumed to be pin-connected. $E = 1.6(10^3)$ ksi. Yielding does not occur.



F13–2

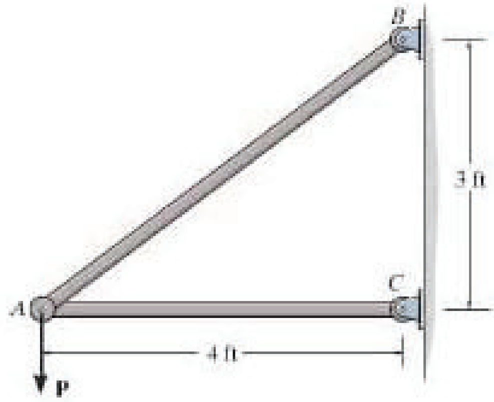
F13–3. The A-36 steel column can be considered pinned at its top and bottom and braced against its weak axis at the mid-height. Determine the maximum allowable force **P** that the column can support without buckling. Apply a F.S. = 2 against buckling. Take $A = 7.4(10^{-3})\ \text{m}^2$, $I_x = 87.3(10^{-6})\ \text{m}^4$, and $I_y = 18.8(10^{-6})\ \text{m}^4$.



F13–3

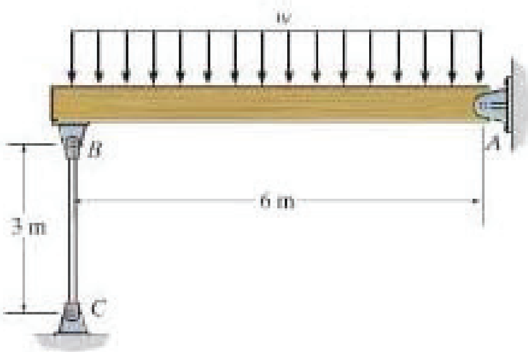
F13–4. A steel pipe is fixed supported at its ends. If it is 5 m long and has an outer diameter of 50 mm and a thickness of 10 mm, determine the maximum axial load P that it can carry without buckling. $E_{st} = 200$ GPa, $\sigma_Y = 250$ MPa.

F13–5. Determine the maximum force **P** that can be supported by the assembly without causing member AC to buckle. The member is made of A-36 steel and has a diameter of 2 in. Take F.S. = 2 against buckling.



F13–5

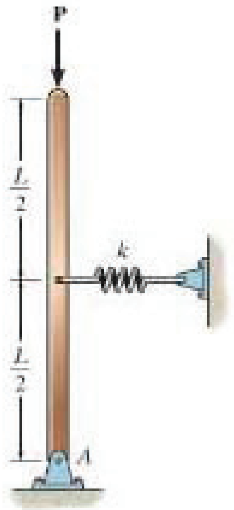
F13–6. The A-36 steel rod BC has a diameter of 50 mm and is used as a strut to support the beam. Determine the maximum intensity w of the uniform distributed load that can be applied to the beam without causing the strut to buckle. Take F.S. = 2 against buckling.



F13–6

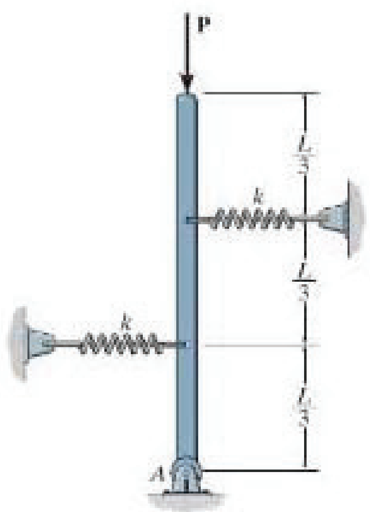
PROBLEMS

•13–1. Determine the critical buckling load for the column. The material can be assumed rigid.



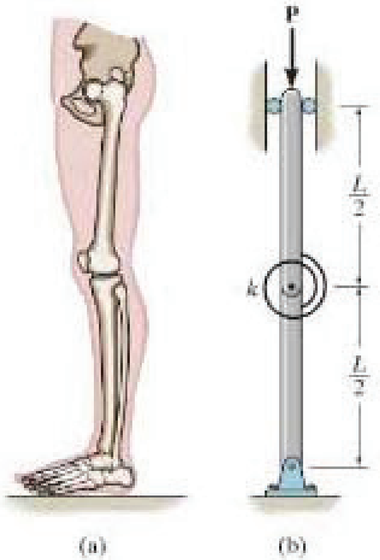
Prob. 13–1

13–2. Determine the critical load P_{cr} for the rigid bar and spring system. Each spring has a stiffness k.



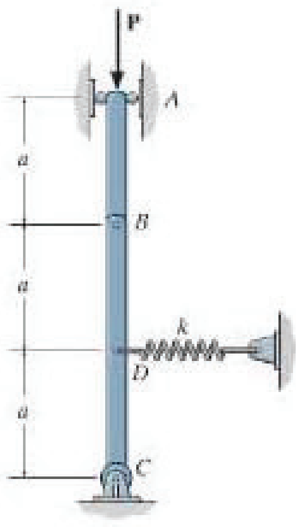
Prob. 13–2

13–3. The leg in (a) acts as a column and can be modeled (b) by the two pin-connected members that are attached to a torsional spring having a stiffness k (torque/rad). Determine the critical buckling load. Assume the bone material is rigid.



Prob. 13–3

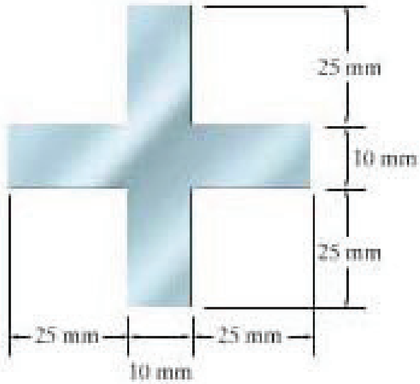
*13–4. Rigid bars AB and BC are pin connected at B. If the spring at D has a stiffness k, determine the critical load P_{cr} for the system.



Prob. 13–4

•**13–5.** An A-36 steel column has a length of 4 m and is pinned at both ends. If the cross sectional area has the dimensions shown, determine the critical load.

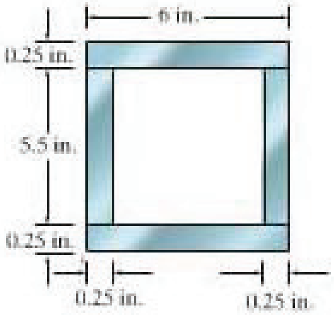
13–6. Solve Prob. 13–5 if the column is fixed at its bottom and pinned at its top.



Probs. 13–5/6

13–7. A column is made of A-36 steel, has a length of 20 ft, and is pinned at both ends. If the cross-sectional area has the dimensions shown, determine the critical load.

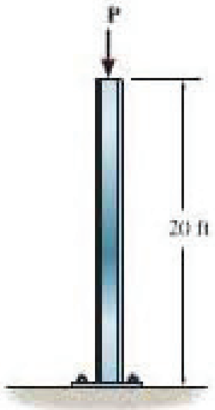
*__13–8.__ A column is made of 2014-T6 aluminum, has a length of 30 ft, and is fixed at its bottom and pinned at its top. If the cross-sectional area has the dimensions shown, determine the critical load.



Probs. 13–7/8

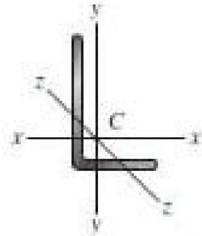
•**13–9.** The W14 × 38 column is made of A-36 steel and is fixed supported at its base. If it is subjected to an axial load of $P = 15$ kip, determine the factor of safety with respect to buckling.

13–10. The W14 × 38 column is made of A-36 steel. Determine the critical load if its bottom end is fixed supported and its top is free to move about the strong axis and is pinned about the weak axis.



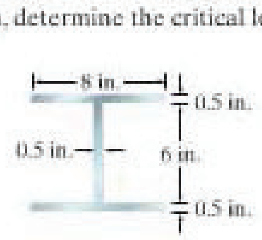
Probs. 13–9/10

13–11. The A-36 steel angle has a cross-sectional area of $A = 2.48\text{ in}^2$ and a radius of gyration about the x axis of $r_x = 1.26$ in. and about the y axis of $r_y = 0.879$ in. The smallest radius of gyration occurs about the z axis and is $r_z = 0.644$ in. If the angle is to be used as a pin-connected 10-ft-long column, determine the largest axial load that can be applied through its centroid C without causing it to buckle.



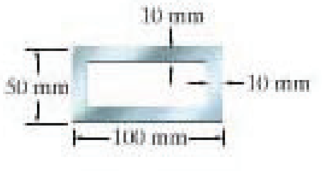
Prob. 13–11

*__13–12.__ An A-36 steel column has a length of 15 ft and is pinned at both ends. If the cross-sectional area has the dimensions shown, determine the critical load.



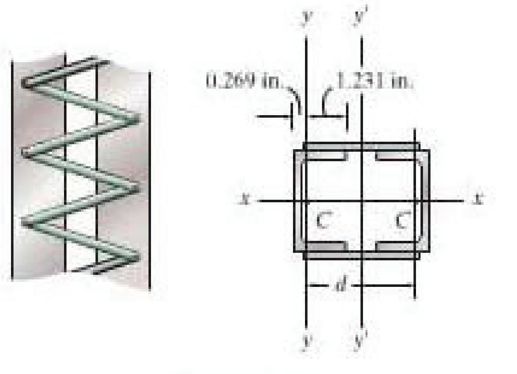
Prob. 13–12

•**13–13.** An A-36 steel column has a length of 5 m and is fixed at both ends. If the cross-sectional area has the dimensions shown, determine the critical load.



Prob. 13–13

13–14. The two steel channels are to be laced together to form a 30-ft-long bridge column assumed to be pin connected at its ends. Each channel has a cross-sectional area of $A = 3.10\ \text{in}^2$ and moments of inertia $I_x = 55.4\ \text{in}^4$, $I_y = 0.382\ \text{in}^4$. The centroid C of its area is located in the figure. Determine the proper distance d between the centroids of the channels so that buckling occurs about the x–x and $y'-y'$ axes due to the same load. What is the value of this critical load? Neglect the effect of the lacing. $E_{st} = 29(10^3)$ ksi, $\sigma_Y = 50$ ksi.



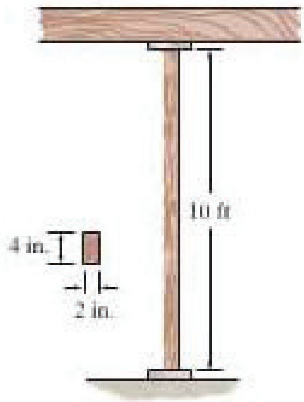
Prob. 13–14

13–15. An A-36-steel W8 × 24 column is fixed at one end and free at its other end. If it is subjected to an axial load of 20 kip, determine the maximum allowable length of the column if F.S. = 2 against buckling is desired.

**13–16.* An A-36-steel W8 × 24 column is fixed at one end and pinned at the other end. If it is subjected to an axial load of 60 kip, determine the maximum allowable length of the column if F.S. = 2 against buckling is desired.

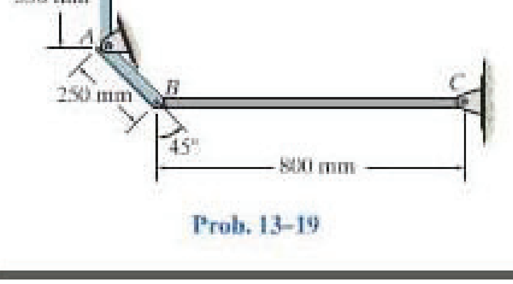
•**13–17.** The 10-ft wooden rectangular column has the dimensions shown. Determine the critical load if the ends are assumed to be pin connected. $E_w = 1.6(10^3)$ ksi, $\sigma_Y = 5$ ksi.

13–18. The 10-ft column has the dimensions shown. Determine the critical load if the bottom is fixed and the top is pinned. $E_w = 1.6(10^3)$ ksi, $\sigma_Y = 5$ ksi.



Probs. 13–17/18

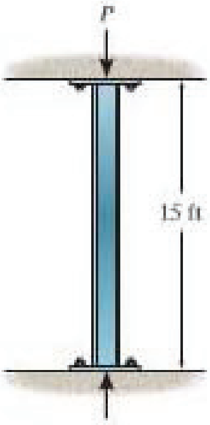
13–19. Determine the maximum force P that can be applied to the handle so that the A-36 steel control rod BC does not buckle. The rod has a diameter of 25 mm.



Prob. 13–19

*13–20. The W10 × 45 is made of A-36 steel and is used as a column that has a length of 15 ft. If its ends are assumed pin supported, and it is subjected to an axial load of 100 kip, determine the factor of safety with respect to buckling.

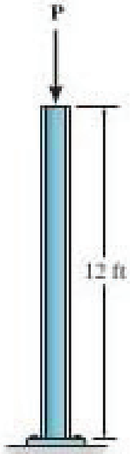
•13–21. The W10 × 45 is made of A-36 steel and is used as a column that has a length of 15 ft. If the ends of the column are fixed supported, can the column support the critical load without yielding?



Probs. 13–20/21

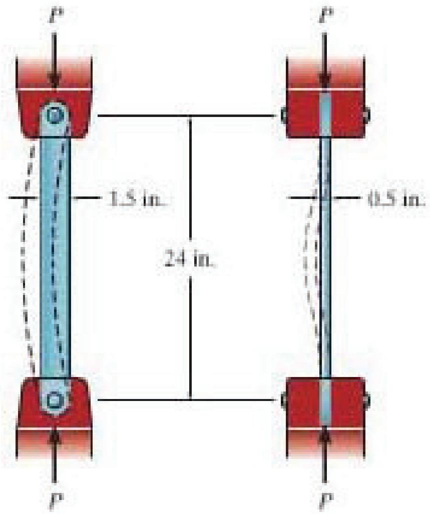
13–22. The W12 × 87 structural A-36 steel column has a length of 12 ft. If its bottom end is fixed supported while its top is free, and it is subjected to an axial load of $P = 380$ kip, determine the factor of safety with respect to buckling.

13–23. The W12 × 87 structural A-36 steel column has a length of 12 ft. If its bottom end is fixed supported while its top is free, determine the largest axial load it can support. Use a factor of safety with respect to buckling of 1.75.



Probs. 13–22/23

*13–24. An L-2 tool steel link in a forging machine is pin connected to the forks at its ends as shown. Determine the maximum load P it can carry without buckling. Use a factor of safety with respect to buckling of F.S. = 1.75. Note from the figure on the left that the ends are pinned for buckling, whereas from the figure on the right the ends are fixed.

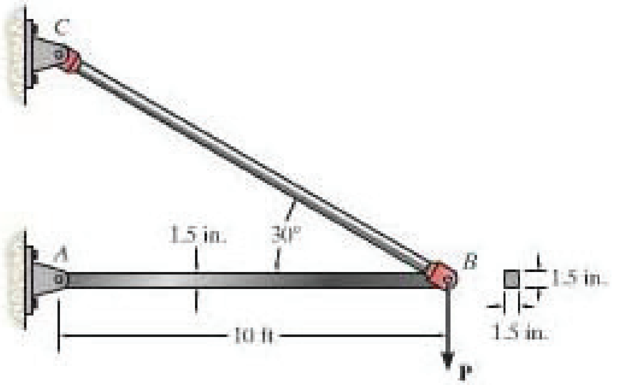


Prob. 13–24

•13–25. The W14 × 30 is used as a structural A-36 steel column that can be assumed pinned at both of its ends. Determine the largest axial force P that can be applied without causing it to buckle.

Prob. 13–25

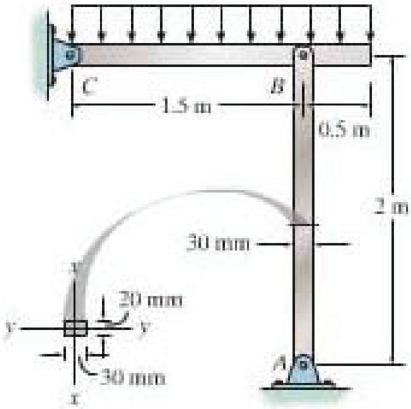
13–26. The A-36 steel bar AB has a square cross section. If it is pin connected at its ends, determine the maximum allowable load P that can be applied to the frame. Use a factor of safety with respect to buckling of 2.



Prob. 13–26

13–27. Determine the maximum allowable intensity w of the distributed load that can be applied to member BC without causing member AB to buckle. Assume that AB is made of steel and is pinned at its ends for x–x axis buckling and fixed at its ends for y–y axis buckling. Use a factor of safety with respect to buckling of 3. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.

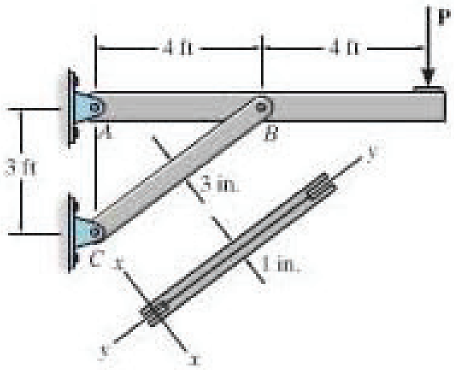
***13–28.** Determine if the frame can support a load of $w = 6$ kN/m if the factor of safety with respect to buckling of member AB is 3. Assume that AB is made of steel and is pinned at its ends for x–x axis buckling and fixed at its ends for y–y axis buckling. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.



Probs. 13–27/28

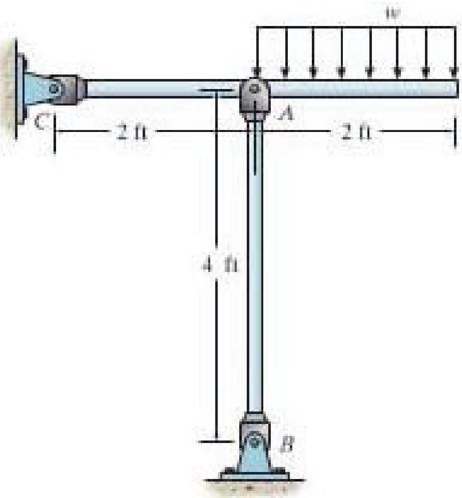
•13–29. The beam supports the load of $P = 6$ kip. As a result, the A-36 steel member BC is subjected to a compressive load. Due to the forked ends on the member, consider the supports at B and C to act as pins for x–x axis buckling and as fixed supports for y–y axis buckling. Determine the factor of safety with respect to buckling about each of these axes.

13–30. Determine the greatest load P the frame will support without causing the A-36 steel member BC to buckle. Due to the forked ends on the member, consider the supports at B and C to act as pins for x–x axis buckling and as fixed supports for y–y axis buckling.



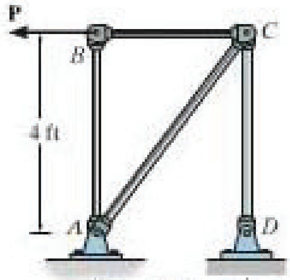
Probs. 13–29/30

13–31. Determine the maximum distributed load that can be applied to the bar so that the A-36 steel strut AB does not buckle. The strut has a diameter of 2 in. It is pin connected at its ends.



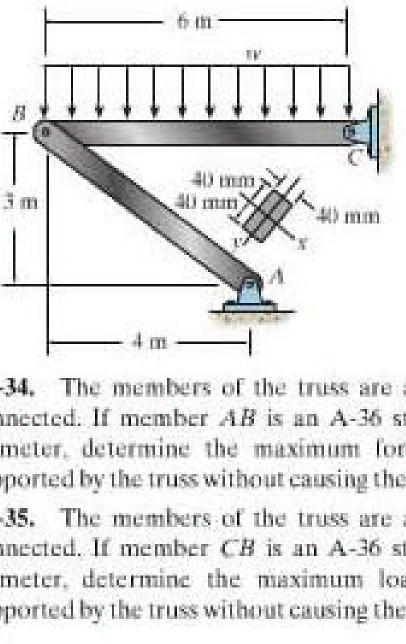
Prob. 13–31

*13–32. The members of the truss are assumed to be pin connected. If member AC is an A-36 steel rod of 2 in. diameter, determine the maximum load P that can be supported by the truss without causing the member to buckle.



Prob. 13–32

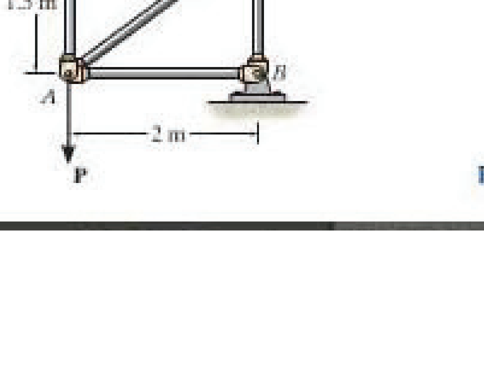
•13–33. The steel bar AB of the frame is assumed to be pin connected at its ends for y–y axis buckling. If $w = 3$ kN/m, determine the factor of safety with respect to buckling about the y–y axis due to the applied loading. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.



Prob. 13–33

13–34. The members of the truss are assumed to be pin connected. If member AB is an A-36 steel rod of 40 mm diameter, determine the maximum force P that can be supported by the truss without causing the member to buckle.

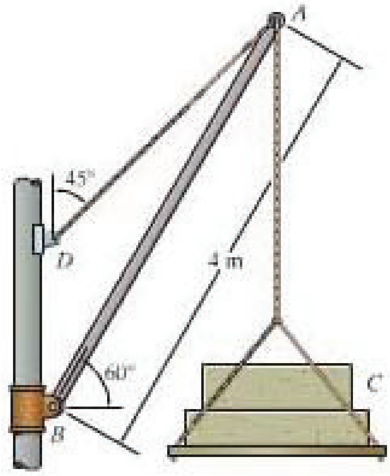
13–35. The members of the truss are assumed to be pin connected. If member CB is an A-36 steel rod of 40 mm diameter, determine the maximum load P that can be supported by the truss without causing the member to buckle.



Probs. 13–34/35

*13–36. If load C has a mass of 500 kg, determine the required minimum diameter of the solid L2-steel rod AB to the nearest mm so that it will not buckle. Use F.S. = 2 against buckling.

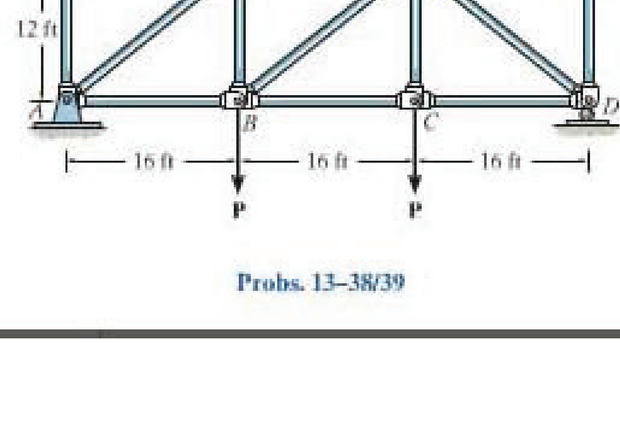
•13–37. If the diameter of the solid L2-steel rod AB is 50 mm, determine the maximum mass C that the rod can support without buckling. Use F.S. = 2 against buckling.



Probs. 13–36/37

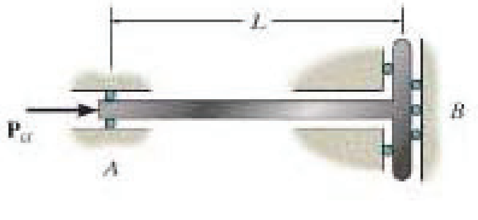
13–38. The members of the truss are assumed to be pin connected. If member GF is an A-36 steel rod having a diameter of 2 in., determine the greatest magnitude of load **P** that can be supported by the truss without causing this member to buckle.

13–39. The members of the truss are assumed to be pin connected. If member AG is an A-36 steel rod having a diameter of 2 in., determine the greatest magnitude of load **P** that can be supported by the truss without causing this member to buckle.



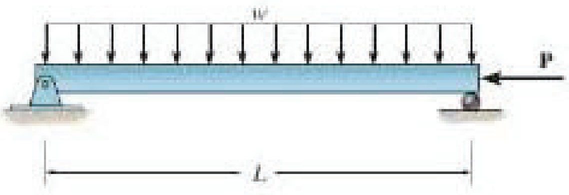
Probs. 13–38/39

*13–40. The column is supported at B by a support that does not permit rotation but allows vertical deflection. Determine the critical load P_{cr}. EI is constant.



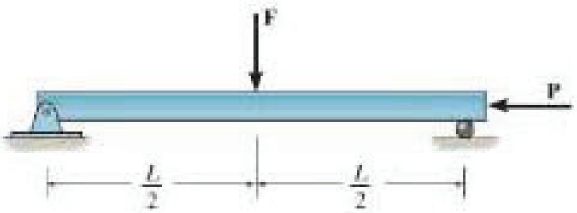
Prob. 13–40

•13–41. The ideal column has a weight w (force/length) and rests in the horizontal position when it is subjected to the axial load **P**. Determine the maximum moment in the column at midspan. EI is constant. *Hint:* Establish the differential equation for deflection, Eq. 13–1, with the origin at the mid span. The general solution is $v = C_1 \sin kx + C_2 \cos kx + (w/(2P))x^2 - (wL/(2P))x - (wEI/P^2)$ where $k^2 = P/EI$.



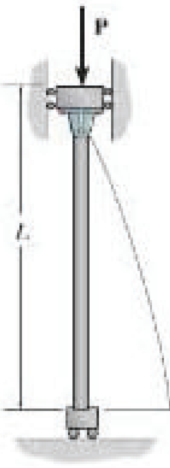
Prob. 13–41

13–42. The ideal column is subjected to the force **F** at its midpoint and the axial load **P**. Determine the maximum moment in the column at midspan. EI is constant. *Hint:* Establish the differential equation for deflection, Eq. 13–1. The general solution is $v = C_1 \sin kx + C_2 \cos kx - c^2x/k^2$, where $c^2 = F/2EI$, $k^2 = P/EI$.



Prob. 13–42

13–43. The column with constant EI has the end constraints shown. Determine the critical load for the column.



Prob. 13–43

*13–44. Consider an ideal column as in Fig. 13–10*c*, having both ends fixed. Show that the critical load on the column is given by $P_{cr} = 4\pi^2 EI/L^2$. *Hint:* Due to the vertical deflection of the top of the column, a constant moment **M′** will be developed at the supports. Show that $d^2v/dx^2 + (P/EI)v = M'/EI$. The solution is of the form $v = C_1 \sin(\sqrt{P/EI}x) + C_2 \cos(\sqrt{P/EI}x) + M'/P$.

•13–45. Consider an ideal column as in Fig. 13–10*d*, having one end fixed and the other pinned. Show that the critical load on the column is given by $P_{cr} = 20.19EI/L^2$. *Hint:* Due to the vertical deflection at the top of the column, a constant moment **M′** will be developed at the fixed support and horizontal reactive forces **R′** will be developed at both supports. Show that $d^2v/dx^2 + (P/EI)v = (R'/EI)(L - x)$. The solution is of the form $v = C_1 \sin(\sqrt{P/EI}x) + C_2 \cos(\sqrt{P/EI}x) + (R'/P)(L - x)$. After application of the boundary conditions show that $\tan(\sqrt{P/EI}L) = \sqrt{P/EI}\,L$. Solve by trial and error for the smallest nonzero root.

13

The column supporting this crane is unusually long. It will be subjected not only to uniaxial load, but also a bending moment. To ensure it will not buckle, it should be braced at the roof as a pin connection.

*13.4 The Secant Formula

The Euler formula was derived assuming the load P is always applied through the centroid of the column's cross-sectional area and that the column is perfectly straight. This is actually quite unrealistic, since manufactured columns are never perfectly straight, nor is the application of the load known with great accuracy. In reality, then, columns never suddenly buckle; instead they begin to bend, although ever so slightly, immediately upon application of the load. As a result, the actual criterion for load application should be limited either to a specified deflection of the column or by not allowing the maximum stress in the column to exceed an allowable stress.

To study this effect, we will apply the load P to the column at a short *eccentric distance* e from its centroid, Fig. 13–13a. This loading on the column is statically equivalent to the axial load P and bending moment $M' = Pe$ shown in Fig. 13–13b. As shown, in both cases, the ends A and B are supported so that they are free to rotate (pin supported). As before, we will only consider small slopes and deflections and linear-elastic material behavior. Furthermore, the x–v plane is a plane of symmetry for the cross-sectional area.

From the free-body diagram of the arbitrary section, Fig. 13–13c, the internal moment in the column is

$$M = -P(e + v) \tag{13–13}$$

The differential equation for the deflection curve is therefore

$$EI\frac{d^2v}{dx^2} = -P(e + v)$$

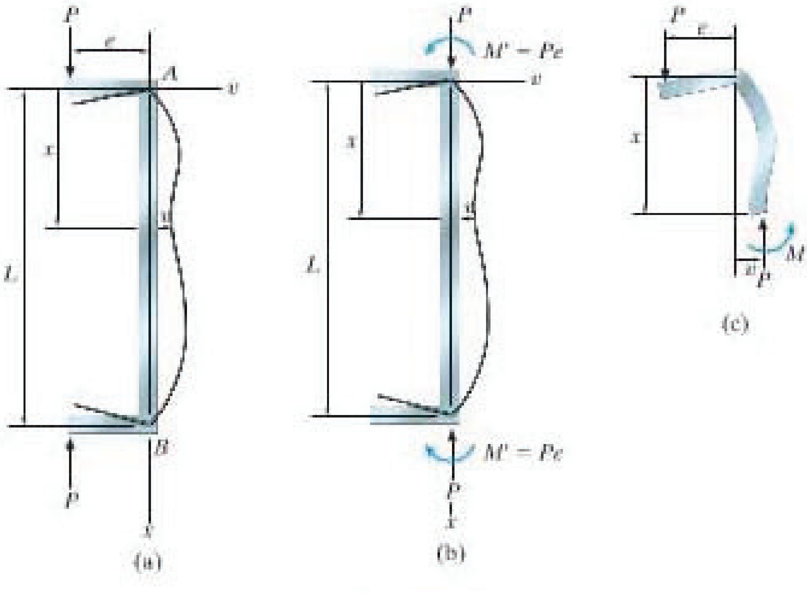


Fig. 13–13

or

$$\frac{d^2v}{dx^2} + \frac{P}{EI}v = -\frac{P}{EI}e$$

This equation is similar to Eq. 13–7 and has a general solution consisting of the complementary and particular solutions, namely,

$$v = C_1 \sin\sqrt{\frac{P}{EI}}x + C_2 \cos\sqrt{\frac{P}{EI}}x - e \qquad (13\text{–}14)$$

To evaluate the constants we must apply the boundary conditions. At $x = 0$, $v = 0$, so $C_2 = e$. And at $x = L$, $v = 0$, which gives

$$C_1 = \frac{e[1 - \cos(\sqrt{P/EI}\,L)]}{\sin(\sqrt{P/EI}\,L)}$$

Since $1 - \cos(\sqrt{P/EI}\,L) = 2\sin^2(\sqrt{P/EI}\,L/2)$ and $\sin(\sqrt{P/EI}\,L) = 2\sin(\sqrt{P/EI}\,L/2)\cos(\sqrt{P/EI}\,L/2)$, we have

$$C_1 = e\tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)$$

Hence, the deflection curve, Eq. 13–14, can be written as

$$v = e\left[\tan\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\sin\left(\sqrt{\frac{P}{EI}}x\right) + \cos\left(\sqrt{\frac{P}{EI}}x\right) - 1\right] \qquad (13\text{–}15)$$

Maximum Deflection. Due to symmetry of loading, both the maximum deflection and maximum stress occur at the column's midpoint. Therefore, when $x = L/2$, $v = v_{max}$, so

$$v_{max} = e\left[\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) - 1\right] \qquad (13\text{–}16)$$

Notice that if e approaches zero, then v_{max} approaches zero. However, if the terms in the brackets approach infinity as e approaches zero, then v_{max} will have a nonzero value. Mathematically, this would represent the behavior of an axially loaded column at failure when subjected to the critical load P_{cr}. Therefore, to find P_{cr} we require

$$\sec\left(\sqrt{\frac{P_{cr}}{EI}}\frac{L}{2}\right) = \infty$$

$$\sqrt{\frac{P_{cr}}{EI}}\frac{L}{2} = \frac{\pi}{2}$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} \qquad (13\text{–}17)$$

which is the same result found from the Euler formula, Eq. 13–5.

If Eq. 13–16 is plotted as load P versus deflection v_{max} for various values of eccentricity e, the family of colored curves shown in Fig. 13–14 results. Here the critical load becomes an asymptote to the curves, and of

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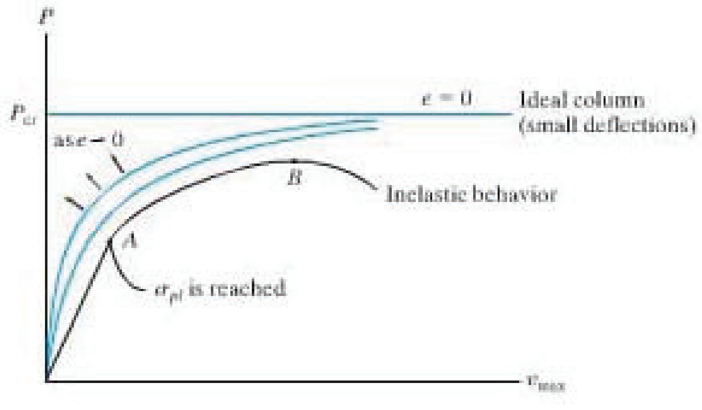


Fig. 13–14

course represents the unrealistic case of an ideal column ($e = 0$). As stated earlier, e is *never zero* due to imperfections in initial column straightness and load application; however, as $e \to 0$, the curves tend to approach the ideal case. Furthermore, these curves are appropriate only for *small deflections*, since the curvature was approximated by d^2v/dx^2 when Eq. 13–16 was developed. Had a more exact analysis been performed, all these curves would tend to turn upward, intersecting and then rising above the line $P = P_{cr}$. This, of course, indicates that a larger load P is needed to create larger column deflections. We have not considered this analysis here, however, since most often engineering design restricts the deflection of columns to small values.

It should also be noted that the colored curves in Fig. 13–14 apply only for linear-elastic material behavior. Such is the case if the column is long and slender. However, if a short or intermediate-length stocky column is considered, then the applied load, as it is increased, may eventually cause the material to yield, and the column will begin to behave in an *inelastic manner*. This occurs at point A for the black curve in Fig. 13–14. As the load is further increased, the curve never reaches the critical load, and instead the load reaches a maximum value at B. Afterwards, a sudden decrease in load-carrying capacity occurs as the column continues to yield and deflect by larger amounts.

Lastly, the colored curves in Fig. 13–14 also illustrate that a *nonlinear* relationship occurs between the load P and the deflection v. As a result, the principle of superposition *cannot be used* to determine the total deflection of a column caused by applying *successive loads* to the column. Instead, the loads must first be added, and then the corresponding deflection due to their resultant can be determined. Physically, the reason that successive loads and deflections cannot be superimposed is that the column's internal moment *depends* on *both* the *load P* and the *deflection v*, that is, $M = -P(e + v)$, Eq. 13–13.

The Secant Formula. The maximum stress in the column can be determined by realizing that it is caused by both the axial load and the moment, Fig. 13–15*a*. Maximum moment occurs at the column's midpoint, and using Eqs. 13–13 and 13–16, it has a magnitude of

$$M = |P(e + v_{\max})| \qquad M = Pe\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) \tag{13–18}$$

As shown in Fig. 13–15*b*, the maximum stress in the column is compressive, and it has a value of

$$\sigma_{\max} = \frac{P}{A} + \frac{Mc}{I}; \qquad \sigma_{\max} = \frac{P}{A} + \frac{Pec}{I}\sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)$$

Since the radius of gyration is defined as $r^2 = I/A$, the above equation can be written in a form called the *secant formula*:

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L}{2r}\sqrt{\frac{P}{EA}}\right)\right] \tag{13–19}$$

Here

$\sigma_{\max}$ = maximum *elastic stress* in the column, which occurs at the inner concave side at the column's midpoint. This stress is compressive

P = vertical load applied to the column. $P < P_{cr}$ unless $e = 0$; then $P = P_{cr}$ (Eq. 13–5)

e = eccentricity of the load P, measured from the centroidal axis of the column's cross-sectional area to the line of action of P

c = distance from the centroidal axis to the outer fiber of the column where the maximum compressive stress $\sigma_{\max}$ occurs

A = cross-sectional area of the column

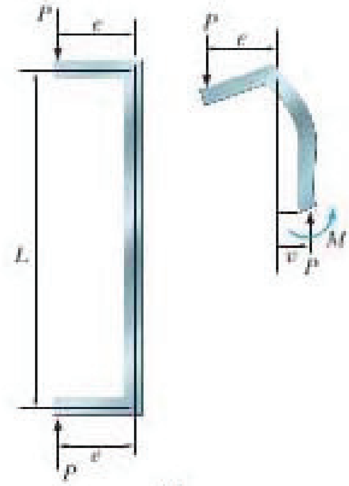
L = unsupported length of the column *in the plane of bending*. For supports other than pins, the effective length $L_e = KL$ should be used. See Fig. 13–10

E = modulus of elasticity for the material

r = radius of gyration, $r = \sqrt{I/A}$, where I is calculated about the centroidal or bending axis

Like Eq. 13–16, Eq. 13–19 indicates that there is a nonlinear relationship between the load and the stress. Hence, the principle of superposition does not apply, and therefore the loads have to be added *before* the stress is determined. Furthermore, due to this nonlinear relationship, any factor of safety used for design purposes applies to the load and not to the stress.

For a given value of $\sigma_{\max}$, graphs of Eq. 13–19 can be plotted as the slenderness ratio KL/r versus the average stress P/A for various values of the *eccentricity ratio* ec/r^2. A specific set of graphs for a structural-grade A-36 steel having a yield point of $\sigma_{\max} = \sigma_Y = 36$ ksi



(a)

(b)

Fig. 13–15

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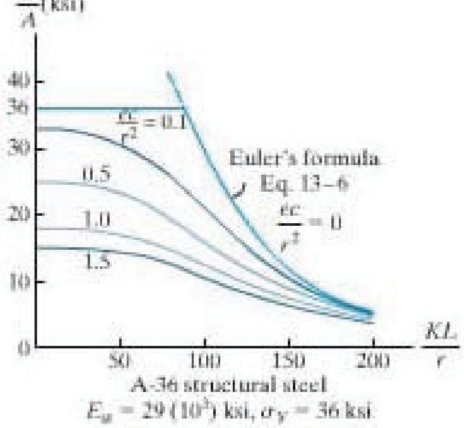


Fig. 13–16

and a modulus of elasticity of $E_{st} = 29(10^3)$ ksi is shown in Fig. 13–16. Note that when $e \rightarrow 0$, or when $ec/r^2 \rightarrow 0$, Eq. 13–19 gives $\sigma_{max} = P/A$, where P is the critical load on the column, defined by Euler's formula. This results in Eq. 13–6, which has been plotted in Fig. 13–7 and repeated in Fig. 13–16. Since both Eqs. 13–6 and 13–19 are valid only for elastic loadings, the stresses shown in Fig. 13–16 cannot exceed $\sigma_Y = 36$ ksi, represented here by the horizontal line.

The curves in Fig. 13–16 indicate that differences in the eccentricity ratio have a marked effect on the load-carrying capacity of columns that have small slenderness ratios. However, columns that have large slenderness ratios tend to fail at or near the Euler critical load regardless of the eccentricity ratio. When using Eq. 13–19 for design purposes, it is therefore important to have a somewhat accurate value for the eccentricity ratio for shorter-length columns.

Design. Once the eccentricity ratio has been determined, the column data can be substituted into Eq. 13–19. If a value of $\sigma_{max} = \sigma_Y$ is chosen, then the corresponding load P_Y can be determined from a trial-and-error procedure, since the equation is transcendental and cannot be solved explicitly for P_Y. As a design aid, computer software, or graphs such as those in Fig. 13–16, can also be used to determine P_Y directly.

Realize that P_Y is the load that will cause the column to develop a maximum compressive stress of σ_Y at its inner concave fibers. Due to the eccentric application of P_Y, this load will always be smaller than the critical load P_{cr}, which is determined from the Euler formula that assumes (unrealistically) that the column is axially loaded. Once P_Y is obtained, an appropriate factor of safety can then be applied in order to specify the column's safe load.

Important Points

- Due to imperfections in manufacturing or specific application of the load, a column will never suddenly buckle; instead, it begins to bend.
- The load applied to a column is related to its deflection in a nonlinear manner, and so the principle of superposition does not apply.
- As the slenderness ratio increases, eccentrically loaded columns tend to fail at or near the Euler buckling load.

EXAMPLE 13.4

The W8 × 40 A-36 steel column shown in Fig. 13–17*a* is fixed at its base and braced at the top so that it is fixed from displacement, yet free to rotate about the *y–y* axis. Also, it can sway to the side in the *y–z* plane. Determine the maximum eccentric load the column can support before it either begins to buckle or the steel yields.

SOLUTION

From the support conditions it is seen that about the *y–y* axis the column behaves as if it were pinned at its top and fixed at the bottom and subjected to an axial load *P*, Fig. 13–17*b*. About the *x–x* axis the column is free at the top and fixed at the bottom, and it is subjected to both an axial load *P* and moment $M = P(9\text{ in.})$, Fig. 13–17*c*.

y–y Axis Buckling. From Fig. 13–10*d* the effective length factor is $K_y = 0.7$, so $(KL)_y = 0.7(12)\text{ ft} = 8.40\text{ ft} = 100.8\text{ in.}$ Using the table in Appendix B to determine I_y for the W8 × 40 section and applying Eq. 13–11, we have

$$(P_{cr})_y = \frac{\pi^2 EI_y}{(KL)_y^2} = \frac{\pi^2[29(10^3)\text{ ksi}](49.1\text{ in}^4)}{(100.8\text{ in.})^2} = 1383\text{ kip}$$

x–x Axis Yielding. From Fig. 13–10*b*, $K_x = 2$, so $(KL)_x = 2(12)\text{ ft} = 24\text{ ft} = 288\text{ in.}$ Again using the table in Appendix B to determine $A = 11.7\text{ in}^2$, $c = 8.25\text{ in.}/2 = 4.125\text{ in.}$, and $r_x = 3.53\text{ in.}$, and applying the secant formula, we have

$$\sigma_Y = \frac{P_x}{A}\left[1 + \frac{ec}{r_x^2}\sec\left(\frac{(KL)_x}{2r_x}\sqrt{\frac{P_x}{EA}}\right)\right]$$

Substituting the data and simplifying yields

$$421.2 = P_x[1 + 2.979\sec(0.0700\sqrt{P_x})]$$

Solving for P_x by trial and error, noting that the argument for the secant is in radians, we get

$$P_x = 88.4\text{ kip} \qquad \textit{Ans.}$$

Since this value is less than $(P_{cr})_y = 1383$ kip, failure will occur about the *x–x* axis.

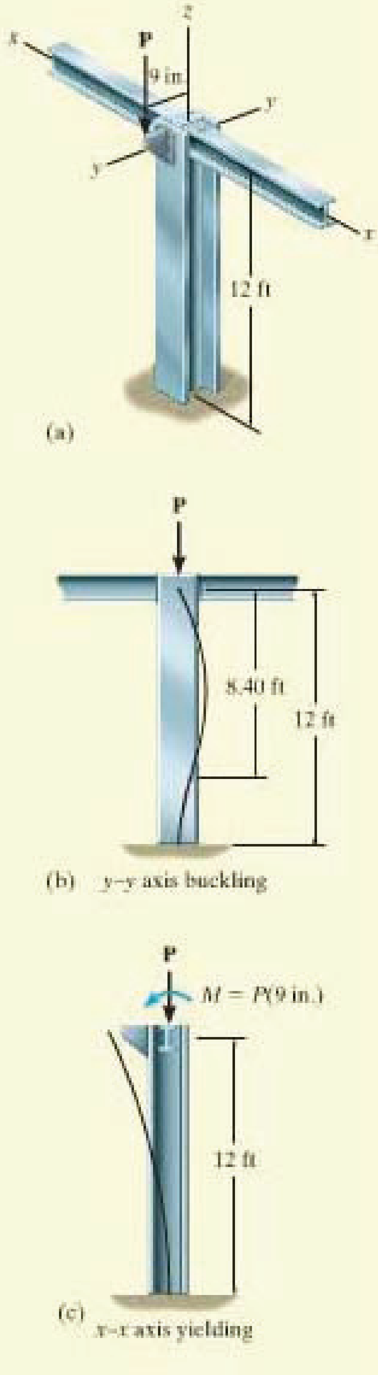
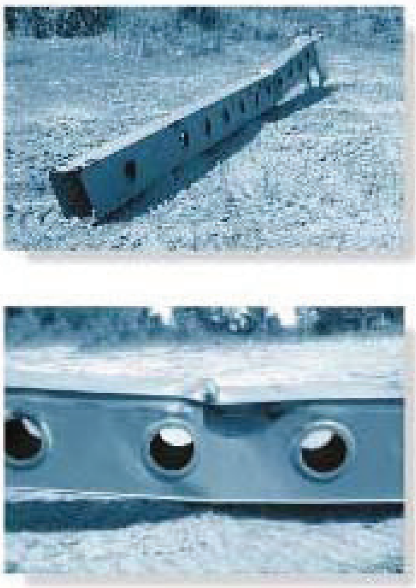


Fig. 13–17

*13.5 Inelastic Buckling



This crane boom failed by buckling caused by an overload. Note the region of localized collapse.

In engineering practice, columns are generally classified according to the type of stresses developed within the column at the time of failure. *Long slender columns* will become unstable when the compressive stress remains elastic. The failure that occurs is referred to as *elastic instability*. *Intermediate columns* fail due to *inelastic instability*, meaning that the compressive stress at failure is greater than the material's proportional limit. And *short columns*, sometimes called *posts*, do not become unstable; rather the material simply yields or fractures.

Application of the Euler equation requires that the stress in the column remain *below* the material's yield point (actually the proportional limit) when the column buckles, and so this equation applies only to long columns. In practice, however, most columns are selected to have intermediate lengths. The behavior of these columns can be studied by modifying the Euler equation so that it applies for inelastic buckling. To show how this can be done, consider the material to have a stress–strain diagram as shown in Fig. 13–18*a*. Here the proportional limit is σ_{pl}, and the modulus of elasticity, or slope of the line AB, is E.

If the column has a slenderness ratio that is *less* than $(KL/r)_{pl}$, then the critical stress in the column must be greater than σ_{pl}. For example, suppose a column has a slenderness ratio of $(KL/r)_1 < (KL/r)_{pl}$, with corresponding critical stress $\sigma_D > \sigma_{pl}$ needed to cause instability. When the column is *about to buckle*, the change in stress and strain that occurs in the column is within a *small range* $\Delta\sigma$ and $\Delta\epsilon$, so that the modulus of elasticity or stiffness for the material can be taken as the ***tangent modulus*** $E_t = \Delta\sigma/\Delta\epsilon$ defined as the slope of the $\sigma-\epsilon$ diagram at point D, Fig. 13–18*a*. In other words, at the time of failure, the column behaves as if it were made from a material that has a *lower stiffness* than when it behaves elastically, $E_t < E$.

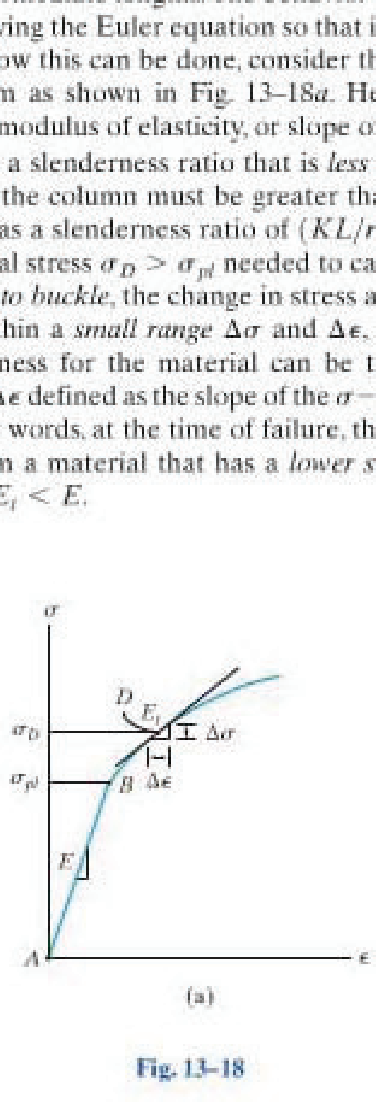


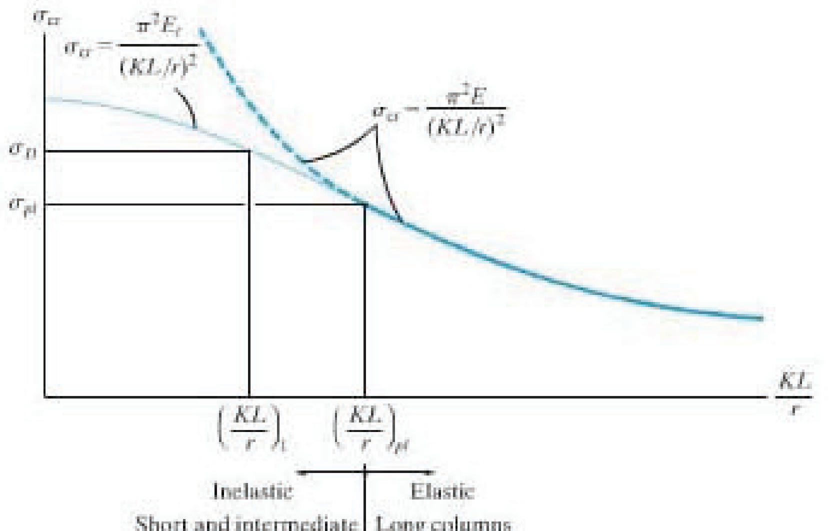
Fig. 13–18

In general, therefore, as the slenderness ratio (KL/r) decreases, the *critical stress* for a column continues to rise; and from the $\sigma-\epsilon$ diagram, the *tangent modulus* for the material *decreases*. Using this idea, we can modify Euler's equation to include these cases of inelastic buckling by substituting the material's tangent modulus E_t for E, so that

$$\sigma_{cr} = \frac{\pi^2 E_t}{(KL/r)^2} \qquad (13\text{–}20)$$

This is the so-called *tangent modulus* or *Engesser equation*, proposed by F. Engesser in 1889. A plot of this equation for intermediate and short-length columns of a material defined by the $\sigma-\epsilon$ diagram in Fig. 13–18*a* is shown in Fig. 13–18*b*.

No *actual column* can be considered to be either perfectly straight or loaded along its centroidal axis, as assumed here, and therefore it is indeed very difficult to develop an expression that will provide a complete analysis of this phenomenon. As a result, other methods of describing the inelastic buckling of columns have been considered. One of these methods was developed by the aeronautical engineer F. R. Shanley and is called the *Shanley theory* of inelastic buckling. Although it provides a better description of the phenomenon than the tangent modulus theory, as explained here, experimental testing of a large number of columns, each of which approximates the ideal column, has shown that Eq. 13–20 is *reasonably accurate* in predicting the column's critical stress. Furthermore, the tangent modulus approach to modeling inelastic column behavior is relatively easy to apply.



(b)

Fig. 13–18 (cont.)

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EXAMPLE 13.5

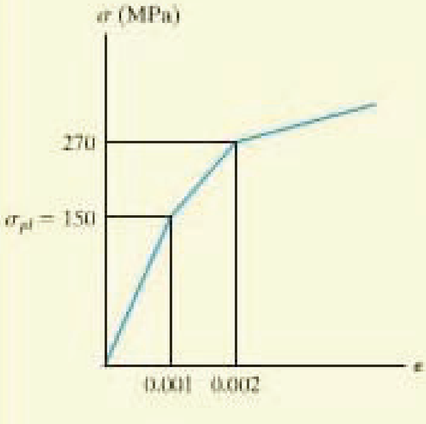


Fig. 13–19

A solid rod has a diameter of 30 mm and is 600 mm long. It is made of a material that can be modeled by the stress–strain diagram shown in Fig. 13–19. If it is used as a pin-supported column, determine the critical load.

SOLUTION

The radius of gyration is

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{(\pi/4)(15\text{ mm})^4}{\pi(15\text{ mm})^2}} = 7.5\text{ mm}$$

and therefore the slenderness ratio is

$$\frac{KL}{r} = \frac{1(600\text{ mm})}{7.5\text{ mm}} = 80$$

Applying Eq. 13–20 we have,

$$\sigma_{cr} = \frac{\pi^2 E_t}{(KL/r)^2} = \frac{\pi^2 E_t}{(80)^2} = 1.542(10^{-3})E_t \qquad (1)$$

First we will assume that the critical stress is elastic. From Fig. 13–19,

$$E = \frac{150\text{ MPa}}{0.001} = 150\text{ GPa}$$

Thus, Eq. 1 becomes

$$\sigma_{cr} = 1.542(10^{-3})[150(10^3)]\text{ MPa} = 231.3\text{ MPa}$$

Since $\sigma_{cr} > \sigma_{pl} = 150$ MPa, inelastic buckling occurs.

From the second line segment of the $\sigma-\epsilon$ diagram, Fig. 13–19, we have

$$E_t = \frac{\Delta\sigma}{\Delta\epsilon} = \frac{270\text{ MPa} - 150\text{ MPa}}{0.002 - 0.001} = 120\text{ GPa}$$

Applying Eq. 1 yields

$$\sigma_{cr} = 1.542(10^{-3})[120(10^3)]\text{ MPa} = 185.1\text{ MPa}$$

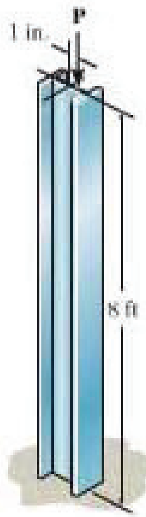
Since this value falls within the limits of 150 MPa and 270 MPa, it is indeed the critical stress.

The critical load on the rod is therefore

$$P_{cr} = \sigma_{cr}A = 185.1(10^6)\text{ Pa}[\pi(0.015\text{ m})^2] = 131\text{ kN} \qquad \textit{Ans.}$$

PROBLEMS

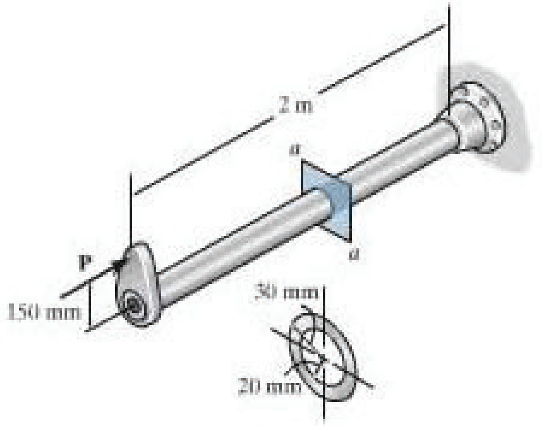
13–46. Determine the load P required to cause the A-36 steel W8 × 15 column to fail either by buckling or by yielding. The column is fixed at its base and free at its top.



Prob. 13–46

13–47. The hollow red brass C83400 copper alloy shaft is fixed at one end but free at the other end. Determine the maximum eccentric force P the shaft can support without causing it to buckle or yield. Also, find the corresponding maximum deflection of the shaft.

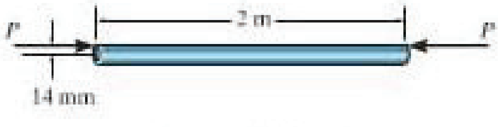
***13–48.** The hollow red brass C83400 copper alloy shaft is fixed at one end but free at the other end. If the eccentric force $P = 5$ kN is applied to the shaft as shown, determine the maximum normal stress and the maximum deflection.



Probs. 13–47/48

•13–49. The tube is made of copper and has an outer diameter of 35 mm and a wall thickness of 7 mm. Using a factor of safety with respect to buckling and yielding of F.S. = 2.5, determine the allowable eccentric load P. The tube is pin supported at its ends. $E_{cu} = 120$ GPa, $\sigma_Y = 750$ MPa.

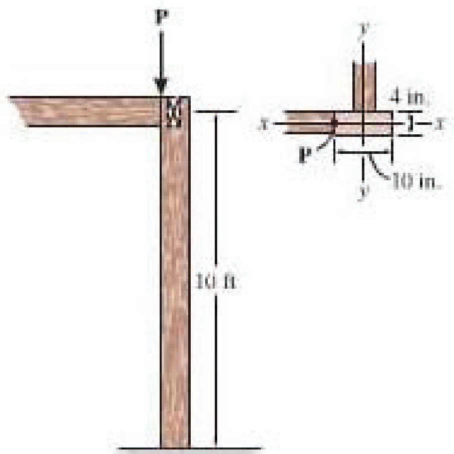
13–50. The tube is made of copper and has an outer diameter of 35 mm and a wall thickness of 7 mm. Using a factor of safety with respect to buckling and yielding of F.S. = 2.5, determine the allowable eccentric load P that it can support without failure. The tube is fixed supported at its ends. $E_{cu} = 120$ GPa, $\sigma_Y = 750$ MPa.



Probs. 13–49/50

13–51. The wood column is fixed at its base and can be assumed pin connected at its top. Determine the maximum eccentric load P that can be applied without causing the column to buckle or yield. $E_w = 1.8(10^3)$ ksi, $\sigma_Y = 8$ ksi.

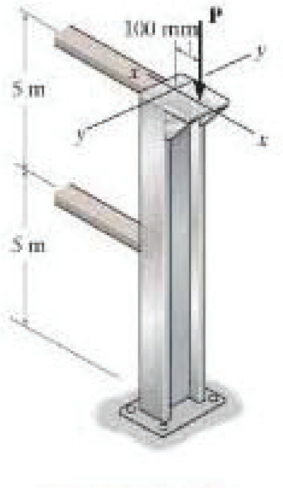
***13–52.** The wood column is fixed at its base and can be assumed fixed connected at its top. Determine the maximum eccentric load P that can be applied without causing the column to buckle or yield. $E_w = 1.8(10^3)$ ksi, $\sigma_Y = 8$ ksi.



Probs. 13–51/52

•**13–53.** The W200 × 22 A-36-steel column is fixed at its base. Its top is constrained to rotate about the y–y axis and free to move along the y–y axis. Also, the column is braced along the x–x axis at its mid-height. Determine the allowable eccentric force P that can be applied without causing the column either to buckle or yield. Use F.S. = 2 against buckling and F.S. = 1.5 against yielding.

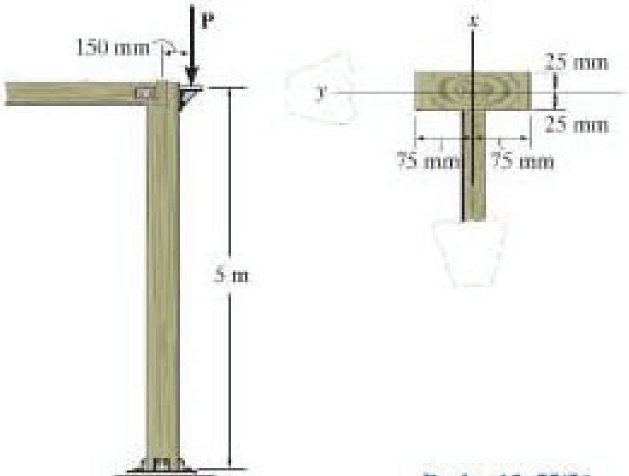
13–54. The W200 × 22 A-36-steel column is fixed at its base. Its top is constrained to rotate about the y–y axis and free to move along the y–y axis. Also, the column is braced along the x–x axis at its mid-height. If $P = 25$ kN, determine the maximum normal stress developed in the column.



Probs. 13–53/54

13–55. The wood column is fixed at its base, and its top can be considered pinned. If the eccentric force $P = 10$ kN is applied to the column, investigate whether the column is adequate to support this loading without buckling or yielding. Take $E = 10$ GPa and $\sigma_Y = 15$ MPa.

*__13–56.__ The wood column is fixed at its base, and its top can be considered pinned. Determine the maximum eccentric force P the column can support without causing it to either buckle or yield. Take $E = 10$ GPa and $\sigma_Y = 15$ MPa.



Probs. 13–55/56

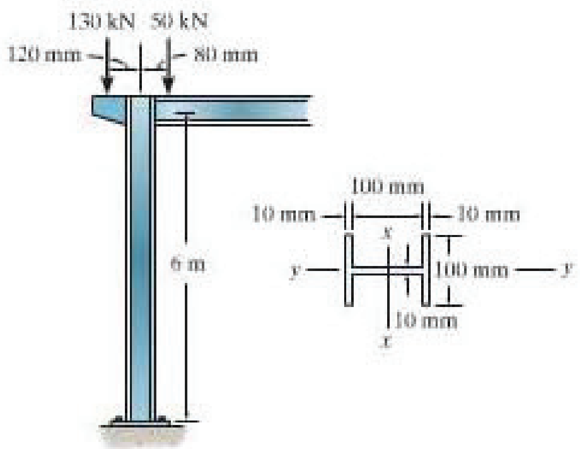
•**13–57.** The W250 × 28 A-36-steel column is fixed at its base. Its top is constrained to rotate about the y–y axis and free to move along the y–y axis. If $e = 350$ mm, determine the allowable eccentric force P that can be applied without causing the column either to buckle or yield. Use F.S. = 2 against buckling and F.S. = 1.5 against yielding.

13–58. The W250 × 28 A-36-steel column is fixed at its base. Its top is constrained to rotate about the y–y axis and free to move along the y–y axis. Determine the force **P** and its eccentricity e so that the column will yield and buckle simultaneously.

Probs. 13–57/58

13–59. The steel column supports the two eccentric loadings. If it is assumed to be pinned at its top, fixed at the bottom, and fully braced against buckling about the y–y axis, determine the maximum deflection of the column and the maximum stress in the column. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.

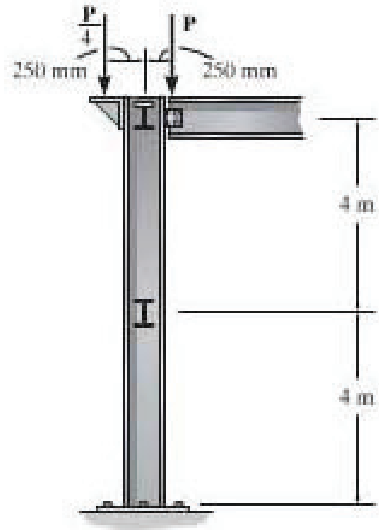
*__13–60.__ The steel column supports the two eccentric loadings. If it is assumed to be fixed at its top and bottom, and braced against buckling about the y–y axis, determine the maximum deflection of the column and the maximum stress in the column. $E_{st} = 200$ GPa, $\sigma_Y = 360$ MPa.



Probs. 13–59/60

13–61. The W250 × 45 A-36-steel column is pinned at its top and fixed at its base. Also, the column is braced along its weak axis at mid-height. If $P = 250$ kN, investigate whether the column is adequate to support this loading. Use F.S. = 2 against buckling and F.S. = 1.5 against yielding.

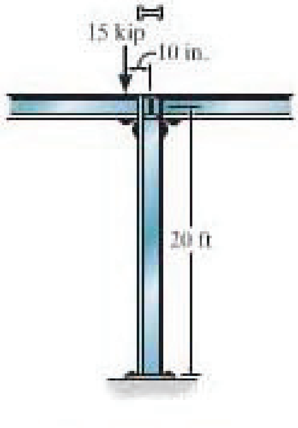
•**13–62.** The W250 × 45 A-36-steel column is pinned at its top and fixed at its base. Also, the column is braced along its weak axis at mid-height. Determine the allowable force P that the column can support without causing it either to buckle or yield. Use F.S. = 2 against buckling and F.S. = 1.5 against yielding.



Probs. 13–61/62

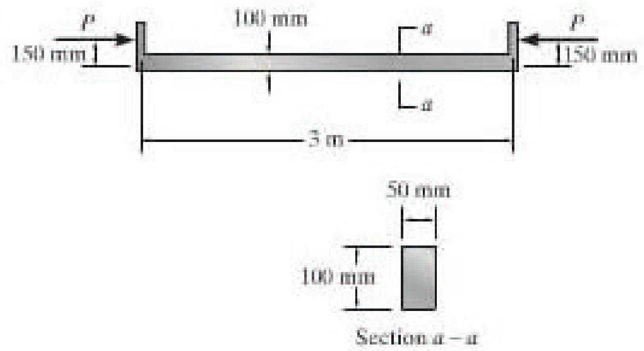
13–63. The W14 × 26 structural A-36 steel member is used as a 20-ft-long column that is assumed to be fixed at its top and fixed at its bottom. If the 15-kip load is applied at an eccentric distance of 10 in., determine the maximum stress in the column.

*__13–64.__ The W14 × 26 structural A-36 steel member is used as a column that is assumed to be fixed at its top and pinned at its bottom. If the 15-kip load is applied at an eccentric distance of 10 in., determine the maximum stress in the column.



Probs. 13–63/64

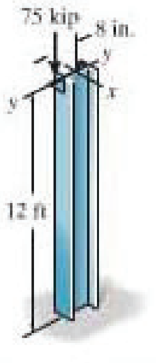
•13–65. Determine the maximum eccentric load P the 2014-T6-aluminum-alloy strut can support without causing it either to buckle or yield. The ends of the strut are pin-connected.



Prob. 13–65

13–66. The W8 × 48 structural A-36 steel column is fixed at its bottom and free at its top. If it is subjected to the eccentric load of 75 kip, determine the factor of safety with respect to either the initiation of buckling or yielding.

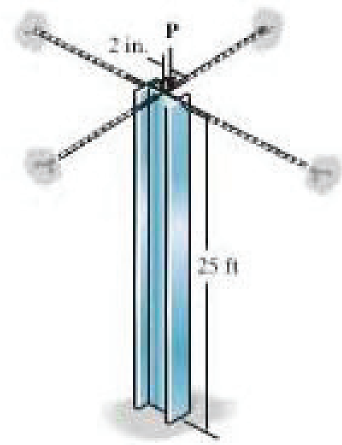
13–67. The W8 × 48 structural A-36 steel column is fixed at its bottom and pinned at its top. If it is subjected to the eccentric load of 75 kip, determine if the column fails by yielding. The column is braced so that it does not buckle about the y–y axis.



Probs. 13–66/67

***13–68.** Determine the load P required to cause the steel W12 × 50 structural A-36 steel column to fail either by buckling or by yielding. The column is fixed at its bottom and the cables at its top act as a pin to hold it.

•13–69. Solve Prob. 13–68 if the column is an A-36 steel W12 × 16 section.

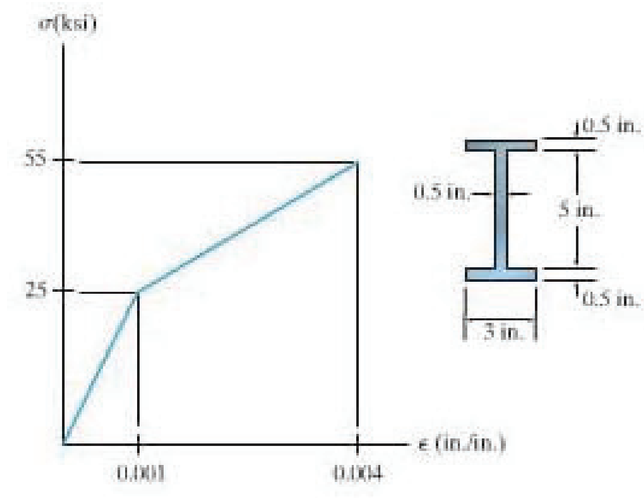


Probs. 13–68/69

13–70. A column of intermediate length buckles when the compressive stress is 40 ksi. If the slenderness ratio is 60, determine the tangent modulus.

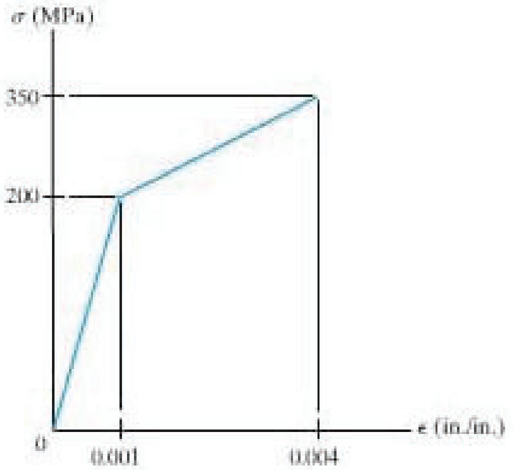
13–71. The 6-ft-long column has the cross section shown and is made of material which has a stress-strain diagram that can be approximated as shown. If the column is pinned at both ends, determine the critical load P_{cr} for the column.

***13–72.** The 6-ft-long column has the cross section shown and is made of material which has a stress-strain diagram that can be approximated as shown. If the column is fixed at both ends, determine the critical load P_{cr} for the column.



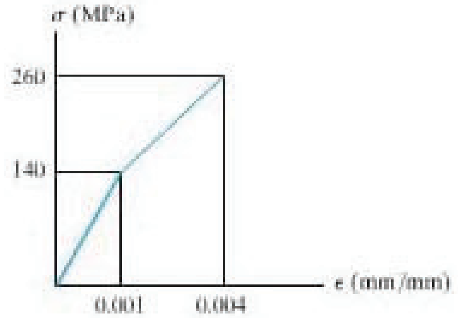
Probs. 13–71/72

•13–73. The stress-strain diagram of the material of a column can be approximated as shown. Plot P/A vs. KL/r for the column.



Prob. 13–73

13–74. Construct the buckling curve, P/A versus L/r, for a column that has a bilinear stress–strain curve in compression as shown. The column is pinned at its ends.

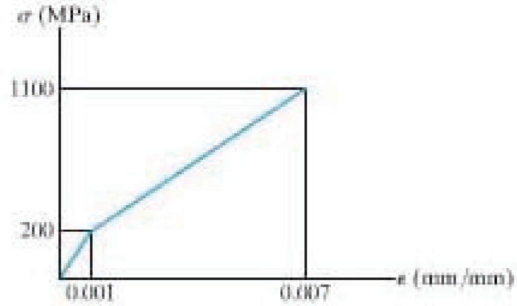


Prob. 13–74

13–75. The stress-strain diagram for a material can be approximated by the two line segments shown. If a bar having a diameter of 80 mm and a length of 1.5 m is made from this material, determine the critical load provided the ends are pinned. Assume that the load acts through the axis of the bar. Use Engesser's equation.

*13–76. The stress-strain diagram for a material can be approximated by the two line segments shown. If a bar having a diameter of 80 mm and a length of 1.5 m is made from this material, determine the critical load provided the ends are fixed. Assume that the load acts through the axis of the bar. Use Engesser's equation.

•13–77. The stress-strain diagram for a material can be approximated by the two line segments shown. If a bar having a diameter of 80 mm and length of 1.5 m is made from this material, determine the critical load provided one end is pinned and the other is fixed. Assume that the load acts through the axis of the bar. Use Engesser's equation.



Probs. 13–75/76/77

*13.6 Design of Columns for Concentric Loading

These long unbraced timber columns are used to support the roof of this building.

The theory presented thus far applies to columns that are perfectly straight, made of homogeneous material, and originally stress free. Practically speaking, though, as stated previously, columns are not perfectly straight, and most have residual stresses in them, primarily due to nonuniform cooling during manufacture. Also, the supports for columns are less than exact, and the points of application and directions of loads are not known with absolute certainty. In order to compensate for these effects, which actually vary from one column to the next, many design codes specify the use of column formulas that are empirical. By performing experimental tests on a large number of axially loaded columns, the results may be plotted and a design formula developed by curve-fitting the mean of the data.

An example of such tests for wide-flange steel columns is shown in Fig. 13–20. Notice the similarity between these results and those of the family of curves determined from the secant formula, Fig. 13–16. The reason for this similarity has to do with the influence of an "accidental" eccentricity ratio on the column's strength. As stated in Sec. 13.4, this ratio has more of an effect on the strength of short and intermediate-length columns than on those that are long. Tests have indicated that ec/r^2 can range from 0.1 to 0.6 for most axially loaded columns.

In order to account for the behavior of different-length columns, design codes usually specify several formulas that will best fit the data within the short, intermediate, and long column range. Hence, each formula will apply only for a specific *range* of slenderness ratios, and so it is important that the engineer carefully observe the KL/r limits for which a particular formula is valid. Examples of design formulas for steel, aluminum, and wood columns that are currently in use will now be discussed. The purpose is to give some idea as to how columns are designed in practice. These formulas should not, however, be used for the design of actual columns, unless the code from which they are referenced is consulted.

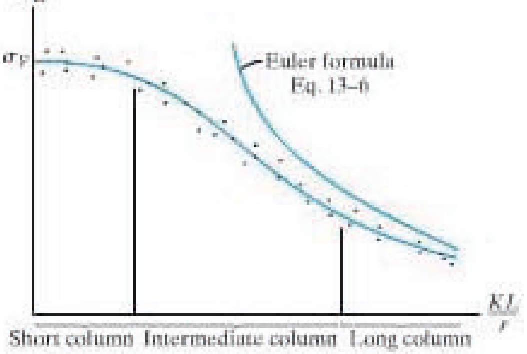


Fig. 13–20

Steel Columns. Columns made of structural steel can be designed on the basis of formulas proposed by the Structural Stability Research Council (SSRC). Factors of safety have been applied to these formulas and adopted as specifications for building construction by the American Institute of Steel Construction (AISC). Basically these specifications provide two formulas for column design, each of which gives the maximum allowable stress in the column for a specific range of slenderness ratios.*

For long columns the Euler formula is proposed, i.e., $\sigma_{max} = \pi^2 E/(KL/r)^2$.

Application of this formula requires that a factor of safety F.S. $= \frac{23}{12} \approx 1.92$ be applied. Thus, for design,

$$\sigma_{allow} = \frac{12\pi^2 E}{23(KL/r)^2} \qquad \left(\frac{KL}{r}\right)_c \leq \frac{KL}{r} \leq 200 \qquad (13\text{–}21)$$

As stated, this equation is applicable for a slenderness ratio bounded by 200 and $(KL/r)_c$. A specific value of $(KL/r)_c$ is obtained by requiring the Euler formula to be used only for elastic material behavior. Through experiments it has been determined that compressive residual stresses can exist in rolled-formed steel sections that may be as much as one-half the yield stress. Consequently, if the stress in the Euler formula is greater than $\frac{1}{2}\sigma_Y$, the equation will not apply. Therefore the value of $(KL/r)_c$ is determined as follows:

$$\frac{1}{2}\sigma_Y = \frac{\pi^2 E}{(KL/r)_c^2} \qquad \text{or} \qquad \left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} \qquad (13\text{–}22)$$

Columns having slenderness ratios less than $(KL/r)_c$ are designed on the basis of an empirical formula that is parabolic and has the form

$$\sigma_{max} = \left[1 - \frac{(KL/r)^2}{2(KL/r)_c^2}\right]\sigma_Y$$

Since there is more uncertainty in the use of this formula for longer columns, it is divided by a factor of safety defined as follows:

$$\text{F.S.} = \frac{5}{3} + \frac{3}{8}\frac{(KL/r)}{(KL/r)_c} - \frac{(KL/r)^3}{8(KL/r)_c^3}$$

Here it is seen that F.S. $= \frac{5}{3} \approx 1.67$ at $KL/r = 0$ and increases to F.S. $= \frac{23}{12} \approx 1.92$ at $(KL/r)_c$. Hence, for design purposes,

$$\sigma_{allow} = \frac{\left[1 - \dfrac{(KL/r)^2}{2(KL/r)_c^2}\right]\sigma_Y}{(5/3) + [(3/8)(KL/r)/(KL/r)_c] - \left[(KL/r)^3/8(KL/r)_c^3\right]} \qquad (13\text{–}23)$$

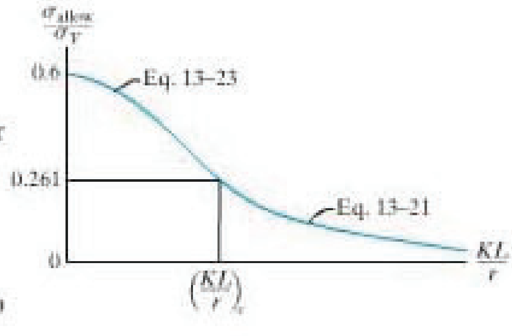


Fig. 13–21

Equations 13–21 and 13–23 are plotted in Fig. 13–21. When applying any of these equations, either FPS or SI units can be used for the calculations.

*The current AISC code enables engineers to use one of two methods for design, namely, Load and Resistance Factor Design, and Allowable Stress Design. The latter is explained here.

Aluminum Columns. Column design for structural aluminum is specified by the Aluminum Association using three equations, each applicable for a specific range of slenderness ratios. Since several types of aluminum alloy exist, there is a unique set of formulas for each type. For a common alloy (2014-T6) used in building construction, the formulas are

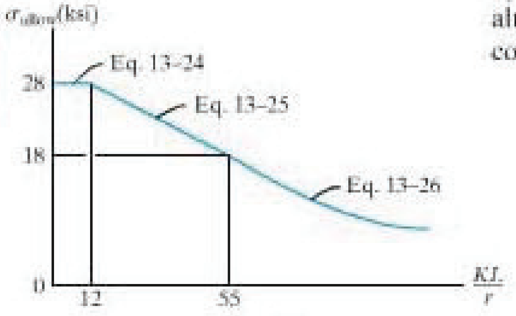


Fig. 13–22

$$\sigma_{\text{allow}} = 28 \text{ ksi} \qquad 0 \le \frac{KL}{r} \le 12 \qquad (13\text{–}24)$$

$$\sigma_{\text{allow}} = \left[30.7 - 0.23\left(\frac{KL}{r}\right)\right] \text{ksi} \qquad 12 < \frac{KL}{r} < 55 \qquad (13\text{–}25)$$

$$\sigma_{\text{allow}} = \frac{54\,000 \text{ ksi}}{(KL/r)^2} \qquad 55 \le \frac{KL}{r} \qquad (13\text{–}26)$$

These equations are plotted in Fig. 13–22. As shown, the first two represent straight lines and are used to model the effects of columns in the short and intermediate range. The third formula has the same form as the Euler formula and is used for long columns.

Timber Columns. Columns used in timber construction are designed on the basis of formulas published by the National Forest Products Association (NFPA) or the American Institute of Timber Construction (AITC). For example, the NFPA formulas for the allowable stress in short, intermediate, and long columns having a rectangular cross section of dimensions b and d, where d is the *smallest* dimension of the cross section, are

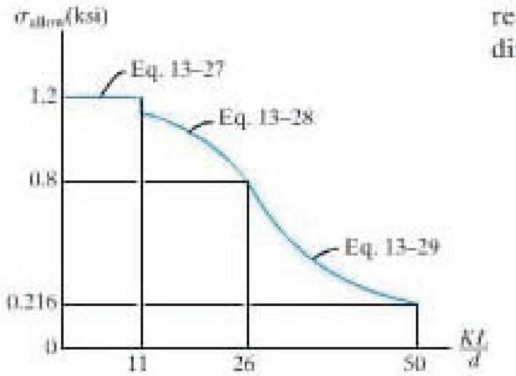


Fig. 13–23

$$\sigma_{\text{allow}} = 1.20 \text{ ksi} \qquad 0 \le \frac{KL}{d} \le 11 \qquad (13\text{–}27)$$

$$\sigma_{\text{allow}} = 1.20\left[1 - \frac{1}{3}\left(\frac{KL/d}{26.0}\right)^2\right] \text{ksi} \qquad 11 < \frac{KL}{d} \le 26 \qquad (13\text{–}28)$$

$$\sigma_{\text{allow}} = \frac{540 \text{ ksi}}{(KL/d)^2} \qquad 26 < \frac{KL}{d} \le 50 \qquad (13\text{–}29)$$

Here wood has a modulus of elasticity of $E_w = 1.8(10^3)$ ksi and an allowable compressive stress of 1.2 ksi parallel to the grain. In particular, Eq. 13–29 is simply Euler's equation having a factor of safety of 3. These three equations are plotted in Fig. 13–23.

Procedure for Analysis

Column Analysis.

- When using any formula to *analyze* a column, that is, to find its allowable load, it is first necessary to calculate the slenderness ratio in order to determine which column formula applies.
- Once the average allowable stress has been calculated, the allowable load on the column is determined from $P = \sigma_{allow}A$.

Column Design.

- If a formula is used to *design* a column, that is, to determine the column's cross-sectional area for a given loading and effective length, then a trial-and-check procedure generally must be followed when the column has a composite shape, such as a wide-flange section.
- One possible way to apply a trial-and-check procedure would be to *assume* the column's cross-sectional area, A', and calculate the corresponding stress $\sigma' = P/A'$. Also, use an appropriate design formula to determine the allowable stress σ_{allow}. From this, calculate the *required* column area $A_{req'd} = P/\sigma_{allow}$.
- If $A' > A_{req'd}$, the design is safe. When making the comparison, it is practical to require A' to be close to but greater than $A_{req'd}$, usually within 2–3%. A redesign is necessary if $A' < A_{req'd}$.
- Whenever a trial-and-check procedure is repeated, the choice of an area is determined by the previously calculated required area. In engineering practice this method for design is usually shortened through the use of computer software or published tables and graphs.

These timber columns can be considered pinned at their bottom and fixed connected to the beams at their tops.

13

13

EXAMPLE 13.6

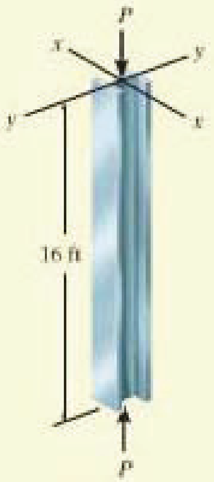


Fig. 13–24

An A-36 steel W10 × 100 member is used as a pin-supported column, Fig. 13–24. Using the AISC column design formulas, determine the largest load that it can safely support.

SOLUTION

The following data for a W10 × 100 is taken from the table in Appendix B.

$$A = 29.4\ \text{in}^2 \quad r_x = 4.60\ \text{in.} \quad r_y = 2.65\ \text{in.}$$

Since $K = 1$ for both x and y axis buckling, the slenderness ratio is largest if r_y is used. Thus,

$$\frac{KL}{r} = \frac{1(16\ \text{ft})(12\ \text{in./ft})}{2.65\ \text{in.}} = 72.45$$

From Eq. 13–22, we have

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[29(10^3)\ \text{ksi}]}{36\ \text{ksi}}} = 126.1$$

Here $0 < KL/r < (KL/r)_c$, so Eq. 13–23 applies.

$$\sigma_{\text{allow}} = \frac{\left[1 - \dfrac{(KL/r)^2}{2(KL/r)_c^2}\right]\sigma_Y}{(5/3) + [(3/8)(KL/r)/(KL/r)_c] - \left[(KL/r)^3/8(KL/r_c)^3\right]}$$

$$= \frac{[1 - (72.45)^2/2(126.1)^2]36\ \text{ksi}}{(5/3) + [(3/8)(72.45/126.1)] - [(72.45)^3/8(126.1)^3]}$$

$$= 16.17\ \text{ksi}$$

The allowable load P on the column is therefore

$$\sigma_{\text{allow}} = \frac{P}{A}; \qquad 16.17\ \text{kip/in}^2 = \frac{P}{29.4\ \text{in}^2}$$

$$P = 476\ \text{kip} \qquad \textit{Ans.}$$

EXAMPLE 13.7

The steel rod in Fig. 13–25 is to be used to support an axial load of 18 kip. If $E_{st} = 29(10^3)$ ksi and $\sigma_Y = 50$ ksi, determine the smallest diameter of the rod as allowed by the AISC specification. The rod is fixed at both ends.

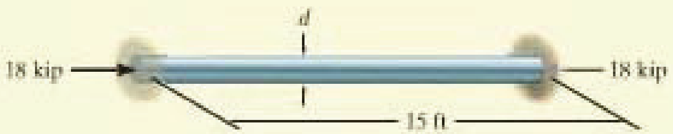


Fig. 13–25

SOLUTION

For a circular cross section the radius of gyration becomes

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{(1/4)\pi(d/2)^4}{(1/4)\pi d^2}} = \frac{d}{4}$$

Applying Eq. 13–22, we have

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[29(10^3)\text{ ksi}]}{50\text{ ksi}}} = 107.0$$

Since the rod's radius of gyration is unknown, KL/r is unknown, and therefore a choice must be made as to whether Eq. 13–21 or Eq. 13–23 applies. We will consider Eq. 13–21. For a fixed-end column $K = 0.5$, so

$$\sigma_{allow} = \frac{12\pi^2 E}{23(KL/r)^2}$$

$$\frac{18\text{ kip}}{(1/4)\pi d^2} = \frac{12\pi^2[29(10^3)\text{ kip/in}^2]}{23[0.5(15\text{ ft})(12\text{ in./ft})/(d/4)]^2}$$

$$\frac{22.92}{d^2} = 1.152d^2$$

$$d = 2.11\text{ in.}$$

Use

$$d = 2.25\text{ in.} = 2\tfrac{1}{4}\text{ in.}$$ *Ans.*

For this design, we must check the slenderness-ratio limits; i.e.,

$$\frac{KL}{r} = \frac{0.5(15\text{ ft})(12\text{ in./ft})}{(2.25\text{ in./4})} = 160$$

Since $107.0 < 160 < 200$, use of Eq. 13–21 is appropriate.

13

EXAMPLE 13.8

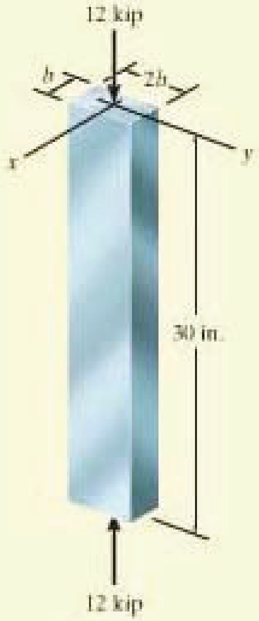


Fig. 13–26

A bar having a length of 30 in. is used to support an axial compressive load of 12 kip, Fig. 13–26. It is pin supported at its ends and made of a 2014-T6 aluminum alloy. Determine the dimensions of its cross-sectional area if its width is to be twice its thickness.

SOLUTION

Since $KL = 30$ in. is the same for both x and y axis buckling, the larger slenderness ratio is determined using the smaller radius of gyration, i.e., using $I_{min} = I_y$:

$$\frac{KL}{r_y} = \frac{KL}{\sqrt{I_y/A}} = \frac{1(30)}{\sqrt{(1/12)2b(b^3)/[2b(b)]}} = \frac{103.9}{b} \qquad (1)$$

Here we must apply Eq. 13–24, 13–25, or 13–26. Since we do not as yet know the slenderness ratio, we will begin by using Eq. 13–24.

$$\frac{P}{A} = 28 \text{ ksi}$$

$$\frac{12 \text{ kip}}{2b(b)} = 28 \text{ kip/in}^2$$

$$b = 0.463 \text{ in.}$$

Checking the slenderness ratio, we have

$$\frac{KL}{r} = \frac{103.9}{0.463} = 224.5 > 12$$

Try Eq. 13–26, which is valid for $KL/r \geq 55$,

$$\frac{P}{A} = \frac{54\,000 \text{ ksi}}{(KL/r)^2}$$

$$\frac{12}{2b(b)} = \frac{54\,000}{(103.9/b)^2}$$

$$b = 1.05 \text{ in.} \qquad \textit{Ans.}$$

From Eq. 1,

$$\frac{KL}{r} = \frac{103.9}{1.05} = 99.3 > 55 \quad \text{OK}$$

NOTE: It would be satisfactory to choose the cross section with dimensions 1 in. by 2 in.

EXAMPLE 13.9

A board having cross-sectional dimensions of 5.5 in. by 1.5 in. is used to support an axial load of 5 kip, Fig. 13–27. If the board is assumed to be pin supported at its top and bottom, determine its *greatest* allowable length L as specified by the NFPA.

Fig. 13–27

SOLUTION

By inspection, the board will buckle about the y axis. In the NFPA equations, $d = 1.5$ in. Assuming that Eq. 13–29 applies, we have

$$\frac{P}{A} = \frac{540\text{ ksi}}{(KL/d)^2}$$

$$\frac{5\text{ kip}}{(5.5\text{ in.})(1.5\text{ in.})} = \frac{540\text{ ksi}}{(1\,L/1.5\text{ in.})^2}$$

$$L = 44.8\text{ in.} \qquad \textit{Ans.}$$

Here

$$\frac{KL}{d} = \frac{1(44.8\text{ in.})}{1.5\text{ in.}} = 29.8$$

Since $26 < KL/d \le 50$, the solution is valid.

PROBLEMS

13

13–78. Determine the largest length of a structural A-36 steel rod if it is fixed supported and subjected to an axial load of 100 kN. The rod has a diameter of 50 mm. Use the AISC equations.

13–79. Determine the largest length of a W10 × 45 structural steel column if it is pin supported and subjected to an axial load of 290 kip. $E_{st} = 29(10^3)$ ksi, $\sigma_Y = 50$ ksi. Use the AISC equations.

***13–80.** Determine the largest length of a W10 × 12 structural A-36 steel section if it is pin supported and is subjected to an axial load of 28 kip. Use the AISC equations.

•13–81. Using the AISC equations, select from Appendix B the lightest-weight structural A-36 steel column that is 14 ft long and supports an axial load of 40 kip. The ends are pinned. Take $\sigma_Y = 50$ ksi.

13–82. Using the AISC equations, select from Appendix B the lightest-weight structural A-36 steel column that is 12 ft long and supports an axial load of 40 kip. The ends are fixed. Take $\sigma_Y = 50$ ksi.

13–83. Using the AISC equations, select from Appendix B the lightest-weight structural A-36 steel column that is 24 ft long and supports an axial load of 100 kip. The ends are fixed.

***13–84.** Using the AISC equations, select from Appendix B the lightest-weight structural A-36 steel column that is 30 ft long and supports an axial load of 200 kip. The ends are fixed.

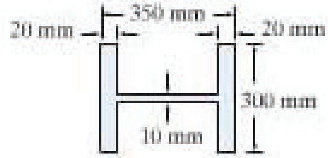
•13–85. A W8 × 24 A-36-steel column of 30-ft length is pinned at both ends and braced against its weak axis at mid-height. Determine the allowable axial force P that can be safely supported by the column. Use the AISC column design formulas.

13–86. Check if a W10 × 39 column can safely support an axial force of $P = 250$ kip. The column is 20 ft long and is pinned at both ends and braced against its weak axis at mid-height. It is made of steel having $E = 29(10^3)$ ksi and $\sigma_Y = 50$ ksi. Use the AISC column design formulas.

13–87. A 5-ft-long rod is used in a machine to transmit an axial compressive load of 3 kip. Determine its smallest diameter if it is pin connected at its ends and is made of a 2014-T6 aluminum alloy.

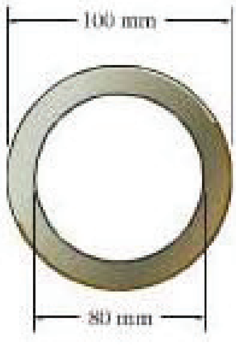
***13–88.** Check if a W10 × 45 column can safely support an axial force of $P = 200$ kip. The column is 15 ft long and is pinned at both of its ends. It is made of steel having $E = 29(10^3)$ ksi and $\sigma_Y = 50$ ksi. Use the AISC column design formulas.

•13–89. Using the AISC equations, check if a column having the cross section shown can support an axial force of 1500 kN. The column has a length of 4 m, is made from A-36 steel, and its ends are pinned.



Prob. 13–89

13–90. The A-36-steel tube is pinned at both ends. If it is subjected to an axial force of 150 kN, determine the maximum length that the tube can safely support using the AISC column design formulas.



Prob. 13–90

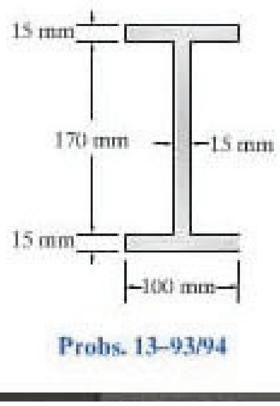
13–91. The bar is made of a 2014-T6 aluminum alloy. Determine its smallest thickness b if its width is $5b$. Assume that it is pin connected at its ends.

***13–92.** The bar is made of a 2014-T6 aluminum alloy. Determine its smallest thickness b if its width is $5b$. Assume that it is fixed connected at its ends.

Probs. 13–91/92

•**13–93.** The 2014-T6 aluminum column of 3-m length has the cross section shown. If the column is pinned at both ends and braced against the weak axis at its mid-height, determine the allowable axial force P that can be safely supported by the column.

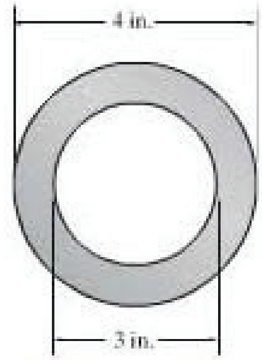
13–94. The 2014-T6 aluminum column has the cross section shown. If the column is pinned at both ends and subjected to an axial force $P = 100$ kN, determine the maximum length the column can have to safely support the loading.



Probs. 13–93/94

13–95. The 2014-T6 aluminum hollow section has the cross section shown. If the column is 10 ft long and is fixed at both ends, determine the allowable axial force P that can be safely supported by the column.

***13–96.** The 2014-T6 aluminum hollow section has the cross section shown. If the column is fixed at its base and pinned at its top, and is subjected to the axial force $P = 100$ kip, determine the maximum length of the column for it to safely support the load.

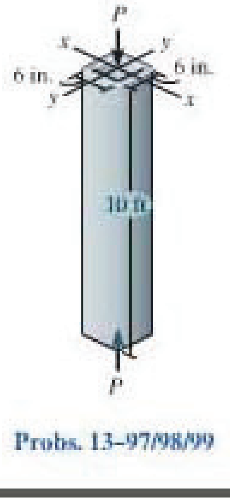


Probs. 13–95/96

•**13–97.** The tube is 0.25 in. thick, is made of a 2014-T6 aluminum alloy, and is fixed at its bottom and pinned at its top. Determine the largest axial load that it can support.

13–98. The tube is 0.25 in. thick, is made of a 2014-T6 aluminum alloy, and is fixed connected at its ends. Determine the largest axial load that it can support.

13–99. The tube is 0.25 in. thick, is made of 2014-T6 aluminum alloy and is pin connected at its ends. Determine the largest axial load it can support.

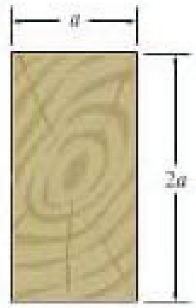


Probs. 13–97/98/99

***13–100.** A rectangular wooden column has the cross section shown. If the column is 6 ft long and subjected to an axial force of $P = 15$ kip, determine the required minimum dimension a of its cross-sectional area to the nearest $\frac{1}{16}$ in. so that the column can safely support the loading. The column is pinned at both ends.

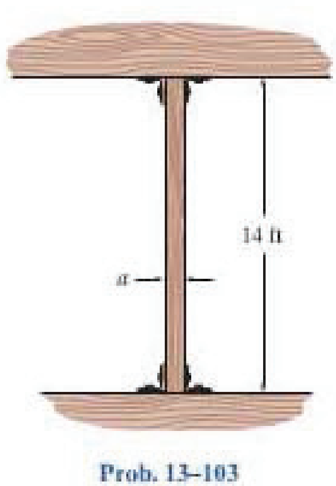
•13–101. A rectangular wooden column has the cross section shown. If $a = 3$ in. and the column is 12 ft long, determine the allowable axial force P that can be safely supported by the column if it is pinned at its top and fixed at its base.

13–102. A rectangular wooden column has the cross section shown. If $a = 3$ in. and the column is subjected to an axial force of $P = 15$ kip, determine the maximum length the column can have to safely support the load. The column is pinned at its top and fixed at its base.



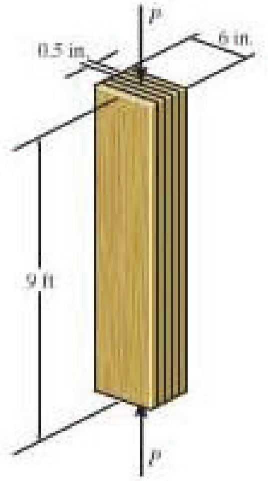
Probs. 13–100/101/102

13–103. The timber column has a square cross section and is assumed to be pin connected at its top and bottom. If it supports an axial load of 50 kip, determine its smallest side dimension a to the nearest $\frac{1}{2}$ in. Use the NFPA formulas.



Prob. 13–103

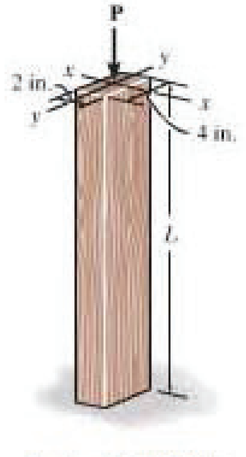
***13–104.** The wooden column shown is formed by gluing together the 6 in. × 0.5 in. boards. If the column is pinned at both ends and is subjected to an axial load $P = 20$ kip, determine the required number of boards needed to form the column in order to safely support the loading.



Prob. 13–104

•13–105. The column is made of wood. It is fixed at its bottom and free at its top. Use the NFPA formulas to determine its greatest allowable length if it supports an axial load of $P = 2$ kip.

13–106. The column is made of wood. It is fixed at its bottom and free at its top. Use the NFPA formulas to determine the largest allowable axial load P that it can support if it has a length $L = 4$ ft.



Probs. 13–105/106

*13.7 Design of Columns for Eccentric Loading

Occasionally a column may be required to support a load acting either at its edge or on an angle bracket attached to its side, such as shown in Fig. 13–28*a*. The bending moment $M = Pe$, which is caused by the eccentric loading, must be accounted for when the column is designed. There are several acceptable ways in which this is done in engineering practice. We will discuss two of the most common methods.

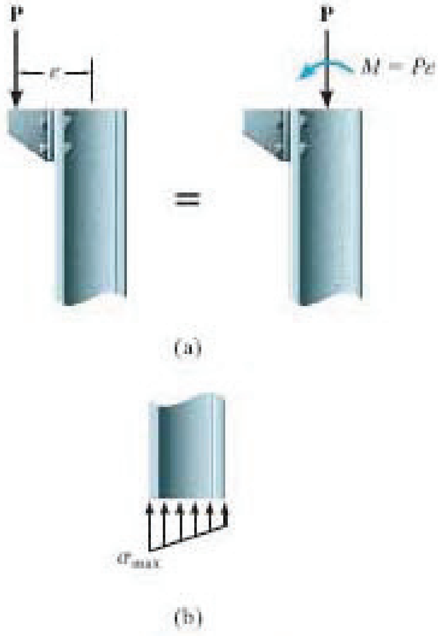


Fig. 13–28

Use of Available Column Formulas. The stress distribution acting over the cross-sectional area of the column shown in Fig. 13–28*a* is determined from a superposition of both the axial force P and the bending moment $M = Pe$. In particular, the maximum compressive stress is

$$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} \qquad (13\text{–}30)$$

A typical stress profile is shown in Fig. 13–28*b*. If we conservatively *assume* that the entire cross section is subjected to the uniform stress σ_{max} as determined from Eq. 13–30, then we can compare σ_{max} with σ_{allow}, which is determined using the formulas given in Sec. 13.6. Calculation of σ_{allow} is usually done using the *largest* slenderness ratio for the column, regardless of the axis about which the column experiences bending. This requirement is normally specified in design codes and will in most cases lead to a conservative design. If

$$\sigma_{max} \leq \sigma_{allow}$$

then the column can carry the specified loading. If this inequality does not hold, then the column's area A must be increased, and a new σ_{max} and σ_{allow} must be calculated. This method of design is rather simple to apply and works well for columns that are short or of intermediate length.

Interaction Formula. When *designing* an eccentrically loaded column it is desirable to see how the bending and axial loads *interact*, so that a balance between these two effects can be achieved. To do this, we will consider the separate contributions made to the total column area by the axial force and moment. If the allowable stress for the axial load is $(\sigma_a)_{allow}$, then the required area for the column needed to support the load P is

$$A_a = \frac{P}{(\sigma_a)_{allow}}$$

Similarly, if the allowable bending stress is $(\sigma_b)_{allow}$, then since $I = Ar^2$, the required area of the column needed to support the eccentric moment is determined from the flexure formula, that is,

13

$$A_b = \frac{Mc}{(\sigma_b)_{\text{allow}} r^2}$$

The total area A for the column needed to resist *both* the axial load and moment requires that

$$A_a + A_b = \frac{P}{(\sigma_a)_{\text{allow}}} + \frac{Mc}{(\sigma_b)_{\text{allow}} r^2} \le A$$

or

$$\frac{P/A}{(\sigma_a)_{\text{allow}}} + \frac{Mc/Ar^2}{(\sigma_b)_{\text{allow}}} \le 1$$

$$\frac{\sigma_a}{(\sigma_a)_{\text{allow}}} + \frac{\sigma_b}{(\sigma_b)_{\text{allow}}} \le 1 \qquad (13\text{–}31)$$

Here

σ_a = axial stress caused by the force P and determined from $\sigma_a = P/A$, where A is the cross-sectional area of the column

σ_b = bending stress caused by an eccentric load or applied moment M; σ_b is found from $\sigma_b = Mc/I$, where I is the moment of inertia of the cross-sectional area calculated about the bending or centroidal axis

$(\sigma_a)_{\text{allow}}$ = allowable axial stress as defined by formulas given in Sec. 13.6 or by other design code specifications. For this purpose, always use the *largest* slenderness ratio for the column, regardless of the axis about which the column experiences bending

$(\sigma_b)_{\text{allow}}$ = allowable bending stress as defined by code specifications

Typical example of a column used to support an eccentric roof loading.

Notice that, if the column is subjected only to an axial load, then the bending-stress ratio in Eq. 13–31 would be equal to zero and the design will be based only on the allowable axial stress. Likewise, when no axial load is present, the axial-stress ratio is zero and the stress requirement will be based on the allowable bending stress. Hence, each stress ratio indicates the contribution of axial load or bending moment. Since Eq. 13–31 shows how these loadings interact, this equation is sometimes referred to as the ***interaction formula***. This design approach requires a trial-and-check procedure, where it is required that the designer *pick* an available column and then check to see if the inequality is satisfied. If it is not, a larger section is then picked and the process repeated. An economical choice is made when the left side is close to but less than 1.

The interaction method is often specified in codes for the design of columns made of steel, aluminum, or timber. In particular, for allowable stress design, the American Institute of Steel Construction specifies the use of this equation only when the axial-stress ratio $\sigma_a/(\sigma_a)_{\text{allow}} \le 0.15$. For other values of this ratio, a modified form of Eq. 13–31 is used.

EXAMPLE 13.10

The column in Fig. 13–29 is made of aluminum alloy 2014-T6 and is used to support an eccentric load **P**. Determine the maximum magnitude of **P** that can be supported if the column is fixed at its base and free at its top. Use Eq. 13–30.

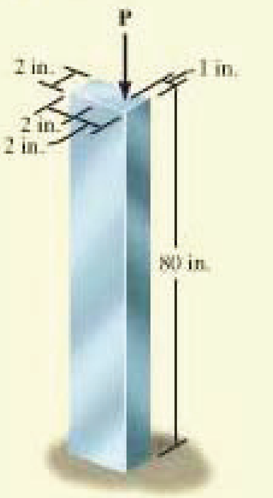


Fig. 13–29

SOLUTION

From Fig. 13–10*b*, $K = 2$. The largest slenderness ratio for the column is therefore

$$\frac{KL}{r} = \frac{2(80\text{ in.})}{\sqrt{[(1/12)(4\text{ in.})(2\text{ in.})^3]/[(2\text{ in.})\,4\text{ in.}]}} = 277.1$$

By inspection, Eq. 13–26 must be used ($277.1 > 55$). Thus,

$$\sigma_{\text{allow}} = \frac{54\,000\text{ ksi}}{(KL/r)^2} = \frac{54\,000\text{ ksi}}{(277.1)^2} = 0.7031\text{ ksi}$$

The maximum compressive stress in the column is determined from the combination of axial load and bending. We have

$$\sigma_{\max} = \frac{P}{A} + \frac{(Pe)c}{I}$$

$$= \frac{P}{2\text{ in.}(4\text{ in.})} + \frac{P(1\text{ in.})(2\text{ in.})}{(1/12)(2\text{ in.})(4\text{ in.})^3}$$

$$= 0.3125P$$

Assuming that this stress is *uniform* over the cross section, we require

$\sigma_{\text{allow}} = \sigma_{\max};$

$$0.7031 = 0.3125P$$

$$P = 2.25\text{ kip}$$ *Ans.*

EXAMPLE 13.11

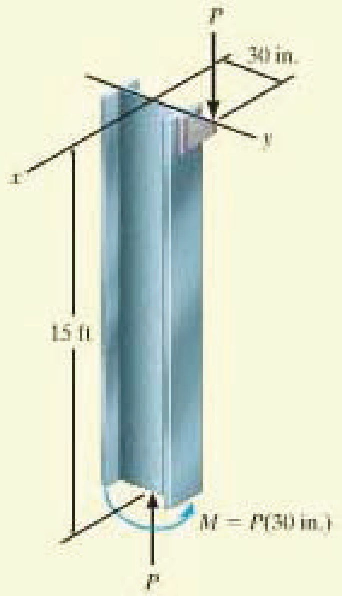


Fig. 13–30

The A-36 steel W6 × 20 column in Fig. 13–30 is pin connected at its ends and is subjected to the eccentric load P. Determine the maximum allowable value of P using the interaction method if the allowable bending stress is $(\sigma_b)_{allow} = 22$ ksi.

SOLUTION

Here $K = 1$. The necessary geometric properties for the W6 × 20 are taken from the table in Appendix B.

$$A = 5.87\text{ in}^2 \qquad I_x = 41.4\text{ in}^4 \qquad r_y = 1.50\text{ in.} \qquad d = 6.20\text{ in.}$$

We will consider r_y because this will lead to the *largest* value of the slenderness ratio. Also, I_x is needed since bending occurs about the x axis ($c = 6.20$ in./2 = 3.10 in.). To determine the allowable compressive stress, we have

$$\frac{KL}{r} = \frac{1[15\text{ ft}(12\text{ in./ft})]}{1.50\text{ in.}} = 120$$

Since

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2[29(10^3)\text{ ksi}]}{36\text{ ksi}}} = 126.1$$

then $KL/r < (KL/r)_c$ and so Eq. 13–23 must be used.

$$\sigma_{allow} = \frac{[1 - (KL/r)^2/2(KL/r)_c^2]\sigma_Y}{(5/3) + [(3/8)(KL/r)/(KL/r)_c] - \left[(KL/r)^3/8(KL/r)_c^3\right]}$$

$$= \frac{[1 - (120)^2/2(126.1)^2]36\text{ ksi}}{(5/3) + [(3/8)(120)/(126.1)] - [(120)^3/8(126.1)^3]}$$

$$= 10.28\text{ ksi}$$

Applying the interaction Eq. 13–31 yields

$$\frac{\sigma_a}{(\sigma_a)_{allow}} + \frac{\sigma_b}{(\sigma_b)_{allow}} \leq 1$$

$$\frac{P/5.87\text{ in}^2}{10.28\text{ ksi}} + \frac{P(30\text{ in.})(3.10\text{ in.})/(41.4\text{ in}^4)}{22\text{ ksi}} = 1$$

$$P = 8.43\text{ kip} \qquad \textit{Ans.}$$

Checking the application of the interaction method for the steel section, we require

$$\frac{\sigma_a}{(\sigma_a)_{allow}} = \frac{8.43\text{ kip}/(5.87\text{ in.})}{10.28\text{ kip/in}^2} = 0.140 < 0.15 \qquad \text{OK}$$

EXAMPLE 13.12

The timber column in Fig. 13–31 is made from two boards nailed together so that the cross section has the dimensions shown. If the column is fixed at its base and free at its top, use Eq. 13–30 to determine the eccentric load **P** that can be supported.

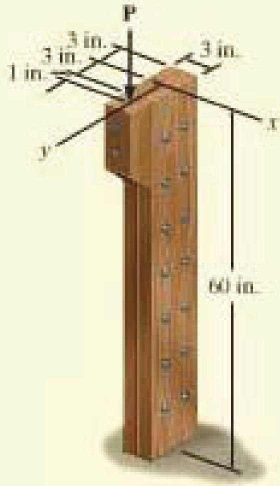


Fig. 13–31

SOLUTION

From Fig. 13–10*b*, $K = 2$. Here we must calculate KL/d to determine which equation from Eqs. 13–27 through 13–29 should be used. Since σ_{allow} is determined using the largest slenderness ratio, we choose $d = 3$ in. This is done to make this ratio as large as possible, and thereby yields the lowest possible allowable axial stress. We have

$$\frac{KL}{d} = \frac{2(60 \text{ in.})}{3 \text{ in.}} = 40$$

Since $26 < KL/d < 50$ the allowable axial stress is determined using Eq. 13–29. Thus,

$$\sigma_{allow} = \frac{540 \text{ ksi}}{(KL/d)^2} = \frac{540 \text{ ksi}}{(40)^2} = 0.3375 \text{ ksi}$$

Applying Eq. 13–30 with $\sigma_{allow} = \sigma_{max}$, we have

$$\sigma_{allow} = \frac{P}{A} + \frac{Mc}{I}$$

$$0.3375 \text{ ksi} = \frac{P}{(3 \text{ in.})(6 \text{ in.})} + \frac{P(4 \text{ in.})(3 \text{ in.})}{(1/12)(3 \text{ in.})(6 \text{ in.})^3}$$

$$P = 1.22 \text{ kip} \qquad \textit{Ans.}$$

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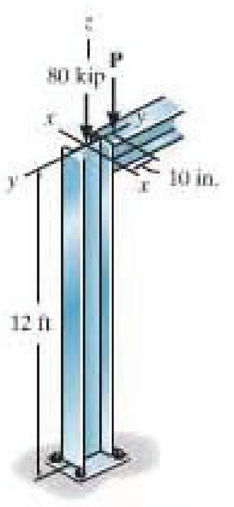
PROBLEMS

13–107. The W14 × 53 structural A-36 steel column supports an axial load of 80 kip in addition to an eccentric load P. Determine the maximum allowable value of P based on the AISC equations of Sec. 13.6 and Eq. 13–30. Assume the column is fixed at its base, and at its top it is free to sway in the x–z plane while it is pinned in the y–z plane.

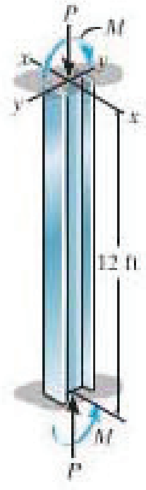
*__13–108.__ The W12 × 45 structural A-36 steel column supports an axial load of 80 kip in addition to an eccentric load of $P = 60$ kip. Determine if the column fails based on the AISC equations of Sec. 13.6 and Eq. 13–30. Assume that the column is fixed at its base, and at its top it is free to sway in the x–z plane while it is pinned in the y–z plane.



Probs. 13–107/108

•**13–109.** The W14 × 22 structural A-36 steel column is fixed at its top and bottom. If a horizontal load (not shown) causes it to support end moments of $M = 10$ kip·ft, determine the maximum allowable axial force P that can be applied. Bending is about the x–x axis. Use the AISC equations of Sec. 13.6 and Eq. 13–30.

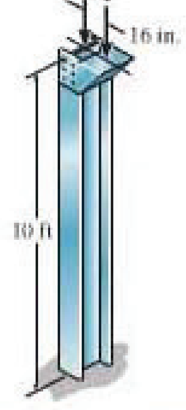
13–110. The W14 × 22 column is fixed at its top and bottom. If a horizontal load (not shown) causes it to support end moments of $M = 15$ kip·ft, determine the maximum allowable axial force P that can be applied. Bending is about the x–x axis. Use the interaction formula with $(\sigma_b)_{allow} = 24$ ksi.



Probs. 13–109/110

13–111. The W14 × 43 structural A-36 steel column is fixed at its bottom and free at its top. Determine the greatest eccentric load P that can be applied using Eq. 13–30 and the AISC equations of Sec. 13.6.

*__13–112.__ The W10 × 45 structural A-36 steel column is fixed at its bottom and free at its top. If it is subjected to a load of $P = 2$ kip, determine if it is safe based on the AISC equations of Sec. 13.6 and Eq. 13–30.



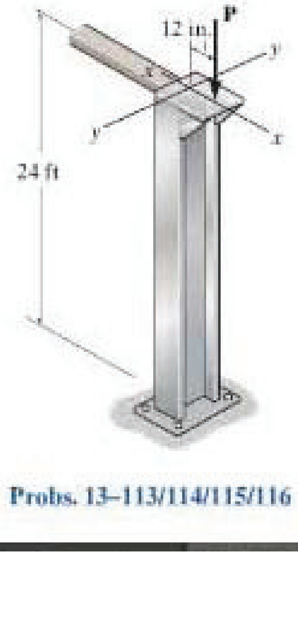
Probs. 13–111/112

•**13–113.** The A-36-steel W10 × 45 column is fixed at its base. Its top is constrained to move along the x–x axis but free to rotate about and move along the y–y axis. Determine the maximum eccentric force P that can be safely supported by the column using the allowable stress method.

13–114. The A-36-steel W10 × 45 column is fixed at its base. Its top is constrained to move along the x–x axis but free to rotate about and move along the y–y axis. Determine the maximum eccentric force P that can be safely supported by the column using an interaction formula. The allowable bending stress is $(\sigma_b)_{allow} = 15$ ksi.

13–115. The A-36-steel W12 × 50 column is fixed at its base. Its top is constrained to move along the x–x axis but free to rotate about and move along the y–y axis. If the eccentric force $P = 15$ kip is applied to the column, investigate if the column is adequate to support the loading. Use the allowable stress method.

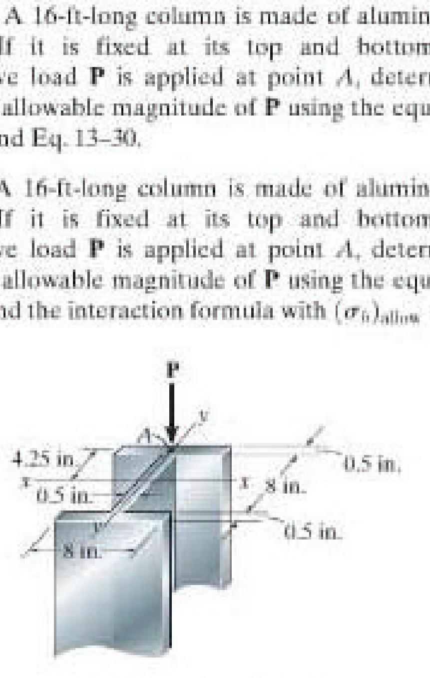
*__13–116.__ The A-36-steel W12 × 50 column is fixed at its base. Its top is constrained to move along the x–x axis but free to rotate about and move along the y–y axis. If the eccentric force $P = 15$ kip is applied to the column, investigate if the column is adequate to support the loading. Use the interaction formula. The allowable bending stress is $(\sigma_b)_{allow} = 15$ ksi.



Probs. 13–113/114/115/116

•**13–117.** A 16-ft-long column is made of aluminum alloy 2014-T6. If it is fixed at its top and bottom, and a compressive load **P** is applied at point A, determine the maximum allowable magnitude of **P** using the equations of Sec. 13.6 and Eq. 13–30.

13–118. A 16-ft-long column is made of aluminum alloy 2014-T6. If it is fixed at its top and bottom, and a compressive load **P** is applied at point A, determine the maximum allowable magnitude of **P** using the equations of Sec. 13.6 and the interaction formula with $(\sigma_b)_{allow} = 20$ ksi.



Probs. 13–117/118

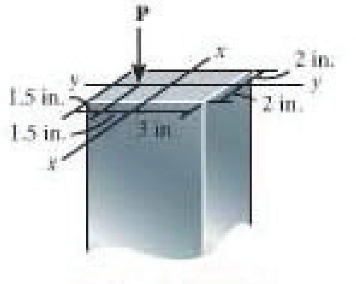
13–119. The 2014-T6 hollow column is fixed at its base and free at its top. Determine the maximum eccentric force P that can be safely supported by the column. Use the allowable stress method. The thickness of the wall for the section is $t = 0.5$ in.

*__13–120.__ The 2014-T6 hollow column is fixed at its base and free at its top. Determine the maximum eccentric force P that can be safely supported by the column. Use the interaction formula. The allowable bending stress is $(\sigma_b)_{allow} = 30$ ksi. The thickness of the wall for the section is $t = 0.5$ in.

Probs. 13–119/120

•**13–121.** The 10-ft-long bar is made of aluminum alloy 2014-T6. If it is fixed at its bottom and pinned at the top, determine the maximum allowable eccentric load **P** that can be applied using the formulas in Sec. 13.6 and Eq. 13–30.

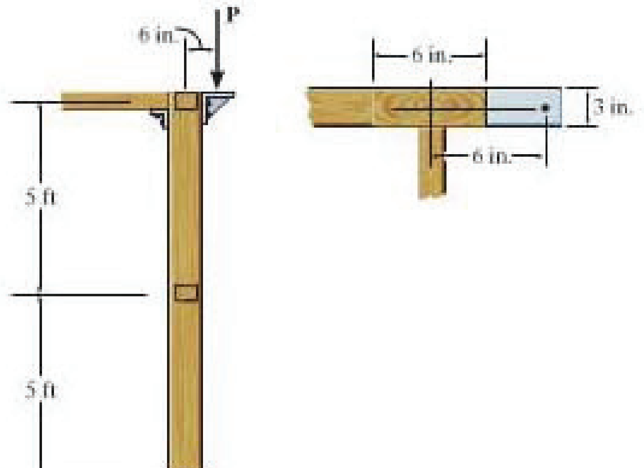
13–122. The 10-ft-long bar is made of aluminum alloy 2014-T6. If it is fixed at its bottom and pinned at the top, determine the maximum allowable eccentric load **P** that can be applied using the equations of Sec. 13.6 and the interaction formula with $(\sigma_b)_{allow} = 18$ ksi.



Probs. 13–121/122

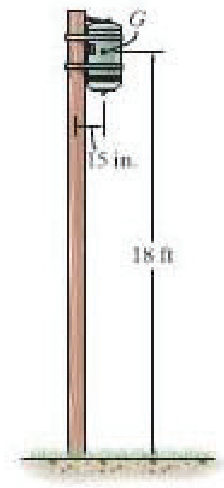
13–123. The rectangular wooden column can be considered fixed at its base and pinned at its top. Also, the column is braced at its mid-height against the weak axis. Determine the maximum eccentric force P that can be safely supported by the column using the allowable stress method.

*__13–124.__ The rectangular wooden column can be considered fixed at its base and pinned at its top. Also, the column is braced at its mid-height against the weak axis. Determine the maximum eccentric force P that can be safely supported by the column using the interaction formula. The allowable bending stress is $(\sigma_b)_{allow} = 1.5$ ksi.



Probs. 13–123/124

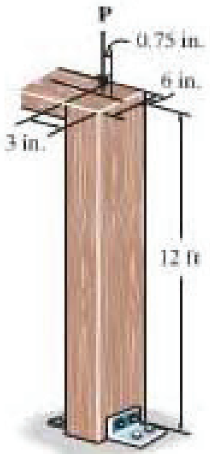
•**13–125.** The 10-in.-diameter utility pole supports the transformer that has a weight of 600 lb and center of gravity at G. If the pole is fixed to the ground and free at its top, determine if it is adequate according to the NFPA equations of Sec. 13.6 and Eq. 13–30.



Prob. 13–125

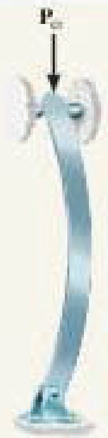
13–126. Using the NFPA equations of Sec. 13.6 and Eq. 13–30, determine the maximum allowable eccentric load P that can be applied to the wood column. Assume that the column is pinned at both its top and bottom.

13–127. Using the NFPA equations of Sec. 13.6 and Eq. 13–30, determine the maximum allowable eccentric load P that can be applied to the wood column. Assume that the column is pinned at the top and fixed at the bottom.



Probs. 13–126/127

CHAPTER REVIEW

Buckling is the sudden instability that occurs in columns or members that support an axial compressive load. The maximum axial load that a member can support just before buckling is called the critical load P_{cr}.		
The critical load for an ideal column is determined from Euler's formula, where $K = 1$ for pin supports, $K = 0.5$ for fixed supports, $K = 0.7$ for a pin and a fixed support, and $K = 2$ for a fixed support and a free end.	$P_{cr} = \dfrac{\pi^2 EI}{(KL)^2}$	
If the axial loading is applied eccentrically to the column, then the secant formula can be used to determine the maximum stress in the column.	$\sigma_{max} = \dfrac{P}{A}\left[1 + \dfrac{ec}{r^2}\sec\left(\dfrac{L}{2r}\sqrt{\dfrac{P}{EA}}\right)\right]$	
When the axial load causes yielding of the material, then the tangent modulus should be used with Euler's formula to determine the critical load for the column. This is referred to as Engesser's equation.	$\sigma_{cr} = \dfrac{\pi^2 E_t}{(KL/r)^2}$	
Empirical formulas based on experimental data have been developed for use in the design of steel, aluminum, and timber columns.		

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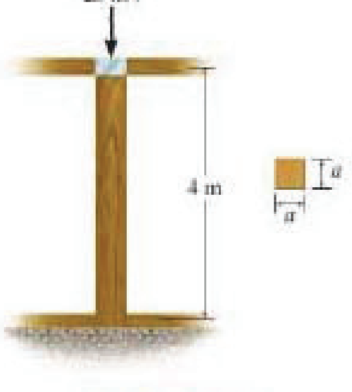
REVIEW PROBLEMS

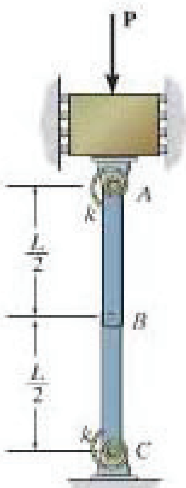
13

*13–128. The wood column is 4 m long and is required to support the axial load of 25 kN. If the cross section is square, determine the dimension a of each of its sides using a factor of safety against buckling of F.S. = 2.5. The column is assumed to be pinned at its top and bottom. Use the Euler equation. $E_w = 11$ GPa, and $\sigma_Y = 10$ MPa.



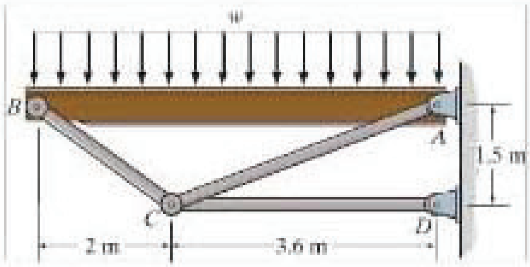
Prob. 13–128

•13–129. If the torsional springs attached to ends A and C of the rigid members AB and BC have a stiffness k, determine the critical load P_{cr}.



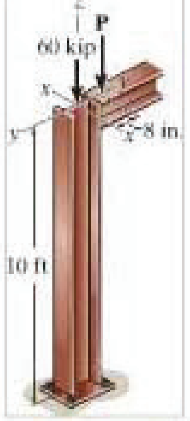
Prob. 13–129

13–130. Determine the maximum intensity w of the uniform distributed load that can be applied on the beam without causing the compressive members of the supporting truss to buckle. The members of the truss are made from A-36-steel rods having a 60-mm diameter. Use F.S. = 2 against buckling.



Prob. 13–130

13–131. The W10 × 45 steel column supports an axial load of 60 kip in addition to an eccentric load **P**. Determine the maximum allowable value of **P** based on the AISC equations of Sec. 13.6 and Eq. 13–30. Assume that in the x–z plane $K_x = 1.0$ and in the y–z plane $K_y = 2.0$. $E_{st} = 29(10^3)$ ksi, $\sigma_Y = 50$ ksi.



Prob. 13–131

*13–132. The A-36-steel column can be considered pinned at its top and fixed at its base. Also, it is braced at its mid-height along the weak axis. Investigate whether a W250 × 45 section can safely support the loading shown. Use the allowable stress method.

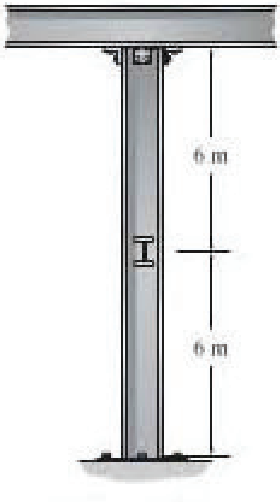
•13–133. The A-36-steel column can be considered pinned at its top and fixed at its base. Also, it is braced at its mid-height along the weak axis. Investigate whether a W250 × 45 section can safely support the loading shown. Use the interaction formula. The allowable bending stress is $(\sigma_b)_{allow} = 100$ MPa.

Probs. 13–132/133

13–134. The member has a symmetric cross section. If it is pin connected at its ends, determine the largest force it can support. It is made of 2014-T6 aluminum alloy.

Prob. 13–134

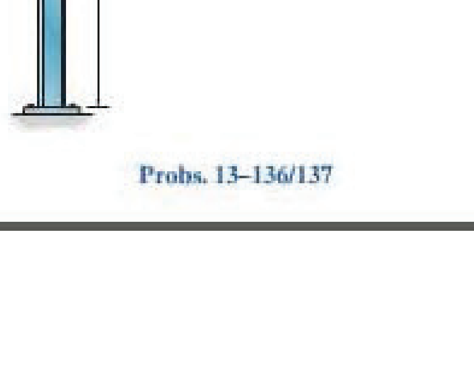
13–135. The W200 × 46 A-36-steel column can be considered pinned at its top and fixed at its base. Also, the column is braced at its mid-height against the weak axis. Determine the maximum axial load the column can support without causing it to buckle.



Prob. 13–135

*13–136. The structural A-36 steel column has the cross section shown. If it is fixed at the bottom and free at the top, determine the maximum force P that can be applied at A without causing it to buckle or yield. Use a factor of safety of 3 with respect to buckling and yielding.

•13–137. The structural A-36 steel column has the cross section shown. If it is fixed at the bottom and free at the top, determine if the column will buckle or yield when the load $P = 10$ kN. Use a factor of safety of 3 with respect to buckling and yielding.



Probs. 13–136/137