WILEY FINANCE

FUNDAMENTALS OF FINANCIAL INSTRUMENTS

An Introduction to
Stocks, Bonds, Foreign Exchange, and Derivatives

SUNIL PARAMESWARAN

Fundamentals of Financial Instruments

Fundamentals of Financial Instruments

An Introduction to Stocks, Bonds, Foreign Exchange, and Derivatives

Sunil Parameswaran

WILEY

John Wiley & Sons (Asia) Pte. Ltd.

Published in 2011 by John Wiley & Sons (Asia) Pte. Ltd.
1 Fusionopolis Walk, #07-01, Solaris South Tower, Singapore 138628

Other Wiley Editorial Offices

John Wiley & Sons, 111 River Street, Hoboken, NJ 07030, USA
John Wiley & Sons, The Atrium, Southern Gate, Chichester, West Sussex, P019 8SQ, United Kingdom
John Wiley & Sons (Canada) Ltd., 5353 Dundas Street West, Suite 400, Toronto, Ontario, M9B 6HB, Canada
John Wiley & Sons Australia Ltd., 42 McDougall Street, Milton, Queensland 4064, Australia
Wiley-VCH, Boschstrasse 12, D-69469 Weinheim, Germany

Library of Congress Cataloging-in-Publication Data

ISBN 978-0-470-82490-0 (Hardcover)
ISBN 978-0-470-82910-3 (ePDF)
ISBN 978-0-470-82909-7 (Mobi)
ISBN 978-0-470-82911-0 (ePub)

Typeset in 10/12 point Palatino-Roman by MPS Limited, a Macmillan Company, Chennai
Printed in Singapore by Markono Print Media Pte. Ltd.
10 9 8 7 6 5 4 3 2 1

Contents

Preface

This book grew out of the lecture notes I used for corporate training programs in India and abroad. The feedback from participants has been invaluable in polishing and refining the exposition. The eventual flow and clarity that I believe I have been able to achieve is in no small measure due to the critical and incisive inputs of my students as well as professional acquaintances.

There is typically no course exclusively on financial instruments in most MBA curricula. Consequently, this topic invariably gets combined with material on financial institutions as a part of a course on financial markets. I believe, however, that there is a strong case for offering a comprehensive course on financial instruments for second-year MBA students. But care should be taken to ensure that material that is covered in traditional courses such as financial derivatives and fixed income securities is not repeated any more than is necessary. Many students who take such a course may not be majoring in finance and consequently may not take specialized courses such as derivatives. For them, therefore, the specter of substantial overlap is less of an issue. Even for students of finance, despite the inevitable repetition of facets of topics such as bonds, futures, and options, a course on instruments is a comprehensive and integrated offering that will serve them in good stead in the future.

The course starts from first principles and builds in intensity. Exposure to one or more courses on financial management will certainly be useful for the reader as he or she navigates through the material. Although the book is a stand-alone treatise on the subject, readers may like to augment it with standard texts on issues in finance such as security analysis, bond markets, futures and options, and international finance.

The issues covered in this book are universal and of relevance for students of finance as well as market professionals irrespective of where they may happen to be located. However, most of the illustrations and examples pertain to markets in the developed world, particularly the United States, and the products that trade in them. Consequently, the book should have appeal for readers in all parts of the world.

Readers are welcome to share their comments and suggestions with me. Constructive advice and criticism are welcome and will be incorporated in future revisions. My e-mail is skp@tarcon.org.

I hope that the book serves as a source of pertinent and comprehensive knowledge for readers everywhere.

Sunil K. Parameswaran
T. A. Pai Management Institute
Manipal, 576104
Karnataka, India
July 2011

Acknowledgments

I am indebted to all my students in India and Australia who went through this material and offered ideas for embellishing the content and polishing the exposition. The participants at my Executive Education programs offered invaluable advice. Because most of them were non-finance professionals, primarily from the information technology field rather than finance, they had unusual and interesting perspectives that augmented the more traditional feedback from business school students and associates.

I am extremely grateful to John Wiley, Singapore, for giving me the opportunity to develop and promote this book. In particular, I owe a tremendous debt to my publisher, Nick Wallwork, for taking a chance with an unknown Indian academic. The production team of Janis Soo and Joel Balbin, based in Singapore, has been very supportive of this project right from the outset, and I owe them an enormous debt. Ms. Nalini and her team at Chennai have been wonderfully co-operative and provided invaluable support at the copyediting and typesetting phases.

And finally I am indebted to my mother for her patience and moral support.

CHAPTER 1

An Introduction to Financial Institutions, Instruments, and Markets

The Role of an Economic System

Economic systems are designed to collect savings in an economy and allocate the available resources efficiently to those who either seek funds for current consumption in excess of what their resources would permit or to invest in productive assets.

The key role of an economic system is to ensure *efficient allocation*. The efficient and free flow of resources from one economic entity to another is a *sine qua non* for a modern economy. This is because the larger the flow of resources and the more efficient their allocation, the greater the chance that the requirements of all economic agents can be satisfied, and consequently the greater the odds that the output of the economy as a whole will be maximized.

The functioning of an economic system entails the making of decisions about both the production of goods and services and their subsequent distribution. The success of an economy is gauged by the extent of wealth creation. A successful economy is one that makes and implements judicious economic decisions from the standpoints of both production and distribution. In an efficient economy, resources will be allocated to those economic agents who are in a position to derive the optimal value of output by employing the resources allocated to them.

Why are we giving the efficiency of an economic system so much importance? Because every economy is characterized by a relative scarcity of resources as compared to the demand for them. In principle, the demand for resources by economic agents can be virtually unlimited, but in practice economies are characterized by a finite stock of resources. Efficient allocation requires an extraordinary amount of information as to what people need, how goods and services can best be produced to cater to these needs, and how the produced output can best be distributed.

Economic systems may be classified as either *command economies* or *free-market economies*. These are the two extreme ends of the economic spectrum. Most modern economies tend to display characteristics of both kinds of systems, and they differ only with respect to the level of government control.

A Command Economy

In a command economy, like that of the former Soviet Union, all production and allocation decisions are made by a central planning authority. The planning authority is expected to estimate the resource requirements of various economic agents and then rank them in order of priority in relevance to social needs. Production plans and resource-allocation decisions are then made to ensure that resources are directed to users in descending order of need. Communist and socialist systems that were based on this economic model ensured that citizens complied with the directives of the state by imposing stifling legal and occasionally coercive measures.

The failure of the command economies was inherent in their structure. As mentioned, efficient economic systems need to aggregate and process an enormous amount of information. Entrusting this task to a central planning authority proves infeasible and also means the quality of information is substandard. The central planning authority was supposed to be omniscient and was expected to have perfect information about available resources and the relative requirements of the socioeconomic system. This was necessary for the authority to ensure that optimal decisions were made about production and distribution.

Command economies were plagued by blatant political interference. The planning authority was often prevented from making optimal decisions because of political pressures. The system gave enormous power to planners that permeated all facets of the social system and not just the economy. One hallmark of such a system was the absence of pragmatism and the presence of a naïve idealism that was out of touch with reality. Planners used their authority to devise and impose stifling rules and regulations. The regulations, which were in principle intended to ensure optimal decision making, at times went to the ridiculous extent of imposing penalties on producers whose output exceeded what was allowed by the permit or license given to them.

Such economies were a colossal failure, characterized by an output that was invariably far less than the ambitious targets that were set at the outset of each financial year. When confronted with the specter of failure, planners tended to place the blame on those who were responsible for implementing the plans. The bureaucrats in charge of implementation passed the buck upward by alleging improper decision making on the part of the planners. Eventually, contradictions in the system led to either the total repeal of such systems or to substantial structural changes that brought in key features of a market economy.

A Market Economy

In principle, a market economy works as follows. Economic agents are expected to make the most profitable use of the resources at their disposal. What is profit? Profit is defined as the revenues from sales minus the costs of production of the goods sold. Profit is a function of the prices of the inputs or the factors of production—such as land, labor, and capital—and the prices of the output. An optimal economic decision is defined as one that maximizes profit. Economic agents who generate surpluses of income in excess of expenditures will be able to attract more and better resources. Failure, as manifested by sustained losses, will result in those economic agents being denied access to the resources they seek.

In such a system, the prices of both inputs and outputs are determined by factors of supply and demand. In contrast to a command economy, a market economy is characterized by decentralized decision making. In principle, every agent is expected to take a rational decision by evaluating competing resource needs based on his or her ability to generate surpluses. Every decision maker will have a required rate of return on investment. The threshold return, or the return above which the venture will be deemed profitable, is the cost of capital for the decision maker. A project is considered to be worth the investment only if its expected rate of return is greater than the cost of the capital being invested.

As we can surmise, the key decision variables in these economies are the prices of inputs and outputs. For such economies to work in an optimal fashion, prices must accurately convey the value of a good or a service from the standpoints of both producers who employ factors of production and consumers who consume the end products. The informational accuracy of prices results in the efficient allocation of resources for the following reasons. If the inputs for the production process such as labor and capital are accurately priced, then producers can make optimal production-related decisions. Similarly, if the consumers of goods and services perceive their prices to be accurate, they will make optimal consumption decisions. The accuracy of input-related costs and output prices will be manifest in the form of profit maximization, which is the primary motivating factor for agents in such economies to engage in economic enterprise.

How do such systems ensure that prices of inputs and outputs are informationally accurate? This is ensured by allowing economic agents to trade in markets for goods and services. If agents have the perception that the price of an asset is different from the value that they place on it, they will seek to trade. If the prevailing price is lower than the perceived value, buyers will seek to buy more of the good than the quantity on offer. If so, the market price will be bid up because of demand being greater than the amount on offer. This demand–supply disequilibrium will persist until the price reaches the optimal level. Similarly, if the price of the good is perceived to be too high relative to the value placed on it by agents, sellers will seek to offload more than what is being demanded. Once again, the supply–demand imbalance will cause prices to decline until equilibrium is restored. Differing perceptions of value will manifest themselves as supply–demand imbalances, the process of resolution of which ultimately helps to ensure that the prices of assets accurately reflect their value.

Opinion

Although free-market economies have largely been more successful than command economies, no one would advocate a total absence of a government's role in economic decision making. Unfettered capitalism, particularly in the aftermath of the current economic crisis, is unlikely to find acceptance anywhere. There are disadvantaged sections of every society whose fate cannot be left to the market and whose well-being has to be ensured by policy makers to promote overall welfare.

Classification of Economic Units

Economic agents are usually divided into three categories or sectors: government, business, and household.

The government sector consists of a nation's central or federal government, state or provincial governments, and local governments or municipalities. The business sector consists of sole proprietorships, partnerships, and both private and public limited companies. Sometimes business units are broadly subclassified as financial corporations and nonfinancial corporations.

Proprietorships

A proprietorship, also known as a *sole proprietorship*, is a business owned by a single person and is the easiest way to start a business. An owner may do business in his or her own name or using a trade name. For instance, a consultant named John Smith may run the business in his name or choose a name such as Business Systems. The owner is fully responsible for all debts and obligations of the business. In other words, creditors—entities to which the business owes money—may stake a claim against all assets of the proprietor, whether they are business-related assets or personal assets. In legal parlance, this is referred to as *unlimited liability*, as opposed to a corporation whose owners have *limited liability*, a point we will shortly explore in greater detail.

The start-up costs of a sole proprietorship are usually fairly low compared to other forms of business. However, unlike a corporation, such businesses face relative difficulties in raising additional capital if and when they choose to expand the scope of their operations. Usually, in addition to the owner's personal investment, the only source of funds is a loan from a commercial bank.

Legally, the proprietorship is an extension of the owner. The owner is permitted to employ other people. The net profits from the business are clubbed with the proprietor's other income, if any, for the purpose of taxation. The life span of these entities is fairly uncertain. If the owner dies, the business ceases to exist.

Partnerships

A partnership is a business entity owned by at least two people or partners. One partner may be a corporation, a concept that we will explain next. The legal extension of the partnership is an extension of the partners. Like a proprietorship, a partnership is allowed to employ others, and it can conduct a business under a trade name. Two lawyers named Joan Smith and Mary Jones may conduct their business as Smith & Jones or under a trade name such as Legal Point. In a general partnership, the partners have unlimited liability and a partner is personally responsible not only for her own acts but also for the actions of her other partners as well as employees.

There are two categories of partnerships in many countries: general partnerships and limited partnerships. A *general partnership* is what we have just discussed. In a *limited partnership*, there are two categories of partners, namely general partners and limited partners. The general partners are usually a corporation and have management control. They are characterized by unlimited liability. The limited partners, on the other hand, are like shareholders in a corporation: their potential loss is limited to the investment they have made.

Like a sole proprietorship, a partnership is also fairly easy to establish. However, unlike a proprietor, who is the sole decision maker, partners must share authority with the others. It is very important to draw up a partnership agreement at the outset, where issues such as profit sharing are clearly spelled out. Compared to corporations, partnerships also find it relatively difficult to raise capital in order to expand their businesses.

Corporations

A corporation or a limited company is a legal entity that is distinct and separate from its owners, who are referred to as *shareholders* or *stockholders*. A corporation may and usually will have multiple owners as well as a number of employees on its payroll. It must necessarily do business under a given trade name. Because a corporation is a separate legal entity, it has the right to sue and be sued in its own name. Shareholders of a corporation enjoy limited liability. Unlike a proprietorship or partnership, the ownership of a company can easily change hands. Each shareholder will possess a number of shares of the company that can be usually bought and sold in a marketplace known as a *stock exchange*. Although such share transfers may result in one party relinquishing majority control in favor of another, the transfers per se have no implications for the corporation's continued existence or its operations. Unlike proprietorships and partnerships, corporations find it relatively easier to raise both debt or borrowed capital, as well as equity or owners' capital. However, in most countries corporations are extensively regulated and are required by statutes to maintain extensive records pertaining to their operations. The cost of incorporation and the costs of raising equity through share issues can also be substantial. Although owners of a corporation may be a part of its management team, very often ownership and management are segregated, entrusting the management of day-to-day activities to a team of professional managers. In some countries, there exist entities known as *private limited companies*. These companies cannot offer shares to the general public, and the shares cannot be traded on a stock exchange. However, the shareholders continue to enjoy limited liability, hence the name. The disclosure norms for public limited companies are generally more stringent than those for private limited companies.

During a given financial year, every economic unit, irrespective of which sector it may belong to, will get some form of income in the course of its operations and will also incur expenditure in some form. Depending on the relationship between the income earned and the expenditure incurred, an economic unit may be classified into one of the following three categories: (1) a balanced budget unit, (2) a surplus budget unit, or (3) a deficit budget unit.

In reality, a balanced budget unit is impossible, so it exists only in the realm of textbooks. It is virtually impossible for a business or government to ensure that its scheduled income during a period is perfectly matched with its scheduled expenditure during the same period. An economic unit may be a *surplus budget unit* (SBU) or a *deficit budget unit* (DBU). A surplus budget unit is one with an income that exceeds its expenditure, whereas a deficit unit is one with expenses that exceed its income. Usually, in most countries, governments and businesses invariably tend to be deficit budget units, whereas households consisting of individuals and families generally tend to be savers—that is, they tend to have budget surpluses. By this we do not mean that all households and individuals are savers or that all governments and businesses have a budget deficit. We mean that even though it is not uncommon for a government or a business to have a surplus in a given financial period, as a group the government and business sectors generally tend to be net borrowers. By the same logic, it is not necessary that all households should save, although the category as a whole generally has a budget surplus in most periods. Finally, a country as a whole may have a budget surplus or a budget deficit.

An Economy's Relationship with the External World

The record of all economic transactions between a country and the rest of the world (ROW) is known as its *balance of payments* (BOP). It is a record of a country's trade in goods, services, and financial assets with the rest of the world. In other words, it is a record of all economic transactions between a country and the outside world. The transaction may be a requited transfer of economic value or an unrequited transfer of economic value. In this context, the term *requited* connotes that the transferor receives a compensation of economic value from the transferee. On the other hand, an *unrequited* transfer represents a unilateral gift made by the transferor.

The BOP is typically broken into three major categories of accounts, each of which is further subdivided into various components. The major categories are as follows:

- The current account: This accounting head includes imports and exports of goods and services, as well as earnings on investments.
- The capital account: Under this heading we have transactions that lead to changes in a country's foreign assets and liabilities.
- The reserve account: This account is similar to the capital account, in the sense that it also deals with financial assets and liabilities, but it deals only with reserve assets—that is, those assets used to settle the deficits and surpluses that arise on account of the other two categories taken together.

A reserve asset is one that is acceptable as a means of payment in international transactions and that is held by and exchanged between the monetary authorities of various countries. It consists of monetary gold, assets denominated in foreign currencies, special drawing rights (SDRs), and reserve positions at the International Monetary Fund (IMF). If there is a deficit in the current and capital accounts taken together, then there will be a depletion of reserves. However, if the two accounts show a surplus when taken together, there will be an increase in the level of reserves.

The balance of payments is an accounting system that is based on the double-entry system of bookkeeping. Every transaction is recorded on both the sides—that is, as both a credit and a debit. All transactions that have led or will lead to an inflow of payments into the country from the ROW will be shown as credits. The payments themselves would be depicted as the corresponding debits. Similarly, all transactions that have led to or will lead to a flow of payments from the country to the ROW will be recorded as debits. The payments will be recorded as the corresponding credits.

A payment received from abroad will increase the country's foreign assets. Thus, an increase in foreign assets or a decrease in foreign liabilities will be shown as a debit. On the other hand, a payment made to an external party will either reduce the country's holding of foreign assets or show up as an increase in its liabilities. This will be shown as a credit.

The accounting principle may also be viewed as follows. Any transaction that leads to an increase in the demand for foreign exchange should be shown as a debit, and any transaction that leads to an increase in the supply of foreign exchange should be shown as a credit. Capital outflows will be debited, whereas capital inflows will be credited.

The balance of payments must always balance. A current account deficit must be matched by a surplus in the capital account or by a depletion of reserves or both. A surplus in the capital account would indicate that the country's foreign liabilities have gone up—in other words, the country has borrowed from abroad. While analyzing the BOP, it is customary to study several subcategories of accounts. We will look at two of the most important subclassifications.

The Balance of Trade

The balance of trade is equal to the sum total of merchandise exports and imports. It consists of all raw materials and manufactured goods bought, sold, or given away. If it shows a surplus, it indicates that exports of goods from the country exceed imports into it; if it shows a deficit, it would indicate that imports exceed exports. The balance of trade is a politically sensitive statistic. If a country's balance of trade shows a deficit, then industries that are being hurt by competition from abroad will typically raise a hue and cry about the need for a level playing field in order to take on the foreign competition.

The Current Account Balance

The current account balance refers to the sum total of the following accounts:

- Exports of goods, services, and income;
- Imports of goods, services, and income; and
- Net unilateral transfers.

Services include tourism, transportation, engineering, and business services. Fees from patents and copyrights are also recognized under this category. Income includes revenue from financial assets, such as dividends from shares and interest from debt securities. Unilateral transfers are one-way transfers of assets, such as worker remittances from foreign countries, and direct foreign aid.

A current account that shows a deficit indicates that the country's liabilities have increased; if it shows a surplus, it means that a country's assets held abroad have increased.

Financial Assets

"A financial asset is a claim against the income or wealth of a business firm, a household, or a government agency, which is represented usually by a certificate, a receipt, a computer record file, or another legal document, and is usually created by or is related to the lending of money."[1]

A financial claim is born in the following fashion. Whenever funds are transferred from a surplus budget unit to a deficit budget unit, the DBU will issue a financial claim. It signifies that the party that is transferring the funds has a claim against the party that is accepting the funds. The transfer of funds from the lender may be in the form of a loan to the borrower or constitute the assumption of an ownership stake in the venture of the borrower. In the case of loans, the claim constitutes a promise to pay the interest either at maturity or at periodic intervals and to repay the principal at maturity. Such claims are referred to as *debt securities* or as *fixed income securities*. In the case of fund transfers characterized by the assumption of ownership stakes, the claims are known as *equity shares*. Unlike debt securities, which represent an obligation on the part of the borrower, equity shares represent a right to the profits of the issuing firm during its operation and to such assets that may remain after all creditors are fully paid as in the event of the venture's liquidation. Remember that claims are always issued by the party that is raising funds, and they are held by parties that are providing the funds.

To the issuer of the claim, the claim is a liability because it signifies that it owes money to another party. To the lender, or the holder of the claim, it is an asset because it signifies that the holder owns an item of value. The sum total of financial claims issued must equal the total financial assets held by investors, and every liability incurred by a party must be an asset for another investor.

Why do investors acquire financial assets? Financial assets are essentially sought after for three reasons.

1. They serve as a store of value or purchasing power.
2. They promise future returns to their owners.
3. They are *fungible*—that is, they can be easily converted into other assets and vice versa.

In addition to debt securities and equity shares, we will also focus on the following assets: money, preferred shares, foreign exchange, derivatives, and mortgages and mortgage-backed securities.

Money

Money is a financial asset because all forms in use today are claims against some institution. Contrary to popular perception, money is not just the coins and currency notes that are handled by economic agents. One of the largest components of money supply today is the checking account balances held by depositors with commercial banks. From the banks' standpoint, these accounts represent a debt obligation. Banks have the capacity to both expand and contract the money supply in an economy. Currency notes and coins also represent a debt obligation of the central bank of the issuing country, such as the Federal Reserve in the United States. In today's electronic age, newer forms of money have emerged, such as credit cards, debit cards, and smart cards.

Money performs a wide variety of important functions, and for that reason it is much sought after. In a modern economy, all financial assets are valued in terms of money, and all flows of funds between lenders and borrowers occur via the medium of money.

Money as a Unit of Account or a Standard of Value

In the modern economy, the value of every good and service is denominated in terms of the unit of currency. Without money, the price of every good or service would have to be expressed in terms of every other good or service. The availability of money leads to a tremendous reduction in the amount of price-related information that has to be processed.

Take the case of a 100 good economy. In the absence of money we would require ${}^{100}C_2$ or 4,950 prices.[2] However, if a currency were to be available, we would require only 100 prices, a saving of almost 98 percent in terms of the required amount of information.

Money as a Medium of Exchange

Money is usually the only financial asset that every business, household, and government department will accept as payment in return for goods and services.

Why is it that everyone is willing to readily accept money as compensation? It is primarily because they can always use it whenever required to acquire any goods or service they desire. Because of the existence of money, it is possible to separate in time the act of sale of goods and services from any subsequent acquisition of goods and services.

In the absence of money, we would have to exchange goods and services for other goods and services, a phenomenon that is termed *barter* or *countertrade*.

Money as a Store of Value

Money also serves as a store of value, or as a reserve of future purchasing power. However, just because an item is a store of value, it does not necessarily mean it is a good store of value. In the case of money, inflation or the erosion of its purchasing power is a virtually constant feature.

Take the example of a good that costs \$2 per unit. If an investor has \$10 with him, he can acquire five units. However, if he were to keep the money with him for a year, he may find that the price in the following year is \$2.50 per unit, so he can only acquire four units.

Money Is Perfectly Liquid

What is a liquid asset? An asset is defined as liquid if it can be quickly converted into cash with little or no loss of value. Why is liquidity important? In the absence of liquidity, market participants will be unable to transact quickly at prices that are close to the true or fair value of the asset. Buyers and sellers will need to expend considerable time and effort to identify each other, and very often they will have to induce a transaction by offering a large premium or discount. If a market is highly liquid at a point in time, it means that plenty of buyers and sellers are available. In an illiquid or thin market, a large purchase or sale transaction is likely to have a major impact on prices. A large purchase transaction will send prices shooting up, whereas a large sale transaction will depress prices substantially. Liquid markets therefore have a lot of depth, as characterized by relatively minimal impact on prices. Liquid assets are characterized by three attributes: (1) price stability, (2) ready marketability, and (3) reversibility.

In these respects, money is the most liquid of all assets because it need not be converted into another form in order to exploit its purchasing power.

However, liquid assets come with an attached price tag. The more liquid the asset in which an investment is made, the lower the interest rate or rate of return from it. Thus, there is a cost attached to liquidity in the form of the interest forgone because of the inability to invest in an asset paying a higher rate of return. Such interest that is forgone is lost forever, and cash is the most perishable of all economic assets.

Equity Shares

Equity shares or shares of common stock of a company are financial claims issued by the firm that confer ownership rights on the investors, who are known as *shareholders*. Every shareholder is a part owner of the company that has issued the shares, and her stake in the firm is equal to the fraction of the total share capital of the firm to which she has subscribed. In general, all companies will have equity shareholders, because common stock represents the fundamental ownership interest in a corporation. Thus, a company must have at least one shareholder. Shareholders will periodically receive cash payments called *dividends* from the firm. In addition, the shareholders are exposed to profits and losses when they seek to dispose off their shares at a subsequent point in time. These profits and losses are referred to as *capital gains* and *losses*.

Equity shares represent a claim on the residual profits after all of the company's creditors have been paid. In other words, a shareholder cannot demand a dividend as a matter of right. The creditors of a firm, including those who have extended loans to it, enjoy priority from the standpoint of payments and are therefore ranked higher in the pecking order.

Equity shares have no maturity date. They continue to exist as long as the firm itself continues to exist. Shareholders have voting rights and a say in the election of the board of directors. If the firm were to declare bankruptcy, then the shareholders would be entitled to the residual value of the assets after the claims of all other creditors have been settled. Creditors enjoy primacy compared to shareholders.

The major difference between the shareholders of a company as opposed to a sole proprietor or the partners in a partnership is that they have limited liability: that is, no matter how serious the financial difficulties facing a company may be, neither it nor its creditors can make financial demands on the common shareholders. The maximum loss that a shareholder may sustain is limited to her investment in the business.

Debt Securities

A debt instrument is a financial claim issued by a borrower to a lender of funds. Unlike equity shareholders, investors in debt securities are not conferred with ownership rights. These securities are merely IOUs (an acronym for "I owe you") that represent a promise to pay interest on the principal amount either at periodic intervals or at maturity, as well as to repay the principal itself at a prespecified maturity date.

Most debt instruments have a finite life span—that is, a stated maturity date—and hence differ from equity shares in this respect. Also, the interest payments that are promised to the lenders at the outset represent contractual obligations on the part of the borrower. In other words, the borrower is required to meet these obligations irrespective of the performance of the firm in a given financial year. It is also the case that, in the event of an exceptional performance, the borrowing entity does not have to pay any more to the debt holders than what was promised at the outset. It is for this reason that debt securities are referred to as *fixed income securities*. The interest claims of debt holders have to be settled before any residual profits can be distributed by way of dividends to shareholders. In the event of bankruptcy or liquidation, the proceeds from the sale of assets of the firm must be used to first settle all outstanding interest and principal. Only the residual amount, if any, can be distributed among the shareholders.

Although debt is important for a commercial corporation, in both the public and the private sectors of an economy, it is absolutely indispensable for central or federal, state, and local (municipalities) governments when they wish to finance their developmental activities. Such entities cannot issue equity shares. For instance, a U.S. citizen cannot become a part owner of the state of Illinois.

Debt instruments can be secured or unsecured. In the case of secured debt, the terms of the contract will specify the assets of the firm that have been pledged as security or collateral. In the event of the failure of the company, the bondholders have a right over these assets. In the case of unsecured debt securities, the investors can only hope that the issuer will have the earnings and liquidity to redeem the promise made at the outset.

Debt instruments can be either negotiable or nonnegotiable. Negotiable securities are instruments that can be endorsed from one party to another, so they can be bought and sold easily in the financial markets. A nonnegotiable instrument is one that cannot be transferred. Equity shares are negotiable securities. Although many debt securities are negotiable, certain loan-related transactions such as loans made by commercial banks to business firms, as well as individuals' savings bank accounts, are examples of assets that are not negotiable.

Debt securities are referred to by a variety of names such as *bills*, *notes*, *bonds*, and *debentures*. In the United States, the word *debenture* is used to refer to unsecured debt securities. U.S. Treasury securities are fully backed by the federal government and have no credit risk associated with them. The term *credit risk* refers to the risk that the issuer may default or fail to honor his commitment. The interest rate on Treasury securities is used as a benchmark for setting the rates of return on other, more risky securities. The U.S. Treasury issues three categories of marketable debt instruments: T-bills, T-notes, and T-bonds. Also known as *zero coupon securities*, T-bills are discount securities—that is, they are sold at a discount from their face value and do not pay any interest. They have a maturity at the time of issue that is less than or equal to one year. T-notes and T-bonds are sold at face value and pay interest periodically. A T-note is akin to a T-bond but has a time to maturity between 1 and 10 years at the time of issue, whereas T-bonds have a life of more than 10 years.

Preferred Shares

Preferred stocks are a hybrid of debt and equity. They are similar to debt in the sense that holders of such securities are usually promised a fixed rate of return. However, such dividends are payable from the post-tax profits of the firm, as in the case of equity shares. On the other hand, interest payments to bond holders are made from pretax profits and therefore constitute a deductible expense for tax purposes.

If a company were to refrain from paying the preferred dividends in a particular year, then the shareholders, unlike the bondholders, cannot take legal recourse as a matter of right. Most preferred shares are *cumulative* in nature. This implies that any unpaid dividends in a financial year must be carried forward, and the accumulated dividends must first be paid before the company can contemplate the payment of dividends to equity shareholders.

Preferred shareholders have restricted voting rights: they usually do not enjoy the right to vote unless the payment of dividends due to them is in arrears. In the event of liquidation of the firm, the preferred shareholders will have to be paid off before the claims of the equity holders can be entertained. The order of priority of the stakeholders of the firm from the standpoint of payments is bondholders first, preferred shareholders second, and then equity shareholders. The term *preferred* arises because such shareholders are given preference over equity shareholders, and not because the shareholders prefer such instruments.

Foreign Exchange

The term *foreign exchange* refers to transactions pertaining to the currency of a foreign nation. Foreign-exchange markets are markets in which foreign currencies are bought and sold. A foreign currency is also a type of financial asset, and it will have a price in terms of another currency. The price of one country's currency in terms of the currency of another is referred to as the *exchange rate*. Foreign currencies are traded among a network of buyers and sellers, comprising mainly commercial banks and large multinational corporations, and not on an organized exchange. The market for foreign exchange is referred to as an *over-the-counter* (OTC) *market*. Physical currency is rarely paid out or received. What happens is that currency is transferred electronically from one bank account to another.

Derivatives

Derivative securities, which are more appropriately termed *derivative contracts*, are assets that confer on their owners certain rights or obligations as the case may be. These contracts owe their availability to the existence of markets for an underlying asset or a portfolio of assets on which such agreements are written. In other words, these assets are derived from the underlying asset. If we perceive the underlying asset as the primary asset, then such contracts may be termed as *derivatives*, because they are derived from such assets.

The three major categories of derivative securities are (1) forward and futures contracts, (2) options contracts, and (3) swaps.

Forward and Futures Contracts

In a typical transaction, where the exchange of cash for the asset being procured takes place immediately—which is referred to as a *cash* or *spot transaction*—the buyer has to hand over the payment for the asset to the seller as soon as the deal is struck; in turn, the seller has to transfer the rights to the asset to the buyer at the same point in time. However, in the case of a forward or a futures contract, the actual transaction does not take place when an agreement is reached between the two parties. What happens in such cases is that, at the time of negotiating the deal, the two parties merely agree on the terms on which they will transact at a future point in time. The actual transaction per se occurs only at a future date that is decided at the outset and at a price that also is decided at the beginning. No money changes hands when two parties enter into such a contract. However, both parties to the contract have an obligation to go ahead with the transaction on the predetermined date as per the agreed-upon terms, as illustrated in Example 1.1. Failure to do so will be tantamount to default.

Example 1.1

WIPRO Technologies has imported products from Frankfurt and has entered into a forward contract with HSBC to acquire €500,000 after 60 days at an exchange rate of 62.50 India rupees (INR) per euro. This is a forward contract, because even though the terms and conditions, including the exchange rate, are fixed at the outset, the currency itself will be procured only 60 days after the date of the agreement. Sixty days hence, WIPRO will be required to pay INR 31,250,000 to the bank and accept the euros. The bank, per the contract, is obliged to accept the Indian currency and deliver the euros.

Forward contracts and futures contracts are similar in the sense that both require (1) the buyer to acquire the underlying asset on a future date and (2) the seller to deliver the asset on that date. And, in the case of either kind of security, both the buyer and the seller have an obligation to perform at the time of expiration of the contract. However, there is one major difference between the two types of contracts. Futures contracts are standardized, whereas forward contracts are customized. The terms *standardization* and *customization* may be understood as follows. In any contract of this nature, certain terms and conditions need to be clearly defined. The major terms that should be made explicit are the following:

1. The number of units of the underlying asset that have to be delivered per contract;
2. The acceptable grade or grades that may be delivered by the seller;
3. The place or places were the seller is permitted to deliver; and
4. The date or, in certain cases, the time interval during which the seller must deliver.

In a customized contract, the above terms and conditions have to be negotiated between the buyer and the seller of the contract. The two parties are at liberty to incorporate any features that they can mutually agree on. Forward contracts come under this category. In standardized contracts, however, a third party will specify the allowable terms and conditions. The two parties to the contract have to design the terms and conditions within the framework specified by the third party and cannot incorporate features other than those that are specifically allowed. The third party in the case of futures contracts is the *futures exchange*, which is the trading arena where such contracts are bought and sold.

Options Contracts

As we have mentioned, both forward as well as futures contracts, despite the differences inherent in their structures, impose an obligation on both the buyer and the seller. The buyer is obliged to take delivery of the underlying asset on the date that is agreed on at the outset, and the seller is obliged to make delivery of the asset on that date and accept cash in lieu.

Options contracts are different. Unlike the buyer of a forward or a futures contract, the buyer of an options contract has the right to go ahead with the transaction subsequent to entering into an agreement with the seller of the option. The difference between a right and an obligation is that a right need be exercised only if it is in the interest of its holder, and if she deems it appropriate. The buyer or holder of the contract does not face a compulsion to subsequently go through with the transaction. However, the seller of such contracts always has an obligation to perform if the buyer were to deem it appropriate to exercise her right. Example 1.2 illustrates this.

Example 1.2

Peter Norton has acquired an options contract that gives him the right to buy 100 shares of GE at a price of $42.50 per share after three months from Mike Selvey. If the price of GE shares after three months were to be greater than $42.50 per share, it would make sense for Peter to exercise his right and acquire the shares. Otherwise, if the share price were to be lower than $42.50, he can simply forget the option and buy the shares in the spot market at a lower price. Notice that he is under no compulsion to exercise the option: it confers a right on the holder but does not impose an obligation. However, if Peter were to decide to exercise his right to buy, Mike would have no choice but to deliver the shares at a price of $42.50 per share. Options contracts always impose a performance obligation on the seller of the option, if the option holder were to exercise his right. The reason is that when two parties enter into an agreement for a transaction that is scheduled for a future date, then at the time of expiration of the contract, a price move that translates into a profit for one of the two parties will lead to a loss for the other. We cannot have a contract that confers to both parties the right to perform, because the party who is confronted with a loss will simply refuse to perform. We can have contracts that impose an obligation on both, such as forward and futures contracts, or else we can have a contract that confers a right on one party and imposes an obligation on the other, which is essentially what an options contract does.

When a person is given a right to transact in the underlying asset, the right can take on one of two forms: he may either have the right to buy the underlying asset or else he may have the right to sell the underlying asset. Options contracts that give the holder the right to acquire the underlying asset are known as *call options*. If the buyer of such an option were to exercise his right, the seller of the option is obliged to deliver the underlying asset as per the terms of the contract. Peter, in the above example, possesses a call option.

There exist options contracts that give the holder the right to sell the underlying asset. These are known as *put options*. In the case of such contracts, if the holder were to decide to exercise his option, the seller of the put is obliged to take delivery of the underlying asset.

Options give the holder the right to buy or sell the underlying asset. If the contract were to permit exercise only at the time of expiration, the option, whether a call or a put, is known as a *European option*. If such an option is not exercised at the time of expiration, then the contract itself will expire. There exists another type of contract in which the holder has the right to transact at any point in time between the time of acquisition of the right and the expiration date of the contract. These are referred to as *American options*. The expiration date is the only point in time at which a European option can be exercised, and it is the last point in time at which an American option can be exercised.

An options contract, whether a call or a put, requires the buyer to pay a price to the seller at the outset for giving him the right to transact. This price is known as the *option price* or *option premium*. This price is nonrefundable if the contract were not to be exercised subsequently. If and when an options contract is exercised, the buyer will have to pay a price per unit of the underlying asset if he is exercising a call option, and he will have to receive a price per unit of the underlying asset if he is exercising a put option. This price is known as the *exercise price* or *strike price*.

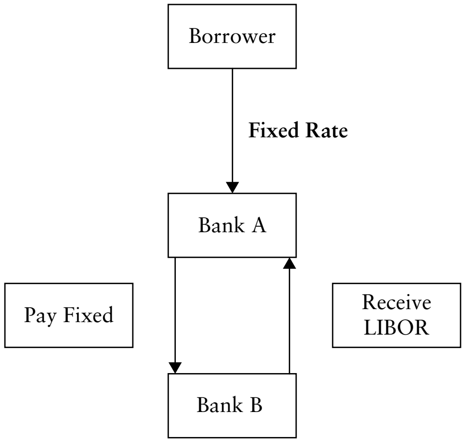
Futures and forward contracts, however, do not require either party to make a payment at the outset, because they impose an equivalent obligation on both the buyer and the seller. The futures price, which is the price at which the buyer will acquire the asset on a future date, will be set in such a way that the value of the futures contract at inception is zero from the standpoint of both the buyer and the seller.

Swaps

A swap is a contractual agreement between two parties to exchange cash flows calculated on the basis of prespecified terms at predefined points in time.

The cash flows being exchanged represent interest payments on a specified principal amount, which are computed using two different yardsticks. One interest payment may be computed using a fixed rate of interest, whereas the other may be based on a variable benchmark such as the T-bill rate.

A swap in which both payments are denominated in the same currency is referred to as an *interest rate swap*. The motivation for such a transaction may be understood as follows. Consider the case of a commercial bank that has entered into a fixed-rate loan with one of its clients. It may now be of the opinion that interest rates are going to rise. Renegotiation of the loan may not be feasible. Even if it were, it would involve substantial legal and administrative efforts. It would be much easier for the bank concerned to enter into a swap transaction with an institution, perhaps another commercial bank, wherein it pays a fixed rate of interest and receives a variable rate based on a benchmark such as the London interbank offer rate (LIBOR). By doing so, it would have converted its fixed-rate income stream to a floating-rate income stream and would stand to benefit if interest rates rise as anticipated. In a nutshell, the objective of a swap is to enable a party to dispose of a cash-flow stream in exchange for another cash-flow stream. Diagrammatically we can depict the above transaction as follows.



There also exist swaps where two parties exchange cash flows denominated in two different currencies. Such swaps are referred to as *currency swaps*.

Mortgages and Mortgage-Backed Securities

A mortgage is a loan that is backed by the collateral of specified real estate property. The borrower of funds, the mortgagor, is obliged to make periodic payments to the lender, the mortgagee, in order to retire the debt. In the event of the mortgagor defaulting, the lender can foreclose the mortgage, which means that she can take over the property in order to recover the balance that is due her.

A mortgage by itself is a fairly illiquid asset for the party that makes the loan to the home buyer. Such lenders are called *originators*. To rotate their capital, lenders will typically pool mortgage loans and issue debt securities that are backed by the underlying pool. Such securities—the cash flows for which arise from the payments made by borrowers of the underling loans—are referred to as *mortgage-backed securities*. The process of converting an illiquid asset such as a home loan into liquid marketable securities is referred to as *securitization*. Although very common in the case of mortgage lending, the process of securitization is not restricted to such loans. Receivables from automobile loans and credit card receivables are also securitized. The securities generated in the process are referred to as *asset-backed* securities.

Hybrid Securities

A hybrid security combines the features of more than one type of basic security. We will discuss two such assets: convertible bonds and warrants.

A *convertible bond* is a debt security that permits the investor to convert the bond into shares of equity at a predecided rate. Until and unless the investor converts the bond, it will continue to trade in the form of a standard debt security. The interest rate on such bonds will be lower than on securities without the option to convert, because the conversion feature will be perceived as a sweetener by potential investors. The rate of conversion from debt into equity will typically be set in such a way that the conversion price is higher than the market price prevailing at the time of issue of the debt. A bond with a principal value of $1,000 may be convertible to 25 shares of equity. In this case, the conversion price is $40. In such a case, the share price that is prevailing at the time of issue of the convertible will be less than $40.

A warrant is a right given to the investor that allows her to subscribe to the equity shares of the company at a future date at a predetermined price. Such rights are usually offered along with debt securities in order to make the bonds more attractive to investors. Once issued, the warrants can be detached from the parent security and traded in the secondary market.

Primary Markets and Secondary Markets

The function of a primary market is to facilitate the acquisition of new financial instruments by investors, both institutional and individual. When a company goes in for an issue of equity shares to the public, it will be termed a *primary market transaction*. Similarly, if the government were to raise funds by issuing Treasury bonds, it will once again be termed a *primary market transaction*.

Once a financial asset has been created and sold to an investor in the primary market, subsequent transactions in that instrument between two investors are said to take place in the secondary market. Assume that GE went in for a public issue of 5 million shares out of which Frank Reitz was allotted 10,000 shares. This would be termed a primary market transaction. Six months hence, Frank sells the shares to Mike Pierce on the New York Stock Exchange. This would constitute a secondary market transaction. Whereas primary markets are used by governments and business entities to raise medium- to long-term capital for making productive investments, secondary markets merely facilitate the transfer of ownership of an asset from one party to another.

Primary markets by themselves are insufficient to ensure the functioning of the free-market system; that is, secondary markets are a sine qua non for the efficient operation of the market economy. Why is this so? Consider an economy without a secondary market. In such an economy, an investor who subscribes to a debt issue would have to hold on to it until its date of maturity. In the case of equity shares, the problem will be more serious, because such securities never mature. The acquirer of shares in a primary market transaction and future generations of his family would have no option but to hold the shares forever. No investor will make an investment unless he is confident there exists an avenue for a subsequent sale if he were to decide that he no longer required the investment.

The ability to trade in a security after acquiring it in a primary market transaction is important for two reasons. First, one of the key reasons for investing in financial assets is that they can always be liquidated or converted into cash. Such needs can never be perfectly predicted, and investors would desire access to markets that facilitate the ready conversion of securities to cash and vice versa. Second, most investors do not hold their wealth in the form of a single asset, but prefer to hold a basket or portfolio of securities. As the old adage says, "Don't keep all your eggs in one basket." A prudent investor would seek to diversify her wealth among various asset classes such as stocks, bonds, real estate, and precious metals such as gold. This kind of diversification will usually be taken a step further in the sense that the entire wealth that an investor has earmarked to be held in stocks will not be invested in the shares of a single company like IBM: a rational investor will diversify across industries, and within an industry she will choose to invest in multiple companies. The logic is that all the companies are unlikely to experience difficulties at the same time. If the workers at GM were to be on strike, it is not necessary that workers at Ford should also be on strike at the same time. If one segment of the portfolio were to be experiencing difficulties, the odds are that another segment would be doing well and will tend to pull up the performance of the portfolio as a whole.

Secondary markets are critical from the standpoint of holding a diversified portfolio of assets. In real life, an investor's propensity to take risk does not stay constant over his life cycle. We know that debt securities promise contractually guaranteed rates of return and are paid off on a priority basis in the event of liquidation. On the other hand, equity shareholders are residual claimants who are entitled to payments only if there were to be a surplus after taking care of the other creditors. The risk of a debt investment will be lower as compared to an equivalent equity investment. Young investors who are having steady and appreciating income usually have a greater capacity to take risk, and they have a tendency to invest more in equity securities. Even from the standpoint of equity shares, young investors are less concerned about steady dividend payments and tend to focus more on the odds of getting significant capital gains in the medium to long term. Senior citizens, on the other hand, want predictable periodic income from their investments and have little appetite for risk. Such investors therefore have a tendency to hold a greater fraction of their wealth in debt securities. If and when such investors acquire equity shares, they display a marked preference for high dividend-paying and less-risky stocks.

As they grow older, investors periodically make perceptible changes in the composition of their portfolios. A young single investor who has recently secured employment may be willing to take more risks and would probably put a greater percentage of her wealth in equities. Later, as her family grows and she approaches middle age with the children ready to go to college, the investor will probably distribute her wealth more or less evenly between debt and equities. A similar redistribution of wealth across asset classes is observed when an investor approaches retirement. Elderly investors tend to have their wealth primarily in the form of debt securities. There is a saying in financial markets that an investor should invest a percentage of his wealth in equity shares that is equal to 100 minus his age—that is, a 30-year-old should have 70 percent of his wealth in equities, whereas a 70-year-old should have 30 percent of his wealth in equities.

From the standpoints of providing liquidity and permitting portfolio rebalancing, it is important to have active secondary markets. In the absence of such markets, the willingness of individuals to save, and the level of investment in the economy, will be severely affected.

Exchanges and OTC Markets

A securities exchange is an organized trading system in which traders interact to buy and sell securities. A securities exchange is a secondary market for securities. Public traders cannot directly trade on these exchanges; they must route their orders through a securities broker who is a member of the exchange. Historically, trading used to take place on a trading floor, where member brokers would congregate and seek to match buy and sell requests received from their clients. These days most exchanges are electronic markets. Traders do not interact face-to-face but are required to key their orders into a computer terminal that conveys the orders to a central processing system. The procedure for matching and executing orders is coded into the software.

Two types of orders can be placed by an investor. In the case of market orders, the investor merely specifies the quantity he seeks with the understanding that he will accept whatever price he is offered. However, investors who are very particular about the price they pay or receive will place what are known as *limit orders*. The limit orders require not only a specified quantity but also a limit price. The limit price is a ceiling in the case of buyers—that is, it represents the maximum amount that the investor is willing to pay. In the case of sell orders, the limit price is a floor that represents the lowest price at which the investor is willing to sell. To ensure that traders are given access to the best available prices, all limit buy orders are ranked in descending order of price while limit sell orders are ranked in ascending order of price. This is known as the *price priority rule*. Potential buyers are given access to the lowest price on the sell side while potential sellers are given access to the highest price on the buy side. If two or more limit buy or sell orders were to have the same limit price, then the order that came in first would be ranked higher. This is known as the *time priority rule*.

Newer exchanges such as Eurex in Frankfurt are fully automated electronic systems. Some of the older exchanges have changed with the times and have abandoned their trading rings or floors and embraced electronic trading platforms. However, some of the other older exchanges are relatively resistant to change. The New York Stock Exchange (NYSE) and the CME Group continue to run floor-based and screen-based trading platforms in parallel.

An OTC network is an informal network of securities brokers and dealers who are linked by phone and fax connections. Most deals on such markets tend to be institutional in nature and are of sizable volume. The foreign-exchange market globally is an OTC market, and most of the trading in bonds also takes place on such markets.

Table 1.1 shows the leading stock exchanges ranked in terms of the value of trades in 2010.

Table 1.2 shows the leading exchange groups by market capitalization.

Table 1.3 shows the leading derivative exchanges by contract volume.

Table 1.1
The World's Leading Stock Exchanges (Value of Trades-Electronic Order Book Trades)

Exchange	Value of Trades ($ Trillion) in 2010
NYSE Euronext (US)	17.80
NASDAQ OMX	12.66
Shanghai Stock Exchange	4.49
Tokyo Stock Exchange Group	3.79
Shenzhen Stock Exchange	3.56
London Stock Exchange Group	2.75
NYSE Euronext (Europe)	2.02
Deutsche Boerse	1.63
Korea Exchange	1.60
Hong Kong Exchanges	1.50
Toronto Stock Exchange (TSX) Group	1.37
BME Spanish Exchanges	1.36
Australian Securities Exchange	1.06

Source: www.world-exchanges.org

Table 1.2
The World's Leading Stock Exchanges (Market Capitalization) as of 31 December 2010

Exchange	Capitalization ($ Billion)
NYSE Euronext	15.97
NASDAQ OMX	4.93
Tokyo Stock Exchange	3.83
London Stock Exchange	3.61
Shanghai Stock Exchange	2.72
Hong Kong Stock Exchange	2.71
Toronto Stock Exchange	2.17
Bombay Stock Exchange	1.63
National Stock Exchange of India	1.60
BM&F Bovespa	1.55
Australian Securities Exchange	1.45
Deutsche Boerse	1.43
Shenzhen Stock Exchange	1.31
SIX Swiss Exchange	1.23
BME Spanish Exchanges	1.17
Korea Exchange	1.09
MICEX	0.95
JSE Limited	0.93

Source: en.wikipedia.org

Table 1.3
The World's Leading Derivatives Exchanges (Contract Volumes)

Exchange	Contracts (Millions) January–December 2010
Korea Exchange	3748.86
CME Group	3080.49
Eurex	2642.09
NYSE Euronext	2154.74
National Stock Exchange of India	1615.79
BM&F Bovespa	1422.10
CBOE	1123.51
NASDAQ OMX	1099.44
Multi Commodity Exchange of India	1081.81
Russian Trading Systems Stock Exchange	623.99
Shanghai Futures Exchange	621.90
Zhengzhou Commodity Exchange	495.90
Dalian Commodity Exchange	403.17
Intercontinental Exchange	328.95
Osaka Securities Exchange	196.35

Source: www.futuresindustry.org

Brokers and Dealers

A broker is an intermediary who arranges trades for his clients by helping them to locate suitable counterparties. His compensation is in the form of a commission paid by the client. Brokers do not finance the transaction—that is, they do not carry an inventory of the assets being sought. They are merely facilitators of a trade who receive a processing fee for services rendered. Brokers are very common in real estate markets. If we were to contemplate the purchase of a house, we would approach a realtor. She will have a list of properties whose owners have evinced interest in selling. However, a realtor does not own an inventory of houses that she has financed.

A dealer, on the other hand, is a market intermediary who carries an inventory of the asset in which he is making a market. Unlike a broker, a dealer has funds locked up in the asset. In effect, a dealer takes over the trading problem of the client. If the client is seeking to sell, the dealer will buy the asset from the client in anticipation of being able to resell it at a higher price. Similarly, when a client wishes to buy the asset, the dealer will sell the asset from his inventory in the hope of being able to replenish his stock subsequently at a lower price. To ensure profits from his trading activities, a dealer has to be a master of the art of trading. In developed countries, dealers will usually specialize in narrow segments of the securities market. Some will handle Treasury bonds, whereas others may choose to specialize in municipal debt securities. This is because, considering the volumes of transactions, skill is of the essence, and even small errors could lead to huge losses given the magnitude of the deals.

How does a dealer make money? The price he quotes for acquiring an asset will be less than the price at which he expresses his desire to sell to another party. The price at which a dealer is willing to buy from a client is called the *bid*, and the price at which he is willing to sell to a client is called the *ask*. The difference between the bid and the ask is called the *bid–ask spread* or simply the *spread*. Dealers seek to make money by rapidly rotating their inventories. A purchase at the bid followed by a subsequent sale at the ask will result in a profit equal to the spread. Such a transaction is termed a *round-trip transaction*.

Many dealers don the mantle of both brokers and dealers. In certain transactions they will act as trade facilitators who provide services in anticipation of a commission, and in other cases they will position themselves on one side of the trade by either buying or selling securities. Such dealers are called *dual traders*. A transaction in which the dealer functions merely as a broker is referred to as an *agency trade*. However, a trade in which the dealer is one of the parties to the transaction is termed a *proprietary trade*.

Dealers who undertake to provide continuous two-way price quotes are referred to as *market makers*. Their role is to create a liquid secondary market. On the NYSE, there is only one market maker for a security, although one dealer may make a market in multiple securities. This monopolist market maker is referred to as a *specialist*. She is also known as an *assigned dealer* because the role has been assigned to her by the exchange. When a company seeks to list its securities on the exchange, a number of potential market makers will express their desire to act as the specialist for the stock being introduced. They will be interviewed by representatives of the company as well as the exchange, and finally one of them will be selected as the specialist for that particular stock.

There are also interdealer brokers who act as intermediaries for trades between market makers. These include ICAP, Tullett Prebon, GFI, Tradition, and Cantor Fitzgerald. Cantor Fitzgerald got global publicity for extremely unfortunate reasons. It was occupying the top floors of the World Trade Center, and 658 employees perished in the attack on September 11, 2001.

The Need for Brokers and Dealers

Why do we require market intermediaries such as brokers and dealers? The reason is that when an investor seeks to buy or sell assets in the secondary market, she has to locate a suitable counterparty. A potential buyer needs to locate a seller and vice versa. Second, a counterparty not only should be available but also there should be compatibility in terms of price expectations of the two parties, and the quantity that each one is seeking to transact. Every trader seeks to trade at a price that is good from her standpoint. Buyers will therefore be on the lookout for sellers who are willing to offer securities at prices that are less than or equal to what they are willing to pay. Similarly, sellers will seek to locate buyers who are willing to offer a price that is greater than or equal to the price at which they are willing to sell. As explained earlier, limit orders are arranged in descending order of the limit price on the buy side and in ascending order of the limit price on the sell side. Buyers are guaranteed access to the lowest prices quoted by sellers, whereas sellers are guaranteed access to the highest prices quoted by buyers. In addition, it is important that the quantity on offer matches the quantity being demanded. Often a large buy or sell order may require more than one buyer to take the opposite position before execution.

On an organized exchange, it is necessary to go through a broker or a dealer for certain reasons. First, access to the exchange is only provided to registered brokers and dealers. For exchanges that are totally automated, only brokers and dealers will have terminals through which orders can be routed to the central processing system of the exchange. It is essential for a public trader to go through a licensed market intermediary. Of course, big institutional investors may be provided with order-routing systems so that they can seamlessly send orders to the exchange via the intermediary. On exchanges that have floor-based trading, only regular traders will be familiar with the jargon and protocol that is required for trading. Allowing a novice to step in would cause unnecessary chaos and confusion.

Second, exchanges insist on dealing with market intermediaries to reduce the possibility of settlement failure. The term *settlement* here refers to the delivery of securities from the seller to the buyer and the delivery of cash from the buyer to the seller. Default on the part of either party to a transaction can substantially affect the public's confidence in the system. To prevent settlement failure, exchanges have elaborate risk-management systems in place. Market intermediaries are required to post performance guarantees or collateral called margins with the exchanges to rule out the possibility of a failed trade. It would make sense for a party to have such a financial relationship with the exchange only if he trades regularly and in large volumes. For public traders who trade relatively infrequently, developing such an arrangement with the exchange is not practical. However, brokers and dealers who either trade regularly on their own account or have a large number of trades routed through them will find it worth the cost and effort to have such a financial relationship with the exchange.

The brokerage industry has been deregulated in most countries. Before deregulation, a minimum brokerage fee was specified by the authorities. What was happening, therefore, was that institutional clients were subsidizing retail clients: institutional clients were paying more than they should have, considering the magnitude of their transactions, whereas retail investors were paying less than what they should have paid. The immediate impact of deregulation was a sharp increase in retail brokerage rates. However, a brand-new industry—*discount brokerage*—was born as a consequence. A regular broker—referred to as a *full-service broker*—will sit one-on-one with his client seeking to ascertain his investment objectives in order to provide suitable recommendations. He will also provide extensive research reports to facilitate decision making. A discount broker, on the other hand, will offer no advice. Her only task is to execute orders placed by her clients. There is also a category of brokers referred to as *deep-discount brokers*. These brokers also provide no investment-related advice. However, they insist on transactions of a substantial magnitude and charge commissions that are even less than those levied by discount brokers.

Table 1.4 lists major full-service brokers.

Table 1.5 lists major discount-brokerage firms.

Table 1.4
Full-Service Brokers

Brokerage House	Number of Brokers	Number of Branches
Raymond James	5,400	2,500
Edward Jones	12,600	11,200
UBS	10,300	700
Merrill Lynch	15,000	950
Morgan Stanley	18,100	920
Wells Fargo Advisors	15,000	5,400

Source: www.smartmoney.com

Table 1.5
Discount-Brokerage Firms

Zecco
TradeKing
E*TRADE
ShareBuilder/ING Direct
OptionsXpress
Scottrade
Fidelity
Vanguard
Charles Schwab
TD Ameritrade

Source: moneybluebook.com

Trading Positions

A trader is said to have a long position when he owns an asset. An investor with a long position will gain if the price subsequently rises and will lose if it subsequently falls. A rise in price will constitute a *capital gain* at the time of sale, whereas a price decline is termed a *capital loss*. The principle behind the assumption of such a position is buy low and sell high. Investors who take long positions in anticipation of rising prices are said to be bullish in nature and are termed *bulls*.

All traders in the market need not be bullish about the future. Some may be of the opinion that prices are going to decline. Such investors will assume what are termed *short positions*. A trader is said to have taken a short position when he has sold an asset that he does not own. This is accomplished by borrowing the asset from another investor. Such a transaction is called a *short sale*. In such cases, the trader will have to eventually purchase the asset and return it to the lender. If his reading of the market is correct, and prices do decline by the time the asset is bought back, he stands to make a profit. When a person with a short position acquires the asset, he is said to be *covering his position*. Short sellers therefore seek to sell high and buy low. Short selling is considered a bearish activity, and such investors are termed *bears*.

The Buy Side and the Sell Side

The trading industry as a whole can be classified into a buy side and a sell side. The buy side consists of traders who seek to buy the services being offered by the exchange. The traders on the sell side are those who are offering the services of the exchange. The terms *buy side* and *sell side* have no implications for the purchase and sale of securities. Traders on both sides of the market regularly buy and sell securities.

Of all the services offered by the exchange, the key is liquidity. Sell-side traders sell liquidity to the buy-side traders by giving them the opportunity to trade whenever they desire. The buy side consists of individuals, investment funds, institutions, and governments that use the markets to achieve objectives such as cash-flow management or risk management. The sell side consists of brokers and dealers who help buy-side traders trade at their convenience.

Investment Bankers

An investment banker is an investment professional who facilitates the issuance of securities in the primary market. These institutions help the issue process in two ways. First, they help the borrower comply with various legal and procedural requirements that are usually mandatory for such issues. A prospectus or an offer document must accompany any solicitation efforts for the issue. Most issues have to be registered with the capital markets regulator of the country, which is the Securities and Exchange Commission (SEC) in the United States. Finally, most issues are listed on at least one stock exchange. Listing is a process by which an exchange formally admits the issue for trading between investors after the securities have been allotted.

Second, investment bankers provide insurance to the issuer by underwriting the issue. This means that they stand ready to buy that portion of the issue that remains unsubscribed if the issue were to be undersubscribed. Underwriting helps in two ways: (1) it reduces the risk for the issuer, and (2) it sends a positive signal to the potential investors about the quality of the issue, because the investment banker stands ready to buy whatever they choose not to subscribe to. To give an example, consider an issue that has been underwritten by UBS. This means that if investors do not subscribe to the entire amount on offer, UBS will accept the remainder. This would reassure a potential investor because a bank like UBS would not give such an undertaking without doing its homework. In certain cases, an investment bank may not desire to take the entire risk of a new security issue on itself, because the issue may be very large or else may be perceived to be extra risky. A group of investment banks may underwrite the issue together, thereby spreading the risk. This is called *syndicated underwriting*. The chief underwriter is referred to as the *lead manager*. The next rung of investment bankers are *co-managers*. There is also a selling group associated with most issues. It consists of relatively smaller investment banks that do not underwrite the issue but that have been roped in because of their expertise in marketing such issues in their zones of influence.

At times, instead of underwriting the issue, the investment bank will offer to sell it on a best-efforts basis. In other words, the bank will merely offer to do its level best to ensure that the issue is fully subscribed. In these cases, the investment bank merely performs a marketing function without providing the insurance that characterizes the process of underwriting. The banker's commission in such cases will be lower.

Most public offerings are usually underwritten because issuers are more comfortable with such arrangements. This is because the investment bank has a greater incentive to sell the securities when there is a risk of devolvement. What is *devolvement risk*? It is the risk that the bank may have to buy the unsold securities in the event of undersubscription. Such an eventuality will inevitably lead to a loss for the bank, in the sense that the shares so acquired will have to be subsequently offloaded in the market at a price that is lower than the issue price. This is because devolvement is a clear sign of negative market sentiments about the issue, and an issue that fails will experience a fall in price on listing.

Global Finance chose Goldman Sachs as the best investment bank globally in 2008. Merrill Lynch was selected as the best equity bank, and Citibank as the best debt bank. Deutsche Bank was chosen as the best investment bank in Western Europe, and Merrill Lynch was voted the best bank for both debt and equities. In Asia, Citibank was voted the best investment bank as well as the best bank for debt. UBS was voted the best equity bank.

Direct and Indirect Markets

In a direct market, the surplus budget units in the economy deal directly with the deficit budget units: funds flow directly from the ultimate lenders to the ultimate borrowers. If AT&T were to be making a public issue of shares and an investor were to purchase them, it would be termed a *direct market transaction*. Similarly, if the government were to issue bonds to individual as well as institutional investors, it also would constitute a direct market transaction. An issuer of equity shares or debt securities can do so either through a public issue or through a private placement. A public issue entails the sale of the issue to a large and diverse body of investors, both retail and institutional. Such issues are usually underwritten by an investment banker. In the case of private placements, which are more common for debt issues, the issuer will sell the entire issue directly to a single institution or a group of institutions.

In the case of indirect financing, the ultimate lender does not interact with the ultimate borrower. A financial intermediary in such markets comes in between the eventual borrower and the ultimate lender. The role of such an intermediary is, however, very different from that of a broker-dealer or an investment banker, who also are financial intermediaries albeit in a different sense.

A classic example of a financial intermediary in an indirect market is a commercial bank. Take the case of a bank such as BNP Paribas. It raises deposits from individual and institutional investors. From the standpoint of the depositor, who is the surplus unit in this case, the bank is the corresponding deficit unit. The bank will issue its financial claims to the depositors for whom these claims will constitute an asset. A bank passbook, or computerized statement, or a certificate issued in lieu of a longer-term deposit is a manifestation of such a claim. The rate of interest on such claims is the rate of return for the depositors. The risk for such depositors is that the bank could fail. If so, they may lose all or a part of their deposits. In the United States, bank deposits up to $250,000 are insured by the Federal Deposit Insurance Corporation (FDIC). This limit will be in effect through December 31, 2013. On January 1, 2014, the standard insurance amount will return to $100,000 per depositor for all account categories except individual retirement accounts (IRAs) and certain other retirement accounts. For these categories, the limit will remain at $250,000 per depositor.

After accumulating funds by way of deposits, the bank will then lend to corporate and noncorporate entities that are in need of funds. For such borrowers, the bank is the surplus budget unit, and for the bank the borrowers constitute the deficit units. The bank will hold claims issued by such borrowers in return for the funds lent to them and will be entitled to all cash flows emanating from them. The bank is exposed to the risk that the borrowing entities could go bankrupt.

Table 1.6 shows the top 10 global retail banks in 2008.

The size of the global retail banking market was in excess of $2 trillion in 2007. Citigroup was the market leader followed by Bank of America and HSBC. Table 1.7 shows the top 10 global banks in terms of assets in 2010.

As can be seen, the link between the ultimate lenders and the ultimate borrowers is broken by a financial intermediary such as a bank in the case of indirect markets. The ultimate lenders—that is, the deposit holders at

Table 1.6
Major Retail Banks (2008)

Citigroup
Bank of America
HSBC
JPMorgan Chase
Bank of Scotland
Banco Santander
Wells Fargo
BNP Paribas
Wachovia
Société Générale

Source: lbslibrary.typepad.com

Table 1.7
The World's Largest Banks in Terms of Assets as of end 2009

Bank	Country	Assets ($ Trillion)
The Royal Bank of Scotland Group PLC	UK	3.48
Deutsche Bank	Germany	3.07
Barclays PLC	UK	2.98
BNP Paribas SA	France	2.89
Credit Agricole SA	France	2.30
UBS AG	Switzerland	1.88
JPMorgan Chase Bank NA	USA	1.75
Societe Generale	France	1.57
The Bank of Tokyo-Mitsubishi UFJ Ltd	Japan	1.49
Bank of America	USA	1.47

Source: finance.mapsofworld.com

the bank—have no claim on the assets of the eventual borrowers, nor do they have a claim on the cash flows being generated by such assets. It is the bank that has a claim on such assets and the corresponding cash flows. The depositors have a claim solely against the bank and are dependent on its performance to get the promised return on their savings. Besides commercial banks, other financial intermediaries in indirect markets include insurance companies, mutual funds, and pension funds.

Mutual Funds

A mutual fund is a financial intermediary in the indirect financial market. It is a collection of stocks, bonds, and other assets that are purchased by pooling the investments made by a large group of investors. The assets of the fund are managed by a professional investment company.

When an investor makes an investment in a mutual fund, her money is pooled with that of other investors who have chosen to invest in the fund. The pooled sum is used to build an investment portfolio if the fund is just commencing its operations or to expand its portfolio if it is already in business. Every investor receives shares of the fund in proportion to the amount of money invested by him or her. When a fund is offering shares for the first time—which is known as an initial public offering or IPO—the shares will be issued at par. Subsequent issues of shares will be made at a price that is based on what is known as the *net asset value* (NAV) of the fund. The net asset value of a fund at any point in time is equal to the total value of all securities in its portfolio minus any outstanding liabilities divided by the total number of shares issued by the fund.

There are two broad categories of mutual funds: open ended and closed ended. Open-ended funds permit investors to acquire as well as redeem shares at any point in time at the prevailing NAV. The capital of these funds is variable. Closed-ended funds make a one-time issue of shares to investors. However, usually such funds are listed on a stock exchange, which ensures that investors have the freedom to trade. Shares of such funds may be priced above or below the prevailing NAV.

The NAV will fluctuate from day to day as the value of the securities held by the fund changes. On a given day, from the perspective of a shareholder, the NAV may be higher or lower than the price he paid per share at the time of acquisition. Just like the shareholders of a corporation, mutual fund owners partake in the profits and losses as well as in the income and expenses of the fund.

Table 1.8 shows the largest asset-management firms in terms of their *assets under management* (AUM).

One advantage of a direct market transaction is that the borrower and lender can save on the margin that would otherwise go to the intermediary to the transaction. After all, how does a depository institution like a bank make profits? The rate of interest it pays to its depositors will be less than the interest rate it charges from borrowers who take its loans. This profit margin is called the *net interest margin*. So if the borrowing firms could directly interact with parties who would otherwise deposit their funds with a bank, then they could profitably share the spread that would

Table 1.8
The World's Largest Asset Management Firms as of December 2009

Manager	AUM ($ Billion)	Country of Origin
Black Rock	3,346.26	US
State Street Global	1,911.24	US
Allianz Group	1,859.35	Germany
Fidelity Investments	1,699.11	US
Vanguard Group	1,509.12	US
AXA Group	1,453.29	France
BNP Paribas	1,326.73	France
Deutsche Bank	1,261.23	Germany
JP Morgan Chase	1,252.92	US
Capital Group	1,179.70	US
Bank of New York Mellon	1,114.51	US
Credit Agricole	918.12	France
UBS	875.73	Switzerland
Goldman Sachs	871.00	US
HSBC Holdings	857.00	UK

Source: www.towerswatson.com

otherwise constitute income for the bank. We will illustrate this with the help of Example 1.3.

Example 1.3

First National Bank is accepting deposits at the rate of 3.50 percent per annum and is lending to companies at 4.50 percent per annum. Thus, the bank has a margin of 1 percent of the transaction amount. Now assume that the borrowing companies opt to directly issue debt securities to the public with an interest rate of 4.00 percent per annum. If so, the investors would be getting 0.50 percent more than what they would were they to deposit their funds with a bank. The issuing companies also stand to incur an interest cost that is 0.50 percent less. Both the parties to the transaction stand to benefit. What we have essentially done is split the profit margin of 1 percent that was going to the bank 50-50 between the lending public and the borrowing firm. If a borrower has a high credit rating, it can directly tap the capital market without going through an intermediary.

Of course, if direct markets were to be all about advantages then indirect markets will fail to exist. There are certain shortcomings of such markets. One major problem in the case of direct markets is that the claims that the borrower wants to issue are often not exactly the type that individual investors want. Such problems could arise with respect to the denomination or the maturity of the issue.

First, take the case of a firm that is issuing securities with a principal value of $5,000. It will automatically lose access to investors who seek to invest less than that amount. This is known as the *denomination problem*. Second, borrowers like to borrow long term. This is because most projects tend to be long term in nature, and entrepreneurs would like to avoid approaching the market at frequent intervals in order to raise funds. But lenders usually prefer to commit their funds for relatively shorter periods. A company issuing debt securities with 20 years to maturity may find that it has few takers if it were to approach the public directly. This is referred to as the *maturity problem*.

Yet another problem with direct markets is that they are highly dependent on active secondary markets for their success. If the secondary market were to be relatively inactive, then borrowers would find it difficult to tap the primary market. This is because most individuals who invest in debt and equity issues place a premium on liquidity and ready marketability as manifested by an active secondary market. In times of recession, the secondary markets will be less active than normal, and such periods are therefore usually characterized by small and less-frequent issues of fresh securities.

Besides, for an issuer of claims, the cost of a public issue can be high. Such issues require a prospectus and application forms to be printed and also require aggressive marketing. This is not cheap. The investment banker has to be paid her fees, which can be substantial. Finally, the issuing firm needs to hire and pay other professionals such as lawyers, certified public accountants, and public relations firms.

As we have discussed, companies that issue financial claims directly to the public find that many potential investors find the denomination or maturity of the securities offered by them to be unsuitable. Financial intermediaries are, however, able to resolve these issues, because they have the ability to invest relatively large amounts and for long periods of time. This is despite the fact that the average deposit may not be for a very large amount for a depository institution such as a bank, and most deposits tend to be for relatively short periods. We say that such intermediaries are able to perform *denomination transformation* and *maturity transformation*.

Intermediaries such as banks are able to effect such transformations because they sell their own claims to the public and, after pooling the funds so garnered, purchase financial claims from the borrowing entities. A large commercial bank will have many depositors ranging from those who deposit a few dollars to those who deposit a few million dollars. Similarly, a mutual fund will have investors ranging from those who seek to buy 100 shares to those who want to acquire 100,000 shares. Because these institutions cumulatively receive funds on a large scale, they can profitably transform the relatively smaller amounts deposited with them into large loans for commercial borrowers. This is the essence of denomination transformation.

As mentioned earlier, most borrowers prefer to borrow long term because of the nature of the projects they are executing. Lenders, on the contrary, like relatively short-term liquid assets that can be converted to cash on demand. Banks can pool short-term deposits and package them into medium- to long-term loans. They are in a position to do so because the deposits will periodically get rolled over either because of renewal by existing depositors or because of new depositors. Similarly, in the case of an open-ended mutual fund, share redemptions during a given day will be accompanied by fresh investments by either existing or new investors.

Financial intermediaries in the indirect market are also able to provide their depositors with risk management and risk diversification. All rational investors dislike risk and are said to be *risk averse*. This does not mean, however, that people will not take risks while investing. After all, every financial market transaction is fraught with a degree of risk. It is just that the magnitude varies from transaction to transaction and from instrument to instrument. The term *risk aversion* connotes that, for a given level of expected return, an investor will prefer that alternative with the least associated risk. Put differently, while considering an investment in assets that have the same degree of associated risk, an investor will choose the security that has the highest expected return.

Intermediaries like banks have considerable expertise in dealing with risk as compared to individual investors. An investor who lends indirectly through a bank can be assured that the bank will only loan after doing a more thorough evaluation of a borrower's creditworthiness, which is more than what he himself could have done had he chosen to lend directly.

There is another dimension to the role played by banks from the standpoint of risk. We have already discussed the principle of diversification: optimally holding one's wealth in a portfolio of securities. However, in reality it is not easy for an individual investor to construct a well-diversified portfolio herself. Most individual investors will have a relatively small corpus of funds at their disposal, and extensive diversification will be neither feasible nor cost effective considering the magnitude of transactions costs that they are likely to incur. An intermediary such as a bank has a large pool of funds at its disposal, so it invests across a spectrum of projects from the standpoint of risk. A depositor is assured that every dollar that he deposits is effectively being lent to multiple borrowers, thereby ensuring diversification.

Financial intermediaries are also able to derive substantial economies of scale—that is, their fixed costs of operation get spread over a vast pool of transactions and assets. They are able to ensure that they are relatively cost efficient, and this benefit will be passed on to the depositors to a degree. Example 1.4 shows the concept of economies of scale.

Example 1.4

Take the case of a business operation that costs $5,000 to mount; it costs $2.50 per unit to process. If 2,000 units were to be processed, the cost per unit will be $5. However, if the number of units processed were to be 20,000, then the cost per unit will be only $2.75. Of course, the cost per unit cannot be reduced beyond a limit. This is because as the production volume increases, after a point in time the fixed costs will have to be incurred again, for a given production structure can support output only up to a point.

Money and Capital Markets

One way to classify financial markets is based on the original term to maturity of the financial claims that are traded. The market in which instruments with one year or less to maturity are traded is called the *money market*, whereas the market in which medium- to long-term instruments are traded is called a *capital market*. All money market securities have to be debt securities because equity shares never mature. Capital market securities can be equity securities or medium- to long-term debt securities.

The two markets differ fundamentally from the standpoint of their roles in a market economy. Money markets provide a means for economic agents to adjust their liquidity positions. As we discussed at the outset, every economic agent will receive some form of income in a financial year and also incur some form of expenditure. However, for any economic unit, it will rarely be the case that cash inflows and outflows are perfectly matched with respect to their timing. The mismatch between inflows and outflows leads to short-term deficits and surpluses, which are bridged by borrowing and lending in the money market.

A capital market on the other hand performs a very different economic function. The purpose of a capital market is to channelize funds from people who wish to save to those who wish to make long-term investments in productive assets in an effort to earn income. When a government or a municipality needs to finance developmental activities that are long term in nature, like building a metro railway or putting up an oil refinery, or when a business wants to expand or diversify it will approach the capital market for the required funds.

The Eurocurrency Market

The development of the Eurocurrency market was one of the early factors in the growth of international investment. A Eurocurrency is a freely traded currency deposited in a bank outside its country of origin. For example, Eurodollars are dollars deposited outside the United States whereas Euroyen are yen deposited outside Japan. The term *euro* refers to the fact that the funds are placed with an institution outside the country to which the currency belongs. The institution need not be located in Europe. Although London, Zurich, and Frankfurt are major financial centers for such deposits, so are Singapore, Tokyo, and Hong Kong in East Asia, and Dubai and Bahrain in the Middle East.

The rapid growth of the Eurodollar market can be attributed to the following factors:

- After World War II, the U.S. dollar became the preferred currency for international trade, displacing the British pound, which was the primary vehicle for commercial transactions before the war.
- All countries sought to keep dollar balances to finance their imports. The former Warsaw Pact countries (satellites of the Soviet Union) were no different. However, they were reluctant to hold dollars with banks in the United States. A cold war was going on, and there was a legitimate fear that such deposits could be impounded by the U.S. government. These countries began depositing their dollars with European banks. As trade grew, the European banks soon discovered

that there was a ready demand for these dollars by parties based outside the United States. As a consequence, an active Eurodollar market came into existence.
- In the 1960s, the U.S. government imposed capital market controls. Through legislation called Regulation Q, the U.S. government imposed low-interest-rate ceilings on U.S. banks and simultaneously imposed significant reserve requirements.

An interest ceiling meant that banks could not offer depositors more than the rate that had been mandated by the law, even if prudent business practices required them to do so.

The implications of bank reserves may be explained as follows: When a unit of currency is deposited with a bank, only a fraction of it can be lent out. The balance has to be kept either in the form of approved securities or as cash. This amount is known as a *reserve*. The lower the reserve ratio, the more money that will be available with the bank for commercial lending. The lower the reserve ratio, the higher will be the rates paid on deposits; and the lower the lending rates, for the larger the amount of funds available, the smaller the net interest margin that banks can afford to operate with.

As a consequence of these two factors, depositors became reluctant to park their funds with U.S. banks because these institutions could not offer attractive interest rates. At the same time, borrowers too were disenchanted, because the rates on loans were very high. The following are among the many reasons for the growth of the Eurodollar market:

- Lack of government regulations on Eurodollar deposits. Eurodeposits are relatively unregulated. The Federal Reserve, which is the central bank of the United States, does not regulate dollar deposits maintained outside the borders of the United States. Besides, such deposits do not suffer from statutory reserve requirements. However, even if there is no statutory reserve requirement, banks by themselves usually maintain voluntary reserves as a measure of caution.
- Because of the large flow of petrodollars into international banks from the Organization of Petroleum Exporting Countries (OPEC): After war in the Middle East in 1973, the oil-exporting countries realized the full worth of their oil reserves as an economic weapon. Rising crude oil prices ensured that these countries were flush with funds, and the Euromarket, because of its lack of reserve requirements and relatively low cost of operation due to economies of scale, was instrumental in recycling these so-called petrodollars.

The International Bond Market

The international bond market provides borrowers with a source of medium- to long-term funds. Borrowers include multinational corporations, domestic corporations, governments, and national as well as supranational financial institutions. It gives investors in the debt markets a way to diversify their portfolios over several different currencies. We have already highlighted the wisdom of not putting all the eggs in one basket. But it is not necessary that all the securities chosen should be from the same country. Transnational diversification enables investors to take the diversification process a step further. The market consists of two broad segments: Eurobonds and foreign bonds.

Eurobonds are bonds denominated in one or more currencies other than the currency of the country in which they are sold. For example, bonds denominated in a currency other than the Japanese yen sold in Japan would be called Eurobonds.

A bond denominated in the currency of the country in which it is sold, but issued by an entity from a foreign country, is called a foreign bond. For example, if a U.S. company were to sell yen denominated bonds in Japan, it would be classified as a foreign bond.

Bonds may be classified into three categories: domestic bonds, foreign bonds, and Eurobonds. If Sony were to issue yen-denominated bonds in Japan, it would be categorized as a domestic bond, because the issuer as well as the currency of issue are local. However, if IBM were to issue yen-denominated bonds in Japan, it would come under the category of a foreign bond. This is because the issuer is from a foreign country even though the currency is local. Finally, if either IBM or Sony were to issue U.S. dollar-denominated bonds in Japan, it would be categorized as a Eurobond issue. This is because the currency is foreign. In this case, the nationality of the issuer is irrelevant.

Foreign bonds are known by nicknames. Bonds sold in the United States are called *Yankee bonds*, those sold in Japan are called *Samurai bonds*, and those sold in the United Kingdom are called *Bulldog bonds*.

The Eurobond market has grown much more rapidly than the foreign bond market for several reasons.

- Eurobond issues are not subject to the regulations of the country in whose currency they are denominated. They can be brought to the market quickly and with less disclosure. This gives the issuer of the bonds greater flexibility to take advantage of favorable market conditions. On the other hand, domestic securities issues are regulated by the market regulator of the country. In the United States, it is the SEC.

The origin of the Eurobond market was fueled by the imposition of a cess called the *interest equalization tax*, which was imposed by the U.S. government in 1963 on the interest received by American investors from Yankee bonds. What had happened was that domestic institutions were unable to pay a high rate of interest to investors because of the interest-rate ceiling in the United States. Foreigners sought to take advantage of this situation by issuing Yankee bonds with relatively higher rates of return. The objective of the interest equalization tax was to ensure that American investors did not perceive Yankee bonds to be attractive, despite their higher interest rates. The motivation for this measure was to arrest the perceived flight of capital from the United States. As a consequence, foreigners were forced to relocate their dollar borrowings to outside the United States.

- Eurobonds offer favorable tax status. They are usually issued in bearer form—that is, the name and address of the owner are not mentioned on the bond certificate. There are two broad categories of securities: registered securities and bearer securities. In the case of registered securities, a record is maintained of the owners at any point in time by an entity known as a *registrar* or *share transfer agent*. Each time the security is transferred from one investor to another, the records are updated. However, in the case of bearer securities, physical possession is the sole evidence of ownership. Such securities are easier to transfer and offer investors the potential freedom to avoid and evade taxes. Holders who desire anonymity can receive interest payments from such securities without revealing their identity. Also, interest on Eurobonds is generally not subject to withholding taxes or tax deduction at source.

  Because of their unique features, investors are willing to accept a lower yield from Eurobonds than from other securities of comparable risk but that lack the favorable tax status.

  Eurobonds are not usually registered with any particular regulatory agency, but they are listed on a stock exchange, typically London or Luxembourg. Listing is done not for the purpose of facilitating trading but to circumvent restrictions imposed on certain institutional investors such as pension funds, which are prohibited from purchasing unlisted securities. Most of the trading in Eurobonds takes place OTC.

Globalization of Equity Markets

Compared to debt markets, equity markets have been relatively slow to globalize. However, the winds of change are blowing across the world, and markets are increasingly becoming modernized and integrated. The doctrine of LPG—liberalization, privatization, and globalization—is gaining currency around the world. New developments in communications technology coupled with deregulatory changes and greater awareness of the benefits of international portfolio diversification on the part of investors have led to rapid integration of equity markets in recent years. The following are among the major deregulatory measures introduced in the past few decades:

- On May 1, 1975, the United States abolished fixed brokerage commissions.
- In 1985, the Tokyo Stock Exchange started admitting foreign brokerage firms as members.
- In 1986, the London Stock Exchange eliminated fixed brokerage commissions and began admitting foreign brokerage houses as full members. This event is known as the "big bang" in financial circles. These changes were designed to give London an open and competitive international market. Until the end of World War II, London was the center of global financial activity. For obvious reasons, the center of postwar economic activities moved across the Atlantic to New York. London is, however, critical for global financial activities, because it lies in an ideal time zone. The city is located between the capital markets of North America and those of East Asia, principally Singapore and Tokyo. It is the middle link for what is effectively a 24-hour market.
- In 1987, financial institutions in London were permitted to participate in both commercial and investment banking.
- In 1999, the Glass-Steagall Act, which sought to segregate commercial and investment banking activities in the United States, was repealed. The act was a product of financial regulation legislated during the Great Depression in an effort to insulate commercial banks from the vagaries of the stock market. Once this act was enacted, institutions were given a clear choice. They could either accept deposits and make loans or provide underwriting and broker-dealer services. U.S. institutions that were in the business of deposit taking and loan making were precluded from trading and market

making in securities. Morgan Stanley was formed as a splinter from J. P. Morgan, which continued as a commercial bank while Morgan Stanley went into the areas of securities dealing and investment banking. In 1999, the Financial Services Modernization Act (also known as the Gramm-Leach-Bliley Act) did away with this restriction and paved the way for giant financial conglomerates that could undertake both investment-banking and commercial-banking activities.

Dual Listing

Dual or *multiple listing* allows the shares of a company to be traded on the exchanges of many different countries. Foreign equity is traded in global markets in the form of *depository receipts* (DRs). On the U.S. exchanges, they are traded in the form of *American depository receipts* (ADRs). Such securities are special shares of foreign equity that are priced in U.S. dollars. The issuance of such assets facilitates the ownership of foreign equity by American residents. An ADR is essentially a receipt issued by a depository bank in the United States that is backed by foreign shares that are deposited with a custodian bank in the country of issue of the original shares. ADRs are quoted and traded in U.S. dollars just like domestic U.S. shares.

The mechanism of issue of ADRs is as follows. A U.S. depository bank will acquire shares of the foreign company in its domestic market. These shares will then be deposited with a local custodian bank in the foreign country. The U.S. depository bank will then issue ADRs to the investors in the United States, where each ADR will correspond to a specified number of foreign shares.

Shareholders receive dividends in U.S. dollars. The depository takes on the task of collecting dividends in the foreign currency, converting the dividends to dollars and making payments.

An ADR may represent either a fraction or a multiple of the underlying shares, packaged in such a way that it will trade at the appropriate price range in the United States, as illustrated in Example 1.5.

The ADR market is growing from the standpoints of both supply and demand. Foreign companies are being increasingly attracted to the U.S. market because it is the largest in the world and arguably the most efficient. Unlike other nations, the United States presents fewer barriers to entry. Compared to other countries, the United States has more high-net-worth individuals who are not only better financially endowed but also more

Example 1.5

Take the case of a company based in India that is being traded at INR 50 per share at a time when the exchange rate is INR 40 per U. S. dollar. If an ADR were set to be equal to one Indian share, then it would trade for approximately 1.25 dollars in the United States. To ensure that the ADRs trade at a more respectable price, one ADR may be made equivalent to 10 domestic shares, thereby ensuring that it trades at approximately $12.50.

On the other hand, a share may be trading at a very high price in its home country. If a share is trading at INR 10,000 in India, it will trade at approximately $200 in the United States, assuming a 1:1 issue ratio and exchange rate of INR 50 per dollar. In such circumstances, one ADR may be set equal to 0.20 Indian shares, thereby ensuring that it trades at approximately $40 in the United States.

aware of the benefits of international diversification and willing to take the attendant risks.

Although an American investor can always acquire a share that is traded on a foreign stock exchange, it is a lot simpler in practice to invest in an ADR. There is no need to locate a foreign broker or be conversant with the systems and practices of a foreign stock exchange and its related institutions. Besides, acquiring a foreign security will expose the investor to exchange rate risk. In most countries, reporting standards are not as stringent as those in the United States. Companies abroad are able to get away with less disclosure, which may not be adequate for a U.S. investor. A practical difficulty from the standpoint of trading foreign shares is that such transactions can be undertaken only when the overseas market is open, and the timings of these trading venues usually do not overlap with market hours in the United States.

One advantage of an ADR is that it may offer greater liquidity in the United States than the underlying shares do in their domestic market. Moreover, certain pension funds and investment managers in the United States have a strong preference for ADRs because they are legally required to invest in ADRs while investing in non-U.S. securities. Many institutional investors prefer investing in securities that are listed for trading in the U.S. market rather than invest directly in foreign equity markets.

To list its shares on the exchange of a developed country, the issuing company must meet the securities market regulations of the foreign country as well as those of the stock exchange on which the shares are to be listed. This frequently requires the company to comply with stringent disclosure norms. To list on the New York Stock Exchange, it is necessary to comply with the *generally accepted accounting principles* (GAAPs) of the United States. For companies based in developing countries, such compliance has ensured more transparency in their operations, leading to the benefit of not just global investors but also the domestic shareholders. However, after events such as Enron's collapse, it is arguable as to whether disclosure norms in the United States are as commendable and effective as once believed. Besides, many other countries have considerably beefed up their security-related regulations as a response to financial scams and scandals.

Foreign listing provides a multinational corporation with indirect advertising for its product brands in the foreign market. It also raises the profile of the company in international capital markets, making it easier for it to raise finances in the future and providing greater mileage for its international marketing efforts. Take the case of an Indian company that is seeking to raise a bank loan in London. It will have greater credibility in the eyes of the international lending community if it were to be listed on an exchange in the developed world. Besides, when a firm's equity is held by shareholders across the globe, the risk of a hostile takeover of the firm may be somewhat reduced.

There could be instances in which foreign investors put a higher premium on the issue than do domestic investors. This could be the case if the foreign market were to have greater experience in dealing with a particular industry. As Geddes (2001) points out, there was a flood of issues by international mining companies on the Toronto Stock Exchange in the mid-1990s because it was felt that Canadian investors had greater experience in evaluating the shares of such firms.

At times, a firm may go in for a global issue because it perceives its domestic market to be too small for an issue of the size it is contemplating. In Europe, for example, this fact has compelled Scandinavian and Eastern European firms to access markets across their borders.

There are two broad categories of ADR programs: sponsored and unsponsored. In the case of a sponsored program, the exercise is initiated by the foreign firm whose shares are sought to be traded in the United States. In the case of an unsponsored issue, on the other hand, the process will typically be initiated by an investment bank in the United States that has acquired shares in a foreign market.

Fungibility

Fungibility means the ability to interchange with an identical item. ADRs may be one-way or two-way fungible. If an ADR is one-way fungible, then a U.S. investor can sell the ADR back to the depository in the United States and have the equivalent number of underlying shares sold in the home country. However, if the ADRs are two-way fungible, then an investor could also surrender shares to the custodian bank in the home country and acquire ADRs in lieu. The problem with one-way fungibility is that it makes ADRs less attractive for American investors, because it has the potential to reduce the liquidity and the floating stock of ADRs in the United States. Besides, two-way fungibility is essential to preclude arbitrage opportunities.

Arbitrage

What is arbitrage? The term *arbitrage* refers to the strategy of making costless, riskless profits by simultaneously transacting in two or more markets. It is one of the fundamental principles underlying modern finance theory. One of the basic tenets of finance is the concept of risk aversion; that is, an investor will demand a risk premium for bearing a certain level of risk, and the higher the risk associated with an investment, the greater will be the premium demanded over and above the riskless rate of interest.

Arbitrage may be defined as the presence of a rate of return greater than the riskless rate on an investment devoid of risk or equivalently as the specter of a positive rate of return from a trading strategy that entails neither an investment nor an assumption of risk.

Take the case of a city such as Mumbai, which has two major stock exchanges, namely, the Mumbai Stock Exchange (BSE) and the National Stock Exchange (NSE). Assume that a share is trading at INR 100 on the BSE and at INR 100.75 on the NSE. An arbitrageur will place a buy order for, say, 500,000 shares on the BSE while simultaneously placing a sell order for an equivalent amount on the NSE. Before accounting for transactions costs, he is assured of a profit of $500{,}000 \times (100.75 - 100) =$ INR 375,000. Considering the fact that he did not have to stake any money and was able to implement the strategy without taking any risk, such an opportunity should not exist.

Certain practical issues must be considered when evaluating what looks like an opportunity for free money. First, can the trader make a profit after factoring in transactions costs such as brokerage commissions?

Most retail investors will have to incur such expenses. However, a firm dealing in securities has a tremendous advantage because it does not have to pay such transaction-related charges. Such strategies that appear infeasible for retail traders may be profitable for institutions.

Second, in this example, both the exchanges have a T+2 settlement cycle—that is, if an investor were to trade on a particular day, the payment of cash to the seller and the delivery of securities to the buyer would take place two business days later. The arbitrageur cannot wait to take delivery on the BSE before giving delivery on the NSE. To capitalize on such opportunities, a potential arbitrageur needs access to a stockpile of cash as well as a long position in the security right at the outset.

Such opportunities will not remain for long. As arbitrageurs start buying on the BSE, the price there will be driven up by the increasing demand. Similarly, as they start selling on the NSE, the increased supply will push prices down. After a brief while, such opportunities will not be apparent.

Such opportunities actually only exist for fleeting moments. They can be exploited by players who are always in the thick of the action such as financial institutions. To ensure that such avenues for profit are seized and exploited, traders increasingly rely on automated systems. One well-known type of arbitrage is stock index arbitrage, which entails the exploitation of deviations from the postulated pricing relationship between stock indexes and futures contracts based on them. This is not easy if we take an index such as the Standard and Poor's 500 (S&P), 500 constituent stocks have to be either bought or sold in the right proportions. The availability of a computer becomes imperative. As a consequence, the implementation of such arbitrage strategies is referred to as *program trading*.

Arbitrage with ADRs

Mispriced ADRs will be exploited by arbitrageurs. We will illustrate it with the help of an example.

Let us assume that shares of the Indian information technology company Infosys Technologies are quoting at INR 1,000 on the NSE in Mumbai and that Infosys ADRs, where each ADR represents 10 domestic shares, are quoting at $260 on the NYSE. The exchange rate is INR 40 per U.S. dollar (USD) at the time of writing.

The ADRs are overvalued: the dollar equivalent of 10 shares should be $250. An arbitrageur will short sell the ADRs in New York. He will acquire 10 shares on the NSE for every ADR that is sold short and deliver them to the custodian bank in Mumbai, which will inform the depository bank in New York. On receiving intimation from Mumbai, the bank in New York will issue an ADR that can be used to cover the short position. The cost of acquisition of 10 shares in Mumbai will be INR 10,000 or $250. The proceeds from the short sale in New York will be $260. An arbitrage profit of $10 can be earned.

Let us examine a situation in which ADRs are undervalued. What if the price of the Infosys ADR were $240? If so, an arbitrageur would acquire an ADR for $240 and deliver it to the depository bank in New York with instructions to sell the underlying shares in Mumbai. The sale proceeds will amount to INR 10,000 or $250. Once again, he will realize an arbitrage profit of $10.

JPMorgan was a pioneer in the creation of ADRs. The company created the first ADR in 1927 to facilitate investment in foreign companies by American investors. An ADR is considered to be an American security and is freely tradable in the United States. It is akin to any other domestic security for the purpose of clearing and settlement.

GDRs

Although ADRs are the most common type of depository receipts, there are other similar securities called *global depository receipts* (GDRs). A GDR differs from an ADR in the sense that it is offered to investors in two or more markets outside the issuer's home country. Most such issues will include a tranche for U.S. investors, as well as a separate tranche for international investors.

Euro depository receipts represent ownership of shares in a corporation that is based in a country outside the European Monetary Union. Although depository receipts are primarily issued to facilitate ownership of overseas equity, they can also be structured to permit investors to take a stake in a foreign debt issue. An *American depository debenture* is a security that is based on a debt security issued by a foreign company.

Risk

Risk is defined as a position with an uncertain outcome that has the potential for loss for the holder. Assume that you are offered a security that will give a 20 percent return with a probability of 50 percent and a 40 percent return with the same probability. This is not a risky position, because there is no possibility of a loss even though the outcome is uncertain. Similarly, take the case of an investment that is guaranteed to give a return of −10 percent. This, too, is not a risky position, because there is no uncertainty regarding the outcome—a loss for the investor. So what is an example of a risky position? Take the case of an investment in a share that will give a return of 0 percent with 25 percent probability, −10 percent with 25 percent probability, and 10 percent with 50 percent probability. This is risky because the outcome can take one of three possible values, and one will lead to a loss for the investor. Financial securities are exposed to multiple types of risks, including the following.

- *Credit risk*. *Credit risk* is the risk that the deficit budget unit, which raises funds, may not make payments as promised to the investor. A business may issue bonds to the public with a face value of $1,000 and an assured interest rate of 10 percent to be paid every year. However, at the end of a financial year, it may be unable to make the promised interest payment. Or else, at the time of maturity of the security, it may be unable to repay the principal in full. Similarly, a financial institution like a bank, which makes loans to borrowers, is also faced with the specter of nonpayment. Such risk is also termed a *default risk*.
- *Price risk*. Also called *market risk*, *price risk* is the risk that the price of the security at the time of purchase may be higher than its price at the point of a subsequent sale. In other words, it is the risk that there could be a capital loss for the investor. All marketable securities are subject to such risk.
- *Reinvestment risk*. Whenever an investor receives a cash flow from a security, she should reinvest it in the market to ensure that she gets an anticipated compounded rate of return. However, there is always a risk that the prevailing market interest rate may be lower than what was expected at the outset. If so, the investor will have to settle for a lower-than-anticipated compounded rate of return. Such risk is termed *reinvestment risk*.
- *Inflation risk*. Inflation or the erosion of purchasing power of money is associated with every investment. Every investor will have a required rate of return that will include a premium for the anticipated rate of inflation. However, if the rate of inflation were to be higher than anticipated, the effective rate of return in terms of the ability to buy goods and services may be lower than expected. The return from a security in the absence of inflation is termed the *real rate*. All investors will demand a positive real rate of return. They will add a markup to this for the expected inflation in order to arrive at what is called the *nominal* or *money rate of interest*.

However, if inflation is higher than anticipated, then the real rate received at maturity may be less than expected and may be negative. The anticipated interest rate is referred to as the *ex ante rate*. The interest rate that is eventually received is termed the *ex post rate*. In the absence of default, the ex-ante nominal rate will be equal to the *ex post* nominal rate. However, because of inflation, the *ex post* real rate may be lower or higher than anticipated and may be negative.

- *Liquidity risk*. We have already expounded on the issue of liquidity risk. Whenever funds are blocked in an investment, there is always the risk that the market for its subsequent sale may not be liquid at the time the sale is affected. If so, the seller may have to make a substantial concession by way of a reduction in price to complete the transaction.
- *Foreign-exchange risk*. A *foreign-exchange risk* is associated with an investment in a foreign country. If the domestic currency were to appreciate with respect to the currency in which the assets are denominated, there could be a loss for the investor.

  Assume that a U.S. investor makes an investment of \$100 in an Indian security when the exchange rate is INR 40 per dollar. At the end of one year, the price of the asset has increased from INR 4,000 to INR 4,400. There is a 10 percent return in rupee terms. However, the exchange rate at the end of the year is INR 50 per dollar; that is, the rupee has depreciated or the dollar has appreciated. If the funds are repatriated to the United States, the American investor will receive only \$88. There is a loss of 12 percent for him in dollar terms.
- *Sovereign risk*. A *sovereign risk* is associated with the structure of foreign economies and governments. Assume that a bank makes a loan to a party in Latin America who subsequently defaults. In such cases, the lender may not have access to the same legal means of redress that prevail in the United States. Besides, foreign governments may not be democratically elected or legally accountable. In certain countries, governments may suddenly impose exchange controls which could preclude a foreign company from repatriating the funds it has invested. Not only do foreign corporate securities pose such risk but also foreign government securities. There is always a likelihood that a Latin American government, which has issued bonds denominated in U.S. dollars, may default. Such parties need not default on debt securities denominated in their own currencies, because they have the freedom to print money. However, such countries cannot print foreign currencies like the dollar or the euro.

After the Trade: Clearing and Settlement

After a trade has been matched by a trading system, a post-trade process called *clearing and settlement* needs to commence in order to ensure that the seller receives the cash that is due her, whereas the buyer receives the securities he has acquired.

Clearing refers to all the post-trade processes other than final settlement, where the term *settlement* refers to the payment of cash to the seller and transfer of ownership of the securities from the seller to the buyer. Settlement is the last step in the post-trade process.

Different security types have different settlement cycles. Money market securities such as negotiable certificates of deposit and commercial paper settle "for cash"—that is, on the same business day. Most U.S. Treasury securities settle on the next business day. Most foreign-exchange transactions settle "for spot"—that is, two business days after the trade date (T+2 settlement). Equity and municipal bond transactions in the United States settle three business days after the trade date (T+3) settlement.

Dematerialization and the Role of a Depository

A depository is a facility for holding securities in electronic form so that the purchase and sale of shares can take place via a process of book entry. The shares may be either *dematerialized* or *immobilized*. The term *dematerialization* refers to the replacement of physical share certificates by electronic records. On the other hand, if the securities were to be immobilized, an electronic record would be created without destroying the original physical certificates, which would continue to be held in secure storage.

The function of a depository is akin to that of a commercial bank. We will illustrate the similarities with the help of a simple tabular format as shown in Table 1.9.

The benefits of holding securities with a depository in dematerialized form are as follows:

1. It offers a safe and convenient way to hold securities.
2. It facilitates immediate transfers of securities.
3. No stamp duty is payable in most cases when securities are transferred in dematerialized form.

Table 1.9
Commercial Banks and Depositories (Similarities and Differences)

Commercial Bank	Depository
1. Holds funds in an account 2. Enables fund transfers between accounts after receiving instructions from the account holder 3. Facilitates transfers of funds without having to handle money 4. Facilitates safekeeping of money	1. Holds securities in an account 2. Enables transfers of securities between accounts after receiving instructions from the account holder 3. Facilitates transfers of share ownership without having to handle securities 4. Facilitates safekeeping of securities

4. Transfer of shares in physical form has certain attendant risks such as:
   a. bad delivery,
   b. fake certificates,
   c. delays, and
   d. thefts.

   These can be eliminated by holding securities in dematerialized form.

5. The paperwork involved in the transfer of securities is considerably reduced.

6. The transactions costs involved in securities trading are reduced.

7. The concept of lot size has no meaning. Even one share can be bought or sold.

8. Nomination facility is available.

9. A change of address of the holder gets registered with all the companies in which she holds securities. This eliminates the need to correspond with each company individually.

10. If there is a corporate action such as a stock dividend or a stock split, the shares are automatically credited to the investor's account.

Custodial Services

A *custodian bank*, or *custodian*, is a financial institution that is entrusted with safeguarding the financial assets of an individual or, more commonly, an institutional investor. The custodian typically performs one or more of the following functions:

1. It holds stocks and bonds in safekeeping.
2. It arranges for settlement whenever securities are purchased or sold. In the event of a purchase, it ensures that securities are credited and cash is debited, whereas in the event of a sale it ensures that securities are debited and that cash is credited.
3. It collects dividends on shares and interest on bonds and ensures that the investor's account is credited.
4. It provides information related to the companies whose securities are being held by the investor, such as the schedule of their annual general meetings.
5. It provides foreign-exchange transactions if required.

Global custodians hold assets for their clients in multiple locations around the world, using their own local branches or other local custodians. The advantage of employing a global custodian is that a settlement instruction only needs to be sent to a single destination. Investors may also opt to use a network of local custodians, one in each financial center in which they undertake trades.

If a custodial facility is used, the shares will be held in the name of the custodian, with the investor continuing to remain as the beneficial owner of the securities. The advantages of employing the facilities of a custodian may be summarized as follows:

1. It makes it easier and quicker to trade securities, particularly international shares.
2. It simplifies the management and reporting of share transactions. At the end of every financial year, the investor will receive a single consolidated statement giving details of purchases and sales and any dividend or interest income received during the period.
3. The custodial facility also makes it easier to track the performance of the investor's portfolio.

Globalization: The New Mantra

The word *globalization* indicates international integration of markets and economies, whereby they become interdependent and interconnected. In today's world, large corporations obtain financing from major money centers around the world in many different currencies to finance their global operations. The major money markets in the world include New York, London, Singapore, and Tokyo. The global nature of operations also forces many corporate treasurers to establish international banking relationships and place short-term funds in several different currencies. The effective management of foreign-exchange risk is therefore an integral part of the duties of a modern treasury department. Individual investors also have started investing in the securities of many different countries to take advantage of the better performance potential from internationally diversified portfolios of financial assets.

The integration of financial markets around the world is due to certain major factors: First, many countries have substantially deregulated their money and capital markets. Second, developed and some developing countries allow foreign brokerage firms to operate in their domestic stock exchanges to facilitate greater competition. Third, the majority of countries have eliminated the structure of fixed brokerage commissions that used to exist. Commissions are now largely negotiable between the brokers and the clients and often are a function of the trading volume and the quality of service that is sought. Fourth, interest-rate ceilings have been largely removed, and offshore banking facilities (international banking facilities, or IBFs, in the United States) are available.

IBFs allow U.S. banks to use domestic branches to service foreign customers with international transactions, deposit, and loan services free of reserve requirements and interest-rate regulations. Other countries have followed suit. The objective was to make U.S. banks competitive with respect to players outside the United States that were both accepting deposits denominated in dollars and making loans in dollars.

Many countries have also sought to do away with the distinction between commercial banking and investment banking and move toward universal banking. Most major banks these days are giant financial conglomerates that serve as one-stop financial solutions providers, as illustrated in Example 1.6.

Example 1.6 (An Illustration from India)

Take the case of ICICI Bank, India's second-largest bank. On the one hand, it is a traditional commercial bank in the sense that it accepts deposits from the public and makes loans. However, it has an asset management company that manages mutual funds in collaboration with Prudential Plc. ICICI Prudential is into life insurance. ICICI Lombard is into general insurance. ICICI Home Finance is a subsidiary that makes real estate loans.

Constant product and process innovations constitute a major feature of the modern financial market. Innovations have manifested themselves in two forms. First, many new products have been created. Second, new methods have been devised to facilitate the transfer of risks.

Of course, none of the observed advances in global markets would have been feasible without the developments in telecommunications, computer hardware, and software that we have witnessed in the recent past. They have helped develop systems in which links can be instantly established, and funds and securities can be transferred safely and quickly. Companies such as Reuters, Bloomberg, and Telerate provide round-the-clock access to prices and news from financial centers around the world. Most exchanges are now fully automated and electronic.

Today's market is also characterized by the increasing sophistication of investors and borrowers. Multinational corporations and, surprisingly, even governments have become increasingly sophisticated. Corporate treasurers, fund managers, and bureaucrats are highly educated and aware. Today's markets also tend to be highly dominated by institutions. These giants can take advantage of economies of scale worldwide. They can also afford to employ large teams of highly qualified experts.

Endnotes

1. See Rose (2000).
2. The price of good A in terms of good B is the reciprocal of the price of good B in terms of good A.

CHAPTER

2

Mathematics of Finance

Interest Rates

Interest on money borrowed and lent is a feature of our daily lives. Most people have paid and received interest at some point in time. It is a common practice to make investments by buying bonds and debentures and by opening checking, savings, and time deposits with institutions such as commercial banks. Bonds and debentures pay interest on their face value or principal. We refer to this as the *coupon*. Banks, although they do not pay interest on checking accounts, pay interest on savings as well as time deposits.

In today's consumer-driven economy, it is also a common practice to buy products and services on loan. Borrowing to buy residential property, which is referred to as a *mortgage loan*, is a major component of debt taken by individuals and families. People also borrow in the form of student loans to fund their academic pursuits. Retail borrowing to finance various purchases such as automobiles and consumer durables (white goods) is a feature of today's society.

Interest may be construed as the compensation that a lender of capital receives. Why should a lender charge for making a loan? In other words, why not give an interest-free loan? Remember: If you part with your money in order to extend a loan, you are deprived of an opportunity to use the funds while it is on loan. The interest you charge is consequently a compensation for this lost opportunity. In economic parlance, we would term this as *rent*. Capital, like land and labor, is a factor of production, and consequently those who seek to use the resources of others must pay a suitable compensation. To give an analogy, take the case of a family that gives its house or apartment to a tenant. It would require the tenant to pay a monthly rental because as long as he or she is occupying the property, the owners are deprived of an opportunity to use it themselves. The same principle is applicable in the event of a loan of funds. The difference is that the compensation in the case of property is termed *rent*, whereas when it comes to capital we term it *interest*. In the language of economics, both constitute rent, albeit for different resources.

The Real Rate of Interest

The price of a factor of production may be set or regulated by the government, or it may be left to be determined by market forces. In a free market, interest rates on loans are determined by the demand for capital and its supply. One key determinant of interest is what is termed the *real* rate of interest.

What exactly is the real rate? The real rate may be defined as the rate of interest that would prevail on a riskless investment in the absence of inflation. What is a riskless investment in practice? A loan to a central or federal government of a country may be termed riskless because these institutions are empowered to levy taxes and print money. As a consequence, there is no risk of nonpayment. In the United States, securities such are Treasury bonds, bills, and notes, which are backed by the full faith and credit of the federal government, may therefore be construed as riskless from the standpoint of nonrepayment.

However, even these Treasury securities are not devoid of risk from the point of view of protection against inflation. What exactly is inflation? *Inflation* refers to the change in the purchasing power of money, or the change in the price level. Usually inflation is positive, which means that the purchasing power of money will be constantly eroding. There could be less common situations in which inflation is negative, a phenomenon termed *deflation*. In such a situation, the value of a dollar, in terms of its ability to acquire goods and services, will actually increase. Example 2.1 illustrates this.

The Fisher Equation

Consider a hypothetical economy which is characterized by the availability of a single good, namely, chocolates. The current price of a box is $\$P_0$. One dollar is adequate to buy $1/P_0$ boxes of chocolates at today's

Example 2.1

Take the case of an investment in a Treasury security with a face value of $1,000. Assume that it will pay $100 by way of interest every year. When the security is acquired, the price of a box of chocolates is $10, and we will assume that the price remains the same untill the end of the year. Consequently, the investor can expect to buy 10 boxes after a year when he receives the interest.

In this example, the rate of interest in terms of dollars is 10 percent per annum, because an investment of $1,000 yields a cash flow of $100. In terms of goods—in this case, chocolates—our return is also 10 percent. The principal corresponds to an investment in 100 boxes of chocolates, and the interest received in dollars facilitates the acquisition of another 10 boxes. The rate of interest as measured by our ability to buy goods and services is termed the *real rate of interest*.

In real life, however, price levels are not constant, and inflation is a constant fact of life. Assume that the price of chocolates after a year is $12.50. If so, the $100 of interest that will be received as cash will be adequate to buy only 8 boxes of chocolates. The principal itself will be adequate to buy only 80 boxes of chocolates, which means that the investor can acquire only 88 boxes in total. Even though the return on investment in terms of money is 10 percent, in terms of the ability to buy goods it is −12 percent.

Assets such as Treasury securities give us returns in terms of money, without any assurance as to what our ability to acquire goods and services will be at the time of repayment. The rate of return yielded by such securities in dollar terms is termed the *nominal* or money rate of return. In our illustration, the investor got a 10 percent return on an investment of $1,000. In the situation in which the price of a box of chocolates remained at $10, the ability to buy chocolates was enhanced by 10 percent, and consequently the real rate was also 10 percent. However, when the price of chocolates rose to $12.50 per box, an initial investment of $1,000, which represented an ability to buy 100 boxes of chocolates at the outset, was translated into an ability to buy only 88 boxes at the end of the year. The real rate of return in this case was negative, or −12 percent to be precise.

The relationship between the nominal and real rates of return is called the *Fisher hypothesis* after the economist who first postulated it.

prices. Let the price of a box after a year be $\$P_1$. Assume that even though P_1 is known with certainty right from the outset, it need not be equal to P_0. In other words, although we are allowing for the possibility of inflation, we are assuming that there is no uncertainty regarding the rate of inflation. If the price of a box at the end of the year is P_1, then one dollar will be adequate to buy $1/P_1$ boxes after a year.

Let us assume two types of bonds are available to a potential investor: a financial bond that will pay $\$(1+R)$ next period per dollar that is invested now, and a goods bond that will return $(1+r)$ boxes of chocolates next period per box that is invested today. An investment of one dollar in the financial bond will give the investor dollars $(1+R)$ next period, which will be adequate to buy $(1+R)/P_1$ boxes. Similarly, an investment of one dollar in the goods bond or $1/P_0$ boxes in terms of chocolates will yield $(1+r)/P_0$ boxes after a year.

For the economy to be in equilibrium, both bonds must yield identical returns. We require that

$$\frac{1+R}{P_1} = \frac{1+r}{P_0}$$

$$\Rightarrow (1+R) = (1+r) \times \frac{P_1}{P_0}.$$

Inflation is defined as the rate of change in the price level. If we denote inflation by π, then

$$\pi = \frac{P_1 - P_0}{P_0} \Rightarrow \frac{P_1}{P_0} = (1+\pi).$$

Therefore

$$(1+R) = (1+r)(1+\pi) \Rightarrow R = r + \pi + r \times \pi.$$

This is the Fisher equation. The return on the financial bond, R, is the nominal rate of return; the rate of return on the goods bond, r, is the real rate of return. The relationship is that 1 plus the nominal interest rate is equal to the product of 1 plus the real interest rate and 1 plus the rate of inflation. If the real rate of interest and the rate of inflation are fairly small, then the product $r \times \pi$ will be of a much smaller order of magnitude. If $r = 0.03$ and $\pi = 0.03$, then $r \times \pi = 0.0009$. If so, we can ignore the product term and rewrite the expression as

$$R = r + \pi.$$

This is called the *approximate Fisher relationship*.

In the preceding example, the nominal rate of return was 10 percent whereas the rate of inflation was 25 percent. The real rate was $(1.10)/(1.25) - 1 = 0.88 - 1.0 = -0.12 \equiv -12$ percent.

Simple Interest and Compound Interest

Before analyzing interest computation techniques, we first define certain key terms.

Measurement Period

The unit in which time is measured for the purpose of stating the rate of interest is called the *measurement period*. The most common measurement period is one year, and we will use a year as the unit of measurement unless otherwise specified. In other words, we will typically state that the interest rate is x percent—say, 10 percent—with the implication that the rate of interest is 10 percent per annum.

Interest Conversion Period

The unit of time over which interest is paid once and is reinvested to earn additional interest is referred to as the *interest conversion period*. The interest conversion period will typically be less than or equal to the measurement period. The measurement period may be a year, whereas the interest conversion period may be three months. Interest is compounded every quarter in this case.

Nominal Rate of Interest

The quoted rate of interest per measurement period is called the *nominal rate of interest*. In the preceding example, the nominal rate of interest is 10 percent.

Effective Rate of Interest

The *effective rate of interest* may be defined as what a dollar invested at the beginning of a measurement period would have earned by the end of the period. The effective rate will be equal to the quoted or nominal rate if the length of the interest conversion period is the same as that of the measurement period. However, if the interest conversion period is shorter than the measurement period or, in other words, if interest is compounded more than once per measurement period, then the effective rate will exceed the nominal rate of interest. Take the case in which the nominal rate is 10 percent per annum. If interest is compounded only once per annum, then an initial investment of \$1 will yield \$1.10 at the end of the year, and we would say that the effective rate of interest is 10 percent per annum. However, if the nominal rate is 10 percent per annum, but interest is credited every quarter, then the terminal value of an investment of one dollar will definitely be more than \$1.10. The relationship between the effective rate and the nominal rate will be derived subsequently. Remember that the term *nominal rate* of interest is being used in a different context than in the earlier discussion, where it was used in the context of the real rate of interest. The potential for confusion is understandable yet unavoidable.

Variables and Corresponding Symbols

$P \equiv$ amount of principal that is invested at the outset
$N \equiv$ number of measurement periods for which the investment is being made
$r \equiv$ nominal rate of interest per measurement period
$i \equiv$ effective rate of interest per measurement period
$m \equiv$ number of interest conversion periods per measurement period.

Simple Interest

Take the case of an investor who makes an investment of \$$P$ for N periods. If interest is paid on a simple basis, then we can state the following:

- The interest that will be earned every period is a constant.
- In every period, interest is computed and credited only on the original principal.
- No interest is payable on any interest that has been accumulated at an intermediate stage.

Let r be the quoted rate of interest per measurement period. Consider an investment of \$$P$. It will grow to \$$P(1 + r)$ after one period. In the second period, if simple interest is being paid, then interest will be paid only on P and not on $P(1 + r)$. Consequently, the accumulated value after two periods will be \$$P(1 + 2r)$. In general, if the investment is made for N periods, then the terminal value of the original investment will be \$$P(1 + rN)$. N need not be an integer—that is, investments may be made for fractional periods. Examples 2.2 and 2.3 illustrate this.

Example 2.2

Katherine Mitchell has deposited \$25,000 with Continental Bank for a period of four years. The bank pays interest at the rate of 8 percent per annum on a simple basis. The growth of Katherine's deposit may be viewed as follows.

After one year, an investment of \$25,000 will become \$27,000:

$$\$25{,}000 \times 1.08 = \$27{,}000.$$

The interest for the year is \$2,000. At the end of the second year, interest for the year will be paid only on the original principal of \$25,000 and not on the previous year's terminal value of \$27,000. Consequently, the accumulated value after two years will be

$$\$25{,}000 \times 1.08 + 2{,}000 = \$29{,}000.$$

Extending the logic, the terminal balance after three years will be \$31,000, and the final balance after four years will be \$33,000:

$$\$33{,}000 = \$25{,}000 \times (1 + 0.08 \times 4) \equiv P(1 + rN).$$

Notice the following:

- The interest paid every year is a constant amount of \$2,000.
- Every year interest is paid only on the original deposit of \$25,000.
- No interest is paid on interest that is accumulated at an earlier stage.

Example 2.3

Alex Gunning deposited \$25,000 with International Bank for four years and nine months and wants to withdraw the balance at maturity. The bank pays 8 percent interest per annum on a simple interest basis. The terminal value in this case is given by

$$P(1 + rN) = \$21{,}000 \times (1 + 0.08 \times 4.75) = \$34{,}500.$$

Notice that N—in this case, 4.75 years—need not be an integer.

Compound Interest

Let us take the case of an investment of $\$P$ that has been made for N measurement periods. However, we will assume this time that interest is compounded at the end of every year. Notice that we are assuming that the interest conversion period is equal to the measurement period, namely, one year. In other words, the quoted rate is equal to the effective rate.

In this case, an original investment of $\$P$ will become $\$P(1 + r)$ dollars after one period. However, the difference as compared to the earlier case is that during the second period the entire amount will earn interest; consequently, the balance at the end of two periods will be $P(1 + r)^2$. Extending the logic, the balance after N periods will be $P(1 + \mathrm{r})^N$. Once again, note that N need not be an integer.

The following observations are valid if interest is paid on a compound interest basis:

- Every time interest is earned, it is automatically reinvested at the same rate for the next conversion period.
- Interest is paid on the accumulated value at the start of the conversion period and not on the original principal.
- The interest earned every period will not be a constant but will steadily increase.

Example 2.4 illustrates these principles.

We mentioned that the number of periods for which interest is compounded need not be an integer. Example 2.5 illusrates this concept.

As can be seen from the examples, compounding yields substantially greater benefits than simple interest. And because the rate of interest is taken to the power of N, the larger the value of N, the greater will be the

Example 2.4

Assume that Katherine Mitchell has deposited \$25,000 with Continental Bank for four years, and that the bank pays 8 percent interest per annum compounded annually.

After one year, the initial investment of \$25,000 will become \$27,000:

$$\$25{,}000 \times 1.08 = \$27{,}000.$$

In contrast to the earlier example where simple interest was considered, in this case the entire accumulated value of \$27,000 will earn interest during the second year. The accumulated value after two years will be

$$\$27{,}000 \times 1.08 = \$29{,}160.$$

By the same logic, the balance after four years will be

$$\$25{,}000 \times 1.08 \times 1.08 \times 1.08 \times 1.08 = \$34{,}012.2240 = 25{,}000 \times (1.08)^4 \equiv P(1+r)^N$$

The growth pattern is illustrated in Table 2.1.

Table 2.1
The Compounding Process

Year	Balance at the Beginning of the Year (\$)	Balance at the End of the Year (\$)	Interest for the Year (\$)
1	25,000.00	27,000.00	2,000.00
2	27,000.00	29,160.00	2,160.00
3	29,160.00	31,492.80	2,332.80
4	31,492.80	34,012.22	2,519.42

Notice the following. In the case of simple interest, the interest paid every year was a constant amount of \$2,000. However, in the case of compound interest, the amount steadily increases, as can be seen from the last column of Table 2.1.

Example 2.5

Alex Gunning deposited $25,000 with International Bank for four years and nine months. The bank has been paying interest at the rate of 8 percent per annum on a compound interest basis. Let us calculate the terminal balance:

$$P(1+r)^N = 25{,}000 \times (1.08)^{4.75} = \$36{,}033.20.$$

Earlier, when we assumed simple interest, we got a value of $34,500.

impact of compounding. In other words, the earlier one starts investing, the greater will be the returns. Example 2.6 illustrate the importance of this.

Example 2.6

Jesus was born approximately 2,000 years ago. Assume that an investment of $1 was made in that year in a bank that has ever since been paying 1 percent interest per annum compounded annually. What will be the accumulated balance in the year 2000?

$$1 \times (1.01)^{2{,}000} = \$439{,}286{,}205.$$

Properties

- If $N = 1$—that is, an investment is made for one period—then both the simple as well as the compound interest techniques will give the same accumulated value.

In the case of Katherine, the value of her initial investment of $25,000 at the end of the first year was $27,000, irrespective of whether simple or compound interest was used.

- If $N < 1$—that is, the investment is made for less than a period—then the accumulated value using simple interest will be higher; that is,

$$(1 + rN) > (1 + r)^N \text{ if } N < 1.$$

Assume that Katherine deposits \$25,000 for nine months at a rate of 8 percent per annum compounded annually. If interest is calculated on a simple interest basis, then she will receive \$26,500:

$$\$25{,}000 \times (1 + 0.08 \times 0.75) = \$26{,}500.$$

On the other hand, compound interest would yield \$26,485.48:

$$\$25{,}000 \times (1.08)^{0.75} = \$26{,}485{,}48.$$

- If $N > 1$—that is, the investment is made for more than a period—then the accumulated value using compound interest will always be greater; that is,

$$(1 + rN) < (1 + r)^N \text{ if } N > 1$$

As can be seen, if Katherine were to invest for four years, simple interest will yield \$33,000 at the end, whereas compound interest will yield \$34,012.22.

Comment 1: The word *period* just used to demonstrate the properties of simple and compound interest should be interpreted as the interest conversion period. In our illustrations, the interest was compounded once per year, so there was no difference between the measurement period and the conversion period. However, take the case where interest is paid at 8 percent per annum compounded quarterly. If so the preceding properties may be stated as follows:

- If the investment is made for one quarter, then both simple and compound interest will yield the same terminal value.
- If the investment is made for less than a quarter, then the simple interest technique will yield a greater terminal value.
- If the investment is made for more than a quarter, then the compound interest technique will yield a greater terminal value.

Simple interest is usually used for short-term or current account transactions, that is, for investments for a period of one year or less. Consequently, simple interest is the norm for money market calculations. The term *money market* refers to the market for debt securities with a time to maturity at the time of issue of one year or less. However, in the case of capital market securities—that is, medium- to long-term debt securities and equities—we use the compound interest principle. Simple interest is also at times used as an approximation for compound interest over fractional periods. Example 2.7 demonstrates this.

Example 2.7

Alex Gunning deposited \$25,000 with International Bank for four years and nine months. Assume that the bank pays compound interest at the rate of 8 percent per annum for the first four years and simple interest for the last nine months.

The balance at the end of four years will be \$34,012.22:

$$\$25{,}000 \times (1.08)^4 = \$34{,}012.22.$$

The terminal balance will be \$36,052.96:

$$\$34{,}012.22 \times (1 + 0.08 \times 0.75) = \$36{,}052.96.$$

In the earlier case, when interest was compounded for four years and nine months, the accumulated value was \$36,033.20. Simple interest for the fractional period yields an additional benefit of \$19.76. We get a higher value in the second case because for a fractional period simple interest will give a greater return than compound interest.

Effective versus Nominal Rates of Interest

Example 2.8 illustrates the difference between nominal rates and effective rates. We will then derive a relationship between the two symbolically.

Example 2.8

ING Bank is quoting a rate of 8 percent per annum compounded annually on deposits placed with it, whereas HSBC is quoting 7.80 percent per annum compounded monthly on funds deposited with it. A naïve investor may be tempted to conclude that ING is offering better returns, because its quoted rate is higher. However, it important to note that the compounding frequencies are different. Whereas ING is compounding on an annual basis, HSBC is compounding every month.

From our earlier discussion, we know that because ING is compounding only once a year, the effective rate offered by it is the same as the rate quoted by it, which is 8 percent per annum. However, because HSBC is compounding on a monthly basis, its effective rate will be greater than the rate quoted by it. The issue is, is the effective rate greater than 8 percent per annum?

An annual rate of 7.80 percent corresponds to $7.80/12 = 0.65$ percent per month. Consequently, if an investor were to deposit \$1 with HSBC for a period of one year, or 12 months, the terminal value would be

$$1 \times (1.0065)^{12} = 1.08085.$$

A rate of 7.80 percent per annum compounded monthly is equivalent to receiving a rate of 8.085 percent with annual compounding. The phrase *effective annual rate* effectively means that the investor who deposits with HSBC receives a rate of 8.085 percent compounded on an annual basis.

When the frequencies of compounding are different, comparisons between alternative investments ought to be based on the effective rates of interest and not on the nominal rates. In our case, an investor who is contemplating a deposit of, say, \$10,000 for a year would choose to invest with HSBC despite the fact that its quoted or nominal rate is lower.

Comment 2: Remember that the distinction between nominal and effective rates is of relevance only when compound interest is being paid. The concept is of no consequence if simple interest is being paid.

A Symbolic Derivation

An investor is being offered a nominal rate of r percent per annum, and that interest is being compounded m times per annum. In the earlier example, because HSBC was compounding on a monthly basis, m was 12. The effective rate of interest i is therefore given by

$$1 + i = (1 + r/m)^m.$$

We can also derive the equivalent nominal rate if the effective rate is given

$$r = m\left[(1 + i)^{1/m} - 1\right].$$

We have already seen how to convert a quoted rate to an effective rate. We will now demonstrate how the rate to be quoted can be derived based on the desired effective rate.

Assume that HSBC Bank wants to offer an effective annual rate of 12 percent per annum with quarterly compounding. The question is, what nominal rate of interest should it quote?

In this case, $i = 12$ percent, and $m = 4$. We have to calculate the corresponding quoted rate r

$$r = m\left[(1 + i)^{1/m} - 1\right]$$

$$\Rightarrow r = 4\left[(1.12)^{0.25} - 1\right] = 11.49\%.$$

A quoted rate of 11.49 percent with quarterly compounding is tantamount to an effective annual rate of 12 percent per annum. HSBC should quote 11.49 percent per annum.

Principle of Equivalency

Two nominal rates of interest compounded at different intervals of time are said to be equivalent if they yield the same effective interest rate for a specified measurement period.

Assume that ING Bank is offering 10 percent per annum with semi-annual compounding. What should be the equivalent rate offered by a competitor if it intends to compound interest on a quarterly basis?

The first step in comparing two rates that are compounded at different frequencies is to convert them to effective annual rates. The effective rate offered by ING is

$$i = (1 + 0.05)^2 - 1 = 0.1025 \equiv 10.25\%.$$

The question is, what is the quoted rate that will yield the same effective rate if quarterly compounding were to be used?

$$r = 4\left[(1.1025)^{0.25} - 1\right] = 0.0988 \equiv 9.88\%.$$

Hence, 10 percent per annum with semiannual compounding is equivalent to 9.88 percent per annum with quarterly compounding, because in both cases the effective annual rate is the same.

Continuous Compounding

We know that if a dollar is invested for N periods at a quoted rate of r percent per period and if interest is compounded m times per period, then the terminal value is given by the expression

$$(1 + r/m)^{mN}.$$

In the limit as $m \to \infty$

$$(1 + r/m)^{mN} \to e^{rN}$$

where e = 2.71828. Known as Euler's number or Napier's constant, e is defined by the expression

$$e = \mathrm{Lim}_{n \to \infty} (1 + 1/n)^n.$$

This limiting case is referred to as *continuous compounding*. If r is the nominal annual rate, then the effective annual rate with continuous compounding is $e^r - 1$.

Example 2.9 illustrates the use of continuous compounding while Example 2.10 demonstrates that continuous compounding is the limit as we go to shorter and shorter compounding intervals.

Example 2.9

Nigel Roberts has deposited $25,000 with Continental Bank for a period of four years at 8 percent per annum compounded continuously. The terminal balance may be computed as

$$\$25{,}000 \times e^{0.08 \times 4} = 25{,}000 \times 1.3771 = \$34{,}428.19$$

Continuous compounding is the limit of the compounding process as we go from annual to semiannual and on to quarterly, monthly, daily, and even shorter intervals. This can be illustrated with the help of an example.

Example 2.10

Sheila Norton has deposited $100 with ING Bank for one year. Let us calculate the account balance at the end of the year for various compounding frequencies. We will assume that the quoted rate in all cases is 10 percent per annum.

The answers are shown in Table 2.2. By the time we reach daily compounding, we have almost reached the limiting value.

Table 2.2
Compounding at Various Frequencies

Compounding Interval	Terminal Balance ($)
Annual	110.0000
Semiannual	110.2500
Quarterly	110.3813
Monthly	110.4713
Daily	110.5156
Continuously	110.5171

Future Value

We have already encountered the concept of future value, although we have not invoked the term in our discussion thus far. What exactly is the meaning of the future value of an investment? When an amount is deposited for a certain time period at a given rate of interest, the amount that is accrued at the end of the designated period of time is called the *future value* of the original investment.

If we were to invest $\$P$ for N periods at a periodic interest rate of r percent, then the future value (FV) of the investment is given by

$$\mathrm{FV} = P(1+r)^N$$

The expression $(1+r)^N$ is the amount to which an investment of \$1 will grow at the end of N periods, if it is invested at a rate r. It is called the *future value interest factor* (FVIF). It depends only on two variables, namely, the periodic interest rate and the number of periods. The advantage of knowing the FVIF is that we can find the future value of any principal amount for given values of the interest rate and time period by simply multiplying the principal by the factor. The process of finding the future value given an initial investment is called *compounding*. Example 2.11 illustrates the computation of the future value of an initial investment.

Example 2.11

Shelly Smith has deposited \$25,000 for four years in an account that pays interest at the rate of 8 percent per annum compounded annually. What is the future value of her investment?

The factor in this case is given by $\mathrm{FVIF}(8,4) = (1.08)^4 = 1.3605$. Thus, the future value of the deposit is $\$25{,}000 \times 1.3605 = \$34{,}012.50$.

Comment 3: Remember that the value of N corresponds to the total number of interest conversion periods, in case interest is being compounded more than once per measurement period. Consequently, the interest rate used should be the rate per interest conversion period. Example 2.12 clarifies this issue.

Example 2.12

Simone Peters has deposited \$25,000 for four years in an account that pays a nominal annual interest of 8 percent per annum with quarterly compounding. What is the future value of her investment?

A nominal interest rate of 8 percent per annum for four years is equivalent to 2 percent per quarter for 16 quarterly periods. Thus, the required factor is FVIF(2,16) and not FVIF(8,4):

$$\text{FVIF}(2,16) = (1.02)^{16} = 1.3728.$$

The future value is $\$25{,}000 \times 1.3728 = \$34{,}320$.

Comment 4: The FVIF is given in the form of tables in most textbooks for integer values of the interest rate and number of time periods. However, if either the interest rate or the number of periods is not an integer, then we cannot use such tables and would have to rely on a scientific calculator or a spreadsheet.

Present Value

Future value calculations entailed the determination of the terminal value of an initial investment. Sometimes, however, we may seek to do the reverse. In other words, we may have a terminal value in mind and seek to calculate the quantum of the initial investment that will result in the desired terminal cash flow, given an interest rate and investment horizon. In this case, instead of computing the terminal value of a given principal, we seek to compute the principal that corresponds to a given terminal value. The principal amount that is obtained in this fashion is referred to as the *present value* (PV) of the terminal cash flow.

The Mechanics of Present Value Calculation

Take the case of an investor who wishes to have \$F after N periods. The periodic interest rate is r percent, and interest is compounded once per period. Our objective is to determine the initial investment that will result in the desired terminal cash flow. So

$$\text{PV} = \frac{\text{F}}{(1+r)^N}$$

where PV is the present value of \$F.

Example 2.13 is an illustration of a present value calculation.

Example 2.13

Patricia wants to deposit an amount of \$P with her bank in order to ensure that she has \$25,000 at the end of four years. If the bank pays 8 percent interest per annum compounded annually, how much does she have to deposit today?

$$P = \frac{25{,}000}{(1.08)^4} = \$\ 18{,}375.75.$$

The expression $1/(1 + r)^N$ is the amount that must be invested today if we are to have \$1 at the end of N periods if the investment were to pay interest at the rate of r percent per period. It is called the *present value interest factor* (PVIF). It, too, depends only on two variables, namely, the interest rate per period and the number of periods. If we know the PVIF for a given interest rate and time horizon, then we can compute the present value of any terminal cash flow by simply multiplying the quantum of the cash flow by the factor. The process of finding the principal corresponding to a given future amount is called *discounting*, and the interest rate that is used is called the *discount rate*. There is a relationship between the present value factor and the future value factor for assumed values of the interest rate and the time horizon. One factor is simply a reciprocal of the other.

Handling a Series of Cash Flows

Let us assume that we wish to compute the present value or the future value of a series of cash flows for a given interest rate. The first cash flow will arise after one period, and the last will arise after N periods. In such a situation, we can simply find the present value of each of the component cash flows and add the terms in order to compute the present value of the entire series. The same holds true for computing the future value of a series of cash flows. Present values and future values are additive in nature as illustrated by Example 2.14.

Example 2.14

Let us consider the vector of cash flows shown in Table 2.3. Assume that the interest rate is 8 percent per annum compounded annually. Our objective is to compute the present value and future value of the entire series.

Table 2.3
Vector of Cash Flows

Year	Cash Flow ($)
1	2,500
2	5,000
3	8,000
4	10,000
5	20,000

The present and future value of the series is depicted in Table 2.4. We have simply computed the present and future value of each term in the series and summed the values.

Table 2.4
Present and Future Values of the Cash Flows

Year	Cash Flow ($)	Present Value ($)	Future Value ($)
1	2,500	2,314.8148	3,401.2224
2	5,000	4,286.6941	6,298.5600
3	8,000	6,350.6579	9,331.2000
4	10,000	7,350.2985	10,800.0000
5	20,000	13,611.6639	20,000.0000
	Total Value	33,914.1293	49,830.9824

While computing the present value of each cash flow, we have to discount the amount so as to obtain the value at time 0. The first year's cash flow has to be discounted for one year, whereas the fifth year's cash flow has to be discounted for five years. On the other hand, while computing the future value of a cash flow, we have to find its terminal value as at the end of five years. Consequently, the cash flow arising after one year has to be compounded for four years, whereas the final cash flow, which is received at the end of five years, does not have to be compounded.

There is a relationship between the present value of the vector of cash flows as a whole and its future value. It may be stated as

$$FV = PV(1 + r)^N$$

In this case,

$$\$49{,}830.9824 = \$33{,}914.1293 \times (1.08)^5.$$

The Internal Rate of Return

Consider a deal in which we are offered the vector of cash flows depicted in Table 2.4 in return for an initial investment of \$30,000. The question is, what is the rate of return that we are being offered? The rate of return r is the solution to the following equation:

$$30{,}000 = \frac{2{,}500}{(1 + r)} + \frac{5{,}000}{(1 + r)^2} + \frac{8{,}000}{(1 + r)^3} + \frac{10{,}000}{(1 + r)^4} + \frac{20{,}000}{(1 + r)^5}$$

The solution to this equation is termed the *internal rate of return* (IRR). It can be obtained using the IRR function in Excel. In this case, the solution is 11.6106 percent.

Comment 5: A Point about Effective Rates.

Let us assume that we are asked to compute the present value or future value of a series of cash flows arising every six months and are given a rate of interest quoted in annual terms without the frequency of compounding being specified. The normal practice is to assume semi-annual compounding. In other words, we would divide the annual rate by two to determine the periodic interest rate for discounting or compounding. In other words, the quoted interest rate per annum will be treated as the nominal rate and not as the effective rate. Example 2.15 illustrates the importance of this comment.

Evaluating an Investment

Kapital Markets is offering an instrument that will pay \$25,000 after four years in return for an initial investment of \$12,500. Alfred is a potential

Example 2.15

Consider the series of cash flows depicted in Table 2.5. Assume that the annual rate of interest is 8 percent.

Table 2.5
Vector of Cash Flows

Period	Cash Flow ($)
6 months	2,000
12 months	2,500
18 months	3,500
24 months	7,000

The present value will be calculated as

$$PV = \frac{2{,}000}{(1.04)} + \frac{2{,}500}{(1.04)^2} + \frac{3{,}500}{(1.04)^3} + \frac{7{,}000}{(1.04)^4} = \$13{,}329.5840$$

Similarly, the future value will be

$$\begin{aligned} FV &= 2{,}000 \times (1.04)^3 + 2{,}500 \times (1.04)^2 + 3{,}500 \times (1.04) + 7{,}000 \\ &= \$15{,}593.7280. \end{aligned}$$

However, if it were to be explicitly stated that the effective annual rate is 8 percent, then the calculations would change. The semi-annual rate that corresponds to an effective annual rate of 8 percent is $(1.08)^{0.5} = 1.039230$. The present value will then be given by

$$\begin{aligned} PV &= \frac{2{,}000}{(1.039230)} + \frac{2{,}500}{(1.039230)^2} + \frac{3{,}500}{(1.039230)^3} + \frac{7{,}000}{(1.039230)^4} \\ &= \$13{,}359.1103. \end{aligned}$$

Similarly the future value will then be given by

$$\begin{aligned} FV &= 2{,}000 \times (1.039230)^3 + 2{,}500 \times (1.039230)^2 + 3{,}500 \\ &\quad \times (1.039230) + 7{,}000 = \$15{,}582.0372. \end{aligned}$$

The present value is higher when we use an effective annual rate of 8 percent for discounting. This is because the lower the discount rate, the higher the present value will be; an effective annual rate of 8 percent is lower than a nominal annual rate of 8 percent with semiannual compounding. Because the interest rate that is used is lower, the future value at the end of four half-years is lower when we use an effective annual rate of 8 percent.

In this case, if we were to calculate the IRR for the given cash flow stream, we would get a semiannual rate of return. We would then have to multiply it by two to get the annual rate of return. The IRR for this cash flow stream, assuming an initial investment of \$12,500, is 6.2716 percent. The IRR in annual terms is therefore 12.5432 percent.

investor who requires a rate of return of 12 percent per annum. The question is, is the offer attractive from his perspective? There are three ways of approaching this problem.

The Future Value Approach

Let us assume that Alfred buys this instrument for \$12,500. If the rate of return received by him were to be 12 percent, he would have to receive a future value of \$19,669. This can be stated as

$$\mathrm{FV} = 12{,}500 \times (1.12)^4 = \$19{,}669.$$

If Alfred were to receive a higher terminal payment, his rate of return would be higher than 12 percent, else it would be lower. Because the instrument offered to him promises a terminal value of \$25,000 which is greater than the required future value of \$19,669, the investment is attractive from his perspective.

The Present Value Approach

The present value of \$25,000 using a discount rate of 12 percent per annum is

$$\mathrm{PV} = \frac{25{,}000}{(1.12)^4} = 25{,}000 \times 0.6355 = \$15{,}888.15.$$

The rate of return, if one were to make an investment of $15,888.15 in return for a payment of $25,000 four years hence, is 12 percent. If the investor were to pay a lower price at the outset, he would earn a rate of return that is higher than 12 percent, whereas if he were to invest more, he would earn a lower rate of return. In this case, Alfred is being asked to invest $12,500, which is less than $15,288.15. Consequently, the investment is attractive from his perspective.

The Rate of Return Approach

If Alfred were to pay $12,500 in return for a cash flow of $25,000 after four years, his rate of return may be computed as

$$12{,}500 = \frac{25{,}000}{(1+r)^4}$$

$$\Rightarrow (1+r)^4 = 2 \Rightarrow r = [2]^{0.25} - 1 = 0.1892 \equiv 18.92\%.$$

Because the actual rate of return obtained by Alfred is greater than the required rate of return of 12 percent, the investment is attractive.

It is no surprise that all the three approaches lead to the same decision.

Annuities: An Introduction

An annuity is a series of payments made at equally spaced intervals of time. If all the payments are identical, then we term it as a *level annuity*. Examples include insurance premiums and monthly installments on housing loans and automobile loans, which are paid off by way of equal installments over a period of time.

If the first payment is made or received at the end of the first period, then we call it an *ordinary annuity*. Examples include a salary, which will be paid only after an employee completes his duties for the month, and house rent, which will be usually paid by the tenant only at the end of the month. The interval between successive payments is called the *payment period*. We will assume that the payment period is the same as the interest conversion period. In other words, if the annuity pays annually, we will assume annual compounding, whereas if it pays semiannually we will assume half-yearly compounding. This assumption is not mandatory; we can easily handle

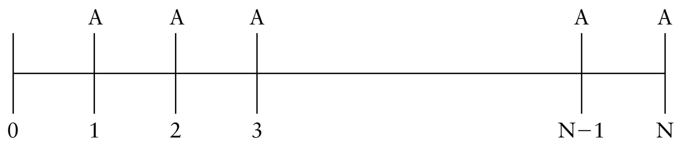


Figure 2.1 Timeline for an Annuity

cases where the payment period is longer than the interest conversion period as well as instances where it is shorter.

Consider a level annuity that makes periodic payments of \$$A$ for N periods. On a timeline, the cash flows can be depicted as shown in Figure 2.1. The point in time where we are is depicted as time 0.

Assume that the applicable interest rate per period is r percent. We can then calculate the present and future values as follows.

Present Value

$$\mathrm{PV} = \frac{A}{(1+r)} + \frac{A}{(1+r)^2} + \frac{A}{(1+r)^3} + - - - - + \frac{A}{(1+r)^N}$$

Therefore,

$$\mathrm{PV}(1+r) = A + \frac{A}{(1+r)} + \frac{A}{(1+r)^2} + - - - - - + \frac{A}{(1+r)^{N-1}}$$

$$\Rightarrow \mathrm{PV}[(1+r)-1] = A - \frac{A}{(1+r)^N}$$

$$\Rightarrow \mathrm{PV} = \frac{A}{r}\left[1 - \frac{1}{(1+r)^N}\right].$$

The value $\frac{1}{r}\left[1 - \frac{1}{(1+r)^N}\right]$ is called the *present value interest factor annuity* (PVIFA). PVIFA(r,N) is the present value of an annuity that pays \$1 at periodic intervals for N periods computed using a discount rate of r percent. Like the factors that we studied earlier, it also depends on the interest rate and the number of periods. The present value of any annuity that pays \$$A$ per period, can therefore be computed by multiplying A by the appropriate value of PVIFA. Example 2.16 illustrates this.

Example 2.16

Alpha Technologies is offering a financial instrument to Alfred that promises to pay \$2,500 per year for 25 years, beginning one year from now. Alfred requires an annual rate of return of 8 percent. The question is, what is the maximum price that he will be prepared to pay?

$$\text{PVIFA}(8,25) = 10.6748.$$

The value of the payments is

$$\$2{,}500 \times \text{PVIFA}(8,25) = \$2{,}500 \times 10.6748 = \$26{,}687.$$

Future Value

Similarly, we can compute the future value of a level annuity that makes N payments by compounding each cash flow until the end of the last payment period.

$$\text{FV} = A(1+r)^{N-1} + A(1+r)^{N-2} + A(1+r)^{N-3} + ----- + A.$$

Therefore,

$$\begin{aligned}\text{FV}(1+r) = {} & A(1+r)^{N} + A(1+r)^{N-1} + A(1+r)^{N-2} \\ & + ----- + A(1+r) \Rightarrow \text{FV}[(1+r)-1] \\ & = A(1+r)^{N} - A \Rightarrow \text{FV} = \frac{A}{r}[(1+r)^{N}-1].\end{aligned}$$

The value $\frac{1}{r}[(1+r)^{N}-1]$ is called the *future value interest factor annuity* (FVIFA). It is the future value of an annuity that pays \$1 per period for N periods, where interest is compounded at the rate of r percent per period. The advantage once again is that if we know the factor, we can calculate the future value of any annuity that pays \$$A$ per period. Example 2.17 illustrates this.

Annuity Due

The difference between an annuity and an *annuity due* is that in the latter case the cash flows occur at the beginning of the period. Figure 2.2 depicts an N period annuity due that makes periodic payments of \$$A$.

Example 2.17

Paula Baker expects to receive \$2,500 per year for the next 25 years starting one year from now. Assuming that the cash flows can be reinvested at 8 percent per annum, how much will she have at the point of receipt of the last cash flow?

$$\text{FVIFA}(8,25) = 73.1059.$$

The future value is

$$\text{FV} = \$2{,}500 \times \text{FVIFA}(8,25) = \$2{,}500 \times 73.1059 = \$182{,}764.75.$$

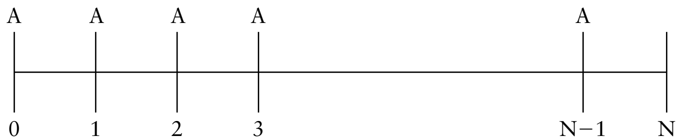


Figure 2.2 Timeline for an Annuity Due

Present Value

$$\text{PV} = A + \frac{A}{(1+r)} + \frac{A}{(1+r)^2} + - - - - - + \frac{A}{(1+r)^{N-1}}.$$

Therefore,

$$\text{PV}(1+r) = A(1+r) + A + \frac{A}{(1+r)} + - - - - - + \frac{A}{(1+r)^{N-2}}$$

$$\Rightarrow \text{PV}[(1+r) - 1] = A(1+r) - \frac{A}{(1+r)^{N-1}}$$

$$\Rightarrow \text{PV} = \frac{A}{r}\left[1 - \frac{1}{(1+r)^N}\right](1+r).$$

Hence

$$\text{PVIFA}_{\text{AD}}(r,N) = \text{PVIFA}(r,N) \times (1+r).$$

The present value of an annuity due that makes N payments is greater than that of a corresponding annuity that makes N payments because, in the case of the annuity due, each of the cash flows has to be discounted for one period less. Consequently, the present value factor for an N period annuity due is greater than that for an N period annuity by a factor of $(1 + r)$.

An example of an annuity due is an insurance policy, because the first premium has to be paid as soon as the policy is purchased. Example 2.18 illustrates this.

Example 2.18

David Mathew has just bought an insurance policy from Met Life. The annual premium is \$2,500, and he is required to make 25 payments. What is the present value of this annuity due if the discount rate is 8 percent per annum?

$$\text{PVIFA}(5{,}25) = 10.6748.$$

$$\text{PVIFA}_{\text{AD}}(8{,}25) = 10.6748 \times 1.08 = 11.5288.$$

The present value of the annuity due is

$$\$2{,}500 \times 11.5288 = \$28{,}822.$$

Future Value

$$\text{FV} = A(1+r)^N + A(1+r)^{N-1} + A(1+r)^{N-2} + - - - - + A(1+r).$$

Therefore,

$$\text{FV}(1+r) = A(1+r)^{N+1} + A(1+r)^N + A(1+r)^{N-1}$$
$$+ - - - - + A(1+r)^2$$
$$\Rightarrow \text{FV}[(1+r) - 1] = A(1+r)^{N+1} - A(1+r)$$
$$\Rightarrow \text{FV} = \frac{A}{r}[(1+r)^N - 1](1+r).$$

Hence,

$$\text{FVIFA}_{\text{AD}}(r,N) = \text{FVIFA}(r,N) \times (1 + r).$$

The future value of an annuity due that makes N payments is higher than that of a corresponding annuity that makes N payments if the future values in both cases are computed at the end of N periods. This is because, in the first case, each cash flow has to be compounded for one period more.

Comment 6: Remember that the future value of an N period annuity due is greater than that of an N period annuity if both the values are computed at time N—that is, after N periods. The future value of an annuity due as computed at time $N - 1$ will be identical to that of an ordinary annuity as computed at time N. Example 2.19 illustrates the computation of the future value of an annuity due.

Example 2.19

In the case of Mathew's Met Life policy, the cash value at the end of 25 years can be calculated as follows:

$$\text{FVIFA}(8{,}25) = 73.1059$$

$$\text{FVIFA}_{\text{AD}}(8{,}25) = 73.1059 \times 1.08 = 78.9544.$$

The cash value of the annuity due is

$$\$2{,}500 \times 78.9544 = \$197{,}386.$$

Perpetuities

An annuity that pays forever is called a *perpetuity*. The future value of a perpetuity is infinite. But it turns out that a perpetuity has a finite present value. The present value of an annuity that pays for N periods is

$$\mathrm{PV} = \frac{A}{r}\left[1 - \frac{1}{(1+r)^N}\right].$$

The present value of the perpetuity can be found by letting N tend to infinity. As $N \to \infty$,

$$\frac{1}{(1+r)^N} \to 0.$$

Thus, the present value of a perpetuity is A/r.

As Example 2.20 demonstrates, a perpetuity may not be as attractive as it initially appears.

Example 2.20

Consider a financial instrument that promises to pay $2,500 per year forever. If an investor requires a 10 percent rate of return, the maximum amount that he would be prepared to pay may be computed as

$$\mathrm{PV} = 2{,}500/0.10 = \$25{,}000.$$

Although the cash flows are infinite, the security has a finite value. This is because the contribution of additional cash flows to the present value becomes insignificant after a certain point in time.

The Amortization Method

Amortization refers to the process of repaying a loan by means of regular installment payments at periodic intervals. Each installment includes payment of interest on the principal outstanding at the start of the period, as well as a partial repayment of the outstanding principal itself. In contrast, an ordinary loan entails the payment of interest at periodic intervals, and the repayment of principal in the form of a single lump-sum payment at maturity. In the case of an amortized loan, the installment payments form an annuity that has a present value equal to the original loan amount. An *amortization schedule* is a table that shows the division of each payment into a principal component and an interest component; it displays the outstanding loan balance after each payment.

Take the case of a loan that is repaid in N installments of \$$A$ each. We will denote the original loan amount by L and the periodic interest rate by r. This is an annuity with a present value of L, which is repaid in N installments

$$L = A \times \text{PVIFA}(r,N) = \frac{A}{r}\left[1 - \frac{1}{(1+r)^N}\right].$$

The interest component of the first installment is

$$\begin{aligned} &= r \times \frac{A}{r}\left[1 - \frac{1}{(1+r)^N}\right] \\ &= A\left[1 - \frac{1}{(1+r)^N}\right]. \end{aligned}$$

The principal component is

$$\begin{aligned} &= A - A\left[1 - \frac{1}{(1+r)^N}\right] \\ &= \frac{A}{(1+r)^N}. \end{aligned}$$

The outstanding balance at the end of the first payment is

$$\begin{aligned} &= \frac{A}{r}\left[1 - \frac{1}{(1+r)^N}\right] - \frac{A}{(1+r)^N} \\ &= \frac{A}{r}\left[1 - \frac{1}{(1+r)^{(N-1)}}\right]. \end{aligned}$$

In general, the interest component of the tth installment is

$$A\left[1 - \frac{1}{(1+r)^{N-t+1}}\right].$$

The principal component of the tth installment is

$$\frac{A}{(1+r)^{N-t+1}}.$$

The outstanding balance at the end of the tth payment is

$$\frac{A}{r}\left[1-\frac{1}{(1+r)^{N-t}}\right].$$

Example 2.21 illustrates the computation of periodic payment for an amortized loan and depicts the corresponding amortization schedule.

Example 2.21

Sylvie has borrowed \$25,000 from First National Bank and has to pay it back in eight equal annual installments. If the interest rate is 8 percent per annum on the outstanding balance, what is the installment amount, and what will the amortization schedule look like?

Let us denote the unknown installment amount by A. We know that

$$25{,}000 = \frac{A}{0.08} \times \left[1 - \frac{1}{(1.08)^8}\right]$$

$$\Rightarrow A = \$\ 4{,}350.37.$$

We will analyze the first few entries in Table 2.6 in order to clarify the principles involved. At time 0, the outstanding principal is \$25,000. After one period, a payment of \$4,350.37 will be made. The interest due for the first period is 8 percent of \$25,000, which is \$2,000. Consequently, the excess payment of \$2,350.37 represents a partial repayment of principal. Once this amount is repaid and adjusted toward the principal, the outstanding balance at the end of the first period will become \$22,649.63. At the end of the second period, the second installment of \$4,350.37 will be paid. The interest due for this period is 8 percent of the outstanding balance at the start of the period, which is \$22,649.63. The interest component of the second installment is \$1,811.97. The balance, which is $ 2,538.40, constitutes a partial repayment of principal. The value of the outstanding principal at the end should be zero. As can be seen, the outstanding principal declines after each installment payment. Because the payments themselves are constant, the interest component will steadily decline while the principal component will steadily increase.

Table 2.6
An Amortization Schedule

Year	Payment ($)	Interest ($)	Principal Repayment ($)	Outstanding Principal ($)
0				25,000.00
1	4,350.37	2,000.00	2,350.37	22,649.63
2	4,350.37	1,811.97	2,538.40	20,111.23
3	4,350.37	1,608.90	2,741.47	17,369.76
4	4,350.37	1,389.58	2,960.79	14,408.97
5	4,350.37	1,152.72	3,197.65	11,211.32
6	4,350.37	896.91	3,453.46	7,757.86
7	4,350.37	620.63	3,729.74	4,028.12
8	4,350.37	322.25	4,028.12	0.00

Amortization with a Balloon Payment

Julie Tate has taken a loan of $25,000 from First National Bank. The loan requires her to pay in eight equal annual installments along with a terminal payment of $5,000. This terminal payment that has to be made over and above the scheduled installment in year eight is termed a *balloon payment*. The interest rate is 8 percent per annum on the outstanding principal. The annual installment may be calculated as follows:

$$25{,}000 = \frac{A}{0.08} \times \left[1 - \frac{1}{(1.08)^8}\right] + \frac{5{,}000}{(1.08)^8}$$

$$\Rightarrow A = \$3{,}880.2950.$$

The larger the balloon, the smaller the periodic installment payment for a given loan amount. An amortization schedule is shown in Table 2.7.

Table 2.7
Amortization with a Balloon Payment

Year	Payment ($)	Interest ($)	Principal Repayment ($)	Outstanding Principal ($)
0				25,000.00
1	3,880.2950	2,000.00	1,880.30	23,119.70
2	3,880.2950	1,849.58	2,030.72	21,088.99
3	3,880.2950	1,687.12	2,193.18	18,895.81
4	3,880.2950	1,511.66	2,368.63	16,527.18
5	3,880.2950	1,322.17	2,558.12	13,969.06
6	3,880.2950	1,117.52	2,762.77	11,206.29
7	3,880.2950	896.50	2,983.79	8,222.50
8	8,880.2950	657.80	8,222.50	0.00

The Equal Principal Repayment Approach

Sometimes a loan may be structured in such a way that the principal is repaid in equal installments. The principal component of each installment will remain constant. However, as in the case of the amortized loan, the interest component of each payment will steadily decline because of the diminishing loan balance. Therefore, the total magnitude of each payment will also decline.

Table 2.8 illustrates the payment stream for an eight-year loan of $25,000, assuming that the interest rate is 8 percent per annum.

Table 2.8
Equal Principal Repayment Schedule

Year	Payment ($)	Interest ($)	Principal Repayment ($)	Outstanding Principal ($)
0				25,000
1	5,125	2,000	3,125	21,875
2	4,875	1,750	3,125	18,750
3	4,625	1,500	3,125	15,625
4	4,375	1,250	3,125	12,500
5	4,125	1,000	3,125	9,375
6	3,875	750	3,125	6,250
7	3,625	500	3,125	3,125
8	3,375	250	3,125	0.00

Types of Interest Computation

Financial institutions employ a variety of techniques to calculate the interest on loans taken from them by borrowers. The interest rate that is effectively paid by a borrower may be very different from what is being quoted by the lender.[1]

The Simple Interest Approach

If the lender were to use a simple interest approach, then a borrower need only pay interest for the actual period of time for which he has used the funds. Each time he makes a partial repayment of the principal, the interest due will decrease for subsequent periods, as Example 2.22 illustrates.

Example 2.22

Michael has borrowed \$8,000 from a bank for a year. The bank charges simple interest at the rate of 10 percent per annum. If the loan is repaid in one lump sum at the end of the year, the amount payable will be

$$1.10 \times \$8{,}000 = 8{,}800.$$

This consists of \$8,000 by way of principal repayment and an interest payment of \$800.

Let us consider a case in which Michael repays the principal in two equal semiannual installments. For the first six months, interest will be computed on the entire principal. So the first installment will be

$$\$8{,}000 \times 0.10 \times 0.5 + \$4{,}000 = \$4{,}400.$$

The second installment will be lower, because it will include interest only on the remaining principal, which in this case is \$4,000. So the amount repayable will be

$$\$4{,}000 \times 0.10 \times 0.5 + \$4{,}000 = \$4{,}200.$$

The sum of the two payments is \$8,600. In the first case, the interest payable was \$800, whereas in the second case it is only \$600. The more frequently principal is repaid, the lower will be the amount of interest.

The Add-On Rate Approach

This approach entails the calculation of interest on the entire principal. The sum total of principal and interest is then divided by the number of installments in which the loan is sought to be repaid. If the loan is repaid in a single annual installment, the total interest payable will be $800 and the effective rate of interest will be 10 percent. However, if Michael were to repay in two equal semiannual installments of $4,400 each, the effective rate of interest may be computed as follows:

$$8{,}000 = \frac{4{,}400}{\left(1+\frac{i}{2}\right)} + \frac{4{,}400}{\left(1+\frac{i}{2}\right)^2}$$

$$\Rightarrow \frac{i}{2} = 6.5965\%$$

$$\Rightarrow i = 13.1930\%.$$

The Discount Technique

In the case of such loans, interest is first computed on the entire loan amount. It is then deducted from the principal, and the balance is lent to the borrower. However, he has to repay the entire principal at maturity. Such loans are usually repaid in a single installment. Let us take the example of Michael.

The interest for the loan amount of $8,000 is $800. So the lender will give him $7,200 and ask him to repay $8,000 after a year. The effective rate of interest is

$$i = \frac{8{,}000 - 7{,}200}{7{,}200} \times 100 = 11.11\%.$$

Loans with a Compensating Balance

Many banks require borrowers to keep a percentage of the loan amount as a deposit with them. Such deposits, referred to as *compensating balances*, earn little or no interest. Such requirements will increase the effective rate of interest, and the higher the required balance, the greater the rate of interest paid by the borrower.

Assume that in Michael's case, the bank required a compensating balance of 12.50 percent. Although he will have to pay interest on the entire loan amount of $8,000, the usable amount is only $7,000.

The effective rate of interest is

$$i = \frac{800}{7{,}000} \times 100 = 11.4286\%.$$

Endnote

1. To paraphrase a famous Microsoft claim, in this case, "What you see is *not* what you get."

CHAPTER 3

Equity Shares, Preferred Shares, and Stock Market Indexes

Introduction

Equity shares or shares of common stock of a company are a type of financial claim issued by the firm to investors who are referred to as *shareholders*. In return for their investment, the shareholders are conferred with ownership rights. A firm must have a minimum of one shareholder, although there is no upper limit on the number of shareholders a firm may have. Correspondingly, there is no restriction on the total number of shares that may be issued by a firm. Large corporations have a large number of shares outstanding, and their ownership is spread over a vast pool of investors. A shareholder is a part owner of the company to whose shares he has subscribed, and his stake is equal to the fraction of the total share capital of the firm to which he has contributed.

At the outset, when a firm is incorporated, a stated number of shares will be authorized for issue by the promoters. The value of such shares is referred to as the *authorized capital* of the firm. However, the entire authorized capital need not be raised immediately. Often a portion of what has been authorized is held for issue at a later date, if and when the firm should require additional capital. What is actually issued is less than or equal to what is authorized, and the amount that is actually raised is referred to as the *issued capital*. The value of the shares that is currently being held by the investors is referred to as the *outstanding capital*. In most cases, the outstanding capital is synonymous with the issued capital. However, in the event of a company buying back shares from the public, the outstanding capital will decline and will be less than what was issued.

Shareholders are entitled to share the profits made by the firm because they represent the owners of the venture. A firm will typically pay out a percentage of its earned profits during the financial year in the form of cash to its shareholders. These cash payouts that shareholders receive from the firm are referred to as *dividends*. The entire profits earned by a firm will usually not be distributed to the shareholders. Most companies will choose to retain a part of what they have earned to meet future requirements of cash on account of activities such as expansion and diversification. The profits that are retained or reinvested in the firm are called *retained earnings*. The earnings that are retained will manifest themselves as an increase in the reserves and surplus account and will show up on the liabilities side of the firm's balance sheet. Retained earnings can be a major source of capital for a corporation.

Shareholders are termed *residual claimants*, and this categorization is valid in two respects. First, every firm will have creditors to whom it owes money on a priority basis. It is a common practice to raise borrowed capital from investors in the form of what are known as *bonds* or *debentures*, a topic we will cover in the next chapter. Creditors always enjoy priority over the owners of the firm when it comes to receiving payments. A firm may declare a dividend only after all payments due to its creditors have been made. Dividends are not contractually guaranteed and can, in principle, fluctuate significantly from year to year. Being a residual claimant, a shareholder cannot demand a dividend as a matter of right. It is up to the board of directors of a firm to take decisions pertaining to dividends. Shareholders, of course, indirectly influence the dividend policy of the firm, because they have the power to elect the board of directors.

Second, at times sustained losses could lead to a situation in which a firm is forced to file for bankruptcy. In such cases, the assets of the firm will be liquidated and the proceeds from the same will be used to compensate the stakeholders. Once again, the order of priority is such that the creditors will have to be paid first. If any funds were to remain after the creditors have been fully paid, then and only then will the shareholders be entitled to a return of capital.

Equity shares have no maturity date—that is, no entrepreneur will incorporate a company with a termination or winding-up date in mind. The shares issued by the firm will continue to exist until and unless the firm itself is wound up. In contrast, most bonds and debentures, as we will see in the next chapter, are issued for a specified term to maturity.

Unlike the owners of a sole proprietorship or a partnership, shareholders of a corporation enjoy limited liability. In other words, if a firm were to face serious financial difficulties and unable to pay what is due to its creditors, neither it nor its creditors can make financial demands on the shareholders, asking them to commit more capital. On the other hand, if a partnership were to go bankrupt, its creditors can come after the partners personally and stake a claim on their personal assets. The maximum financial loss that a shareholder may suffer is limited to the investment she made in the process of acquisition of the shares at the outset; that is, the market price of a share has a lower limit of zero in principle, and if this limit were to be reached, the shares will be totally worthless, and the investor would have lost the entire amount she invested.

Par Value versus Book Value

Common stock usually has a *par value*, which is also known as *face value* or *stated value*. The par value has no significance and in countries such as the United States it can be fixed at a low and arbitrary level. Many companies in the United States choose to issue stocks with very low par values because, per the regulations of certain states, the cost of incorporating a firm is based on the par value of the shares being registered. Such fees can be minimized by assigning low par values.

In the case of a no-par stock, the board of directors will assign a value to the stock each time it raises capital. Such stocks are popular with small organizations in which the owners issue themselves a number of shares and simply infuse money into the corporation when needed. Such stock gives more flexibility to the corporation. Because there is no stated price for the stock, the directors can raise the price when the firm becomes more valuable.

The issue price of a share need not be equal to its par value, and it will often be in excess of its par value. This is true for companies that are already established at the time of issue. The excess of the issue price over the par value is referred to as the *share premium*. Assume that Alpha Corporation is issuing 100,000 shares with a par value of \$5 at a price of \$12.50 per share. If the issue is successful, the company will raise \$1,250,000 from the market. In the balance sheet, \$500,000 would be reported as share capital and \$750,000 would be reported as the share premium.

The *book value* is the value of a firm as obtained from the balance sheet or books of account, hence the name. The term refers to the value of the assets behind a share per the balance sheet. It is derived by adding the par value, the share premium, and the retained earnings and dividing by the number of shares issued by the firm. The book value can be very different from the par value of the shares. A third term that is used is the *market value* of a firm. This is the value assigned to the shares of the company by the stock market, and it is determined by multiplying the number of shares issued by the firm with the current market price per share.

Voting Rights

Investors who choose to acquire equity shares of a firm are conferred with voting rights, which includes the right to elect the directors of the company. The most common arrangement is to give a shareholder one vote per every share of stock he holds. However, shares with differential voting rights can be issued.

If a firm were to issue multiple classes of shares, which differ from the standpoint of the voting rights bestowed on the holders, the categories of shares are usually otherwise similar to each other. In other words, all shareholders, irrespective of their voting privileges, have an unlimited right to participate in the earnings of the corporation. They also have an equal right on the assets of the company on liquidation after all the other creditors and prioritized security holders have been paid off. All shares in a given class or category have equal standing, regardless of the point in time or the price at which they are issued. If Alpha Corporation were to issue three years after its IPO another 100,000 shares at $17.50 per share, then the newly issued shares will rank at par with the 100,000 shares issued earlier. In legal terms, the two tranches are said to rank *pari passu*.

At times, equity shares are divided into two or more classes with differential voting rights. One or more categories may have subordinated voting rights, and at times a category may be issued with no voting rights. The purpose of such an exercise is to vest the voting powers with a minority of shareholders who can control the company with less than a 50 percent equity stake. For example, the Ford Motor Company has two classes of shares. The shares available to the public have voting rights. As of 1998, there were more than 1 billion Ford shares outstanding, but 70.9 million of these were class B shares that are owned by the Ford family and certain key officers. These shares have weighted voting rights that allow them to control nearly 40 percent of the votes. The holders of these shares have always voted as a unified block. The majority group of shareholders cannot force a decision easily on the company.[1]

A group of shareholders can exert considerable influence over the affairs of a company if they satisfy one of the following criteria:

- They own more than 50 percent of the voting shares.
- They have one or more representatives on the board of directors.
- They themselves are directors of the company.

Minority shareholders have little say in the affairs of their company, despite the fact that they do enjoy voting rights. This is particularly true when the company is controlled by a majority shareholder.

Statutory versus Cumulative Voting

Every share of stock held by an investor gives her one vote for each director position that is up for voting. However, the votes may be apportioned in two different ways. Assume that a shareholder has 1,000 shares and that there are four vacancies on the board. If statutory voting were to be applicable, then the shareholder can cast a total of 4,000 votes in all. However, not more than 1,000 votes can be cast in favor of any one candidate. On the other hand, if cumulative voting were to be applicable, then the votes could be apportioned in any way that the shareholder chooses. In this case, too, a total of 4,000 votes can be cast. One shareholder may decide to cast 3,000 votes in favor of one candidate and 1,000 in favor of a second without giving any votes to the remaining candidates. On the other hand, another shareholder in a similar situation might opt to cast 1,000 votes in favor of each of four candidates. Cumulative voting is designed to give minority shareholders the opportunity to elect at least one candidate of their choosing, because it gives them the power to concentrate the votes on a candidate.

Proxies

To enjoy the right to vote, an investor must be a shareholder *of record*. What this means is the following. The registrar of a firm will be maintaining a record of its current shareholders. Only a person who is listed on the corporation's register of shareholders as of a date known as the *record date* is an eligible shareholder from the standpoint of being eligible to cast his vote. The *record date* is usually a few days before the date of the meeting at which the actual voting will take place. Therefore, it is conceivable that a person who happens to be a shareholder of record (by virtue of his name appearing in the register on the record date) may have sold his shares before the date of the meeting. In such cases, the new owner who has acquired the shares cannot in principle vote, because his name will not be reflected in the register. To get over this problem, a seller of shares can give a proxy to the buyer.

It is not realistic to expect a large percentage of the shareholders of large companies to attend the annual meetings in order to be physically present to cast their votes. Companies choose to send a proxy statement to absentee shareholders along with a ballot before the scheduled date of the meeting. The shareholders are expected to mark their preferences and return the ballot before the meeting. A typical proxy statement will include information on the individuals seeking appointment or reappointment as directors, as well as details of any resolutions for which the opinions of the shareholders is being sought, which is the *raison d'être* for the vote. Once the ballots are received from the absentee shareholders, they will be collated, and a person appointed by the firm will cast the votes as directed by the shareholders who have submitted the ballots.

There is a critical reason why companies require shareholders to attend meetings or to send proxies if they are unable to be physically present. This is because a *quorum* is required before any business can be transacted—that is, a minimum number of shares must be represented at the meeting either by the holders in person or in the form of proxies.

Dividends

As explained earlier, shareholders are residual claimants. They cannot demand dividends from the firm—or, in other words, dividends, unlike interest payments to creditors, are not a contractual obligation. The payment of dividends is not mandatory, and the decision to pay or not pay is entirely at the discretion of the company's board of directors. Companies usually declare a dividend when they announce their results for a period. In the United States, because results are typically declared on a quarterly basis, dividends are also announced every quarter. In the United Kingdom, most companies pay their annual dividends in two stages. In India, too, companies typically declare an interim dividend and a final dividend.

A dividend declaration is a statement of considerable importance. It is an affirmation by the company that its affairs are on track and that it has adequate resources to reinvest in its operations as well as reward its shareholders.

In the context of a dividend payment, four dates are important. The first is what is termed the *declaration date*. It is the date on which the decision to pay a dividend is declared by company directors and the amount of the dividend is announced. The dividend announcement will mention a second date called the *record date*. The significance of this date is the same as we have seen earlier for voting; that is, only those shareholders whose names appear as of the record date on the register of shareholders will be eligible to receive the forthcoming dividend.

A third and extremely critical date is what is termed the *ex-dividend date*, which is specified by the stock exchange on which the shares are traded. The relationship between the record date and the ex-dividend date depends on the settlement cycle. The term *settlement cycle* refers to the duration between the date of a sale transaction and the date on which securities are received by the buyer and cash is received by the seller. The import of the ex-dividend date is that an investor who purchases shares on or after the ex-dividend date will not be eligible to receive the forthcoming dividend. The ex-dividend date will be set a few days before the share-transfer book is scheduled to be closed in order to help the share registrar complete the administrative formalities.

The ex-dividend date will be such that transactions before that date will be reflected in the register of shareholders as on the record date, whereas transactions on or after that date will be reflected in the books only after the record date. It is therefore easy to surmise the relationship between the two dates. Assume that an exchange follows a T+3 settlement cycle—that is, it takes three business days after the trade date to fully consummate the trade. If the trade were to take place three days before the record date or earlier, then the transfer will be reflected as on the record date. The ex-dividend date in such cases must be two business days before the record date.

The New York Stock Exchange (NYSE), for instance, follows a T+3 settlement cycle. On the NYSE, the ex-dividend date for an issue is specified as two business days before the record date announced by the firm.

Before the ex-dividend date, the shares are said to be traded on a *cum-dividend* basis: the right to receive the dividend is inherent in the shares. Therefore, an investor who acquires the share on a cum-dividend basis is entitled to receive the dividend. On the ex-dividend date, the shares begin to trade ex-dividend, which connotes that a potential buyer will no longer be eligible to receive the next dividend if she were to acquire the share. This is because it is too late for her name to figure in the register as of the record date, and the dividend will go to the party who is selling the shares.

On the ex-dividend date, in theory the shares ought to decline by the amount of the dividend, if we assume that there is no other information with implications for the share value, which permeates the market. If the cum-dividend price is \$75 per share at the close of trading on the day prior to the ex-dividend date, and the quantum of the dividend is \$3.50 per share, then from a theoretical standpoint the market should open with a price of \$71.50 on the ex-dividend date.

Finally, we have a date called the *distribution date* that is purely of academic interest. This is the date on which the dividends are actually paid or distributed.

Dividend Yield

The annual *dividend yield* is defined as the annual dividend amount divided by the current share price, expressed in percentage terms

$$\text{Dividend Yield} = \frac{\text{Dividend}}{\text{Price}} \times 100\%.$$

Take the case of a company that has reported a dividend of \$2.50 per quarter over the past financial year. Assume that the current market price of the shares is \$80. The dividend yield is

$$\frac{2.50 \times 4}{80} \times 100 = 12.50\%.$$

The dividend yield is a function of the share price as well as the quantum of the dividend. Yields are generally lower for profitable companies compared to companies in financial difficulty. If the profits and dividends of a company are expected to increase over time, then the yield will be relatively lower. This is because the market will place a higher value on the shares of such companies. On the other hand, companies perceived as having a lower potential for growth will have a relatively higher dividend yield. Yields are also a function of the liquidity of a scrip. Potential buyers of a stock that is thinly traded are likely to require a higher rate of return that will manifest itself as a lower price. The yield for such stocks will be relatively higher. It is not necessary that a company that promises a steady growth pattern of dividends will be characterized by a dividend yield that will steadily increase over time. This is because a growing dividend stream will also lead to higher share prices. The net result on the dividend yield will therefore be ambiguous.

It is not necessary that the dividends declared in a year be less than the profits earned by the firm. Companies are allowed to pay dividends out of profits retained in earlier years. A company with relatively low profits can declare a high dividend. Similarly, a loss-making firm also can declare a dividend. Companies generally try to maintain a steady growth rate in dividends. Ideally, the growth rate of dividends should be greater than or equal to the prevailing rate of inflation. In years of financial hardship, a company may be forced to cut its dividend payout. However, such cuts can send unwarranted distress signals to shareholders, and if the current financial situation is perceived as a temporary aberration, the firm, despite its difficulties, may opt to keep the dividend at a steady level by dipping into profits retained in earlier years.

Dividend Reinvestment Plans

Dividend reinvestment plans (DRIPS) are schemes that allow shareholders to opt to have cash dividends automatically reinvested in additional shares of stock. Members can directly acquire shares from the company and do not need to route the order through a broker.[2] The plans are typically administered by large banks that usually send a report every quarter to the participating investors stating how much dividend has been paid, how many additional shares have been acquired in the process of reinvestment, and the total number of shares held in the shareholder's account. The plan administrator may levy an annual fee that will, in some cases, be paid by the company that is issuing the shares.

There are two basic types of DRIPs. First, in some cases the company will acquire shares in the open market and reissue them each time a shareholder expresses a desire to reinvest dividends. These plans are administered by a bank acting in the capacity of a trustee who will actually buy the shares from the market for the investing shareholder. In these cases, the shares are acquired at the prevailing market price. Brokerage costs are often paid by the company. The second type of reinvestment plans entails the issue of additional shares directly by the company. In such cases, the issuing firms may offer the shares at a discount. Firms can afford to issue shares at below the market price because, despite the discount, the shares can be sold at a higher price as compared to an underwritten public issue.

Stock Dividends

A stock dividend is a dividend that is distributed in the form of shares of stock rather than in the form of cash. The issue of additional shares does not require any monetary contribution from the investors, so such a corporate action is referred to as a *bonus share issue* in some markets. A stock dividend entails the transfer of funds from the reserves and surplus account to the share capital account. This is known as the *capitalization of reserves*. The net result is that funds are transferred from an account that belongs to the shareholders to another account that also is theirs.

From a theoretical standpoint, stock dividends do not create any value for an existing shareholder. Assume that a shareholder owns 1,000 shares of a firm that has issued a total of 500,000 shares. So this individual owns 0.20 percent of the firm. Assume that the firm announces a 20 percent stock dividend or one additional share for every five existing shares. If so, it will have to issue 100,000 shares; of this, the investor will receive 200. After the issue of the additional shares, the investor will be in possession of 1,200 shares, which is 0.20 percent of the 600,000 total shares issued by the firm. The investor's percentage stake in the company remains unaltered.

From the perspective of the company, the issue of the new shares neither connotes any changes in the asset base of the firm nor signals any enhancements to the earnings capacity of the firm. All that has happened is an accounting transaction. It is as if an investor has two bank accounts, one with Citibank and the other with JPMorgan Chase. What has happened is that we have debited one account and credited the other without increasing or decreasing the investor's wealth in the process.

The share price should theoretically decline after a stock dividend is declared. Returning to our illustration, assume that the share price before the stock dividend was \$60 per share. The ex-dividend price, P, should be such that

$$500{,}000 \times 60 = 600{,}000 \times P$$
$$\Rightarrow P = \$50.$$

A stock dividend is usually declared when a firm wants to reward its shareholders without having to face an external outflow of cash. A firm might wish to conserve cash for various reasons. One reason could be that the firm is short of funds. Or it could be the case that the available cash is required for productive investments.

Sometimes a company may declare a stock dividend before the payment of a cash dividend. The implications for the share price may be analyzed with the help of a numerical example.

Silverline Technologies has 500,000 shares outstanding, and the price is \$60 per share. The firm announces a stock dividend of 20 percent and a cash dividend of \$4 per share. The company also declares that the new shares that come into existence by virtue of the stock dividend will be eligible for the cash dividend.

The cum-stock dividend cum-cash dividend price is \$60. The market value of the firm is

$$500{,}000 \times 60 = \$30{,}000{,}000.$$

Because the stock dividend in theory is value neutral, in principle the market capitalization of the firm after the stock dividend should remain at \$30,000,000. The theoretical price of ex-stock dividend ex-cash dividend shares will be

$$(30{,}000{,}000 - 4.00 \times 600{,}000) \div 600{,}000 = \$46.$$

Treasury Stock

The term *treasury stock* refers to shares that were at one point in time issued to the public but that have subsequently been reacquired by the firm.

These shares are held by the company and can subsequently be reissued, if and when employees were to exercise their stock options. Unlike the shares issued by the firm that are held by the shareholders, treasury shares have no voting rights, are ineligible for dividends, and are not included in the denominator used for computation of the earnings per share (EPS).

Why do companies repurchase shares? One motivation could be that the directors of the firm are of the opinion that the market is undervaluing the stock, and they would like to prop up the share price by creating greater demand. For a given level of profitability a buyback program will increase the earnings per share. And if the dividends per share (DPS) were to be kept constant, it will also reduce the total amount of dividends that the company needs to declare. Buyback is also a potent tool for fighting a potential takeover by corporate raiders. By reducing the shares in circulation, the current management can acquire greater control.

There are also situations in which a company is generating a lot of cash but is unable to identify profitable avenues for investment or is unable to identify projects with a positive *net present value* (NPV). One way to deal with such a situation is by declaring an extraordinary dividend. But in many countries cash dividends are taxed at the hands of the shareholder at the normal income tax rate, which could be significant for investors in higher tax brackets. On the other hand, if an investor were to sell her shares back to the firm at a price that is higher than what she paid to acquire them, the profits will be construed as capital gains, which in most countries are taxed at a lower rate.

Splits and Reverse Splits

An n:1 stock split means that n new shares will be issued to the existing shareholders in lieu of one existing share. A 5:4 split means that a holder of four existing shares will receive five shares after the split without having to invest any more funds. From a mathematical standpoint, this is exactly analogous to a 25 percent stock dividend. However, despite this equivalence, a stock split is operationally different from a stock dividend. Stock dividends entail the capitalization of reserves, but stock splits do not. What happens in the case of a split is that the par value of existing shares is reduced and the number of shares outstanding is correspondingly increased. The net result is that the issued capital remains unchanged. Consider a company that has issued 100,000 shares with a face value of $100. If it were to announce a 5:4 stock split, the number of shares issued will increase to 125,000, while the par value will stand reduced to $80.

Why do companies split their shares? Companies generally go in for such a course of action if, in the perception of management, the share price of the firm has become too high. Although in theory there is nothing wrong with the stock price being at a relatively high level, companies that would like to attract a broad class of investors will try to ensure that their scrip is within an affordable price range for small and medium investors. It is difficult to categorically state as to what a high price is and what constitutes an affordable price range. Though, it is believed that most managers have a feel for the popular price range for their stock. In other words, they are believed to be aware of the price range within which their stock should trade if it is to attract adequate attention from investors. Historically, investors have normally traded in *round lots*, or *board lots*, which is usually defined as a bundle of 100 shares. Anything less than a round lot is referred to as an *odd lot*. Round lots had a lot of significance when physical share certificates were the norm. In these circumstances, special procedures were often required for odd lots because only an investor with an odd lot was likely to be interested in another trader with a similar lot. However, we must point out that with the advent of dematerialized or scripless trading, this distinction does not carry any significance. Psychologically however, many investors are still comfortable trading in round lots. At very high share prices, small and medium investors may not be able to afford round lots. One of the motives that has been expounded to justify stock splits is the desire to ensure that even small investors can buy round lots of the shares of the firm.

The opposite of a stock split is a reverse split or a consolidation. The difference between an *n:m* split and an *n:m* reverse split is that, in the first case, *n* will be greater than *m*; in the second case, it will be less. Assume that the company, which has 100,000 shares outstanding with a par value of $100, announces a 4:5 reverse split. It will subsequently have 80,000 shares outstanding, each of which will have a par value of $125. Companies are likely to go in for such a course of action if their managements were to perceive that their stock prices are too low. Exchanges such as the NYSE discourage the listing of securities that are consistently trading at very low prices. This is because such prices have a tendency to attract inexperienced traders with unrealistic price expectations who could get their fingers burned. This is referred to as the *penny stock trap*. Suppose a share is trading at $1. A naïve investor may buy it in the belief that a mere 50 cent increase in the price will amount to a 50 percent return on investment. What such investors fail to realize is that such a gain is highly improbable for a company in the doldrums.

Stock splits will usually result in a reduced dividend per share. This is because, given the fact that the number of shares on which dividends have to be paid is a multiple of the number of shares outstanding before the split, most companies will have little option but to reduce the magnitude of the dividend per share. However, in many cases, companies will increase the aggregate dividends. This may be best understood with the help of an example. Take the case of a company that was paying a dividend of $2.50 per share. Assume it announces a 5:4 split. The post-split dividend may be fixed at, say, $2.20 per share so that the holder of four shares before the split, who would ordinarily have been entitled to a dividend of $10, will now be entitled to an aggregate dividend of $11 on the five shares that he possesses as a consequence of the split.

Example 3.1 illustrates the impact of stock splits and consolidations on the prevailing market price.

Example 3.1

Assume that a company has 1,000,000 shares outstanding that are currently trading at \$80 per share. The market capitalization is \$80,000,000. If it were to announce a 5:4 split, then the market capitalization of 1,250,000 shares after the split will, in theory, be \$80 million. The theoretical post split price should be

$$1{,}250{,}000 \times P = 1{,}000{,}000 \times 80$$
$$\Rightarrow P = \$64.$$

Consider a 4:5 reverse split. If so, the theoretical market capitalization of 800,000 shares ought to be \$80 million. Thus,

$$1{,}000{,}000 \times 80 = 800{,}000 \times P$$
$$\Rightarrow P = \$100.$$

Costs Associated with Splits and Stock Dividends

Research by Copeland has found that two types of transaction costs increase after a stock split. Such costs serve to ultimately reduce the liquidity of the stock. First, brokerage fees, measured in percentages, increase after a split. Generally, fees for low-priced securities are a larger percentage of the sales price than they are for high-priced securities. An investor will pay a higher commission when he buys 100 shares trading at \$100 each than when he buys 40 shares trading at \$250 each. Second, Copeland finds that the bid–ask spread, expressed as a percentage of the sales price, rises after a stock split. It is debatable whether the benefits associated with such corporate actions outweigh the corresponding costs.

Preemptive Rights

The directors of a company must obtain the approval of existing shareholders if they wish to issue shares beyond what was already issued. As per the charter or articles of association of some corporations, existing shareholders must be given the first right to buy the additional shares being issued in proportion to the shares they already own. Of course, the right to acquire the shares confers them with an option and does not require a mandatory course of action. This requirement ensures that existing shareholders have a preemptive right to acquire new shares as and when they are issued.[3] In other words, they have an opportunity to maintain their proportionate ownership in the company. The merits of giving such rights to existing shareholders are debatable. Many people have argued that preemption rules prevent new investors from achieving meaningful stakes in companies, thereby narrowing the shareholder base and increasing dependence on a few investors. Others argue that the right to maintain one's proportional investment is just and fair for the existing shareholders.

Usually, the rights issue is made at a price that is lower than the prevailing market price of the share. If so, then the right acquires a value of its own. The existing shareholders in this case can either exercise their rights and acquire additional shares or sell the rights to someone else. In most cases, the discount from the prevailing market price is set between 10 and 15 percent. Readers may be surprised to know that rights issues also are usually underwritten. One may wonder why there is a need for underwriting, considering that the firm has a captive audience. The reason is that the rights issue must remain open for 21 days. During this period, it is conceivable that the market price may fall below the issue price set by the company. In such circumstances, no investor will subscribe to the issue because he can always acquire the shares for a lower price in the secondary market.

What is the value of a right? Let us suppose that a company has 1,000,000 shares outstanding and that shareholders are entitled to purchase one new share for every five shares that they hold. So the firm will be issuing 200,000 additional shares. Let us assume that the prevailing market price is \$85 per share and that the additional shares are being issued at \$55 per share. The market capitalization of the firm before the issue is \$85 million. The rights issue is infusing an additional \$11 million into the firm. The postissue firm value in theory ought to be \$96 million. This would imply that, considering the fact that 1.2 million shares will be outstanding after the issue, the ex-rights price ought to be such that

$$P \times 1{,}200{,}000 = 96{,}000{,}000$$
$$\Rightarrow P = \$80.$$

The current shareholders are getting a share worth \$80 at \$55. This implies that the value of the right to acquire one share is \$25. Because the shareholder needs five shares to acquire the right to buy one share, the value of a right is \$5.

Prima facie, it would appear that the issue of additional shares at a discount to their current value would amount to a loss for the existing shareholders. This is because although the cum-rights price is \$85, the ex-rights price is \$5 lower. But there is a fallacy in this perspective. It is important to remember that the shareholders have been given the opportunity to buy new shares at \$55, and that this opportunity compensates for the decline in the price of the share.

Consider an investor who is in possession of 100 shares. The value of these shares before the rights issue will be \$8,500. If he were to decide to exercise his rights, he can acquire 20 additional shares by paying an additional \$55 for each. The value of his portfolio if he were to do so would be

$$120 \times 80 = 9{,}600 = 8{,}500 + 1{,}100.$$

The portfolio is worth the original \$8,500 plus the additional amount of \$1,100 that he has pumped in. There is no loss of value or dilution.

If the investor were to decide not to exercise his rights, he can renounce them in favor of another investor. The rights can in this case be sold for \$5 per right. The investor's wealth in the event of such renunciation would be

$$80 \times 100 + 5 \times 100 = \$8{,}500$$

which is nothing but the original value of the portfolio. In this case as well there is no loss of value.

The ex-rights price may be higher than what we would expect from theory. This could be because the rights issue may be perceived as a signal of information emanating from the firm. In reality, the very fact that the company has chosen to issue additional shares may be construed as a signal of enhanced future profitability. A plausible reason could be that investors believe that the new funds raised will be used for more profitable projects. Another line of argument could be that, considering the fact that cash dividends are usually maintained at existing levels in the medium term, the additional shares from the perspective of the shareholders are an indication of greater profitability from the existing operations of the firm. Both these factors could serve to push up the demand for the shares of the firm, and they may lead to a situation where the ex-rights price, although lower than the cum-rights price, is higher than what is predicted by theory.

When a stock with rights is trading on an exchange it is said to be trading cum-rights. The rights certificates are issued to the shareholders on the *rights record date*. A rights certificate gives the shareholder the right, but not the obligation, to buy additional shares of stock at the subscription price. On an exchange that follows a T+3 settlement cycle, the ex-rights date will be two days before the rights record date. From the time of announcement of the rights issue until the ex-rights date, the rights are attached to the stock. During the ex-rights period, the rights are sold separately just like a share of stock. The rights usually expire four to six weeks after the ex-rights date. After this point, the rights are worthless.

A shareholder is given one right for each share of stock she owns. The number of rights required to acquire one share of stock is obtained by dividing the number of outstanding shares by the number of shares that the firm proposes to issue. Assume that a company has 100,000 shares outstanding and wants to raise $500,000 of capital by issuing rights with a subscription price of $25. It needs to issue 20,000 new shares. The number of rights required to acquire one new share is

$$\frac{100{,}000}{20{,}000} = 5.$$

Interpreting Stated Ratios

Consider a rights issue of one share for every four shares that are being held. In Europe and Asia, this will be indicated as a 1:4 rights issue. The first number indicates the additional quantity of the security to be distributed, and the second number indicates the quantity of the underlying security relative to which additional securities are being issued. If an investor were to have 2,500 shares, she will be entitled to an additional 625 shares. So, in total she will have 3,125 shares if she were to exercise her right.

In the United States, however, this will be indicated as a 5:4 rights issue. The first number indicates the total quantity held after the event, and the second number indicates the quantity relative to which the additional securities are being issued. An investor who is holding 2,500 shares will have $(2{,}500 \times 5)/4 = 3{,}125$ shares if she were to subscribe to the issue.

Handling Fractions

Assume that an investor is holding 945 shares of a company that has announced a 5:8 reverse split. This will entitle him to 590.625 shares. He cannot be allocated 0.625 shares of stock. The issuer must specify a method to deal with this eventuality. The commonly prescribed methods include the following:[4]

- Round up the fraction to the next higher whole number. In this case, it will be 591 shares.
- Round up the fraction to the next higher whole number if it is greater than or equal to 0.50. If it is less than 0.50, then round down to the previous whole number. In this case, it will be 591 shares.
- Round up the fraction to the next higher whole number if it is greater than 0.50. If it is less than or equal to 0.50, then round down to the previous whole number. In this case, it will be 591 shares.
- Round down the fraction to the previous whole number. In this case, it will be 590 shares.
- Distribute cash in lieu of the fractional security. If we assume that the company pays out $20 per share, then the investor will receive $12.50 in lieu of 0.625 shares.

In those cases in which the rounding procedure leads to the previous whole number, the investor may be given an option to buy a fraction of a share so that she is entitled to the next whole number. If the practice is to round down to the previous whole number, she may in this case be given the option to buy 0.375 shares so as to be entitled to 591 shares.

Physical Certificates versus Book Entry

Traditionally, equity shares have been represented by physical certificates, with each certificate representing ownership of a stated quantity of shares. These days however, investors in many countries can hold shares as a computer entry or in book-entry form. This facility is provided by organizations known as *securities depositories*. In the United States, the major depository is the Depository Trust and Clearing Corporation (DTCC). In the United Kingdom, it is Crest, and in Europe it is Clearstream in Luxembourg and Euroclear in Brussels. When shares are held in book-entry form, they are said to be either dematerialized or immobilized. In the case of a dematerialized share, there is no physical certificate and the book entry is the sole means of record. However, in the case of immobilized shares, although a book entry is created, the original physical certificates are kept in safe custody and are not destroyed.

As can be surmised, book entry makes share transfers a lot easier. In the case of physical shares, when a sale is made, the seller's scrip would have to be canceled and one or more new scrips would have to be created. In the case of dematerialized trading, however, all that is required is a debit to the seller's account at the depository and a credit to the buyer's account.

Tracking Stock

Tracking stocks, also known as *targeted stocks*, are issued by many companies in addition to the usual equity shares. A tracking stock tracks the performance of a specific business unit or operating division of a company. The value of such stock is determined by the performance of this division rather than that of the company as a whole. If the unit or division does well, the tracking stock will rise in value, even though the company as a whole may have underperformed. The converse is also true. Such stocks are traded as separate securities.

Unlike normal equity, tracking stocks usually have limited or no voting rights. If the stock were to pay dividends, then the quantum of the dividend would depend on the financial performance of the concerned business unit or division.

Report Cards

Companies issue quarterly and annual reports as mandated by the regulator. Such reports are intended to provide relevant information to the shareholders. Quarterly reports provide shareholders with a summary of the company's performance over the preceding three months. The reports often contain the firm's balance sheet as at the end of the quarter and an income statement for the same period. A copy has to be filed with the market regulator. In the United States, quarterly reports that are filed with the Securities and Exchange Commission (SEC) are called *10Q reports*. In addition, at the end of every financial year, U.S.-based companies are required to file an annual report with the SEC called the *10K report*.

Types of Stocks

Most stocks are classified as either growth stocks or cyclical stocks. A *cyclical stock* is one whose fortunes rise and fall in tandem with the business cycle; that is, the stock price rises during an economic boom and falls during a recession. Although this is a feature to some extent of all traded stocks, cyclical stocks are characterized by extreme sensitivity to changes in the business cycle. Two examples of cyclical stocks are automobile stocks and stocks of real estate developers. The reason is that when the economy is on an upward trend, people tend to invest in new cars and new houses. However, during a recession, investors tend to postpone the purchase of such durable assets.

The term *growth stocks* is used for the stocks of companies whose sales and earnings have grown faster than those of an average firm and that can be reasonably expected to display a similar trend in the future. Investors tend to acquire such stocks in anticipation of capital appreciation. This is because even though such firms do pay dividends periodically, they exhibit a marked tendency to retain profits in order to fuel further growth. Unlike cyclical stocks, the performance of such stocks depends more on the quality of the products and the capability of the management teams and less on economywide factors. Stocks of pharmaceuticals and food and beverage companies usually fall in this category.

Interest-Sensitive Stocks

The performance of the equity markets is generally better when interest rates are low. There are two reasons for this. First, most businesses rely to a large extent on borrowed money. In a low-interest environment, the financing costs for such firms will go down, and this has obvious implications for their profitability. Second, when rates are low, consumers are likely to borrow more to fund their purchases.

Interest-sensitive stocks are those whose performance is very closely linked to movements in interest rates. These include utility firms, banks, brokerage houses, and insurance companies.

Risk and Return and the Concept of Diversification

Consider two securities whose returns are given by the probability distribution shown in Table 3.1.

The expected return is a probability-weighted average of returns.

The expected return of asset A is

$$E(r_A) = (0.10 \times -10) + (0.15 \times -6) + (0.10 \times 0) + (0.25 \times 8) + (0.20 \times 12) + (0.20 \times 20) = 6.50\%.$$

Table 3.1
Asset Returns

Probability	Return on Asset A (%)	Return on Asset B (%)
0.10	−10	−12
0.15	−6	3
0.10	0	8
0.25	8	4
0.20	12	6
0.20	20	12

Similarly, the expected return of asset B is given by

$$E(r_B) = (0.10 \times -12) + (0.15 \times 3) + (0.10 \times 8) + (0.25 \times 4) + (0.20 \times 6) + (0.20 \times 12) = 4.65\%.$$

The variance is a probability-weighted average of squared deviations from the mean or the expected value. The variance of asset A's return is given by

$$\sigma_A^2 = [0.10(-10 - 6.50)^2] + [0.15(-6 - 6.50)^2] + [0.10(0 - 6.50)^2] + [0.25(8 - 6.50)^2] + [0.20(12 - 6.50)^2] + [0.20(20 - 6.50)^2] = 97.95.$$

Similarly, the variance of asset B's return is given by

$$\sigma_B^2 = [0.10(-12 - 4.65)^2] + [0.15(3 - 4.65)^2] + [0.10(8 - 4.65)^2] + [0.25(4 - 4.65)^2] + [0.20(6 - 4.65)^2] + [0.20(12 - 4.65)^2] = 40.5275.$$

The square root of the variance is known as the *standard deviation*. In this case, $\sigma_A = 9.8970$ and $\sigma_B = 6.3661$. The return, the expected return, and the standard deviation have the same unit of measure—in this case, percentage.

When we have more than two assets, we need to assess the way the assets move with respect to each other. For this purpose, we compute two statistics, *covariance* and *correlation*. The covariance is the probability-weighted average of the product of the deviation from the mean for the two variables.

In this case,

$$\begin{aligned}\text{Covariance}(r_A, r_B) = {} & [0.10(-10 - 6.50)(-12 - 4.65)] \\ & + [0.15(-6 - 6.50)(3 - 4.65)] \\ & + [0.10(0 - 6.50)(8 - 4.65)] \\ & + [0.25(8 - 6.50)(4 - 4.65)] \\ & + [0.20(12 - 6.50)(6 - 4.65)] \\ & + [0.20(20 - 6.50)(12 - 4.65)] = 49.4750.\end{aligned}$$

The correlation is obtained by dividing the covariance by the product of the two standard deviations. In this case, the correlation is given by $\rho_{\mathrm{A,B}} = 49.4750/(9.897 \times 6.3661) = 0.7853$. The value of the correlation will always lie between -1.00 and $+1.00$.

While studying stocks in isolation, we use the variance as a measure of risk. However, as we saw earlier, no rational investor will hold a security in isolation. She will typically hold a well-diversified portfolio. The underlying rationale is that although there is no option but to tolerate exposure to economywide risk—which is termed *systematic risk* in finance—it is possible to eliminate idiosyncratic or firm specific risk by holding a well-diversified portfolio. The relevant measure of risk of an asset is not its variance but its covariance with other assets.

From the *capital asset pricing model* (CAPM), the relevant measure of risk in a portfolio context is beta (β). Beta is defined as the covariance of an asset's return with the rate of return on the market portfolio, divided by the variance of the return on the market portfolio. The market portfolio is a value-weighted portfolio of all assets in the economy.

If we denote the expected return from asset i as $E(r_i)$, the expected return on the market portfolio as $E(r_m)$, and the riskless rate of interest as r_f, then from the CAPM

$$E(r_i) = r_f + \beta_i[E(r_m) - r_f].$$

The expected return of a portfolio is a weighted average of the expected returns of the constituent assets. The variance of a portfolio's return is not, however, a weighted average of the variance of its components.

Consider three assets with returns r_1, r_2, and r_3. Consider a portfolio with weights of w_1, w_2, and w_3 in the three assets, where the weights will sum to 1.00.

The expected portfolio return is given by

$$E(r_p) = w_1 E(r_1) + w_2 E(r_2) + w_3 E(r_3).$$

The portfolio variance is given by

$$\sigma_p = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + w_3^2\sigma_3^2 + 2w_1w_2\sigma_{1,2} + 2w_2w_3\sigma_{2,3} + 2w_1w_3\sigma_{1,3}.$$

The covariance of the returns on assets i and j is $\sigma_{i,j}$.

The beauty of the portfolio beta is that it also is a linear combination of the betas of the components

$$\begin{aligned}\beta_p &= \mathrm{Cov}(r_p, rm)/\mathrm{Var}(r_m) = \mathrm{Cov}(w_1r_1 + w_2r_2 + w_3r_3, r_m)/\mathrm{Var}(r_m)\\ &= [w_1\mathrm{Cov}(r_1, r_m) + w_2\mathrm{Cov}(r_2, r_m) + w_3\mathrm{Cov}(r_3, r_m)]/\mathrm{Var}(r_m)\\ &= w_1\beta_1 + w_2\beta_2 + w_3\beta_3.\end{aligned}$$

Preferred Shares

Preferred shares are similar to equity shares in the sense that they are financial claims that confer ownership rights on the shareholders. The term *preferred* connotes the fact that such shares have certain associated privileges. In other words, they get preference over equity shareholders in certain respects.

Current dividends due on the preferred shares must be paid before any dividends for the year can be declared for the equity shareholders.

If a company were to file for bankruptcy and be liquidated, the preferred shareholders have to be paid their due before the balance, if any, can be paid to the equity shareholders. The pecking order in the event of bankruptcy is creditors and bondholders, preferred shareholders, and equity shareholders.

In general, preferred shares do not carry voting rights. Such shareholders may be temporarily conferred with the right to vote if any dividends outstanding have not been paid.

Preferred stocks usually carry a fixed rate of dividend, although there is a variant with variable dividends referred to as *adjustable rate preferreds* (ARPs). The rate may be expressed in dollar terms or as a percentage. The phrase "$7.50 preferred" denotes shares carrying a dividend of $7.50 per share. Another way of expressing the same is to describe the shares as *7.50-percent preferred stock*, assuming that the par value is $100.

From the standpoint of income distribution, preferred shares are similar to debt securities in the sense that the rate of dividends is fixed and not a function of the profits earned during the year. However, unlike bonds, preferred stocks represent ownership of the firm, and the dividend is not a legal liability, which makes preferred stocks similar to equity shares. Such securities have features of both debt and equity. As in the case of equity dividends, preferred dividends must be declared by the directors of the firm.

From the standpoint of capital appreciation, such shares can rise in value. But this is usually caused by declining interest rates in the economy, which leads to a lower required rate of return and does not result from the anticipation of enhanced profitability for the firm, which is the prime driver of equity share prices. The price of a preferred share is inversely related to the prevailing rate of interest, and it behaves like the price of fixed-income securities such as bonds and debentures.

Preferred shares offer companies the opportunity to lock in a fixed yet flexible expense; that is, although a fixed dividend is payable on such stocks, such as the coupon interest on a bond, the dividends can be deferred if required, if financial compulsions were to preclude the firm from paying them as scheduled. The flexibility from the standpoint of dividends makes such securities similar to equity shares, which can be critical in times of financial distress.

Just like bonds, preferred shares provide leverage to common stockholders. *Leverage* refers to the ability to magnify returns on equity by using fixed-income securities to partly fund the firm. In the event of higher-than-anticipated profits, the additional earnings accrue entirely to the equity holders, because the holders of bonds and preferred shares are eligible for fixed payouts. However, in this connection it must be reiterated that although debt securities carry a fixed rate of interest, the bond holders have a legal right to be paid interest and principal when such payments fall due. Unlike in the case of preferred shares, deferral of such payments is not an option for the firm, no matter how difficult the conditions facing a firm may be.

Callable Preferred Stock

Companies that issue preferred shares in an economic environment characterized by high interest rates must necessarily offer a high rate of dividends to make them attractive to investors. At times, management may be of the opinion that, although it is imperative to offer a high dividend under present circumstances, the forecast for the future is such that interest rates are headed downward. If so, the company could issue *callable* preferred stock. As the name suggests, such shares can be prematurely recalled or retired by the company at a predetermined price, unlike conventional or plain vanilla preferred shares, which are *noncallable*.

The presence of a call provision benefits the issuing firm because it can recall the existing issue if the rates were to decline and reissue fresh shares carrying a lower rate of dividends. However, if rates were to decline, the existing shareholders will be extremely reluctant to part with the shares, because they are earning a higher rate of return. The call provision works in favor of the issuing firm and against the investors. To sweeten the deal, companies offer callable shares that cannot be recalled for the first few years. The presence of such a feature helps ensure an assured return for the shareholders for a specified period, while at the same time allowing the issuer to build in the call feature.

Convertible Preferred Shares

A conversion option offers another tool to the issuing firm to sweeten the issue of the preferred shares and lock in a lower rate of dividends. Assume that a company believes that, although the current atmosphere is not conducive for the issue of equity shares at an attractive price, its prospects are very bright in the medium to long term and that its equity shares are therefore likely to appreciate in value. It has the option of issuing preferred shares with an option to convert subsequently into equity at a prespecified conversion ratio. If investors are convinced that the conversion option is likely to be beneficial, then they may accept a lower rate of dividends on the preferred shares than they would otherwise. Example 3.2 is an example of convertible preferred shares and the implications of the prevailing share price for the conversion decision.

Example 3.2

Trigyn Solutions has issued convertible preferred stock. Each share is convertible into four equity shares; in other words, the conversion ratio is 4:1. Let the price of the preferred shares be $\$P_P$ and that of the equity shares be $\$P_E$.

If $P_P = 4P_E$, then the two types of stock are said to be *at parity*. In this case, assume that the preferred share is selling at \$80 and that equity shares are selling at \$20 each. So we have a situation in which the two types of shares are at parity. If parity were to prevail and the dividend of the equity shares obtained on conversion is greater than the dividend on the preferred share, then the preferred shareholders will normally convert under such circumstances. Assume that the preferred share is paying a dividend of \$7.50

while the equity shares are paying a dividend of $2 per share. So if a preferred shareholder were to convert, he would receive a total dividend of $8 from the four equity shares that he would obtain on conversion. In contrast, if he were to hold on to the preferred shares, he would receive only $7.50.

If the preferred share is worth more than the value of the converted equity shares, then we say that the preferred shares are trading above parity. If the preferred share were to trade at $80 and the equity shares at $18.50 each, then the preferred share is trading above parity.

If the preferred shares were to be trading below parity, then it would be tantamount to the availability of an arbitrage opportunity. Assume that the preferred share is trading at $80 and that the equity shares are selling at $22 each. An arbitrageur will buy a preferred share for $80 and immediately convert it to equity shares. The four shares she will receive can be sold for a consideration of $88, which will lead to an arbitrage profit of $8. As arbitrageurs engage in this activity, the price of the preferred stock will rise because of greater demand, whereas that of the equity shares will fall because everyone will be selling them. Eventually, parity will be restored, which will preclude further arbitrage.

Cumulative Preferred Shares

In the case of noncumulative preferred shares, if the issuing firm were to skip a dividend, then the dividend lost is lost forever from the standpoint of the shareholder. However, if the preferred shares were to be cumulative in nature, then all outstanding dividends, including the current dividend, must be paid before the management can contemplate declaring dividends for the equity shareholders. Example 3.3 illustrates the difference between cumulative and noncumulative shares.

Example 3.3

A firm has issued a preferred share with a dividend of $7.50 to Holly and an equity share to Sandy. We will assume that they are the only two stakeholders in the firm and that the company has a policy of paying out the entire earnings for the year as dividends.

The earnings for the company over a five-year horizon are given in Table 3.2.

Table 3.2
Earnings Record

Year	Earnings ($)
2000	15.00
2001	2.50
2002	6.50
2003	12.00
2004	22.00

If the preferred share is noncumulative, then the dividends will be distributed as shown in Table 3.3.

Table 3.3
Dividend Distribution: The Case of Noncumulative Shares

Year	Earnings ($)	Preferred Dividends ($)	Common Dividends ($)
2000	15.00	7.50	7.50
2001	2.50	2.50	0.00
2002	6.50	6.50	0.00
2003	12.00	7.50	4.50
2004	22.00	7.50	14.50

However, if the preferred share is cumulative, the dividends will be distributed as shown in Table 3.4.

Table 3.4
Dividend Distribution: The Case of Cumulative Shares

Year	Earnings ($)	Preferred Dividends ($)	Common Dividends ($)
2000	15.00	7.50	7.50
2001	2.50	2.50	0.00
2002	6.50	6.50	0.00
2003	12.00	12.00	0.00
2004	22.00	9.00	13.00

Let us examine the entries in Tables 3.3 and 3.4. First, consider the case of the noncumulative preferred share. In the year 2000, the earnings are $15.00. Of that, $7.50 will be paid to the preferred shareholder, which is what is due for the year, and the remaining half ($7.50) will be paid to the equity shareholder because, as we have assumed, the firm does not have a policy of retaining any earnings. In 2001, the earnings are $2.50. The entire amount will go to the preferred shareholder. He is eligible for $7.50, but the company does not have adequate funds to pay him his dues in full. The equity holder will receive nothing under the circumstances. In 2002, earnings are $6.50. The entire amount will go to the preferred shareholder, and once again the equity holder will receive nothing. In 2003, the company earns $12.00. The preferred shareholder will receive $7.50. There is a cumulative deficit of $6.00 from the previous two years, but this need not be factored in because the shares are noncumulative. The equity holder will receive $4.50. In 2004, the company has earnings of $22. It will pay $7.50 to the preferred shareholder, and the balance of $14.50 will be paid to the equity holder.

Let us turn to the case of the cumulative preferred share. In 2000, the earnings of $15 will be adequate to pay the preferred shareholder what he is due: $7.50. The balance of $7.50 will once again go to the equity holder. In 2001, the entire earnings of $2.50 will go to the preferred shareholder, and nothing will be paid to the equity shareholder. In 2002, the earnings are $6.50. There is a backlog of $5.00 from the previous year. The amount due to the preferred shareholder is $12.50. Because only $6.50 is available, the deficit will be carried forward to the next year. The equity holder will receive nothing. In 2003, the earnings are $12.00. The preferred shareholder is entitled to $7.50 for the year and $6.00 on account of the deficit carried over from the earlier years. He will receive $12.00, and the equity holder will not receive anything. Finally, out of the earnings of $22 for 2004, $9 will go the preferred holder. This represents $7.50 for the current year and the arrears of $1.50 carried over from the previous year. The remaining $13 will go to the equity shareholder.

Adjustable Rate Preferred Shares

In the case of such securities, the dividend rate is not fixed, but it is subject to periodic revision based on a prespecified formula. The dividend rate may be specified as the T-bond rate plus a spread.

Participating Preferred Shares

Holders of such preferred shares may receive additional dividend payouts over and above what is fixed at the outset. The extra payments may be linked to the performance of the firm based on a predetermined formula, or they may be based on the decision of the board of directors, or both. The presence of such a feature can enhance the value of the preferred shares. However, most preferred shares are nonparticipatory in nature.

Dividend Discount Models

The value of an asset is the present value of all the cash flows that an investor expects to receive from it. The value of a financial asset is a function of (1) the size of the cash flows, (2) the timing of the cash flows, and (3) the risk of the cash flows.

The magnitude of the cash flow determines the numerator in the pricing equation. The timing of the cash flow is crucial because, as we are aware, money has time value, so it matters not only how much we get but also when we get it. The risk of the cash flows has implications for the discount rate that constitutes the denominator in the pricing equation. For a given cash flow, the greater the risk associated with it, the larger will be the rate at which it is discounted, and consequently the smaller will be the present value.

When it comes to the valuation of a stock, the first question we need to ask is what the cash flows are for a person who is contemplating the acquisition of a share. Irrespective of his planning horizon, an investor will receive a dividend for each period that she chooses to hold the stock.[5] Second, at the end of her investment horizon, she will have an inflow on account of the sale of the stock. This will tantamount to either a capital gain or a capital loss.

A General Valuation Model

We will use the following symbols. Additional variables will be defined as we go along.

$P_0 \equiv$ Price of the stock at the outset
$d_t \equiv$ Expected dividend per share at the end of period t
$P_t \equiv$ Expected price of the stock at the end of period t
$r \equiv$ Required rate of return for the asset class to which the stock belongs.

Let us first take the case of an investor who plans to hold the stock for one period.

$$P_0 = \frac{d_1}{1+r} + \frac{P_1}{1+r}.$$

If we assume that the person who buys the stock after one period also has a one-period horizon, then

$$P_1 = \frac{d_2}{1+r} + \frac{P_2}{1+r}$$
$$\Rightarrow P_0 = \frac{d_1}{1+r} + \frac{d_2}{(1+r)^2} + \frac{P_2}{(1+r)^2}.$$

By extending the logic, we arrive at the conclusion that

$$\begin{aligned} P_0 &= \frac{d_1}{1+r} + \frac{d_2}{(1+r)^2} + \frac{d_3}{(1+r)^3} + - - - - - - - - - - - \\ &= \sum_{t=1}^{\infty} \frac{d_t}{(1+r)^t}. \end{aligned}$$

From the above expression, we conclude that the stock price is the present value of an infinite stream of expected dividends.

The Constant Growth Model

No one can forecast an infinite stream of dividends, so we need to make an assumption about how the dividends are expected to evolve over time. The simplest approach is to assume that dividends grow at a constant rate year after year.

Let us assume that dividends grow at a constant rate of g percent per annum, and that the last declared dividend was d_0. The current price can then be expressed as

$$P_0 = \frac{d_0(1+g)}{1+r} + \frac{d_0(1+g)^2}{(1+r)^2} + \frac{d_0(1+g)^3}{(1+r)^3} + - - - - - - - - - - -$$
$$\Rightarrow \frac{P_0(1+r)}{(1+g)} = d_0 + \frac{d_0(1+g)}{1+r} + \frac{d_0(1+g)^2}{(1+r)^2} + - - - - - - - - - - -$$
$$\Rightarrow \frac{P_0}{1+g}[r-g] = d_0$$
$$\Rightarrow P_0 = \frac{d_0(1+g)}{r-g} = \frac{d_1}{r-g}.$$

This is called the constant growth model or the Gordon growth model. For the cash-flow stream to converge, the required rate of return must be higher than the assumed constant growth rate.

Example 3.4 illustrates where this model may be applied.

Example 3.4

Technora has just paid a dividend of \$7.50 per share. The required rate of return on the stock is 8 percent per annum, and dividends are expected to grow at the rate of 5 percent per annum. What should be the stock price?

The price per the Gordon model is

$$P_0 = \frac{d_0(1+g)}{r-g} = \frac{7.50(1.05)}{0.08-0.05} = \$262.50.$$

The Two-Stage Model

The constant growth model assumes a growth rate that stays constant forever. One way of building in a touch of realism is to assume that the stock will display a high growth rate of g_a percent per annum for the first A years, and that the growth rate will then settle down to a more modest level of g_n percent that will last forever. In other words, at the end of the high-growth period, the stock is similar to one that pays dividends growing at a constant rate. The price of the stock under these assumptions may be then determined as follows:

$$\begin{aligned} P_0 &= \frac{d_1}{1+r} + \frac{d_2}{(1+r)^2} + \cdots + \frac{d_A}{(1+r)^A} \\ &\quad + \frac{d_{A+1}}{(1+r)^A(r-g_n)} \\ &= \frac{d_0(1+g_a)}{1+r} + \frac{d_0(1+g_a)^2}{(1+r)^2} + \cdots + \frac{d_0(1+g_a)^A}{(1+r)^A} \\ &\quad + \frac{d_0(1+g_a)^A}{(1+r)^A}\frac{(1+g_n)}{r-g_n}. \end{aligned}$$

$$\text{Let } S = \frac{d_0(1+g_a)}{1+r} + \frac{d_0(1+g_a)^2}{(1+r)^2} - - - - - - - - - - - + \frac{d_0(1+g_a)^A}{(1+r)^A}$$

$$\Rightarrow S = \frac{d_1}{r-g_a}\left[1 - \frac{(1+g_a)^A}{(1+r)^A}\right]$$

$$\Rightarrow P_0 = \frac{d_1}{r-g_a}\left[1 - \frac{(1+g_a)^A}{(1+r)^A}\right] + \frac{d_1(1+g_a)^A}{(1+r)^A}\frac{(1+g_n)}{r-g_n} \times \frac{1}{(1+g_a)}$$

$$= \frac{d_1}{r-g_a}\left[1 - \left(\frac{1+g_a}{1+r}\right)^{A-1}\frac{(g_a-g_n)}{r-g_n}\right].$$

Example 3.5 illustrates this model.

Example 3.5

Technora has just paid a dividend of \$7.50 per share. Analysts expect that dividends will grow at the rate of 10 percent per annum for the first eight years and will then settle down to a constant rate of 5 percent per annum. The required rate of return on the stock is 8 percent. What should the price of the stock be per the two-stage model?

$$P_0 = \frac{7.50(1.10)}{0.08-0.10}\left[1 - \left(\frac{1.10}{1.08}\right)^7\left(\frac{0.10-0.05}{0.08-0.05}\right)\right]$$

$$= -412.50[1 - 1.8951] = \$369.23.$$

The Three-Stage Model

A major shortcoming of the two-stage model is that the growth rate suddenly declines at the end of the initial high-growth phase. A more plausible assumption would be that the growth rate gradually declines after the high-growth phase until it reaches its long-run value, and this is precisely what the three-stage model does. It avoids a discrete jump in the growth rate by postulating that dividends grow at a high rate during an initial period and that the growth rate then declines linearly year after year during an intermediate phase until it reaches the long-run value at which it remains stable thereafter. We will denote the duration of the high-growth phase as A years and the duration of the declining growth phase as B-A years. The model may then be stated as

$$P_0 = \sum_{t=1}^{A} \frac{d_1(1+g_a)^{t-1}}{(1+r)^t} + \sum_{t=A+1}^{B} \frac{d_{t-1}(1+g_t)}{(1+r)^t} + \frac{d_B(1+g_n)}{(1+r)^B(r-g_n)}$$

where

$$g_t = g_a - (g_a - g_n)\frac{t-A}{B-A}$$

during the period $(A+1) \leq t \leq B$.

Example 3.6 illustrates the three-stage model in detail.

Example 3.6

Technora has just paid a dividend of \$ 7.50 per share. Dividends are expected to grow at a rate of 10 percent per annum for the first eight years. The growth rate is then expected to decline linearly for four years till it reaches a constant growth rate of 5 percent per annum. The required rate of return on the stock is 8 percent per annum. The question is what should be the value of the stock?

The computation of the first 12 dividend payments and their present values is depicted in Table 3.5.

Table 3.5
Valuation of Cash Flows per the Three-Stage Model

Year	Dividend (\$)	Present Value (\$)
1	8.2500	7.6389
2	9.0750	7.7804
3	9.9825	7.9244
4	10.9808	8.0718
5	12.0788	8.2206
6	13.2867	8.3729
7	14.6154	8.5279
8	16.0769	8.6859
9	17.4837	8.7462
10	18.7949	8.7057
11	19.9696	8.5646
12	20.9681	8.3267

$$\frac{d_{12}(1+g_n)}{(r-g_n)(1+r)^{12}} = \frac{20.9681 \times 1.05}{(0.03)(1.08)^{12}}$$

$$= 291.4350$$

$$P_0 = \$391.00.$$

The H Model

The H model was developed as an alternative to the three-stage model. In this model, growth begins at a high rate of g_a percent per annum. However, unlike in the earlier case, there is no initial high-growth period of A years. Instead, the growth rate starts declining linearly from the outset over a period of 2H years until it reaches a value of g_n percent per annum. Thereafter, the growth rate remains constant at the long-run value. The value of the share as per this model is given by

$$P_0 = \frac{d_0}{r-g_n}[(1+g_n)+H(g_a-g_n)].$$

Example 3.7 is a simple illustration of the H model.

Example 3.7

Technora has just declared a dividend of \$7.50 per share. The initial growth rate is 10 percent per annum, and it is expected to declare linearly over a period of 10 years until it reaches 5 percent per annum. The required rate of return on the stock is 8 percent. What is the price of the stock per the H model?

$$2\text{H} = 10 \text{ years} \Rightarrow \text{H} = 5 \text{ years}$$

$$P_0 = \frac{7.50}{0.08-0.05}[(1.05)+5(0.10-0.05)]$$

$$= \frac{7.50}{0.03} \times 1.30 = \$325.00.$$

Stock Market Indexes

A stock market index is constructed by considering the prices, or the market capitalization, of a predefined basket of securities. The objective of constructing an index is to enable us to track the performance of the securities market. We cannot meaningfully assess the market by looking at a vector of asset prices or returns. We need one number that will enable us to gauge or track the performance of the market. This explains the faith of the securities industry in a tracking index such as the Dow Jones Industrial Average (DJIA). An index may be constructed to represent an entire market or a market segment. It is important to select the constituent assets such that they are representative of the market or market segment.

Price-Weighted Indexes

One way to construct a market index is by considering only the prices of the constituent stocks. The first step in the construction of an index is the decision regarding the number of securities to be included and the selection of specific securities.

In the case of a price-weighted index, the current prices of all the component stocks are added and then divided by a number known as the *divisor*.

On the base date, or the date on which the index is being computed for the first time, the divisor can be set equal to any arbitrary value. A logical way to set the initial value of the divisor is by setting it equal to the number of stocks that have been chosen for inclusion in the index. The divisor does not have to be adjusted on a day-to-day basis. However, whenever there is a corporate action such as a split−reverse split, a stock dividend, or a rights issue, the divisor will be adjusted as described later. An adjustment is also required when there is a change in the composition of the index—that is, one or more stocks are substituted.

Let us assume that we are standing at the end of day t, and the closing price of the ith stock on the day is $P_{i,t}$. The index level I_t, is given by

$$I_t = \sum_{i=1}^{N} \frac{P_{i,t}}{Div_t}$$

where Div_t is the applicable value of the divisor for the day, and N is the number of stocks comprising the index. Example 3.8 is a detailed numerical example to illustrate how such an index may be computed.

Example 3.8

Let us assume that we are standing on the base date of an index, which has been defined to include five stocks. The starting value of the divisor has been chosen to be 5.0. Let the closing prices of these five stocks at the end of the day be as shown in Table 3.6.

Table 3.6
Prices of the Constituent Stocks on the Base Date

Stock	Price ($)
3M	80
American Express	55
Coca-Cola	45
IBM	80
Merck	35
Total	295

The end-of-the-day index value will therefore be $295/5 = 59.00$.

On the following day, the prices of the component stocks are assumed to be as shown in Table 3.7.

Table 3.7
Prices of the Constituent Stocks on the Following Day

Stock	Price ($)
3M	85
American Express	60
Coca-Cola	48
IBM	85
Merck	40
Total	318

The value of the index on this day will be $318/5 = 63.60$, and we will conclude that the market has moved up. If we compare Tables 3.6 and 3.7, we will find that every stock has risen in value. It is not necessary that all the stocks must rise in price for the index to rise or that all of them should decline in price if the index falls.

Changing the Divisor

The divisor has to be adjusted if one or more of the following corporate actions were to occur:

- A split or a reverse split in one or more of the constituent stocks;
- A rights issue if the issue is at a discount to the prevailing market price;
- A stock dividend on one or more of the constituent stocks; or
- A change of composition—that is, a replacement of an existing stock by a new stock.

We will illustrate the mechanics of adjusting the divisor by considering a situation in which one of the constituent stocks undergoes a split.

We will assume that Coca-Cola undergoes a 3:1 split at the end of the base date. The prices of the constituent stocks at the end of the following day are assumed to be as shown in Table 3.8.

When we compare Table 3.8 with Table 3.7 we find that all the other stocks have the same value at the end of the next day as before, except for Coca-Cola, which has one-third of the value it would have had in the absence of the split.

Let us first compute the index without making an adjustment to the divisor. We will get an index value of 286/5 = 57.20 and will conclude that the market has gone down as compared to the base date. However, this is an erroneous deduction because every stock, including Coca-Cola, has risen in value compared to the base date. The perceived decline in the index is entirely because of our failure to take the split into account.

Table 3.8
Prices of the Constituent Stocks on the Following Day, Assuming a Stock Split in Coca-Cola

Stock	Price ($)
3M	85
American Express	60
Coca-Cola	16
IBM	85
Merck	40
Total	286

Table 3.9
Theoretical Postsplit Stock Prices

Stock	Price ($)
3M	80
American Express	55
Coca-Cola	15
IBM	80
Merck	35
Total	265

If the index has to continue to be an accurate barometer of the market, then an adjustment needs to be made. The variable that needs to be adjusted is the divisor. We would proceed to adjust the divisor as follows. First, we would list the theoretical postsplit values at the end of the day on which the split is declared, as shown in Table 3.9. The split will affect only the price of the stock whose shares have been split. In this case, the theoretical postsplit price of Coca-Cola will be one-third of its presplit value of $45.

The new divisor, Div_N, should be such that the index value is the same, whether we use the presplit prices and the old divisor or the postsplit prices and the new divisor. In this case, $265/Div_N$ should equal 59.00. The new divisor is therefore 4.4915. If we were to use this value of the divisor to compute the index level on the following day, we will get a value of 63.68, which is consistent with our earlier observation that the market has risen.

We will continue to use the new divisor until there is another corporate action. The adjustment procedure for a stock dividend is identical to that for a stock split because the two are mathematically equivalent. If a firm were to declare a stock dividend of 20 percent, it would be equivalent to a 6:5 split, and we would proceed to adjust the divisor accordingly.

We will illustrate the adjustment procedure in the event of a change in composition with the help of an example. Assume that at the end of the day following the base date, Merck, which has a prevailing market price of $40, is replaced with General Electric, which has a price of $50. The index level before the change is 63.68 as computed earlier. The prices of the stocks contained in the reconstituted index will be as shown in Table 3.10.

Table 3.10
Prices of the Component Stocks of the Reconstituted Index

Stock	Price ($)
3M	85
American Express	60
Coca-Cola	16
IBM	85
General Electric	50
Total	296

The new divisor, Div_N, should be such that $296/Div_N = 63.68 \Rightarrow Div_N = 4.6482$.

The Importance of Price

A feature of a price-weighted index, which finance theorists opine to be one of its weaknesses, is that higher-priced stocks tend to have a greater impact on the index level than lower-priced components. Let us take the data given in Table 3.11 for our five-stock index on a particular day. Assume that the divisor is 5.0.

The index level is $310/5 = 62.00$.

Consider two possible situations for the following day, as depicted in Table 3.12. In case A, IBM's price has gone up by 20 percent, whereas Merck's price has gone up by 20 percent in case B.

In the first case, the index value is 65.60, which represents an increase of 5.81 percent as compared to the previous day. However, in the second

Table 3.11
Prices of the Constituent Stocks on a Given Day

Stock	Price ($)
3M	85
American Express	60
Coca-Cola	45
IBM	90
Merck	30
Total	310

Table 3.12
Prices on the Following Day: Two Different Scenarios

Stock	Case A Price ($)	Case B Price ($)
3M	85	85
American Express	60	60
Coca-Cola	45	45
IBM	108	90
Merck	30	36
Total	328	316

case, the index value is 63.20, which represents an increase of only 1.94 percent compared to the previous day. A change of 20 percent in the price of IBM, which is a high-priced stock, has had a greater impact than a similar change in the price of Merck, which is priced considerably lower.

In finance, the importance accorded to a company is based on its market capitalization, not on its price. A model such as CAPM is consistent with this viewpoint, because it defines the *market portfolio* as a market capitalization–weighted portfolio of all assets. A price-weighted index can in a sense be construed as a less-than-perfect barometer of the stock market.

Value-Weighted Indexes

A value-weighted index is theoretically sounder because it takes into consideration the market capitalization of a component stock and not merely its price.

Let us assume that we are standing on day t and that the starting or base date of the index is day b. We will use $P_{i,t}$ and $P_{i,b}$ to denote the market prices of the ith stock on days t and b, respectively, and $Q_{i,t}$ and $Q_{i,b}$ to denote the number of shares outstanding on those two days.

On the base date, the index can be assigned any arbitrary value. We will assign a value of 100. The level of the index on day t is then defined as

$$\frac{1}{Div_t}\left(\frac{\sum_{i=1}^{N} P_{i,t}Q_{i,t}}{\sum_{i=1}^{M} P_{i,b}Q_{i,b}}\right) \times 100$$

Div_t represents the value of the divisor on day t. In the case of a value-weighted index, the divisor is always assigned a value of 1.0 on the base date. Subsequently it will be adjusted as and when required. However, unlike the case of the divisors used for price-weighted indexes, the divisor need not be adjusted for a stock split or a stock dividend as we shall shortly see. Example 3.9 illustrates the computation of a value-weighted index.

Example 3.9

We will consider the same five companies that we used earlier; their prices are given in Table 3.13. The difference is that we will now also consider the number of shares issued by each firm.

Table 3.13
Prices, Number of Shares Outstanding, and Market Capitalization of the Components of a Value-Weighted Index on the Base Date

Stock	Price(P) ($)	Number of Shares (Q)	Market Capitalization ($)
3M	85	780,000,000	66,300,000,000
American Express	60	1,250,000,000	75,000,000,000
Coca-Cola	45	2,425,000,000	109,125,000,000
IBM	85	1,635,000,000	138,975,000,000
Merck	35	2,220,000,000	77,700,000,000

The total market value is

$$\sum_{i=1}^{5} P_i Q_i = 467{,}100{,}000{,}000.$$

Suppose that on the following day, the prices and number of shares are as depicted in Table 3.14.

Table 3.14
Prices, Number of Shares Outstanding, and Market Capitalization of the Components of a Value-Weighted Index on the Following Day

Stock	Price (P) ($)	Number of Shares (Q)	Market Capitalization ($)
3M	90	780,000,000	70,200,000,000
American Express	65	1,250,000,000	81,250,000,000
Coca-Cola	50	2,425,000,000	121,250,000,000
IBM	90	1,635,000,000	147,150,000,000
Merck	40	2,220,000,000	88,800,000,000

The total market value is

$$\sum_{i=1}^{5} P_i Q_i = 508{,}650{,}000{,}000.$$

The value of the index on this day is therefore

$$\frac{508{,}650{,}000{,}000}{467{,}100{,}000{,}000} \times 100 = 108.8953.$$

We can conclude that the market has moved up.

Changing the Divisor

In the case of a value-weighted index, there is no need to adjust the divisor if one of the components of the index were to undergo a split or a reverse split or if a firm that is present in the index were to declare a stock dividend. This is because, from a theoretical standpoint, such corporate actions are value neutral and do not have any impact on the market capitalization.

In the case of a stock split or a stock dividend, the share price will decline, whereas in the case of a reverse split the share price will rise. In the first two cases, the number of shares outstanding will rise whereas in the last case it will decline. The changes in the number of shares outstanding will exactly offset the price change such that there is no impact on the market capitalization. Example 3.10 illustrates this phenomenon.

The divisor would, however, have to be modified whenever there is a change in the composition of the index. Let us assume that the prices and number of shares of the companies constituting the index are as shown in Table 3.14 at the end of a particular day. Assume that Merck with a price of $40 and number of shares outstanding equal to 2,220,000,000 is replaced with General Electric, which has a price of $50 and number of shares outstanding equal to 10,500,000,000. The market capitalization of the component stocks after the change will be as depicted in Table 3.15.

Example 3.10

Assume that 3M has a market price of \$90, and that the number of shares outstanding is 780,000,000. Consider the case of a 20 percent stock dividend. The share price will immediately decline to $\$90 \times 780{,}000{,}000/936{,}000{,}000 = \$75$. The market capitalization before the stock dividend is

$$90 \times 780{,}000{,}000 = 70{,}200{,}000{,}000$$

which is the same as the market capitalization after the dividend, which is

$$75 \times 936{,}000{,}000 = 70{,}200{,}000{,}000.$$

Table 3.15
Market Capitalization of the Component Stocks of the Reconstituted Index

Stock	Price (P) (\$)	Number of Shares (Q)	Market Capitalization (\$)
3M	90	780,000,000	70,200,000,000
American Express	65	1,250,000,000	81,250,,000,000
Coca-Cola	50	2,425,000,000	121,250,000,000
IBM	90	1,635,000,000	147,150,000,000
General Electric	50	10,500,000,000	525,000,000,000

The total market capitalization after the change is 944,850,000,000. The new divisor, Div_N, should be such that

$$\frac{1}{Div_N} \times \frac{944{,}850{,}000{,}000}{467{,}100{,}000{,}000} \times 100 = 108.8953$$

$$\Rightarrow Div_N = 1.8576.$$

Changing the Base-Period Capitalization

In the event of a change in its composition, one way to handle the situation, as we have just seen, is to change the divisor. In such cases, the base-period market capitalization will always stay frozen at its initial level.

A different but equivalent approach is, however, adopted in the case of certain value-weighted indexes. In such cases, the divisor is not recomputed in the event of a change in composition. In other words, it is always maintained at its initial level of 1.00. What is done in such cases is that the base-period market capitalization is adjusted to reflect the change in the composition of the index.

Let us take the data depicted in Table 3.15. If we denote the modified base-period capitalization by BPC_N, then it should be such that

$$\frac{944{,}850{,}000{,}000}{BPC_N} \times 100 = 108.8953$$

$$\Rightarrow BPC_N = 867{,}668{,}301{,}600.$$

For all subsequent calculations, the new value will be used for the base-period capitalization until there is another change in the composition of the index. Mathematically, the two approaches are equivalent because the denominator in the index computation formula consists of the product of the divisor and the base-period market capitalization. We can keep either one constant and adjust the other.

Equally Weighted Indexes

An equally weighted index offers yet another alternative for tracking the performance of a market. In the case of such indexes, like the case of price-weighted indexes, we take cognizance of only the prices of the component stocks.

Let us consider an index consisting of N component stocks. The value of the index on day t is defined as

$$I_t = I_{t-1} \times \frac{1}{N} \sum_{i=1}^{N} \frac{P_{i,t}}{P_{i,t-1}}.$$

The ratio of the prices, $P_{i,t}/P_{i,t-1}$ may be expressed as $(1 + r_{i,t})$, where $r_{i,t}$ is the arithmetic rate of return on the ith stock between day t and day $t - 1$. Therefore,

$$\begin{aligned}\frac{1}{N}\sum_{i=1}^{N}\frac{P_{i,t}}{P_{i,t-1}} &= \frac{1}{N}\sum_{i=1}^{N}(1 + r_{i,t}) \\ &= 1 + \frac{1}{N}\sum_{i=1}^{N} r_{i,t} = 1 + \bar{r}_t\end{aligned}$$

where $\bar{r}_t$ is the arithmetic average of the returns on all the component stocks between day $t-1$ and day t. Thus

$$I_t = I_{t-1} \times (1 + \bar{r}_t).$$

In other words, the value of the index on a particular day is its value at the point in time at which it was previously computed, multiplied by one plus the average arithmetic rate of return on all the stocks constituting the index for the period.

Tracking Portfolios

Investors often seek to hold a basket of securities that imitates or mirrors the behavior of a stock market index. The method of forming such a tracking or mimicking portfolio would depend on the method adopted to compute the index being tracked.

Imitating an equally weighted index is fairly simple. The investor has to put an equal fraction of his wealth in all the assets that constitute the index. Consider an investor with an initial wealth of $\$W$, who is seeking to track an index consisting of N stocks. He will have to invest an amount of W/N in each security.

Tracking a price-weighted index too is fairly straightforward. An investor who seeks to do so must acquire an equal number of shares of each company present in the index.

Finally, we come to the task of forming a portfolio to track a value-weighted index. This is the most complex task. To imitate such an index, we must invest a fraction of our wealth in each asset that is equal to the ratio of the market capitalization of that particular asset to the total market capitalization of all the assets that constitute the index.

Rebalancing a Tracking Portfolio

Once we set up a tracking portfolio, we cannot just sit back and watch the portfolio automatically track the index in question. In the case of every index, irrespective of its mode of computation, there will arise circumstances in which the tracking portfolio will have to be rebalanced.

Equally Weighted Portfolios

An equally weighted tracking portfolio needs to be rebalanced very frequently—that is, virtually on a daily basis. This is because, unless none of the component stocks were to undergo a change in price from one day to the next, an index that is equally weighted on a particular day will no longer be equally weighted the following day. We will give a detailed example as to why an equally weighted portfolio will require rebalancing with the sheer passage of time.

Take the case of an investor who has a corpus of $600,000 and decides to form an equally weighted portfolio consisting of the four stocks whose prices are shown in Table 3.16.

Because she has $600,000, she will have to invest $150,000 in each of the four stocks. She needs to acquire 3,000 shares of General Electric, 1,500 shares of 3M, 3,750 shares of Merck, and 1,200 shares of IBM. If we assume that the index value is 100, then the portfolio is worth 6,000 times the index.

Let us assume that the prices of the companies are as shown in Table 3.17 on the next day.

The amounts invested in the stocks will be $120,000, $187,500, $187,500, and $120,000, respectively. The total portfolio value is $615,000, of which 19.51 percent each is in General Electric and IBM, and 30.49 percent each is in 3M and Merck.

Table 3.16
Prices of the Stocks Constituting an Equally Weighted Index at the Time of Formation of the Portfolio

Stock	Price ($)
General Electric	50
3M	100
Merck	40
IBM	125

Table 3.17
Prices of the Stocks Constituting an Equally Weighted Index on the Following Day

Stock	Price ($)
General Electric	40
3M	125
Merck	50
IBM	100

The portfolio is no longer equally weighted; if it were, the investor would have 25 percent of her wealth in each asset. If the weights have to be reset to 0.25 each, then she would need to rebalance by selling off a part of her holdings in 3M and Merck and investing the proceeds in General Electric and IBM.

She would need to proceed as follows. The total value of the portfolio is \$615,000, which means that the investor needs to have \$153,750 in each stock. For this, she would need to buy 843.75 shares of General Electric and 337.50 shares of IBM. She would also need to sell 270 shares of 3M and 675 shares of Merck.

The inflow on account of what is being sold is $270 \times 125 + 675 \times 50 =$ \$67,500.

The outflow on account of what is being bought is $843.75 \times 40 + 337.50 \times 100 = \$67,500$.

We have demonstrated the investor can rebalance at zero net cost. It can be verified that the amount invested in each company is \$153,750.

The new index level is $100 \times (1 + \bar{r}_t)$.

$$\bar{r}_t = \frac{(-0.20 + 0.25 + 0.25 + -0.20)}{4} = \frac{0.10}{4} = 0.025.$$

Therefore, the new value of the index is 102.50. The total value of the portfolio is \$615,000, which is 6,000 times the value of the index. The portfolio continues to mimic the index.

Price-Weighted Portfolios

Let us turn our attention to a portfolio that is constructed to track a price-weighted stock market index. Such a portfolio has to be rebalanced whenever there is a split or a reverse split, a stock dividend, a rights issue, or a change in the index composition. In other words, whenever circumstances warrant a change in the divisor that it used to compute the index, we need to rebalance the tracking portfolio.

Let us consider a price-weighted index consisting of the four stocks depicted in Table 3.16. Assume that the divisor is equal to 4.0, which would mean that the index level is

$$\frac{50 + 100 + 40 + 125}{4.0} = 78.75.$$

Let us form a tracking portfolio by buying 2,000 shares of each of the constituent stocks. The value of our tracking portfolio will be

$$2{,}000 \times (50 + 100 + 40 + 125) = \$630{,}000 = 8{,}000 \times 78.75.$$

Our portfolio is worth 8,000 times the index.

Consider a situation in which 3M undergoes a 2:1 split, which means that its postsplit theoretical value will be \$50. The new divisor, Div_N, should be such that

$$\frac{50 + 50 + 40 + 125}{Div_N} = 78.75$$
$$\Rightarrow Div_N = 3.3651.$$

To ensure that our portfolio continues to mimic the index, we need to rebalance in such a way that our portfolio value remains unchanged. Let the number of shares of each stock held before the split be denoted by N_O, and the number of shares required after the split by N_N. If the value of our tracking portfolio is to remain unchanged, it should be the case that

$$N_0 \times Div_o \times I_t = N_N \times Div_N \times I_t$$
$$\Rightarrow N_N = \frac{N_o \times Div_o}{Div_N}.$$

In our example, the number of shares of each stock required after the split is

$$\frac{2{,}000 \times 4.0}{3.3651} = 2{,}377.3439.$$

If we were to assume that fractional shares can be bought and sold, we will have to buy 377.3439 shares of General Electric, Merck, and IBM and sell 1,622.6561 shares of 3M.[6]

The inflow on account of what is sold is $1{,}622.6561 \times 50 = \$811{,}32.805$.

The outflow on account of what is being bought is $377.3439 \times (50 + 40 + 125) = \$811{,}28.938$.

Once again, if we ignore transactions costs, we can rebalance at zero net cost. The difference that we get between the inflow and the outflow entirely results from rounding errors.

Rights Issues

Let us consider the case of rebalancing a price-weighted portfolio to factor in a rights issue. Consider the four stocks in Table 3.17. We will assume that the divisor at the outset is 4.0 and the index level is 78.75. To form a tracking portfolio, the investor has acquired 2,000 shares of each company.

Assume that Merck announces a 1:10 rights issue at a price of \$39 per share. The ex-rights price will decline from 50 to 49, and the aggregate value of the four stocks will decline from 315 to 314. The new number of shares required to track the price-weighted index is given by

$$N_N = (No \times Div_O)/Div_N = (2000 \times 4) \times 78.75/314 = 2{,}006.3694.$$

There are two possibilities. The investor may renounce his rights. Because he owns 2,000 shares, he has the right to acquire 200 shares, and the right to acquire a share is worth \$10. He will receive \$2,000 if he were to renounce his rights. The outflow on account of the extra shares acquired is

$$6.3694 \times (49 + 100 + 40 + 125) = \$2{,}000.$$

The inflow from the renounced rights is exactly equal to the outflow.

The other possibility is that he acquires 200 shares of Merck by paying \$7,800. He can keep 6.3694 shares and sell the balance at a price of \$49 each. The net inflow $= 49 \times 193.6306 - 7{,}800 = 1{,}687.90$. This will be exactly sufficient to acquire 6.3694 shares of each of the other three companies

$$6.3694 \times (100 + 40 + 125) = 1{,}687.89.$$

Value-Weighted Portfolios

Consider four stocks, whose prices and number of shares outstanding are as shown in Table 3.18.

Table 3.18
Components of a Value-Weighted Index

Stock	Price (P) (\$)	Number of Shares(Q)	Market Capitalization (\$)
MRF	20	200,000	4,000,000
J.K. Tyres	80	50,000	4,000,000
Apollo Tyres	50	200,000	10,000,000
Viking Tyres	20	100,000	2,000,000

The total market capitalization is \$20,000,000. Assume that the base-period market capitalization is \$16,000,000 and that the current divisor is 1. The index level is therefore 125.00.

Let us consider the case of an investor who has an endowment of \$500,000 and wishes to create a portfolio to track the index. For his portfolio to mimic the index, he must have

$$\frac{4{,}000{,}000}{20{,}000{,}000} \times 500{,}000 = \$100{,}000 \text{ in MRF stock,}$$

$$\frac{4{,}000{,}000}{20{,}000{,}000} \times 500{,}000 = \$100{,}000 \text{ in J.K. Tyres stock,}$$

$$\frac{10{,}000{,}000}{20{,}000{,}000} \times 500{,}000 = \$250{,}000 \text{ in Apollo Tyres stock, and}$$

$$\frac{2{,}000{,}000}{20{,}000{,}000} \times 500{,}000 = \$50{,}000 \text{ in Viking Tyres stock.}$$

He must buy 5,000 shares of MRF, 1,250 shares of J.K. Tyres, 5,000 shares of Apollo Tyres, and 2,500 shares of Viking Tyres. In general, the number of shares of the *i*th stock is given by

$$\frac{Q_i}{\sum P_i Q_i} \times W$$

where W is the initial wealth. The total portfolio value in this case is 4,000 times the index value of 125.00

Assume that Viking Tyres is replaced by Ceat, which has a share price of \$70 and 100,000 shares outstanding. The total market capitalization of the four components of the index will be \$25,000,000. The divisor will have to be adjusted in such a way that the index level remains unchanged; that is,

$$\frac{1}{\text{Div}_N} \times \frac{25{,}000{,}000}{16{,}000{,}000} \times 100 = 125$$

$$\Rightarrow \text{Div}_N = 1.25.$$

For the portfolio to remain value weighted, the investor must have

$$\frac{4{,}000{,}000}{25{,}000{,}000} \times 500{,}000 = \$80{,}000 \text{ in MRF stock,}$$

$$\frac{4{,}000{,}000}{25{,}000{,}000} \times 500{,}000 = \$80{,}000 \text{ in J.K. Tyres stock,}$$

$$\frac{10{,}000{,}000}{25{,}000{,}000} \times 500{,}000 = \$200{,}000 \text{ in Apollo Tyres stock, and}$$

$$\frac{7{,}000{,}000}{25{,}000{,}000} \times 500{,}000 = \$140{,}000 \text{ in Ceat stock.}$$

The investor requires 4,000 shares of MRF, 1,000 shares of J.K, 4,000 shares of Apollo, and 2,000 shares of Ceat. This means that he will have to sell 1,000 shares of MRF, 250 shares of J.K, 1,000 shares of Apollo, and 2,500 shares of Viking. He will also have to buy 2,000 shares of Ceat. The number of shares of the *i*th stock, is once again given by

$$\frac{Q_i}{\sum P_i Q_i} \times W.$$

The inflow $= 1{,}000 \times 20 + 250 \times 80 + 1{,}000 \times 50 + 2{,}500 \times 20 = \$140{,}000$.

The outflow $= 2{,}000 \times 70 = \$140{,}000$.

If we ignore transactions costs, then once again we can rebalance at zero net cost. The portfolio value after rebalancing will be \$500,000, which is 4,000 times the index level of 62.50.

Handling a Rights Issue

Consider the data given in Table 3.18. Assume that Apollo Tyres announces a 1-for-4 rights issue at a price of \$25 per share. The ex-rights price will be

$$\frac{(50 \times 200{,}000 + 25 \times 50{,}000)}{250{,}000} = \$45.$$

The price and market capitalization data, postrights, is as shown in Table 3.19.

The aggregate market capitalization is \$21.25 million.

Let us first assume that the investor renounces his rights. He will get \$20 for the right to acquire a share. Because he has 5,000 shares of Apollo, he has the right to buy 1,250 shares.

Table 3.19
Components of a Value-Weighted Index

Stock	Price (P)	Number of Shares (Q) ($)	Market Capitalization ($)
MRF	20	200,000	4,000,000
J.K. Tyres	80	50,000	4,000,000
Apollo Tyres	45	250,000	11,250,000
Viking Tyres	20	100,000	2,000,000

The value of the portfolio after the rights issue is

$$5{,}000 \times 20 + 1{,}250 \times 80 + 5{,}000 \times 45 + 2{,}500 \times 20 = \$475{,}000.$$

The inflow from the sale of rights is

$$1{,}250 \times 20 = \$25{,}000.$$

The total capital available is $500,000.

The amount of capital required to be invested in each constituent firm after the rights issue can be computed as follows:

$$\frac{4{,}000{,}000}{21{,}250{,}000} \times 500{,}000 = \$94{,}117.645 \text{ in MRF stock,}$$

$$\frac{4{,}000{,}000}{21{,}250{,}000} \times 500{,}000 = \$94{,}117.645 \text{ in J.K. Tyres stock,}$$

$$\frac{11{,}250{,}000}{21{,}250{,}000} \times 500{,}000 = \$264{,}705.88 \text{ in Apollo Tyres stock, and}$$

$$\frac{2{,}000{,}000}{21{,}250{,}000} \times 500{,}000 = \$47{,}058.82 \text{ in Viking stock.}$$

The outflow on account of what is being bought is

$$250{,}000 - 264{,}705.85 = \$14{,}705.88.$$

The inflow from the three firms whose shares are being sold is

$$(100{,}000 - 94{,}117.645) + (100{,}000 - 94{,}117.645) + (50{,}000 - 47{,}058.82) = \$14{,}705.90.$$

The portfolio can be rebalanced at zero net cost.

What if he were to exercise his rights and buy 1,250 shares at \$25 each? The amount of funds required is \$31,250. The value of the shares will be $(5{,}000 + 1{,}250) \times 45 = \$281{,}250$.

The investment required in Apollo Tyres is \$264,705.88. Thus, $281{,}250 - 264{,}705.88 = 16{,}544.12$ will be released when the surplus shares are sold.

The inflow from the other three firms whose shares are being sold is

$$(100{,}000 - 94{,}117.645) + (100{,}000 - 94{,}117.645) + (50{,}000 - 47{,}058.82) = \$14{,}705.90.$$

The total funds released $= 16{,}544.12 + 14{,}705.90 = \$31{,}250.02$, which is exactly equal to the money required to subscribe to the rights issue.

The Free-Floating Methodology

The traditional method of computing a value-weighted index entails the use of the market capitalization of the constituent stocks, where the market capitalization is defined as the price of the share multiplied by the total number of shares issued by the company. This approach has come under criticism in recent years from those who propound that the appropriate number of shares used for the purpose of computing the market capitalization should be those that are *freely floating*. The free-floating methodology entails the calculation of the market capitalization of a stock as the share price multiplied by the number of shares that are freely available for trading in the market.

For a widely held firm, the free-floating market capitalization will be high and close to or even equal to its total market value. However, for closely held companies, the free-floating market capitalization may be considerably less than the full market value. One argument advanced against the use of the full market capitalization to compute the index value is that those index constituents that have a relatively smaller percentage of shares available for trading may be vulnerable to sharp price fluctuations. Not only will a price move in such an illiquid stock cause the index to move sharply but also such stocks are vulnerable to manipulation by speculators. Therefore, defining the index on the basis of free-floating shares can help ensure that the impact of underlying market movements is reflected in a better fashion.

Table 3.20
Constituents of the Dow Jones Industrial Average on March 28, 2010

AT&T	General Electric	Microsoft
Bank of America	Hewlett-Packard	Pfizer
Boeing	Home Depot	Proctor & Gamble
Caterpillar	Intel	Travelers
Chevron	IBM	United Technologies
Cisco Systems	Johnson & Johnson	Verizon
Coca-Cola	JPMorgan Chase	Communications
E. I. DuPont	Kraft Foods	WalMart Stores
de Nemours	McDonald's	Walt Disney
Exxon Mobil	Merck	

Well-Known Global Indexes

The most famous index is undoubtedly the Dow Jones Industrial Average, which is popularly known as the *Dow*. It is a price-weighted average of 30 stocks. Table 3.20 lists its constituent stocks.

The Nikkei Index, which is a barometer of the Japanese stock market, is also price weighted and includes 225 large Japanese companies. The Standard & Poor's 500 Index (S&P 500) and the NASDAQ 100 index are both value weighted.

Margin Trading and Short Selling

The term *buying stock on margin* refers to a process of investment in which an investor funds the acquisition of the stock by borrowing a part of the funds required from the broker. As can be surmised, the ability to raise a loan to fund a share purchase will enable the investor to acquire more shares than what her own funds will permit.

Terminology

The minimum percentage of the market value of the securities that has to be deposited by the customer is called the *margin rate*. The difference between the market value of the securities and the minimum amount that has to be deposited by the customer is called the *loan value*. The loan value represents the maximum amount that can be borrowed from the broker. The loan value expressed as a fraction of the market value is referred to as the *loan rate*. The margin rate, which is the minimum that the investor has to deposit, and the loan rate, which is the maximum that he can borrow, will always be equal to 100 percent. The actual amount that is borrowed from the broker is called the *broker's loan* or is referred to as a *debit balance* to indicate that the investor is indebted to the broker and owes him money. The difference between the market value and the debit balance is called the *owner's equity*. The owner's equity represents the amount that the investor will be entitled to if the securities are immediately liquidated and the broker is repaid what is due him. Example 3.11 illustrates these terms.

Example 3.11

A client wants to purchase 200 shares of GE stock, which is currently trading at $45 per share. The margin rate is 50 percent—that is, regulations stipulate that she should put up at least 50 percent of the cost from her own funds. We will assume that she invests the minimum amount required.

The market value of 200 shares is $9,000. The margin rate is 50 percent, which implies that a minimum of $4,500 has to be deposited by the investor. The maximum amount that she can borrow from the broker is $4,500. The loan value of the position is $4,500. The loan rate which is the loan value expressed as a fraction of the market value is 50 percent. The debit balance, which represents the amount borrowed from the broker, is $4,500.

The position may be depicted in a T-account format as shown in Figure 3.1.

Liabilities	Assets
Broker's Loan $4,500	200 Shares @ $45 = $9,000
Owner's Equity $4,500	

Figure 3.1 Initial Equity Position

Let us assume that the share price increases to $50. The market value of the securities will be $10,000. The debit balance will continue to remain at $4,500 because nothing further has been borrowed. If the position is immediately liquidated, the investor will be eligible to withdraw $5,500. The owner's equity is $5,500. As per the requirement, the owner's equity must be at least 50 percent of the market value, which in this case is $5,000. In this case, we say that the investor has excess equity of $500. There is another way to perceive the excess equity. The loan value of the position is 50 percent of 10,000, which amounts to $5,000. However, the debit balance is only $4,500. The difference between the loan value and the debit balance will also be equal to the excess.

What can the investor do with the excess? She can either withdraw it as cash or use it to acquire more securities. If the client were to withdraw the excess, the funds will be provided by the broker. Once the broker has done so, the debit balance will increase by that amount and the owner's equity will decline by an equivalent amount. In this example, the owner's equity will decline to $5,000 and the debit balance will increase to $5,000. This is consistent with the stipulation that a maximum of 50 percent of the market value of the securities can be borrowed. The situation is depicted in Figure 3.2.

However, what if the investor were to desire to use her excess to acquire more securities? The excess equity of $500 can support an acquisition of $1,000 worth of shares, because the requirement is that at least 50 percent of the value of what is acquired must be by way of the owner's contribution. In this case, the broker will advance $1,000 to enable her to purchase the securities. The market value of the securities will be $11,000. The owner's equity will remain at $5,500, and the debit balance will increase to $5,500. The requirement that not more than 50 percent of the value of the securities can be borrowed has been complied with. Figure 3.3 illustrates this situation.

Liabilities	Assets
Broker's Loan $5,000	200 Shares @ $50 = $10,000
Owner's Equity $5,000	

Figure 3.2 Position If Cash Is Withdrawn

Liabilities	Assets
Broker's Loan $5,500	220 Shares @ $50 = $11,000
Owner's Equity $5,500	

Figure 3.3 Position If the Excess Equity Is Used to Acquire Shares

In this case an excess equity of $500 represents the ability to buy additional securities worth $1,000. This is referred to as the *buying power* of the excess equity. The buying power of a position may be defined as

$$\text{Buying power} = \text{excess equity}/\text{margin rate}.$$

Let us analyze as to how margin trading magnifies the return of investment for a trader (see Example 3.12).

Example 3.12 (Magnifying Returns)

Consider the case of an investor named Martin, who has $5,000 at his disposal and wants to acquire shares of GE, which are currently available at a price of $50 each. We will assume that the margin rate is 50 percent.

If Martin were to purely trade on a cash basis—that is, if he were to acquire securities only up to the limit permitted by his own funds—he could acquire 100 shares of stock. However, if he were to trade on margin, he could borrow up to a maximum of $5,000, because the margin rate is given to be 50 percent, and can purchase 200 shares.[7]

What is the advantage of margin trading? Just as the assumption of debt enables the equity holders of a firm to obtain leverage, margin trading allows the investor to obtain *leverage*—that is, by using borrowed capital to partly fund the transaction, investors can magnify the returns on investment. However, as we have seen earlier, leverage is a double-edged sword. This can be illustrated by continuing with our example on Martin.

We will first analyze a pure cash trade with the help of different scenarios for the price at the end of a week, which we will assume is Martin's investment horizon.

Case A: The Market Rises

The market price of GE at the time of disposal is \$62. The 100 shares that Martin owns will therefore be worth \$6,200. The return on investment is

$$\frac{(6{,}200 - 5{,}000)}{5{,}000} \times 100 = 24\%.$$

Case B: The Market Declines

The market price of GE a week is \$42. The shares are worth only \$4,200, and the rate of return is

$$\frac{(4{,}200 - 5{,}000)}{5{,}000} \times 100 = -16\%.$$

Let us consider the situation in which Martin chooses to exert the full borrowing power of the capital at his disposal and indulge in margin trading. With \$10,000 in hand, he can acquire 200 shares of GE. Let us examine the consequences from the standpoint of the rate of return.

Case A: The Market Rises

Assume that the share price rises to \$62. The value of 200 shares will be \$12,400. After returning the amount borrowed from the broker, Martin will be left with \$7,400. The rate of return is

$$\frac{(7{,}400 - 5{,}000)}{5{,}000} \times 100 = 48\%.$$

Case B: The Market Declines

But what if the price was to decline to \$42 per share? The value of 200 shares will be \$8,400. After returning the amount borrowed, Martin will be left with \$3,400. The rate of return is

$$\frac{(3{,}400 - 5{,}000)}{5{,}000} \times 100 = -32\%.$$

Trading on the margin has doubled the rate of return. This is true irrespective of whether the trade has led to a profit or a loss.

Interest and Commissions

In the preceding illustration, we ignored two major real-life issues. First, the broker will charge interest on the amount borrowed by Martin. Second, investors will have to pay commissions both when the shares are acquired and when they are sold. When these facets are factored in, the result will be a reduction in the percentage of profit in the event of a rise in the market price, and a magnification of the percentage loss in the event of an adverse price change.

Margin interest income is a significant source of revenue for brokers/dealers. The normal practice is for the broker to obtain a loan from a bank, by pledging the securities as collateral. The rate charged by commercial banks for such loans is called the *broker call money rate*, which is usually about 1 percent less than the *prime rate*, which is the rate at which a commercial bank lends to its best clients. The reason why the loan rates are lower for such transactions is that the borrower, in this case the broker, is providing very liquid or marketable securities. The broker will usually add a mark-up to the call money rate, before extending a loan to a client. The lowest rates ordinarily go to clients who have the largest debit balances, or in other words, bigger borrowers pay less. The level of trading undertaken by the borrower is also a factor while determining the rate of interest charged by the broker. A customer with a very active account may be able to negotiate a lower rate of interest, because the additional income generated by way of commissions will more than compensate the broker for the concession made by way of a lower rate of interest.

Let us assume that the broker charges Martin an interest of 10 percent per annum on the amount borrowed. We will reexamine the consequences of a price increase and a price decline, assuming that the account is kept open for a period of three months.

Case A: The Market Rises

Once again consider a situation in which the terminal stock price is \$62. The 200 shares bought by Martin are worth \$12,400. Because he has borrowed \$5,000, he needs to repay \$5,125, which represents the repayment of principal with interest for three months. The rate of return is

$$\frac{(12{,}400 - 5{,}125 - 5{,}000)}{5{,}000} \times 100 = 45.50\%.$$

The assumption of an interest payment, which is consistent with a real-life transaction, leads to a reduction of 2.50 percent in returns.

Case B: The Market Declines

Consider a situation in which the terminal stock price is $42. After repaying the lender, the customer will be left with $3,275. The rate of return is

$$\frac{(8{,}400 - 5{,}125 - 5{,}000)}{5{,}000} \times 100 = -34.50\%.$$

Compared to the earlier case, our loss has been increased by 2.50 percent. Interest charges mitigate profits and magnify losses.

Commissions also have a similar impact on the rate of return obtained by the investor—that is, they mitigate profits and magnify losses.

Maintenance Margin

As we have seen, if the stock price were to rise, it will lead to an increase in the owner's equity. The investor can either withdraw the excess equity as cash or use it to acquire more shares. However, it is not necessary that the stock price should always rise. The price can always decline, and we need to consider the consequences of the same. Losses will lead to a reduction in the owner's equity, and sustained losses can lead to a significant reduction in the equity level. To protect himself, a broker will fix a threshold level called the maintenance margin level. If the account were to become undermargined because of adverse market movements, the investor may eventually receive a notice—a *margin call*—for additional margin from the broker. If the investor were to fail to respond to such a call, which in essence is a demand for additional collateral, the broker is at liberty to liquidate all or some of the securities in the account.[8] The objective of prescribing a maintenance margin level is to protect the broker's investment.

Let us take the case of an investor who acquires 200 shares of GE at $75 each by borrowing $7,500. Assume that the broker has fixed a maintenance margin level of 25 percent. The issue is, how low must the price of the stock fall before a margin call is triggered? Let us denote this price by P. At this price, the owner's equity must be exactly 25 percent. So, it must be that

$$(200P - 7{,}500)/200P = 0.25$$
$$\Rightarrow 200\text{P} = 50P + 7{,}500$$
$$\Rightarrow p = 7{,}500/150 = \$50.$$

At this point, the T-account would look as shown in Figure 3.4.

Liabilities	Assets
Broker's Loan \$7,500	200 Shares @ \$50 = \$10,000
Owner's Equity \$2,500	

Figure 3.4 Equity at the Maintenance Margin Level

If the stock price were to decline below \$50, a call would be issued demanding an immediate deposit of new funds or securities in order to raise the equity to a level higher than 25 percent. A margin call must be met on demand.

Regulation T and NYSE and NASD Rules

Margin trading is regulated by Regulation T, popularly known as *Reg T*. Per regulation, an investor must deposit a minimum of 50 percent of the purchase price. The margin rate is 50 percent.[9] The margin requirement may be met by depositing cash or fully paid-up securities. This requirement of 50 percent is referred to as a *Fed requirement*, a *Reg T requirement*, or an *initial requirement*.

As per Reg T, a margin call will be issued when an investor first acquires securities on the margin, and she must deposit a minimum of 50 percent of the value of the assets in response. Subsequent declines in the market may cause the equity to dip below the maintenance level fixed by the broker. As a consequence, a margin call may be issued that demands additional collateral. Such additional calls for collateral are based on the requirements of self-regulatory organizations such as the NYSE and the National Association of Securities Dealers (NASD), but they are not mandated by Reg T. To reiterate, federal regulations mandate only the initial margin call made at the outset, which is termed a *Fed call*. It must also be mentioned that it is a common industry practice to impose a higher maintenance margin requirement than what has been specified by NYSE or NASD.

Short Selling

Investors who anticipate a bull phase—that is, expect the stock market to rise—will seek to acquire stocks in anticipation of being able to sell them subsequently at a higher price. The principle being followed is therefore *buy low and sell high*. Some investors may decide to take additional risk by using leverage, which amounts to margin trading.

Short sellers have an opposite viewpoint regarding the market. They are bearish and anticipate a decline in the market. Their objective therefore is to *sell high and buy low*. Short selling requires such investors to sell stocks that they do not own. You may wonder how an investor can sell something that he does not possess? The answer is that he can always borrow shares from a third party and sell them. Short sellers borrow securities from brokers. Short selling is also a form of margin trading. The difference is that margin trading entails the borrowing of cash whereas short selling requires the borrowing of securities.

A broker obtains the right to borrow securities from a client by appropriately wording the margin agreement. Such a clause is known as a *customer loan consent*. Before executing a short-sale transaction, the broker has to seek and receive an authorization from his stock loan department stating that the security is indeed available for the purpose of the short sale. Security availability can change on a day-to-day basis. Short-sale orders are accepted only as *day orders*—that is, they are valid only for the day on which they are placed. If such an order were to remain unexecuted at the end of the trading day on which it is placed, it would be canceled automatically. A client who seeks to keep the order active on the following day would have to rebook the order.

When an investor borrows and sells a share, the proceeds will be credited to the investor's account. At some point in time, the shares will have to be purchased and returned to the broker. This is called *covering the short position*. If the price at this point of time is lower than the original purchase price, then the investor stands to make a gain. Otherwise, if the price has risen, he will have to countenance a loss.

In principle, traders have the freedom to keep the position open for as long as they desire. However, at times the lenders of the shares may seek to call away the stock they have lent. If such a situation were to arise, the broker would try to borrow the securities from a fresh source. However, if he were to be unable to do so, he would be left with no option but to request the investor to close out or cover his short position.

Let us analyze the transaction from the standpoint of the trader who lends the share. The investor or broker who parts with his shares to facilitate the sale by the short seller does not actually sell the shares. He merely lends to another party to facilitate a sale. Take a situation in which a cash dividend is declared on the underlying stock during the period of the short sale. The firm will pay the dividend to the party who has acquired the shares from the short seller. However, the lender of the share will demand that he be paid an equivalent amount because he has merely lent the shares, and should not lose the entitlement to the payout. As a result, the short seller has to compensate the lender for such lost income from his personal funds. Similarly, if the stock were to undergo an *n*:1 split during the period of the short sale, the lender would be entitled to receive *n* shares when the position is closed, per share that was lent at the outset. There is one right, however, that is irretrievable from the standpoint of the lender. If there is a meeting of shareholders during the period of the short sale, the lender loses his right to vote: this is one entitlement that the borrower of securities cannot compensate. It must be remembered that a short sale creates two long positions, the "real" long position held by the buyer who takes delivery of the stock from the short seller and a "phantom" long position held by the person who has lent the stock.

As we explained earlier, short selling is also a form of margin trading, because the investor is borrowing shares from the broker. Rules similar to margin trading apply to short sales, and such transactions also are governed by Regulation T.

A trader who engages in margin trading must deposit at least 50 percent of the cost of the shares from his pocket. This margin is required because the market may subsequently decline and the broker must have adequate assets to recover his loan. In the case of a short sale, the investor borrows a share and sells it. She is required to have enough funds to reacquire the stock subsequently in order to cover the position. The account must at all times have adequate funds to facilitate the reacquisition of the shares. Because the price may rise after the short sale, an investor who is selling short must deposit additional funds over and above what she receives in the form of the sale proceeds. As per Reg T, a short seller must make the same deposit that would be required if the stock were to be purchased on margin: he must offer at least 50 percent of the sale proceeds as additional collateral. This deposit, plus the proceeds of the short sale, would show up as a credit balance in his account. Example 3.13 illustrates this.

Example 3.13

Nick decides to sell short 100 shares of GE, which is currently trading at $50 per share. He is anticipating a falling market. He will receive $5,000 when he sells the shares. He will have to deposit this amount, plus an additional 50 percent—in this case, $2,500—with the broker. His account position is represented as shown in Figure 3.5.

Liabilities	Assets
100 Shares @ \$50 = \$5,000	Credit Balance \$7,500
Owner's Equity \$2,500	

Figure 3.5 Initial Equity Position After the Short Sale

Assume that the value of the shares drops to \$40. The credit balance will remain at \$7,500, whereas the value of the shares will be only \$4,000. The position will have an equity of \$3,500. As per the requirement, the necessary equity is \$2,000. There is an excess equity of \$1,500. The excess equity can either be withdrawn in the form of cash or used for additional shorting. If the balance is withdrawn, the account position will look as shown in Figure 3.6.

Liabilities	Assets
100 Shares @ \$40 = \$4,000	Credit Balance \$6,000
Owner's Equity \$2,000	

Figure 3.6 Equity Position If Excess Equity Is Withdrawn

The other possibility is that the excess equity of \$1,500 can be used to short additional shares. Because the margin requirement is 50 percent, the shorting power of the excess is \$3,000. In this case 75 more shares can be shorted. The position will then look as shown in Figure 3.7.

The credit balance in a short account provides collateral for the shares that have been sold short. They are not *free credit balances* in the sense that they cannot be removed from the account. So typically they will not earn any interest for the short seller. Any interest earned will accrue to the brokerage firm. Stock loan departments of brokerage firms are usually very profitable. However, institutional clients may demand that the brokerage firm share the interest that it will earn on the credit balance. Such a payment by a brokerage firm is referred to as a *short interest rebate*.

Liabilities	Assets
175 Shares @ \$40 = \$7,000	Credit Balance \$10,500
Owner's Equity \$3,500	

Figure 3.7 Equity Position If Additional Shares Are Shorted

Maintenance of a Short Position

A short position is inherently more risky than a long position. This is because the worst thing that can happen when one buys a security is that the securities can become totally worthless. Because of the limited liability feature, the price of a share has a lower bound of zero.

In the case of a short sale, however, what has been loaned is a stock. Although stocks have a lower bound, they do not have an upper bound. In other words, there is no limit on how high the share price can rise. Because the short seller is required to acquire the stock at the prevailing market value, in principle, at the time of covering the short sale the maximum possible loss is infinite.

Brokers take cognizance of this higher risk aspect of short sales by setting the maintenance margin requirement for such transactions at higher levels.

Let us assume that the maintenance level is 30 percent. How do we compute the corresponding trigger for a margin call? Assume that Nick has sold 100 shares of GE at \$65 and deposited an initial margin of \$3,250. Let the price corresponding to a 30 percent margin level be denoted by P. Therefore,

$$(9{,}750 - 100P)/100P = .30$$
$$\Rightarrow 9{,}750 - 100P = 30P$$
$$\Rightarrow P = 975/13 = 75.00.$$

Shorting Against the Box

This entails the establishment of a short position in a security that the investor already owns. In other words, the investor has no intention of delivering the stock that she is holding and instead seeks to borrow the security to facilitate delivery. Why would a trader resort to such a course of action? Typically, traders go in for such a strategy to defer any unrealized capital gains for tax purposes, while at the same continuing to protect the magnitude of the capital gain. Example 3.14 illustrates this.

Example 3.14

Let us assume that we are on December 15, 20XX, where 20XX denotes a year in the twenty-first century. In March of the same year we had acquired a stock for $800, and it is currently worth $1,200. If we were to sell it immediately, we would have a capital gain of $400. If we assume that the tax rate is 15 percent, we would have a post-tax profit of $340. Consider a situation in which we are anticipating a capital loss after a few months from another source, which in principle can be set off against this capital gain to reduce or even avoid our tax liability. If so, we may like to defer the sale. But this entails a risk because, if we postpone the sale, we may end up with a reduced capital gain or even a capital loss, depending on how the share price moves in the interim. In such a situation, we can defer the capital gain while continuing to protect its quantum by engaging in a short sale.

For if we were to engage in a short sale, any change in the value of the existing long position would be exactly offset by an equal and opposite change in the value of the newly established short position. The net result would be a freeze on the capital gain of $400. If the share price were to rise to $1,500 after six months, then the capital gain on the long position would increase to $700. However, the short position would lead to a loss of $300. The net result would be an overall capital gain of $400.

The Risk Factor

In general, over a medium- to long-term horizon, stock prices have a tendency to drift upward because, even if the performance of the company were to remain stagnant, its share price should rise over a period of time to compensate owners for the effects of inflation. The long-run direction of the market is upward. Short selling represents an effort to profit from a falling market. It amounts to betting against the overall direction of the market.

A long position resulting from the limited liability feature entails finite losses and potentially infinite profits. Short sales, however, entail finite profits and infinite losses. Although profits are capped at 100 percent, potential losses cannot be capped.

In a rising market, a situation could arise in which a large number of short sellers attempt to close out their positions after witnessing a rapid price increase. This can cause prices to quickly spiral upward. This phenomenon is known as a *short squeeze*. Usually, the root cause of such an event is the availability of news relevant to the security. But, at times, a trader who notices a high short interest will attempt to induce a squeeze by placing a large buy order. If the corresponding price rise were to lead to the covering of positions by short sellers, the trader who placed the buy order may be able to exit the market with a handsome profit.

The Economic Role of Short Sales

Many market analysts have held short sales as a major cause of market downturns. However, short selling undoubtedly contributes positively to the functioning of a free-market system. It provides liquidity and drives down the prices of overvalued securities to realistic levels. If a security is perceived to be undervalued, then traders will assume long positions to take advantage of the anticipated price rise. On the other hand, if a security were to be perceived to be overvalued, then the freedom to short sell is imperative to ensure that prices can be driven down to a fair or true value.

The Uptick Rule

A *tick* is the minimum price increment or variation observable in the market. In other words, it is the smallest amount by which two prices can differ. It is usually set by exchange regulations. In the United States, until the year 2000, the tick size was one-sixteenth of a dollar, or 6.25 cents. Now the system has been decimalized, and the tick size is 0.01 dollar, or 1 cent.

The minimum tick size on the Tokyo Stock Exchange is a function of the share price; it is depicted in Table 3.21.

Traders classify prices by their relation to previous prices. The price is said to be an uptick if the current price is higher than the last price, on a downtick if it is lower, and on a zero tick if it is the same. Zero tick

Table 3.21
Tick Sizes on the TSE

Price	Tick Size
$P \leq 2000$ yen	1 yen
$2000 < P \leq 3000$ yen	5 yen
$3000 < P \leq 30{,}000$ yen	10 yen
$30{,}000 < P \leq 50{,}000$ yen	50 yen
$50{,}000 < P \leq 100{,}000$ yen	100 yen
$100{,}000 < P \leq 1{,}000{,}000$ yen	1,000 yen
$P > 1{,}000{,}000$ yen	10,000 yen

Table 3.22
Illustration of Uptick, Downtick, and Zero Tick

Previous to Last Price	Last Price	Current Price	Term
72	72	72.10	Uptick
72	72	71.90	Downtick
72.10	72	72	Zero Downtick
71.90	72	72	Zero Uptick

prices are further classified by the last different price observed. A zero tick price is said to be on a zero downtick if the last different price observed was higher and on a zero uptick if the last different price observed was lower. We will illustrate these concepts using an example as shown in Table 3.22.

On U.S. exchanges, short sales are permitted only on an uptick or a zero uptick. This is because sustained short selling in a declining price environment can cause the market to crash.

Endnotes

1. See Teweles and Bradley (1998).
2. They stand to gain by way of a saving in brokerage commissions.
3. In this case, we are talking about shares being issued for a monetary consideration.
4. See Simmons and Dalgleish (2006).

5. Although this is true in general, there could be periods where the firm decides not to pay a dividend.
6. Remember that we would have 4,000 shares of 3M after the split.
7. Because of the 50 percent margin requirement, Martin can borrow an amount up to the funds at his disposal.
8. Remember that the securities have been pledged as collateral by the investor.
9. Deposits over and above the prescribed minimum are permissible.

CHAPTER 4

Bonds

Bonds and debentures, which we have already introduced, are referred to as *fixed-income securities*. The reason is that once the rate of interest is set at the onset of the period for which it is due to be paid, it is not a function of the profitability of the firm. It is for this reason that all bonds—including floating-rate bonds, which entail the resetting of interest at the commencement of each interest-payment period—are referred to as *fixed-income instruments*. Interest payments are therefore contractual obligations, and failure to pay what was promised at the start of an interest-computation period will amount to default.

A bondholder is a stakeholder in a business but is not a part owner of the business. She is entitled to only the interest that was promised and to the repayment of principal at the time of maturity of the security and does not partake in the profits of the firm. As we discussed earlier, bonds may be unsecured or secured. Unsecured debt is referred to as a *debenture* in the United States, and the term connotes that no specific assets have been earmarked as collateral for the securities. In the case of secured debt, however, the issuer sets aside specific assets as collateral on which investors have a claim in the event of default. Debt securities may be negotiable or nonnegotiable. A negotiable security is one that can be traded in the secondary market, whereas a nonnegotiable security cannot be endorsed by the holder in favor of another investor. Bank accounts are classic examples of nonnegotiable securities, because if an investor were to open a term deposit with a commercial bank, although she can always withdraw the investment and pay a third party, she cannot transfer the ownership of the deposit.

The most basic form of a debt security is referred to as a *plain vanilla bond*. It is an IOU that promises to pay a fixed rate of interest every period, which is usually every six months in the United States, and to repay the principal at maturity. Floating-rate bonds are similar except that the interest rate does not remain constant from period to period but fluctuates with changes in the benchmark to which it is linked. There are also bonds with embedded options. *Convertible bonds* can be converted to shares of stock by the investor. *Callable bonds* can be prematurely retired by the issuing company, whereas *putable bonds* can be prematurely surrendered by the bondholders in return for the repayment of the principal. These bonds are discussed in greater detail later in the chapter.

Bonds provide equity shareholders with leverage. Here is a detailed illustration. Company ALPHA is entirely equity financed and has issued shares worth $1,000,000. Company BETA has raised the same amount of capital, half in the form of debt and the other half in the form of equity. The debt carries interest at the rate of 6 percent per annum.

Let us consider two situations. In the first, both companies make a profit of $250,000 from operations; in the second. they both make a loss of $250,000. To keep matters simple, we will assume that the firm does not have to pay tax.

From Table 4.1, the presence of debt in the capital structure creates leverage. The profit for the shareholders is magnified from 25 percent to 44 percent, when a firm is financed 50 percent with debt. On the other hand, the loss if incurred is also magnified, in this case from −25 percent to −56 percent. As we saw in the case of margin trading, leverage is a double-edged sword. It can also be seen from the case of BETA that the incurrence of a loss does not give the flexibility to the firm to avoid or postpone the interest due to bondholders. Interest on bonds is indeed a contractual obligation.

Bonds also provide the issuing firm with a tax shield because, although interest on debt is a deductible expense for figuring the tax liability of a company, dividends on equity shares are not. Interest payments reduce the tax liability for the firm or, in other words, give it a

Table 4.1
Illustration of Leverage

	Case A: Firms Make a Profit		Case B: Firms Make a Loss	
	ALPHA	BETA	ALPHA	BETA
Profits from operations	250,000	250,000	(250,000)	(250,000)
Less interest	0	(30,000)	0	(30,000)
PAT	250,000	220,000	(250,000)	(280,000)
ROI (%)	25	44	−25	−56

PAT stands for *profit after tax* and represents what the shareholders are entitled to. ROI is the *return on investment* for shareholders.

Table 4.2
Illustration of a Tax Shield

	ALPHA	BETA
Profit from operations	250,000	250,000
Interest	NIL	30,000
PBT	250,000	220,000
Tax	(75,000)	(66,000)
PAT	175,000	154,000

PBT stands for *profits before tax*; PAT stands for *profits after tax*.

tax shield. Table 4.2 illustrates this using the same data as in Table 4.1. Both firms are assumed to have a profit of $250,000, and the applicable tax rate is assumed to be 30 percent.

Let us analyze the last row of Table 4.2. If company BETA had been a zero-debt company like ALPHA, its shareholders would have been entitled to an income of $175,000. However, because it has paid interest, the shareholders have received only $154,000, which is $21,000 less. From the standpoint of shareholders of BETA, they have effectively paid an interest of $21,000. So what explains the missing $9,000? After all, we know that BETA did pay $30,000 to its bondholders. The answer is that, by allowing the firm to deduct the interest paid as an expense before the computation of tax, the Internal Revenue Service (IRS) has forgone taxes to the extent of $9,000. The IRS has effectively provided a subsidy to the company. The *tax shield*, as we term it, is equal to the product of the tax rate and the interest paid. In this case, it is $0.30 \times 30{,}000 = \$9{,}000$.

If the rules were to be amended and interest on debt were no longer to be tax deductible, BETA would have to pay tax on $250,000, and the *profit after tax* (PAT), which is what belongs to the shareholders, would be only $145,000. In this situation, the shareholders will feel the full burden of the interest paid by the firm.

In many countries, interest received from bonds is taxable at the hands of the receiver, whereas dividends received from shares are not. The reason is that because the interest is being paid out of pretax profits, it is not being taxed at the level of the firm. However, dividends on equity are not a tax-deductible expense for the firm; to prevent it from being taxed twice, it is not taxed at the hands of the shareholders.

Terms Used in the Bond Market

- **Face value:** Face value is also known as *par value*, *redemption value*, and *maturity value*.
- **Principal value:** Principal value is the principal amount underlying a bond. It was the amount raised by the issuer from the first holder,

and it is the amount repayable by the issuer to the last holder. We will denote it by the symbol M.

- **Term to maturity:** Term to maturity is the time remaining in the life of a bond as measured at the point of evaluation. It may be perceived as the length of time after which the debt shall cease to exist and the point at which the borrower will redeem the issue by repaying the holder. Equivalently, it may be perceived as the length of time for which the borrower has to service the debt in the form of periodic interest payments. The words *maturity*, *term*, and *term to maturity* are used interchangeably. We will assume that we are stationed at time zero and will denote the point of maturity by T. The number of periods until maturity is T, which is normally measured in years.
- **Coupon:** The contractual interest payment made by the issuer is called a *coupon payment*. The name came about because in earlier days bonds were issued with a booklet of postdated coupons. On an interest-payment date, the holder was expected to detach the relevant coupon and claim his payment.

  The coupon may be denoted as a rate or as a dollar value. We will denote the coupon rate by c. The dollar value, C, is therefore given by $c \times M$. Most bonds pay interest on a semiannual basis, and the semiannual cash flow is $c \times M/2$.

  Consider a bond with a face value of \$1,000 that pays a coupon of 8 percent per annum on a semiannual basis. The annual coupon rate is 0.08. The semiannual coupon payment is $0.08 \times 1{,}000/2 = \$40$.
- **Yield to maturity:** Like the coupon rate, the *yield to maturity* (YTM) is also an interest rate. The difference is that although the coupon rate is the rate of interest paid by the issuer, the YTM is the rate of return required by the market. At a given point in time, the yield may be greater than, equal to, or less than the coupon rate. The YTM will be denoted by y, and it is the rate of return that a buyer will get if he were to acquire the bond at the prevailing price and hold it to maturity. We will shortly see that the YTM of a bond is equivalent to the concept of the *internal rate of return* (IRR) used in capital budgeting. As in the case of the IRR, the YTM computation assumes that all intermediate cash flows are reinvested in the YTM itself.

Valuation of a Bond

To value a bond, we will first assume that we are standing on a coupon payment date; that is, we will assume that a coupon has just been received and that the next coupon is exactly six months, or one period, away. If T is the term to maturity, we have N coupons remaining where $N = 2T$.

We will receive N payments during the life of the bond, where each payment or cash flow is equal to $C/2$. This payment stream constitutes an annuity. The present value of this annuity is

$$\frac{\frac{C}{2}}{\frac{y}{2}} \times \left[1 - \frac{1}{\left(1+\frac{y}{2}\right)^N}\right].$$

The terminal payment of the face value is a lump-sum payment. Because we are discounting the cash flows from the annuity on a semi-annual basis, this payment also needs to be discounted on a similar basis. The present value of this cash flow is

$$\frac{M}{\left(1+\frac{y}{2}\right)^N}.$$

The price of the bond is therefore given by

$$P = \frac{\frac{C}{2}}{\frac{y}{2}} \times \left[1 - \frac{1}{\left(1+\frac{y}{2}\right)^N}\right] + \frac{M}{\left(1+\frac{y}{2}\right)^N}.$$

As can be seen, the YTM is the discount rate that makes the present value of the cash flows from the bond equal to its price. The price of a bond is like the initial investment in a project. The remaining cash flows are similar to the inflows from the project. The YTM is exactly analogous to the IRR.

Example 4.1 demonstrates the valuation of a bond given the other variables.

Example 4.1

Infosys Technologies has issued bonds with 10 years to maturity and a face value of \$1,000. The coupon rate is 8 percent per annum payable on a semiannual basis. If the required yield in the market is equal to 9 percent, what should be the price of the bond?

The periodic cash flow is $0.08 \times 1{,}000/2 = \$40$. The present value of all the coupons is

$$\frac{\frac{80}{2}}{\frac{0.09}{2}} \times \left[1 - \frac{1}{\left(1 + \frac{0.09}{2}\right)^{20}}\right] = \$520.3175.$$

The present value of the face value is

$$\frac{1{,}000}{\left(1 + \frac{0.09}{2}\right)^{20}} = \$414.6429.$$

Therefore, the price of the bond is 520.3175 + 414.6429 = \$934.9604.

Par, Premium, and Discount Bonds

In the preceding illustration, the price of the bond is less than its face value. Such bonds are said to be "trading at a discount to the par value" and are therefore referred to as *discount* bonds. If the price of the bond were to be greater than its face value, then it would be said to be "trading at a premium to its face value" and would be referred to as a *premium* bond. If the price is equal to the face value, then the bond is said to be trading at *par*.

The relationship between the price and the face value depends on whether the YTM is greater or less than the coupon. Consider the pricing equation

$$\begin{aligned}
p &= \frac{\frac{C}{2}}{\frac{y}{2}} \times \left[1 - \frac{1}{\left(1 + \frac{y}{2}\right)^{N}}\right] + \frac{M}{\left(1 + \frac{y}{2}\right)^{N}} \\
&= \frac{\frac{cM}{2}}{\frac{y}{2}} \times \left[1 - \frac{1}{\left(1 + \frac{y}{2}\right)^{N}}\right] + \frac{M}{\left(1 + \frac{y}{2}\right)^{N}} \\
&= \frac{cM}{y} + \frac{M}{\left(1 + \frac{y}{2}\right)^{N}} \times \left[1 - \frac{c}{y}\right].
\end{aligned}$$

If $c = y$, $P = M$. In other words, if the coupon is equal to the YTM, the bond will always trade at par.

Let us differentiate the price with respect to the coupon.

$$\frac{\partial P}{\partial c} = \frac{M}{y} - \frac{M}{y} \times \frac{1}{\left(1 + \frac{y}{2}\right)^N}$$

$$= \frac{M}{y}\left[1 - \frac{1}{\left(1 + \frac{y}{2}\right)^N}\right].$$

Price is a monotonically increasing function of the coupon. If the coupon is equal to the yield, then the price is equal to the face value. If the coupon is less than the yield, then the price will be less than the face value; if the coupon is greater than the yield, then the price will exceed the par value.[1]

Why would a bond sell at a premium or at a discount? The price of a bond is the present value of all the cash flows emanating from it. If the yield is equal to the coupon, then the rate of return being demanded by investors is exactly equal to the rate of return being offered by the issuer, and the bond will sell at par.

Suppose the yield is less than the coupon. Assume that the coupon is 10 percent per annum while the yield is 8 percent per annum. An investor will be willing to pay more than its face value for it. The price would be bid up to a level at which the YTM is exactly equal to 8 percent per annum. Finally, consider a case in which the YTM is greater than the coupon. Assume that the coupon is 8 percent per annum while the yield is 10 percent per annum. Investors in these circumstances will be willing to pay only a price that is less than the face value. The price would be driven down to a level at which the YTM is exactly equal to 10 percent.

Evolution of the Price

Let us consider the change in the price of a bond from one coupon date to another, assuming that the YTM remains constant.

The price of a bond with N coupons remaining is

$$P = \frac{\frac{C}{2}}{\frac{y}{2}} \times \left[1 - \frac{1}{\left(1 + \frac{y}{2}\right)^{N}}\right] + \frac{M}{\left(1 + \frac{y}{2}\right)^{N}}.$$

The price when there are $N - 1$ coupons left is given by

$$P = \frac{\frac{C}{2}}{\frac{y}{2}} \times \left[1 - \frac{1}{\left(1 + \frac{y}{2}\right)^{(N-1)}}\right] + \frac{M}{\left(1 + \frac{y}{2}\right)^{(N-1)}}.$$

The price change between successive coupon periods is given by

$$\begin{aligned}\Delta P &= \frac{\frac{C}{2}}{\frac{y}{2}} \times \left[\frac{1}{\left(1 + \frac{y}{2}\right)^{N}} - \frac{1}{\left(1 + \frac{y}{2}\right)^{(N-1)}}\right] \\ &\quad + M \times \left[\frac{1}{\left(1 + \frac{y}{2}\right)^{(N-1)}} - \frac{1}{\left(1 + \frac{y}{2}\right)^{N}}\right] \\ &= \frac{M}{\left(1 + \frac{y}{2}\right)^{N}} \times \left[\frac{y}{2} - \frac{c}{2}\right].\end{aligned}$$

If $y = c$, then $\Delta P = 0$. If the YTM were to remain constant, the price of a par bond would continue to remain at par, as we move from one coupon date to the next. If the yield is greater than the coupon, then $\Delta P > 0$. A discount bond will steadily increase in price as we go from one coupon date to the next. In the case of a premium bond, the price will steadily decline as we go from one coupon date to the next.

Example 4.2 illustrates the change in the price from one coupon date to the next, keeping the yield constant, for both premium as well as discount bonds.

Example 4.2

Consider a bond with 10 years to maturity and a face value of \$1,000. Assume that the YTM is 8 percent per annum and the coupon is 6 percent per annum. The bond will sell at a discount. The price can be calculated to be \$864.10. Let us move one period ahead to a time when only nine and a half years are left. The price can be calculated to be \$868.66. As expected, the price has increased.

Consider the same bond but assume that the coupon is 10 percent per annum. The bond will sell at a premium. The price when there are 20 coupons left is \$1,135.90. Six months later, if the yield were to remain constant, the price would be \$1,131.34. As expected, the price has declined.

Zero-Coupon Bonds

Unlike a plain vanilla bond that pays coupons at periodic intervals, *zero-coupon bonds*, also known as *deep discount bonds*, do not pay any interest. Such instruments are always traded at a discount to the face value, and the holder at maturity will receive the face value. Consider a zero-coupon bond with a face value of \$1,000 and 10 years to maturity. Assume that the required yield is 8 percent per annum. The price may be computed as follows:

$$\frac{1{,}000}{(1.04)^{20}} = \$456.39.$$

Notice that we have chosen to discount at a rate of 4 percent for 20 half-yearly periods, and not at 8 percent for 10 annual periods. The reason is that a potential investor will have a choice between plain vanilla bonds and zero-coupon bonds. To draw meaningful inferences, it is imperative that the discounting technique be common. Because the cash flows from plain vanilla bonds are usually discounted on a semiannual basis, we choose to do the same for zero-coupon bonds.

A zero-coupon bond will never sell at a premium. It will always trade at a discount except at the time of maturity when it will trade for the face value. That does not mean that a buyer of such a bond will always experience a capital gain. If she were to buy and hold it to maturity, then she will have a capital gain. But if she chooses to sell it before maturity, she may well end up with a capital loss as we shall demonstrate in Example 4.3.

Example 4.3

Alexis bought a zero-coupon bond when there were 10 years to maturity. The prevailing yield was 10 percent per annum. Today, a year later, the YTM is 12 percent per annum. The purchase price was $\dfrac{1{,}000}{(1.05)^{20}} = \$376.89$. The price at the time of sale can be shown to be \$350.34. In this case, the investor has a capital loss of \$26.55.

Valuing a Bond in between Coupon Dates

We have assumed that we are valuing the bond on a coupon date—that is, the next coupon is exactly one period away. Let us consider a more realistic situation in which the price of the bond is to be calculated between coupon dates.

Consider the timeline depicted in Figure 4.1.

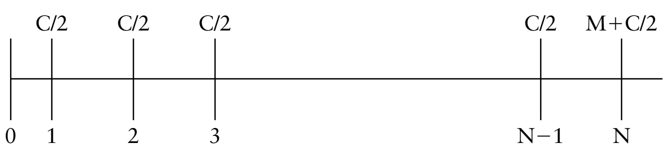


Figure 4.1 Cash Flows from a Plain Vanilla Bond

As you can see, the length of time between 0 and 1 is less than one period, whereas the other coupon dates are spaced exactly one period apart.

To value the bond at time 0, we will proceed in two steps. First, let us value the bond at time 1. At this point in time, we will get a cash flow of $C/2$. There are $N - 1$ coupons left after this, and the face value is scheduled to be received $N - 1$ periods later. The present value of the remaining cash flows at this point in time is

$$\frac{\frac{C}{2}}{\frac{y}{2}} \times \left[1 - \frac{1}{\left(1+\frac{y}{2}\right)^{(N-1)}}\right] + \frac{M}{\left(1+\frac{y}{2}\right)^{(N-1)}}.$$

The price of the bond at time '1' is

$$\begin{aligned} P_1 &= \frac{C}{2} + \frac{\frac{C}{2}}{\frac{y}{2}} \times \left[1 - \frac{1}{\left(1+\frac{y}{2}\right)^{(N-1)}}\right] + \frac{M}{\left(1+\frac{y}{2}\right)^{(N-1)}} \\ &= \frac{\frac{C}{2}}{\frac{y}{2}} \left[\frac{\left(1+\frac{y}{2}\right)^{N} - 1}{\left(1+\frac{y}{2}\right)^{N-1}}\right] + \frac{M}{\left(1+\frac{y}{2}\right)^{N-1}}. \end{aligned}$$

Let us denote the length of time between 0 and 1 by the symbol k where, $k < 1$. Discounting back P_1 for k periods, we get

$$P_o = \left[\frac{\frac{c \times M}{2}}{\left(1+\frac{y}{2}\right)^{k}} \times \frac{\left[\left(1+\frac{y}{2}\right)^{N} - 1\right]}{\left(\frac{y}{2}\right)\left(1+\frac{y}{2}\right)^{N-1}}\right] + \frac{M}{\left(1+\frac{y}{2}\right)^{N-1+k}}.$$

Day-Count Conventions

When valuing a bond between coupon dates the key issue is the calculation of the fractional first period. Unfortunately, there is no unique method for computing this period. Different markets, and at times different products in the same market, follow different conventions. A method of computation of the fractional period is called a *day-count convention*.

Actual–Actual

We will illustrate a convention known as the *actual–actual approach*. This is used for Treasury bonds in the United States. Let us go back and analyze the fractional period. To compute the fraction, we need to define the numerator and the denominator. The numerator is the number of days between the valuation date and the next coupon date. The denominator is the number of days between the previous coupon date and the next coupon date.

With the actual–actual method, we have to count the exact number of days in both the numerator and the denominator. We will illustrate it with the help of Example 4.4.

Example 4.4

Let us assume that we are standing on May 21, 20XX, and that we have bond maturing on May 15 (20XX + 20). The face value is $1,000, the coupon rate is 6 percent per annum payable semi-annually, and the YTM is 8 percent per annum. The coupon dates are May 15 and November 15.

The numerator in this case is the period between May 21 and November 15. The number of days is calculated monthwise as shown in Table 4.3. The principle is that we include either a starting date or an ending date, but not both.

Table 4.3
Calculation of the Numerator

Month	Number of Days
May	10
June	30
July	31
August	31
September	30
October	31
November	15
Total	178

The denominator is the period between May 15 and November 15. In this case, it is 184 days as shown in Table 4.4.

If we were to count the actual number of days between two dates, which are six months apart, we will always get a number between 181 and 184. There are only four possibilities: 181, 182, 183, and 184.

Table 4.4
Calculation of the Denominator

Month	Number of Days
May	16
June	30
July	31
August	31
September	30
October	31
November	15
Total	184

The actual–actual method is denoted in short form as Act/Act and is pronounced as "ack ack" by Wall Street traders. Let us use it to derive the price of the bond on May 21, 20XX. The number of coupons remaining as of this day is 40. The value of the fractional first period is 178/184 = 0.9674.

The price of the bond is therefore:

$$P_o = \left[\frac{\frac{0.06 \times 1{,}000}{2}}{\left(1 + \frac{0.08}{2}\right)^{0.9674}} \times \frac{\left[\left(1 + \frac{0.08}{2}\right)^{40} - 1\right]}{\left(\frac{0.08}{2}\right)\left(1 + \frac{0.08}{2}\right)^{39}} \right] + \frac{1{,}000}{\left(1 + \frac{0.08}{2}\right)^{40-1+0.9674}}$$

$$= \$803.0939.$$

The Treasury's Approach

The Treasury uses a slightly different method to compute the prices of its bonds, with the difference being that it uses simple interest for the fractional period. The Treasury's formula may be stated as

$$P_o = \left[\frac{\frac{c \times M}{2}}{\left(1 + k \times \frac{y}{2}\right)} \times \frac{\left[\left(1 + \frac{y}{2}\right)^{N} - 1\right]}{\left(\frac{y}{2}\right)\left(1 + \frac{y}{2}\right)^{N-1}} \right] + \frac{M}{\left(1 + k \times \frac{y}{2}\right)\left(1 + \frac{y}{2}\right)^{N-1}}.$$

In this case, the price obtained by the Treasury will be \$803.0738. The price that is computed by the Treasury for a given bond will always be lower than the value determined by Wall Street. This is because simple interest gives a higher discount rate for fractional periods as compared to compound interest.

Corporate Bonds

Corporate bonds in the United States are priced using a different day-count convention known as 30/360 PSA. In accordance with this convention, the denominator or the time period between two successive coupon dates is always taken to be 180 days—that is, every month is assumed to consist of 30 days. To compute the fractional period, the numerator is determined as follows.

Let us define the start date as $D_1 = (\text{month}_1, \text{day}_1, \text{year}_1)$ and the end date as $D_2 = (\text{month}_2, \text{day}_2, \text{year}_2)$. The numerator is calculated as

$$360(\text{year}_2 - \text{year}_1) + 30(\text{month}_2 - \text{month}_1) + (\text{day}_2 - \text{day}_1).$$

Some additional rules may have to be applied, depending on the circumstances.

1. If $\text{day}_1 = 31$, then set it equal to 30.
2. If day_1 is the last day of February, whether 28 or 29, then set it equal to 30.
3. If day_1 is 30 or it has been set equal to 30 based on the two rules listed above, then if $\text{day}_2 = 31$, set it equal to 30.

Let us take the case of the bond maturing on May 15 (20XX+20). The valuation date as per our assumption is May 21, 20XX. The numerator for the fractional period is given by

$$360(20XX - 20XX) + 30(11 - 5) + (15 - 21) = 174.$$

The value of the fractional period is $174/180 = 0.9667$.

The value of the bond is therefore

$$P_o = \left[\frac{\frac{0.06 \times 1{,}000}{2}}{\left(1 + \frac{0.08}{2}\right)^{0.9667}} \times \frac{\left[\left(1 + \frac{0.08}{2}\right)^{40} - 1\right]}{\left(\frac{0.08}{2}\right)\left(1 + \frac{0.08}{2}\right)^{39}} \right] + \frac{1{,}000}{\left(1 + \frac{0.08}{2}\right)^{40 - 1 + 0.9667}}$$
$$= \$803.1150.$$

The day-count convention does have an impact, albeit a minor one, on the price.

Accrued Interest

The value of the bond as calculated by us using the appropriate day-count convention is referred to as the *full* or the *dirty* price. This is the price that is payable by the buyer to the seller. The price includes a component called the *accrued interest*, which we will explain.

When a bond is sold between two coupon dates, the next coupon will go to the buyer of the bond. However, because the seller has held the security for a fraction of the coupon period, he is entitled to a part of the next coupon payment. This fraction of the next coupon that belongs to the seller is termed the *accrued interest*. The bond-pricing equation that we used earlier automatically factors in the accrued interest, because it discounts all the forthcoming cash flows, from the perspective of the buyer, until the date of sale. The partial derivative of the dirty price with respect to the length of the fractional period is negative. The shorter the time remaining until the next coupon, the higher the accrued interest, keeping all other variables constant, and hence the higher is the dirty price.

The method for computing the accrued interest is a function of the day-count function. Let us take the case of the Treasury bond. On May 21, 20XX, it has accrued interest for 6 days. The total length of the coupon period is 184 days.[2]

The accrued interest is therefore

$$\frac{0.06 \times 1{,}000}{2} \times \frac{6}{184} = \$0.9783.$$

The dirty price minus the accrued interest is referred to as the *clean price*. In this case, it is \$803.0939 − 0.9783 = \$802.1156.

The significance of the clean price may be demonstrated as follows. Consider the value of the bond on the previous coupon date—that is, May 15, 20XX—assuming that the YTM is 8 percent per annum. The price comes out to be \$802.0722.

On May 15, there is no difference between the clean price and the dirty price, because interest accrual for the next period is yet to commence. If the YTM is assumed to be constant, then the dirty price changes by \$0.9783, which amounts to almost a dollar, whereas the clean price changes only by \$0.0434. In the absence of a change in yield, the clean price remains relatively constant in the short run.

Look at the situation from the perspective of a bond-market analyst. She knows that yield changes induce price changes. However, if she were to analyze dirty prices, she would be unable to discern how much of the perceived change is the result of a movement in the required yield, and what is resulting from the change in accrued interest. Analysts look at prices that are not contaminated with accrued interest, or what we term *clean prices*. For this reason, quoted prices in bond markets are always clean prices. However, a trader who buys a bond has to pay the full price or the dirty price. When a bond is bought, the accrued interest has to be computed and added to the quoted price in order to determine the amount payable.

Negative Accrued Interest

One interesting question is, can the accrued interest be negative? Let us analyze the possible reason why the accrued interest may be negative. Accrued interest represents the amount that the seller of the bond is entitled to by virtue of the fact that he has held the bond for a part of the coupon period. The buyer has to compensate him with this amount. Negative accrued interest would correspond to a situation in which the seller has to compensate the buyer, and such a case can arise when the bond trades *ex-dividend*.

Certain bonds trade on an ex-dividend basis close to the coupon date. We know that bonds pay coupons and not dividends, and the term ex-dividend is a bit of a misnomer. However the implication is the same. Until the ex-dividend day, a bond will trade cum-dividend—that is, the buyer will be entitled to the next coupon. On the ex-dividend day, however, the bond will begin to trade ex-dividend, which implies that the next coupon will go to the seller and not the buyer. But from the perspective of a buyer who acquires the bond on or after the ex-dividend day, he is entitled to the pro rata interest for the number of days remaining until the next coupon date. In such a situation the seller has to share a part of the interest received with the buyer, which leads to a situation in which the accrued interest is negative.

Let us consider a corporate bond maturing on May 15 (20XX+20). Assume that we are on November 8, 20XX, which is an ex-dividend date. The dirty price just before the bond goes ex-dividend is

$$P_o = \left[\frac{\frac{0.06 \times 1{,}000}{2}}{\left(1 + \frac{0.08}{2}\right)^{0.0380}} \times \frac{\left[\left(1 + \frac{0.08}{2}\right)^{40} - 1\right]}{\left(\frac{0.08}{2}\right)\left(1 + \frac{0.08}{2}\right)^{39}} \right] + \frac{1{,}000}{\left(1 + \frac{0.08}{2}\right)^{40 - 1 + 0.0380}}$$
$$= \$832.9078.$$

The moment the bond goes ex-dividend, the price will drop by the present value of the next coupon because the buyer is no longer entitled to it. The ex-dividend dirty price is

$$832.9078 - \frac{30}{(1.04)^{0.0380}} = \$802.9525.$$

Notice that the dirty price will fall by the present value of the coupon and not by the coupon itself. This is because there is a week remaining until the next coupon, so we need to discount the cash flow.

The accrued interest as of November 8, 20XX, is

$$\frac{0.06 \times 1{,}000}{2} \times \frac{177}{184} = \$28.8587.$$

The ex-dividend clean price is $832.9078 - 28.8587 = \$804.0491$. The ex-dividend clean price is greater than the ex-dividend dirty price by \$1.0966. This is the negative accrued interest, and it corresponds to the accrued interest for the remaining week, which is

$$\frac{0.06 \times 1{,}000}{2} \times \frac{7}{184} = \$1.1413.$$

Yields

The yield or the rate of return from a bond can be computed in a variety of ways. We have already referred to the YTM, which we will discuss in further detail shortly. We will also examine a number of other yield measures.

The Current Yield

The current yield is very simple to compute, although technically it leaves a lot to be desired. However, it is commonly reported in practice, because of the ease with which it can be calculated.

To compute the current yield, we simply take the annual coupon payment (irrespective of the frequency with which the coupon is paid) and divide it by the current market price. Thus,

$$CY = \frac{\text{Annual Coupon Interest}}{\text{Price}}.$$

One question is, should the price used be the clean or the dirty price? If the clean price were to be used, then the current yield would change only when there is a change in the YTM. However, if the dirty price were to be used, the current yield using the dirty price would be lower than the value obtained using the clean price if we were to calculate it before the ex-dividend date. This is because the dirty price in this period will be higher than the clean price. Besides, even if the YTM were to remain constant, the current yield will steadily decline along with the increase in the dirty price. In the ex-dividend period, however, the value of the current yield that we get with the dirty price will be higher than what we would get with the clean price because the dirty price will be lower than the clean price. In this period, too, if the YTM were to remain constant, the dirty price would steadily increase, and the current yield would steadily decline. If we were to plot the current yield versus the dirty price, for a given YTM, we would get a saw-tooth pattern.

Why do we say that the current yield is an unsatisfactory yield measure? Consider the case of an investor who has a one-year investment horizon. She will get a coupon of $\$C$ in the course of the year. If we assume that P_0 was the price she paid at the outset to acquire the bond, then her interest yield for the year is the same as the current yield as defined by us. But even if this investor were to have a one-year horizon, she will sell the bond at the end of the year, unless, of course, it were to mature at that time. She will experience a capital gain or a loss.[3]

If the investor were to have a longer-term horizon, she will get additional coupons. These can be reinvested to earn income. Besides, when multiple cash flows are involved, the time value of money will enter the picture. All these facets of yield computation are totally ignored by the current yield measure.

Example 4.5 illustrates how the current yield is computed.

Example 4.5

Consider a bond with a face value of \$1,000 and a coupon of 8 percent per annum payable semiannually. Assume that the current price is \$950.

The annual coupon is \$80. The current yield $= 80/950 = 0.084211 \equiv 8.4211\%$.

Simple Yield to Maturity

The simple yield to maturity, also known as the *Japanese yield* because it is a key yield measure used in Japan, factors in the capital gain or loss that an investor will get. However, it assumes that the bond will be held to maturity and that capital gains and losses occur evenly over the life of the bond. In other words, it builds in the assumption that capital gains and losses occur on a straight-line basis.

Consider the case of an investor who buys the bond at a price P. If he holds it to maturity, he will get back the face value, M. The capital gain or loss over the investment horizon is given by $M - P$. If the bond is bought at a discount, there will be a capital gain; if it is bought at a premium, there will be a capital loss. If we assume that we are standing on a coupon date and that there are N coupons left, then there are $N/2$ years remaining until maturity. The capital gain or loss amortized on a straight-line basis is

$$\frac{M-P}{\frac{N}{2}}.$$

The simple YTM is given by

$$\text{Simple YTM} = \frac{C}{P} + \frac{M-P}{\frac{N}{2} \times P}.$$

One shortcoming of the Japanese yield is that it fails to take into account the fact that investors in bonds can reinvest the coupons received by them in order to earn interest on such interest. This has the potential to significantly increase the returns from holding a bond, as we shall demonstrate in Example 4.6.

Example 4.6

Consider a bond with a face value of \$1,000 that pays coupons at the rate of 7.25 percent per annum. Assume that the current price is \$925 and that there are 10 years to maturity. The Japanese yield is given by

$$\begin{aligned}\text{Japanese Yield} &= \frac{0.0725 \times 1{,}000}{925} + \frac{1{,}000 - 925}{10 \times 925} \\ &= 0.07838 + .0081 = 0.086486 \equiv 8.6486\%.\end{aligned}$$

Yield to Maturity

The yield to maturity is the internal rate of return of a bond. It is that single discount rate[4] that makes the present value of the cash flows from a bond equal to its price. The bond-pricing equation is a nonlinear equation. Because there is only a single change of sign in the cash flows, from Descartes' rule of signs we know that there will be a single positive real root. The pricing formula gives us a polynomial of degree N, assuming that the bond has N coupons remaining. To solve it, we require a computer program in general, although we can get fairly close with the *approximate yield to maturity* (AYM) approach, which we shall demonstrate. We will show subsequently that it is a very simple matter to compute the YTM for zero-coupon bonds and bonds with only two coupons remaining.

Approximate Yield to Maturity

Consider a bond with a face value of M, a current price of P, an annual coupon of C, and N coupons remaining until maturity.

The current investment in the bond is \$$P$. An instant before the bond matures, the investment in the security is \$$M$. The average investment is $(M + P) \div 2$.

The annual capital gain or loss computed by amortizing on a straight-line basis is

$$\frac{M - P}{\frac{N}{2}}$$

as we have seen in the case of the Japanese yield.

The approximate YTM is defined as

$$\text{AYM} = \frac{\dfrac{C + \dfrac{M-P}{N}}{2}}{\dfrac{M+P}{2}}.$$

The numerator in the expression represents the annual income for the investor, assuming, of course, that capital gains or losses are realized on a straight-line basis. As we have explained, the denominator is the average investment during the life of the bond.

Once the approximate yield is computed, we need to choose an interest-rate band such that the upper limit gives a lower price than the observed price, whereas the lower limit gives a higher price. The exact YTM is then obtained using linear interpolation.

Example 4.7 demonstrates the computation of the yield to maturity using an approximate value followed by interpolation.

Example 4.7

Consider a bond with a price of \$925. The coupon is 7 percent per annum, and the face value is \$1,000. There are 10 years to maturity, and the bond pays coupons on a semiannual basis.

$$\begin{aligned}\text{AYM} &= \frac{\dfrac{70 + \dfrac{1{,}000 - 925}{20}}{2}}{\dfrac{1{,}000 + 925}{2}} \\ &= 0.08052 \equiv 8.052\%.\end{aligned}$$

Let us choose a band of 7.75–8.25 percent. The prices at the two extremes are 948.4675 and 915.9929. In this case, the actual price lies between the two extreme prices. However, at times we may end up choosing the interest-rate band in such a way that the actual price is either higher than both the extreme values or is lower than both. In such cases, we have to redefine the band such that one of the corresponding prices is higher than the actual price while the other is lower.

Let us interpolate. 8.25–7.75 percent corresponds to a price difference of 915.9929 − 948.4675. So, 8.25% − y^* should correspond to a price difference of 915.9929 − 925, where y^* is the true YTM.

$$\begin{aligned} 0.0825 - 0.0775 &\equiv 915.9929 - 948.4675 \\ 0.0825 - y^* &\equiv 915.9929 - 925. \end{aligned}$$

If we take the ratio of both and solve for y^*, we get

$$\frac{0.005}{0.0825 - y^*} = \frac{-32.4746}{-9.0071}$$
$$\Rightarrow y^* = 0.081113 \equiv 8.1113\%.$$

We can calculate the price corresponding to a YTM of 8.1113 percent to verify the accuracy. It comes out to be $924.86.

Zero-Coupon Bonds and the YTM

It is a very simple task to determine the YTM for a zero-coupon bond given its price, as the following example will illustrate.

Consider a bond with 10 years to maturity and a face value of $1,000. Assume that the current price is $475. The pricing equation is[5]

$$475 = \frac{1{,}000}{\left(1 + \frac{y}{2}\right)^{20}}$$
$$\Rightarrow \left(1 + \frac{y}{2}\right)^{20} = 2.105263.$$
$$\frac{y}{2} = 0.037923 \Rightarrow y \equiv 7.5847\%.$$

Analyzing the YTM

To better appreciate the mathematics underlying the YTM, let us consider the sources of return for an investor who buys the bond at the prevailing market price and holds it to maturity.

She will get N coupons, with each coupon equal to \$$C/2$. Each time she gets a coupon, she can reinvest at the market yield prevailing at that time in an asset of the same risk class. And, finally, when the bond matures, she will be repaid the face value. The cash flows from the coupon payments constitute an annuity. When these cash flows are reinvested, they will earn interest. The important thing is the future value of these cash flows at the time the bond matures, which can be computed by using the standard annuity formula. Let us assume that each cash flow can be reinvested at a periodic (six-monthly) rate of $r/2$ percent. For ease of exposition. we will assume that r is the same for each cash flow or, in other words, that the reinvestment rate is a constant. This is not a necessary assumption and is made purely to facilitate the presentation of the argument.

The future value of the coupons as calculated at the time of maturity is given by

$$\frac{\frac{C}{2}}{\frac{r}{2}}\left[\left(1+\frac{r}{2}\right)^N-1\right].$$

The total cash flow at maturity is

$$M+\frac{\frac{C}{2}}{\frac{r}{2}}\left[\left(1+\frac{r}{2}\right)^N-1\right].$$

From the bond-pricing equation

$$P=\frac{\frac{C}{2}}{\frac{y}{2}}\times\left[1-\frac{1}{\left(1+\frac{y}{2}\right)^N}\right]+\frac{M}{\left(1+\frac{y}{2}\right)^N}.$$

When can we make a claim that we have received a yield of $y/2$ per period over N periods? Only if the terminal cash flow is equal to the initial investment compounded at $y/2$ for N periods.

In other words, we can claim that we have obtained a semiannual yield of $y/2$ if

$$P \times \left(1+\frac{y}{2}\right)^N = \frac{\frac{C}{2}}{\frac{y}{2}} \times \left[\left(1+\frac{y}{2}\right)^N - 1\right] + M$$
$$= \frac{\frac{C}{2}}{\frac{r}{2}}\left[\left(1+\frac{r}{2}\right)^N - 1\right] + M.$$

The implication is that to receive a periodic yield of $y/2$ over N periods, we need to satisfy two conditions. First, the investor must hold the bond until the maturity date. Second, $y/2$ must equal $r/2$. In other words, each intermediate cash flow obtained during the life of the bond (coupon) must be reinvested at the YTM prevailing at the time of acquisition of the bond.

Let us analyze the consequences of relaxing this assumption.

The Realized Compound Yield

We will relax one of the two preceding assumptions. Although we will continue to assume that the bond continues to be held until maturity, we will no longer take it for granted that each coupon is reinvested at the YTM. Instead, we will assume a specific rate r at which the coupons are assumed to be reinvested.

Example 4.8 illustrates the approach adopted to compute what is termed the *realized compound yield*.

Example 4.8

Consider the bond with a face value of $1,000 and 10 years to maturity. The price is $925, and the coupon is 7 percent per annum paid semiannually. The YTM obtained by us was 8.1113 percent.

Assume that each cash flow can be reinvested at 7 percent per annum, or at 3.50 percent per six-month period.

The future value of the coupons is

$$\frac{\frac{70}{2}}{\frac{0.07}{2}}\left[\left(1+\frac{0.07}{2}\right)^{20} - 1\right] = 989.7889.$$

The terminal cash flow is $1{,}000 + 989.7889 = \$1{,}989.7889$.

The initial investment was \$925. The rate of return over 20 periods is given by

$$925\left(1+\frac{i}{2}\right)^{20} = 1{,}989.7889 \Rightarrow i \equiv 7.8085\%.$$

This return is called the *realized compound yield*. It can be done on an ex-ante basis, as we have just done, by assuming a rate at which the reinvestment will take place, or on an ex-post basis by plugging in the actual rate at which the cash flows were reinvested.

In this illustration, the realized compound yield is less than the YTM because the reinvestment rate assumed by us is less than the YTM. If we had assumed a reinvestment rate higher than the YTM, we would have obtained an RCY greater than the YTM.

Reinvestment and Zero-Coupon Bonds

The inability to reinvest at a rate of return assumed at the outset is referred to as *reinvestment risk*. For coupon-paying bonds, reinvestment risk is a critical feature. Zero-coupon bonds are devoid of reinvestment risk for there are no coupons to be reinvested. To obtain a rate of return equal to the YTM for a zero-coupon bond, we have to satisfy only one condition: we should hold the bond to maturity.

The Holding-Period Yield

We will relax both the assumptions that we made earlier in order to compute the YTM. The first change, as we did in the case of the RCY, is that we will explicitly assume a rate at which the coupons are reinvested. Second, we will no longer assume that the bond is held to maturity but will assume an investment horizon that is shorter than the time to maturity. The corresponding yield measure is termed the *horizon yield* or *holding-period yield*.

The consequence of the second assumption is that it is no longer necessary for an investor to receive the face value at the end of his investment horizon. He will receive the market price at that time, which may be more or less than the face value and will depend on the YTM prevailing at that time.

Because we need to compute the sale price before maturity, we need to make an assumption about the YTM that is likely to prevail at that time, if we are computing the holding period yield on an ex-ante basis.[6]

Let us consider the data that we used to compute the RCY. However, we will assume that the investor has a horizon of eight years or 16 semiannual periods. We will assume that the bond can be sold at a YTM of 7.50 percent at the end of eight years. At that point in time, it will be a two-year bond. The corresponding price is \$990.8715.

The future value of the reinvested coupons is given by[7]

$$\frac{\frac{70}{2}}{\frac{0.07}{2}}\left[\left(1+\frac{0.07}{2}\right)^{16}-1\right]=733.9860.$$

The terminal cash flow is 990.8715 + 733.9860 = 1,724.8575.

The rate of return over the eight year period is given by

$$925\left(1+\frac{i}{2}\right)^{16}=1{,}724.8575 \Rightarrow i \equiv 7.9425\%.$$

Taxable-Equivalent Yield

Certain bonds are exempt from income taxes. In the United States, both the federal and state governments are empowered to levy such taxes. To deal with a situation in which a comparison is sought between a bond with taxable interest and another with tax-free income, we need to compute the *taxable-equivalent yield* (TEY) of the tax-free bond.

The computation of the TEY depends on the applicable taxes. Let us first consider a municipal bond that gives a yield of 6.00 percent. The bond is exempt from federal income tax, which we will assume to be 25 percent. The TEY is given by

$$\text{TEY}=\frac{6.00}{1-0.25}=8.00\%.$$

The implication is that an investor should be indifferent between a taxable bond that yields 8 percent and the tax-free municipal bond, which yields 6 percent. If the taxable bond were to yield more than 8 percent, he would prefer it. On the other hand, if it were to yield less than 8 percent, he would prefer the municipal bond.

To make the computation more precise, we need to account for the fact that both bonds attract state income tax. This per se does not warrant an adjustment, because both the bonds will be equally impacted. However, for a bond that attracts both federal and state income taxes, the state tax can be deducted from the federal tax bill. This calls for the following adjustment.

Let us assume that the federal rate is 25 percent while the state tax rate is 8 percent. The adjusted federal rate is given by

$$0.25 - 0.25 \times 0.08 = 0.23.$$

The TEY of the municipal bond is therefore

$$\text{TEY} = \frac{6.00}{1 - 0.23} = 7.7922\%.$$

The rationale for this correction is the following. If a person were to earn \$100 of income, he will have to pay \$8 by way of state tax. The federal tax is applicable only on \$92. So the effective federal tax rate is: $0.92 \times 0.25 = 0.23 \equiv 23\%$.

Consider the municipal bond is exempt from both federal and state taxes. To make the necessary adjustment, we need to compute the combined tax rate for a bond that is subject to both taxes. In our case, it is $23\% + 8\% = 31\%$.

The TEY of the municipal bond in such a situation is given by

$$\text{TEY} = \frac{6.00}{1 - 0.31} = 8.6957\%.$$

There can be a situation in which the taxable bond that is being compared with the municipal bond is subject to federal taxes but not to state taxes. If so, we need to compute the TEY of the taxable bond and compare it with that of the municipal bond.

Assume that a T-bond with a coupon of 6.90 percent is being compared with a municipal bond that is exempt from both state and federal taxes. The TEY of the T-bond is

$$\text{TEY} = \frac{6.90}{1 - 0.08} = 7.50\%.$$

This TEY should be compared with the TEY of the municipal bond, and the bond that offers the higher TEY will be deemed to be superior.

Credit Risk

Credit or default risk is the risk that the coupons or principal or both may not be paid as scheduled. Except for Treasury securities, which are backed by the full faith and credit of the federal government, all debt securities are subject to default risk and differ only with respect to the degree of the associated risk. When a bond is issued, the borrower will release a prospectus with detailed information about its financial health and creditworthiness. However, most investors lack the required skill sets to draw meaningful conclusions from analyzing such documents. To give confidence to potential investors about the quality of the issue, issuers get the securities rated by credit-rating agencies. These agencies specialize in evaluating the credit quality of a security issue. They not only provide a rating before the issue but also continuously monitor the health of the issuer throughout the life of the security, modifying their recommendations as and when required. The rating accorded to a particular issue is based on the financial health of the issuer and the quality of its management team. In the case of secured bonds, it also depends on the quality of collateral that has been specified.

The three main rating agencies in the United States are Moody's Investors Service, Standard and Poor's Corporation, and Fitch Ratings.

There are two categories of rated securities: investment grade and speculative grade. Investment grade–rated bonds are of higher quality and carry a lower credit risk. Noninvestment grade, which is also known as *speculative grade* or *junk bond*, carries a higher risk of default. Because a riskier security must offer a higher rate of return to compete with securities that are better rated, junk bonds carry a high coupon. Investors are cautioned that they are investing at their own risk. If nothing goes wrong, investors will walk away with higher returns.

The ratings scales adopted by the three rating agencies are depicted in Tables 4.5 and 4.6.

Table 4.5
Investment Grade Ratings

Credit Risk	Moody's Ratings	S&P's Ratings	Fitch's Ratings
Highest quality	Aaa	AAA	AAA
High quality	Aa	AA	AA
Upper medium	A	A	A
Medium	Baa	BBB	BBB

Table 4.6
Speculative Grade Ratings

Credit Risk	Moody's Ratings	S&P's Ratings	Fitch's Ratings
Somewhat speculative	Ba	BB	BB
Speculative	B	B	B
Highly speculative	Caa	CCC	CCC
Most speculative	Ca	CC	CC
Imminent default	C	C	C
Default	C	D	D

As discussed, the agencies may revise their ratings during the life of the security. If a rating change is being contemplated, they will signal their intentions. S&P will place the security on *credit watch*. Moody's will place the security *under review*, and Fitch will place it on *rating watch*.

Bond Insurance

A company that seeks a better rating can have its issue insured in order to enhance the credit quality. In such a case, the issuer will have to pay a premium to the insurance company. But this cost will be passed on to investors in the form of a lower coupon. In the case of insured bonds, the timely payment of promised cash flows is guaranteed by the insurance company. The rating of such issues would depend on the financial health of the insurer. It would make sense to get an issue insured only by a company that enjoys a better reputation than the issuer.

Equivalence with Zero-Coupon Bonds

Consider a bond with a face value of $1,000 and two years to maturity. Assume that the coupon is 7 percent per annum payable on a semiannual basis. This bond will give rise to five cash flows per the schedule shown in Table 4.7.

Consider the first cash flow. It is like the maturity value of a zero-coupon bond with a face value of $35 that matures after six months. Similarly, the second cash flow is like the maturity value of a zero-coupon bond with a face value of $35 maturing after 12 months. The plain vanilla bond is like a portfolio of five zero-coupon bonds. This is true for any

Table 4.7
Cash Flows from a Two-Year Bond

Time Period (Months)	Cash Flow ($)
6	35
12	35
18	35
24	35
24	1,000

plain vanilla bond. If the bond has N coupons remaining until maturity, then it is equivalent to a portfolio of $N+1$ zero-coupon bonds.

Spot Rates

Consider the same two-year bond. It is equivalent to a portfolio of five zeros. Each zero must have its own yield to maturity. The yield to maturity of a zero-coupon bond, for a given time to maturity, is referred to as the *spot rate* for that period. The price of the plain vanilla bond may be expressed as

$$P_0 = \frac{35}{(1+s_1)} + \frac{35}{(1+s_2)^2} + \frac{35}{(1+s_3)^3} + \frac{1{,}035}{(1+s_4)^4}.$$

The traditional pricing equation, based on the YTM, states that

$$P_0 = \frac{35}{\left(1+\frac{y}{2}\right)} + \frac{35}{\left(1+\frac{y}{2}\right)^2} + \frac{35}{\left(1+\frac{y}{2}\right)^3} + \frac{1{,}035}{\left(1+\frac{y}{2}\right)^4}.$$

Because a plain vanilla bond is a portfolio of zeros, the correct way to price it is by discounting each cash flow at the corresponding spot rate. When we value the cash flows by discounting at the YTM, we are using an average discount rate to capture the effect of the various spot rates. The YTM of a bond is a complex average of the underlying spot rates.

The Coupon Effect

The YTM is subject to what we call a *coupon effect*. Consider two one-year bonds, both with a face value of $1,000. Bond A pays coupons at the rate of 7 percent per annum, whereas bond B pays a coupon at the rate of 10 percent per annum. Assume that the six-month spot rate is 6 percent per annum, while the one-year spot rate is 7.25 percent per annum.

The price of bond A is

$$\frac{35}{(1.03)} + \frac{1{,}035}{(1.03625)^2} = 33.9806 + 963.8540 = \$997.8346.$$

The YTM may be calculated as

$$997.8346 = \frac{35}{\left(1 + \frac{y}{2}\right)} + \frac{1{,}035}{\left(1 + \frac{y}{2}\right)^2}.$$

If we solve the quadratic equation, we get $y = 7.2283\%$.

Consider bond B. The price is given by

$$\frac{50}{(1.03)} + \frac{1{,}050}{(1.03625)^2} = 48.5437 + 977.8229 = \$1{,}026.3666.$$

The YTM comes out to be 7.2197 percent.

Why is it that bond A has a higher YTM than bond B? The price of one-period money is 6 percent per annum, whereas that of two-period money is 7.25 percent per annum. Two-period money is more expensive. Bond A has

$$\frac{35}{(1.03) \times 997.8346} = 3.4054\%$$

of its value locked up on one-period money. Bond B, on the other hand, has

$$\frac{50}{(1.03) \times 1{,}026.3666} = 4.7297\%$$

of its value locked up in one-period money. Because bond B has a greater percentage of its value locked up in one-period money, which in this case is cheaper, it is not surprising that its YTM is lower than that of bond A. The YTM, which is a complex average of spot rates, is a function of the coupon rate of a bond for a given term to maturity, which is what we term the *coupon effect*.

Bootstrapping

We will not have price data for zero-coupon bonds expiring exactly one period apart. In other words, we will not be in a position to compute the

Table 4.8
Prices of Plain Vanilla Bonds with Varying Times to Maturity

Term to Maturity (Months)	Price ($)
6	985
12	950
18	925
24	900

spot rates directly from the observable prices. Bootstrapping is a technique for obtaining spot rates from price data for plain vanilla bonds.

Consider the information in Table 4.8. All the bonds have a face value of $1,000 and pay a coupon of 7 percent per annum on a semiannual basis.

The one-period spot rate is given by

$$985 = \frac{1{,}035}{\left(1 + \frac{s_1}{2}\right)} \Rightarrow s_1 = 10.1523\% \text{ per annum.}$$

We know that

$$950 = \frac{35}{\left(1 + \frac{s_1}{2}\right)} + \frac{1{,}035}{\left(1 + \frac{s_2}{2}\right)^2}.$$

Substituting for s_1, we get $s_2 = 12.5146$ percent per annum. Similarly, using the pricing equation for the 18-month bonds and substituting for s_1 and s_2, we can compute s_3 and, by extending the logic, find s_4. This is the essence of bootstrapping.

Forward Rates

Consider a person with a two-period investment horizon. She can directly invest in a two-period zero-coupon bond or she can invest in a one-period bond and roll over her investment. The second approach is fraught with risk because we cannot predict at the outset what the one-period rate will be after one period. However, it may be possible to enter into a forward contract at the outset that locks in a rate for the one-period investment after one period. The rate implicit in such a contract is referred to as the *forward rate of interest*. We will denote it as f_1^1. The subscript indicates that it is for a loan to be made after one period, and the superscript indicates that the loan is for one period. To rule out arbitrage, it must be the case that

$$\left(1+\frac{s_2}{2}\right)^2=\left(1+\frac{s_1}{2}\right)\left(1+\frac{f_1^1}{2}\right).$$

If the one-period rate is 8 percent per annum while the two-period rate is 8.50 percent per annum, then

$$(1.0425)^2=(1.04)\left(1+\frac{f_1^1}{2}\right)\Rightarrow f_1^1=0.090012.$$

The one-period forward rate is 9.0012 percent per annum. In general, the forward-rate symbol for an $m - n$ period loan to be made after n periods is f_n^{m-n}.

The relationship between the n period spot rate and the m period spot rate is given by

$$\left(1+\frac{s_m}{2}\right)^m=\left(1+\frac{s_n}{2}\right)^n\times\left(1+\frac{f_n^{m-n}}{2}\right)^{m-n}.$$

The Yield Curve and the Term Structure

The plot of the YTM versus the time to maturity of the bond is referred to as the *yield curve*. On the other hand, a plot of the spot rate versus the term to maturity of the bond is called the *term structure of interest rates*. The term structure is also known as the *zero-coupon yield curve*. The two will coincide if the yield curve is flat. A flat yield curve means that all the spot rates are identical. Because the YTM is an average of spot rates, the YTM in such circumstances will be the same as the spot rate.

While plotting the yield curve or the term structure, it is important to ensure that the data being used pertain to bonds of the same risk class. In other words, if one of the bonds is AAA, then all the bonds in the data set must be AAA—that is, we should compare apples with apples and not mix up data for AAA bonds with those for T-bonds.

Shapes of the Term Structure

The term structure may take a variety of shapes. The commonly observed shapes are upward sloping or rising, downward sloping or inverted, humped, and U-shaped.

Rising yield curves will have a positive slope—that is, short-term yields will be lower than long-term yields. On the other hand, inverted yield curves are characterized by a negative slope—that is, short-term yields are higher than long-term yields. Humped yield curves tend to have lower rates at the short and long ends of the spectrum, and higher rates in between: the curve initially rises, peaks at the middle of the maturity spectrum, and then gradually slopes downward. U-shaped yield curves have the opposite shape—that is, the rates are higher at the short and long ends of the spectrum and tend to be low in between. Such curves are characterized by a curve that declines initially, reaches a trough at the middle of the maturity spectrum, and then gradually slopes upward.

Theories of the Term Structure

A variety of theories have been expounded to explain the various observed shapes of the yield curve. One popular theory is the *pure* or *unbiased expectations hypothesis*. It states that the implied forward rates computed using current spot rates are nothing but unbiased estimators of future spot rates. According to this theory, then, long-term spot rates are geometric averages of expected future short-term rates. This hypothesis can be used to explain any shape of the yield curve. Let us take an upward-sloping curve. The expectations hypothesis would explain it with the argument that the market expects spot rates to rise. If rates are expected to rise, then holders of long-term bonds will be perturbed because the prices of such bonds are expected to decline, and they are confronted with the possibility of a capital loss. Such investors will start selling long-term bonds and buying short-term bonds. This will lead to an increase in long-term yields and a decrease in short-term yields. The net result would be an upward-sloping yield curve.

Consider an inverted yield curve. This would be consistent with the view that the market expects future spot rates to fall. If so, investors will seek to sell short-term bonds and invest in long-term bonds, which will push up short-term yields and lead to declining long-run yields. The net result will be an inverted yield curve.

A humped yield curve would be consistent with the expectations that investors expect short-term rates to rise and long-term rates to fall.

The expectation of the future direction of the market is primarily a function of the expected rate of inflation. If the market expects inflationary pressures in the long run, then the yield curve will be upwardly sloping. However, if inflation rates are expected to decline in the long run, then the curve will be negatively sloped.

The Liquidity Premium Hypothesis

This hypothesis argues that the forward rate is not equal to the expected spot rate but is greater than the expectation of the future spot rate. The difference between the forward rate and the expectation of the future spot rate is termed the *liquidity premium*. Theory argues that because lenders generally prefer to lend short term, they must be suitably compensated if they are to be induced to lend for longer terms. This compensation takes the form of a premium for a loss of liquidity. Per this hypothesis, the yield curve should invariably be upwardly sloping, reflecting the investors' preference for liquidity. However, a declining yield curve can be explained by postulating that future spot rates may sharply decline, causing long-term rates to be lower than short-term rates, the liquidity premium not withstanding.

The Money Substitute Hypothesis

According to this theory, short-term bonds are essentially a substitute for cash. Investors generally hold only short-term bonds because of the lower perceived risk. This drives up the demand for such securities and pushes down the yield. This explains low yields at the short end of the maturity spectrum. As far as the other end is concerned, borrowers tend to issue long-term debt, which implies that they will have to access the capital market less frequently, because it minimizes the costs associated with borrowing. This leads to an excess supply at the longer end of the maturity spectrum and pushes up the yield.[8]

The Market-Segmentation Hypothesis

The theory postulates that the market is made up of a wide variety of investors and issuers. Each class of investors or issuers has its own requirements and tends to focus on a particular range of the maturity spectrum. The theory argues that the segments of the market are compartmentalized and there are no interrelationships between them.

The observed shape of the yield curve, in accordance with this theory, depends on the demand–supply dynamics within the market segment, and activities in a given segment have no implications for any other part of the curve. The need to constantly manage their cash leads commercial banks to primarily focus on the short end of the curve. On the other hand, institutions such as insurance companies and pension funds, whose liabilities are primarily long term, tend to focus on the long end of the market.[9] There is relatively less demand for medium-dated bonds. According to this theory, yields will be relatively low at the short and long ends of the maturity spectrum and high in the middle of the term structure, which is consistent with what we call a *humped yield curve*.

The Preferred Habitat Theory

This theory is a slight modification of the market-segmentation hypothesis. This proposes that even though it is true that investors tend to concentrate on their chosen market segment, they can be persuaded to hold securities from other segments by offering them suitable inducements. Whereas banks generally operate at the lower end of the spectrum, an increase in yields of long-dated bonds may sometimes be adequate to persuade them to hold such securities. Similarly, an increase in short-term rates may encourage pension funds and insurance companies to invest in that segment, an area of the market they would otherwise avoid.

The Short Rate

A short rate of interest is a future spot rate of interest that may evolve over time. It is usually represented as a single period rate for the shortest period of time considered by the model on the basis of which it is postulated to evolve. In our preceding discussions, we have assumed that the shortest time period corresponds to six months.

At a given point in time, we will have a vector of spot rates corresponding to various intervals of time, and this vector can be used to derive a unique vector of current forward rates as explained earlier. However, we cannot with certainty state as to what the spot rate will be at a future point in time, because it may take on one of several values, each having an associated probability of occurrence.

At the current instant, the one-period spot rate will be equal to the one-period forward rate, which will be equal to the short rate. However, if we look at a longer time horizon, these rates will in general not be equal to each other.

Example 4.9 illustrates the evolution of short rates over time based on the Ho and Lee model.

Example 4.9

The tree for the evolution of short rates that we describe as follows is based on the Ho and Lee model and was originally presented in Parameswaran (2009).[10] The details behind the derivation are beyond the scope of this book. The derivation assumes the term structure depicted in Table 4.9.

Table 4.9
Vector of Spot Rates

Period	Spot Rate (%)
1	6.00
2	5.80
3	6.05
4	5.90

Evolution of the Short Rate

In Figure 4.2, at each node there is a 50 percent probability of reaching the upper state at the end of the current period, as well as

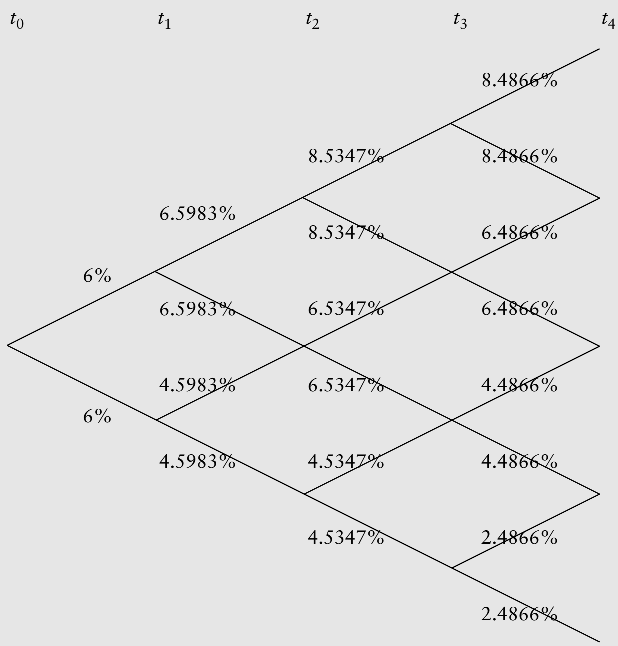


Figure 4.2 Evolution of the Short Rate

an equal probability of reaching the lower state. The current one-period spot rate is 6 percent, which is equal to the current short rate. At the end of the period, there is a 50 percent chance that the short rate will be 6.5983 percent and an equal probability that it will be 4.5983 percent. Similarly, at the end of two periods, the short rate can take on one of three possible values, whereas at the end of three periods it can take on one of four possible values.

Floating-Rate Bonds

Floating-rate notes and bonds, also referred to as *floaters*, are debt securities whose coupons are reset periodically based on a reference or benchmark rate. Typically, the coupon on such a security is defined as

$$\text{Benchmark rate} + \text{quoted margin.}$$

Consider a security whose coupon is specified as

$$\text{The 5-year T-note rate} + 75 \text{ basis points.}$$

In this case, the reference rate is the yield on a five-year T-note, and the quoted margin is 75 basis points (bp). Note that the quoted margin need not always be positive. A floater may have a coupon rate specified as

$$\text{The 5-year benchmark rate} - 50 \text{ bp.}$$

In the case of a default risk-free floating-rate bond, the price of the security will always reset to par on a coupon date, although it may sell at a premium or at a discount between two coupon dates.

Consider a floater with a coupon equal to the five-year T-bond rate and assume there are two periods to maturity. The price of the bond at the end of the first coupon period will be given by

$$P_{T-1} = \frac{M + \frac{c_{T-1}}{2} \times M}{\left(1 + \frac{y_{T-1}}{2}\right)}$$

where c_{T-1} is the coupon rate one period before maturity, and y_{T-1} is the required yield one period before maturity. On the coupon-reset date, the YTM will be equal to the coupon because we have assumed that there is no default risk implicit in the security. Any change in the required yield, as reflected by the YTM at that point in time, will also be reflected in the coupon that is set on that day. We know that if the yield is equal to the coupon, then the bond should sell at par. Thus $P_{T-1} = M$.

The price at the outset is given by

$$P_{T-2} = \frac{P_{T-1} + \frac{c_{T-2}}{2} \times M}{\left(1 + \frac{y_{T-2}}{2}\right)} = \frac{M + \frac{c_{T-2}}{2} \times M}{\left(1 + \frac{y_{T-2}}{2}\right)}.$$

Once again, at $T-2$, $c_{T-2} = y_{T-2}$, and $P_{T-2} = M$. This logic can be applied to a bond with any number of coupons remaining to maturity.

Between two coupon dates, however, the price of such a floater may not be equal to par. Consider the valuation of the note at time $T-2+k$. The price is given by

$$P_{T-2+k} = \frac{P_{T-1} + \frac{c_{T-2}}{2} \times M}{\left(1 + \frac{y_{T-2+k}}{2}\right)} = \frac{M + \frac{c_{T-2}}{2} \times M}{\left(1 + \frac{y_{T-2+k}}{2}\right)}.$$

Although c_{T-2} was set at time $T-2$ and is equal to y_{T-2}, y_{T-2+k} is determined at time $T-2+k$ and will reflect the prevailing five-year T-note yield at that point in time. In general, y_{T-2+k} need not equal c_{T-2} and may be higher or lower. Between two coupon dates, a floater may sell at a premium or at a discount.

Let us consider the case of floaters characterized by default risk. In this case, it is not necessary that the risk premium required by the market be constant over time. Assume that when the bond was issued the required return was equal to the five-year T-note rate + 75 bp. The coupon was set equal to this rate and the bond was sold at par. 6-M hence the issue is perceived to be more risky, and the required return in the market is the five-year T-note rate + 95 bp. The coupon will however be reset at the prevailing T-note rate + 75 bp. This issue will not reset to par at the next coupon date.

Bonds with Embedded Options

We will consider three types of bonds with embedded options: callable, putable, and convertible bonds. All of these bonds give either the issuer or the lender an option. As we have seen, an option gives the holder the right to take a course of action. Because no one will give a right away for free, the holder has to pay a price or a premium to the seller of the option. In the case of bonds with embedded options, if the option is with the issuer, then he must pay for it, and the price of the bond will be less than that of a plain vanilla bond. However, if the option were to be with the lender, then it will manifest itself as a higher price compared to that of a plain vanilla bond.

Callable Bonds

Such bonds contain a call option; that is, they give the issuer the right to call away the bond from the lender before maturity. The issuers can in the case of such bonds change the maturity of the bonds by prematurely recalling them. When will such a bond be recalled? When market interest rates are declining. Under such circumstances, the issuer can recall the existing bonds and replace them with a fresh issue that can be issued with a lower coupon because of the changed circumstances.

The call provision works against the lender because she may have to part with the bond when the market rates are falling, which is precisely a situation in which she will desire to hold on to the bonds and keep earning a high coupon. To compensate for this, she will demand a higher yield as compared to what she would from a plain vanilla bond of the same credit quality. This will manifest itself as a lower price. Whether we view it from the issuer's perspective or the lender's, a callable bond must sell at a lower price compared to an otherwise similar plain vanilla bond.

Such bonds may be *discretely callable* or *continuously callable*. A discretely callable bond may be recalled only at certain prespecified dates—for instance, at the coupon dates over a portion of the bond's life. A continuously callable bond may be called at any time after it becomes callable.

As we have just mentioned, a bond may be recalled only when it becomes callable. This implies that it may not be callable right from the outset. Issuers generally specify a call-protection period, which is a period of time during which the bond may not be recalled regardless of what happens to the market rate of interest. A discretely callable bond can usually be recalled on any coupon payment date after the call-protection period ends, whereas a continuously callable bond may be recalled at any time from the end of the call-protection period until the maturity date of the bond. Bonds with a call-protection period are referred to as *deferred callable bonds*, and they serve to provide holders with relatively greater certainty.

The price at which the bond can be recalled is referred to as the *call price*. In many cases, a bond may be recalled at par by an issuer. At times, however, the issuer may specify a call premium—that is, he will pay a value higher than the face value of the bonds if and when the issue is recalled. The call premium is usually set equal to one year's coupon.

Holders of callable bonds are extremely vulnerable to reinvestment risk. First, it is increasingly likely that they will experience a return of cash in a falling interest-rate environment, in a falling interest-rate environment, which means they will have to face the specter of reinvesting their corpus at a lower rate of interest. Second, the potential for price appreciation in a falling rate environment is relatively limited as compared to a plain vanilla bond. Why do bond prices increase in value? Because yields are declining in the market. But in the case of callable bonds, this is precisely the situation in which the bond can be recalled, which means the buyer of such bonds in a falling interest-rate environment is constantly exposed to the risk of having to part with it at the call price. This aspect is referred to as *price compression*.

Yield to Call

In the case of callable bonds, it is a normal practice to compute the *yield to call* (YTC). For a given call date, all the cash flows from the current point in time until the call date are specified, and the discount rate that makes the present value of these cash flows equal to the dirty price of the bond is computed. The cash flows will be the coupons scheduled to be paid on or before the call date and the call price. The price at which the bond is recalled may in certain cases vary with the call date—that is, the call price may be a function of the call date.

The formula for the YTC, assuming that the bond is callable after N^* coupons have been paid, may be stated as

$$P = \frac{\frac{C}{2}}{\frac{y_c}{2}} \times \left[1 - \frac{1}{\left(1 + \frac{y_c}{2}\right)^{N^*}}\right] + \frac{M^*}{\left(1 + \frac{y_c}{2}\right)^{N^*}}.$$

This looks similar to the pricing equation that corresponds to the YTM calculation. But there are two key differences. First, M^*, which is the call price, need not equal the face value. Second, N^*, the number of coupons until the call dates, will be $\leq N$.

Example 4.10 illustrates the computation of the yield to call for a callable bond.

Example 4.10

Consider a bond with 20 years to maturity and a face value of \$1,000. Assume that the coupon rate is 7 percent per annum and that the current price is \$925. The first call date is eight years away; if the bond is called, the issuer will pay one year's coupon as the call premium. The YTC can be computed from the following equation:

$$925 = \sum_{t=1}^{16} \frac{35}{\left(1 + \frac{y_c}{2}\right)^t} + \frac{1{,}070}{\left(1 + \frac{y_c}{2}\right)^{16}}.$$

The solution comes out to be 8.9503 percent.

Putable Bonds

In the case of a putable bond, the holders have a put option: they can prematurely return the bond to the issuer and claim the face value. Such an option will be exercised when the market interest rates have risen. Under such circumstances, the holders can return the old bonds, which are yielding a relatively lower coupon, and use the proceeds to buy bonds yielding a higher coupon. Because the option in these cases is with the holders, they have to pay for it. This will manifest itself as a higher price as compared to that for a plain vanilla bond carrying the same coupon. A putable bond will sell for a lower yield as compared to a plain vanilla bond.

Consider the relative coupons for plain vanilla, callable, and putable bonds for a given risk class. The callable will have to offer the highest coupon, whereas the putable can be issued with the lowest coupon. After the issue, if we were to compare bonds with and without call and put options for a given coupon rate, the callable will have the lowest price or the highest yield, and the putable will have the highest price or the lowest yield.

The yield to put is typically defined as the discount rate that makes the present value of the cash flows from the bond equal to its price, assuming that the bond is held to the first put date.

Convertible Bonds

Convertible bonds allow holders to convert the debt securities into shares of stock of the issuer. The number of shares that an investor will receive if he not we. were to convert the bond is known as the *conversion ratio*. Consider a bond with a face value of $1,000 that can be converted to 50 shares of stock. The conversion ratio is 50. The *conversion price* is the face value divided by the conversion ratio—in this case, $20. The conversion value is the value of the shares if the bond were to be converted immediately. If we assume that the current share price is $22.50, then the conversion value is $1,125.

The minimum value of a convertible is the greater of the conversion value and the value that will be obtained if the bond were to be valued under the assumption that it is a plain vanilla bond. The latter value is termed the *straight value* of the bond.

The computation of the conversion and straight values for a convertible bond is illustrated in Example 4.11.

Example 4.11

Consider a convertible bond with a face value of $1,000 and five years to maturity. Assume that the coupon is 7 percent per annum payable on a semiannual basis, and that the conversion ratio is 50.

If the current stock price is $21, then the conversion value is $1,050. If we assume that the YTM of a plain vanilla bond with the same risk is 8 percent per annum, then the straight value will be $959.45. Because 1,050 is greater than 959.45, the price of the convertible must be $\geq$ $1,050.

But what if the YTM of a comparable bond were to be 5 percent? The straight value will be \$1,087.52. In this case, the value of the convertible must be $\geq$ \$1,087.52.

Using Short Rates to Value Bonds

Let us consider a segment of the interest-rate evolution tree (Figure 4.3) that we referred to earlier.

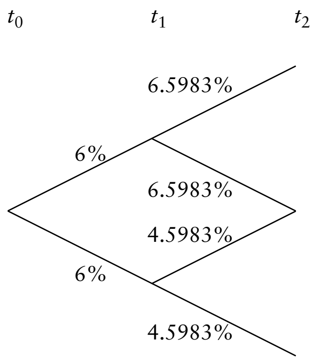


Figure 4.3 A Segment of the Short-Rate Tree

A Segment of the Short-Rate Tree

Consider a plain vanilla bond maturing at t_2, with a face value of \$100, paying a coupon of 6 percent per annum on a semiannual basis. At time t_2, there are three possible nodes. At each node, the payoff will be \$103.

The value of the bond at the upper node at t_1 is

$$\frac{103}{\left(1+\frac{0.065983}{2}\right)} = 99.7104.$$

Similarly, the value at the lower node at t_1 is

$$\frac{103}{\left(1+\dfrac{0.045983}{2}\right)} = 100.6851.$$

Because, given a node, the probability on an up move is equal to that of a down move, which in turn is equal to 50 percent, we can compute the value of the bond at t_0 as

$$\frac{0.5 \times (3+99.7104)}{1.03} + \frac{0.5 \times (3+100.6851)}{1.03} = 100.1920.$$

Consider a callable bond, which we will assume can be called back at the face value.[11] Because the value at the upper node at t_1 is less than the call price, the bond will not be recalled. However, at the lower node the value is higher than the call price and the issuer would like to exercise his call option. The price of the callable bond at t_0 may be computed as

$$\frac{0.5 \times (3+99.7104)}{1.03} + \frac{0.5 \times (3+100)}{1.03} = 99.8594.$$

As can be seen, the presence of the call option leads to a reduction in the bond value. Assume a bond with a put option. The holder will exercise it only if the value of the bond is less than its face value, assuming that he has the right to put it back at the face value. If so, the value at t_0 is given by

$$\frac{0.5 \times (3+100)}{1.03} + \frac{0.5 \times (3+100.6851)}{1.03} = 100.3326.$$

For obvious reasons, the put option makes the bond more valuable.

Price Volatility

It is common knowledge that long-term bonds are more price sensitive to a change in yield than comparable short-term bonds. This can be easily explained. The present value of a cash flow is given by $\dfrac{CF_t}{(1+r)^t}$.

The further away the cash flow—that is, the larger the value of t—the greater will be the impact of a change in the discount rate. Because long-term bonds have more cash flows coming at distant points in time, it can be concluded that their prices were more volatile. However, a second fact was subsequently noticed. For a given term to maturity, a zero-coupon bond was more price sensitive than any coupon-paying bond with the same term to maturity. This was perplexing because the bonds have the same term to maturity.

Frederick Macaulay came up with the concept of *duration* to explain this phenomenon. The crux of the idea is the following. A plain vanilla bond, as we have seen earlier, is a portfolio of zero-coupon bonds. Each component zero will have its own term to maturity. When we state that a plain vanilla bond has a term to maturity of T years, we are taking cognizance of the term to maturity of only the last of the component zeros. Macaulay argued that the effective term to maturity of a plain vanilla bond ought to be a weighted average of the terms to maturity of each component zero. The weight attached to a cash flow, he postulated, should be the present value of the cash flow divided by the price of the bond. Because the price of a bond is the sum of the present values of all the cash flows received from it, the weights defined by Macaulay will add up to 1. The Macaulay duration of a bond may be defined as

$$\sum_{t=1}^{N} \frac{CF_t \times t}{P \times \left(1 + \frac{y}{2}\right)^t}.$$

Example 4.12 demonstrates the computation of the duration for a plain vanilla bond.

Example 4.12

Consider a bond with a term to maturity of five years and a face value of \$1,000. Assume that the coupon is 7 percent per annum and that the YTM is 9 percent per annum. The computation of the duration is shown in Table 4.10.

The price of the bond is \$920.8728. The weighted average term to maturity, which we term as the *duration* of the bond, is 8.537804 semiannual periods, or 4.26892 years. Although the bond has

Table 4.10
Computation of Duration

Time Period	Cash Flow	Present Value of Cash Flow	Weight	Weight × Time
1	35	33.49282	0.036371	0.036371
2	35	32.05055	0.034805	0.069609
3	35	30.67038	0.033306	0.099917
4	35	29.34965	0.031872	0.127486
5	35	28.08579	0.030499	0.152495
6	35	26.87635	0.029186	0.175114
7	35	25.71900	0.027929	0.195503
8	35	24.61148	0.026726	0.213810
9	35	23.55165	0.025575	0.230178
10	1035	666.4652	0.723732	7.237320
	TOTAL	920.8728	1.000000	8.537804

a stated term to maturity of five years, its effective term to maturity is only 4.26892 years. It should be obvious why a five-year zero-coupon bond will be more price sensitive: because in the case of the zero, with a single cash flow, there is no difference between its stated term to maturity and its effective term to maturity. Irrespective of the value of T, a T-year zero will always have a higher duration than a T-year plain vanilla bond and will therefore be more price sensitive.

A Concise Formula

For a plain vanilla bond, we can derive a concise expression for the duration. Let us first redefine a few variables.

$$c \equiv \textbf{semiannual} \text{ coupon rate}$$

$$y \equiv \textbf{semiannual} \text{ YTM}.$$

It can be shown that the duration of a bond is given by

$$D = \frac{1+y}{y} - \frac{(1+y) + N(c-y)}{c[(1+y)^N - 1] + y}.$$

Using the data that we considered for the preceding illustration,

$$D = \frac{1.045}{0.045} - \frac{(1.045) + 10(0.035 - 0.045)}{0.035[(1.045)^{10} - 1] + 0.045} = 8.5378.$$

Duration and Price Volatility

The rate of change of the percentage change in price with respect to yield is a function of the duration of the bond. The relationship may be expressed as

$$\frac{\frac{dp}{P}}{dy} = -\frac{D}{\left(1 + \frac{y}{2}\right)}.$$

In this expression, D is the duration of the bond expressed in years.

The expression $\frac{D}{\left(1 + \frac{y}{2}\right)}$ is referred to as the modified duration of the bond, D_m.

We know that for a finite price change, the percentage change in price is $\frac{\Delta P}{P}$. In the limit we can express it as dP/P. The rate of change of the percentage change in the price with respect to the yield is equal to the modified duration of the bond. It is duration, and not the term to maturity, that is an accurate measure of interest-rate sensitivity.

We must point out that the duration of a bond technically should be perceived as a measure of interest-rate sensitivity. It cannot always be perceived as a measure of the effective average life of the bond.

Properties of Duration[12]

1. The duration of a bond increases with its term to maturity. There are two reasons for this. First, the principal repayment is a major component of the bond's present value and has a significant impact on its duration. As the time to maturity is increased, the repayment of principal is postponed, which serves to increase the duration. Second, as compared to a short-term bond, a long-maturity bond has cash flows arising at later points in time, which serves to increase the duration.
2. The duration of a bond is inversely related to its coupon rate. There are two reasons for this. First, high coupon bonds have greater amounts of cash flow occurring before the maturity date. This serves

to reduce the relative impact of the principal repayment on duration. Second, the impact of discounting is less on the earlier cash flows as compared to greater cash flows. The greater the coupon, the more the relative present value of the earlier cash flows, which serves to reduce the duration of the security.

3. Duration is inversely related to the YTM of the bond. Duration is computed by weighting the times to maturity of each coupon payment by its contribution to the present value of the bond. The higher the discount rate, the lower the present value of a cash flow. However, increasing the discount rate has a greater impact on long-term cash flows as compared to shorter-term cash flows. The relative weightage of shorter term cash flows is increased as we increase the YTM, which serves to bring down the duration of the bond.

Dollar Duration

The *dollar duration* of a bond is defined as the product of the modified duration of the bond and its price. In the preceding illustration, the price of the bond was $920.8728, and its modified duration was 4.0851 years. The dollar duration is 920.8728 × 4.0851 = 3,761.8574.

Convexity

The price−yield relationship for a plain vanilla bond is convex in nature. Duration is a measure of the first derivative and varies along with yield. To factor in the convex nature or curvature of the bond, we need to compute the second derivative.

Figure 4.4 illustrates the price−yield relationship for a plain vanilla bond. The bond was assumed to have 10 years to maturity, a face value of $1,000, and a coupon of 7 percent per annum payable semiannually.

From the Taylor series expansion, we can state that

$$
\begin{aligned}
dP &= \frac{dP}{dy}dy + \frac{1}{2}\frac{d^2P}{dy^2}(dy)^2 + \text{h.o.t} \\
\Rightarrow \frac{dP}{P} &= \frac{dP}{dy}\frac{dy}{P} + \frac{1}{2}\frac{d^2P}{dy^2}\frac{(dy)^2}{P} + \frac{\text{h.o.t}}{P}
\end{aligned}
$$

where h.o.t. stands for *higher-order terms*.

The first term on the right-hand side captures the duration effect. The second expression is the convexity effect. The convexity of a bond is defined as $\frac{d^2P}{dy^2}\frac{1}{P}$. The percentage price change because of the convexity effect is

$$\frac{1}{2} \times \text{convexity} \times (dy)^2.$$

The convexity can be computed as $\sum_{t=1}^{N} \frac{CF_t}{\left(1+\frac{y}{2}\right)^{t+2} P} \times t(t+1)$.

The calculation of the convexity for the five-year bond with a coupon of 7 percent is depicted in Table 4.11.

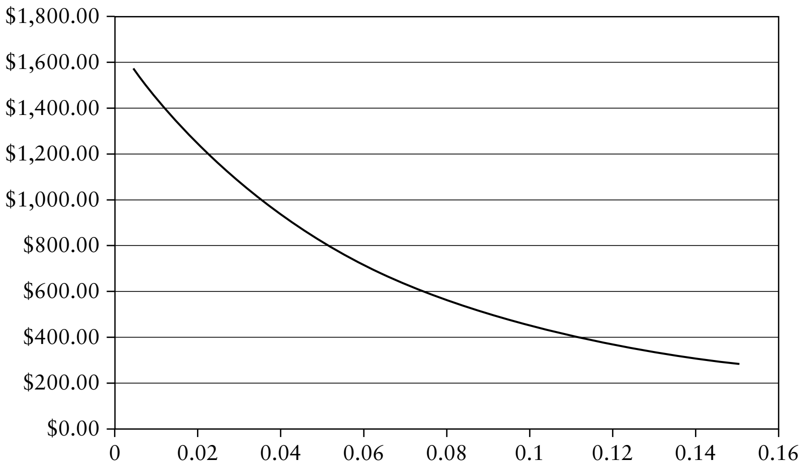


Figure 4.4 The Price–Yield Relationship

Table 4.11
Computation of Convexity

Time Period	Cash Flow (CFt)	$\frac{CF_t}{\left(1+\frac{y}{2}\right)^{t+2} p}$	t × (t+1)	$\frac{CF_t}{\left(1+\frac{y}{2}\right)^{t+2} p} \times t(t+1)$
1	35	0.033306	2	0.066612
2	35	0.031872	6	0.191229
3	35	0.030499	12	0.365989
4	35	0.029186	20	0.583715
5	35	0.027929	30	0.837868
6	35	0.026726	42	1.122503
7	35	0.025575	56	1.432220
8	35	0.024474	72	1.762130
9	35	0.023420	90	2.107811
10	1,035	0.662743	110	72.90174
			TOTAL	81.37182

The convexity in half years is 81.37182. The convexity in annual terms can be obtained by dividing this number by 4.[13] In this case, the annual convexity is 20.3430.

Consider a finite change of 100 bp in the annual YTM. The price at a YTM of 10 percent is \$884.1740. The percentage price change is

$$\frac{884.1740 - 920.8728}{920.8728} = -0.039852 \equiv -3.9852\%.$$

The percentage price change as captured by duration is $-D_m \times dy$. The modified duration of the bond is

$$\frac{8.5378}{2 \times 1.045} = 4.0851.$$

The price change as captured by duration is $-4.0851 \times 0.01 = -0.040851 \equiv -4.0851\%$.

The price change because of the convexity effect is $0.5 \times 20.3430 \times (0.01)^2 = 0.001017 \equiv 0.1017\%$

The percentage price change as captured by both the factors combined is $-4.0851\% + 0.1017\% = -3.9834\%$. The result is fairly close to the exact percentage price change, and a combination of duration and convexity does a better job than duration alone.

A Concise Formula

There is a closed-form expression for computing the convexity of a plain vanilla bond. It may be stated as

$$\frac{1}{P}\left\{\frac{2C}{y^3}\left[1 - \frac{1}{(1+y)^N}\right] - \frac{2CN}{y^2(1+y)^{N+1}} + \frac{N(N+1)\left(M - \dfrac{C}{y}\right)}{(1+y)^{N+2}}\right\}$$

where, as before, $C \equiv$ periodic (in our case, semiannual), coupon $y \equiv$ semiannual YTM, and $N \equiv$ number of coupons remaining until maturity.

In our illustration, $C = \$35$, $y = 0.045$, $N = 10$, and $P = \$920.8728$. Substituting for the variables, we get convexity = 81.3718.

Dollar Convexity

The dollar convexity of a bond is equal to its convexity multiplied by its price. In our illustration, the price is 920.8728 and the convexity is 20.3430. The dollar convexity is 18,733.3150.

Properties of Convexity

The following properties are valid for the convexity of plain vanilla bonds:[14]

1. As the YTM increases, the convexity decreases and vice versa. This property of option-free bonds is referred to as *positive convexity*.
2. For a given value of the YTM and time to maturity, the lower the coupon, the greater the convexity of the bond. For a given yield and time to maturity, a zero-coupon bond will have the highest convexity.
3. For a given value of the YTM, *and modified duration*, the lower the coupon, the lower the convexity of the bond. For a given yield and modified duration, a zero-coupon bond will have the lowest convexity.

Immunization

Consider the case of a pension fund that promises to pay a return of 4.9688 percent per annum compounded semiannually on an initial investment of \$100,000 after eight years. If it were to invest the corpus in a bond, it is exposed to two types of risks. The first is reinvestment risk, or the risk that the cash flows received at intermediate stages may have to be invested at lower rates of interest. The second, termed *price* or *market risk*, is the risk that interest rates could increase, causing the price of the bond to fall at the end of the investment horizon. The two risks work in opposite directions. The issue, therefore, is whether there is a bond that will ensure that the terminal cash flow is equal to the amount required to satisfy the liability, irrespective of whether rates rise or fall. The process of protecting a bond portfolio against a change in the interest rate is termed *immunization*.

We are considering a simple immunization strategy in which we have to immunize a portfolio required to satisfy a single liability. We need to satisfy two conditions in such cases. First, the present value of the liability should be equal to the amount invested in the bond at the outset. Second, the duration of the bond should be equal to the investment horizon.

Consider a bond with a face value of \$1,000 and 10 years to maturity. Assume that the coupon rate is equal to the YTM and equal to 4.9688 percent per annum. It can be shown that the duration is eight years.

Assume a one-time change in interest rates right at the outset. Assume that the interest rate increases from 4.9688 percent per annum to 5.50 percent per annum. The terminal value of the coupons from the bond at the end of eight years is given by

$$100 \times \frac{24.844}{.0275}[(1.0275)^{16} - 1] = \$49{,}101.6306.$$

If the YTM for two-year bonds after eight years is 5.50 percent per annum, the price at which 100 bonds can be sold is $100 \times 990.0681 = \$99{,}006.8080$. The total terminal cash flow is $148,108.4386. Table 4.12 gives the terminal cash flow for various values of the interest rate.

Analysis

The income from reinvested coupons steadily increases with the interest rate, whereas the price of the bond at the time of sale steadily decreases with the interest rate. When there is no change in the rate, the terminal cash flow from the bond is exactly adequate to meet the liability. In all other cases, there is a surplus. Under all circumstances, the terminal cash flow from the bond is adequate to meet the liability. If the pension fund were to invest in an asset, whose duration is equal to the time to maturity of its liability, then the funds received will always be adequate to meet the contractual outflow. To meet this requirement, four conditions must be satisfied. First, the amount invested in the bonds must be equal to the present value of the liability. In this case, because the bond is assumed to be selling at par, we need to invest in 100 bonds. Second, the duration of the asset must equal the maturity horizon of the liability. Third, there must be a one-time change in the interest rate right at the very outset. Fourth, there must be a parallel shift in the yield curve.

Treasury Auctions

The Treasury issues T-bills, T-notes, and T-bonds by way of auctions. The auction procedure is described in detail in the next chapter on *money market securities*. In Examples 4.13 and 4.14 we will illustrate two recent auctions of notes and bonds to describe the salient features.

Example 4.13 (Issue of a Five-Year T-Note)

Consider the following details:

- Interest rate = 2.5000 percent
- High yield = 2.6050 percent
- Price = $99.510730

Table 4.12
An Immunized Portfolio

Interest Rate (annualized)	Cash Flow from Reinvested Coupons	Proceeds from Sale of Bonds	Terminal Cash Flow per Bond	Terminal Cash Flow from 100 Bonds	Deviation
0.010	412.7639	1078.394	1491.1576	149,115.76	1026.28
0.015	420.6656	1068.094	1488.7601	148,876.01	786.53
0.020	428.7544	1057.921	1486.6752	148,667.52	578.04
0.025	437.0349	1047.871	1484.9056	148,490.56	401.08
0.030	445.5118	1037.943	1483.4544	148,345.44	255.96
0.035	454.1900	1028.134	1482.3244	148,232.44	142.96
0.040	463.0744	1018.445	1481.5190	148,151.90	62.42
0.045	472.1701	1008.871	1481.0415	148,104.15	14.67
0.049688	480.8948	1000.000	1480.8954	148,089.48	0.00
0.050	481.4823	999.4131	1480.8948	148,089.54	0.06
0.055	491.0163	990.0681	1481.0844	148,108.44	18.96
0.060	500.7776	980.8346	1481.6122	148,161.22	71.74
0.065	510.7716	971.7112	1482.4829	148,248.29	158.81
0.070	521.0043	962.6962	1483.7005	148,370.05	280.57
0.075	531.4813	953.7881	1485.2694	148,526.94	437.46
0.080	542.2087	944.9853	1487.1940	148,719.40	629.92
0.085	553.1925	936.2864	1489.4789	148,947.89	858.41
0.090	564.4392	927.6898	1492.1290	149,212.90	1123.42
0.095	575.9551	919.1942	1495.1493	149,514.93	1425.45
0.100	587.7467	910.7981	1498.5448	149,854.48	1765.00

- Accrued interest = None
- Issue date = March 31, 2010
- Maturity date = March 31, 2015
- Original issue date = March 31, 2010
- Dated date = March 31, 2010.

This is an issue of a brand-new security with a maturity of five years. The issue date is the same as the original issue date: March 31, 2010. The maturity date is five years after the issue date because the security has a life span of five years. The dated date is the first date from which coupons begin to accrue; in this case, it is the same as the issue date.

The auction cleared at a YTM of 2.605 percent; that is, the quotes were arranged in ascending order of bids (yields), and the highest bid at which some securities were awarded was 2.605 percent. The coupon was unknown at the outset because this was a new issue. The Treasury will set the coupon after rounding down the market yield to the nearest multiple of 0.125, which in this case is 2.5 percent. The issue price may be determined as follows:

$$P = \frac{1.25}{0.013025}\left[1 - \frac{1}{(1.013025)^{10}}\right] + \frac{100}{(1.013025)^{10}} = 99.510730.$$

Example 4.14 (Reopening of a 30-Year T-Bond)

Consider the following details.

- Interest rate = 4.6250 percent
- High yield = 4.7700 percent
- Price = \$97.692939
- Accrued interest = 7.53798
- Issue date = April 15, 2010
- Maturity date = February 15, 2040
- Original issue date = February 16, 2010
- Dated date = February 15, 2010.

This is a reopening of an existing treasury security. In other words, on February 16, 30-year T-bonds were issued, and on April 15 a fresh issue was made of 29-year, 10-month bonds with the same coupon; that is, the auction added to the supply of securities that were already available in the market. This is the import of the word *reopening*.

The issue date is two months after the original issue date. February 15 was a market holiday. Although the dated date is February 15, the original issue date is February 16.

The auction cleared at 4.77 percent. The coupon was known from the outset, because the security was already in existence. The accrued interest and price may be computed as follows. The number of days from the dated date until the issue date is 59. The total number of days in the coupon period is 181. The accrued interest per $1,000 of face value is given by

$$AI = \frac{46.25}{2} \times \frac{59}{181} = \$7.537983.$$

The clean price may be determined as follows. The fractional period until the next coupon is 122/181 = 0.6740. The dirty price is given by

$$P_o = \left[\frac{\frac{0.04625 \times 100}{2}}{\left(1 + 0.6740 \times \frac{0.0477}{2}\right)} \times \frac{\left[\left(1 + \frac{0.0477}{2}\right)^{60} - 1\right]}{\left(\frac{0.0477}{2}\right)\left(1 + \frac{0.0477}{2}\right)^{59}} \right]$$

$$+ \left(\frac{100}{\left(1 + 0.6740 \times \frac{0.0477}{2}\right)\left(1 + \frac{0.0477}{2}\right)^{59}} \right) = \$98.4467.$$

The clean price is = $98.4467 − $0.7538 = $97.6929.

Frequency of Issue

Various Treasury notes—2, 3, 5, and 7 years—are auctioned on a monthly basis. Ten-year T-notes are auctioned on a quarterly basis in February, May, August, and November. Reopenings take place in March, April, June, July, September, October, and December. Issues are reopened one month after original issuance and, once again, two months after original issuance.

Thirty-year T-bonds are auctioned in February, May, August, and November. The reopening schedule is identical to that for 10-year T-notes.

When Issued Trading

The *when issued* (WI) market is a market for forward trading of a bond whose issue has been announced but has not yet taken place. Trades in this market take place from the date of announcement of a forthcoming auction of government securities until the actual issue date of the securities. The WI market helps potential bidders to gauge the market's interest in the security before the actual auction. This will have ramifications for the bidding strategies of market participants and will have a bearing on the outcome of the auction.

Traders can take both long and short positions in the WI market, with settlement being scheduled for the issue date. Trades in this market represent forward contracts, with the issue date being the settlement date. On and before the date of the auction, the issues trade on a yield basis. The actual price in dollars can be established only after the Treasury sets the coupon. A five-year T-note is quoting in the WI market at 5 percent. If the coupon is set at 4.875, then the price of the security will be 99.4530. However, if the coupon is set at 5.125, the price will be 100.547. Starting with the day after the auction, securities in the WI market are quoted on a price basis because the coupon has already been established.

Price Quotes

Bond prices are always quoted per \$100 of face value. The reason is the following. Consider two bonds, one with a face value of \$1,000 and another with a par value of \$2,000. Assume that both securities are quoting at \$1,500. The first is a premium bond and the second a discount bond. However, this cannot be discerned from the observed price unless the corresponding face value is also specified. On the other hand, if the price were to be expressed for a par value of \$100, then any value less than 100 would signify a discount bond whereas a value more than 100 would connote a premium bond.

In the United States, market prices are quoted as a percentage of par plus thirty-seconds. Consider a quote of 97-08. This does not mean a price of \$97 and 8 cents per \$100 of face value. What it means is that the price per \$100 of face value is 97 + 8/32 or \$97.25. So, if we were to buy bonds with a face value of \$1 million, the amount payable will be $1{,}000{,}000 \times 97.25/100 = \$972{,}500$.

The quoted prices are always clean prices. To compute the exact amount payable by the buyer, the accrued interest has to be calculated per the prescribed convention and added to the clean price.

Bond Futures

Futures contracts are available on 2-, 5-, and 10-year T-notes and on 30-year T-bonds. The underlying asset for the 2-year note contract is a T-note with a face value of \$200,000. Multiple grades are allowable for delivery with the stipulation that the deliverable grade must have an original maturity of not more than 5 years and 3 months, an actual maturity of not less than 1 year and 9 months from the first day of the delivery month, and not more than 2 years from the last day of the delivery month. At any point in time, 5 contract months from the March quarterly cycle are listed. The last trading day is the last business day of the calendar month.

The underlying asset for the 5-year note contract is a T-note with a face value of \$100,000. Multiple grades are allowable for delivery with the stipulation that the deliverable grade must have an original maturity of not more than 5 years and 3 months and an actual maturity of not less than 4 years and 2 months from the first day of the delivery month. At any point in time, 5 contract months from the March quarterly cycle are listed. The last trading day is the last business day of the calendar month.

The underlying asset for the 10-year note contract is a T-note with a face value of \$100,000. Multiple grades are allowable for delivery with the stipulation that the deliverable grade must have an actual maturity of not less than 6 years and 6 months from the first day of the delivery month and not more than 10 years from that date. At any point in time, 5 contract months from the March quarterly cycle are listed. The last trading day is the seventh business day preceding the last business day of the calendar month.

The underlying asset for the 30-year bond contract is a T-bond with a face value of \$100,000. Multiple grades are allowable for delivery with the stipulation that the deliverable grade must, if it is callable, not be callable for at least 15 years from the first day of the delivery month. Noncallable bonds must have a maturity of at least 15 years from the first day of the delivery month. At any point in time, 5 contract months from the March quarterly cycle are listed. The last trading day is the seventh business day preceding the last business day of the calendar month. All note and bond futures contracts are delivery settled.

STRIPS

STRIPS is an acronym for *separate trading of registered interest and principal of securities*. As the name suggests, these represent zero-coupon debt securities created by separately selling the entitlement to each cash flow from the mother security. As we saw earlier, a conventional bond is nothing but a portfolio of zero-coupon bonds. A 10-year T-note can be stripped into 21 zero-coupon securities, where the first 20 represent the respective coupon payments and the last represents the face value. The U.S. Treasury does not strip eligible securities. However, a trader who holds a Treasury note or bond can seek to have the issue converted into the coupon and principal components. The request can be made at any point in time before the maturity date of the security. Once stripped, each zero-coupon component can be traded separately. Each component will have a unique CUSIP number.[15] All interest-based strips that mature on the same day are assigned a common CUSIP number, regardless of the mother bond from which they are created. However, principal strips that mature on a given day are assigned different CUSIP numbers.

A Treasury note or a bond can be not only stripped but also reconstituted; that is, a trader who holds the entire set of component zeros can have the original or parent Treasury security regenerated.

Why is the STRIPS program popular? As we have seen, immunization strategies require the matching of the duration of the assets of the institution with that of its liabilities. Unlike coupon-paying bonds, which have constantly changing durations, zero-coupon bonds always have a duration equal to their remaining term to maturity. Zeros are invaluable for implementing such a strategy. Besides, whereas a 30-year note will have a duration that is substantially less than 30 years, by stripping it into 61 component securities, we can create zeros with durations ranging from 6 months to 30 years.[16]

Endnotes

1. The same can be explained by demonstrating that the partial derivative of the price with respect to the yield is negative. This is shown in the appendix.
2. Remember we are using the actual−actual day-count convention.

3. Even if the bond were to mature after a year, there will be a capital gain or a loss.
4. The key term is *single*.
5. As explained earlier, we are pricing the bond based on 20 half years to maturity and not on 10 years.
6. On an *ex post* basis, we will know the selling price.
7. Notice that we have taken the coupons only for the first 16 periods.
8. See Choudhry (2004).
9. See Choudhry (2004).
10. Sunil Parameswaran, *Futures and Options: Concepts and Applications* (New Delhi: McGraw-Hill, 2010).
11. In other words, there is no call premium.
12. See Livingston (1990).
13. In general, we have to divide by the square of the frequency of compounding.
14. See Fabozzi (1996).
15. CUSIP is the acronym for Committee on Uniform Securities Identification Procedures.
16. See Sundaresan (2009).

CHAPTER 5

Money Markets

Introduction

The money market is an important segment of the debt market of countries with active financial markets. Debt markets exist so that parties in need of funds can interact with those who are seeking to deploy surplus funds. So there are two key constituents of such markets: *borrowers* or *issuers*, or those who are ready to borrow money by issuing securities; and *lenders* or *holders*, or those who are willing to lend in the process of acquiring securities. However, all debt-market transactions are not comparable. There are differences between a 20-year term loan negotiated by a company with its bank, a 30-year mortgage loan negotiated by a home buyer with a savings-and-loan institution, and a 3-month bank loan that is arranged by a corporation with its bank to fund its inventories.

The purpose for which money is borrowed varies from borrower to borrower. And, in the case of the same borrower, it may vary from transaction to transaction. A company may be negotiating a term loan to facilitate the construction of a factory. The nature of the requirement is such that it needs capital for a relatively long period of time. At the same time, however, the company may need an overdraft from its bank to pay outstanding wages. The first is a capital market transaction, and the second is a short-term or current account transaction.

The different motives for borrowing funds led to the creation of different types of debt securities that differ with respect to their magnitude, tenor, and risk features. Our focus here is on the money market or the market for short-term credit. Loans in the money market have an original term to maturity of one year or less. In the case of debt securities, we need to distinguish between the original term to maturity of a security and its actual or current term to maturity. The *original term to maturity* is the time to maturity of the instrument at the time of issue. It cannot change after the issue is made. On the other hand, the *actual* or *current term to maturity* is the term remaining to maturity at the point of consideration. With the passage of time, the actual term to maturity will keep declining. The *capital market*, in contrast to the money market, is an arena in which securities with an original term to maturity of more than one year are issued and traded.

The money market consists of transactions to meet short-term cash needs. It is an arena in which parties with temporary cash surpluses are given an opportunity to interact with those who have temporary cash deficits. The first category of people are potential investors in money market securities, whereas the second category represents potential issuers of such securities. Participants in the money market include corporations, financial institutions, and governments. The nature of transactions ranges from overnight deals to those with a maturity of as long as one year, although some markets are occasionally characterized even by intraday transactions.

Why does an economy require a money market? The reason is that, in real life, for most of us, both individuals and institutions, inflows and outflows of cash will rarely be synchronized. Take the case of the federal or central government of a country. Such entities raise income primarily in the form of tax revenues. However, such income is said to be "lumpy" in nature and is never received in the form of uniform or predictable cash flows. The problem is that a government has to make payments throughout the year in the form of wages, pension payments, and other budgeted expenditure. There will be periods when the government is flush with tax revenues. At such times, the government needs to deploy the surplus funds meaningfully until they are required. At other points in time, when the availability of cash is low relative to budgeted expenses, the same government will need to raise funds on a short-term basis. The same is true for a corporation. When there is a surge in sales-related revenues, the firm's checking account with its bank will show a healthy credit balance. However, as it starts spending on supplies, wages, taxes, and other annual items of expenditure, it may find that it suddenly needs funds urgently to cover short-term deficits. At such points in time, it will need to negotiate for a temporary line of credit such as an overdraft, or it must resort to the issue of other money market securities. The key feature of a money market is its ability to help governments and businesses bridge the gap between their receipts and expenditures. In other words, it facilitates what is termed *liquidity management*.

Why is so much significance attached to the ability of these entities to borrow and lend for relatively short durations? The reason is that money is a highly perishable commodity. When idle cash is not invested, the holder incurs an opportunity cost in the form of forgone interest income, and such income that is forgone is lost forever. When large amounts of funds are involved, the lost income can be substantial. Take, for instance, the case of an institution that has $12 million dollars available overnight. If we assume that the interest rate is 12 percent per annum and that the year consists of 360 days, which is a common assumption in the case of money market securities, the loss if the funds are kept idle is

$$12{,}000{,}000 \times 0.12 \times \frac{1}{360} = \$4{,}000.$$

For one week, that amounts to $28,000 of lost income. Similar analogies can be given from the hotel and airline industries. If the New York Hilton has 20 unoccupied rooms on a given day, then the revenue that is lost is lost forever. The hotel cannot accommodate two different guests in the same room on a subsequent day. Similarly, if British Airways were to fly from New York to London with 10 empty seats, then the income lost is lost forever.

It must be mentioned that no entity is usually a permanent borrower or lender in the money market. A corporation may issue commercial paper to borrow a large amount of money. However, it is conceivable that it may re-enter the market subsequently as a lender, because of a sudden surge of cash receipts. Also, there are institutions that simultaneously operate on both sides of the market. Take the case of a large commercial bank such as Citibank. At any point in time, it will be issuing negotiable certificates of deposit and borrowing federal funds, activities that warrant its classification as a borrower. However, at the same time it is likely that it is making short-term loans to satisfy the working capital needs of its corporate clients. One institution that almost always is on the demand side of the money market is the government. The U.S. Treasury is the largest of all money market borrowers worldwide at any point in time.

Investors in the money market seek to keep their temporary surpluses gainfully invested, without compromising credit risk or liquidity. Because cash inflows and outflows cannot be perfectly anticipated, liquidity becomes paramount because players may seek to enter and exit the market in a sudden and unpredictable fashion. It is imperative from the standpoint of such parties that the market be characterized by the presence of plenty of active buyers and issuers of securities who can facilitate transactions without a major price impact. Safety is important because investments are made for short periods and may need to be liquidated at any time to meet future cash requirements. For this reason, money market investors are especially sensitive to default risk. Even the slightest perception of a decline in credit quality can bring the market to a grinding halt. Rose (2000) gives two significant examples from the U.S. money market. In 1970, the Penn Central Transportation Company defaulted on its commercial paper. Immediately, the paper market came to a grinding halt because of the reluctance of investors to subscribe to the paper of even highly rated companies. A second significant incident was the failure of the Continental Illinois Bank and its subsequent need for government support. This led to an immediate increase in the rates of all bank negotiable certificates of deposit because of a fear among market players that all banks had become more risky.

Money market securities, like other debt securities, are in principle vulnerable to interest-rate risk. However, the prices of such securities are relatively more stable over time. Whereas such securities do not offer prospects for significant capital gains, they also do not raise the specter of significant capital losses for essentially two reasons. First, interest-rate movements over relatively short periods of time usually tend to be moderate. Second, the price impact due to a given interest-rate change is greater, the longer the term to maturity of the cash flow being impacted.

Similarly default risk is minimal in the money market. This is because the ability to borrow in such markets is restricted to well-established institutions with impeccable credit ratings, usually from multiple rating agencies.

The money market is extremely liquid and can absorb large volumes of transactions with only relatively small effects on security prices and interest rates. It is possible for borrowers to issue large quantities of securities at short notice, often in a matter of minutes. The market consists of a wide network of securities dealers, brokers, and banks. There is a large volume of interbank trading that is often brokered. Money market dealers also assume large positions in securities and access the market both to acquire such securities and to fund such acquisitions. Brokers in these markets have access to good and accurate information, and they provide traders with the latest information on the best available prices and rates. Such intermediaries also facilitate the maintenance of confidentiality in the market because large institutions would be unwilling to disclose their identity before a trade, fearing that such information may tend to influence the eventual outcome. In the case of brokered transactions, however, the intermediary will reveal the identity of the principal that it is representing to the counterparty only after the deal has been wrapped up.

Such brokers provide up-to-date indicative interest rates to the market via information-service providers such as Reuters. Typically, rates are quoted for all freely convertible currencies. They charge a standard commission for their services. But large clients can often negotiate the rates payable.

The market is dominated by active traders who are consistently looking out for arbitrage opportunities. They will move money in an instant from a corner of the market that is offering a lower yield to another corner that is offering a higher yield. The market is overseen by the Federal Reserve in the United States and by other central banks in their respective countries. There is no central trading arena for money market securities, and the market is entirely over the counter. Participants negotiate trades over the phone or through computer networks. Speed is of the essence because money, as we have seen, is highly perishable. Most trades are consummated in a matter of minutes, and at times even in seconds.

Money markets may be securities market dominated or bank dominated. In a securities-dominated market most borrowing and lending is through open-market trading of financial instruments. In a bank-dominated market, bank borrowing and lending is at the center of most transactions. Markets in Western economies are largely securities dominated whereas Asian markets, like those in Japan and Korea, tend to be largely bank dominated. Bank-dominated money markets have a potential weakness: they are more vulnerable to pressures from the government, particularly in countries where the banking sector is characterized by substantial government ownership or control. This raises the possibility of bad loans, and it has the potential to impede the development of financial markets in such societies.

Market Supervision

Money markets are overseen by the respective central banks. The major central banks in the world are listed in Table 5.1.

The Federal Reserve System

The Federal Reserve system, commonly known simply as the Fed, consists of 12 district Federal Reserve banks. These are identified by the cities in which they are located. The 12 banks are listed in Table 5.2.

Table 5.1
Major Central Banks

The Federal Reserve
The European Central Bank
The Bank of England
The Bank of Japan
The Swiss National Bank
The Bank of Canada
The Reserve Bank of Australia
The Reserve Bank of New Zealand

Table 5.2
The District Federal Reserve Banks

Federal Reserve Bank of Boston
Federal Reserve Bank of New York
Federal Reserve Bank of Philadelphia
Federal Reserve Bank of Cleveland
Federal Reserve Bank of Richmond
Federal Reserve Bank of Atlanta
Federal Reserve Bank of Chicago
Federal Reserve Bank of St. Louis
Federal Reserve Bank of Minneapolis
Federal Reserve Bank of Kansas City
Federal Reserve Bank of Dallas
Federal Reserve Bank of San Francisco

The Fed sets and implements the monetary policy of the United States, which has implications for the level of interest rates in the economy and for the rate of inflation. Considering the importance of the United States in global financial markets, the actions of the Fed resonate beyond the borders of the country.

The Fed sets and targets two important interest rates in the economy. The first is the discount rate, which is actually set by the Fed. This is the rate at which banks borrow from the central bank. The greater the discount rate, the higher the rates charged by commercial banks for extending credit. The second rate, which is targeted by the Fed, is the federal funds rate. This is the rate at which banks extend loans, typically overnight, to other depository institutions.

Key Dates in the Case of Cash Market Instruments

There are three key dates in the case of such instruments: transaction date, value date, and maturity date.

The transaction date is the date on which the terms and conditions of a financial instrument such as the term to maturity, transaction amount, and price are agreed upon. In other words it is the date on which the counterparties enter into a contract with each other.

The value date is the date on which the instrument starts to earn or accrue a return. The value date may or may not be the same as the transaction date. If the two dates were to be the same, then the transaction is said to be for *same-day value* or *value today*. In other cases, the value date will be the following business day. Transactions with such a feature are said to be for *next-day value* or *value tomorrow*. Finally, there are markets and transactions in which the value date will be two business days after the transaction date. A transaction with such a feature is referred to as a *spot transaction* and is said to be for *spot value*.

The maturity date is the date on which the instrument ceases to accrue a return. The maturities of money market instruments are often fixed not by agreeing to a specific date of maturity but by agreeing to a term to maturity, which will be a number of weeks or months after the value date. Assume that today is June 21, 20XX. A bank will usually quote an interest rate for a one-month deposit rather than for a deposit with a specific maturity date in the following month. The date on which the one-month deposit will mature is then fixed according to market convention. Many money markets follow two conventions[1]: the modified following business day convention and the end–end rule.

The Modified Following Business Day Convention

This convention consists of the following three rules.

1. Maturity is set for the same date as the value date. If the value date is June 21, 20XX, the maturity date for a three-month deposit will be September 21, and for a six-month deposit will be December 21.
2. However, if the maturity date per the first rule happens to be a nonbusiness day, then it will be moved to the following business day. Take the case of a three-month deposit with a value date of June 21, 20XX. Under normal circumstances, the maturity date will be September 21, 20XX. However, if September 21 were to be a Saturday, then the maturity date would be taken as Monday, September 23, 20XX, assuming, of course, that this day is not a market holiday.
3. If the following business day, according to the second rule, were to fall in the next calendar month, then the maturity date will be moved back to the last business day of the maturity month. Consider a one-

month deposit made on July 31, 20XX. The maturity date per the first rule will be August 31, 20XX. Assume that August 31 is a market holiday. So, according to the previous rule, the maturity date should be taken as the following business day. In this case, however, the following business day will fall in September, the next calendar month. The maturity date will be moved back to the last business day of August, which we will assume to be August 29.

The End–End Rule

This rule states that if the value date is the last business day of the current calendar month, then the maturity date will be the last business day of the relevant calendar month. For example, take a one-month deposit with a value date of May 31. It will mature on June 30, assuming that it is a business day. Similarly, a one-month deposit with a value date of June 30 will mature on July 31, assuming once again that it is a business day.

A one-month deposit with a value date of January 31 will mature on February 28 or 29, depending on whether or not it is a leap year. But a one-month deposit with a value date of February 28 or 29 will mature on March 31. However, if the maturity date per this rule were to be a holiday, then the modified following business day convention would apply—that is, the maturity date will be taken as the last business day of the maturity month.

The Interbank Market

The interbank market is a market for large or wholesale loans and deposits. As the name suggests, it is primarily an arena for borrowing and lending deals between commercial banks, with tenors less than or equal to one year.

Why is there a need for an interbank market? On any given day, certain commercial banks will have a surplus of funds and others will be confronted with the specter of a deficit. For an institution whose inflow of deposits exceeds its outflow of loans, there will be a surge in its reserves. On the contrary, banks whose demand for funds on account of loan requests by their clients exceeds the availability of funds by way of deposits will face a depletion of reserves. The term *reserves* refers to the balance that a commercial bank has on deposit with the regional Federal Reserve bank in the United States and in general refers to the balance that a bank must maintain with its central bank as a result of statutory requirements. Such reserve balances do not earn any interest for banks with a surplus. It is prudent for banks with surpluses to lend them to those with a deficit, despite the fact that interest rates for such transactions tend to be relatively low. Although the loan of funds is a manifestation of the desire of lending institutions to keep idle resources productively invested on the part of the borrowers, it reflects the fact that they cannot leave the shortfall in reserves uncovered because of statutory regulations. Loans made in the interbank market are unsecured.

Types of Loans

Loans in the interbank market can take several forms.

- **Overnight money:** This term refers to money that is borrowed or loaned on a given banking day and is scheduled to be repaid on the next banking day. *Weekend money* is the term used to describe loans that are made on Friday, with repayment being scheduled for the following Monday. Interest on weekend money is, however, payable for a period of three days.
- **Call money:** This term refers to deposits for an unspecified term on a day-to-day basis. The lender can call back the funds at any time, and they will be repaid on the same day.
- **Notice money:** This is money that is loaned with a short notice of withdrawal—for example, with seven days' notice.
- **Term money:** This is money that is loaned or deposited for a fixed period such as a week or a month.
- **Intraday money:** This term refers to money that is loaned and repaid on the same day. Borrowing takes place in the morning of the business day, and repayment is scheduled for the same afternoon.

LIBOR

LIBOR is an acronym for London Inter-Bank Offer Rate. It may be defined as the rate at which a top-rated bank in London is prepared to lend to a similar bank. It is the main *benchmark* rate in the London interbank market. LIBOR is quoted for different lengths of time: 1 month, 2 months, 3 months, 6 months, and 12 months. There are several LIBOR rates that are quoted at any point in time, and any quotation must be prefixed by its term to maturity. Every bank in London will quote its own LIBOR rate for each tenor, but usually the rates quoted by competing banks are identical, with some minor differences being observed occasionally.

Although LIBOR is the globally recognized benchmark for loans granted by a top-rated depository institution to parties with a high credit rating, highly rated banks and companies can often borrow short term at a rate below the prevailing LIBOR. On the other hand, institutions with a lower credit rating may have no option but to borrow at a rate above LIBOR. Indicative LIBOR rates for the London market as a whole are produced daily by the British Bankers Association (BBA). The rate is computed from the LIBOR rates quoted at 11 A.M. London time every day by a number of top London banks for loans with different tenors.

BBA LIBOR

BBA LIBOR is the most widely used benchmark or reference rate for short-term interest rates. It is compiled by the BBA in conjunction with Reuters and released shortly after 11 A.M. London time each day. The BBA maintains a reference panel of at least eight contributor banks. The objective is to provide a reference panel that reflects the balance of the market by country and type of institution. Individual banks are selected on the basis of reputation, scale of market activity, and perceived expertise in the currency concerned. The BBA publishes the quotes of the panel on screen. The top-quartile and bottom-quartile market quotes are disregarded, and the middle two quartiles are averaged to arrive at the BBA LIBOR rate. The quotes from all panel banks are published on screen to ensure transparency. BBA LIBOR rates are provided for 10 currencies, which are listed in Table 5.3.

Table 5.3
Currencies for which BBA LIBOR is Reported

Currency Name	Symbol
Pound sterling	GBP
U.S. dollar	USD
Japanese yen	JPY
Swiss franc	CHF
Canadian dollar	CAD
Australian dollar	AUD
Euro	EUR
Danish krona	DKK
Swedish krona	SEK
New Zealand dollar	NZD

There are 15 different maturities for each currency. The shortest maturity is overnight (O/N) for euro, U.S. dollar, pound sterling, and Canadian dollar and spot/next (s/n) for all other currencies. The quotation for one week is 1w, and 1m stands for one month. The longest maturity for which BBA LIBOR is fixed is 12 months. An overnight rate that is quoted on a day has a value date of the same day and matures on the following day. The s/n rate that is quoted on a given day has a value date that is two days later and a maturity date the day after that.

LIBID

LIBID is an acronym for the London Inter-Bank Bid Rate. It is the rate that a bank in London with a good credit rating is prepared to pay for funds deposited with it by another highly rated London bank. Just like LIBOR, LIBID is quoted for a number of tenors. Whereas LIBOR represents the rate that a bank seeking to borrow in the interbank market has to pay on a loan it seeks, LIBID is the rate that a bank with surplus funds will have to accept on funds deposited by it with another bank. LIBID will be lower than LIBOR. Although the size of the spread between the two rates can vary, the difference is usually just a few basis points for the same tenor.

In an interbank transaction, the rate that is agreed on will often be somewhere between LIBID and LIBOR and is often the average of the two rates. Some depository institutions therefore use the London Inter-Bank Mean Rate (LIMEAN), which is an arithmetic average of the LIBID and the LIBOR, as the reference rate for their interbank transactions.

SONIA and EURONIA

SONIA is the acronym for the Sterling Overnight Index Average, and EURONIA is an abbreviation for Euro Overnight Index Average. These indexes track actual market overnight funding rates. SONIA is the weighted-average rate to four decimal places of all unsecured sterling overnight transactions brokered in London by member firms of the Wholesale Markets Brokers' Association (WMBA) between midnight and 4:15 P.M. with all counterparties in a minimum deal size of £25 million. EURONIA is the weighted average to four decimal places of all unsecured euro overnight cash transactions brokered in London by WMBA members between midnight and 4 P.M. London time with all counterparties. The indexes are weighted-average overnight deposit rates for each business day. Each rate in the average is weighted by the principal amount of deposits that were taken on that day. Both indexes are published at 5 P.M. London time every day.

EURIBOR and EONIA

The Euro Inter-Bank Offered Rate (EURIBOR) is a benchmark rate used by the international market for the euro. It is produced by the European Banking Federation based in Brussels. It is compiled from the offer rates of commercial banks in the euro-zone countries for loans in euros. Although the BBA computes a euro LIBOR rate from rates quoted by London banks, the EURIBOR rate is more widely accepted as the standard benchmark rate for the market.

The European Banking Federation takes quotes from 57 banks. Forty-seven of these are selected by national banking associations to represent the euro markets in the participating states. Ten are international banks that are active in the euro market and have an office in the euro zone. The quotes are ranked, and the top 15 percent and the bottom 15 percent are discarded, and the remainder are averaged. The EURIBOR is reported at 11 A.M. Brussels time every day. The rates are spot rates—that is, they start two working days after the measurement day. The interest is computed on an actual (ACT)/360-day basis. Rates are reported for maturities ranging from one week to one year. At any time, 15 rates are quoted for 1 week, 2 weeks, 3 weeks, and 1 to 12 months.

EONIA stands for Euro Over Night Index Average and is the effective overnight reference rate for the euro. It is compiled by taking the weighted average of all overnight unsecured interbank lending transactions within the euro area. The banks that are polled to compute the value of EONIA on a given day are the same as the panel banks whose quotes are used in computing the EURIBOR.

Interest-Computation Methods

For interbank loans and some money market instruments, interest is payable on the principal value of the instrument—that is, interest is paid at the end of the loan period, along with the repayment of principal to the investor. On the other hand, securities such as Treasury bills and commercial paper are termed *discount securities*. In other words, they are issued at a discount to their principal value and repay the principal at maturity. They are analogous to zero-coupon bonds.

For interbank loans, interest is computed and paid along with the principal. The method of calculating the interest differs according to the currency under consideration. Interest on most currencies, including the U.S. dollar and the euro, is calculated on the assumption that the year has 360 days and is based on the ACT/360-day count convention.

This means that the interest payable on a loan for T days with a principal of P and carrying an interest rate of r is

$$P \times \frac{r}{100} \times \frac{T}{360}.$$

For certain currencies such as the sterling, however, interest is calculated on the assumption of a 365 days per year and is said to be based on an ACT/365-day day-count convention. The formula for computing interest is

$$P \times \frac{r}{100} \times \frac{T}{365}.$$

Examples 5.1, 5.2 and 5.3 illustrate the computation of interest for loans of different tenors as well as currencies.

Example 5.1

A bank makes a loan of €10 million from July 15 until October 15 at an interest rate of 5.75 percent per annum. The number of days = 16 + 31 + 30 + 15 = 92. Notice that although the loan is for three months, we consider the actual number of days in the period and do not automatically take the period as consisting of 90 days. The interest is given by

$$10{,}000{,}000 \times \frac{5.75}{100} \times \frac{92}{360} = €146{,}944.44.$$

Example 5.2

A bank makes a loan of \$7.5 million for a period of one year (365) days at an interest rate of 5.25 percent per annum. The interest is given by

$$7{,}500{,}000 \times \frac{5.25}{100} \times \frac{365}{360} = \$399{,}218.75.$$

Notice that because the actual number of days exceeds 360, although the quoted rate is 5.25 percent, the rate paid on a simple interest basis is

$$\frac{399{,}218.75}{7{,}500{,}000} \times 100 = 5.3229\%.$$

Example 5.3

A bank makes a loan of £7.5 million for a period of 180 days at an interest rate of 4.95 percent per annum. The interest is given by

$$7{,}500{,}000 \times \frac{4.95}{100} \times \frac{180}{365} = \text{£}183{,}082.19.$$

Term Money Market Deposits

A term money market deposit is a short-term deposit with a maximum maturity of one year that is placed with a bank or a security house. Such deposits carry a fixed rate of interest, and the rate is linked to the prevailing LIBOR for the same term. Interest is usually paid along with the principal when the deposit matures. These deposits are nonnegotiable—that is, they cannot be sold to another party before maturity by the depositor. Depositors can usually break or terminate the deposit prematurely; in other words, they have the option to withdraw their money before expiration. However, this feature carries an attached penalty.

Such deposits can be placed in any of the major currencies, and settlement happens on the same day for domestic deposits, and in two working days for Eurocurrency deposits.

Federal Funds

Federal funds are the principal means of making payments in the money market in the United States. By definition, the term *federal funds* refers to money that can be immediately transferred from the buyer to the seller or from the lender to the borrower. The term arose because, in the earlier years, the principal way of effecting an immediate fund transfer was by debiting the reserve account held by the lender at its regional Federal Reserve bank and crediting the reserve account maintained by the borrower at its regional Federal Reserve bank. Such transactions are instantaneous and can be effected in a matter of seconds. Today, however, the Fed funds market is broader in scope than just the funds that are available with Federal Reserve banks in the form of reserves held by depository institutions. It is a common practice for all banks to maintain deposits with large correspondent banks in major financial centers. These deposits may also be transferred readily from the account of one bank to that of another.

Large corporate borrowers tend to route their financial dealings through large banks in the major money market centers, especially New York City. For such money center banks, the demand for loans and investments made by their clients inevitably exceeds the quantum of funds at their disposal. On the other hand, for smaller banks located in rural or semiurban areas in states such as Alabama, the availability of funds from deposits by the local population usually exceeds the demand for funds at the local level.

Banks and other depository institutions must hold in a special reserve account liquid assets equal to a fraction of the funds deposited with them by the public. Only vault cash held within the bank and reserve balances kept with the local Federal Reserve bank count in meeting a U.S. bank's requirement to hold legal reserves. An increase in deposits leads to an increase in the availability of fed funds, whereas loans made and securities purchased manifest themselves as a reduction in the availability of such funds. Frequently, some banks hold more legal reserves than what the law requires. Because excess reserves yield nil returns, these banks tend to keep their surpluses gainfully invested by lending to the larger banks. Most lenders tend to make overnight loans of fed funds. This is because the availability of excess reserves tends to vary daily and in a fairly unpredictable fashion. The lending of such funds is termed as a *sale*, whereas the borrowing of such funds is referred to as a *purchase*.

Federal Funds versus Clearinghouse Funds

Transfers of federal funds operate very differently from the payment mechanism used in the capital market. When a money market dealer acquires securities from another party, the dealer immediately instructs its bank to transfer funds from its account to the account held by the selling institution at its bank. The buyer's bank will contact its regional Federal Reserve bank and request the Fed to debit its reserve account and credit the reserve account held by the seller's bank at the same or possibly a different district Federal Reserve bank. All 12 Federal Reserve banks maintain an *interdistrict settlement account* in Washington, D.C. Transfers between regional Federal Reserve banks take place by decreasing the share of the buyer's bank and augmenting the share of the seller's bank. These transactions are so quick that the seller of securities has funds available for immediate use on the day of the transaction. Federal funds are referred to as *immediately available funds*.

In contrast, consider the conventional method of payment that is used in most transactions. The buyer will either issue a check favoring the seller or for personal transactions will pay by a credit card and then subsequently issue a check. Funds that are transferred by means of checks are referred to as *clearinghouse funds* because once the buyer writes a check, it goes to the seller's bank, which eventually forwards that check to the depository institution on which it was drawn. Transfers between the two depository institutions are processed and reconciled through a central processing center known as a *clearinghouse*. Clearinghouse funds are not acceptable in money markets because speed is of the essence. Compared to federal funds, it takes at least a day to clear local checks and two to three days for outstation checks in the United States, which is far too slow for a money market transaction, because no interest can be earned until the check is collected.

Correspondent Banks: Nostro and Vostro Accounts

It is a common practice for banks to maintain accounts with other banks. Foreign banks have accounts with banks in the United States. Within the United States, banks located in smaller towns and cities tend to have relationships with larger banks in cities such as New York and Chicago. Such arrangements are referred to as *correspondent banking relationships*. The bank that maintains an account is referred to as the *respondent bank*, and the bank that offers the account maintenance facility is referred to as the *correspondent bank*. Correspondent banks facilitate check clearing and collection, and foreign-exchange transactions, among other activities. They also participate in loan syndication. For many small banks, the demand for loans at the local level often exceeds available deposits and at times may exceed the bank's legal lending limit. Syndication helps surmount these barriers, besides helping in risk diversification.

From the standpoint of a bank, a *nostro* is its account of money being held by another bank, whereas a *vostro* is its account of money of another bank that it holds. Assume that Barclays Bank is holding an account with

Citibank in New York City. Barclays would refer to this account as its nostro. Citibank, on the other hand, would term it a vostro account. A nostro account with a credit balance is an asset, and in this case it will show up on the asset side of Barclay's balance sheet. Seen from the perspective of a vostro account, a credit balance signifies that the bank is holding the assets of another institution, and it is the manifestation of a liability. On the contrary, a vostro with a debit balance would signify that the bank has made a loan to the counterparty and would therefore be tantamount to an asset.

Payment Systems

There are two networks for large-value fund transfers in the United States. One of them is Fedwire Funds Service, which is operated by the Federal Reserve banks. Fedwire not only facilitates interbank payments but also enables the safekeeping and transfer of government and other securities. The second network is known as the *Clearing House Interbank Payments System* (CHIPS).

Fedwire

Fedwire is a *real-time gross settlement system* (RTGS). An RTGS system is a fund-transfer mechanism that enables transfers between two institutions in both a *real-time* and *gross* basis. An RTGS system represents the fastest possible channel for the transfer of money through the banking system. The term *real-time settlement* means that there is no waiting period for the payee. The phrase *gross settlement* means the transaction is settled on a one-to-one basis and without bunching with any other transaction. Payments over Fedwire are final and irrevocable. All Fedwire participants are required to maintain an account with a Federal Reserve bank.

CHIPS

CHIPS is a privately owned real-time payments system. The system continuously matches, nets, and settles payment orders. To settle positions on a real-time basis, CHIPS requires as many as two rounds of prefunding. This requires members to jointly maintain a prefunded account with the New York Fed bank. Each participating institution has an initial prefunding requirement that, once satisfied, is used to settle payments throughout the day. In the course of the day, institutions will submit payment orders to a centralized queue. When the system finds that it is possible to settle one or more payment orders, its will release the concerned orders from the queue. Simultaneously, accounting entries will be passed to debit the payer and credit the receiver.

However, this process of settlement will be unable to process all the payment orders during the course of the day. After 5 P.M. Eastern time, CHIPS will tally unprocessed orders on a multilateral netting basis. The net position for an institution will be combined with its present position, which will either be zero or positive, to calculate the final net position. Institutions whose final position is negative must transfer the amount to the CHIPS account. This represents the second round of prefunding. The funds received will be used to credit the accounts of institutions that are due to receive the funds. Once the second round of prefunding is fully over, CHIPS will release and settle all the remaining payment orders and transfer the amounts due to participating institutions. The CHIPS account will have a zero balance at the end of the day.

If on a day, one or more participants with negative balances do not transfer the required amounts, CHIPS will settle as many payments as possible, subject to the positive balance requirement, and will delete the remaining messages from the central queue. Participants whose messages were deleted will be provided with details of those messages that were not settled.

Fed Funds and Reserve Maintenance

The legal reserve requirement of banks is calculated on a daily average basis over a two-week period known as the *reserve-computation period*, which stretches from a Tuesday through a Monday two weeks later. The Federal Reserve calculates the daily average of transaction deposits held by each depository institution over this two-week period and then multiplies that average by the required reserve percentage to determine the amount of legal reserves that must be held by each institution. These legal reserves must average the required amount over a two-week period known as the *reserve-maintenance period*. This period starts on a Thursday, 17 days after the transaction deposit reserve-computation period ends, and it ends on a Wednesday two weeks later. While computing the required reserves, the amount of vault cash available is adjusted. Vault cash may be adjusted up to the amount of the reserve requirement. If in any maintenance period, the vault cash held is inadequate to meet the reserve requirement, then the depository institution has a reserve-balance requirement. The requirement for any maintenance period may be computed as the reserve requirement for the computation period minus the vault cash held for the same 14-day period.

The federal funds market is an indispensable tool for this kind of daily reserve management, especially for larger and more aggressive banks that hold few reserves of their own. Many large U.S. banks today borrow virtually all of their legal reserves behind their deposits from the federal funds market.

Treasury Bills

Treasury bills are short-term debt securities issued by the federal government. The U.S. Treasury bill rates set the benchmark for the rates on other money market securities. T-bills carry the lowest yield for a given tenor for the following reasons. First, they are totally devoid of credit risk, because they are backed by the full faith and credit of the federal government. Second, they are highly liquid. Third, income from such securities is exempt from state income tax. In the United States, unlike in many other countries, state governments are empowered to levy income taxes. The governments follow a guideline of mutual reciprocity: federal securities are exempt from state income tax and vice versa.

By law, T-bills in the United States must have an original maturity of one year or less. Regular series bills are issued routinely every week or month by way of competitive auctions. Four-week, three-month, and six-month bills are auctioned every week, and one-year bills are sold usually once a month. Of the four maturities, the six-month bills provide the largest amount of revenue for the U.S. Treasury. On the other hand, cash-management bills are issued only when the Treasury has a special need on account of low cash balances. Such bills have maturities ranging from as short as a few days to as long as six months. They give the maximum flexibility to the Treasury, because they can be issued as and when required. The money raised through these issues is used by the Treasury to meet any temporary shortfalls.

T-bills are sold by an auction process. Four-week bills are normally auctioned on Tuesdays, as are one-year bills. The issue of new regular three-month and six-month bills is announced by the Treasury on the Thursday of every week, with bids from investors being due on the following Monday before 1 P.M. New York time. T-bills are usually issued on the Thursday following Monday's auction. The Treasury entertains both competitive and noncompetitive bids. Large investors submit competitive bids in which they indicate not only the quantity sought but also the minimum yield they are prepared to accept. The *yield* in this case refers to the discount yield for the bill being auctioned. Noncompetitive tenders, on the other hand, are submitted by small investors who agree to accept the yield at which securities are auctioned off, whatever that yield may be. Such bidders have to only indicate the quantities sought by them. Generally, the Treasury fills all noncompetitive tenders.

Besides being traded in the United States, T-bills issued by the U.S. Treasury are actively traded in other major financial centers such as London and Tokyo. The market for such securities virtually operates around the clock. The globalization of the market is characterized by the presence of non-U.S. dealers in the U.S. market, as well as by the activities of American dealers in markets outside the country.

Reopenings

Every T-bill issue is identified with a unique CUSIP number.[2] Some issues are, however, a reopening of an existing issue. In other words, the new issue is identical in all respects to an issue that is already trading in the secondary market. The three-month bill issued three months after the issue of a six-month bill is considered a reopening of the six-month bill. The new issue in this case is given the same CUSIP number. For a given maturity, the most recently issued securities are referred to as *on the run*, whereas those issued earlier are referred to as *off the run*. On-the-run securities generally trade at slightly lower yields because they are more liquid. The reason is that, for some time after the issue of such bills, there tends to be active trading in the secondary market. Thereafter, most securities pass into the hands of investors who choose to hold them until maturity. Off-the-run securities are less liquid, which explains the higher yield.

Cash-management bills are issued via a standard auction process. However, they are irregular with respect to their term to maturity and auction schedule. If a cash-management bill were to mature on the same day as a regular bill, which is usually a Thursday, then it is said to be *on cycle*. In this case, the issue is considered to be a reopening, so it is allotted the same CUSIP. However, if it were to mature on a different day, it would be said to be *off cycle*, and it would carry a different CUSIP number.

Yields on Discount Securities

In the market for fixed-income securities, there are different ways of computing the yield on an instrument. It is very important for an investor to be conversant with the various methods used for calculation and the relationship between the corresponding yields to be able to understand the various instruments that are available.

Notation

We will define the following symbols. The corresponding terms will become clear as we proceed

$d \equiv$ quoted yield on a discount security such as a T-bill
$D \equiv$ discount from the face value in dollars
$V \equiv$ face value of the discount security
$T_m \equiv$ time left to maturity in days
$P \equiv$ price of the discount security
$y_{365} \equiv$ bond equivalent yield of a discount security, with $T_m < 182$ days
$y_{360} \equiv$ money market yield of a discount security, with $T_m < 182$ days
$y_c \equiv$ bond equivalent yield of a discount security, with $T_m > 182$ days

Discount Rates and T-Bill Prices

The quoted yield on a T-bill is a discount rate, which is used to determine the difference between the price of a T-bill and its face value. For the purpose of calculation, the year is treated as if it has 360 days.

If d is the quoted yield for a T-bill, with a face value of $\$V$ and having T_m days to maturity, the dollar discount D is given by

$$D = V \times d \times \frac{T_m}{360}.$$

Given the discount, the price may be calculated as

$$P = V - D.$$

Alternately, the price may be directly computed from the discount rate as follows:

$$P = V - D = V - V \times d \times \frac{T_m}{360} = V\left[1 - \frac{d \times T_m}{360}\right].$$

We will illustrate these concepts with the help of Examples 5.4 and 5.5.

Example 5.4

A T-bill with 90 days to maturity and a face value of \$1,000,000 has a quoted yield of 4.8 percent. What is the price in dollars?

$$D = 1{,}000{,}000 \times 0.048 \times \frac{90}{360} = \$12{,}000.$$

The price is given by

$$P = V - D = \$1{,}000{,}000 - 12{,}000 = \$988{,}000.$$

Example 5.5

A one-year bill (364 days) has just been issued at a quoted yield of 5.4 percent. What is the corresponding price in dollars?

$$D = 1{,}000{,}000 \times .054 \times \frac{364}{360} = \$54{,}600.$$

The price is given by

$$P = V - D = \$1{,}000{,}000 - 54{,}600 = \$945{,}400.$$

The Bond Equivalent Yield

The objective of calculating the *bond equivalent yield* (BEY), also known as the *coupon equivalent yield*, is to compute a yield measure that facilitates comparisons between the rate of return on discount securities such as T-bills and capital market debt instruments such as coupon-paying bonds. The procedure that is adopted to compute the BEY depends on whether the discount instrument under consideration has less than or more than six months left to maturity.

Case A: $T_m < 182$ days

The bond equivalent yield for a T-bill with less than 182 days to maturity is just the equivalent rate of return on a simple interest basis computed under the assumption that the year has 365 days.

The bond equivalent yield for a T-bill with a given discount rate will always be greater than the stated discount rate. The reason is the following. Yield, or the rate of return obtained by an investor, is always computed with respect to the price that is paid for the security. The rate of return on an asset that is bought for P and that pays the face value V at maturity is given by

$$\frac{(V-P)}{P}.$$

For a discount security, the yield that is computed in this manner will always be greater than

$$\frac{(V-P)}{V} = \frac{D}{V}$$

which is the dollar discount expressed as a fraction of the face value. If a T-bill is bought at a price corresponding to a given discount rate and is held to maturity after acquisition, then the rate of return earned by the investor will always be greater than the quoted discount rate.

In this case, the BEY is defined as

$$y_{365} = \frac{(V-P)}{P} \times \frac{365}{T_m}.$$

It can be directly computed given the quoted yield, using the following equation

$$\begin{aligned} y_{365} &= \frac{(V-P)}{P} \times \frac{365}{T_m} = \left(\frac{V}{P} - 1\right) \times \frac{365}{T_m} \\ &= \left[\frac{V}{V\left[1 - \dfrac{d \times T_m}{360}\right]} - 1\right] \times \frac{365}{T_m} \\ &= \left[\frac{360}{360 - d \times T_m} - 1\right] \times \frac{365}{T_m} \\ &= \frac{d \times 365}{360 - d \times T_m}. \end{aligned}$$

Example 5.6 provides an illustration for computing the BEY.

Example 5.6

A T-bill with a face value of $1,000,000 and 90 days to maturity has a quoted yield of 4.75 percent. What is the bond equivalent yield?

The price is given by

$$\begin{aligned} P &= V\left[1 - \frac{d \times T_m}{360}\right] \\ &= 1{,}000{,}000 \times \left[1 - \frac{0.0475 \times 90}{360}\right] = \$988{,}125. \end{aligned}$$

The bond equivalent yield is given by

$$\begin{aligned} i_{365} &= \frac{(V - P)}{P} \times \frac{365}{T_m} \\ &= \frac{(1000000 - 988{,}125)}{988{,}125} \times \frac{365}{90} \\ &= 0.048738 \equiv 4.8738\%. \end{aligned}$$

The Money Market Yield

The BEY can be converted to a money market yield—that is, the yield on a 360-day basis—by multiplying by 360/365. Thus,

$$\begin{aligned} y_{360} &= \frac{d \times 365}{360 - d \times T_m} \times \frac{360}{365} \\ &= \frac{d \times 360}{360 - d \times T_m}. \end{aligned}$$

Example 5.7 demonstrates the computation of the money market yield for a bill.

Example 5.7

Take the case of the T-bill with a face value of $1,000,000 and 90 days to maturity, which has a quoted yield of 4.75 percent. The money market yield is

$$\frac{0.0475 \times 360}{360 - 0.0475 \times 90} = 0.048071 \equiv 4.8071\%.$$

Case B: $T_m > 182$ days

A conventional coupon-paying bond with a time to maturity greater than 182 days (half a year) will make a coupon payment before it matures. It is inappropriate to compare the simple yield on a discount security with the yield to maturity (YTM) of a bond, because the bond will yield an interest payment before maturity but the discount security will not. To facilitate a comparison between the BEY for a discount instrument, such as a T-bill, with more than 182 days to maturity and the YTM for the bond, the T-bill must be treated as if it also would pay interest after six months, and that interest is paid on this intermediate interest for the remaining period left to maturity.

Let us denote the BEY in this case by y. The following equation gives y:

$$P\left(1+\frac{y}{2}\right)\left(1+\frac{y}{2}\left\{\frac{T_m-\frac{365}{2}}{\frac{365}{2}}\right\}\right)=V.$$

The logic is as follows. The future value of P at the end of six months will be

$$P\left(1+\frac{y}{2}\right).$$

The future value of this expression, as calculated on the date of maturity, must equal the face value. The compounding factor for the remaining period on a simple interest basis is

$$\left(1+\frac{y}{2}\times\frac{T_m-\frac{365}{2}}{\frac{365}{2}}\right).$$

Therefore,

$$P\left(1+\frac{y}{2}\right)\left(1+\frac{y}{2}\left\{\frac{T_m-\frac{365}{2}}{\frac{365}{2}}\right\}\right)=V.$$

The expression for y is:[3]

$$\frac{-\frac{2T_m}{365} \pm 2\sqrt{\left(\frac{T_m}{365}\right)^2 - \left(\frac{2T_m}{365} - 1\right)\left(1 - \frac{V}{P}\right)}}{\frac{2T_m}{365} - 1}.$$

We will discard the negative root and retain the positive. The calculation of the BEY for a T-bill with a time to maturity greater than six months can be illustrated with the help of Example 5.8.

Example 5.8

Consider a T-bill with 240 days to maturity and a face value of \$1,000,000 and a quoted yield of 6 percent. What is the bond equivalent yield?

The price of the bill is

$$1{,}000{,}000 - 1{,}000{,}000 \times 0.06 \times \frac{240}{360} = \$960{,}000.$$

Therefore,

$$y_c = \frac{-\frac{2 \times 240}{365} \pm 2\sqrt{\left(\frac{240}{365}\right)^2 - \left(\frac{2 \times 240}{365} - 1\right)\left(1 - \frac{1000000}{960{,}000}\right)}}{\frac{2 \times 240}{365} - 1}$$

$$= \frac{-1.315068 + 2\sqrt{(.432351) - (.315068)(-.041667)}}{.315068} = .0629 \equiv 6.29\%.$$

Holding Period Return

Take the case of an investor who buys a bill at a discount rate of d_1 when there are T_{m1} days left to maturity and sells it at a discount rate of d_2 when there are T_{m2} days left to maturity. The holding period return is given by

$$\begin{aligned}\text{HPR} &= \frac{(P_2 - P_1)}{P_1} \times \frac{365}{(T_{m1} - T_{m2})} \\ &= \frac{\left(1 - d_2 \times \dfrac{T_{m2}}{360}\right) - \left(1 - d_1 \times \dfrac{T_{m1}}{360}\right)}{\left(1 - d_1 \times \dfrac{T_{m1}}{360}\right)} \times \frac{365}{(T_{m1} - T_{m2})} \\ &= \frac{(d_1 T_{m1} - d_2 T_{m2})}{360 - d_1 T_{m1}} \times \frac{365}{(T_{m1} - T_{m2})}.\end{aligned}$$

On a 360-day basis, we would state the equivalent return as

$$= \frac{(d_1 T_{m1} - d_2 T_{m2})}{360 - d_1 T_{m1}} \times \frac{360}{(T_{m1} - T_{m2})}.$$

Example 5.9 illustrates the calculation of the holding period return for a bill.

Example 5.9

A bill with 180 days to maturity is bought at a yield of 6 percent. It is sold 30 days later at a yield of 5.80 percent. The holding period return is

$$\text{HPR} = \frac{(0.06 \times 180 - 0.0580 \times 150)}{360 - 0.06 \times 180} \times \frac{365}{30} = 0.0732 \equiv 7.32\%.$$

Value of an 01

Money market participants are always interested to know the price sensitivity of a security with respect to changes in the yield. In particular, they seek to know how much the price would change if the yield were to change by 1 basis point (bp).

For a T-bill, this is given by

$$P_{01} = 0.0001 \times V \times \frac{T_m}{360}.$$

Consider the bill with 90 days to maturity that is quoting at 5 percent:

$$P_{01} = 0.0001 \times 1{,}000{,}000 \times \frac{90}{360} = \$25.$$

Concept of Carry

The term *carry* may be defined as "The interest income received on the security being financed minus the interest expense incurred in financing the security."[4]

The carry may be positive or negative.

Concept of a Tail

What is a tail? Trading-glossary.com defines a tail as follows: "Calculating the yield at which a future money market security (one available some period hence) is purchased when that future security is created by buying an existing instrument and financing the initial portion of its life with a term repo [repurchase agreement]."

We will illustrate it with the help of Example 5.10.

Example 5.10

Stanley & Co., a brokerage house in New York, is funding the acquisition of a 108-day bill using a 45-day term repo. The quoted rate for the bill is 4.8 percent, and the rate for the repo is 4.56 percent. The initial cost of the bill is

$$1{,}000{,}000\left[1 - 0.048 \times \frac{108}{360}\right] = 985{,}600.$$

The funding cost is

$$985{,}600 \times 0.0456 \times \frac{45}{360} = 5{,}617.92.$$

At the end of 45 days, the brokerage house will be holding 63-day bills. The cost of this bill is

$$985{,}600 + 5{,}617.92 = \$991{,}217.92.$$

This corresponds to a dollar discount of \$8,782.08, which translates into a discount rate of

$$\frac{8{,}782.08}{1{,}000{,}000} \times \frac{360}{63} = 0.050183 \equiv 5.0183\%.$$

The transaction would be attractive from the brokerage house's standpoint only if it were to be of the view that the 63-day bill could be sold 45 days hence at a discount rate that is lower than 5.0183 percent.

Example 5.11 is a detailed example of a Treasury auction.

Example 5.11 (A Treasury Auction)

Bills are arranged in descending order from the highest price if it is a price-based auction or equivalently in ascending order from the lowest yield if it is a yield-based auction. All competitive bids are required to be submitted to three decimal places. The minimum denomination for T-bills is \$100, and bills are issued in multiples of \$100 thereafter. Noncompetitive bidders cannot submit bids for bills with a face value in excess of \$5 million. No competitive bidder can be awarded more than 35 percent of the amount on offer. The highest yield at which at least some bills are awarded is called the *stop-out yield* or *high yield*. No investor who has bid more than the stop-out yield will be awarded any bills. However, once bills are acquired by successful bidders, many of them will be sold right away in the secondary market. This will afford the unsuccessful bidders a second chance to acquire the bills. All bills are these days being issued only in book-entry form. In other words, a computerized record of ownership is maintained in Washington, D.C., and at the Federal Reserve banks, and no physical certificates are issued.

Types of Auctions

There are two types of auctions: uniform price or yield auctions, and discriminatory price or yield auctions. We will illustrate the difference between the two with the help of an example. Assume that four bids (shown in Table 5.4) have been received by the Treasury, which is auctioning $10 billion worth of securities. Assume that no noncompetitive bids have been received.

Table 5.4
Bids in Ascending Order of Yield

Bidder	Bid	Amount ($ Billion)
Alfred	1.245	4.00
Christine	1.260	4.00
David	1.275	4.00
Stephanie	1.280	2.50

Alfred and Christine will get 100 percent of the quantities they seek. When it comes to David, however, the Treasury will have only $2 billion left to allot. He will get 50 percent of the quantity sought by him. The stop-out yield or high yield in this case is 1.275 percent. Stephanie will get no securities and therefore is said to be *shut out* of the auction.

In the case of a discriminatory auction, the successful bidders will be allotted securities at their bids. Alfred will have to pay based on a discount rate of 1.245 percent, Christine at a rate of 1.260 percent, and David at a rate of 1.275 percent. However, in the case of a uniform price or yield auction, all successful bidders will be allotted at the market clearing yield, which in this case is 1.275 percent. Those who have bid less will have no objections to paying a lower price.

Results of a Recent Auction

On April 23, 2009, the Treasury issued 91-day bills with a maturity date of July 23, 2009. The high yield was 0.135 percent. All bids at lower yields were accepted in full. The high yield corresponds to a price of

$$100 - 100 \times 0.00135 \times \frac{91}{360} = \$99.965875.$$

The bond equivalent yield corresponding to this price is

$$\frac{(100 - 99.965875)}{99.965875} \times \frac{365}{91} = 0.00137 \equiv 0.137\%.$$

The median rate was 0.11 percent: that is, 50 percent of the amount of accepted competitive tenders was tendered at or below this rate. The low rate was 0.050 percent. In other words, 5 percent of the amount of accepted competitive tenders was tendered at or below this rate. At the high rate, 59.26 percent of the quantity demanded at that rate was awarded.

The total competitive bids were for \$85,359,617,000. The amount accepted was \$26,110,257,000. The total noncompetitive bids were for \$1,889,862,600. All noncompetitive bids were accepted in full. The total bids, both competitive and noncompetitive taken together, were for \$87,249,479,600, and the total amount accepted was \$28,000,119,600. The ratio of what was sought to what was eventually awarded is called the *bid to cover ratio*. In this case, it was

$$87{,}249{,}479{,}600/28{,}000{,}119{,}600 = 3.116.$$

Of the total competitive bids, bids for \$72,064,000,000 were received from primary dealers, a topic to which we now turn.

Primary Dealers and Open-Market Operations

The money market depends heavily on the buying and selling activities of securities dealers. The term *primary dealer* simply means that the dealer firm is qualified to trade securities directly with the Federal Reserve Bank of New York.

The significance of the New York Fed may be understood as follows. The Federal Reserve periodically conducts open-market operations. Such operations refer to the buying and selling of securities in the secondary market as an instrument of monetary policy. The acquisition of government securities by the Fed adds reserves to the banking system, whereas the sale of such securities amounts to a depletion of reserves. The decisions pertaining to such operations are taken by a body known as the Federal Open Market Committee (FOMC). The New York Fed is permanently represented on this body because New York is the world's largest money market, and all decisions taken by the FOMC are ultimately implemented by the New York Fed.

Primary dealers may be commercial banks or brokerage houses. Foreign-owned institutions are permitted to be primary dealers in the United

Table 5.5
List of Primary Dealers (as of May 31, 2011)

BNP Paribas Securities Corp.
Barclays Capital Inc.
Cantor Fitzgerald & Co.
Citigroup Global Markets Inc.
Credit Suisse Securities (USA) LLC
Daiwa Capital Markets America Inc.
Deutsche Bank Securities Inc.
Goldman, Sachs & Co.
HSBC Securities (USA) Inc.
Jefferies & Company, Inc.
J.P. Morgan Securities LLC
MF Global Inc.
Merrill Lynch, Pierce, Fenner & Smith Incorporated
Mizuho Securities USA Inc.
Morgan Stanley & Co. LLC
Nomura Securities International, Inc.
RBC Capital Markets, LLC
RBS Securities Inc.
SG Americas Securities, LLC
UBS Securities LLC

Source: www.newyorkfed.org/markets

States. Bank-related dealers must have at least $100 million of tier I capital, and brokerage houses must have at least $50 million in regulatory capital. These norms are imposed to ensure that such dealers are in a position to undertake large-scale transactions with the Fed. Primary dealers are required to participate actively in both open-market operations and fresh Treasury auctions. They are also expected to serve as a source of information for the Fed from the standpoint of formulating and implementing monetary policy. Table 5.5 is a list of primary dealers in May 2011.

Repurchase Agreements

Securities dealers need to fund their security purchases, which have values far in excess of what their own funds will permit. One source of funds is demand loans from the banking sector. Another source is the repo or repurchase agreement. Under such an arrangement, the dealer sells securities to a lender with a concomitant commitment to buy back the securities at a later date at the original sale price plus interest. A repo is nothing but a borrowing arrangement that is collateralized by marketable securities. Many repos are overnight transactions: the funds are returned in exchange for the collateral on the following business day. Longer-term repos for a fixed length of time are referred to as *term repos*. *Continuing contracts* are repos that carry no explicit maturity date but may be terminated at short notice by either party. Lending institutions find the repo market to be a relatively convenient and low-risk outlet for investing short-term cash surpluses.

Repos were first introduced by the Federal Reserve as a tool for conducting open-market operations and to regulate the supply of money in the economy.[5] Although securities dealers undertake such transactions to fund their positions, the FED uses them as a monetary policy tool. Today repos are a major constituent of money market operations worldwide in both developed capital markets and markets in the emerging economies. The securities used in repo transactions are typically government securities such as T-bills and T-bonds. However, other highly rated debt securities may also be pledged as collateral.

Example 5.12 provides a detailed illustration of a repurchase transaction.

Example 5.12

A dealer wishes to borrow by pledging government securities with a face value of \$5 million. The current market price is 100-05, and the accrued interest is \$2.50 per 100 dollars of face value. The dirty price of the bond is

$$5{,}000{,}000 \times \frac{\left[100 + \dfrac{5}{32} + 2.5\right]}{100} = \$5{,}132{,}812.50.$$

The loan carries an interest of 4.8 percent per annum and is repayable after 45 days. The amount payable at maturity when the collateral is taken back is

$$5{,}132{,}812.50 \times \left[1 + \frac{4.8}{100} \times \frac{45}{360}\right] = \$5{,}132{,}812.50 + 30{,}796.875$$
$$= \$5{,}163{,}609.375.$$

Reverse Repos

Buyers of bonds enter into repo agreements to fund their purchases. The lender receives what is known as *general collateral*, which implies that the securities pledged are indistinguishable from the point of view of their function as collateral. The advantage for the lender is that the transaction constitutes a secured loan, with a return that is sometimes higher than what is available on a bank deposit. Because repos constitute a secured transaction, the general collateral rate is below the prevailing LIBOR and often below the prevailing LIBID.

The motivation for the party that lends money is the following. When a short-sale transaction is entered into, the party will require bonds for delivery and also have cash for investment. A reverse repo is a mechanism for investing the cash and simultaneously acquiring the security required for delivery. If the bond desired by the lender of cash is in short supply, or is what is termed in market parlance as *being on special*, then the repo rate, or the rate paid by the borrower of funds, may be considerably below the rate for transactions backed by general collateral. From the standpoint of the seller of securities, the cash received can usually be invested at a rate that is higher than the repo rate, particularly when the collateral is on special.

From the perspective of the borrower of cash, such transactions are termed *repos*, whereas from the point of view of the lender of cash they are termed *reverse repos*. Market makers engage in both kinds of transactions. A dealer in search of cash will do a repo, and a dealer in search of securities will do a reverse repo.

A bond-trading house may acquire bonds by financing its purchase via a repurchase transaction. The acquired securities will be sold with an agreement for future repurchase, and the proceeds obtained will be used to pay off the party who sold the bonds to the trading house. The scheduled repurchase of the bonds can be timed to coincide with the sale of bonds to other investors at a profit by the trading house, assuming that the anticipated decline in yield were to materialize.

General Collateral versus Special Repos

The rate on a repo is a function of the underlying security that is pledged as collateral. Most issues are viewed as what is termed *general collateral*—that is, they are perceived as being close substitutes for each other—and the repo rate for transactions backed by such securities is the same for all securities in this category. However, there could be other securities for which the repo rate may be lower, and, as discussed, such securities are said to be on special. Unlike the general collateral rate, which is common for all securities in that class, special repo rates are issue specific. Different issues on special will have their own associated repo rates.

Why would a security be on special? A party may require a specific security because she needs to cover a short position in it. Or else it could be the case that she requires it for delivery under a futures contract if the security in question has become the cheapest to deliver. In such cases, the security being offered is not just collateral, it is the prime motivation for the transaction. The repo rate in such cases is a function of the demand and supply for the security; if it is in short supply, then the repo rate could be considerably less than the general collateral rate.

Margins

In principle, both the parties to a repo face credit risk. From the standpoint of the lender, the risk is that the collateral may decline in value because of an increase in the yield, and it may prove to be insufficient to recover the investment if the borrower were to default. From the borrower's perspective, the risk is that the collateral may appreciate in value because of a fall in the yield, and the lender may be reluctant to return it in order to reclaim his cash.

There is no strategy that will simultaneously reduce the risk for both parties. Lenders can protect themselves by seeking margins—that is, they may lend an amount that is less than the current market value of the securities. The higher the margin, the greater the protection. Borrowers, in principle, can protect themselves by seeking a reverse margin: they can demand a loan amount that is in excess of the market value of the collateral. Both parties cannot protect themselves at the same time. It is the lender who receives margin. The rationale is that the lender is giving away a more liquid asset, because cash is always more liquid than any security that is offered as collateral for the loan.

To protect the lender from the specter of an under-collateralized loan, the borrower is required to provide securities with a market value in excess of the amount of money borrowed. The difference between the market value of the securities and the loan amount is called the *margin* or *haircut*. The size of the haircut depends on the following factors:

- The credit quality of the collateral;
- The time to maturity of the collateral; and
- The term to maturity of the repo.

The lender will set a threshold level for the value of the collateral termed as the *maintenance margin* level. If the securities were to fall in value and a situation were to arise in which their current market value was lower than the maintenance level, then the borrower would be required to provide additional securities with a market value that is sufficient to make up the deficit. On the other hand, if the securities were to rise in value, the borrower might ask for extra cash or a partial return of collateral.

Example 5.13 illustrates two ways in which the haircut can be applied.

Example 5.13 (Initial Margins)

A pension fund that is prepared to offer bonds with a face value of \$25 million enters into a three-day repurchase agreement with a bank. The collateral is T-bonds with a coupon of 6 percent and a market price of \$95 per \$100 of face value. The haircut is 1.25 percent. Accrued interest is \$2 per \$100 of face value. The repo rate is 7.50 percent per annum.

The loan amount may be calculated in either of two ways.

1. Method 1:

$$\text{Loan amount} = 25{,}000{,}000 \times \frac{97}{100} \times 0.9875 = \$23{,}946{,}875.$$

2. Method 2:

$$\text{Loan amount} = 25{,}000{,}000 \times \frac{97}{101.25} = \$23{,}950{,}617.28.$$

Assuming that method 1 is used to compute the loan amount, the amount repayable at maturity is

$$23{,}946{,}875\left[1 + \times 0.075 \times \frac{3}{360}\right] = \$23{,}961{,}841.80.$$

Sale and Buyback

In a standard repurchase transaction, or what is termed a *classic repo*, the price at which the collateral is sold is the same as the price at which it is repurchased, because the interest is separately computed and paid. Of course, the amount that is repaid at maturity will be greater than the loan amount, because it will incorporate the interest in addition to the principal. Any coupons that are paid on the underlying collateral during the life of the repo will be handed over to the borrower whenever they are received by the lender. This is because, although technically the borrower ceases to be the legal owner, he continues to be the beneficial owner.

In a sale and buyback transaction, too, the securities are sold at the outset and accompanied with an agreement for forward repurchase. Such a transaction also entails the transfer of ownership. However, there are some critical differences between the two transactions. Unlike a repo, which is technically a single transaction, a sale and buyback is a combination of two distinct cash-market trades. Sale and buyback does not involve the maintenance of margins. There is no requirement on the part of the borrower to provide additional collateral if the security were to decline in value. There is greater credit risk for the lender in such circumstances. Second, unlike in the case of a classic repo in which any coupons paid during the life of the transaction are immediately passed on to the borrower, in a sale and buyback transaction, the coupon is factored into the price at which the security is repurchased. Third, classic repos often allow substitution of securities—that is, the borrower can substitute the collateral that was initially offered with a similar security. Sale and buyback transactions do not have a clause for substitutions. Finally, repos are covered by standard legal contracts developed by the industry. There is, however, no such standard documentation for sale and buyback deals. From the standpoint of the borrower, such a transaction is referred to as a *sale and buyback*, whereas from the perspective of the lender it is termed a *buy and sell-back* transaction.

Collateral

The securities that are offered as collateral should be placed in a custodial account at a bank. When the loan is repaid, the borrower's liability will be canceled and the securities will be returned to her. This method ensures safety for the party who is lending. However, it was found that dealers were not always following this norm. This led to a situation in the United States in which many depository institutions lost money because the dealers who borrowed from them went out of business, and they could not easily take possession of the collateral. Subsequently, federal authorities have imposed strict guidelines for dealers entering into such transactions.

Repos and Open-Market Operations

The Federal Reserve undertakes repurchase and reverse repurchase transactions with primary dealers. These transactions constitute an important component of the monetary policy instruments employed by the Fed. A repo represents a collateralized loan made by the Fed to primary dealers, so they are used by dealers to borrow funds from the Fed. A reverse repo transaction entails the borrowing of funds by the Fed from primary dealers. Typically, these are overnight transactions, but the Fed can undertake operations with as long as 65 business days to maturity.

From a monetary policy standpoint, a repo temporarily adds reserve balances to the banking system, whereas a reverse repo transaction temporarily drains such balances from the system. However, this only true for transactions involving the central bank. Transactions not involving the central bank do not affect total reserves in the banking system.

Repurchase and reverse agreements are operations that are undertaken by the trading desk at the New York Fed, which in turn implements monetary policy decisions taken by the FOMC. When the Fed does a repo, funds are credited to the dealer's commercial bank. This enhances the level of reserves in the banking system. At maturity, in return for the amount loaned plus interest, the Fed transfers the collateral back to the dealer concerned. This automatically neutralizes the extra reserves that were created by the original transaction. Such operations undertaken by the Fed have a short-term, self-reversing effect on bank reserves.

Reverse repos are similar from the standpoint of their impact on reserves. In such transactions, the Fed sends collateral to the dealer's clearing bank in return for the funds. This action reduces the availability of reserves in the banking system. At the time of maturity of the contract, the dealer returns the collateral to the Fed, which in turn returns with interest the funds that were initially borrowed. This automatically restores the reserves that were reduced by the original transaction.

Negotiable CD*s*

A *certificate of deposit* (CD) is an interest-bearing receipt for funds left with a depository institution for short periods of time. CDs are usually issued at par and pay interest explicitly, although banks also issue discount CDs that are similar to T-bills in structure. Payment is made in federal funds on the day of maturity. Banks and other institutions issue many types of CDs, but true money market CDs are negotiable instruments that can be resold before maturity, and they are typically issued with a face value of

\$1 million. CDs issued to retail investors are nonnegotiable, and these are not classified as money market instruments. CDs issued in the United States and Eurodollar CDs usually have an original maturity that is less than or equal to one year, and they pay interest at maturity. The day-count convention is Actual/360, and the rate of interest is referred to as the *coupon rate*. The interest rate on a money market CD is slightly less than that on a standard time deposit of the same tenor, because the instrument is negotiable.

Notation

$N \equiv$ the number of days from the time of issue until the time of maturity
$T_m \equiv$ the number of days from the settlement date until the maturity date
$c \equiv$ is the coupon rate
$y \equiv$ quoted yield
$V \equiv$ the face value of the CD or the principal amount
$P \equiv$ the dirty price of the CD.

The amount payable at maturity is given by

$$V \times \left[1 + c \times \frac{N}{360}\right].$$

Example 5.14 illustrates the computation of the maturity value of a CD.

Example 5.14

Consider a CD with 180 days to maturity that has just been issued with a face value of \$1 MM and a coupon rate of 4.5 percent. The amount that the holder will receive at maturity is equal to

$$1{,}000{,}000 \times \left[1 + 0.045 \times \frac{180}{360}\right] = 1{,}022{,}500.$$

Quoted yields on CDs are yields on a simple interest basis. If the dirty price of the bond is \$$P$, then the quoted yield y is given by

$$y = \left(\frac{V \times \left[1 + c \times \frac{N}{360}\right] - P}{P} \right) \times \frac{360}{T_m}.$$

Example 5.15 demonstrates the computation of the yield on a CD.

Example 5.15

A CD with an original term to maturity of 180 days and a face value of \$1,000,000 was issued with a coupon rate of 4.80 percent. The current term to maturity is 144 days, and the CD is quoting at a price of \$1,000,000. What is the corresponding yield?

The maturity value is given by

$$1{,}000{,}000 \times \left[1 + 0.048 \times \frac{180}{360}\right] = \$1{,}024{,}000.$$

The yield is given by

$$\frac{(1{,}024{,}000 - 1{,}000{,}000)}{1{,}000{,}000} \times \frac{360}{144} = 0.06 \equiv 6\%.$$

If we are given the yield, we can compute the price. The expression for the price is given by

$$y \times \frac{T_m}{360} = \frac{V \times \left[1 + c \times \frac{N}{360}\right]}{P} - 1 \Rightarrow P = \frac{V \times \left[1 + c \times \frac{N}{360}\right]}{\left[1 + y \times \frac{T_m}{360}\right]}.$$

Example 5.16 illustrates the computation of the price of a CD given its yield.

Example 5.16

A CD with an original term to maturity of 180 days, and a face value of $1,000,000 was issued with a coupon of 5.2 percent. The current term to maturity is 108 days, and the quoted yield is 5.6 percent. What is the dirty price?

$$P = \frac{1{,}000{,}000 \times \left[1 + 0.052 \times \frac{180}{360}\right]}{\left[1 + 0.056 \times \frac{108}{360}\right]} = \frac{1{,}026{,}000}{1.0168} = \$1{,}009{,}048.$$

The price of the CD can be decomposed into the clean price plus the accrued interest. The accrued interest on CDs is given by

$$\mathrm{AI} = V \times c \times \frac{N - T_m}{360}.$$

In the preceding example, the accrued interest is given by

$$1{,}000{,}000 \times 0.052 \times \frac{(180 - 108)}{360} = \$10{,}400.$$

The clean price is 1,009,048 − 10,400 = $998,648.

Cost of a CD for the Issuing Bank

The actual cost of a CD from the standpoint of an issuing bank is usually greater than the coupon rate. There are two reasons for this. First, in most cases banks have to maintain reserves. Second, bank deposits are often insured up to specified amounts. The effect of these factors may best be illustrated with the help of Example 5.17.

Term CDs

Term CDs have a maturity in excess of one year at the time of issue, and they usually pay coupons on a semiannual basis. The price of the instrument is the present value of all the cash flows to be received from it, as of the settlement date, where the discounting is done using the quoted

Example 5.17

Bank Alpha has issued a CD with a coupon of 6.3 percent. The bank has to maintain 10 percent reserves with the central bank; these do not bear interest. If the bank were to raise a deposit of \$100, it would be paying 6.3 percent interest on \$90 of usable money. This automatically raises the effective cost of the deposit to

$$\frac{6.30}{90} = 0.07 \equiv 7\%.$$

Assume that the bank has to pay an insurance premium of 7.5 bp. The overall cost of the deposit is therefore

$$0.07 + 0.00075 = 0.07075 \equiv 7.075\%.$$

yield. The procedure is analogous to the valuation of a bond, in which the coupons and the face value are discounted at the YTM, although with certain differences.

First, the amount of interest in a coupon period is not $\frac{c \times V}{2}$ as in the case of a bond. The interest in this case is given by

$$\text{Interest} = c \times V \times \frac{N_i}{360}.$$

where N_i is the number of days in the ith coupon period. The coupon will vary from period to period.

The second difference is that we do not discount the kth cash flow by $(1 + \frac{y}{2})^k$ where y is the YTM of the security. In the case of term CDs, the discounting is based on the actual number of days in the coupon period, and it is effected on a simple interest basis.

Example 5.18 illustrates the valuation of a term CD.

Example 5.18

A CD with a coupon rate of 4.8 percent was issued on July 1, 2009. It is scheduled to mature on June 30, 2011. On September 1, 2009, the quoted yield was 5 percent. What should be its current price?

The first step is to calculate the number of days in each coupon period as well as the number of days from the date of settlement to the first coupon. The length of each period is given in Table 5.6.

Table 5.6
Day Counts for Valuing the Term CD

Start Date	End Date	Number of Days
July 1, 2009	January 1, 2010	184
January 1, 2010	July 1, 2010	181
July 1, 2010	January 1, 2011	184
January 1, 2011	June 30, 2011	180
September 1, 2009	January 1, 2010	122

To value the security, we start with the cash flow at maturity. This is discounted back to the penultimate coupon date. The coupon payment at this point of time is added to the present value obtained in the previous step. The sum is then discounted back to the previous coupon date. This procedure is repeated until we reach the settlement date.

In our example, the cash flow at maturity is

$$1{,}000{,}000 \times \left[1 + 0.048 \times \frac{180}{360}\right] = \$1{,}024{,}000.$$

The present value of this on January 1, 2011, is

$$\frac{1{,}024{,}000}{\left[1 + 0.05 \times \frac{180}{360}\right]} = 999{,}024.39.$$

The coupon due on January 1, 2011, is

$$1{,}000{,}000 \times 0.048 \times \frac{184}{360} = 24{,}533.33.$$

The amount to be discounted to the previous coupon date is

$$999{,}024.39 + 24{,}533.33 = \$1{,}023{,}557.72.$$

The present value on July 1, 2010, is

$$\frac{1{,}023{,}557.72}{\left[1 + 0.05 \times \frac{184}{360}\right]} = 998{,}051.95.$$

The coupon due on July 1, 2010, is

$$1{,}000{,}000 \times 0.048 \times \frac{181}{360} = 24{,}133.33.$$

The amount to be discounted to the previous coupon date is

$$998,051.95 + 24,133.33 = \$1,022,185.28.$$

The present value on January 1, 2010, is

$$\frac{1{,}022{,}185.28}{\left[1 + 0.05 \times \dfrac{181}{360}\right]} = 997{,}118.82.$$

The coupon due on January 1, 2010, is

$$1{,}000{,}000 \times 0.048 \times \frac{184}{360} = 24{,}533.33.$$

The amount to be discounted to the settlement date is

$$997{,}118.82 + 24{,}533.33 = \$1{,}021{,}652.15.$$

The present value on September 1, 2009, is

$$\frac{1{,}021{,}652.15}{\left[1 + 0.05 \times \dfrac{122}{360}\right]} = 1{,}004{,}629.27.$$

This is the dirty price of the CD as of September 1, 2009. The accrued interest as of this day is

$$1{,}000{,}000 \times 0.048 \times \frac{62}{360} = \$8{,}266.67.$$

The clean price is

$$1,004,629.27 - 8,266.67 = \$996,362.60.$$

The interest rate on a large negotiable CD is set by negotiations between the issuing institution and its customer and generally reflects prevailing market conditions. Like the rates on other money market securities, rates rise in period where funds are hard to come by and fall in periods characterized by easy availability of funds.

CDs increase the average cost of funds as well as their volatility for the issuing banks. But banks have no choice but to offer such instruments because otherwise they face the specter of a loss of business. The cash-management departments of major corporations have become increasingly aware of the various profitable outlets for their short-term surpluses. Before the introduction of the negotiable CD, banks found that their biggest corporate customers were reducing their deposits and buying T-bills, repos, and other money market instruments. The negotiable CD was developed to attract the lost deposits back into the banking system.

CDs versus Money Market Time Deposits

In the case of a conventional time deposit, an investor will deposit a sum of money with a bank for a stated period of time. He will be paid interest at a specified rate. At the end of the deposit period, he can withdraw the original sum deposited plus the interest. In the case of a CD, however, the depositor will typically be issued a bearer security. Although he is entitled to claim the deposit with interest at the end of the deposit period, he can always sell the security before maturity in the secondary market. In the process, ownership of the underlying deposit will be transferred from the seller to the buyer. CDs have one major advantage over a conventional money market deposit, namely, liquidity. A money market deposit cannot be easily terminated until it matures. If the investor wishes to withdraw the funds before maturity, he will typically have to pay a penalty in terms of a lower rate of interest. In contrast, CDs can be liquidated at any time at the prevailing market rate, so these instruments are attractive for investors who seek the high interest rates offered by time deposits but who are reluctant to commit their money for the full term of the deposit.

Commercial Paper

Every year, several large corporations borrow billions of dollars in the money market through the sale of unsecured promissory notes known as *commercial paper*. Such instruments are issued by large corporations and bought principally by other large corporations. In terms of volumes issued and traded, such paper is the single largest instrument issued in the U.S. money market.

By definition, such paper consists of short-term unsecured promissory notes issued by well-known companies that are financially strong and carry high credit ratings. The word *unsecured* connotes that the issuer is not pledging any collateral to back these securities and that buyers can only depend of the liquidity and earning power of the issuer from the standpoint of repayment. The funds raised in this manner are normally used for current transactions such as purchasing raw materials, paying accrued taxes, meeting wage and salary obligations, and other short-term expenditures rather than for capital transactions or, in other words, long-term investments. However, these days a substantial number of paper issues are used to provide *bridge financing* for long-term projects. In these instances, issuing companies usually plan to convert their short-term paper into more permanent financing when the capital market looks more favorable.

Most issuers of paper enjoy a high credit rating. However, to reduce investor risk, borrowers nearly always secure a line of credit at a commercial bank for a small fee or in return for a compensating deposit. But the line of credit cannot be used to directly guarantee payment if the company goes bankrupt, and the lender may renege on the credit line if it perceives that the credit quality of the borrower has significantly deteriorated. Many issuers also take out irrevocable letters of credit prepared by their banks. Such a letter of credit makes a bank unconditionally responsible for repayment if the corporation defaults on its paper.

Letters of Credit and Bank Guarantees

A *letter of credit* (LC) is a document issued by a bank, called the *issuer* or *opener*, at the behest of a client, referred to as the *applicant*, in favor of a stated beneficiary, stating that it will effect payment of a stated sum of money for certain specified good or services against presentation of a predefined set of documents. Such instruments are very common in international trade transactions, although they may be occasionally used for domestic deals as well. There are two categories of LCs: commercial and standby. Commercial LCs are used to cover the movement of goods, whereas standby LCs are used for other financial arrangements. Although a commercial LC is a direct instrument of payment, a standby LC is in the nature of a guarantee, and it is invoked only if there is default in the primary settlement process or performance, including the failure to pay.[6]

Another instrument that is used to provide reassurance to a beneficiary is what is known as a *bank guarantee*. There is, however, a difference between the two instruments from the standpoint of the position of the bank relative to the applicant and the beneficiary. A letter of credit is a bank's direct undertaking to the beneficiary: when the shipment of goods occurs under a letter of credit, the issuing bank does not wait for the buyer to default or for the seller to invoke the undertaking before making the payment to the beneficiary. On the other hand, a bank guarantee is an undertaking stating that in the event of the inability or unwillingness of the buyer to make payments to the seller, the bank, as the guarantor to the transaction, would pay the supplier. A bank guarantee comes into play only when the buyer fails to pay the supplier. In this case, the supplier is expected to first seek to have her claim honored by the buyer and not the bank. So a bank guarantee is a contingent guarantee. In the case of an LC, however, the principal liability is that of the issuing bank. It must first pay on submission of the predefined set of documents and then collect the amount from its client. An LC substitutes the bank's creditworthiness for that of its client.

A bank guarantee is therefore more risky for the supplier and less risky for the bank. An LC is less risky for the seller but more risky for the bank. In both cases, however, the bank accepts full liability for the specified amount.

What, then, is a standby letter of credit? A standby LC is a written obligation on the part of the issuing bank to pay the beneficiary on behalf of its client if the client does not pay the beneficiary. This sounds a lot like the definition of a conventional bank guarantee. So is there a difference between the two? The answer is that U.S. banking legislation forbids depository institutions in the United States from assuming guarantee obligations of third parties. To overcome this legal hurdle, banks in the United States created the instrument that has come to be known as the standby LC, which is based on the *uniform customs and practices for documentary credits* (UCPDC), the protocol that applies for standard LCs.

There are two major types of commercial paper: direct paper and dealer paper. The main issuers of direct paper are large finance companies and bank holding companies that deal directly with the investors rather than using securities dealers as intermediaries. Such companies, which regularly extend installment credit to consumers and large working capital loans and leases to business firms, announce the rates they are currently paying on various maturities of their paper. Investors select maturities that most closely match their expected holding periods and buy the securities directly from the issuer. Such borrowers have an ongoing need for huge amounts of short-term money, possess top credit ratings, and have established working relationships with major institutional investors in order to place new issues regularly.

Directly placed paper must be sold in large volume to cover the substantial costs of distribution and marketing. Issuers of direct paper do not have to pay dealers' commissions, but they must operate a marketing division to maintain constant contact with active investors. These companies also have to pay fees to banks for supporting lines of credit, to rating agencies that rate their issues, and to agents such as trust companies that collect funds and disburse payments.

The other major variety of commercial paper is dealer paper, which is issued by security dealers on behalf of their corporate customers. Dealer paper is issued mainly by firms that borrow less frequently than companies that issue direct paper. The issuing company may sell the paper directly to the dealer, who will buy it minus a discount and commissions, and then resell it at the highest possible price in the market. Alternatively, the issuing company may bear all the risk, with the dealer agreeing only to sell the issue at the best price available minus commissions. This is referred to as a *best-efforts transaction*.

One reason for the market's rapid growth is the high quality of most paper. Many investors regard such paper as a high-quality substitute for T-bills and other money market instruments. The second factor is the expanding use of credit enhancements in the form of standby letters of credit, indemnity bonds, and other irrevocable payment guarantees. Such paper usually carries the higher credit rating of the guarantor rather than the lower credit rating of the issuer. Despite paying the guarantor's fee, the issuer can still save on interest costs.

Yankee Paper

Paper that is issued in the United States by foreign firms is called *Yankee paper*. Foreign issuers find that they can often issue Yankee paper at a cheaper rate than what it would cost them to borrow outside the United States. However, foreign borrowers must generally pay higher interest costs than U.S. companies of comparable credit rating. In other words, Yankee paper generally carries a higher yield than U.S. paper of the same credit quality. This is because an American investor will demand higher returns in consideration of the difficulty of gathering information on a foreign firm.

There are several advantages for companies with good ratings that are in a position to tap the paper market. Generally, rates on high-quality paper are less than on bank loans. Moreover, the effective rate on bank loans is even higher than what is quoted because of the fact that corporate borrowers usually are required to keep a percentage of the loan in the form of a deposit with the bank. This compensating balance is generally 15 to 20 percent of the loan amount. If a company borrows US$100 million at 8 percent with a compensating balance of 20 percent, the effective rate on the loan will be

$$\frac{8{,}000{,}000}{80{,}000{,}000} \equiv 10\%.$$

Credit Rating

The major credit rating agencies in the world are listed in Table 5.7. It is extremely difficult to market unrated paper. Generally, commercial paper bearing ratings from at least two agencies are preferred by investors.

Table 5.7
Prominent Rating Agencies

Moody's Investors Service
Standard & Poor's Corporation
Fitch Investor's Service
Canadian Bond Rating Service
Japanese Bond Rating Institute
Dominion Bond Rating Service
IBCA Ltd.

Moody's Ratings Scale

Moody's uses the following rating scale for short-term taxable instruments such as commercial paper.

- Prime-1: Superior ability to repay short-term debt obligations.
- Prime-2: Strong ability to repay short-term debt obligations.
- Prime-3: Acceptable ability to repay short-term obligations.
- Not prime: These do not fall within any of the prime rating categories.

Prime-1, Prime-2, and Prime-3 are all investment-grade ratings.

S&P's Rating Scale

Standard and Poor's rates issues on a scale from A-1 to D. Within the A-1 category, an issue can be designated with a plus sign. This indicates that the issuer's ability to meet its obligation is extremely strong. Country risk and the obligor's currency of repayment in meeting its obligation are factored into the credit analysis and are reflected in the issue rating.

- A-1: Issuer's capacity to meet its financial commitment on the obligation is strong.

- A-2: The issuer is susceptible to adverse economic conditions. However, the issuer's capacity to meet its financial commitment is satisfactory.
- A-3: Adverse economic conditions are likely to weaken the issuer's capacity to meet its financial commitment on the obligation.
- B: Has significant speculative characteristics. The issuer currently has the capacity to meet its financial obligation but faces major ongoing uncertainties that could impact its financial commitment.
- C: The issue is currently vulnerable to nonpayment and is dependent on favorable business, financial, and economic conditions for the obligor to meet its financial commitment.
- D: The issue is in payment default. Obligation not made on due date and grace period may not have expired. The rating is also used on the filing of a bankruptcy petition.

A-1, A-2, and A-3 are all investment-grade ratings.

Fitch's Rating Scale

Fitch's short-term ratings indicate the potential level of default within a 12-month period. A plus (+) or minus (−) sign may be appended to the F1 rating to denote relative status within the category.

- F1: Best-quality grade. Indicates strong capacity of obligor to meet its financial commitment.
- F2: Good-quality grade. Indicates satisfactory capacity of obligor to meet its financial commitment.
- F3: Fair-quality grade: Adequate capacity of obligor to meet its financial commitment, but near-term adverse conditions could impact the obligor's commitments.
- B: Of speculative nature. Obligor has minimal capacity to meet its commitment and is vulnerable to short-term adverse changes in financial and economic conditions.
- C: Possibility of default is high. Financial commitment of the obligor is dependent upon sustained, favorable business and economic conditions.
- D: The obligor is in default as it has failed on its financial commitments.

Short-term debt issues that are given a top-grade credit rating by both S&P and Moody's are referred to as A1/P1 debt. A1/P1 paper will sell at the finest rates.

Bills of Exchange

Before we go on to study the nature of a bankers' acceptance, which is a key constituent of the money market, let us first analyze the concept of a bill. Per the English law of contracts, a bill of exchange is defined as:[7]

> "An unconditional order in writing, addressed by one person to another, signed by the person giving it, requiring the person to whom it is addressed to pay on demand or at a fixed or determinable future time a sum certain in money or to the order of a specified person, or to the bearer."

The parties to a bill are the following:

- **Drawer:** The drawer is the party who makes out the bill of exchange. A bill of exchange is drawn by the person who will be paid.
- **Drawee:** The drawee is the party who is directed to pay by the drawer. A bill of exchange is always drawn on the person who is ordered to pay. The drawee is the party who owes the money and to whom the bill is addressed.

There are two categories of bills of exchange: bank bills and trade bills. A trade bill is drawn by one nonbank company on another, typically demanding payment for a trade debt. A bank bill, on the other hand, is a bill that is drawn on and payable by a commercial bank.

Bills may be classified as sight bills and time, term, or usance bills. A sight bill has to be honored on demand—that is, the drawer is expected to pay on sight. In the case of a term bill, however, the specified amount is payable on a future date, which is mentioned on the bill. There are three ways in which the due date for payment of a term bill may be specified:

1. Payment to be made on a stated date;
2. Payment to be made after a stated period after the bill is sighted, where the date of sighting is the date on which the drawee signs on it, indicating his acceptance; and
3. Payment to be made after a stated period from the date of the bill.

For a bill with a future payment date, the drawee will sign her acceptance across the face of the bill and return it either to the drawer or her bank. When such a bill is accepted by the debtor, it becomes a promise to pay, or an IOU. Because a bill represents an undertaking to pay a stated amount at a future date, it is a form of short-term finance for the debtor. The holder of the bill therefore effectively gives credit to the debtor.

The drawer may hold the bill until maturity and present it to the drawee for payment or he may sell it in the money market at any time before its maturity date. A bill of exchange can be transferred by a simple endorsement. An authorized signatory of the drawer will sign the back of the bill and give it to the buyer. The seller of the bill in the secondary market gives up the right to eventual payment on the maturity date in return for an immediate payment from the person who is acquiring the bill. The purchaser of the bill acquires the right to receive payment on the maturity date. A common feature of the secondary market for all categories of bills is that the market price of the instrument will always be less than the amount that is payable at maturity. Bills are discount instruments that are always traded at a discount to their face values.

The ability of a holder of a bill to sell it at a reasonable price depends on the credit quality of the drawee and on the existence of a liquid secondary market. In the case of trade bills, it would depend on the financial status of the two companies concerned. Financial institutions will buy only the finest trade bills in the market—that is, bills in which both companies have a high credit standing. For obvious reasons, bank bills are more easily marketable than most trade bills. The secondary market is provided by banks and other institutions dealing in the money market. Bills are purchased extensively and held as short-term investments by banks and nonbanking institutions, and market makers are able to operate a liquid two-way market for accepted bills.

A bill of exchange is doubly secured. In other words, if the drawee fails to honor it on presentation, the holder may look to the drawer for payment. A common form of a bank bill is a banker's acceptance, whereby a bank accepts a bill on behalf of a customer and promises to pay the bill at maturity.

Eligible and Noneligible Bank Bills

Bank bills may be classified as eligible or noneligible. The term *eligibility* refers to the willingness of the government to deal in the bills in the secondary market. An eligible bill is one which the government or government agency will agree to buy. Eligibility depends partly on the bill's original term to maturity. The government will not buy bills with an original term to maturity greater than a specified number of days. Eligibility also depends on the identity of the bank. The government will buy bills of only those banks that are present on its list of eligible banks. Eligible bank bills are more highly rated and attract smaller and finer rates of discount. In the United States, eligible bank bills are those that the Federal Reserve will be willing to purchase.

Buying and Selling Bills

A company has drawn a bill on HSBC for \$5,000,000 with a maturity of 150 days. The bank has accepted it, and the drawer has sold it to Barclays at a discount of 5.25 percent. Barclays has sold the bill to ABN Amro 30 days hence at a discount of 4.75 percent. The rate of return for Barclays may be computed as follows.

The purchase price is given by

$$5{,}000{,}000\left[1-\frac{5.25}{100}\times\frac{150}{360}\right]=\$4{,}890{,}625.$$

The sale price is given by

$$5{,}000{,}000\left[1-\frac{4.75}{100}\times\frac{120}{360}\right]=\$4{,}920{,}833.33.$$

The profit is

$$4,920,833.33-4,890625 = \$30,208.33.$$

The return on investment on a 360-day year basis is

$$\frac{30{,}208.33}{4{,}890{,}625}\times\frac{360}{30}\equiv 7.41\%.$$

Banker's Acceptance

A *banker's acceptance* is a term bill that is drawn on a bank by a seller of merchandise. If the bank honors the bill, it will stamp "Accepted" on it face, thereby signaling its willingness to pay at the end of the stated credit period. A banker's acceptance is a very popular instrument in international trade transactions, because most exporters are uncertain of the credit standing of the importer to whom the goods are being shipped. However, such exporters are usually quite content to rely on acceptance financing by a well-reputed domestic or foreign bank.

Example 5.19 illustrates the creation of a bankers' acceptance.

Example 5.19

IBM is shipping six mainframe computers to Kim and Company in Seoul and is prepared to grant three months' credit to the buyer. The Korean importer will approach his bank in Seoul, the First National Bank of Korea, for a line of credit. Once the bank approves a line of credit, it will issue an LC in favor of IBM. The bank in Seoul will send it to its correspondent bank in the United States, which is known as the *advising bank*. The advising bank may or may not confirm the LC. If it were to confirm the LC, then IBM can be assured that it will make the payment, even if the Seoul-based exporter or the Korean bank were to default. As per the LC, IBM can draw a term bill for a specified amount against the bank, provided that the shipping documents are sent to the bank. On receipt, the bank will check to see that the draft and any accompanying documents are correctly drawn and will then stamp "Accepted" on its face. In this way, a banker's acceptance is created.

In this case, IBM must wait for three months in order to be paid. However, a situation may arise when it needs the funds immediately. It may therefore seek to have it discounted by its bank in the United States. The American bank that has acquired the bill from IBM may hold it as an asset, or else sell it in the secondary market. At the end of the stipulated period, which is three months in the case, the party who is holding it at that point in time will present it to the bank in Seoul in return for payment.

When a bank wishes to trade an acceptance in the secondary market, the rate of discount applicable is determined by the current rate on acceptances of similar maturity. The yield on bankers' acceptances is usually only slightly higher than the rate on T-bills because banks that issue such acceptances are normally large and have a good reputation from the standpoint of credit risk. The discount rate on an acceptance is also comparable with the rate on a CD, because both instruments are unconditional obligations of the issuing bank.

Eurocurrency Deposits

What is a Eurocurrency? It is a freely convertible currency deposited outside the country to which it belongs. Dollars deposited outside the United States are Eurodollars. Japanese yen deposited outside Japan are

Euroyen. The large majority of such deposits are held in Europe, but these deposits have spread worldwide; Europe's share of the total is declining. Among the most important non-European centers for Eurocurrency trading are the Bahamas, Canada, the Cayman Islands, Hong Kong, Panama, and Singapore.

We will illustrate how a Eurocurrency deposit arises. Siemens has exported equipment to a party in Dallas and has invoiced in dollars. Subsequently, a bill for US$1 million is sent to the American party, which pays by issuing a check in U.S. dollars. Let us assume that the check is deposited with Citibank in New York, where Siemens has an account. After the check clears, Siemens's account in New York will be credited with US$1 million. Siemens has a dollar-denominated asset of the same magnitude. This is not, however, a Eurodollar deposit because the deposit of dollars has been placed with a bank in the United States, where the dollar is the official unit of currency.

Assume that a few days later, Siemens decides to transfer the funds to Deutsche Bank in Frankfurt, where it is being offered a better rate of return on a term deposit. However, instead of converting the funds into euros, it decides to keep them in U.S. dollars. Once the funds are transferred, Siemens will receive a confirmation for a dollar deposit of US$1 million from the bank in Frankfurt. However, the dollars per se will not be transferred to Europe. Deutsche Bank will have a correspondent bank in the United States—say, Wells Fargo—that will be credited with the dollars once the funds are transferred from Citibank.

We have a Eurodollar deposit in the form of a US$1 million term deposit with Deutsche Bank in Frankfurt. This is because Siemens's deposit has been accepted and recorded on the books of Deutsche Bank in U.S. dollars, although the official currency of Germany is the euro.

The total amount of dollar deposits in the U.S. banking system has not changed, nor has the quantum of reserves maintained by U.S. banks with the Federal Reserve. The creation of a Eurodollar deposit does not lead to a situation in which the money actually leaves the United States. All that happens is that the ownership of money is transferred across international borders.

Most Eurocurrency deposits are short-term deposits, ranging from call money to term deposits with as long as a year to maturity. Call deposits are analogous to demand deposits placed with banks in the United States; euro CDs are also common. The difference between a term deposit and a CD is that the latter is a negotiable instrument. For a U.S. bank, euro CDs often offer a lower-cost alternative to domestic CDs because they are exempt from reserve requirements and the need to have them insured. There is also a market for Eurocurrency interbank loans that is the equivalent of the Fed funds market.

Money Market Futures

Futures contracts are available on T-bills, Eurodollars, Euroyen, and Federal funds. Eurodollar futures are based on a time deposit with a principal of US$1 million and three months to maturity. The underlying interest rate is LIBOR, and the contracts are cash settled to the BBA three-month LIBOR. Contracts referred to as LIBOR futures are also available for time deposits with a principal of US$3 million and one month to maturity.

Euroyen futures are of two types. The first contract is based on a time deposit of ¥100 million and three months to maturity. The contract cash settles into the Tokyo Interbank Offer Rate (TIBOR). The second type of contract is based on a similar time deposit, the difference being that it settles to the LIBOR.

In the case of the T-bill futures contract, the underlying asset is a 13-week or 91-day bill with a face value of US$1 million. The contract cash settles into the auction rate of the weekly 91-day bill auction in the week of the third Wednesday of the contract month.

Fed funds futures contracts are on federal funds with a face value of US$5 million. All the futures contracts discussed above are cash settled.

Endnotes

1. See Comotto (1998).
2. As discussed in Chapter 4, CUSIP is a securities identification code used in the United States and Canada. It is the acronym for Committee on Uniform Securities Identification Procedures.
3. The derivation is given in the Appendix for this chapter.
4. See Stigum and Robinson (1996).
5. See Choudhry (2001).
6. See Bose (2006).
7. See Palat (2006).

CHAPTER

6

Forward and Futures Contracts

Introduction

To understand forward and futures contracts, we begin our discussion with an illustration of a spot transaction. Assume that it is May 15, 20XX, and we wish to buy 100 shares of IBM. We will go to an exchange such as the New York Stock Exchange (NYSE) and place a buy order for 100 shares of IBM stock. The order will be matched with a sell order placed by another party, assuming that a suitable counterparty is available, and a trade will be executed for 100 shares. Such a transaction is termed a *cash* or *spot* transaction. This is because as soon as a deal is struck between the buyer and the seller, the buyer has to immediately make arrangements for the cash to be transferred to the seller, while the seller has to ensure that the right to the asset in question—in this case, shares of IBM—is immediately transferred to the buyer.

Consider a slightly different situation. Assume that on May 15, 20XX, we wish to enter into a contract to acquire 100 shares of IBM on August 15, 20XX, and that a counterparty exists who is willing to sell the shares on that day. If so, we can enter into a contract that entails the payment of cash on August 15 against delivery of shares on the same day. The difference as compared to the earlier transaction is that no money need be paid by the buyer at the time of negotiating the contract, nor does the seller need to transfer rights to the underlying asset on that day. The actual exchange of cash for the asset will take place on the transaction date specified in the contract: 15 August, in this case. Such a contract is termed a *forward contract*, because it entails the negotiation of terms and conditions in advance for a trade that is scheduled to take place on a future date.

In a forward contract at the time of negotiating the deal, the two parties have to agree on the terms by which they will transact on the future date. The following need to be clearly spelled out: the underlying asset (in this case, shares of IBM) and the contract size (in this case, 100 shares). The delivery date and the transaction venue should also be agreed on. In our illustration, the delivery date is August 15, and we will assume that the venue for the exchange is New York City. Finally, the price at which the deal will be consummated on August 15 should also be fixed at the time of negotiating on May 15. Let us assume that the agreed-on price is $80 per share.

A forward contract is termed a *commitment contract*, because it represents an unconditional commitment to buy the underlying asset on the part of the buyer and an equivalent commitment to sell on the part of the seller. If the buyer were to renege by not paying the contractual amount on August 15, it would be tantamount to default on his part. Similarly, if the seller were to refuse to part with the shares on that day, it would be construed as default.

In every forward contract, there will be a party who agrees to buy the underlying asset as well as a party who agrees to sell the underlying asset. The first party is termed the *buyer* or the *long*, and the counterparty is designated as the *seller* or the *short*.

A forward contract is a customized, private, or over-the-counter (OTC) contract. OTC connotes that such trades are not executed on an organized exchange. In other words, a forward contract such as the one that we have previously described will not take place on an exchange like the NYSE. Such contracts are negotiated privately between a potential buyer and a potential seller. They are referred to as customized agreements because the two parties are at liberty to specify any terms and conditions on which they can mutually agree. Let us consider the forward contract on shares of IBM. We have specified the underlying asset as shares of IBM. The asset could be shares of any other stock or even a nonfinancial product such as gold or wheat. The contract size has been assumed to be 100 shares. This, too, was arbitrarily fixed. Although two parties—say, Alfred and Colin—may agree to transact in 100 shares of IBM stock, two others—say, Molly and Patsy—may agree on 175 shares of IBM stock. The transaction date has been set as August 15, 20XX. There is no sanctity about this date, and the potential buyer and seller are at liberty to arrive at any transaction date by consensus. Finally, we have assumed that the location for the trade is New York City. This was chosen arbitrarily as well, and the two parties may agree on any location of their choice. In the case of a forward contract, all features such as the underlying asset, the quantum of the underlying asset, the date of delivery, and the place of delivery are arrived at by bilateral discussions between the potential buyer and the seller. Needless to say, the transaction price that is fixed at the outset is also determined by a process of negotiation between the two parties.

In these contracts, there is a major risk of default. Let us analyze the rationale. Assume that Alfred agrees to acquire 500 shares of IBM from Bob on August 15, 20XX. The agreement is entered into on May 15, 20XX, and the price is fixed at $80 per share. On August 15 there are two possibilities: the market price for a cash or spot transaction may be less than $80 or more than $80. If the price were to be less than $80 per share, then Alfred would be unwilling to pay $80 per share if he can get away with it. If the spot price on the scheduled transaction date were to be less than the price specified in the forward contract, the long will have an incentive to renege. However, what if the spot price on August 15 were to be more than $80 per share? If so, the seller would be unwilling to sell the asset for the contracted price. In other words, if the spot price on the scheduled date were to be greater than the price specified in the forward contract, the short has an incentive to default. Default risk is an integral part of such contracts. It is for this reason that forward contracts are primarily between institutional parties such as commercial banks, investment banks, large corporations, and insurance companies. Such parties have the resources and skills to assess the credibility of a potential counterparty. Forward contracts are extremely popular in the foreign-exchange market, where importers and exporters routinely enter into such transactions with commercial banks.

Let us look at another product known as a *futures contract*. It also is a commitment contract that is written on an underlying asset, but it is offered by an organized futures exchange and is not an OTC product. The most famous futures exchange in the world is the CME Group based in Chicago, and we will illustrate such contracts with an example from this exchange.

Consider the corn futures contract on this exchange. Each contract is for 5,000 bushels of the commodity. The contract can be used only by traders who wish to transact in multiples of 5,000 bushels. The exchange specifies three deliverable grades: number 1 yellow, number 2 yellow, and number 3 yellow. Transactions are feasible in only the specified grades. The contract months available are March, May, July, September, and December. If two traders wish to transact on August 15, 20XX, as we assumed in the preceding illustration, they cannot use these contracts. The last delivery date per the specifications of the exchange is the second business day following the last trading day of the delivery month, and the last trading date is the business day before the fifteenth calendar day of the contract month. Consider the futures contracts that expires in July

2010. The last trading day is Wednesday, July 14, and the last delivery date is Friday, July 16. If two parties desire to effect delivery on July 25, 2010, then such contracts are unsuitable.

As can be deduced from the preceding example, futures contracts are standardized contracts. In other words, unlike the case of forward contracts in which the terms and conditions are set based on bilateral negotiations, in futures contracts a third party specifies the permitted terms and conditions. This third party is the futures exchange. Such an exchange may be a stand-alone entity such as the CME Group or it may be a division of a stock exchange. The exchange will specify the following features in the case of a futures contract.

- **The contract size**: In our illustration, the contract size is 5,000 bushels. Traders can take positions only in multiples of this amount.
- **The allowable grades**: In our illustration, the exchange has specified three allowable grades for corn. Transactions in any other variety of the product are ruled out.
- **The allowable locations**: In some cases, the exchange will specify more than one location where the underlying asset can be delivered. Delivery has to be made at only the locations specified in the contract.
- **The delivery date or window**: The exchange may specify a specific date on which delivery can be made. It is more common to specify a delivery window. Delivery can be made only between the start and end dates specified by the exchange. In the case of corn futures, the first delivery date is the first business of the expiration month, which is July 1, 2010, for the July 2010 contract. The last delivery date is the second business day following the last trading day of the delivery month, which is July 16, 2010.

What are the consequences of such standardization? First, it reduces transactions costs. The cost of negotiating a customized agreement in which each clause has to be drafted after painstaking negotiation will be much higher. Second, the trading volumes and the liquidity of futures contracts are much higher because only a few types of contracts are eligible for trading for each underlying product.

The futures exchange will have an associated entity known as a *clearinghouse*. The clearinghouse will position itself as the counterparty for each of the two original parties to the trade. In other words, once a trade is consummated between a buyer and a seller, the clearinghouse will intervene and position itself as the effective seller from the standpoint of the buyer and as the effective buyer from the perspective of the seller. The link between the two original parties is severed by the clearinghouse. It must be noted that neither party to the trade actually trades with the clearinghouse. The clearinghouse intervenes and positions itself as a central counterparty only after the trade is consummated. The clearinghouse essentially reduces the risk of default for the two parties to the trade. Once the clearinghouse enters the picture, each of the two parties has to only worry about the strength and integrity of the clearinghouse and not about the original counterparty. The clearinghouse also makes it easier to offset the transaction.

What is *offsetting*? Also referred to as the *assumption of a counter-position*, offsetting entails the taking of a short position by a trader who has previously gone long or by the taking of a long position by an investor who has earlier gone short.

A forward contract is a customized contract between two parties. So, to offset or abrogate an existing position, the party who is seeking to do so has to approach the original counterparty and have the agreement canceled. Assume that Sterling Infotech, a software company based in Bangalore, has entered into a forward contract with HSBC to sell US$250,000 to the bank after three months at a rate of 45 India rupees (INR) per dollar. However, one month after entering into the contract, the party finds that its customer in the United States has gone bankrupt and that the receivable in dollars will not materialize. In such a situation, the company has to approach the bank and have the original agreement canceled.

However, the cancellation of a futures position is a lot easier and does not require the party who is seeking to cancel to approach the party with which it traded initially. Assume that Ray, a trader in Kansas City, has gone long in 100 futures contracts on wheat with a counterparty in Des Moines. In this case, the order would have been routed through an exchange such as the CME Group, and the nature of the bilateral auction process on such exchanges is such that the parties who are trading would not even be aware of the identity of the counterparty. Assume that two weeks after the transaction, Ray wishes to offset his long position. All he has to do is contact a broker and have a sell order placed for 100 futures contracts on wheat. This time, the counterparty with whom the trade is effected need not be the party in Des Moines with whom he had originally traded, and it could be anybody who wishes to go long at that instant. Let us assume that this time the party who trades with Ray is based in Bloomington. As far as the records of the clearinghouse are concerned, they will reflect the fact that Ray had initially gone long in 100 contracts on wheat and has subsequently gone short in 100 contracts on the same asset. His net position after the second transaction will be nil, and he has no further obligations from the standpoint of giving or taking delivery.

To offset a futures position, the offsetting order must be on the same underlying asset and for a contract expiring in the same month. If Ray had initially gone long in wheat futures expiring in September, he should subsequently go short in futures on the same underlying commodity, which are scheduled to expire in the same expiration month, namely, September. However, the price at which the original trade was consummated need not be the same as the price at which the offsetting order is placed. Ray may exit the market with either a profit or a loss. If we assume that he went long at \$4.25 per bushel and that he subsequently offset at \$4.95 per bushel, his profit or loss will be

$$100 \times 5{,}000 \times (4.95 - 4.25) = \$350{,}000.$$

There are two reasons why futures contracts are easy to offset. First, contracts on the underlying product—in this case, wheat—have to be designed per the specifications of the exchange. In this case, each contract has to be for 5,000 bushels with the same allowable grade or grades and the same delivery schedule. The nature of Ray's initial contract with the party in Des Moines will be identical to that of the subsequent offsetting contract with the party in Bloomington. Second, once Ray's initial trade is consummated, the link between him and the counterparty in Des Moines is broken because the clearinghouse has subsequently positioned itself as the effective counterparty to the trade. On the other hand, forward contracts are typically for nonstandard sizes, products, and delivery dates. They can be offset only with the approval of the original counterparty. Besides, the identity of this party remains critical because there is no intervention by an entity such as a clearinghouse.

The clearinghouse provides a guarantee to both the original counterparties against the specter of default. The question is, how can such a guarantee be implemented? To ensure that neither party has to worry about the possibility of default on the part of the other, the clearinghouse will estimate the potential loss for each party and collect it in advance. This deposit is referred to as a *margin*, and it is a performance guarantee or a good-faith deposit. The rationale is that if a party were to default after providing such a margin, then the funds available with the clearinghouse would be adequate to take care of the interests of the counterparty. In a futures contract, both parties have a commitment to perform, and *a priori* the possibility of default exists for both. Both the long and the short are required to deposit margins immediately after the trade. The margin that is deposited at the outset is referred to as the *initial margin*.

A margin collected at the very outset does not mean that both parties will be protected against default until the very end. What would happen if a substantial price move significantly eroded one of the two parties' margin, thereby putting the counterparty at risk once again? There is a need to periodically assess the level of margin posted by a party and make adjustments for prior profits and losses. To understand how the mechanism works, we need to introduce the concepts of *marking to market* and *maintenance and variation margins*.

The loss or gain accruing to a party in a futures trade arises over a period of time, so there is a need to periodically compute the gains or losses for a party and adjust the margin position accordingly. Futures exchanges do so at the close of trading every day, a procedure that is termed *marking to market*. While marking to market, the clearinghouse will compute the profit for a party from the time the contract was previously marked to market. Gains will be credited to the margin account, and losses will be debited. Futures contracts are marked to market for the first time on the day that the position is entered into. Subsequently, they are marked to market on every business day until the contract expires or is offset by the party, if that were to happen earlier. The principle for computing the profit and loss is the following.

Assume that a trader has gone long in a contract at a price of F_0. Assume that the price at the end of the day is F_1. If he were to offset at the end of the day, it would amount to a situation in which he agrees to sell the underlying asset, which he had earlier agreed to buy at F_0 at a price of F_1. If $F_1 > F_0$, he will have a profit else he would have a loss. While marking to market, the clearinghouse will compute the profit or loss, assuming that the party is offsetting and credit or debit the margin account accordingly. However, because the party has not actually expressed a desire to offset, the clearinghouse will reopen the position at the price at which the offset was assumed to have taken place.

Rising futures prices lead to profits for the long, and declining prices lead to losses. It can be logically deduced that the situation for the shorts is exactly the opposite: rising futures prices will lead to losses, and falling prices will imply profits. All futures contracts are marked to market every day. The price that is used for this purpose is referred to as the *settlement price*. This may be the closing price for the day, although many exchanges use a volume-weighted average of prices observed during the last phase of trading for the day. If a contract is inactive, then the clearinghouse may set the settlement price equal to the average of the bid and ask quotes for the product.

Once a contract is marked to market, the gain for one of the counterparties is realized from the standpoint of price movements until that instant. If the other party were to subsequently default, the clearinghouse has the resources to protect the interests of the party who has realized a gain. However, a series of negative price movements caused by rising prices from the standpoint of the shorts or falling prices from the point of view of the longs may erode the amount of funds available in the margin account significantly, thereby defeating the very purpose of requiring the party to post margins. To protect the counterparty against this eventuality, exchanges specify a threshold level termed as the *maintenance margin*. This is set at a level below that of the initial margin.

If adverse price movements were to lead to a situation where the balance in the margin account declines below the maintenance level, the clearinghouse will issue what is termed a *margin call*. This is a request to the party to top up the funds in the margin account to take the balance back to the initial level. The funds deposited in response to a margin call are referred to as *variation margins*. A margin call always has a negative connotation, because it is an indication that the party has suffered a significant loss. Although initial margins may be deposited in the form of cash or cashlike securities, variation margins must always be deposited in the form of cash. A cash like security is a liquid marketable security. The broker will apply a haircut while evaluating the value of the security to protect himself against a sharp unanticipated loss. Variation margins must always be in the form of cash is because, unlike initial margins, which represent a performance guarantee, a variation margin represents an actual loss suffered by the trader. The traders have to maintain a margin account with their respective brokers while the brokers have to maintain margin accounts with the clearinghouse. The margin that a broker deposits with the clearinghouse is referred to as a *clearing margin*. These accounts are adjusted on a daily basis for gains and losses in exactly the same manner that a client's account with a broker is adjusted.

We will provide a detailed illustration to depict how the marking to market mechanism works. Assume that the initial margin is $5,000, and the maintenance margin is $3,750. The two traders entered into the contract at a price of $4.50 per bushel of wheat, with each contract representing 5,000 bushels. The observed prices over the next four days are shown in Table 6.1.

The impact on the margin accounts of the long and the short, respectively, are given in Tables 6.2 and 6.3.

Table 6.1
Observed Futures Prices

Time	Contract Price
0	4.50
1	4.35
2	4.15
3	4.20
4	4.25

Table 6.2
Margin Account of the Long

Time	Price ($)	Account Balance	Day's Gain or Loss	Total Gain or Loss	Variation Margin
0	4.50	5,000	—		
1	4.35	4,250	(750)	(750)	
2	4.15	3,250	(1,000)	(1,750)	1,750
3	4.20	5,250	250	(1,500)	
4	4.25	5,500	250	(1,250)	

Table 6.3
Margin Account of the Short

Time	Price ($)	Account Balance	Day's Gain or Loss	Total Gain or Loss	Variation Margin
0	4.50	5,000	—		
1	4.35	5,750	750	750	
2	4.15	6,750	1,000	1,750	
3	4.20	6,500	(250)	1,500	
4	4.25	6,250	(250)	1,250	

Let us analyze Table 6.2. At the end of the first day, the change in price as compared to the initial transaction price is -0.15. Falling prices lead to a loss for the long. In this case, the loss is $0.15 \times \$5{,}000 = \$750$. This is the loss for the day as a consequence of which the margin account balance at the end of the day declines to $4,250 after the loss is debited. The price at the end of the following day is $4.15. As compared to the previous day's settlement price, the price has declined by a further 20 cents per bushel. This amounts to a further loss of $1,000 per contract. When this loss is debited, the balance declines further to $3,250. This leads to a situation in which the account balance is below the threshold level of $3,750. A margin call will go out. To comply with it, the trader must deposit an additional $1,750 to take the balance back to the initial level of $5,000. The following day the price increases by 5 cents per bushel. The long will make a profit of $250. The cumulative loss will be $1,500. The margin account balance will be $5,250. The following day, the price again rises by 5 cents a bushel. The day's profit will once again be $250. The cumulative loss will be $1,250, and the account balance will be $5,500. This can be deduced as follows:

$$\$5{,}000 \text{ (initial deposit)} + \$1{,}750 \text{ (variation margin)} \\ - \$1{,}250 \text{ (cumulative loss)} = \$5{,}500.$$

The margin account for the short is depicted in Table 6.3. The corresponding entries are self-explanatory.

The cumulative gain or loss for the long is exactly equal to the loss or gain for the short. Futures contracts are *zero-sum games*; that is, taken together, the profit or loss for the long and the short is always zero.

Delivery Options

The right to choose the deliverable grade and location are given to the short. The short also has the right to initiate the process of delivery. What this means is that a long cannot seek delivery of the underlying asset. He will have to wait until he is chosen by the exchange to take delivery in response of a declaration of intent by a short. What this also means is that longs who do not desire to take delivery will quit the market by offsetting before the commencement of the delivery period. If they were to keep their positions open, they could be called on to take delivery without having the right to refuse.

Profit Diagrams

Rising futures prices lead to profits for holders of long positions, and falling futures prices lead to profits for holders of short positions. Take the case of two parties who traded at time t and hold their positions until time T, the point of expiration. The profit for the long is $F_T - F_t$. In general, if a trade is initiated at a price of F_t and offset at time t^* at a price of F_t^*, the profit for the long is $F_t^* - F_t$. Once a position is set up, the profit for the long increases dollar for dollar with an increase in the terminal futures price.

The profit for the short may be expressed as $F_t - F_T$ or, in general, as $F_t - F_t{}^*$. Once a short position is set up, the profit increases dollar for dollar with a decrease in the terminal futures price. The profit diagrams for both long and short positions are therefore linear. We depict the diagrams in Figures 6.1 and 6.2.

In Figure 6.1, π represents the profit, which is shown along the Y-axis. The terminal futures price is shown along the X-axis. The maximum loss

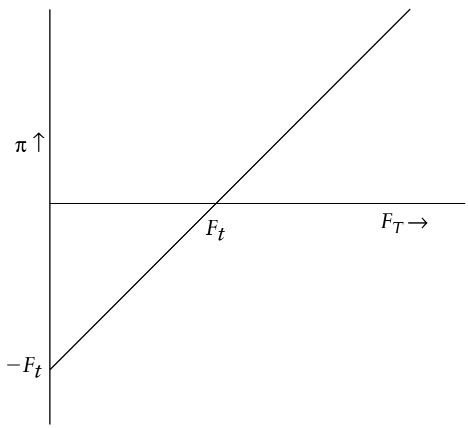


Figure 6.1 Profit Profile: Long Futures

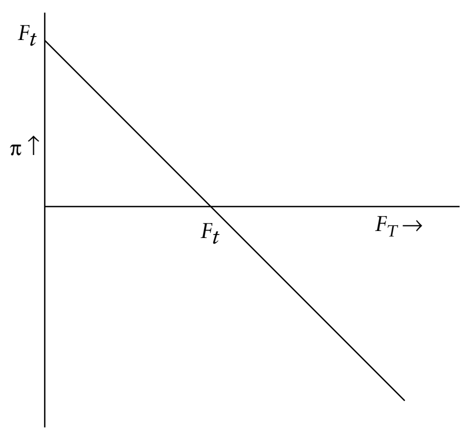


Figure 6.2 Profit Profile: Short Futures

occurs when the terminal futures price is zero and is equal in magnitude to the initial futures price F_t. Like it is for a holder of a long spot position, the maximum loss is limited. In this case, it is equal to the futures price prevailing at the outset. The maximum profit, as in the case of a long spot position, is unlimited because the terminal futures price has no upper bound. The position breaks even if the terminal futures price is equal to the initial futures price or, in other words, the price remains unchanged. The holder of a long futures position faces the specter of potentially infinite profits. However, his loss is capped at the initial futures price. As with the case of trades in the cash market, long futures positions are intended for investors who are bullish about the market.

The profit diagram for a short futures position is also linear but downward sloping as depicted in Figure 6.2.

In this case, the maximum profit occurs when the terminal futures price is zero, and it is equal in magnitude to the initial futures price. The maximum loss is unlimited, because the terminal futures price has no upper bound. Holders of short positions, similar to investors who establish short positions in the spot market, are confronted with the prospect of finite profits but infinite losses. Such investors also are bearish in nature, because they stand to make profits if the prices decline.

Value at Risk

To estimate the required margin, the clearinghouse needs to assess the potential loss for a party. The loss from a futures contract does not arise all of a sudden but is accumulated over a period of time as the futures price fluctuates from trade to trade in the market. Exchanges use a statistical technique known as *value at risk* (VaR) to estimate the potential loss over a given time horizon. The value at risk of a portfolio is an estimate of the loss over a specified time horizon for a given probability level. If the 95 percent value at risk of a portfolio over a one-day horizon is $7,500, there is only a 5 percent chance that the loss from the portfolio will exceed this amount over a one-day horizon. The estimate of the loss is a function of the specified probability level as well as the time horizon over which the loss is sought to be measured. In other words, for a given probability level, the estimated loss over a three-day horizon will be different from the estimate for a one-day horizon. And for a given time horizon, the 95 percent VaR estimate will be different from the 99-percent VaR estimate. A VaR number is meaningless until it is accompanied by the corresponding probability percentage and the time horizon. It must be remembered that the value at risk of a portfolio is not the maximum loss that it can suffer over a given time period. The value of a portfolio can always go to zero over any specified horizon however small. The maximum loss that a portfolio can suffer is its entire current value.

To illustrate how the VaR is computed, let us demonstrate the application of a method called RiskMetrics developed by JPMorgan. The volatility of the underlying asset on a given day is computed as an exponential moving average. The formula is given by

$$\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)r_t^2.$$

The variable σ is the value of the standard deviation or the volatility of the rate of return, and r is the return for the day. A value of 0.94 is recommended for λ. Let us consider futures contracts on a stock index such as the Standard & Poor's (S&P) 500 Index. The return for day t is defined as $r_t = \ln(I_t/I_{t-1})$, where I_t is the level of the index on day t. Consider a short futures position. We know that shorts will lose if the futures price were to rise. We can state with 99 percent confidence that the return will lie within +2.33 standard deviations. To account for the fat tails of the observed return distribution, a limit of +3 standard deviations is often taken. For a short position the upper limit is taken as $+3\sigma$.

Therefore,

$$ln\left(\frac{I_{t+1}}{I_t}\right) = 3\sigma_t{}^*$$

$$\Rightarrow \left(\frac{I_{t+1}}{I_t}\right) = e^{3\sigma_t{}^*}$$

$$\Rightarrow I_{t+1} = I_t[1 + e^{3\sigma_t{}^*} - 1].$$

If we know I_t, we can say with the stated level of confidence that the percentage change in the futures price will be less than or equal to $[e^{3\sigma_t{}^*} - 1] \times 100$.

Similar logic will show that the percentage decline in the futures price, which has pertinence for a long futures position, will be less than or equal to $[1 - e^{-3\sigma_t{}^*}] \times 100$.

Example 6.1 illustrates the application of this methodology.

Example 6.1

Assume that $\sigma_{t-1} = 3.25$ percent, while $r_t = 2.5$ percent. If so, then

$$\begin{aligned}\sigma_t^2 &= 0.94 \times 0.0325 \times 0.0325 + (1 - 0.94) \times 0.0250 \times 0.0250 \\ &= 0.001030375 \\ \Rightarrow \sigma_t &= 0.032099 \equiv 3.2099\%.\end{aligned}$$

So the percentage margin for a short position is

$$[e^{3 \times 0.032099} - 1] \times 100 = 10.1086\%.$$

Assume that the futures price at the end of the day is \$5,000 and that the contract multiplier is 50. The required margin for the following day is

$$50 \times \$5{,}000 \times 0.101086 = \$25{,}271.50.$$

Spot–Futures Equivalence

Consider a futures contract that is entered into an instant before the specified delivery time. It is no different from a spot contract, because it requires the short to make delivery immediately and the long to accept immediate delivery. The futures price at the time of expiration of the contract must be exactly equal to the spot price to preclude arbitrage: as the time of expiration approaches, the spot and futures prices must converge to each other.

Let us examine the consequences if the futures price were not equivalent to the spot price as the contract expires. There are two possibilities: the futures price may be higher or it may be lower.

We will first consider the case in which the futures price is higher. Assume that IBM is trading at \$80 per share on the stock exchange and that IBM futures are trading at \$82 on the futures exchange. Each futures contract is for 100 shares of stock.

An arbitrageur will acquire 100 shares in the spot market and immediately go short in a futures contract. She can take delivery at \$80 in the spot market and simultaneously give delivery at \$82 in the futures market. She has a costless assured profit of \$2 per share or \$200 per contract awaiting her. To preclude such an arbitrage opportunity, we require that the futures price must be less than or equal to the spot price at expiration.

The futures price, however, cannot be less than the spot price, because this situation also will give rise to an arbitrage opportunity. Assume that the spot price of IBM is $80 and the futures price is $77.50. An arbitrageur will short sell the stock and simultaneously go long in the futures contract. He will get $80 per share when he takes the short position. He can immediately cover the short position by taking delivery under the futures contract at $77.50. There is an arbitrage profit of $2.50 per share or $250 per contract. To rule out both forms of arbitrage, it is imperative that the futures price converge to the spot price at the time of expiration.

Products and Exchanges

Futures contracts are available on a variety of financial assets as well as commodities. There was a time when the bulk of trading activity was in commodity based products. Since then, trading action has picked up considerably in contracts based on financial assets such as stock indexes, bonds, and foreign currencies, although many commodity derivative contracts continue to be very active. The newest derivatives on financial assets are single-stock or individual-stock futures.

Commodities, metals, and energy products, on which futures contracts are traded on the CME Group are listed in Tables 6.4, 6.5, and 6.6.

Financial products on which contracts are traded include bonds such as T-notes and T-bonds, as well as money market securities such as T-bills and Eurodollars. Contracts are traded on a variety of stock indexes such as the Dow Jones Industrial Average, the S&P 500, and the

Table 6.4
Commodity-Based Derivative Products

Category		
Grain and oilseed	**Dairy**	**Livestock**
Corn	Milk	Live cattle
Wheat	Dry whey	Lean hogs
Soybean meal	Butter	Feeder cattle
Soybean oil	Cheese	Frozen pork bellies
Oats		
Rough rice		

Table 6.5
Futures Contracts on Metal Products on the CME Group

Gold
Silver
Platinum
Palladium
Copper
HRC Steel
Uranium

Table 6.6
Futures Contracts on Energy Products on the CME Group

Crude Oil
Ethanol
Natural Gas
Electricity
Coal
Heating Oil
Gasoline

Table 6.7
The Leading Derivative Exchanges in the World

CME Group
EUREX
Korea Exchange
NYSE Euronext
CBOE
BM&F Bovespa
NASDAQ OMX Group
National Stock Exchange of India
JSE South Africa
Dalian Commodity Exchange

Nikkei 225. Foreign-exchange products, based on the currencies of both G-10 countries and emerging markets, are also actively traded.

Derivative exchanges in the developed countries are characterized by large trading volumes for most of their products, so they attract attention from traders around the world. However, emerging markets have picked up considerably in recent years and offer attractive trading opportunities for both domestic market participants and cross-border traders. The derivatives markets have been characterized by a series of high-profile mergers in recent years. Prominent mergers include CME with CBOT, NYSE with Euronext, NASDAQ with OMX, and BM&F with Bovespa. The top derivative exchanges in the world are listed in Table 6.7.

Cash-and-Carry Arbitrage

An arbitrage strategy that is implemented for taking advantage of an overpriced futures contract is termed *cash-and-carry arbitrage*. It entails the assumption of a long position in the spot market coupled with a short position in the futures market. If the futures contract is scheduled to expire immediately, the profit is $F_T - S_T$. However, if there is time left for the maturity of the contract, the arbitrage profit may be expressed as $F_t - S_t \times (1 + r)$, where r is the interest rate for the period from t until T.[1] The interest cost is the market rate of interest if the arbitrageur is borrowing to fund the acquisition in the spot market, or else it is an opportunity cost if he is deploying his own funds.

Reverse Cash-and-Carry Arbitrage

A *reverse cash-and-carry arbitrage* strategy comes into play when the futures contract is underpriced. It entails the assumption of a short position in the spot market coupled with a long position in the futures market. If the futures contract is slated to expire immediately, the profit is $S_T - F_T$. However, if the maturity date of the contract is in the future, the profit may be expressed as $S_t \times (1 + r) - F_t$.

Repo and Reverse Repo Rates

Consider a cash-and-carry arbitrage strategy. It requires the arbitrageur to go long at a price S_t in order for an assured payoff of F_t at the time of expiration. The rate of return is given by

$$\frac{F_t - S_t}{S_t}$$

and is termed the *implied repo rate* (IRR). If the contract were to be fairly priced, then this rate will be exactly equal to the riskless rate. However, if the contract were to be overpriced, then this will exceed the riskless rate. Cash-and-carry arbitrage is profitable if the contract is overpriced or if the IRR is greater than the borrowing rate.

Consider reverse cash-and-carry arbitrage. The arbitrageur will indulge in a short sale at a price S_t, knowing at the outset that he will eventually have to cover at a price F_t. His funding rate is

$$\frac{F_t - S_t}{S_t}$$

which is referred to as the *implied reverse repo rate* (IRRR). If the contract were to be fairly priced, then this rate will be exactly equal to the riskless rate. But if the contract were to be underpriced, then this rate will be less than the riskless rate. Reverse cash-and-carry arbitrage is profitable if the contract is underpriced or if the IRRR is less than the lending rate.

Synthetic Securities

Cash-and-carry arbitrage entails an initial investment in return for an assured payoff at the time of expiration of the futures contract. It is akin to an investment in a zero-coupon security such as a Treasury bill. The artificial security created by such an arbitrage strategy is referred to as a *synthetic T-bill*. Reverse cash-and-carry arbitrage is equivalent to a short position in a synthetic T-bill.

We can therefore state that

$$\text{Spot} - \text{futures} = \text{synthetic T-bill}.$$

In other words, a long position in the spot market coupled with a short position in a futures contract is equivalent to a long position in an artificial T-bill. If we have a natural position in any two of the three assets, the third can be synthetically created.

Valuation

The futures price is a function of the spot price, the interest rate, the time until expiration, and any payouts made by the underlying asset. We will start by analyzing the futures price of an asset that is not scheduled to make any payouts during the life of the contract.

We know that cash-and-carry arbitrage is ruled out if $F_t \le S_t(1+r)$ and that reverse cash-and-carry arbitrage is precluded if $F_t \ge S_t(1+r)$. To rule out both forms of arbitrage, it must be the case that $F_t = S_t(1+r) = S_t + rS_t$. So the futures price in an arbitrage-free setting should be equal to the cost of acquisition of the asset and the funding cost for the duration of the contract. This relationship is referred to as the *cost of carry*.

Example 6.2 illustrates the computation of the futures price for an asset that does not make any payouts.

Example 6.2

Assume that shares of IBM are currently trading at $80 and that futures contracts are available with six months to expiration. The borrowing and lending rate for a six-month period is 6 percent per annum. The no-arbitrage futures price is therefore

$$\$80 \times (1.03) = \$82.40.$$

The Case of Assets Making Payouts

Assume that IBM is expected to make a payout of $2.50 after three months. Let us analyze the impact on both forms of arbitrage. Cash-and-carry arbitrage requires the trader to borrow and acquire the asset and go short in the futures contract. If the asset were to make a payout during the life of the contract, the effective carrying cost will stand reduced. The cost of funding the purchase of the security is rS_t. If we denote the future value of the payout as computed at the point of expiration of the futures contract by I, then the effective carrying cost is $rS_t - I$. We need to compute the future value of the payout because the interest cost is incurred at the point of expiration of the futures contract. To be consistent with the principles of time value of money, the magnitude of the payout also should be measured at this point. If the payout has occurred before the expiration date of the contract, then it will need to be compounded until the point of expiration. To rule out cash-and-carry arbitrage, we require that $F_t \le S_t \times (1+r) - I$.

Let us analyze reverse cash-and-carry arbitrage. The arbitrageur would have short sold the asset and invested it in the market while simultaneously going long in the futures contract. The income from the investment proceeds of the short sale is rS_t. However, if there were to be a payout from the asset, then the interest income would stand reduced because the short seller, as explained earlier, will have to compensate the party who is lending the asset for any lost income. The net proceeds from the investment is $rS_t - I$. To rule out reverse cash-and-carry arbitrage, we require that

$$F_t \geq S_t \times (1 + r) - I.$$

To rule out both forms of arbitrage in the case of contracts on assets that make a payout, we require that $F_t = S_t \times (1 + r) - \mathrm{I}$. Examples 6.3 and 6.4 provide numerical illustrations of how arbitrage profits can be earned if this condition is violated.

Example 6.3

Assume that IBM is trading at \$80 in the spot market and that six-month contracts on the stock are trading at \$81. The stock is scheduled to pay a dividend of \$2 after three months, and the riskless rate of interest is 6 percent per annum. The no-arbitrage futures price is $80 \times (1.03) - 2 \times (1.015) = \$80.37$. The contract is overpriced.

An arbitrageur will buy the stock by borrowing \$80 and will simultaneously go short in a futures contract. After three months, she will receive a dividend that can be reinvested until the expiration date of the contract. The cash inflow at expiration is

$$\$81 (\text{from delivery under the futures contract}) \\ + \$2.03 \ (\text{from the reinvested dividend}).$$

The cash outflow at the same point of time is \$82.40 from repayment of the amount borrowed with interest. There is an arbitrage profit of \$0.63.

Example 6.4

Assume that all the other variables have the same values as in the preceding illustration except for the futures price, which we will assume is \$79.50. An arbitrageur will short sell the stock and invest the proceeds. After three months, he will be required to compensate the lender of the security with the amount of the dividend. We will assume that this amount can be borrowed at the riskless rate. At the point of expiration, he will need to take delivery under the futures contract and repay the amount borrowed to pay the dividend.

$$\text{Cash inflow} = \$80 \times 1.03 = \$82.40$$
$$\text{Cash outflow} = 2 \times 1.015 + \$79.50 = \$81.53$$
$$\text{Profit} = \$82.40 - \$81.53 = \$0.87.$$

Physical Assets

Unlike financial assets, physical assets do not make payouts. However, the acquirers of such assets have to expend resources for storing and possibly insuring such assets. Let us denote the total cost incurred in holding such assets by Z. A cost is nothing but a negative income. If we replace the variable I in the pricing equation for assets making a payout with Z, then we get the relationship for physical assets:

$$F_t = S_t \times (1 + r) + Z.$$

However, the issue is not so straightforward. First, we need to distinguish assets that are held for investment from those that are held for consumption. This has implications for the ability to short sell the asset. Financial assets such as stocks and bonds are held for investment reasons. Holders of such assets will have no problems in lending such assets to facilitate a short sale if the asset is returned at the end of the period for which they intend to hold it, and if they are compensated for any income that they would have received had they continued to hold it. Precious metals such as gold and silver are also usually held for investment purposes. However, an agricultural product such as wheat or corn may be held for other reasons. The owner of a mill may choose to hold wheat in stock to avoid problems of short supply from unanticipated events such as a natural calamity or lack of rainfall. Such assets will in these circumstances not be parted with to facilitate short sales. We term such assets as *consumption* or *convenience assets* and say that the holders are receiving a convenience value. The convenience value is like an implicit dividend. However, it cannot be quantified because it cannot be objectively measured, and it varies from holder to holder. Before we proceed to discuss the pricing of futures contracts on physical assets, let us introduce the concept of quasi-arbitrage.

We know that a long spot position plus a short futures position is equivalent to a long position in a security that behaves like a T-bill, which we have termed a *synthetic T-bill*. The variables can be repositioned to show that a synthetic position in any of the three products—the underlying asset, the futures contract, or the riskless asset—can be taken if we have natural positions in the remaining assets. A strategy that is designed to take a synthetic position in a product, because the perceived returns are greater than what the trader would get if he took a natural position in it, is termed a *quasi-arbitrage strategy*.

Let us consider cash-and-carry arbitrage. It requires the trader to go long in spot and short in futures. This per se should pose no problems irrespective of whether the asset is a pure asset such as gold or silver or a convenience asset such as wheat. Overpricing of the futures contract is ruled out. However, the reverse cash-and-carry argument may be needed to be supplanted with a quasi-arbitrage argument to rule out an underpriced futures contract as Example 6.5 will demonstrate.

Example 6.5

A metal product is trading at \$250 per ounce. Futures contracts are available with six months to maturity at a price of \$260. The riskless rate is 8 percent per annum. The storage cost for six months is \$10, which is assumed to be payable at the end of the contract.

Take the case of an arbitrageur who goes short in the spot market and long in a futures contract. The future value of the proceeds from the short sale will be \$260, which will be just adequate to take delivery under the futures contract. So for her to make an arbitrage profit, the party who loaned the asset to facilitate the short sale must share some of the storage costs that she has saved due to the fact that she chose not to hold the asset. The rationale is that under normal circumstances the short seller would have been required to compensate the lender of the assets for any income forgone. Logically, therefore, when it comes to saving expenses, the lender of the asset should share the gain with the short seller. However, this is extremely unlikely. Therefore, to rule out reverse cash-and-carry arbitrage. We need to consider the activities of quasi-arbitrageurs.

Take the case of a party who already owns an ounce of the metal. He can sell it for $250 and invest the proceeds at 8 percent per annum for six months. Simultaneously, he can go long in a futures contract. At the end of six months, the asset will be back in his possession and he will have saved $10 in the process by saving storage costs. Quasi-arbitrage is indeed a profitable activity in such circumstances, and a combination of conventional cash-and-carry arbitrage and reverse cash-and-carry quasi-arbitrage will ensure that the asset is neither underpriced nor overpriced.

However, the situation can be different in the case of a convenience asset. Conventional cash-and-carry arbitrage will help ensure that the contract is not overpriced. But traders may be unwilling to indulge in either reverse cash-and-carry arbitrage or quasi-arbitrage because they are getting a convenience value from the asset. All that we can state in the case of such assets is that $F_t \leq S_t \times (1 + r) + Z$—that is, although the contract cannot be overpriced, what looks like underpricing may not be exploitable.

Net Carry

The net carry for an asset that makes no payouts may be expressed as $NC = r$; for assets making a payout, it is $r - \frac{I}{S_t}$. For assets that entail the payment of storage costs, we can express it as $NC = r + \frac{Z}{S_t}$. In all cases we can state that $F_t = S_t \times (1 + NC)$. However, if the asset were to yield a convenience value, we could state the futures price as $F_t = S_t \times (1 + NC) - Y$. The variable that equates the two sides, Y, is termed the *marginal convenience value*.

Backwardation and Contango

If the futures price is in excess of the spot price or if the price of the nearby futures contract is lower than that of the more distant contract, then we say that the market is in *contango*. However, if the futures price is less than the spot price or if the nearby contract is priced higher than the more distant contract, then we say that the market is in *backwardation*. Let us first consider financial assets.

$$F_t = S_t \times (1 + NC).$$

The net carry may be positive or negative, depending on whether the interest cost dominates the income from the security or is dominated by it. The market may be in backwardation or in contango.

For physical assets held for investment purposes, the net carry will always be positive. Such assets will always be in contango. Convenience assets may be in backwardation or in contango. For such assets,

$$F_t = S_t \times (1 + NC) - Y.$$

The net carry will be positive. If the effect of the net carry dominates, the market will be in contango. However, if the convenience yield is substantial, then the market may be in backwardation. Tables 6.8 and 6.9 represent two illustrations showing markets in backwardation and contango, respectively. It is not necessary that a market should steadily be in backwardation or contango. Whereas a three-month contract may be priced higher than the spot, which is what we would associate with a contango market, a six-month contract may be priced lower than the three-month contract, thereby displaying backwardation.

Table 6.8
Illustration of a Contango Market

Contract	Price ($)
Spot	100.00
3-M Futures	107.50
6-M Futures	112.50
9-M Futures	118.00
12-M Futures	122.00

Table 6.9
Illustration of a Backwardation Market

Contract	Price ($)
Spot	100.00
3-M Futures	97.50
6-M Futures	92.50
9-M Futures	88.00
12-M Futures	82.00

The Case of Multiple Deliverable Grades

We have shown that the futures price must converge to the spot price of the underlying asset. But in the case of contracts on certain commodities, multiple grades are permitted for delivery. Because each eligible grade of the underlying asset will have its own spot price, the question is, which spot price will the futures price converge to?

Before we answer this question, let us deal with the issue of the spot–futures ratio. In the case of contracts in which there is only one eligible grade for delivery, any arbitrage strategy that entails the assumption of a futures position until the time of expiration requires spot–futures positions in the ratio of 1:1; that is, the number of units in which a long or a short position is taken in the spot market must be identical to the number of units of the underlying asset represented by the position taken in the futures market. However, if multiple grades are permitted for delivery, then this will typically not be the case. There are two possible ways of price compensation when multiple grades have been specified for delivery. These are referred to as *multiplicative price adjustment* and *additive price adjustment*, respectively.

In the case of contracts in which a multiplicative price-adjustment system has been specified, the short will receive a_iF_T if he were to deliver grade i, where a_i is the adjustment factor for the grade. One grade will be designated as the par grade for which the adjustment factor will be 1.0. For premium grades, the adjustment factor will be greater than 1.0; for discount grades, it will be less than 1. The cash inflow for an arbitrageur who chooses to initiate a cash-and-carry arbitrage strategy at time t with a short position in h futures contracts will be $a_iF_T + h(F_t - F_T)$. To make the cash flow from this strategy riskless, we need to eliminate the term involving T, which implies that $h = a_i$. The number of futures contracts required is equal to the adjustment factor of the grade in which the trader has taken a long spot position.

The profit from the arbitrage strategy is $a_iF_t - S_t(1 + r)$. To rule out arbitrage, we require that the profit be zero, which implies that $F_t = \frac{S_t \times (1+r)}{a_i}$. Because there are multiple grades, this condition cannot prevail for all grades simultaneously. The most preferred grade, for which this condition will prevail, is called the *cheapest to deliver* (CTD) grade. We say in technical terms that the futures price before expiration will be equal to the delivery-adjusted, no-arbitrage futures price of the CTD grade, where the no-arbitrage futures price is $S_t \times (1 + r)$. For all the other grades, which we will denote in general by j, it must be the case that $F_t < \frac{S_t \times (1+r)}{a_j}$. The no-arbitrage condition at expiration is

$F_T = S_T/a_i$. The futures price will converge at the time of expiration of the contract to the delivery-adjusted spot price of the CTD grade where the delivery-adjusted spot price of a grade is S/a. For all other grades j, the delivery-adjusted spot price will be higher. The CTD grade at expiration is the grade with the lowest delivery-adjusted spot price.

The multiplicative system of price adjustment is used for contracts such as Treasury bond futures. Consider the September 2008 futures contract. The CTD bond is a 5 percent coupon bond maturing on May 15, 2037. Its adjustment factor is 0.8642.

On September 15, which is an allowable date for delivery, the futures price (per dollar 100 of face value) is 96-08. The accrued interest per dollar 100 of face value is \$1.6984. The price payable by the long is

$$96.25 \times 0.8642 + 1.6984 = \$84.87765.$$

Consider the system of additive price adjustment that is used for contracts on agricultural commodities with multiple deliverable grades. In the case of such contracts, the short will receive $F_T + a_i$ if he were to deliver grade i where a_i is the adjustment factor for the grade. One grade will be designated as the par grade for which the adjustment factor will be 0.0. For premium grades, the adjustment factor will be positive, whereas for discount grades it will be negative. The cash inflow for an arbitrageur who chooses to initiate a cash-and-carry arbitrage strategy at time t with a short position in h futures contracts will be $F_T + a_i + h(F_t - F_T)$. To make the cash flow from this strategy riskless, we need to eliminate the term involving T, which implies that $h = 1$. The appropriate spot–futures ratio for such contracts is 1:1, as it is for contracts that do not offer a choice with respect to the eligible grade.

The profit from the arbitrage strategy is $F_t + a_i - S_t(1 + r)$. To rule out arbitrage, we require that the profit be zero, which implies that $F_t = S_t \times (1 + r) - a_i$. Because there are multiple grades, this condition once again cannot prevail for all grades simultaneously. The most preferred grade, for which this condition will prevail, is the CTD grade. As in the case of multiplicative adjustment, the futures price before expiration will be equal to the delivery-adjusted, no-arbitrage futures price of the CTD grade, where the no-arbitrage futures price is $S_t \times (1 + r)$. For all other grades, which we will denote in general by j, it must be the case that $F_t < S_t \times (1 + r) - a_j$. The no-arbitrage condition at expiration is $F_T = S_T - a_i$. The futures price will converge at the time of expiration of the contract to the delivery-adjusted spot price of the CTD grade where the delivery-adjusted spot price of a grade is $S - a$. For all other grades j, the delivery-adjusted spot price will be higher. The CTD grade at expiration is once again the grade with the lowest delivery-adjusted spot price.

Risk Arbitrage

In the preceding case, we stated that the futures price before expiration will converge to the delivery-adjusted, no-arbitrage futures price of the CTD grade. For all other grades, we can state that $F_t < \frac{S_t \times (1+r)}{a_j}$ or $F_t < S_t \times (1+r) - a_j$. The question is, if the futures price is lower than the delivery-adjusted, no-arbitrage futures price for a grade, then why can we not implement a reverse cash-and-carry arbitrage strategy to earn a costless, riskless profit? The answer is that such strategies are not riskless.

Let us analyze the cash-and-carry strategy for contracts on a commodity for which additive or multiplicative price-adjustment systems are used. The arbitrageur would have gone long in the required number of units in the spot market and assumed a short position in the futures market. Because she holds the short position in futures, she has the prerogative to decide which grade she should choose to deliver. If the maximum profit is likely to be obtained by delivering the grade in her possession, then she will choose to deliver that grade. However, if she were of the opinion that any other grade is likely to lead to a larger cash flow if delivered, then she will choose to dispose of the grade in her possession in the spot market and acquire the required number of units of the grade that will lead to optimal profits if delivered. Although the arbitrageur may realize the profits that she has been anticipating from the outset, she can only earn more if circumstances change.

However, reverse cash-and-carry arbitrage is different. It requires the arbitrageur to go long in futures and short in the spot. In this case, the right to choose the deliverable grade is with the counterparty and not the arbitrageur. There is no guarantee that the counterparty will choose to deliver the grade that the arbitrageur has sold short. If she were to choose to deliver a different grade because it is optimal for her, then the arbitrageur will realize a lower profit than anticipated. While implementing a reverse cash-and-carry strategy, an arbitrageur may end up realizing a lower profit that may even be a loss and can never earn more than what she was anticipating at the outset. It is for this reason that reverse cash-and-carry arbitrage under such circumstances is termed *risk arbitrage*. We will demonstrate the preceding arguments, first with the multiplicative-adjustment system and then with the additive-adjustment system.

The Case of Multiplicative Adjustment

Consider the case of an arbitrageur who has gone long in one unit of grade i of the asset and short in a_i futures contracts. Assume that, at the point of expiration, another grade j has become more profitable to deliver. If so, he will sell the unit of grade i that he possesses in the spot market and acquire a_i units of grade j to satisfy his delivery commitments under the futures contract. Let us consider his cash flows:

Inflow from sale of grade $i = S_{i,T}$
Marking to market profit or loss $= a_i(F_t - F_T)$
Cost of acquisition of grade $j = a_iS_j,T$
Inflow from delivery under the futures contract $= a_i \times a_jF_T$

$$\begin{aligned}\text{Net cash flow} &= \mathrm{S}_{i,T} + a_i \times a_jF_T + a_i(F_t - F_T) - a_iS_{j,T} \\ &= a_iF_t + (S_{i,T} - a_iF_T)\end{aligned}$$

$$\begin{aligned}&a_iF_t + a_i\left[\frac{S_{i,T}}{a_i} - \frac{S_{j,T}}{a_j}\right] \\ &> a_iF_t.\end{aligned}$$

We have used the fact that $a_jF_T = S_{j,T}$ and that because j is the CTD grade $\frac{S_{i,T}}{a_i} > \frac{S_{j,T}}{a_j}$.

The arbitrageur will never earn a profit that is less than what he anticipated at the outset.

Let us turn to reverse cash-and-carry arbitrage. It would have been initiated by short selling one unit of grade i and going long in a_i futures contracts. Assume that at the point of expiration, another grade j has become more profitable to deliver. If so, the arbitrageur will acquire one unit of grade i to cover his short position and sell a_i units of grade j that he would receive under the futures contract in the spot market. Let us consider his cash flows:

Outflow on account of acquisition of grade $i = S_{i,T}$
Marking to market profit or loss $= a_i(F_T - F_t)$
Proceeds from sale of grade $j = a_iS_{j,T}$
Outflow because of acquisition under the futures contract $= a_i \times a_jF_T$

$$\begin{aligned}\text{Net cash flow} &= a_i S_{j,T} + a_i(F_T - F_t) - S_{i,T} - a_i \times a_j F_T \\ &= a_i(F_T - F_t) - S_{i,t} \\ &= a_i\left[\frac{S_{j,T}}{a_j} - \frac{S_{i,T}}{a_i}\right] - a_i F_t.\end{aligned}$$

We know that $\frac{S_{i,T}}{a_i} > \frac{S_{j,T}}{a_j}$. The net outflow can be greater than but not less than what was anticipated at the outset. Reverse cash-and-carry arbitrage is fraught with danger and is termed *risk arbitrage*.

The Case of Additive Adjustment

Consider the case of cash-and-carry arbitrage: the arbitrageur has gone long in one unit of grade i of the asset and short in one futures contract. Assume that at the point of expiration, another grade j has become more profitable to deliver, and that he decides to sell the unit of grade i that he possesses in the spot market and acquire one unit of grade j to satisfy his delivery commitments under the futures contract. Let us consider his cash flows:

Inflow from sale of grade $i = S_{i,T}$

Marking to market profit or loss $= (F_t - F_T)$

Cost of acquisition of grade $j = S_{j,T}$

Inflow from delivery under the futures contract $= F_T + a_j$

$$\begin{aligned}\text{The net cash flow} &= \mathrm{S_{i,T}} + F_T + a_j + (F_t - F_T) - S_{j,T} \\ &= F_t + a_i + [S_{i,T} - a_i - (S_{j,T} - a_j)].\end{aligned}$$

Because grade j is the CTD, we know that $(S_{i,T} - a_i) > (S_{j,T} - a_j)$. The net inflow may be higher than anticipated but never less. Using similar arguments, Example 6.6 demonstrates that reverse cash-and-carry arbitrage in the case of a contract for which additive adjustment has been prescribed is also a case of risk arbitrage.

Example 6.7 will demonstrate that a cash and carry arbitrage strategy will always yield a profit that is greater than or equal to what was anticipated at the outset. However the profit from a reverse cash and carry arbitrage strategy will always be less than or equal to what was expected at the outset.

Example 6.6

Futures contracts on a commodity permit three grades for delivery: red, white, and blue. Each contract is for 5,000 bushels of the underlying asset. White is the par grade. Red can be delivered at a discount of 2 cents per bushel, and blue can be delivered at a premium of 2 cents per bushel.

Assume that futures contracts are available with six months to maturity. The riskless rate is 8 percent per annum. The spot prices for the three grades are depicted in Table 6.10.

Table 6.10
Spot Prices Before Expiration

Grade	Price ($)
Red	3.2500
White	3.2800
Blue	3.3000

We compute the delivery-adjusted, no-arbitrage futures price for each of the three grades. For red, it is $3.2500 \times 1.04 - (-0.02) = 3.4000$. We can similarly show that it is 3.4112 for white and 3.4120 for blue. Red is the CTD grade, and the futures price must be equal to 3.4000 to rule out arbitrage.

Assume that the spot prices and the delivery-adjusted spot prices at expiration are as shown in Table 6.11.

Table 6.11
Spot Prices, and Delivery-Adjusted Spot Prices at Expiration

Grade	Price ($)	Delivery-Adjusted Spot Price ($)
Red	3.3900	3.4100
White	3.4000	3.4000
Blue	3.4500	3.4300

White is the CTD grade at expiration.

Example 6.7

Consider the data in the preceding illustration. Assume that the futures price six months before expiration is 3.4075. The contract is overpriced. Take the case of an arbitrageur who goes long in 5,000 bushels of red and goes short in one futures contract. The futures price at expiration will be the delivery-adjusted spot price of the CTD grade, which in this case is 3.4000.

If the arbitrageur delivers the grade in his possession, he will receive $3.4000 - 0.02 = 3.3800$. His profit or loss from marking to market will be $(3.4075 - 3.4000)$ per bushel. The net amount receivable for 5,000 bushels is

$$5{,}000 \times [3.3800 + 0.0075] = \$16{,}937.50.$$

The amount repayable on account of the loan taken to acquire the 5,000 bushels at the outset is $5{,}000 \times 3.2500 \times 1.04 = \$16{,}900$.

The profit from the strategy is \$37.50, which is equal to $5{,}000 \times 0.0075$, where 0.0075 represents the extent to which the contract was mispriced at the outset.

Consider a situation in which the arbitrageur decides to sell the 5,000 bushels of red in his possession and acquire 5,000 bushels of white to fulfill his commitments under the contract.

The inflow from the sale of 5,000 bushels of red $= 5{,}000 \times 3.3900 = \$16{,}950$
The cost of acquisition of 5,000 bushels of white $= 5{,}000 \times 3.4000 = \$17{,}000$
The profit from marking to market $= 5{,}000 \times (3.4075 - 3.4000) = 37.50$
The amount receivable when delivery is made under the contract $= 5{,}000 \times 3.4000 = \$17{,}000$
Thus, the net inflow $= 16{,}950 - 17{,}000 + 37.50 + 17{,}000 = \$16{,}987.50$.

The amount repayable on account of the loan taken at the outset is \$16,900. The profit is \$87.50. Consequently, the profit from a cash-and-carry strategy in the case of commodities for which multiple grades are permitted for delivery can be greater than what was anticipated at the outset, but it cannot be less.

Let us turn to reverse cash-and-carry arbitrage. Assume that the futures price at the outset is 3.3925 per bushel. The trader will have to short sell 5,000 bushels of red and go long in a futures contract. We will first compute his profits under the assumption that he is asked to take delivery of red at maturity.

The inflow from the initial investment of the short-sale proceeds $= 5{,}000 \times 3.2500 \times 1.04 = \$16{,}900$
The cost of taking delivery under the futures contract $= 5{,}000 \times 3.3800 = \$16{,}900$
The profit or loss from marking to market $= 5{,}000 \times (3.4000 - 3.3925) = \$37.50$.

The overall profit is \$37.50, which corresponds to the mispricing of the contract at the outset.

Let us turn to a situation were the arbitrageur is asked to take possession of White.

He will have to acquire 5,000 bushels of Red which will cost: $5{,}000 \times 3.3900 = \$16{,}950$.

The proceeds from the sale of 5,000 bushels of white acquired under the contract $= 5{,}000 \times 3.4000 = \$17{,}000$
The cost of taking delivery under the futures contract $= 5{,}000 \times 3.4000 = \$17{,}000$
The profit or loss from marking to market $= 5{,}000 \times (3.4000 - 3.3925) = \$37.50$
The proceeds from the investment of the short-sale proceeds $=$ \$16,900
The overall profit $= -16{,}950 + 17{,}000 - 17{,}000 + 16{,}900 + 37.50 =$ \$(12.50).

Instead of the anticipated profit of \$37.50, he has ended up with a loss of \$12.50. This is why reverse cash-and-carry arbitrage under such circumstances is said to be fraught with risk.

Trading Volume and Open Interest

Trading volume refers to the number of contracts traded during a given time interval. We are usually concerned with the volume for a given trading day from the commencement of trading in the morning until the market closes. Every trade that is consummated during the day adds to the day's trading volume. Take the case of Mitch, who goes long in 250 contracts on wheat with Doug. The addition to the trading volume for the day is +250. A trade will always serve to increase the observed volume for the time period.

Open interest refers to the number of open positions at any point in time. Every long position must be matched by a corresponding short position. Open interest may be measured as the total number of open long positions at a given point in time or, equivalently, as the total number of open short positions. A trade may lead to an increase or a decrease in the open interest or leave it unchanged, as we will demonstrate.

Assume that Mitch took a long position in 250 contracts on wheat last week and that Doug also had a prior short position in 200 contracts on the commodity. The two parties execute a trade in 100 contracts in which Mitch takes a long position and Doug takes a short position. The number of open long positions increases by 100 as a consequence of the trade. Equivalently, the number of open short positions also increases by 100. The impact of the trade is to increase the open interest by 100 contracts. So if a trade results in an establishment of new positions by both the parties who enter into it, the open interest will rise.

Consider a situation in which Mitch goes long in 100 contracts with Kirk, who takes a short position. Assume that Kirk previously had a long position in 200 contracts. The impact of the trade is to increase the number of total open long positions by 100 as seen from Mitch's perspective. However, the trade simultaneously results in a decline of the open interest by 100 contracts because Kirk is partially offsetting his position. The net impact on open interest is nil. So if a trade results in the opening of a position by a party by trading with a counterparty who is offsetting, the open interest will remain unchanged.

Finally, consider a situation in which Mitch takes a short position in 100 contracts by trading with Doug, who takes a long position. The consequence of the trade is that Mitch's long position is reduced by 100 contracts. Simultaneously, the trade has resulted in a decrease of 100 in Doug's short position. The open interest will decline by 100 contracts in this case. So if a trade results in the offsetting of existing positions by both the parties, then open interest will decline.

Even though a trade will always lead to an increase in the trading volume for a day, its impact on the open interest will depend on the previous positions held by the counterparties.

A high trading volume for a day is symptomatic of a highly liquid market. Open interest, on the other hand, is an indicator of high future liquidity because the higher the open interest, the greater the potential for offsetting transactions.

Delivery

Both forward and futures contracts may set forth terms for delivery, but the two types of contracts differ in several respects. First, the majority of forward contracts that come into existence are actually settled by delivery. On the other hand, the majority of futures contracts are offset before maturity. A small number of futures contracts, from 2 to 5 percent, are actually settled by delivery.

Second, in the case of a forward contract, the party who went short at the outset will end up delivering to the party with whom he traded initially. As explained earlier, there is no intervention in the case of such contracts by an agency such as a clearinghouse, and the link between the two original counterparties remains very much intact. In the case of a futures contract, however, the link between the two counterparties is snapped by the clearinghouse immediately after the trade. Subsequently, one or both counterparties may exit the market by taking a counter-position. When a short expresses his desire to make delivery, the exchange has to locate a party to whom he can deliver. The long who is chosen to accept delivery is the party with the oldest outstanding long position.

Third, futures contracts are marked to market on a daily basis. The cash flow for a long position on a given day is $F_t - F_{t-1}$. If we aggregate the cash flows, we get $\sum_{t=1}^{T} [F_t - F_{t-1}] = F_T - F_0$. To ensure that the price paid by the long is what was contracted at the outset, he should pay a price P such that, after accounting for the cash flows from marking to market, he effectively ends up paying F_0:

$$P - [F_T - F_0] = F_0 \Rightarrow P = F_T.$$

The price at which delivery is made under a futures contract is the terminal futures price; as we saw earlier, this is no different from the terminal spot price.

In the case of a forward contract, however, there is no marking to market. So the price that is paid by the long at the time of delivery is the price that was contracted at the outset.

Cash Settlement

All futures contracts do not mandate the delivery of the underlying asset. Stock-index futures contracts do not because it is extremely cumbersome to deliver a large basket of securities (the S&P 500 is composed of 500 stocks) in exactly the same proportions in which they are present in the index. Contracts such as index futures are cash settled—that is, they are marked to market one last time at the close of trading on the expiration day, and all positions are declared closed. Both the long and the short will exit the market with their profit or loss but without taking delivery. The cash flow for the long will be $F_T - F_0$, while that for the short will be $F_0 - F_T$. Cash settlement may at times be the prescribed mechanism in markets where there is a perceptible risk of manipulation or the potential to create artificial shortages. Exchanges in many developing economies therefore prescribe cash settlement as the norm for many contracts.

Hedging and Speculation

There are two categories of traders in futures contracts: hedgers and speculators. The former take positions in futures contracts to reduce if not eliminate their exposure to risk on account of a spot-market exposure. The latter consciously seek to take an exposure to risk in the futures market in anticipation of gains.

The rationale for hedging may be explained as follows. Take the case of a trader who owns 5,000 bushels of wheat and is planning to sell the commodity after a month. Her worry is that the price of the product may fall in the spot market by the time the delivery date arrives. She can go short, however, in a futures contract at the prevailing futures price F_t. If so, she can be assured of being able to sell at this price, no matter what the spot price of wheat may be on the expiration date of the contract. In this argument, we are assuming that the planned date of sale of the commodity coincides with the expiration date of the futures contract. This may not be the case. If the wheat were to be sold before the delivery date of the contract on a date t^*, the trader will have to sell the asset in the spot market at a price $S_t{}^*$ and collect her profit or loss from marking to market in the futures market. The net inflow will be

$$S_{t^*} + (F_t - F_{t^*}) = F_t + (S_{t^*} - F_{t^*}).$$

$S_{t^*} - F_{t^*}$ is referred to as the basis. The risk for the hedger is the risk of variation in the basis. If the trader had not hedged, she would have received S_{t^*}. The risk of variation in the spot price is termed *price risk*. Hedging replaces price risk with the more acceptable risk of variation in the price difference between the spot and the futures price, which is termed *basis risk*.

Let us take the case of a trader who is short in the spot market. The term *short* connotes that he has a prior commitment to buy. A company in Bangalore may have imported mainframe computers from the United States, and it may have a commitment to pay in U.S. dollars after a month. The risk is that the rupee price of the U.S. dollar may rise—in other words, the U.S. dollar may appreciate. Such a trader can go long in the futures market in order to hedge. If the date of acquisition of dollars were to coincide with the expiration date of the futures contracts, the dollars can be acquired at the initial futures price F_t. If not, the dollars can be bought in the spot market, and the profit or loss from marking to market can be collected. In this case, the outflow will be

$$-S_{t^*} + (F_{t^*} - F_t) = -F_t - (S_{t^*} - F_{t^*}).$$

The magnitude of the cash inflow for a short hedger is $F_t + b_{t^*}$, whereas the magnitude of the outflow for a long hedger is also $F_t + b_{t^*}$. The short hedger therefore stands to benefit from a rising basis, whereas the long hedger stands to profit from a falling basis. We know that traders with short futures positions stand to benefit from falling futures prices while those with long futures positions stand to gain from rising futures prices. The basis itself may be construed as a price, because it is nothing but the difference of two prices. We say that the short hedger is long the basis whereas the long hedger is short the basis.

On the day of expiration, the basis will be zero because $b_T = S_T - F_T = 0$ and the futures price must converge to the spot price when the contract itself is scheduled to expire. A hedge without any inherent basis risk is termed as a *perfect hedge*. One key requirement for a perfect hedge is that the date of termination of the hedge must coincide with the expiration date of the futures contract. In the case of a perfect hedge, a short hedger can be assured of receiving the futures price that was prevailing when the hedge was set up, whereas a long hedger can be guaranteed that he can acquire the asset by paying the futures price that was prevailing when he took a long position.

There is another key requirement for a perfect hedge. Let us consider the cash inflow for a short hedger, who is long in Q units of the asset and short in N futures contracts where each contract is for C units of the underlying asset. The futures position represents Q_f units of the underling asset, where $Q_f = NC$, The cash inflow is

$$QS_T + NC(F_t - F_T).$$

For this to be equal to QF_t, we require that $Q = NC = Q_f$. The hedge ratio must be 1:1, which implies that the quantity being hedged must be an integer multiple of the size of the futures contract.

Let us summarize the three conditions for a perfect hedge:

1. The first and most basic condition is that futures contracts must be available on the asset whose price is being sought to be hedged.
2. The date of sale or purchase of the underlying asset must coincide with the expiration date of the futures contract.
3. The number of units of the underlying asset must be an integer multiple of the contract size.

Condition 1 is likely to be satisfied in many cases. However, condition 2 is not. Suppose a commodity on which contracts expiring in March, May, July, September, and December are available. Assume that the contracts expire on the twenty-first of the respective months, whereas the hedger would like to terminate the hedge on July 15. The May contract is not appropriate because it will leave the trader open to price risk for the period from May until July. July contracts are not usually suitable because futures contracts can exhibit erratic price movements in the expiration month. A hedger who seeks to buy or sell in July would like to avoid exposure to this contract. The choice is between the September and December contracts. The further away the expiration month, the greater the basis risk. The reason is as follows.

The spot price, as well as the futures price, is being influenced by the same economic factors. The spot price represents the current market price, whereas the futures price is the price for a transaction at a future point in time. If the termination date of the hedge is close to the expiration date of the futures contract, then both markets will be discovering the price of the product for virtually the same point in time. As a result, the prices will converge so that the difference or the basis is more predictable.[2] The September contract is likely to expose the trader to a lower basis risk than the December contract. When the transaction date does not coincide with the expiration date of the futures contract, the hedger will choose a contract that is slated to expire soon after the month in which the hedge is being terminated.

Rolling a Hedge

In the preceding argument we assumed that the hedger would choose the September futures contract. Let us assume that the futures position was taken on April 21. On that day, however, the September contracts may not have commenced trading. Or they may have been so illiquid that they deterred investors who seek to enter and exit the market at a price that is close to the true or fair value of the asset. In such cases, it is conceivable that the hedger will first go short in the May contract. Because he would like to avoid exposure to the contract in its expiration month, he is likely to offset the position at the end of April and go short in July contracts. Finally, toward the end of June, he is likely to offset the July contracts and take a short position in September contracts. On July 15, he will sell the underlying asset in the spot market and offset the September futures position. Such a hedging strategy is known as a *rolling hedge*.

Tailing a Hedge

Consider a perfect hedge. We know that a short hedger will receive the spot price prevailing at the time of termination of the hedge and that he can collect the total profit or loss from marking to market. The profit or loss from marking to market on a given day t is $F_{t-1} - F_t$. This can be reinvested if it is a profit or financed if it is a loss until the maturity date so as to yield $(F_{t-1} - F_t)(1 + r)^{t^*-t}$. The cumulative profit or loss from a futures position equivalent to Q_f units is $Q_f \sum_{t=1}^{t^*} [F_{t-1} - F_t](1 + r)^{t^*-t}$. In general, this will not be equal to $Q_f[F_0 - F_{t^*}]$. To make it so, we should have a position in

$$Q_{f,t} = \frac{Q_f}{(1 + r)^{t^*-t}}$$

futures contracts at the beginning of day t. We would have to start with a quantity less than Q_f and increase our position steadily at the end of each day. If we were to start with Q_f units rights from the outset, we would be overhedging. Overhedging will inflate the profit when there is a profit but will magnify the loss when there is a loss. The process of periodically adjusting the futures position in order to account for the effect of marking to market is termed *tailing*. The futures position is not usually adjusted daily but only periodically, because each adjustment entails the incurrence of transactions costs.

The Minimum Variance Hedge Ratio

We know that the cash flow for a short hedger is

$$QS_{t^*} + Q_f(F_t - F_{t^*}).$$

If $t^* = T$, then $S_{t^*} = S_T = F_{t^*} = F_T$, and we should set $Q_f = Q$ such that the total revenue is QF_t, an amount about which there is no uncertainty. To obtain a perfect hedge under such circumstances, we need to set $Q_f = Q$; in other words, we need a hedge ratio of 1:1.

In general, however, the revenue from the hedged position is

$$\begin{aligned} R &= QS_{t^*} + Q_f(F_t - F_{t^*}) \\ &= QS_{t^*} + hQ(F_t - F_{t^*}) \\ &= QS_t + (S_{t^*} - S_t)Q - (F_{t^*} - F_t)hQ \\ &= QS_t + (\Delta S - h\Delta F)Q \end{aligned}$$

where h is the hedge ratio.

At the outset, we know Q, S_t, and h. To minimize risk, we need to minimize the variance of the revenue using the only variable in our control, which is h.

$$\begin{aligned} \mathrm{Var}[R] &= \mathrm{Var}[QS_t + (\Delta S - h\Delta F)Q] \\ &= Q^2\mathrm{Var}[\Delta S - h\Delta F]. \end{aligned}$$

Let us denote Var(ΔS) by ${\sigma_s}^2$ and Var(ΔF) by σ_f^2. The correlation between the two variables is ρ. If we minimize the variance of R with respect to h, we get $h = \rho\frac{\sigma_s}{\sigma_f}$.

Estimation of the Hedge Ratio and the Hedging Effectiveness

Consider the following statistical relationship:

$$\Delta S = \alpha + \beta\Delta F + \varepsilon.$$

To estimate the parameters, we can run a linear regression with ΔF as the independent variable and ΔS as the dependent variable. The minimum variance hedge ratio is given by the slope coefficient β. The R^2 of the regression is a measure of the hedging effectiveness. If the basis risk is equal to the price risk, there will be no risk reduction and the R^2 will be zero. However, if there is a perfect hedge, then the R^2 will be 1.0. We will get a value between 0 and 1.0.

Cross Hedging

Sometimes, it may so happen that there are no suitable futures contracts on the product whose price is sought to be hedged. It could be because there is no contract on the commodity in which we are interested. Otherwise, it could be the case that even though contracts are available, they are so illiquid that the hedger does not have to confidence to take a position. Under such circumstances, the trader may choose to hedge by taking a futures position in a closely related commodity, where the term *closely related* means a commodity whose price is highly positively correlated with that of the commodity that we are interested in. The use of futures contracts on a closely related commodity is termed *cross hedging*.

The revenue for the short hedger under such circumstances is

$$S_{t^*} + [F^*_t - F^*_{t^*}]$$

where S and F represent the spot and futures prices of the commodity the revenue from whose risk is being hedged, and S^* and F^* represent the spot and futures prices of the commodity that is used to cross-hedge.

This can be expressed as $F^*_t + (S^*_{t^*} - F^*_{t^*}) + (S_{t^*} - S^*_{t^*})$. The basis therefore consists of two components. The term $(S^*_{t^*} - F^*_{t^*})$ would have been the basis if the asset being hedged had been the same as the asset underlying the futures contract. The second term, $(S_{t^*} - S^*_{t^*})$, is the basis that arises because of the fact that the two assets are different. If $t^* = T$, then the first term will drop out and the basis for a cross hedge will be $(S_{t^*} - S^*_{t^*})$.

Speculation

Futures contracts can be used for speculation. Take the case of a trader who is bullish about the market. If so, she can go long in a futures contract at the prevailing market price. If the market were to rise, she can either take delivery at expiration and sell the commodity at the prevailing spot price, which, by assumption, is higher, or she can offset her position before expiration and exit the market with a cumulative profit from marking to market.

Examples 6.8–6.13 illustrate the profits and losses from speculative strategies using futures contracts under different circumstances.

Example 6.8

Rob is of the opinion that corn will trade at least at \$3.95 per bushel after a month. The current futures price for a one-month contract is

$3.40. Let us assume that he goes long in five futures contracts with each contract representing 5,000 bushels.

Assume that the price after one month is $4.05 per bushel. Rob can take delivery under the contract at $3.40 and sell the commodity immediately at $4.05.

His profit is

$$5{,}000 \times 5 \times (4.05 - 3.40) = \$16{,}250.$$

Example 6.9

Assume that two weeks after Rob went long, the spot price is $4.00. The futures price—assuming that the cost of carry model is applicable and that the interest rate is 6.5 percent per annum—is $4.00 \times (1 + 0.065 \times 2/52) = \$4.01$.[3]

Rob will have a cumulative profit from marking to market of

$$5{,}000 \times 5 \times (4.01 - 3.40) = \$15{,}250.$$

Futures contracts can also be used by bears for speculation. To put into operation their viewpoint regarding the market, they need to go short in futures contracts as the following two examples illustrate.

Example 6.10

Holly is of the opinion that wheat will decline in price over the next two months. Futures contracts with two months to expiration are currently quoting at $4.95 per bushel. Assume that Holly goes short in five contracts, each of which is for 5,000 bushels.

Consider a situation in which the price after two months is $4.25 per bushel. Holly can buy spot at this price and immediately deliver under the futures contract at $4.95. Her profit is

$$5{,}000 \times 5 \times (4.95 - 4.25) = \$17{,}500.$$

Example 6.11

Assume that one month after Holly goes short, the spot price is \$4.00 and that the futures price[4] is $4.00 \times (1 + 0.06 \times 1/12) =$ \$4.02.

If Holly were to offset at this point in time, she can walk away with a profit of

$$5{,}000 \times 5 \times (4.95 - 4.02) = \$23{,}250.$$

Futures contracts can yield substantial profits for both bulls and bears if they make the right calls on the movement of the market. However, if they make an error of judgment, their losses can be substantial, as the following examples with bulls illustrate.[5]

Example 6.12

Assume that Rob goes long in five corn futures contracts when the futures price is \$3.40 per bushel. A month later when the contract expires, the prevailing spot price is \$2.50 per bushel. If Rob were to take delivery under the futures contract and offload the corn in the spot market, he would incur a loss of

$$5{,}000 \times 5 \times (2.50 - 3.40) = \$22{,}500.$$

Example 6.13

Assume that Rob goes long in corn futures when the futures price is \$3.40. However, instead of going up, the spot market crashes to \$2.00 after two weeks, which corresponds to a futures price of \$2.005 per bushel. If Rob were to exit at this point in time, he would have incurred a loss of

$$5{,}000 \times 5 \times (2.005 - 3.400) = \$34{,}875.$$

Leverage

Bulls have a choice between a long spot position and a long futures position from the standpoint of speculation, whereas bears have a choice between a short spot position and a short futures position.

Futures positions offer the benefit of leverage to a speculator. As we previously saw, a position may be said to be levered if a small price movement in the underlying asset has a disproportionately large impact on the rate of return.

Why do we say that a futures position is levered? Take the case of a trader who goes long in a futures contract on a commodity by depositing $5,000 as the initial margin. Let the initial futures price be $5.00 per bushel and assume that the contract size is 5,000 bushels. Consider a price increase of $0.40 per bushel. The profit per contract will be $2,000, which amounts to a return of 40 percent on the initial margin deposit. However, if the trader had taken a spot position at a price equal to the initial futures price, then he would have had to invest $25,000, and a profit of $2,000 would amount to a return of only 8 percent. The leverage inherent in a futures contract helps magnify the rate of return. As we are aware, though, leverage is a double-edged sword: losses, if any, will also be magnified. A price decline of $0.40 per bushel would amount to a −40 percent return on the futures position but to a loss of only 8 percent on a spot position.

Contract Value

When a forward contract is agreed to, neither the short nor the long has to pay a price to enter it. The price at which the underlying asset is scheduled to be bought or sold at a future date per the contract is referred to as the *delivery price* of the contract. The delivery price is set in such a way that the contract value at inception is zero. In other words, the two equal and offsetting obligations ensure that neither party has to pay the other to take a position.

The delivery price remains fixed once a contract is sealed. However, the price for a fresh contract will keep fluctuating over time. The price that is applicable if a forward contract were to be sealed at a particular instant is referred to as the *forward price*. If a contract were to be sealed, the prevailing forward price will be the delivery price of the contract. An instant later, however, the forward price will typically change.

Although the value of a contract will be zero at inception, such contracts will accrue value as time elapses. Consider a long forward position that was taken at a point of time in the past when the forward price was K. This forward price would have been set as the delivery price of the contract. A few periods later, the prevailing forward price is F. If the party who had originally gone long were to offset, he would have to do so by going short at a price F. The two offsetting positions would lead to a cash flow of $F - K$ at the time of expiration of the contract. The current value of the contract is the present value of this cash flow. As we have seen earlier, rising prices will lead to profits for the longs, and falling prices will lead to losses. If the forward price at a point in time is higher than the delivery price of the contract, then a long forward position will have a positive value. However, if the price were to have declined after a contract was entered into, a long position will have a negative value. The situation will be exactly the opposite for shorts. Falling prices will lead to positive contract values, and rising prices will lead to negative values.

Example 6.14 illustrates the computation of the value of an outstanding contract.

Example 6.14

A four-month forward contract was taken a month ago with a delivery price of 100. Today, the forward price for a three-month contract is 106.15. Assume that the rate of interest is 10 percent per annum. The value of a long position is

$$\text{Value} = \frac{F - K}{(1 + r)} = \frac{106.15 - 100}{1.025} = \$6.$$

The value of a short position is

$$\text{Value} = \frac{K - F}{(1 + r)} = \frac{100 - 106.15}{1.025} = -\$6.$$

A contract is always a zero-sum game. The value for a short is exactly the negative of the value for a long.

What about the value of a futures contract? Such contracts also will have a zero value at inception. At the end of each day, starting with the trade date, the contract will be marked to market. Marking to market is nothing but the settlement of built-up value since the time that the contract was previously marked to market. The party with a positive value will have his margin account credited while the counterparty will have his margin account debited. The only time that a futures contract acquires value is in the intervening period between two successive marking to market dates. Each time the contract is marked to market, the value will revert to zero.

Forward versus Futures Prices

In general, the forward price will not be equal to the futures price for contracts on the same underlying asset and with the same expiration date. The reason is that futures contracts are periodically marked to market, but forward contracts are not.

We can show that if the interest rate is a constant and is equal for all maturity periods, then there will be no difference between the forward price and the futures price. However, in reality, interest rates are a random variable because they are stochastic in nature. Therefore the forward price will differ from the futures price.

The rationale is the following. If interest rates were to be positively correlated with futures prices, then rising prices will lead to a situation in which the long, who will have a profit under such circumstances, can invest his profits from marking to market at high rates of interest. Falling futures prices will lead to a situation in which the longs have to finance their losses at a lower rate of interest. An interest rate that is positively correlated with futures prices is beneficial for the long. Therefore, longs will have to pay more under such circumstances, which means that the futures price will be higher than the forward price. The positive correlation has no consequences for a forward contract because there is no concept of marking to market. Under such circumstances, futures prices will be higher than forward prices.

On the other hand, if futures prices are negatively correlated with interest rates, then rising futures prices will correspond to a situation in which the longs are investing their profits at lower rates of interest, whereas declining prices will mean that they are financing their losses at higher rates of interest. Under such circumstances, the futures price will be lower than the forward price.

Locking in Borrowing and Lending Rates

Eurodollar futures contracts can be used to lock in the rate payable or receivable on a loan in advance. Assume that a company is planning to borrow US$5 million one month hence for a period of 90 days. We will also assume that the borrowing date coincides with the expiration date of the Eurodollar futures contract. The firm will be worried about the possibility of an increase in interest rates and would like to lock in a rate in advance. Assume that the current futures price is 95.60. This corresponds to an implicit interest rate of $100 - 95.60 = 4.40$ percent. The company is confident that it can borrow at the prevailing LIBOR[6] plus 40 basis points (bp) on the day of borrowing. Because it is going to borrow, it needs a short hedge. The rationale is that if interest rates were to rise, there should be a profit from the futures contracts. Because each ED futures contract is for US$1 million, the firm needs to go short in five contracts.

Examples 6.15 and 6.16 illustrate the outcome of the hedged position under two different assumptions about the terminal value of the LIBOR.

Example 6.15

Let us first consider a situation in which the LIBOR one month after is at 5.2 percent. The interest payable by the company on a loan of US$ 5 million is

$$0.056 \times 5{,}000{,}000 \times \frac{90}{360} = \$70{,}000.$$

The profit or loss from the futures market is

$$\begin{aligned} &5 \times 1{,}000{,}000 \times \frac{(F_0 - F_1)}{100} \times \frac{90}{360} \\ &= 5 \times 1{,}000{,}000 \times \frac{(95.60 - 94.80)}{100} \times \frac{90}{360} \\ &= \$10{,}000. \end{aligned}$$

The net interest paid is $70{,}000 - 10{,}000 = 60{,}000$.

Example 6.16

Consider a situation in which the LIBOR after a month is 3.2 percent.

The gross interest payable is

$$0.036 \times 5{,}000{,}000 \times \frac{90}{360} = \$45{,}000.$$

The profit loss from the futures position is

$$\begin{aligned} & 5 \times 1{,}000{,}000 \times \frac{(F_0 - F_1)}{100} \times \frac{90}{360} \\ & = 5 \times 1{,}000{,}000 \times \frac{(95.60 - 96.80)}{100} \times \frac{90}{360} \\ & = \$15{,}000. \end{aligned}$$

The net interest payable is $45{,}000 + 15{,}000 = \$60{,}000$.

Irrespective of the prevailing LIBOR, the company can lock in an interest payment of $60,000. This corresponds to an annual rate of

$$\frac{60{,}000}{5{,}000{,}000} \times 100 \times \frac{360}{90} = 4.80\%.$$

This corresponds to the rate implicit in the initial futures price (4.40 percent) plus the premium of 40 bp payable by the borrower.

Lenders can also use Eurodollar futures contracts to lock in interest rates. The principle is the same with the difference being that they will have to go long in futures contracts.

Hedging the Rate of Return on a Stock Portfolio

Index futures can be used to lock in the rate of return on a stock portfolio. The number of futures contracts required to set up a risk-minimizing hedge can be shown to be

$$\beta_p \frac{P_t}{I_t}.$$

β_p is the beta of the portfolio whose risk is sought to be hedged. P_t is the current market value of the portfolio being hedged, and I_t is the present index level in dollars.

Example 6.17 illustrates the computation of the number of futures contracts for a given beta, index level, and portfolio value.

Example 6.17

Consider a stock portfolio that is currently worth US$5 million. The S&P 500 is quoted at 400, and the lot size (contract multiplier) is 250. The value of the index in terms of dollars is $400 \times 250 =$ \$100,000.

If the portfolio beta is 1.40, then the hedger needs to go short in $1.40 \times \frac{5{,}000{,}000}{100{,}000} = 70$ contracts.

Assuming that the stocks in the index are not scheduled to pay any dividends during the life of the futures contract, the pricing condition required to preclude both forms of arbitrage is

$$F_t = I_t\left[1 + r \times \frac{(T-t)}{360}\right]$$

where $T - t$ is the time remaining until the expiration date of the futures contract.

If we were to assume that there are 90 days until expiration and that the interest rate is 7.20 percent per annum, the corresponding futures price is

$$F_t = 400 \times [1 + 0.072 \times 0.25] = 407.20.$$

We will consider two illustrations: in Example 6.18, the index rises; in Example 6.19, it declines.

Example 6.18

The index value 90 days after is at 425. The return on the index is $\frac{(425-400)}{400} \equiv 6.25\%$.

The riskless rate for 90 days is $7.20 \times 0.25 = 1.80$ percent.
The rate of return on the portfolio is

$$1.80 + 1.40(6.25 - 1.80) = 8.03\%.$$

The terminal value of the portfolio is $5{,}000{,}000 \times 1.0803 =$ \$5,401,500.

The profit or loss from the futures position is

$$70 \times 250 \times (407.20 - 425) = \$(311{,}500).$$

The net value of the asset is $5{,}401{,}500 - 311{,}500 = 5{,}090{,}000$.

The rate of return is

$$\frac{90{,}000}{5{,}000{,}000} \times 100 \times \frac{360}{90} = 7.20\%.$$

The portfolio has earned the riskless rate of return over a period of 90 days, because the futures contracts have completely eliminated the exposure to market risk.

Example 6.19

Assume that the index is at 375 after 90 days. The rate of return is -6.25 percent. The portfolio return is

$$1.80 + 1.40(-6.25 - 1.80) = -9.47\%.$$

The terminal value of the portfolio is $5{,}000{,}000 \times (1.00 - 0.0947) = \$4{,}526{,}500$.

The futures profit or loss is $70 \times 250 \times (407.20 - 375.00) =$ \$563,500.

Thus, the net value of the asset is $4{,}526{,}500 + 563{,}500 =$ \$5,090,000.

As expected, we have once again locked in the riskless rate for 90 days.

Changing the Beta

Stock-index futures can be used to change the beta of a stock portfolio. Assume that a portfolio has a beta of β_0 and that we wish to change it to β_T. The number of futures contracts required is given by

$$N = (\beta_T - \beta_0)\frac{P_t}{I_t}.$$

If we wish to increase the beta, we should go long in futures contracts; if we wish to decrease the beta, a short position will be required.

Example 6.20 demonstrates the futures position required to change the beta.

Example 6.20

An investor is holding a portfolio worth $5 million. The current beta is 1.40, and he wishes to increase it to 1.80. The index is currently at 400, and the index multiplier is 250. Because the beta is being sought to be increased, a long futures position is required. The number of contracts required is

$$(1.80 - 1.40)\frac{5{,}000{,}000}{400 \times 250} = 20.$$

Program Trading

Program trading is a market term for stock-index arbitrage. In other words, it refers to cash-and-carry and reverse cash-and-carry arbitrage strategies that are implemented with the help of index futures contracts. If a potential arbitrageur were to be of the opinion that the futures contract is overpriced, she will go in for a cash-and-carry strategy by buying the stocks contained in the index and taking a short position in index futures. On the other hand, if she were to be of the opinion that the contract is underpriced, then she will short sell the stocks contained in the index and take a long position in index futures.

Index arbitrage therefore requires traders to take simultaneous long or short positions in a large basket of securities.[7] Given the magnitude of the task, computer-based programs are indispensible for implementing such strategies, hence the origin of the term *program trading*.

Example 6.21 is a detailed illustration of program trading.

Example 6.21

A value-weighted index is based on four stocks, whose current prices and numbers of shares outstanding are given in Table 6.12.

Table 6.12
Prices, Number of Shares Outstanding, and Market Capitalization

Company	Price ($)	Number of Shares Outstanding	Market Capitalization
Avery	25	400,000	10,000,000
Avon	40	100,000	4,000,000
Aliph	50	200,000	10,000,000
Alize	100	160,000	16,000,000

Assume that the base period market capitalization is 25,000,000 and that the divisor is 1.0. The index level is

$$\frac{40,000,000}{25,000,000} \times 100 = 160.$$

We will assume that the contract multiplier is 25.

Consider a futures contract that is scheduled to expire after 90 days. Assume that none of the constituent stocks is scheduled to pay a dividend during this period, and that the riskless rate is 8 percent per annum.

The no-arbitrage futures price is given by

$$F = 160 \times \left[1 + 0.08 \times \frac{90}{360}\right] = 163.20.$$

Assume that the futures contract is priced at $167.50. The contract is overpriced. An arbitrageur decides to invest $1,000,000 in the stocks constituting the index and go short in the required futures contracts. He will have to invest

$$\frac{10,000,000}{40,000,000} \times 1,000,000 = \$250,000 \text{ in Avery}$$

$$\frac{4{,}000{,}000}{40{,}000{,}000} \times 1{,}000{,}000 = \$100{,}000 \text{ in Avon}$$

$$\frac{10{,}000{,}000}{40{,}000{,}000} \times 1{,}000{,}000 = \$250{,}000 \text{ in Aliph}$$

$$\frac{16{,}000{,}000}{40{,}000{,}000} \times 1{,}000{,}000 = \$400{,}000 \text{ in Alize.}$$

He will acquire 10,000 shares of Avery, 2,500 shares of Avon, 5,000 shares of Aliph, and 4,000 shares of Alize. The number of futures contracts required is[8]

$$\frac{1{,}000{,}000}{160 \times 25} = 250.$$

At the time of contract expiration, the futures price will be set equal to the spot index value at that time, because index futures are always cash settled.

Assume that the share prices after 90 days are as shown in Table 6.13.

Table 6.13
Prices, Number Of Shares Outstanding, and Market Capitalization on the Expiration Date

Company	Price ($)	Number of Shares Outstanding	Market Capitalization
Avery	30.00	400,000	12,000,000
Avon	45.00	100,000	4,500,000
Aliph	57.50	200,000	11,500,000
Alize	137.50	160,000	22,000,000

The index value on this date will be:

$$\frac{50{,}000{,}000}{25{,}000{,}000} \times 100 = 200.$$

The gain or loss from the futures position is

$$(167.50 - 200) \times 25 \times 250 = \$(203{,}125).$$

The cash inflow when the shares are sold is

$$10{,}000 \times 30.00 + 2{,}500 \times 45 + 5{,}000 \times 57.50 + 4{,}000 \times 137.50 = \$1{,}250{,}000.$$

The repayment of the amount borrowed at the outset with interest entails an outflow of

$$1{,}000{,}000 \times [1 + 0.08 \times 0.25] = 1{,}020{,}000.$$

The net cash flow is

$$1{,}250{,}000 - 1{,}020{,}000 - 203{,}125 = 26{,}875.$$

This is equivalent to $250 \times 25 \times 4.30$, where 4.30 is the difference between the quoted futures price of \$167.50 and the no-arbitrage price of \$163.20. The profit will always be independent of the stock prices prevailing at expiration because we have assumed that the arbitrageur can sell the shares in the market at the same prices as those used to compute the index value at expiration.

Let us turn to reverse cash-and-carry arbitrage. Assume that all the variables have the same value as previously assumed except for the initial futures price, which we will assume is \$158.50. An arbitrageur will short sell the shares required to realize a cash flow of \$1 million, which he will lend out at 8 percent per annum. Simultaneously, he will go long in 250 futures contracts. At expiration, he will buy back the shares required to cover the short position.

Cash inflow from the initial investment $= 1{,}000{,}000 \times [1 + 0.08 \times 0.25] = 1{,}020{,}000$.
Profit or loss from the futures market $= 250 \times 25 \times [200 - 158.50] = \$259{,}375$.
Cost of acquisition of the shares required to cover the short position is \$1,250,000.
The net cash flow $= 1{,}020{,}000 + 259{,}375 - 1{,}250{,}000 = \$29{,}375$.

This is equal to $250 \times 25 \times 4.70$, where 4.70 is the difference between the no-arbitrage futures price of \$163.20 and the quoted price of \$158.50.

Stock Picking

Stock pickers are traders who believe that they have the uncanny ability to spot underpriced and overpriced stocks. The rate of return on an asset may be expressed as

$$r_i = r_f + \beta_i[r_m - r_f] + \varepsilon_i + \alpha_i.$$

Per the capital asset pricing model, ε_i, which is the return from unsystematic risk, has an expected value of zero; α_i, is what is termed the *abnormal return*. If the asset is fairly priced, then α_i will have a value of zero. However, if the asset were to be underpriced or overpriced, then it will have a nonzero value. For underpriced stocks, the abnormal return will be positive, whereas for overpriced stocks it will be negative.

Stock pickers who take positions based on their hunch about the stock being mispriced need to guard against adverse movements in the market as a whole. There is always a risk that, even if the anticipated abnormal return were to materialize, adverse market movements could wipe out the profits, leading to a lower profit or even a loss.

Take the case of a trader who believes that a stock is underpriced and that she will get an abnormal return of 0.75 percent if she bought it. Assume that the nonsystematic risk is zero, that the beta of the stock is 1.25, and that the riskless rate is 2.75 percent.

It could so happen that even though the expected abnormal return is translated into reality, the market moves down by, say, 4 percent. The return from the stock will be

$$2.75 + 1.25[-4.00 - 2.75] + 0.75 = -4.9375\%.$$

The overall market movement has resulted in a situation in which the trader has ended up with a negative rate of return even though she made the right call.

Examples 6.22 and 6.23 illustrate how stock pickers can use index futures to capture abnormal returns.

Example 6.22

Assume that the trader invests \$10 million in the stock and goes short in index futures. If we assume that the current index value is 160, the corresponding futures price will be \$164.40.

Assuming a contract multiplier of 25, the required number of futures contracts is

$$1.25 \times \frac{10{,}000{,}000}{160 \times 25} = 3{,}125.$$

If we assume that none of the component stocks are scheduled to pay a dividend during the life of the contract, a −4 percent rate of return on the market corresponds to an index level of 153.60. The rate of return on the stock is −4.9375 percent, which corresponds to a terminal portfolio value of $10{,}000{,}000 \times [1 - 0.049375] =$ \$9,506,250.

The profit or loss from the futures market is

$$3{,}125 \times 25 \times [164.40 - 153.60] = \$843{,}750.$$

The total net worth of the trader is $\$9{,}506{,}250 + \$843{,}750 =$ \$10,350,000, which implies a return of 3.50 percent. This return corresponds to the riskless return of 2.75 percent plus the abnormal return of 0.75 percent.

Example 6.23

Assume that the trader believes that the abnormal return will be −0.75 percent. He will have to short sell the stock and go long in futures contracts. We will assume that all the other variables have the same values as in the preceding illustration.

Assume that the market rises by 4 percent. The rate of return on the stock will be

$$2.75 + 1.25[4.00 - 2.75] - 0.75 = \$3.5625\%.$$

The terminal value of the portfolio will therefore be

$$10{,}000{,}000 \times [1.00 + 0.035625] = 10{,}356{,}250.$$

A 4-percent increase in the market means that the terminal index level will be 166.40. The profit or loss from the futures market will be

$$3{,}125 \times 25 \times [166.40 - 164.40] = \$156{,}250.$$

The proceeds of the short sale can be invested at the outset to yield

$$10{,}000{,}000 \times [1 + 0.0275] = \$10{,}275{,}000.$$

The net cash flow will be

$$10.275{,}000 + 156{,}250 - 10{,}356{,}250 = \$75{,}000.$$

This amount represents the abnormal return of 0.75 percent.

Portfolio Insurance

In principle, index futures contracts can be used to remove the entire risk inherent in a portfolio, thereby ensuring that it earns the riskless rate of return. However, a portfolio manager may not like to convert the entire portfolio into a riskless position. He may opt instead to convert a fraction of the portfolio into an effective riskless security using index futures contracts while continuing to hold the balance in its current form. The riskless fraction will earn the riskless rate of return. The risky component's value will be market determined and in a worst case scenario can go to zero. Therefore, there will be a floor on the overall value of the portfolio. The returns on the portfolio may exceed this floor value, but they cannot be less.

An investment strategy using futures contracts to partially hedge the risk of a stock portfolio is referred to as *portfolio insurance*. Fund managers will constantly monitor the market and will either increase or decrease the percentage of investment in the riskless component—that is the proportion of equity to synthetic debt will be periodically changed. At any point in time, if a greater level of insurance were to be sought, then more futures contracts will be sold. Such an asset management strategy is referred to as *dynamic hedging*.

Example 6.24 illustrates the mechanics of portfolio insurance.

Example 6.24

Assume that a fund manager is holding a stock portfolio worth US$5 million, with a beta of 1.20. The current level of the index is 160, and the contract multiplier is 25. Futures contracts are available with 90 days to expiration, and the riskless return for this period is 2.75 percent.

The no-arbitrage futures price is

$$160 \times [1 + 0.0275] = 164.40.$$

Assume that the manager wants to use a short futures position to convert US$3 million worth of equity to synthetic debt. The number of futures contracts required is

$$1.20 \times \frac{3{,}000{,}000}{160 \times 25} = 900.$$

Assume that 18 days later, the index is at 200. The corresponding futures price will be

$$200 \times \left[1 + 0.11 \times \frac{72}{360}\right] = 204.40.$$

Because the market has increased by 25 percent, the rate of return on the portfolio over 18 days is

$$0.55 + 1.20 \times [25 - 0.55] = 29.89\%.$$

The value of the portfolio is

$$\$5{,}000{,}000 \times [1 + 0.2989] = \$6{,}494{,}500.$$

The profit or loss from the futures position is

$$900 \times 25 \times [164.40 - 204.40] = \$(900{,}000).$$

The total value of the hedged portfolio is

$$\$6{,}494{,}500 - 900{,}000 = \$5{,}594{,}500.$$

If the US$3 million had been perfectly hedged, the value of the overall position after 18 days would have been

$$\$2{,}000{,}000 \times [1 + 0.2989] + \$3{,}000{,}000 \times 1.0055 = \$5{,}614{,}300.$$

Why were we unable to obtain a perfect hedge? Because even though the futures contracts had 90 days to expiration at the outset, we opted to terminate the hedge after 18 days.

Because the portfolio value has increased by $594,500, the manager may opt to expose a greater proportion of his funds to market risk. Let us assume that he opts to insure the value of US$2.75 million. He will require

$$1.20 \times \frac{2{,}750{,}000}{200 \times 25} = 660 \text{ contracts.}$$

The Importance of Futures

Futures contracts provide economic benefits in a number of ways. First, they facilitate reallocation of risk. All traders in the market do not have an identical appetite for risk. Hedgers seek to avoid risk, whereas speculators consciously seek to take risk. Futures contracts enable the transfer of risk from those who do not want to bear it to those who consciously seek to take risky positions in anticipation of gains.

Second, futures markets facilitate price discovery. The informational accuracy of asset prices is imperative for ensuring the success of a free-market economy. When new information enters the market, it typically percolates into the futures markets a lot faster. This is because taking a long futures position requires the deposit of a relatively small margin as compared to a long stock position where the entire value has to be deposited up front. Second, from the standpoint of bears, taking a short futures position is a lot easier than short selling the underlying asset. Trading volumes in derivative markets tends to be high. This has two major consequences. First, transactions costs in such markets tend to be much lower as compared to spot markets. Second, these markets are characterized by a high degree of liquidity.

Futures contracts play a critical economic role by enabling markets to be more efficient. Because trading in such markets is relatively easier, new information with an implication for prices typically permeates such markets first. In addition, because there is a pricing relationship between spot and futures markets, this will initially manifest itself as an arbitrage opportunity. The activities of arbitrageurs, however, will quickly ensure that the spot markets also reflect the new information.

Finally, futures markets are a potent tool for speculators, both bulls and bears. The freedom to speculate is imperative for the effective functioning of the free-market system. Futures contracts are attractive for speculators because positions can be taken by depositing relatively smaller amounts. Besides, as explained earlier, they are a vital tool for those who seek to go short.

Endnotes

1. That is, r is not an annualized rate.
2. See Koontz and Purcell (1999).
3. The contract at this point in time has two weeks remaining until expiration.
4. We have assumed that the interest rate is 6 percent per annum.
5. Similar illustrations can be designed for bears who choose to speculate with futures.
6. The London Inter-Bank Offer Rate.
7. The S&P 500 is composed of 500 stocks.
8. The index has a beta of 1.0.

CHAPTER 7

Options Contracts

The forward and futures contracts that we examined in Chapter 6 are commitment contracts. In other words, once an agreement is reached to transact at a future point in time, noncompliance on the part of either party is tantamount to default. Having committed himself to the transaction, the long, who we have defined as the party who has agreed to acquire the underlying asset, must buy the asset by paying the prefixed price to the short. At the same time, the short is obliged to deliver the underlying asset in return for the payment.

Let us turn our attention to contracts that give the buyer the right to transact in the underlying asset. The difference between a right and an obligation is that a right needs to be exercised if it is in the interest of the holder, and it need not be exercised if it is not beneficial for him. An obligation, on the other hand, mandates him to take the required action, irrespective of whether or not he stands to benefit. Contracts to transact at a future point in time, which give the buyer the right to transact, are referred to as *options contracts*, because he is being conferred with an option to perform. When it comes to bestowing a right, there are two possibilities. The buyer can be given the right to acquire the underlying asset or be given the right to sell the underlying asset. There are two categories of options: call options and put options. A *call option* gives the buyer of the option the right to buy the underlying asset at a predecided price, and a put option gives him the right to sell the underlying asset at a prefixed price. The prespecified price in the contract is termed the *strike price* or *exercise price*.

We first consider call options. The holder will exercise his right to purchase the underlying asset only if the prevailing spot price at the time of exercise were to be higher than the exercise price. However, in such a situation, the counterparty, who has sold the option, would be unwilling to transact if he could get away with it. The only way that we can ensure compliance on the part of the seller of the option is by imposing an obligation on him. Call options give the buyer the right to buy the underlying asset and impose a contingent obligation on the seller. The term *contingent* indicates that the seller will have an obligation to perform if and when the buyer were to exercise his right.

The same rationale applies for put options. The buyer of the option will choose to sell the underlying asset only if the prevailing spot price at the time of exercise is less than the exercise price of the option. Once again, the seller of the put option will be unwilling to take delivery under such circumstances if he can avoid it. So once again there is a need to impose an obligation on the seller of the put. Both call and put options give a right to the option buyer and impose a contingent obligation on the seller. Contracts for transactions scheduled at a future point in time may therefore entail the imposition of an obligation on both parties, as do forward and futures contracts, or they may provide for the grant of a right to one party while imposing an obligation on the counterparty as options contracts do.

The party who buys the option contract, whether it is a call or a put, is referred to as the *option buyer* or the *long*. The counterparty who sells the option contracts is termed the *option writer* or *short*. The implication is that he has written the options contract.

From the standpoint of the date of exercise, there are two possibilities. The scheduled transaction date may be either the only point in time at which the holder of the option can exercise his right or the last point in time at which he can exercise the option. As a consequence, we have two types of call options and two types of put options. Call and put options that permit the holder to exercise only at the point of contract expiration are referred to as *European options*. Contracts that give the holder the flexibility of exercising on or before the expiration date of the option are referred to as *American options*. If we were to compare options contracts that are equivalent in all other respects, an American option will be more valuable than a European option.

The strike price or the exercise price represents the price at which the underlying asset can be bought if a call is exercised, and the price at which it can be sold if a put option is exercised. However, the option itself carries an attached price tag, which is referred to as the *option price* or the *option premium*. This is the price that an option buyer has to pay to the option seller at the outset for conferring him with the right to transact. Unlike forward and futures contracts, where the two equal and opposite obligations ensure that neither party has to pay the other at the outset, options contracts require buyers to pay an upfront price. This is required for both calls and puts, irrespective of whether the option is European or

American. Although the strike price enters the picture only if the option is exercised, the premium that is payable at the outset is nonrefundable and represents a sunk cost.

Forward contracts are over-the-counter (OTC) products, and futures contracts are exchange traded.

Options can be of two types. Options exchanges offer standardized contracts, but customized contracts can be privately negotiated by corporations and financial institutions. The terms *standardized* and *customized* refer to the terms and conditions of the contract. In the case of an options contract, the critical terms are the exercise price, the expiration date, and the contract size. In the case of OTC options, the two parties are at liberty to fix any strike price and contract size and to choose any desirable expiration date. As in the case of a forward contract, these terms will have to be finalized by way of bilateral negotiations. In the case of exchange-traded options contracts, however, the exchange will fix these parameters. On the Chicago Board Options Exchange (CBOE) a stock options contract is for 100 shares. Transactions are possible only in multiples of 100. Contracts expire on the Saturday following the third Friday of the month—that is, the fourth Saturday of the month if the first day of the month were to be a Saturday (otherwise, it refers to the third Saturday of the month). The last day of trading is the third Friday of the month or the preceding business day if the third Friday were to be a holiday. Stock options contracts expiring on any other day of the month cannot be traded on the exchange. The exchange will also specify the allowable exercise prices. Traders are at liberty to enter into contracts with one of the allowable exercise prices, but they cannot choose a price that is not permitted by the exchange. Exchange-traded stock options are American in nature, whereas options based on stock indexes may be either European or American.

Notation

We will use the following symbols to depict the various variables.

$t \equiv$ today, a point in time before the expiration of the options contract

$T \equiv$ the point of expiration of the options contract

$S_t \equiv$ the stock price at time t

$S_T \equiv$ the stock price at the point of expiration of the options contract

$I_t \equiv$ the index value at time t

$I_T \equiv$ the index value at the point of expiration of the options contract
$X \equiv$ the exercise price of the option
$C_t \equiv$ a general symbol for the premium of a call option at time t when we do not wish to make a distinction between European and American options
$P_t \equiv$ a general symbol for the premium of a put option at time t when we do not wish to make a distinction between European and American options
$C_T \equiv$ a general symbol for the premium of a call option at time T when we do not wish to make a distinction between European and American options
$P_T \equiv$ a general symbol for the premium of a put option at time T, when we do not wish to make a distinction between European and American options
$C_{E,t} \equiv$ the premium of a European call option at time t
$P_{E,t} \equiv$ the premium of a European put option at time t
$C_{A,t} \equiv$ the premium of an American call option at time t
$P_{A,t} \equiv$ the premium of an American put option at time t
$C_{E,T} \equiv$ the premium of a European call option at time T
$P_{E,T} \equiv$ the premium of a European put option at time T
$C_{A,T} \equiv$ the premium of an American call option at time T
$P_{A,T} \equiv$ the premium of an American put option at time T
$r \equiv$ the riskless rate of interest per annum.

Exercise

Let us first consider contracts that are delivery settled. For the purpose of exposition, we will focus on European options, although the underlying rationale applies to American options as well, with the difference being that such options sometimes may be exercised before the expiration date.

Take the case of a call options contract on XYZ stock. Each contract is for 100 shares. Let the exercise price be $\$X$ and the option premium be C_t. Option premiums are always quoted on a per-share basis and the premium for the contract is $100C_t$. At the time of entering into the trade, the buyer of the contract will have to pay $\$100C_t$ to the seller for giving him the right to transact. On the expiration date of the contract, there could be two possibilities. The prevailing stock price S_T may be greater than X or less than X. The buyer will choose to exercise only if $S_T > X$. If he decides to exercise, he will pay $-\$100X$ to the seller who will, in turn, deliver 100 shares to him.

Consider put options on the same stock. Let us denote the premium per share by $\$P_t$. So the buyer will have to pay the writer $\$100P_t$ at the outset. It would make sense to sell a share at a price X only if the prevailing market price were to be lower. A European put option will be exercised only if the terminal stock price S_T is less than X. If the holder were to exercise, he would have to deliver 100 shares to the writer who, in turn, will have to pay $\$100X$ to him. In the case of both contracts—that is, calls and puts—the initial premium is nonrefundable if market movements were to be such that the option holder chooses not to exercise his right. Let us consider numerical illustrations as shown in Examples 7.1, 7.2 and 7.3 to reinforce the concepts.

Example 7.1

Consider a call options contract on IBM with an exercise price of \$85, which is scheduled to expire on July 21, 2010. Assume that today is June 1, 2010, and that the premium is \$6.25 per share. The option is bought by Ron from Penelope.

Right at the outset, Ron will have to pay $6.25 \times 100 = \$625$ to Penelope. This represents the option premium, which is nonrefundable. On July 21, there are two possibilities. The stock price may be more than \$85 (say, \$95) or it may be less than \$85 (say, \$70). If the terminal price is \$95, then it would make sense for Ron to exercise the option and acquire the shares at \$85. However, if the terminal price is \$70, it will make no sense to exercise the option, and Ron will simply allow it to expire without exercising it. The last action demonstrates the difference between a futures contract and an options contract. Because an options contract is not a commitment contract, the holder need not perform if it is not in his interest.

Assuming that the terminal stock price is \$95, Ron will pay $\$85 \times 100 = \$8{,}500$ to Penelope in return for the 100 shares. If he does not want to hold the shares, he can immediately sell them to realize a profit of $\$9{,}500 - \$8{,}500 = \$1{,}000$. After taking into account the premium paid at the outset, the profit is \$375. However, if the terminal stock price is \$70, the options will not be exercised and the loss for Ron is \$625, which represents the premium paid at the outset.

Example 7.2

Let us consider the information in the preceding illustration but assume that the options are puts with an option premium of \$3.00. If Ron were to buy the options from Penelope, then he would have to pay \$300 at the outset. This is a nonrefundable payment. On July 21, we will assume that the stock price can be either \$95 or \$70. It would not make sense to sell the shares for \$85 if the market price per share were to be \$95. In this case Ron would allow the options to expire worthless and forfeit the initial payment of \$300. However, if the terminal stock price were to be \$70, then it would make sense to sell the shares at \$85. If Ron has the shares, he would be willing to sell. Otherwise, he would be willing to buy the shares from the market and exercise the option. His profit in this case would be \$8,500 − \$7,000 = \$1,500, or \$1,200 if we were to take the upfront premium paid into account.

Example 7.3

Consider call options on GE with an exercise price of \$45. Assume that the premium is \$3.25 per share. If the terminal stock price is \$52, the options will be exercised. The payoff for the writer is

$$100 \times (45 - 52) = (700).$$

The profit is

$$100 \times (45 - 52) + 100 \times 3.25 = (375).$$

In symbolic terms, the payoff can be expressed as $(X - S_T)$ or $-(S_T - X)$. The profit is $-(S_T - X) + C_t$.

However, if the terminal spot price were to be \$40, then the options will not be exercised and the writer will get to keep the premium of \$375. His profit in this case is C_t.

A call will be exercised if $S_T > X$; otherwise, it would be allowed to expire worthless. The payoff at expiration therefore is $\text{Max}[0, S_T - X]$. The profit is $\text{Max}[0, S_T - X] - C_t$. On the other hand, a put will be exercised if $S_T < X$, and it would otherwise be allowed to expire. So the payoff at expiration is Max[0, $X - S_T$], whereas the profit is Max[0, $X - S_T$] $- P_t$.

We look at the cash flows from the perspective of the writer.

The payoff for a call writer may be expressed as −Max[0, ($S_T - X$)] = Min[0, $X - S_T$]. The profit for the writer is therefore Min[0, $X - S_T$] + C_t.

Using similar logic, we can demonstrate that the payoff for a put writer is Min[0, $S_T - X$], and the profit is Min[0, $S_T - X$] + P_t.

Options like futures contracts are zero-sum games. The profit or loss for the holder is exactly equal to the loss or profit for the writer. Let us consider the profit for a call holder, which is Max[0, $S_T - X$] $- C_t$. The maximum profit is unbounded because the terminal stock price has no upper bound. There is no cap on the potential profit for a call holder. However, her maximum loss is restricted to the premium paid at the outset—that is, the worst-case scenario from the standpoint of the holder is that the options are not exercised and she loses the entire premium. This once again illustrates the difference between an options contract and a commitment contract. For the call writer, the maximum loss is unbounded because the stock price has no upper bound. However, his maximum profit is restricted to the premium received at the outset, so the best possibility from the standpoint of the writer is that the options are not exercised and that he gets to retain the premium paid by the holder. Therefore, although call holders face the specter of infinite profits and finite losses, call writers face the possibility of infinite losses and finite profits.

The profit for a put holder is Max[0, $X - S_T$] $- P_t$, so the holder stands to gain if the asset price declines. The lowest possible stock price, because of the limited liability feature, is zero. The maximum profit for a put holder is $X - P_t$. Her maximum possible loss is once again the premium that she pays at the outset. A put holder faces finite profits as well as finite losses. For a writer, the profit is

$$\text{Min}[0, S_T - X] + P_t.$$

From his standpoint, therefore, the maximum loss is $P_t - X$, and the maximum profit is P_t. Once again, as in the case of the call writer, the best situation that a put writer can hope for is that the options are not exercised, thereby enabling him to retain the premium received at the outset.

Moneyness

If the current asset price is greater than the exercise price of a call option, the option is said to be *in the money* (ITM). However, if the spot price is less than the exercise price, the call is said to be *out of the money* (OTM). If the two are exactly equal, then the option is said to be *at the money* (ATM).

Symbolically,

- $S_t > X \Rightarrow$ ITM call and OTM put
- $S_t = X \Rightarrow$ ATM call and ATM put
- $S_t < X \Rightarrow$ OTM call and ITM put.

For put options, it is just the opposite. If the spot price of the asset is greater than the exercise price of the option, the put is said to be *out of the money*. But if the spot price is less than the strike price of the option, we say that the put is *in the money*. Once again, if the spot price equals the strike price, the put option is *at the money*.

An option, irrespective of whether it is a call or a put, would be exercised only if it is in the money. However, before the expiration date, it is not necessary that an option will be exercised if it is in the money because the holder may like to wait in anticipation of the contract going deeper into the money. At expiration, of course, an in-the-money contract will be exercised.

Example 7.4 illustrates options with varying degrees of moneyness.

Example 7.4

Call and put options on IBM are available with exercise prices of \$45, \$50, and \$55. The current stock price is \$50. In the case of calls, the contract with a strike of 45 is in the money, a contract with a strike of 50 is at the money, and one with a strike of 55 is out of the money. For puts, it is just the reverse. The put with an exercise price of 45 is out of the money, that with an exercise price of 50 is at the money, and the contract with an exercise price of 55 is in the money. So if a call with a given exercise price and expiration date is in the money, a put with the same exercise price and expiration date will be out of the money and vice versa.

Exchange-Traded Options

An exchange-traded options contract is standardized as discussed earlier: the contract size, allowable exercise prices, and allowable expiration dates are specified by the exchange. So, even though an infinite number of OTC options contracts can be designed, there will only be a finite number of exchange-traded products available at any point in time. We will discuss how the allowable exercise prices and expiration dates are fixed by the exchange.

When the exchange declares a new asset to be eligible to have options written on it, it will assign the asset to a quarterly cycle. There are three possible cycles: January, February, and March. The January cycle comprises the following months: January, April, July, and October. The February cycle comprises February, May, August, and November. The March cycle comprises March, June, September, and December.

At any point in time, the allowable contract months are:

- the current month,
- the following month,
- and the next two months of the cycle to which the underlying asset has been assigned.

Example 7.5 illustrates the available expiration months at a given point in time for a stock that has been assigned to the January cycle.

Example 7.5

Assume that today is July 28, 2010, which is the fourth Friday of the month. The July 2010 contracts would have ceased trading on July 21. Assume that the asset has been assigned to the January cycle. The available contracts on July 28 will be August 2010 contracts, which represent the current month; the September 2010 contracts, which represent the following month; and October 2010 and January 2011, which represent the two months from the January cycle after September 2010. When the August contracts expire, September will automatically become the current month. October will be the following month. As far as the next two months from the cycle are concerned, January 2011 will already exist. Contracts expiring in April 2011 will be declared to be eligible for trading. When the September contracts expire, October will automatically become the current month. So November contracts, which represent the next month, will need to be introduced. The next two months from the cycle—namely, January 2011 and April 2011—will already exist. Similarly, when the October contracts expire, December 2010 contracts will have to be introduced. And when the November contracts expire, July 2011 contracts will have to be permitted.

In addition to these contracts, the CBOE offers contracts on both individual stocks and stock indexes with as long as three years to maturity. These contracts are termed *long-term equity anticipation securities* (LEAPS). These contracts expire on the Saturday following the third Friday of January each year.

Let us turn our attention to the permitted exercise prices. At any point in time, there will always be a contract that is at the money or near the money. In addition, there will be other contracts that are in or out of the money to various extents. When contracts for a new calendar month are listed, two strike prices that are closest to the prevailing price of the stock will be chosen. The CBOE usually follows the rule that the interval formed by these two strike prices should be \$2.50 if the stock price is less than \$25, \$5.00 if the stock price is between \$25 and \$200, and \$10.00 if the stock price exceeds \$200. Subsequently, based on the movement in the price of the underlying asset, additional exercise prices may be permitted for trading. At a given point in time, contracts with many different exercise prices will be trading for a particular expiration month. Of course, not all contracts will be equally active.

Option Class and Option Series

All options contracts on a given asset, which are of the same type—that is, calls or puts are said to belong to the same option class, regardless of the exercise price or expiration date. An IBM call with $X = \$80$ and a maturity of January 20XX would be said to belong to the same class as an IBM call with $X = \$85$ and a maturity of February 20XX.

All the contracts that belong to the same class and have the same exercise price and expiration date are said to belong to the same option series. An IBM call option contract with $X = \$80$ and a maturity of January 20XX that is executed at 10:30 A.M. on a given day is said to belong to the same series as another call with $X = \$80$ and a maturity of January 20XX that is executed at 11 A.M. on that day.

FLEX Options

Before the CBOE introduced exchange-traded options, the only way to trade options was in the OTC market. Although standardized options contracts have their own benefits, for many institutional traders OTC options are more attractive because they provide greater flexibility from the standpoint of contract design. The exchanges were required to design contracts specifically targeted at such investors in order to prevent them from opting for OTC products. The exchanges' response was to create so-called FLEX (for *flexible exchange*).

Such options combine the features of exchange-traded and OTC products. On the one hand, as in the case of OTC products, such contracts permit traders to choose their own contract terms such as the exercise price and expiration date. On the other hand, such contracts are cleared by the clearing corporation, which eliminates the counterparty risk that is associated with OTC products.

Contract Assignment

When a trader with a long position declares his intention to exercise, a counterparty has to be found to give delivery in the case of calls and to take delivery in the case of puts. As in the case of futures contracts, it is not necessary that the party who originally traded with the holder of the option should still have an open short position. This is because he could have offset the market at a prior point in time by taking a counterposition.

When a holder wishes to exercise, she will inform her broker, who will convey her declaration of intent to the clearinghouse by placing an exercise order. In most cases, there will be many different brokers through whom traders would have taken short positions in the contract being exercised. The clearinghouse will have to choose one of them. Typically, the clearing firm chosen by the clearinghouse will have multiple clients who have written options with the same exercise price and expiration date. The clearing firm will have to choose one such writer to deliver the stock in the case of calls or to accept delivery in the case of puts. The writer who is chosen to perform, in such a fashion, is said to be *assigned*.

Margining

Options confer a right on the holder and impose an obligation on the writer. As discussed, the risk of nonperformance is only with respect to the writer. If an option were to be in the money and the holder were to declare her intent to exercise, the writer will end up with a loss. In the case of calls there is a risk that he may not deliver the required shares, whereas in the case of puts there is a risk that he may not pay the cash and accept delivery. In the case of options contracts writers must post a performance guarantee or a margin.

The initial margin for a call option is given by the expression

$$\text{Max}[C_t + 0.10S_t, C_t + 0.20S_t - \text{Max}(0, X - S_t)].$$

The initial margin is the greater of the following amounts:

- call premium at the time of trade + 10 percent of the stock price or
- call premium at the time of trade + 20 percent of the stock's value minus the amount by which the option is out of the money.

The relevant stock price is the market price prevailing at the time of the option trade.

If $S_t > X$, the call is in the money, and the extent to which it is out of the money is zero.

If $S_t < X$, the call is out of the money, and the extent to which it is out of the money is $X - S_t$.

The extent to which the call is out of the money is Max$[0, X - S_t]$.

The formula for computing the maintenance margin at the end of each subsequent day is the same as the one just given except that the option premium is the price prevailing at the end of the day for a contract with the same exercise price and expiration date. Similarly the stock price is the closing price for the day.

Examples 7.6 and 7.7 illustrate applications of contract value margining.

Example 7.6

Ralph has written a call-option contract on a stock with an exercise price of \$80. The premium per share is \$5.75, and the prevailing stock price is \$75.

Call premium plus 10 percent of the stock price is $5.75 + 0.10 \times 75 = \$13.25$.

Call premium plus 20 percent of the stock price is $5.75 + 0.20 \times 75 = \$20.75$.

The extent to which the option is out of the money is $80 - 75 = \$5$.

The initial margin per share is Max$[13.25, 20.75 - 5] = \$15.75$.

The margin per contract is $100 \times 15.75 = \$1{,}575$.

Assume that the following day the call premium is \$3.75 and that the stock price is \$72.50.

Call premium plus 10 percent of the stock price is $3.75 + 0.10 \times 72.50 = \$11$.
Call premium plus 20 percent of the stock price is $3.75 + 0.20 \times 72.50 = \$18.25$.
The extent to which the option is out of the money is $80 - 72.50 = \$7.50$.
The margin per share = Max[11, 18.25 − 7.50] = \$11.
So the required margin per contract is $100 \times 11 = \$1{,}100$.

Example 7.7

Assume that Ralph has written a put option on a stock with an exercise price of \$80. The option premium is \$2.75, and the stock price is \$82.50.

The initial margin is given by

$$\begin{aligned}&\text{Max}[2.75 + 0.10 \times 80; 2.75 + 0.20 \times 82.50 - 2.50]\\&\quad = \text{Max}[10.75; 16.75] = \$16.75.\end{aligned}$$

The required margin per contract is $100 \times 16.75 = \$1{,}675$.

The required margin for a put option is given by the greater of the following amounts:

- Put premium + 10 percent of the exercise price—that is, $P_t + 0.10X$; or
- Put premium + 20 percent of the stock value minus the amount by which the option is out of the money.

The extent to which a put option is out of the money is given by Max[0, $S_t - X$]. The margin in the second case is $P_t + 0.20S_t -$ Max[0, $S_t - X$].

Adjusting for Corporate Actions

Exchange-traded options have to be adjusted for corporate actions such as stock splits and stock dividends. Take a 20 percent stock dividend. If the current share price were to be S_t, then after the split the price, in theory, ought to be $5/6 \times S_t$. The terms of the options contract would be adjusted as follows. The exercise price would be modified to $5/6 \times X$, and the contract size would be changed to $6/5 \times 100 = 120$. Similarly, if a firm were to announce an n:m split, the contract size will be increased from 100 to $n/m \times 100$, and the exercise price would be modified to $m/n \times X$.

Examples 7.8 and 7.9 demonstrate changes in the contract terms for various corporate actions.

Example 7.8

Consider an equity option contract with a contract size of 100 and an exercise price of \$75. Assume that the underlying stock pays a 20 percent stock dividend. If so, the terms of the contract will be altered as follows. The contract size will be increased to 120, and the exercise price will be changed to \$62.50.

Example 7.9

Consider an equity-option contract with a contract size of 100 and an exercise price of \$75. Assume that the firm announces a 5:4 split. If so, the following adjustments will be made to the terms of the contract. The contract size will be increased to 125, and the exercise price will be reduced to \$60.

Nonnegative Option Premiums

Neither a European nor an American option can ever have a negative premium: no rational trader will pay you for giving you a right. The acquirer of the right has to pay the party who is offering the right. A negative option premium, which would imply that the writer has to pay the holder, will be a classic case of arbitrage because if an option were to have a negative premium, a rational trader would buy the option and pocket the premium. If the option were to subsequently end up in the money, the trader stands to make an additional profit. Otherwise, if the option were to end up out of the money, he need not take any action and does not face the specter of a cash outflow. This is a case of arbitrage.

Intrinsic Value and Time Value

The intrinsic value (IV) of an option is the extent to which it is in the money if it is in the money (otherwise, it is equal to zero). In other words, in-the-money options will have a positive intrinsic value, whereas at-the-money and out-of-the-money options will have a zero intrinsic value. This is true for calls as well as puts. For calls, IV = Max[0, $S_t - X$]; and for puts, IV = Max[0, $X - S_t$].

The difference between the current option premium and the current intrinsic value is referred to as the *time value* or the *speculative value* of the option. Because out-of-the-money options have a zero intrinsic value, their entire premium represents their time value. And because an option cannot have a negative premium, such options will always have a nonnegative time value. In-the-money options may or may not have a negative time value, as we will demonstrate shortly. Examples 7.10 and 7.11 illustrate the concepts of intrinsic and time values.

Example 7.10

Call options on IBM are available with a premium of \$7.25. The current stock price is \$105, and the exercise price of the contracts is \$100. The intrinsic value is $(105 - 100) = \$5$. The time value is $7.25 - 5.00 = \$2.25$.

Example 7.11

Consider a put-option contract with an exercise price of \$80 and a premium of \$2.75. Assume that the current stock price is \$84. The IV is Max[0, 80 − 84] = 0.

The entire premium of \$2.75 represents the speculative or time value of the option.

Time Value of American Options

American calls and puts cannot have a negative time value because it would be tantamount to arbitrage. This is because if the time value is negative, a trader can acquire the option and immediately exercise it to realize a riskless profit. The rationale does not apply to European options because they cannot be exercised immediately. For an American call,

$$C_{A,t} \geq \text{Max}[0, S_t - X]$$

and for an American put,

$$P_{A,t} \geq \text{Max}[0, X - S_t].$$

Out-of-the-money American options cannot have a negative time value because an options contract cannot have a negative premium. Examples 7.12 and 7.13 are numerical examples that in-the-money American calls and puts also cannot have negative time values.

Example 7.12

Consider a stock that is current trading at \$100. Call options with an exercise price of \$95 are trading at \$4. The intrinsic value is \$5. The time value is −1. We will demonstrate that this amounts to an arbitrage opportunity.

Consider the following strategy. Buy a call and immediately exercise it. The initial outflow is $4 \times 100 = \$400$. The options can immediately be exercised to yield a cash inflow of $5 \times 100 = \$500$. There is an arbitrage profit of \$100.

Example 7.13

Consider a stock that is currently trading at \$80. Put options with an exercise price of \$85 are currently trading at \$3.50. This, too, is a manifestation of arbitrage, because the time value is −1.50.

Consider the following strategy. Buy a put contract. The initial outflow will be \$350. Exercise the options immediately. The inflow will be \$500. There is a riskless profit of \$150.

Time Value at Expiration

All options, European or American, calls or puts, must have a zero time value at expiration. In other words, the premium of an option at expiration must be equal to its intrinsic value. Violation of this condition will amount to arbitrage, as Examples 7.14 and 7.15 show.

Example 7.14

Take the case of a stock that is trading at \$100. Call options with an exercise price of \$95, which are scheduled to expire immediately, are quoting at \$5.50. Because the intrinsic value is \$5, the option has a positive time value that can be exploited by an arbitrageur as follows. A trader who notices these prices will write a call option that will lead to a cash inflow of \$5.50 for him. Because the option is in the money and is scheduled to expire immediately, the counterparty will exercise on account of which the arbitrageur will have an outflow of \$5.00. The difference of \$0.50 per share is an arbitrage profit. Options cannot have a positive time value at the time of expiration.

Example 7.15

A stock is trading at \$100. Call options with an exercise price of \$90, which are scheduled to expire immediately, are quoting at \$8.50. Because the intrinsic value of the option is \$10, the time value is −1.50. Such a situation also can be exploited by an arbitrageur.

A trader who notices these prices will buy a call option, which will entail a cash outflow of \$8.50. Because it is in the money, she will immediately exercise it for an inflow of \$10. The difference of \$1.50 is an arbitrage profit. Therefore, options cannot have a negative time value at expiration.

Put-Call Parity

The put-call parity relationship for European options on a nondividend-paying stock may be stated as follows:

$$C_{E,t} - P_{E,t} = S_t - \frac{X}{(1+r)^{T-t}}.$$

To prove this, consider the following strategy. Buy the stock at the prevailing price, sell a call with an exercise price of X and a time to maturity of $T - t$, buy a put with the same exercise price and expiration date, and borrow the present value of the exercise price. The strategy and

Table 7.1
Illustration of Put-Call Parity: Strategy 1

Action	**Initial Cash Flow**	**Terminal Cash Flow**	
		If $S_T > X$	If $S_T < X$
Buy the Stock	$-S_t$	S_T	S_T
Sell the Call	$C_{E,t}$	$-(S_T - X)$	0
Buy the Put	$-P_{E,t}$	0	$X - S_T$
Borrow the PV of X	$\dfrac{X}{(1+r)^{T-t}}$	$-X$	$-X$
Total	$-S_t + C_{E,t} - P_{E,t} + \dfrac{X}{(1+r)^{T-t}}$	0	0

its implications in terms of current and future cash flows are depicted in Table 7.1.

Let us analyze the entries in the preceding table. The first column represents the transactions that form components of the overall strategy. The second column depicts the cash flow corresponding to each transaction. If an asset results in a cash inflow, then the initial cash flow will be shown with a positive sign; if it were to result in a cash outflow, the initial cash flow will have a negative sign. When an asset is bought, there will be an outflow; when an asset is sold, there will be a cash inflow. Similarly, when funds are borrowed, there will be an inflow; when funds are loaned, there will be a cash outflow.

The third and fourth columns depict the cash flows at the time of expiration of the option. The variable of relevance is the terminal stock price and its level with respect to the exercise price of the options. There are two possibilities: the stock price may be more than the exercise price or it can be less than it. Regardless of the stock price, the stock can be sold at the prevailing stock price. If the call is in the money, it will be exercised by the counterparty. His cash flow will be $S_T - X$, so the arbitrageur's cash flow will be $-(S_T - X)$. If the put is in the money, it will be exercised by the arbitrageur, who will receive $X - S_T$. Finally, because the present value of X has been borrowed at the outset, there will be an outflow of \$$X$ at the time of the expiration of the option, because the borrowed amount has to be returned with interest.

As can be seen from Table 7.1, the net cash flow at the point of expiration of the options is zero, irrespective of whether the terminal stock price is above the exercise price or below it. So to rule out arbitrage, we require that the initial cash flow be nonpositive because if we have a cash inflow at the outset, followed by no further outflows, then it would be a case of arbitrage. Therefore, to rule out arbitrage, we require that

$$-S_t + C_{E,t} - P_{E,t} + \frac{X}{(1+r)^{T-t}} \leq 0.$$

However, if $-S_t + C_{E,t} - P_{E,t} + \frac{X}{(1+r)^{T-t}} < 0$, then we can make an arbitrage profit by simply reversing the preceding strategy, as demonstrated in Table 7.2.

If, as assumed earlier, $-S_t + C_{E,t} - P_{E,t} + \frac{X}{(1+r)^{T-t}} < 0$, then the initial cash flow in this case will be positive, whereas the terminal cash flows irrespective of the value of the stock price will continue to be zero.

To ensure the total absence of arbitrage, we require that

$$-S_t + C_{E,t} - P_{E,t} + \frac{X}{(1+r)^{T-t}} = 0$$

$$\Rightarrow C_{E,t} - P_{E,t} = S_t - \frac{X}{(1+r)^{T-t}}.$$

This relationship is referred to as *put-call parity*. The expression that we have derived is valid only for European options on a nondividend-paying stock. We will subsequently derive the equivalent expression for European options on a dividend-paying stock.

Table 7.2
Illustration of Put-Call Parity: Strategy 2

Action	Initial Cash Flow	Terminal Cash Flow	
		If $S_T > X$	If $S_T < X$
Short sell the Stock	S_t	$-S_T$	$-S_T$
Buy the Call	$-C_{E,t}$	$(S_T - X)$	0
Sell the Put	$P_{E,t}$	0	$-(X - S_T)$
Lend the PV of X	$-\frac{X}{(1+r)^{T-t}}$	X	X
Total	$S_t - C_{E,t} + P_{E,t} - \frac{X}{(1+r)^{T-t}}$	0	0

Example 7.16 and 7.17 illustrate the arbitrage strategies that can yield a profit if this relationship were to be violated.

Example 7.16

IBM is currently trading at $80. European call and put options with a year to maturity are selling at $7.75 and $1.25, respectively. The stock is not scheduled to pay any dividends during the life of the option contracts. The exercise price of the options is $80, and the riskless rate is 8 percent per annum.

The difference between the call and put premiums is $6.50. However, the stock price minus the present value of the exercise price is only $5.9259. There is a violation of put-call parity. We can exploit the situation by adopting the first arbitrage strategy that we considered previously: buy the stock, sell the call, buy the put, and borrow the present value of the exercise price.

The initial cash flow is $-80 + 7.75 - 1.25 + 74.0741 = 0.5741$. This is an arbitrage profit because the subsequent cash flows are guaranteed to be zero, irrespective of the terminal value of the stock price.

Example 7.17

Consider the data in the preceding illustration. Assume that the calls are priced at $6.25 each whereas the other assets—namely, the put and the stock—have the same values as assumed earlier. The difference between the call and put premiums is $\$6.25 - \$1.25 = \$5$. The difference between the stock price and the present value of the exercise price is 5.9259 as before. The arbitrage strategy used previously will not yield a profit in this case, but the second strategy used to demonstrate put-call parity will. In other words, we need to short sell the stock, buy the call, sell the put, and lend the present value of the exercise price.

The initial cash flow is $80 - 6.25 + 1.25 - 74.0741 = \$0.9259$. This is an arbitrage profit because the subsequent cash flows, if the strategy were to be implemented, are guaranteed to be zero.

Put-Call Parity with Dividends

Let us consider European call and put options on a stock with an exercise price of \$$X$ and $T - t$ periods to maturity. Assume that the stock will pay a dividend of \$$D$ at a time t^*, where $t < t^* < T$. The put-call parity relationship for such options may be stated as

$$C_{E,t} - P_{E,t} = S_t - \frac{D}{(1+r)^{t^*-t}} - \frac{X}{(1+r)^{T-t}}.$$

To prove this, consider the strategy depicted in Table 7.3.

It is very similar to the strategy used earlier to demonstrate the put-call parity relationship for a nondividend-paying stock. The difference is that the stock that we have bought is scheduled to pay a known dividend at time t^*. To make the net cash flow at this point in time equal to zero, we assume that the trader borrows the present value of this dividend at the outset. Because the dividend is scheduled to be paid at t^*, the present value is $\frac{D}{(1+r)^{t^*-t}}$.

From Table 7.3, the cash flows at all subsequent points in time are guaranteed to be zero. To rule out arbitrage, we require that the initial cash flow be nonpositive. However, if the initial cash flow is negative, we

Table 7.3
Illustration of Put-Call Parity for a Dividend Paying Stock

Action	Initial Cash Flow	Intermediate Cash Flow	Terminal Cash Flow	
			If $S_T > X$	If $S_T < X$
Buy the Stock	$-S_t$	D	S_T	S_T
Sell the Call	$C_{E,t}$		$-(S_T - X)$	0
Buy the Put	$-P_{E,t}$		0	$X - S_T$
Borrow the PV of D	$\frac{D}{(1+r)^{t^*-t}}$	-D		
Borrow the PV of X	$\frac{X}{(1+r)^{T-t}}$		$-X$	$-X$
Total	$-S_t + C_{E,t} - P_{E,t} + \frac{D}{(1+r)^{t^*-t}} + \frac{X}{(1+r)^{T-t}}$	0	0	0

can reverse the preceding strategy and make arbitrage profits. To rule out all forms of arbitrage, we require that

$$C_{E,t} - P_{E,t} = S_t - \frac{D}{(1+r)^{t^*-t}} - \frac{X}{(1+r)^{T-t}}.$$

A Very Important Property for American Calls

We will demonstrate that an American call on a nondividend-paying stock will never be exercised early—that is, before expiration. In other words, an American call on a nondividend-paying stock is exactly equivalent to a European call on the same stock.

Consider the strategy outlined in Table 7.4.

Assume that the initial cash flow is positive—that is,

$$S_t - \frac{X}{(1+r)^{(T-t)}} - C_t > 0$$
$$\Rightarrow C_t < S_t - \frac{X}{(1+r)^{(T-t)}} > 0.$$

In the table, the initial cash flow has been assumed to be positive. The cash flows at the time of expiration of the option will be nonnegative, regardless of whether the option is in the money or out of it. However, the

Table 7.4
Lower Bound for Call Options

Action	Initial Cash Flow	Intermediate Cash Flow	Terminal Cash Flow	
		If $S_{t^*} > X$	If $S_T > X$	If $S_T < X$
Short sell the Stock	S_t	$-S_{t^*}$	$-S_T$	$-S_T$
Buy the Call	$-C_t$	$(S_{t^*} - X)$	$(S_T - X)$	0
Lend the PV of X	$-\frac{X}{(1+r)^{T-t}}$	$\frac{X}{(1+r)^{T-t^*}}$	X	X
Total	$S_t - C_t - \frac{X}{(1+r)^{T-t}}$	$\frac{X}{(1+r)^{T-t^*}} - X < 0$	0	$X - S_T > 0$

cash flow at the intermediate point in time t^* looks to be negative. Let us analyze this.

First, why are we concerned with the situation at a time before expiration? The reason is that this proof is intended to be common for both European and American options. And whenever we seek to present a result for American options, we need to factor in the consequences of early exercise. In this strategy, the arbitrageur has bought the call. If she were to prematurely exercise it, it will lead to an overall cash outflow. However, because the option is in her control, she will never exercise it early in such circumstances. We do not have to consider the possibility of a negative cash flow before expiration. So, irrespective of the subsequent movements in the stock price, this strategy will give rise to nonnegative cash flows. To rule out arbitrage, the initial cash flow must therefore be nonpositive. Thus,

$$C_t \geq S_t - \frac{X}{(1+r)^{T-t}}.$$

But what if $S_t - \frac{X}{(1+r)^{T-t}} < 0$? Then we can state that $C_t > 0$, because we know that an option cannot have a negative premium. So combining the two conditions, we can state that

$$C_t \geq \text{Max}\left[0,\, S_t - \frac{X}{(1+r)^{T-t}}\right].$$

We know that an American option must trade at least for its intrinsic value. In other words,

$$C_t > \text{Max}[0, \mathrm{S_t} - X].$$

But if $C_t \geq \text{Max}\left[0,\, S_t - \frac{X}{(1+r)^{T-t}}\right]$, then this condition will be automatically satisfied. The condition that we have just derived gives us a tighter lower bound for the premium of an option before expiration. Based on this result, we can draw important conclusions about an American call on a nondividend-paying stock.

- An in-the-money American call will always have a positive time value except at expiration. We know that the time value of all options must be zero at expiration. However, from our result, if we assume that $S_t > X$, before expiration,

$$C_{A,t} \geq S_t - \frac{X}{(1+r)^{T-t}} \geq S_t - X.$$

- An American call on a nondividend paying stock will never be exercised early. If a holder were to exercise such an option when it is in the money, he will get the intrinsic value of the option. However, if he were to offset his long position by taking a counterposition, he will get the premium, which as we have just demonstrated will be greater than the intrinsic value. A trader who desires to get out of a long call position would rather offset than exercise before maturity.
- An American call on a nondividend-paying stock must have the same premium as a European call on the same asset, with the same exercise price and expiration date. Why is an American call generally priced higher than a European call on the same asset and with the same features? It is because the American call gives the trader the flexibility to exercise early. However, if an option is never going to be exercised before maturity, then the early exercise feature will have no value, and no rational investor will pay for it. An American call on a nondividend paying stock must sell for the same premium as an otherwise similar European call on the same stock.

Early Exercise of Options: An Analysis

Consider the position of an investor who owns a call option on a stock and has cash equal to the exercise price of the option. If she were to exercise at time t, she will get one share of stock. Let us move forward to a point in time denoted by t^*. The value of the share will be S_{t^*}. If the holder had not exercised at t, she would have an amount of cash equal to $X(1+r)^{t^*-t}$ plus an option worth C_{A,t^*}.

At t^*, there are two possibilities: the option may be in or out of the money. If it is in the money, it can be exercised to acquire the stock, leaving a balance of $X(1+r)^{t^*-t} - X$ in cash. The trader would have been better off waiting. But what if the option was to be out of the money? If so, then $S_{t^*} < X$, which implies that $S_{t^*} < X(1+r)^{t^*-t} + C_{A,t^*}$. Once again, the decision to wait would be vindicated. So under no circumstances is it beneficial to exercise an American call on a nondividend-paying stock before expiration. A trader who acquires such an option will either offset it before expiration or hold it to maturity and exercise it if it is in the money.

American puts may, however, be exercised before expiration. Assume that a put is substantially in the money and that the market is convinced that it will finish in the money. A holder will rather exercise than wait because if he exercises he will get the exercise price, which can be immediately invested to earn the riskless rate. Waiting is suboptimal because it will delay the cash inflow and amount to a loss of interest income. In such a situation, offsetting will not be beneficial because no potential counterparty will be willing to pay a positive time value for the option.

Profit Profiles

We will depict the profit diagrams for the four basic option positions: long call, short call, long put, and short put. π denotes the profit and π_{max} is the maximum profit while π_{min} is the maximum loss. S_{T^*} denotes the breakeven price. If there is a second breakeven price for a strategy it is denoted as $S_{T^{**}}$.

The payoff from a long call is Max$[0, S_T - X]$, and the profit is Max $[0, S_T - X] - C_t$. Consider a stock that is currently trading at \$100. Call and put options with an exercise price of \$100 are currently trading at \$9.50 and \$5.60, respectively. The payoffs and profits for the options for various values of the terminal stock price are given in Table 7.5.

The profit diagrams are as depicted in Figures 7.1 and 7.2.

The payoffs and profits for short call and put positions are as shown in Table 7.6. See also Figures 7.3 and 7.4.

Table 7.5
Payoffs and Profits for Long Call and Put Positions

Terminal Stock Price	Payoff for Call	Profit for Call	Payoff for Put	Profit for Put
50	0	−950	5,000	4,440
60	0	−950	4,000	3,440
70	0	−950	3,000	2,440
80	0	−950	2,000	1,440
90	0	−950	1,000	440
94.40	0	−950	560	0
100	0	−950	0	−560
109.50	950	0	0	−560
110	1,000	50	0	−560
120	2,000	1,050	0	−560
130	3,000	2,050	0	−560
140	4,000	3,050	0	−560
150	5,000	4,050	0	−560

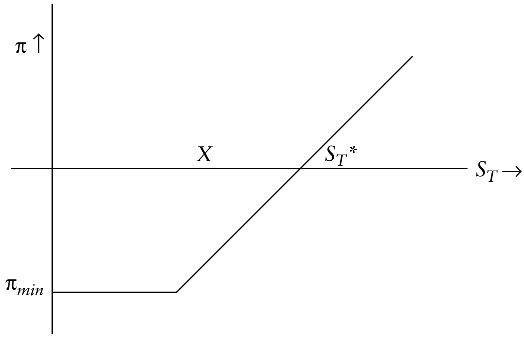


Figure 7.1 Profit Diagram for a Long Call Position

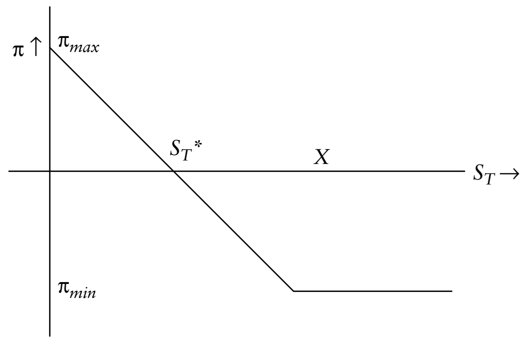


Figure 7.2 Profit Diagram for a Long Put Position

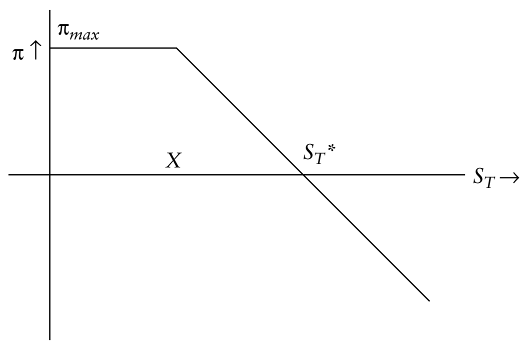


Figure 7.3 Profit Diagram for a Short Call Position

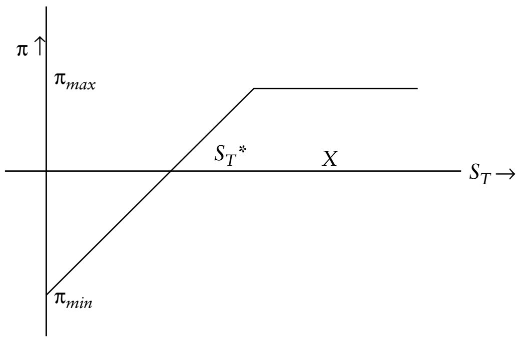


Figure 7.4 Profit Diagram for a Short Put Position

Table 7.6
Payoffs and Profits for Short Call and Put Positions

Terminal Stock Price	Payoff for Call	Profit for Call	Payoff for Put	Profit for Put
50	0	950	−5,000	−4,440
60	0	950	−4,000	−3,440
70	0	950	−3,000	−2,440
80	0	950	−2,000	−1,440
90	0	950	−1,000	−440
94.40	0	950	−560	0
100	0	950	0	560
109.50	−950	0	0	560
110	−1,000	−50	0	560
120	−2,000	−1,050	0	560
130	−3,000	−2,050	0	560
140	−4,000	−3,050	0	560
150	−5,000	−4,050	0	560

Speculation with Options

Traders can use options contracts for speculation. Bulls can take a long position in call options or short positions in put options. In either case, they stand to make a profit if they are right and the market does rise. Examples 7.18–7.22 illustrate this.

Example 7.18

A stock is currently trading at \$100. Call options with six months to expiration and an exercise price of \$100 are trading at \$10.90. A trader is of the view that the market will rise over the next six months and decides to speculate by taking a long position in five contracts.

Assume that the stock price after six months is 122. If so, the options will be exercised and the profit from five contracts will be

$$5 \times 100 \times (122 - 100) - 5 \times 100 \times 10.90 = \$5{,}550.$$

But if the stock price were to fall and reach a value of \$90 after six months, the contracts will be allowed to expire worthless. The loss in this case would be the premium paid at the outset:

$$5 \times 100 \times 10.90 = \$5{,}450.$$

The difference between speculating with options versus speculating with futures is that, in the case of an options contract, a speculator who goes long in an option cannot lose more than the premium he paid at the outset.

Example 7.19

Consider the data used in the previous illustration. Assume that the stock price rises to \$110 after a few weeks and that the corresponding value of the option is \$16.65. The trader can offset his position and exit with a profit. In this case, the profit will be

$$5 \times 100 \times (16.65 - 10.90) = \$2{,}875.$$

Example 7.20

Bulls can also use a short position in put options to speculate. Assume that put options with an exercise price of \$100 and six months to expiration are trading at \$6.00. A trader decides to sell five contracts. If she reads the market correctly and the stock price after six months is more than \$100, the counterparty will not exercise the options. The profit for the speculator is the premium received at the outset, which is

$$5 \times 100 \times 6.00 = \$3{,}000.$$

Example 7.21

Consider the data used in the previous illustration. Assume that a few weeks after the position is taken the stock price rises to \$110 and the put premium falls to \$2.70. The trader can offset her position by going long in puts and exit the market with a profit. The profit in this case is

$$5 \times 100 \times (6.00 - 2.70) = \$1{,}650.$$

In the case of speculators who take short positions in options, the losses can, however, be significant as the following example illustrates.

Example 7.22

Richard is bullish about the market and takes a short position in five put options contracts when the premium is \$6.00. A few weeks later, the stock price falls dramatically to \$25, and the option premium rises to \$71. The loss for the trader if he were to offset would be

$$5 \times 100 \times (6.00 - 71.00) = -32{,}500.$$

Bears can also use options to speculate. In their case, they will have to use a long position in put options or a short position in call options.

Hedging with Options

Call and put options can be used to hedge the risk associated with spot positions. We will study two hedging strategies, using calls to hedge a short spot position, and using puts to hedge a long spot position.

Using Call Options to Protect a Short Position

Call options can be used to hedge the risk associated with a short spot position. To implement the strategy, the trader needs to acquire a call option for every share that is sold short. The profit from the short spot position is $S_t - S_T$. The profit from the long call is Max$[0, S_T - X] - C_t$. The total profit is equal to

$$S_t - S_T + \text{Max}[0, S_T - X] - C_t.$$

If the option expires in the money, the call will be exercised. The total profit in this case will be $S_t - X - C_t$. This represents the minimum profit from the position. However, if the option were to expire out of the money, then the call will expire worthless and the profit will be $S_t - S_T - C_t$. The breakeven stock price is given by

$$S_t - S_{T^*} - C_t = 0 \Rightarrow S_{T^*} = S_t - C_t.$$

The maximum profit will arise if the stock price were to attain a value of zero and will be equal to $S_t - C_t$. The hedged position puts a cap on the loss in the event of an adverse market movement but enables the trader to benefit from a positive market movement.

Example 7.23 illustrates the use of a call option to hedge a short stock position.

Using Put Options to Protect a Long Spot Position

In a similar way, put options can be used to protect a long spot position from price risk. In this case, a hedger has to buy one put option for every stock that she acquires. The profit from the long stock position is $S_T - S_t$. The profit from the long put position is

$$\text{Max}[0, X - S_T] - P_t$$

Example 7.23

Assume that the initial stock price is \$100. Call options with an exercise price of \$100 and six months to expiration are available at a premium of \$10.90. A trader decides to hedge his short stock position by buying a call option. The profit for various values of the terminal stock price is shown in Table 7.7.

Table 7.7
Profit from the Hedged Position for Various Values of the Terminal Stock Price

Terminal Stock Price	Cash Flow from Short Stock	Cash Flow from Option	Profit from Strategy
10	9,000	0	7,910
20	8,000	0	6,910
30	7,000	0	5,910
40	6,000	0	4,910
50	5,000	0	3,910
60	4,000	0	2,910
70	3,000	0	1,910
80	2,000	0	910
89.10	1,090	0	0.00
90	1,000	0	−90
100	0	0	−1,090
110	−1,000	1,000	−1,090
120	−2,000	2,000	−1,090
130	−3,000	3,000	−1,090
140	−4,000	4,000	−1,090
150	−5,000	5,000	−1,090
160	−6,000	6,000	−1,090

The maximum loss is capped at \$1,090. The breakeven point is 89.10, which is $100 - 10.90 = S_t - C_t$. The position leaves the trader with the opportunity to take advantage of price movements on the downside.

The profit from the overall position is $S_T - S_t + \text{Max}[0, X - S_T] - P_t$. If the terminal stock price were to be below the exercise price, the puts will be exercised and the overall profit will be $X - S_t - P_t$. This represents the maximum loss from the position. If the option ends up out of the money, the profit will be $S_T - S_t - P_t$. The breakeven point is given by $S_T{}^* = S_t + P_t$. There is no upper limit on the profit. Once again, the hedge puts a cap on the loss while permitting the trader to take advantage of favorable market movements.

Example 7.24 illustrates the use of a put option to hedge a long stock position.

Example 7.24

Consider a stock that is trading at \$100. Put options with an exercise price of \$100 and six months to expiration are trading at \$6.00. A trader decides to hedge his long stock position by buying a put option. The profit for various values of the terminal stock price is shown in Table 7.8.

Table 7.8
Profit from the Hedged Position for Various Values of the Terminal Stock Price

Terminal Stock Price	Cash Flow from Long Stock	Cash Flow from Option	Profit from Strategy
10	−9,000	9,000	−600
20	−8,000	8,000	−600
30	−7,000	7,000	−600
40	−6,000	6,000	−600
50	−5,000	5,000	−600
60	−4,000	4,000	−600
70	−3,000	3,000	−600
80	−2,000	2,000	−600
90	−1,000	1,000	−600
100	0	0	−600
106	600	0	0.00
110	1,000	0	400
120	2,000	0	1,400
130	3,000	0	2,400
140	4,000	0	3,400
150	5,000	0	4,400
160	6,000	0	5,400

The maximum loss is capped at \$600. The breakeven point is 106, which is $100 + 6 = S_t + P_t$. The position leaves the trader with the opportunity to take advantage of price movements on the upside.

Valuation

Unlike futures contracts, options cannot be priced by ruling out simple cash-and-carry and reverse cash-and-carry arbitrage strategies. The value of an option depends on the probability that it will finish in the money and the payoff if it does so. To price an option, we need to make an assumption about the process for the evolution of the price of the underlying asset over time. The pricing formula obtained would depend on the price process that is postulated. In some cases, we will arrive at precise mathematical formulas, or what we term *closed-form solutions*, whereas in other cases the best we can do is to come up with a numerical approximation.

The price of an option, irrespective of the price process that is assumed, is a function of the following variables.

The Price of the Underlying Asset

Because the option is based on the underlying asset, its price will depend on the asset's price. Keeping all other variables constant, the higher the price of the underlying asset, the higher the value of a call option, and the lower the value of a put option.

The Exercise Price

The partial derivative of the option premium with respect to the exercise price will be negative for call options and positive for put options, as is to be expected. In other words, the higher the exercise price, all else being equal, the lower will be the call premium and the higher will be the put premium.

Dividends

When a stock goes ex-dividend, the share price will decline immediately. Because the call premium is positively related to the stock price, the larger the magnitude of the dividend, the lower the call premium. Similar logic tells us that the larger the dividend, the higher the put premium. It must be clarified that the only dividends of consequence are those that are scheduled to be paid during the life of the options contract.

Volatility

In finance, we typically assume that investors are risk averse—that is, for a given level of expected returns, the greater the risk as measured by the standard deviation or variance, the lower the utility. Options are different. Both calls and puts, being contingent contracts, protect the holder from price movements on one side while permitting him to take full advantage of movements on the other side. This is because the worst that can happen to a call holder is that he will lose the option premium, no matter how low the stock price may go. Similarly, irrespective of the increase in the stock price, a put holder can never lose more than the premium. The higher the volatility, the greater the values of both call and put options.

Time to Maturity

Most options are what are termed as *wasting assets*: keeping all other variables constant, their value steadily erodes over time. Most options, with the exception of certain deep-in-the-money European puts, have a positive time value before expiration. At the point of expiration, as we have seen earlier, all options must have a zero time value to rule out arbitrage. Most options will experience a steady decline of value over time if the other variables that influence the price remain constant.

The Riskless Interest Rate

Take the case of an investor who has adequate funds to acquire a stock at its prevailing price. One alternative would be to buy a call option and invest the balance at the riskless rate. The higher the interest rate, the more attractive the second course of action. The higher the interest rate, the greater the premium of a call option.

To draw a conclusion for put options, let us look at the issue from the perspective of an investor who owns the underlying asset and is contemplating its sale. One alternative is to buy a put option and thereby lock in a minimum price for the asset. The higher the prevailing rate of interest, the more enticing the prospect of an immediate sale followed by reinvestment of the sale proceeds. The higher the interest rate, the lower the value of a put option.

The Binomial Option Pricing Model

This model assumes that given a state of nature characterized by a price S_t, at the end of the next period, the asset price may either go up to uS_t or down to dS_t, where u is greater than 1.0 and d is less than 1.0. Because the asset can only take on one of two possible values at the end of every time period, the model is termed the *binomial model*.

We will first consider the one-period case—that is, we will assume that we are time $(T-1)$, where T is the scheduled expiration date of the option. The evolution of the asset price is depicted in Figure 7.5.

Our objective is the find the option premium at $(T-1)$. However, we have to commence our computation from the point of expiration, because that is the instant at which we know what the payoff from the contract will be. So in the binomial model, we always start by considering the cash flows from the option at the point of expiration.

If the up state is reached, the payoff from a call option will be $C_u = \text{Max}[0, uS_t - X]$; if the down state is reached, it will be $C_d = \text{Max}[0, dS_t - X]$.

Consider a portfolio consisting of α shares of stock and a short position in a call option. The term α is the hedge ratio. The current value of the portfolio is $\alpha S_t - C_t$, where C_t is the unknown call premium that we are seeking to derive. Next period, in the up state, the portfolio will be worth $\alpha uS_t - C_u$, whereas in the down state it will be worth $\alpha dS_t - C_d$. We will choose the hedge ratio in such a way that the payoff in the up state is the same as that in the down state; that is

$$\begin{aligned}
&\alpha uS_t - C_u = \alpha dS_t - C_d \\
&\Rightarrow \alpha S_t(u-d) = C_u - C_d \\
&\Rightarrow \alpha = \frac{C_u - C_d}{S_t(u-d)}.
\end{aligned}$$

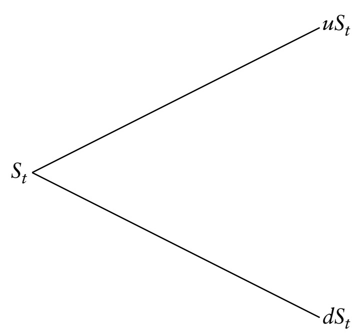


Figure 7.5 Stock-Price Tree for the One-Period Case

By construction, this portfolio is riskless because the payoff is identical in both states of nature. To rule out the prospect of arbitrage, it must therefore earn the riskless rate of return; that is

$$\alpha u S_t - C_u = \alpha d S_t - C_d = (\alpha S_t - C_t) r$$

where r is one plus the riskless rate per period.[1]

Therefore,

$$\alpha u S_t - C_u = (\alpha S_t - C_t) r$$

$$\Rightarrow \left(\frac{C_u - C_d}{S_t(u-d)}\right) \times u S_t - C_u = \left(\frac{C_u - C_d}{S_t(u-d)}\right) \times r S_t - C_t r$$

$$\Rightarrow \left(\frac{C_u - C_d}{u-d}\right) \times (u - r) - C_u = -C_t r$$

$$\Rightarrow C_t = \left(\frac{C_u\left(\frac{r-d}{u-d}\right) + C_d\left(\frac{u-r}{u-d}\right)}{r}\right).$$

Let $\dfrac{r-d}{u-d} = p$. Therefore, $\dfrac{u-r}{u-d} = 1 - p$

$$\Rightarrow C_t = \left(\frac{pC_u + (1-p)C_d}{r}\right).$$

The terms p and $(1 - p)$ are referred to as *risk-neutral probabilities*. The value of the option per this model is a probability-weighted average of the option values in the up and down states.

Example 7.25 demonstrates the use of the binomial model to price an option.

The Two-Period Model

In the preceding exposition, we assumed that the stock price will move just once before the expiration of the option. In general, the asset price will move many times before the expiration of the options contract. The

Example 7.25

Assume that the current stock price is \$80 and that every period the price may go up by 25 percent or down by 25 percent; that is, $u = 1.25$ and $d = 0.75$. We will assume that the riskless rate is 10 percent per period.

Consider a call option with an exercise price of \$80 and one period to expiration:

$$C_u = \text{Max}[0{,}100 - 80] = 20 \text{ and } C_d = \text{Max}[0{,}60 - 80] = 0$$

$$p = \frac{r - d}{u - d} = \frac{1.10 - 0.75}{1.25 - 0.75} = 0.70;\ 1 - p = 0.30$$

$$C_t = \frac{0.70 \times 20 + 0.30 \times 0}{1.10} = \$12.7273.$$

arguments used previously can be extended to value an option in a multiperiod framework.

Let us assume that the current stock price is S_t and that two periods are left until the option's expiration date. The evolution of the stock price is depicted in Figure 7.6.

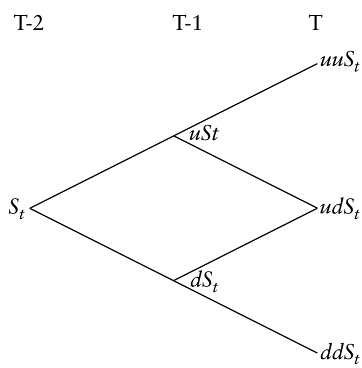


Figure 7.6 Stock-Price Tree for the Two-Period Case

We know as usual the payoffs at expiration. Let us step back one period from T to $T-1$ to compute the option price. This is a one-period problem. We can therefore state that

$$C_u = \frac{pC_{uu} + (1-p)C_{ud}}{r}$$
$$C_d = \frac{pC_{ud} + (1-p)C_{dd}}{r}.$$

We know the values of C_{uu}, C_{ud}, and C_{dd}, because they represent the payoffs at the three nodes corresponding to the expiration date of the option. Once we have computed the values of C_u and C_d, we can once again invoke the one-period model to evaluate C_t, which is the value of the option at $T-2$. This is the basis on which we will solve the multi-period option-pricing problem. In other words, start at expiration and repeatedly invoke the one-period logic to compute the value of the option at a node corresponding to a previous point in time. We will present the solution for the two-period problem using data in Example 7.26.

Example 7.26

Assume that the current stock price is \$80 and that every period the stock price may move up or down by 25 percent. The riskless rate will be assumed to be 10 percent. The terms p and $(1-p)$ are 0.70 and 0.30. Consider a call option with two periods to expiration. Figure 7.7 depicts the stock-price tree.

$$C_{uu} = \text{Max}[0, 125 - 80] = 45$$

$$C_{ud} = \text{Max}[0, 75 - 80] = 0$$

$$C_{dd} = \text{Max}[0, 45 - 80] = 0$$

$$C_u = \frac{0.70 \times 45 + 0.30 \times 0}{1.10} = 28.6364$$

$$C_d = \frac{0.70 \times 0 + 0.30 \times 0}{1.10} = 0$$

$$C_t = \frac{0.70 \times 28.6364 + 0.30 \times 0}{1.10} = \$18.2231.$$

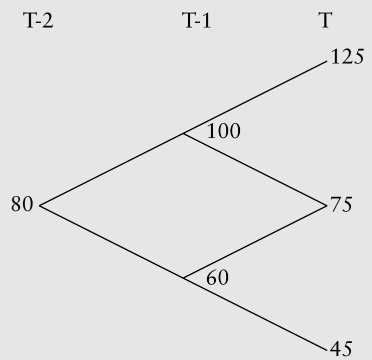


Figure 7.7 Stock-Price Tree for the Two-Period Example

Valuation of European Put Options

Let us consider the one-period case first. Define $P_u = \text{Max}[0, X - uS_t]$ and $P_d = \text{Max}[0, X - dS_t]$. The option premium at a prior point in time may be computed as a probability-weighted average of the payoffs at expiration, where the risk neutral probabilities have the same definition as before. When we extend the theory to the multi-period case, we once again start at the expiration date of the option and work backwards, one step at a time by repeatedly invoking the one-period model. Example 7.27 demonstrates the valuation process for a two-period put option.

Example 7.27

Consider the data used in the previous illustration but assume here that we are dealing with a put option with two periods to expiration:

$$P_{uu} = \text{Max}[0, 80 - 125] = 0$$

$$P_{ud} = \text{Max}[0, 80 - 75] = 5$$

$$P_{dd} = \text{Max}[0, 80 - 45] = 35$$

$$P_u = \frac{0.70 \times 0 + 0.30 \times 5}{1.10} = 1.3636$$

$$P_d = \frac{0.70 \times 5 + 0.30 \times 35}{1.10} = 12.7273$$

$$P_t = \frac{0.70 \times 1.3636 + 0.30 \times 12.7273}{1.10} = \$4.3388.$$

Valuing American Options

The binomial model can be easily extended to value American options. At each node, we have to compare the model value, which is obtained by working backward from the following point in time, with the intrinsic value of the option. If the intrinsic value is greater, then we use it to compute the option premium for subsequent calculations. The rationale is that if the model value is lower than the intrinsic value at a particular node, then the holder will choose to exercise it early. If she does not wish to hold the option, then early exercise will occur. This is because although offsetting is an alternative for such an investor, it will not be optimal because a potential buyer will not pay more than the model value for the option. This is because a buyer can always replicate the option at the model value. Early exercise is superior to taking a counterposition under such circumstances. Even if the holder were to desire to continue to hold the option, it would be more profitable to exercise it, realize the intrinsic value, and replicate it at a cost that is lower than the intrinsic value.

We demonstrate the valuation of an American put option in Example 7.28.[2]

Example 7.28

Let us use the data we considered in the previous example. We will assume, however, that we are dealing with an American put option with two periods to expiration:

$$P_{uu} = \text{Max}[0, 80 - 125] = 0$$

$$P_{ud} = \text{Max}[0, 80 - 75] = 5$$

$$P_{dd} = \text{Max}[0, 80 - 45] = 35$$

$$P_u = \frac{0.70 \times 0 + 0.30 \times 5}{1.10} = 1.3636.$$

The intrinsic value is zero, so we will use the model value while working backward:

$$P_d = \frac{0.70 \times 5 + 0.30 \times 35}{1.10} = 12.7273.$$

The intrinsic value is \$20, which is greater than the model value. Early exercise is optimal at this node, and we need to use the intrinsic value to work backward.

$$P_t = \frac{0.70 \times 1.3636 + 0.30 \times 20}{1.10} = \$6.3223.$$

As we might expect, the American put is priced higher than the European put. We need to check for the possibility of early exercise at time $T - 2$ as well. In this case, the option is at the money, so early exercise is not an issue.

Implementing the Binomial Model in Practice

While illustrating the binomial model, we have arbitrarily assumed values for u, d, and p. These parameters are chosen such that the moments of the discrete-time asset-price process correspond to the moments of the lognormal distribution. The significance of the lognormal distribution is that it is the price process assumed by Black and Scholes in order to derive their option-pricing model.

If σ is the standard deviation of the rate of return on the underlying asset, i is the annual riskless rate, and Δt is the length of a period in the binomial process being modeled, as measured in years, then the normal practice is to set

$$u = e^{\sigma\sqrt{\Delta t}}; d = e^{-\sigma\sqrt{\Delta t}}; r = e^{i\sqrt{\Delta t}}.$$

Example 7.29 illustrates the application of the binomial model using a given value for the volatility.

Example 7.29

A stock is currently priced at \$80. A six-month call option is available with an exercise price of \$80. The riskless rate is 8 percent per annum, and the volatility of the rate of return is 25 percent. If we model the stock-price evolution process as a four-period binomial model, then $\Delta t = 0.125$. Consequently,

$$u = e^{0.25\sqrt{0.125}} = 1.0924;\ d = e^{-0.25\sqrt{0.125}} = 0.9154;\ r = e^{0.08\sqrt{0.125}} = 1.0287.$$

Thus,

$$p = \frac{1.0287 - 0.9154}{1.0924 - 0.9154} = 0.6401 \text{ and } 1 - p = 0.3599.$$

The Black-Scholes Model

Black and Scholes derived closed-form solutions for the values of European call and put options on nondividend-paying stocks by assuming that stock prices follow a lognormal process. The Black-Scholes formula for a European call option may be stated as follows:

$$C_{E,t} = S_t N(d_1) - Xe^{-r(T-t)}N(d_2).$$

The formula for European put options is

$$P_{E,t} = Xe^{-r(T-t)}N(-d_2) - S_t N(-d_1).$$

In these formulas,

$$d_1 = \frac{ln\left(\frac{S_t}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{(T-t)}}$$

and

$$d_2 = d_1 - \sigma\sqrt{(T-t)}.$$

$N(X)$ is the cumulative probability distribution function for a standard normal variable.

Example 7.30 demonstrates the use of the Black-Scholes model to price an option.

Example 7.30

A stock is currently priced at \$80. European call and put options are available with 4½ months to expiration and an exercise price of \$80. The stock is not scheduled to pay any dividends during the life of the options contract. The riskless rate of interest is 8 percent per annum, and the volatility of the rate of return is 25 percent.

Thus $S_t = X = \$80$; $T - t = 0.375$; $r = 0.08$; $\sigma = 0.25$.

$$d_1 = \frac{ln\left(\frac{80}{80}\right) + \left(0.08 + \frac{(0.25)^2}{2}\right)0.375}{0.25\sqrt{0.375}} = \frac{0.04172}{0.1531} = 0.2725$$

$$d_2 = 0.2725 - 0.1531 = 0.1194.$$

$N(d_1)$ and $N(d_2)$ have to be computed using linear interpolation. $N(0.27) = 0.6064$, and $N(0.28) = 0.6103$. Thus,

$$N(0.2725) = 0.6064 + (0.6103 - 0.6064)\frac{0.0025}{0.01} = 0.6074$$

$$N(0.11) = 0.5438 \text{ and } N(0.12) = 0.5478.$$

Therefore,

$$N(0.1194) = 0.5438 + (0.5478 - 0.5438)\frac{0.0094}{0.01} = 0.5476$$

$$C_{E,t} = 80 \times 0.6074 - 80e^{-0.08 \times 0.375} \times 0.5476 = \$6.0787$$

$$N(-X) = 1 - N(X).$$

Thus,

$$N(-0.2725) = 1 - 0.6074 = 0.3926 \text{ and } N(-0.1194) = 1 - 0.5476 = 0.4524.$$

Thus,

$$P_{E,t} = 80e^{-0.08 \times 0.375} N(-0.1194) - 80N(-0.2725) = \$3.7144.$$

Put-Call Parity

Put-call parity is a relationship that must be satisfied if arbitrage is to be ruled out. It is independent of the model that is used to price the options, so we would expect the Black-Scholes model to satisfy it. We will demonstrate that the formula indeed satisfies the put-call parity condition.

$$\begin{aligned} C_{E,t} &= S_t N(d_1) - Xe^{-r(T-t)} N(d_2) \\ &= S_t[1 - N(-d_1)] - Xe^{-r(T-t)}[1 - N(-d_2)] \\ &= Xe^{-r(T-t)} N(-d_2) - S_t N(-d_1) + S_t - Xe^{-r(T-t)} \\ &= P + S_t - Xe^{-r(T-t)}. \end{aligned}$$

Interpretation of the Black-Scholes Formula

The option price as per the formula is independent of the risk preferences of traders. If the attitude toward risk is indeed irrelevant, then the simplest approach to valuation would entail the assumption that all investors are risk neutral. A risk-neutral investor would value any asset as the present value of the expected payoff, where the discount rate used will be the riskless rate.

From the Black-Scholes formula for call options,

$$\begin{aligned} C_{E,t} &= S_t N(d_1) - Xe^{-r(T-t)} N(d_2) \\ &= e^{-r(T-t)}[S_t e^{r(T-t)} N(d_1) - XN(d_2)]. \end{aligned}$$

$[S_t e^{r(T-t)} N(d_1) - XN(d_2)]$ is the expected payoff from the option at the time of expiration as perceived by a world of risk-neutral investors.

The expression $S_t e^{r(T-t)} N(d_1)$ is the expected value of a variable, and once again in a risk-neutral world that has a value equal to S_T if the option is exercised and a value of zero otherwise. $XN(d_2)$ is the expected outflow on account of the exercise price. Because the exercise price has to be paid only if the option is exercised, $N(d_2)$ is the probability that the option will be exercised. In other words $N(d_2)$ is the probability that the option will end up in the money—that is, the odds that $S_T > X$.

For a put option,

$$\begin{aligned} P_{E,t} &= Xe^{-r(T-t)}N(-d_2) - S_t N(-d_1) \\ &= e^{-r(T-t)}[XN(-d_2) - S_t e^{r(T-t)} N(-d_1)]. \end{aligned}$$

Using similar logic $S_t e^{r(T-t)} N(-d_1)$ is the expected value of a variable that will be equal to S_T if $S_T < X$ and zero otherwise. $N(-d_2)$ is the probability that the put will end up in the money and will be exercised.

The Greeks

The option price is a function of many different variables that we discussed earlier. The rate of change of the option premium with respect to these variables is denoted by Greek symbols. This part of option valuation theory is referred to as *the Greeks*.

The partial derivative of the option price with respect to the price of the underlying asset is termed *delta*. For a call option, delta will always be between 0 and 1. For deep-out-of-the-money options, delta will be close to zero; for deep-in-the-money options, it will be close to 1. Delta itself is a function of several variables, including the price of the underlying asset. Delta will change as the asset price changes, and a given value of delta is valid only for an infinitesimal change in the price of the asset.

Even if the asset price were to remain a constant, delta will change with the passage of time. As the time to expiration nears, delta will tend toward 1 if the call is in the money and toward zero if the option is out of the money. Put options have a delta between 0 and −1.

The rate of change of delta with respect to the price of the underlying asset is termed the *gamma* of the option. Gamma is positive for both calls, and puts and tends to be at its highest when the option is close to the money.

The rate of change of the option premium with respect to the volatility of the rate of return is termed *vega*. Because volatility will be perceived positively by both call and put holders, vega will be positive for both types of options.

The time decay of the option is denoted by *theta*—that is, theta is the negative of the partial derivative of the option premium with respect to the time remaining until maturity. Most options, with the exception of certain deep-in-the-money European put options, are wasting assets: their time values will steadily approach zero as expiration approaches. Because the time value at expiration must be zero for all options, theta will be negative for most options. Certain deep-out-of-the-money options may have a negative time value before expiration. In such cases, theta will be positive because the time value must increase so as to attain a value of zero at expiration.

The rate of change of the option premium with respect to the riskless rate of interest is termed *rho*. As discussed earlier, rho will be positive for call options and negative for put options.

From the put-call parity relationship for European options on a nondividend-paying stock:

$$C_{E,t} = P_{E,t} + S_t - Xe^{-r(T-t)}.$$

We can make the following assertions:

1. The delta of a call will be equal to 1 plus the delta of a put.
2. The gamma of call and put options will be equal.
3. The vega of both call and put options will be equal.

If options were to be priced according to the Black-Scholes model, the expressions for the Greeks will be as follows.

1. The call delta will be $N(d_1)$, and the put delta will be $-N(-d_1)$.
2. The gamma for both calls and puts will be $\frac{n(d_1)}{S_t\sigma\sqrt{T-t}}$.
3. Vega for both calls and puts will be $S_t n(d_1)\sqrt{T-t}$.
4. The theta for a call will be $-rXe^{-r(T-t)}N(d_2) - S_t n(d_1)\frac{\sigma}{2\sqrt{T-t}}$, and that for a put will be $rXe^{-r(T-t)}N(-d_2) - S_t n(d_1)\frac{\sigma}{2\sqrt{T-t}}$.
5. The rho for a call will be $X(T-t)e^{-r(T-t)}N(d_2)$, and that for a put will be $-X(T-t)e^{-r(T-t)}N(-d_2)$.

Option Strategies

Options contracts can be used to implement various trading strategies. We will discuss two types of strategies: spreads and combinations. We will look at three categories of spreads (bull spreads, bear spreads, and butterfly spreads) and two types of combinations (straddles and strangles).

Bull Spreads

Bull spreads lead to a profit for the trader if the stock price were to rise, hence the name. It can be implemented using both call and put options. A bull spread, whether set up with calls or with puts, puts a cap on both the loss and the profit from the strategy.

Let us first analyze a spread using calls. We will denote the two exercise prices by X_1 and X_2 where $X_1 < X_2$. The strategy requires the trader to buy an option with a lower exercise price and sell an option with a higher exercise price. Because the lower the exercise price, the higher the option premium, the strategy involves a net investment. The payoff from the strategy at expiration for various values of the terminal stock price is depicted in Table 7.9.

The minimum payoff is zero while the maximum payoff is $X_2 - X_1$. If we denote the initial investment by ΔC, then the maximum loss is $-\Delta C$, whereas the maximum profit is $X_2 - X_1 - \Delta C$. The breakeven stock price is $X_1 + \Delta C$.

The profit diagram may be depicted as shown in Figure 7.8.

Table 7.9
Payoffs from a Bull Spread with Calls

Terminal Price Range	Payoff from Long Call	Payoff from Short Call	Total Payoff
$S_T < X_1$	0	0	0
$X_1 < S_T < X_2$	$S_T - X_1$	0	$S_T - X_1$
$S_T > X_2$	$S_T - X_1$	$-(S_T - X_2)$	$X_2 - X_1$

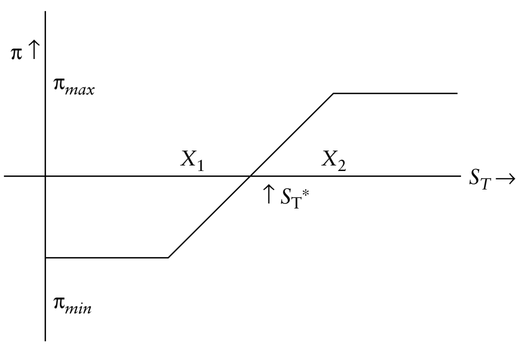


Figure 7.8 Profit Profile: Bull Spread

Table 7.10
Payoffs from a Bull Spread with Puts

Terminal Price Range	Payoff from Long Put	Payoff from Short Put	Total Payoff
$S_T < X_1$	$X_1 - S_T$	$-(X_2 - S_T)$	$X_1 - X_2$
$X_1 < S_T < X_2$	0	$-(X_2 - S_T)$	$S_T - X_2$
$S_T > X_2$	0	0	0

A bull spread can also be created using puts. The trader has to buy the put with the lower exercise price and sell the put with the higher exercise price. Because the put premium increases with the exercise price, the inflow from the option that is sold will be higher than the outflow on account of the option that is bought. A bull spread with puts will lead to a cash inflow at inception. The payoff from the strategy at expiration for various values of the terminal stock price is depicted in Table 7.10.

The minimum payoff is $X_1 - X_2$, and the maximum payoff is zero. If we denote the initial inflow as ΔP, then the maximum loss is $X_1 - X_2 + \Delta P$ and the maximum profit is ΔP. The breakeven stock price is $X_2 - \Delta P$.

Examples 7.31 and 7.32 illustrate bull spreads using call and put options respectively.

Example 7.31

Denise has bought a call option on IBM with an exercise price of \$95 for \$9.68 and sold an option on the same stock with an exercise price of \$100 for \$6.68. Both options have 110 days to maturity.

The initial investment is $100 \times (9.68 - 6.68) = \$300$. The maximum loss is \$300. The maximum profit is $100 \times (100 - 95) - 300 = \$200$. The breakeven stock price is $95 + 3 = 98$. The payoffs and profits for various values of the terminal stock price are depicted in Table 7.11.

Table 7.11
Profit from the Bull Spread

Stock Price	Payoff from Long Call	Payoff from Short Call	Payoff from the Spread	Total Profit or Loss
75	0	0	0	(300)
80	0	0	0	(300)

Table 7.11
Continued

Stock Price	Payoff from Long Call	Payoff from Short Call	Payoff from the Spread	Total Profit or Loss
85	0	0	0	(300)
90	0	0	0	(300)
95	0	0	0	(300)
96	100	0	100	(200)
97	200	0	200	(100)
98	300	0	300	0
100	500	0	500	200
105	1,000	(500)	500	200
110	1,500	(1,000)	500	200
115	2,000	(1,500)	500	200
120	2,500	(2,000)	500	200
125	3,000	(2,500)	500	200

Example 7.32

Chris has bought a put option on IBM with an exercise price of \$95 for \$2.42 and sold a put on the same stock with an exercise price of \$100 for \$4.30. Both options have 110 days to expiration. The initial cash inflow is $100 \times (4.30 - 2.42) = \$188$. This is also the maximum profit. The maximum loss is $100 \times (95 - 100) + 188 = -\$312$. The breakeven stock price is $100 - 1.88 = 98.12$.

Bear Spreads

Bear spreads lead to a profit for the trader if the stock price declines, hence the name. They can be implemented using both call and put options. A bear spread, whether set up with calls or with puts, puts a cap on both the loss and the profit from the strategy.

Let us first analyze a spread using calls. We will denote the two exercise prices by X_1 and X_2 where $X_1 < X_2$. The strategy requires the trader to sell an option with a lower exercise price and buy an option with a higher exercise price. Since the lower the exercise price, the higher will be the option premium the strategy involves a cash inflow at inception. The payoff from the strategy at expiration for various values of the terminal stock price is depicted in Table 7.12.

The maximum payoff is zero, and the minimum payoff is $X_1 - X_2$. If we denote the initial inflow by ΔC, then the maximum profit is ΔC and the maximum loss is $X_1 - X_2 + \Delta C$. The breakeven stock price is $X_1 + \Delta C$.

The profit diagram is depicted in Figure 7.9.

A bear spread can also be created using puts. The trader has to sell the put with the lower exercise price and buy the put with the higher exercise price. Because the put premium increases with the exercise price, the outflow on account of the option that is bought will be higher than the inflow on account of the option that is sold. A bear spread with puts will lead to a cash outflow at inception. The payoff from the strategy at expiration for various values of the terminal stock price is depicted in Table 7.13.

The minimum payoff is zero, and the maximum payoff is $X_2 - X_1$. If we denote the initial investment as ΔP, then the maximum loss is $-\Delta P$

Table 7.12
Payoffs from a Bear Spread with Calls

Terminal Price Range	Payoff from Long Call	Payoff from Short Call	Total Payoff
$S_T < X_1$	0	0	0
$X_1 < S_T < X_2$	0	$-(S_T - X_1)$	$X_1 - S_T$
$S_T > X_2$	$S_T - X_2$	$-(S_T - X_1)$	$X_1 - X_2$

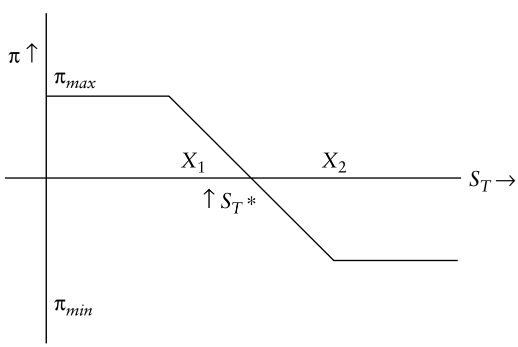


Figure 7.9 Profit Profile: Bear Spread

Table 7.13
Payoffs from a Bear Spread with Puts

Terminal Price Range	Payoff from Long Put	Payoff from Short Put	Total Payoff
$S_T < X_1$	$X_2 - S_T$	$-(X_1 - S_T)$	$X_2 - X_1$
$X_1 < S_T < X_2$	$(X_2 - S_T)$	0	$X_2 - S_T$
$S_T > X_2$	0	0	0

and the maximum profit is $X_2 - X_1 - \Delta P$. The breakeven stock price is $X_2 - \Delta P$.

Examples 7.33 and 7.34 illustrate bear spreads using call and put options respectively.

Example 7.33

Denise has sold a call option on IBM with an exercise price of \$95 for \$9.68 and bought an option on the same stock with an exercise price of \$100 for \$6.68. Both options have 110 days to maturity. The initial cash inflow is $100 \times (9.68 - 6.68) = \$300$. This is the maximum profit from this strategy. The maximum loss is $100 \times (95 - 100) + 300 = -\$200$. The breakeven stock price is $95 + 3 = 98$. The payoffs and profits for various values of the terminal stock price are depicted in Table 7.14.

Table 7.14
Profit from the Bear Spread

Stock Price	Payoff from Long Call	Payoff from Short Call	Payoff from the Spread	Total Profit or Loss
75	0	0	0	300
80	0	0	0	300
85	0	0	0	300
90	0	0	0	300
95	0	0	0	300
96	0	(100)	(100)	200
97	0	(200)	(200)	100
98	0	(300)	(300)	0
100	0	(500)	(500)	(200)

Table 7.14
Continued

Stock Price	Payoff from Long Call	Payoff from Short Call	Payoff from the Spread	Total Profit or Loss
105	500	(1,000)	(500)	(200)
110	1,000	(1,500)	(500)	(200)
115	1,500	(2,000)	(500)	(200)
120	2,000	(2,500)	(500)	(200)
125	2,500	(3,000)	(500)	(200)

Example 7.34

Chris has sold a put option on IBM with an exercise price of \$95 for \$2.42 and bought a put on the same stock with an exercise price of \$100 for \$4.30. Both options have 110 days to expiration. The initial investment is $100 \times (4.30 - 2.42) = \$188$. This is also the maximum loss. The maximum profit is $100 \times (100 - 95) - 188 = \$312$. The breakeven stock price is $100 - 1.88 = 98.12$.

Butterfly Spread

A butterfly spread requires options with three different exercise prices. To set up a long butterfly spread with calls, the trader takes a long position in an in-the-money as well as an out-of-the-money call, and also takes a short position in two at-the-money calls. A butterfly spread can also be set up using put options, as we shall shortly demonstrate.

We will denote the exercise prices by X_1, X_2, and X_3, where $X_1 < X_2 < X_3$. The exercise prices are generally chosen so that the middle price is an arithmetic average of the other two prices: that is, $X_2 = (X_1 + X_3)/2$. We can demonstrate that a butterfly spread will always require an initial investment. which we will denote by ΔC. The payoffs from the strategy at expiration are depicted in Table 7.15.

The minimum payoff is zero and arises if the terminal stock price is either below the lowest of the exercise prices or above the highest. In either case, the profit is $-\Delta C$. This is the maximum potential loss from the strategy. The maximum profit is realized when $S_T = X_2$. There are two breakeven prices: $X_1 + \Delta C$ and $X_3 - \Delta C$.

Figure 7.10 shows the profit diagram.

Table 7.15
Payoffs from a Long Butterfly Spread

Terminal Price Range	Payoff From Call with $X = X_1$	Payoff from Call with $X = X_3$	Payoff from Calls with $X = X_2$	Total Payoff
$S_T < X_1$	0	0	0	0
$X_1 < S_T < X_2$	$S_T - X_1$	0	0	$S_T - X_1$
$X_2 < S_T < X_3$	$S_T - X_1$	0	$-2(S_T - X_2)$	$X_3 - S_T$
$S_T > X_3$	$S_T - X_1$	$S_T - X_3$	$-2(S_T - X_2)$	0

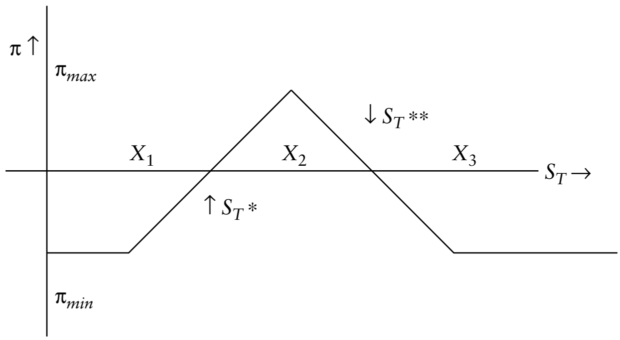


Figure 7.10 Profit Profile: Butterfly Spread

A butterfly spread can also be set up using put options. The strategy requires an investor to buy an in-the-money and an out-of-the-money put and sell two at-the-money puts, where the exercise prices are such that $X_1 < X_2 < X_3$ and $X_2 = (X_1 + X_3)/2$. The strategy will entail an initial investment of ΔP, which also represents the magnitude of the maximum loss. Once again, the maximum loss will be realized if the terminal stock price is below the lowest exercise price or above the highest. The two breakeven prices are $X_1 + \Delta P$ and $X_3 - \Delta P$.

Example 7.35 is an illustration of a butterfly spread using call options.

Example 7.35

Meg Foster decides to set up a butterfly spread using call options on IBM. The exercise prices and corresponding premiums of the options chosen by her are given in Table 7.16.

Table 7.16
Premiums for Call Options

Exercise Price	Option Premium
\$80	\$12.80
\$90	\$6.01
\$100	\$2.20

Table 7.17
Profit from the Butterfly Spread

Stock Price	Payoff from Call with $X = 80$	Payoff from Call with $X = 100$	Payoff from Calls with $X = 90$	Total Payoff	Total Profit or Loss
70	0	0	0	0	(298)
75	0	0	0	0	(298)
80	0	0	0	0	(298)
82.98	298	0	0	298	0
85	500	0	0	500	202
90	1,000	0	0	1,000	702
95	1,500	0	(1,000)	500	202
97.02	1,702	0	(1,404)	298	0
100	2,000	0	(2,000)	0	(298)
105	2,500	500	(3,000)	0	(298)
110	3,000	1,000	(4,000)	0	(298)
115	3,500	1,500	(5,000)	0	(298)
120	4,000	2,000	(6,000)	0	(298)

The initial investment is $100 \times (12.80 + 2.20 - 2 \times 6.01) = \$298$. The maximum loss is $-\$298$. The maximum profit is realized when the stock price is \$90 and is equal to $100 \times (90 - 80 - 2.98) = \$702$. The two breakeven prices are \$82.98 and \$97.02. The payoffs and profits for various values of the terminal stock price are depicted in Table 7.17.

A Straddle

A long straddle requires the investor to buy a call as well as a put option on the same asset. Both options must have the same exercise price and

Table 7.18
Payoffs from a Long Straddle

Terminal Price Range	Payoff from Call	Payoff from Put	Total Payoff
$S_T < X$	0	$X - S_T$	$X - S_T$
$S_T > X$	$S_T - X$	0	$S_T - X$

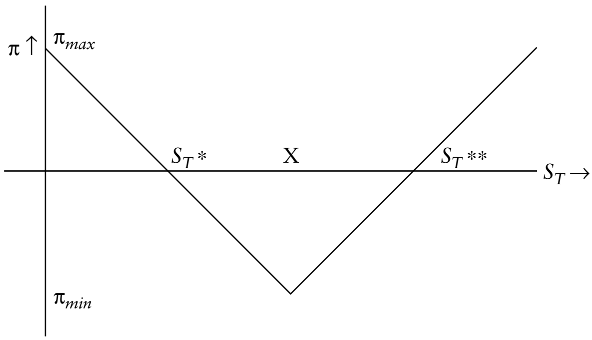


Figure 7.11 Profit Profile: Straddle

expiration date. The initial investment is $C_t + P_t$. The call option will be in the money if the stock rises in value, whereas the put will be in the money if the stock declines in value. So a straddle will pay off in both a bull and a bear market. It is a suitable strategy for an investor who is anticipating a large price move but is unsure about its direction. The payoff at expiration is depicted in Table 7.18.

The maximum payoff is unlimited if $S_T > X$, and so is the maximum profit. The profit increases dollar for dollar with the stock price. In the other direction, the maximum payoff is X, because the stock price cannot decline below zero. The maximum profit in this price range is $X - C_t - P_t$. The lowest payoff is zero, which occurs if $S_T = X$. The maximum loss is $-(C_t + P_t)$. There are two breakeven points: $X - C_t - P_t$ and $X + C_t + P_t$. The profit diagram is depicted in Figure 7.11.

Example 7.36 is an illustration of a long straddle.

A Strangle

A strangle is similar to a straddle in the sense that it requires the trader to buy a call and a put on the same asset and with the same expiration date. The difference is that the two options are chosen with different exercise prices. If we denote the exercise price of the call as X_1 and that of the put

Example 7.36

Gene Altman buys a call and a put option on IBM with an exercise price of $90 and 110 days to expiration. The respective premiums are $6.01 and $3.87. The initial investment is $988, which is also the maximum potential loss from the strategy. In the price range $S_T > X$, the profit is $S_T - 99.88$. The maximum profit is unbounded, and the breakeven stock price is 99.88. In the price range $S_T < X$, the profit is $80.12 - S_T$. The maximum profit in this region is 80.12. The breakeven price is 80.12. The payoffs and profits for various values of the terminal stock price are depicted in Table 7.19.

Table 7.19
Profit from a Long Straddle

Stock Price	Payoff from Call	Payoff from Put	Total Payoff	Total Profit or Loss
70	0	2,000	2,000	1,012
75	0	1,500	1,500	512
80	0	1,000	1,000	12
80.12	0	988	988	0
85	0	500	500	(488)
90	0	0	0	(988)
95	500	0	500	(488)
99.88	988	0	988	0
100	1,000	0	1,000	12
105	1,500	0	1,500	512
110	2,000	0	2,000	1,012
115	2,500	0	2,500	1,512

as X_2, there are two possibilities: $X_1 > X_2$ and $X_1 < X_2$. So, there are two types of strangles: out-of-the-money strangles and in-the-money strangles.

Let us first consider an out-of-the-money strangle. The payoff is depicted in Table 7.20.

If $S_T < X_2$, then the profit is $X_2 - S_T - C_t - P_t$. The maximum profit in this region is $X_2 - C_t - P_t$, which is also the magnitude of the breakeven stock price. If $X_2 < S_T < X_1$, then the payoff is zero and the loss is equal to the initial investment. This represents the maximum possible loss for this strategy. If $S_T > X_1$, then the profit is $S_T - X_1 - C_t - P_t$. The maximum profit in this region is unbounded. The second breakeven price is $X_1 + C_t + P_t$. The profit diagram is depicted in Figure 7.12.

Let us consider an in-the-money strangle. The payoffs are shown in Table 7.21.

If $S_T < X_1$, the profit is $X_2 - S_T - P_t - C_t$. The maximum profit in this region is $X_2 - P_t - C_t$ which is also the magnitude of the breakeven

Table 7.20
Payoffs from an Out-of-the-Money Long Strangle

Terminal Price Range	Payoff from Call	Payoff from Put	Total Payoff
$S_T < X_2$	0	$X_2 - S_T$	$X_2 - S_T$
$X_2 < S_T < X_1$	0	0	0
$S_T > X_1$	$S_T - X_1$	0	$S_T - X_1$

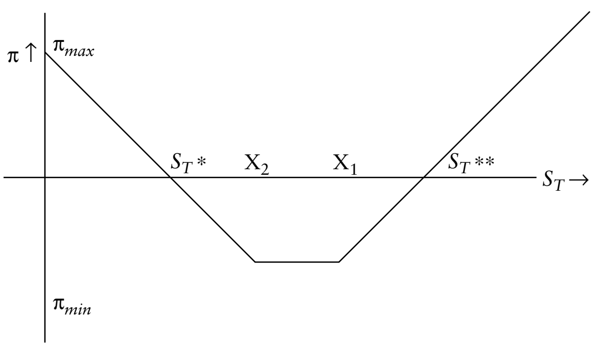


Figure 7.12 Profit Profile: Strangle

Table 7.21
Payoffs from an In-of-the-Money Long Strangle

Terminal Price Range	Payoff from Call	Payoff from Put	Total Payoff
$S_T < X_1$	0	$X_2 - S_T$	$X_2 - S_T$
$X_1 < S_T < X_2$	$S_T - X_1$	$X_2 - S_T$	$X_2 - X_1$
$S_T > X_2$	$S_T - X_1$	0	$S_T - X_1$

stock price. If $X_1 < S_T < X_2$, the profit is $X_2 - X_1 - P_t - C_t$. This represents the maximum loss from the strategy. Finally if $S_T > X_2$, the profit is $S_T - X_1 - P_t - C_t$. The profit is unbounded is this region. The corresponding breakeven stock price is $X_1 + P_t + C_t$.

Example 7.37 is a numerical illustration of an out-of-the-money strangle.

Example 7.37

Gene Altman buys a call and a put option on IBM with 110 days to expiration. The call has an exercise price of \$100, and the put has an exercise price of \$90. The respective premiums are \$2.20 and \$3.87. The initial investment is \$607, which is also the maximum potential loss from the strategy. In the price range $S_T > 100$, the profit is $S_T - 106.07$. The maximum profit is unbounded, and the breakeven stock price is 106.07. In the price range $S_T < 90$, the profit is $83.93 - S_T$. The maximum profit in this region is 83.93. The breakeven price is 83.93. The payoffs and profits for various values of the terminal stock price are depicted in Table 7.22.

Table 7.22
Profit from a Long Strangle

Stock Price	Payoff from Call	Payoff from Put	Total Payoff	Total Profit or Loss
70	0	2,000	2,000	1,393
75	0	1,500	1,500	893
80	0	1,000	1,000	393
83.93	0	607	607	0
85	0	500	500	(107)
90	0	0	0	(607)
95	0	0	0	(607)
100	0	0	0	(607)
105	500	0	500	(107)
106.07	607	0	607	0
110	1,000	0	1,000	393
115	1,500	0	1,500	893
120	2,000	0	2,000	1,393

Futures Options

A futures option is an option that is written on a futures contract. A call futures option gives the holder the right to assume a long position in a futures contract if she were to exercise the option. When such a contract is exercised, a long position is established in the futures contract and the position is immediately marked to market. At this stage, the holder has two options. She can either deposit the margin required to support the long futures position or offset the futures contract. When a call futures option is exercised by the holder, a short position in the futures contract is established for the writer of the contract. His position will also be marked to market, and he will be confronted with the same two choices: either deposit the margin required to maintain the futures position or offset it.

If we denote the current futures price by F_t and the exercise price of the option by X, the contract will be exercised only if $F_t > X$. When the contract is marked to market, there will be an inflow of $F_t - X$, which is nothing but the intrinsic value of the contract. What is the rationale for marking the contract to market upon exercise? The options contract gives the holder the right to assume a long futures position upon exercise. When a futures position is established, it will be at the prevailing futures price F_t. To ensure that the position is effectively at the exercise price, the price difference must be paid to the holder.

Example 7.38 illustrates the mechanics of a call option on a futures contract.

Example 7.38

Benny has acquired a call futures option with an exercise price of \$100. Each contract is for 100 units of the underlying asset and the current futures price is \$106. If the option is to be exercised, then a long position will be established in the futures contract and the holder will receive a cash flow of $100 \times (106 - 100) = \$600$.

A put futures option gives the holder the right to assume a short position in a futures contract if he exercises the option. Simultaneously a long position in the futures contract will be established for the writer. Both contracts will be marked to market; as in the case of the call, they can either post the required margin or offset their positions. If we denote the current futures price by F_t, the put will be exercised only if $F_t < X$. As in the case of the call, there will be an immediate cash inflow of $X - F_t$.

Example 7.39 illustrates the mechanics of a put option on a futures contract.

Example 7.39

Michelle has bought a put futures option. The exercise price is \$100, and the current futures price is \$93.50. Each contract is for 100 units of the underlying asset. If the put is exercised, a short position will be established in the futures contract and the holder will receive a cash flow of $100 \times (100 - 93.50) = \$650$.

Put-Call Parity

The put-call parity condition for European futures options is

$$C_{E,t} = P_{E,t} + (F_t - X)e^{-r(T-t)}.$$

The Black Model

The Black model is applicable for pricing European options on futures contracts. In the case of European options, which are scheduled to expire at the same time as the underlying futures contracts, the Black model states that

$$C_{E,t} = e^{-r(T-t)}[F_t N(d_1) - XN(d_2)]$$

and

$$P_{E,t} = e^{-r(T-t)}[XN(-d_2) - F_t N(-d_1)]$$

The variables d_1 and d_2 may be expressed as follows:

$$d_1 = \frac{ln\left(\frac{F}{X}\right) + \left(\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}.$$

Example 7.40 is an illustration of the Black model.

Example 7.40

A futures contract with 4½ months to expiration is currently trading at \$95. European call and put options with the same time to expiration are available with an exercise price of \$90. The riskless rate is 10 percent per annum, and the volatility is 25 percent per annum:

$$d_1 = \frac{ln\left(\frac{95}{90}\right) + \left(\frac{0.25 \times 0.25}{2}\right)0.375}{0.25\sqrt{0.375}} = \frac{0.0658}{0.1531} = 0.4298$$

$$d_2 = 0.4298 - 0.1531 = 0.2767$$

$$N(0.4298) = 0.6663; N(0.2767) = 0.6090$$

$$C_{E,t} = e^{-0.10 \times 0.375}[95 \times 0.6663 - 90 \times 0.6090] = 8.1761$$

$$P_{E,t} = e^{-0.10 \times 0.375}[90 \times 0.3910 - 95 \times 0.3337] = 3.3601.$$

Endnotes

1. The symbol r is generally used to denote the riskless rate of return. In the binomial model, however, the standard practice is to use it to denote 1 plus the riskless rate. In this context, r represents the periodic rate of interest, which need not be an annual rate.
2. An American call on a nondividend-paying stock will never be exercised early.

CHAPTER 8

Foreign Exchange

Introduction

The market for the sale and purchase of currencies is an over-the-counter market: there is no organized exchange on which currencies are traded. The largest players in the market are commercial banks. These banks typically provide two-way quotes for a number of currencies. They will quote a bid rate for buying a particular currency as well as an ask rate for selling the currency. The difference between the two rates, which is termed the *spread*, is a source of profit for the dealer. In the major money centers of the world, some nonbank dealers as well as large multinational corporations may also don the mantle of dealers. The market in which these entities operate is referred to as the *interbank market*. The transactions sizes are typically very large, usually of the magnitude of several million U.S. dollars. The retail market for foreign exchange, in which tourists typically transact in the form of currency notes and travelers checks, is characterized by transactions of much smaller magnitudes, and the corresponding bid−ask spreads are also larger.

An exchange rate is the price of one country's currency in terms of the currency of another country. In any bilateral trade, there has to be a buyer and a seller. In the foreign-exchange market, the words *buy*, *sell*, *purchase*, and *sale* are always used from the dealer's perspective. When a dealer buys a foreign currency from a client, she will pay the equivalent in terms of the domestic currency. On the other hand, when a dealer sells a foreign currency, she will take the equivalent amount in terms of domestic currency from the client.

Examples 8.1 and 8.2 provide illustrations of purchase and sale transactions respectively.

Example 8.1

Golden Circle in Sydney has exported canned food to Singapore and has been paid in Singapore dollars. The company presents the check to Commonwealth Bank seeking the equivalent amount in Australian dollars (AUD). From the standpoint of the bank, this represents the acquisition of a foreign currency: the Singapore dollar. This is a purchase transaction.

Example 8.2

An expatriate manager working in Sydney has saved AU$250,000 and wants to remit the amount to New York in the form of U.S. dollars (USD). He goes to the bank seeking a transfer to the United States. From the standpoint of the bank, this represents a sale of foreign currency: the U.S. dollar. This is a sale transaction.

Currency Codes

Every currency has a three-character code that has been assigned by the International Standards Organization (ISO). The codes for some of the globally important currencies are given in Table 8.1.

Base and Variable Currencies

When quoting the rate of exchange between two currencies, the practice is to keep the number of units of one currency fixed while making changes in the number of units of the other to reflect changes in the rate of exchange. The currency that has unvarying units is referred to as the *base currency* and the other is termed the *variable currency*. The practice in the foreign-exchange market is to show the ISO codes for the two currencies involved as ABC−XYZ or ABC/XYZ, with the first code referring to the base currency and the second to the variable currency. A quote of 0.8125 USD/EUR or 0.8125 USD−EUR denotes a quote of 0.8125 euros (€) per U. S. dollar. Thus, in this quote the U.S. dollar is the base currency and the euro is the variable currency.

Table 8.1
Symbols for Major Currencies

Country	Currency	Symbol
Australia	Dollar	AUD
Brazil	Real	BRL
Canada	Dollar	CAD
China	Renminbi Yuan	CNY
Czech Republic	Koruna	CAK
European Monetary Union	Euro	EUR
Hong Kong	Dollar	HKD
Hungary	Forint	HUF
India	Rupee	INR
Israel	Shekel	ILS
Japan	Yen	JPY
Malaysia	Ringgit	MYR
Mexico	Peso	MXN
New Zealand	Dollar	NZD
Norway	Krone	NOK
Poland	Zloty	PLN
Russia	Rouble	RUB
Singapore	Dollar	SGD
South Africa	Rand	ZAR
South Korea	Won	KRW
Sweden	Krona	SEK
Switzerland	Franc	CHF
Thailand	Baht	THB
UK	Pound Sterling	GBP
USA	Dollar	USD

Direct and Indirect Quotes

When quoting the exchange rate for a currency, it is possible to quote the rate by designating either the domestic or foreign currency as the base currency. Exchange quotes where the domestic currency is the variable currency and the foreign currency is the base currency are referred to as *direct quotes*. Thus, in London 0.6750 USD−GBP is a direct quote for the U.S. dollar and the British pound (GBP). On the other hand, if the exchange rate were to be specified with the domestic currency as the base currency and the foreign currency as the variable currency, it would be deemed an *indirect quote*. Thus, a quote of 1.125 GBP−USD in London would be an example of an indirect quote.

The reason why we have two systems may be explained as follows. Take the case of a commodity such as wheat. We typically quote the price as dollars per unit of the commodity and not as the number of units of the commodity per dollar. So the standard quote will be something like $5.00 per bushel of wheat and not 0.2000 bushels per dollar. In the currency market, however, we are dealing with two currencies. Both quoting conventions are equally valid—that is, a quote in London may be USD−GBP or GBP−USD. We will briefly consider the indirect method because it is important that readers understand the principle. However, all our illustrations in this chapter will use the direct method.

European Terms and American Terms

If an exchange rate were to be quoted with the U.S. dollar as the base currency, then it would be referred to as a quote as *per European terms*. So a quote of 0.6750 USD−GBP in London would represent a rate as per European terms. However, an exchange rate with the U.S. dollar as the variable currency will be categorized as a quote as *per American terms*. A quote of 1.2500 EUR−USD in New York City (NYC) would represent a quote as per the American convention. Direct quotes in the United States would be as per American terms and indirect quotes will be as per European terms. In other countries, quotes for the U.S. dollar, in which the dollar is represented as the base currency, would be termed European-style quotes and those in which the dollar is represented as the variable currency would amount to American-style quotes.

Bid and Ask Quotes

Let us first consider direct quotes. Consider a quote of 1.2250−1.2375 EUR−USD. The first term represents the rate at which the dealer is willing to buy euros from a client, and the second is the rate at which he is willing to sell euros to the client. If the dealer has to remain profitable, the rate at which he is prepared to buy the foreign currency, the bid, should be lower than the rate at which he is willing to sell the foreign currency, the ask or the offer. Thus, in the case of direct quotes, the bid rate will always be lower than the ask rate.

Consider a quote of 0.8195−0.8075 USD−EUR. Once again, the first term represents the rate at which the dealer is prepared to buy euros from the client, and the second term is the rate at which he is willing to sell euros to a party. When he is acquiring euros from a client he would want to acquire more per dollar. On the other hand, when he is selling euros he would like to part with less per dollar. Thus, it is not surprising that the bid is higher than the ask in the indirect quotation system.

Remember that in a direct quote the bid is the rate for buying the base currency, and the ask is the rate for selling the base currency. In an indirect quote, however, the bid is the rate for buying the variable currency and the ask is the rate for selling the variable currency.

Appreciating and Depreciating Currencies

If the value of a currency increases in terms of another currency, then it has *appreciated*. On the other hand, if the value of a currency declines in terms of another, then it has *depreciated*. Note that appreciation or depreciation of a currency is with respect to another currency, so statements such as "the U.S. dollar has appreciated" have no meaning. To be meaningful, the currency of comparison also should be specified—as "the U.S. dollar has appreciated with respect to the Australian dollar."

Consider a quote of 1.2250 EUR−USD in NYC. It is a direct quote. If the rate were to increase to 1.2305 EUR−USD it would mean that a euro is worth more in terms of dollars. However if the rate were to decrease to 1.2180 EUR−USD it would mean that a euro is worth less in terms of dollars. An increase in the quote would imply that the euro has appreciated or that the U.S. dollar has depreciated. On the contrary if the rate were to decline it would signify that the euro has depreciated or that the U.S. dollar has appreciated. Thus in the case of direct quotes a rising value signifies a depreciating home currency and an appreciating foreign currency while a decline in the rate signifies the opposite.

Consider indirect quotes. Take a quote of 0.8125 USD−EUR. If the rate increases, it signifies that the dollar is worth more in terms of euros and therefore the dollar has appreciated—or, equivalently, the euro has depreciated. On the other hand, a fall in the rate would signify that the dollar is worth less in terms of the euro, which would be construed as a depreciation of the dollar with respect to the euro. In the case of indirect quotes, a rising value signifies an appreciating home currency, whereas a decline in the rate connotes a depreciating home currency.

This appears confusing at first glance because an increase in the quoted rate connotes a depreciating U.S. dollar in NYC if rates were to be quoted directly, whereas it signifies an appreciating dollar if rates were to be quoted indirectly. The best way to clear the confusion is to perceive the two currencies involved as base and variable currencies. An increase in the quote always means that the variable currency has depreciated, whereas a decrease in the quote always means that the variable currency has appreciated. In the case of a direct quote in NYC, the U.S. dollar is the variable currency. So a rising quote with respect to a currency like the euro signifies a depreciating dollar and an appreciating euro, and a declining rate connotes the opposite. On the other hand, in the case of an indirect quote in NYC, the U.S. dollar is the base currency. In this case, a rising quote with respect to a currency like the euro signifies a depreciating euro or an appreciating dollar. A declining value in the case of an indirect quote in NYC would imply a depreciating dollar and an appreciating foreign currency.

What are the implications of an appreciation or depreciation of the U.S. dollar with respect to the euro? An appreciation of the dollar would mean that the price of the euro in terms of dollars has gone down. Imports from the European Union (EU) would be more attractive for Americans just as imports from the United States would be less attractive for consumers in the EU. A depreciating dollar would lead to a decline in imports from the EU into the United States while boosting U.S. exports to the EU.

Converting Direct Quotes to Indirect Quotes

Consider a quote of 1.2250−1.2375 EUR−USD offered by Citibank in NYC. It is a direct quote. Consider the bid of 1.2250. If the dealer is prepared to buy euros at dollars 1.2250 per euro, then it means she is prepared to sell dollars at $1/1.2250 = 0.8163$ euros per dollar. Similarly, the ask rate of 1.2375 is equivalent to a rate of 0.8081 euros per dollar. So the equivalent indirect quote is 0.8163−0.8081 USD−EUR.

The direct quote of 1.2250−1.2375 will have an equivalent direct quote in Frankfurt, where a direct quote will be in terms of euros per dollar. However, the bid for a dealer in Frankfurt will be a rate for buying U.S. dollars, whereas the ask rate will be a rate for selling dollars. Thus, the equivalent direct quote in Frankfurt is 0.8081−0.8163 USD−EUR.

Points

Consider a quote of 1.2235−1.2295 EUR−USD. Exchange-rate quotations in interbank markets are given up to four decimal places, so the last digit corresponds to 1/10,000th of the variable currency. The last two digits in such quotes are called *points* or *pips*. The first three digits of a quote are known as the *big figure*. In this case, the big figure is 1.22 and the spread is 60 points.

In the case of certain currencies, the quotes are given to two decimal places only. As such, a point or pip is 1/100th of the variable currency. Consider a quote of 95.75–96.95 USD–JPY (i.e., U.S. dollars to Japanese yen). The spread is 120 points.

Rates of Return

Let S_0 be the exchange rate for U.S. dollars in terms of the euro as quoted in Frankfurt. Assume that a year later the exchange rate is S_1 USD–EUR. The rate of return for a European trader who acquires a dollar is given by

$$r_d = \frac{(S_1 - S_0)}{S_0}.$$

Consider the issue from the perspective of an American trader who acquires a euro. The price of a euro in terms of the dollar is $1/S_0$ EUR–USD. Thus, the rate of return for the trader is

$$\begin{aligned} r_f &= \frac{\left(\frac{1}{S_1} - \frac{1}{S_0}\right)}{\frac{1}{S_0}} \\ &= \frac{(S_0 - S_1)}{S_1} \\ &= \frac{-r_d}{(1 + r_d)}. \end{aligned}$$

The percentage change in return therefore depends on the perspective we take. We have considered the European trader to be the domestic investor and the American trader the foreign investor. We can demonstrate that

$$(1 + r_d) = \frac{1}{(1 + r_f)}.$$

Therefore, 1 plus the rate of return for a domestic investor is the reciprocal of 1 plus the rate of return for a foreign investor.

Example 8.3 illustrates the return for investors who acquire a currency as an investment.

Example 8.3

Consider the following quotes for the U.S. dollar on two different dates:

January 1, 20XX: 0.8000 USD–EUR
December 31, 20XX: 0.8125 USD–EUR

The rate of return for a European investor is

$$r_d = \frac{(0.8125 - 0.8000)}{0.8000} = 0.015625 \equiv 1.5625\%.$$

The rate of return for a European investor is positive because the dollar has appreciated against the euro.

The rate of return for an American trader is

$$r_f = \frac{-0.015625}{1 + 0.015625} = -0.015385 \equiv -1.5385\%.$$

The return is negative because the euro has depreciated against the dollar.

The Impact of Spreads on Returns

Assume that the quoted rates for the U.S. dollar in Frankfurt at the beginning and end of a year are:

January 1, 20XX: $S_{b,0}$–$S_{a,0}$ USD–EUR
December 31, 20XX: $S_{b,1}$–$S_{a,1}$ USD–EUR

A European trader can acquire a dollar on January 1 at a price of $S_{a,0}$ and can sell it after a year at $S_{b,1}$. The rate of return is

$$r_d = \frac{\left(S_{b,1} - S_{a,0}\right)}{S_{a,0}}.$$

An American trader can acquire a euro on January 1 at $1/S_{b,0}$. A year later, the euro can be sold to realize $1/S_{a,1}$ dollars. The rate of return is

$$
\begin{aligned}
r_f &= \frac{\left(\dfrac{1}{S_{a,1}} - \dfrac{1}{S_{b,0}}\right)}{\dfrac{1}{S_{b,0}}} \\
&= \frac{\left(S_{b,0} - S_{a,1}\right)}{S_{a,1}}.
\end{aligned}
$$

Example 8.4 demonstrates the computation of the return for a trader who acquires a foreign currency in a market characterized by a bid-ask spread.

Example 8.4

Consider the following rates in the Frankfurt interbank market:

January 1, 20XX: 0.8000–0.8075 USD–EUR
December 31, 20XX: 0.8100–0.8175 USD–EUR

The rate of return for a European investor is

$$r_d = \frac{(0.8100 - 0.8075)}{0.8075} = 0.003096 \equiv 0.3096\%.$$

The rate of return for an American investor is

$$r_f = \frac{(0.8000 - 0.8175)}{0.8175} = -0.021407 \equiv -2.1407\%.$$

Arbitrage in Spot Markets

We will consider three types of arbitrage in foreign-exchange spot markets: one-point arbitrage, two-point arbitrage, and three-point arbitrage. We look first at one-point arbitrage.

One-Point Arbitrage

Consider the following quotes in NYC on a given day:

Citibank: 1.2225–1.2280 EUR–USD

HSBC: 1.2285–1.2340 EUR–USD

Take the case of an arbitrageur who has US$1,000,000 with her. She can buy 1,000,000/1.2280 or 814,332.25 euros from Citibank. This can immediately be sold to HSBC at the bid rate of 1.2285 to yield 1.2285 × 814,332.25 = $1,000,407.17. There is an arbitrage profit of 407.17 dollars.

Consider another situation:

Citibank: 1.2225–1.2280 EUR–USD

HSBC: 1.2180–1.2220 EUR–USD

In this case, an arbitrageur can acquire 1,000,000/1.2220, or €818,330.61, from HSBC. This can immediately be sold to Citibank at its bid rate of 1.2225 to yield $1,000,409.17. Thus, once again there is an arbitrage profit of $409.17.

Such potential for arbitrage will exist as long as the quotes from competing banks do not overlap by at least one point. Consider the following quotes:

Citibank: 1.2225–1.2280 EUR–USD

HSBC: 1.2280–1.2340 EUR–USD

Or

Citibank: 1.2225–1.2280 EUR–USD

HSBC: 1.2185–1.2225 EUR–USD

There is no scope for arbitrage in either of these two cases.

Two-Point Arbitrage

Consider the following quote in NYC on a given day:

Citibank: 1.2225–1.2280 EUR–USD

On the same day BNP Paribas in Paris is quoting 0.8185–0.8220 USD–EUR.

This also represents an arbitrage opportunity, although it is not obvious at first glance. Consider the following strategy. Borrow €1,000,000 and acquire US$1,222,500 in NYC. The currency can immediately be sold in Paris to yield €1,000,616.25. There is an arbitrage profit of €616.25. What is the cause in this case?

The equivalent direct quote in Paris for Citibank's quote in NYC is 0.8143–0.8180. This is not overlapping with the quote given by BNP, and we know that the two quotes must overlap by at least one point to rule out arbitrage. This kind of an arbitrage opportunity is termed *two-point arbitrage*. Two-point arbitrage is a manifestation of one-point arbitrage executed across two markets.

Triangular Arbitrage

Consider the following quotes in Tokyo and NYC, respectively:

Bank of Tokyo: 125.00 EUR–JPY

112.50 USD–JPY

Citibank: 1.1250 EUR–USD

Consider the following strategy: Borrow ¥1 million in Tokyo and acquire €8,000. The euros can be sold in NYC to yield US$9,000. The USD can then be sold in Tokyo to yield ¥1,012,500. The strategy yields an arbitrage profit of ¥12,500. This kind of arbitrage, which entails the use of three currencies, is termed *triangular arbitrage*.

There are two ways to acquire a currency. In this case, USD can be acquired in Tokyo by paying Japanese yen, or euros can be acquired in Tokyo using Japanese yen, which can subsequently be sold in NYC to yield dollars. The first transaction, which is based on a direct yen-for-dollar exchange, can be termed the *natural rate*. The second may be termed the *synthetic rate*. To rule out arbitrage, the natural rate must equal the synthetic rate. If we assume that the Tokyo market is fairly priced, then the rate that must prevail in NYC to rule out arbitrage is

$$112.50 = 125/X \Rightarrow X = 125/112.50 = 1.1111 \text{ EUR–USD}.$$

Let us make matters more realistic by introducing bid–ask spreads. Consider the following rates in Tokyo:

124.25–125.50 EUR–JPY

111.75–112.50 USD–JPY

The issue is what the rates in NYC should be to rule out triangular arbitrage.

Let us depict the two rates as S_b–S_a EUR–USD, where the subscripts denote bid and ask, respectively.

Consider an arbitrageur who borrows US\$1 million and sells it in Tokyo to yield ¥111.75 million. This can be used to acquire 111,750,000/125.50 = €890,438.25 in Tokyo. The euros can be sold in NYC to yield $S_b \times$ US\$890,438.25. The condition to preclude arbitrage is therefore

$$S_b \times 890{,}438.25 \leq 1{,}000{,}000 \Rightarrow S_b \leq 1.1230 \text{ EUR}-\text{USD}$$

S_b may be termed as the natural bid rate.

The no-arbitrage condition is that

$$1{,}000{,}000 \times (\text{USD}-\text{JPY})_{\text{bid}} \times 1/(\text{EUR}-\text{JPY})_{\text{ask}} \times S_b \leq \times 1{,}000{,}000$$

$$\Rightarrow S_b \leq (\text{EUR}-\text{JPY})_{\text{ask}} \times 1/(\text{USD}-\text{JPY})_{\text{bid}} = (\text{EUR}-\text{JPY})_{\text{ask}} \times (\text{JPY}-\text{USD})_{\text{ask}}.$$

Consider the right-hand side (RHS). It represents an indirect way of acquiring euros using U.S. dollars because it requires the acquisition of Japanese yen using dollars followed by the acquisition of euros using yen. Thus. it can be termed a *synthetic ask rate* for euros in terms of USD. The no-arbitrage condition is

$$S_b \equiv (\text{EUR} - \text{USD})_{\text{bid}} \leq (\text{EUR} - \text{USD})_{\text{synthetic ask}}.$$

In other words, the natural bid should be less than or equal to the synthetic ask.

We can derive a similar condition for the natural ask. Take the case of a trader who borrows ¥1 million and acquires dollars in Tokyo. He will get 1,000,000/112.50 = US\$8,888.8889. This can be used to acquire 8,888.8889 $\times 1/S_a$ euros in NYC. The euros can be sold in Tokyo to yield 8888.8889 $\times$ 124.25 $\times 1/S_a$ Japanese yen. The no arbitrage condition is that

$$8888.8889 \times 124.25 \times 1/S_a \leq 1{,}000{,}000$$
$$\Rightarrow S_a \geq 1.1044\text{EUR} - \text{USD}.$$

Thus, the no-arbitrage condition is that

$$[1{,}000{,}000/(\text{USD} - \text{JPY})_{\text{ask}}] \times (\text{EUR} - \text{JPY})_{\text{bid}} \times 1/S_a \leq 1{,}000{,}000$$

$$\Rightarrow S_a \geq (\text{EUR} - \text{JPY})_{\text{bid}} \times (\text{JPY} - \text{USD})_{\text{bid}}.$$

The RHS represents an indirect way of acquiring U.S. dollars using euros because it requires the acquisition of Japanese yen using euros followed by the acquisition of U.S. dollars using yen. Thus, it can be termed a *synthetic bid rate* for euros in terms of USD. The no-arbitrage condition is

$$S_a \equiv (\mathrm{EUR} - \mathrm{USD})_{\mathrm{ask}} \geq (\mathrm{EUR} - \mathrm{USD})_{\mathrm{synthetic\ bid}}.$$

In other words, the natural ask should be greater than or equal to the synthetic bid. Of course, the natural ask must be greater than the natural bid.

Cross Rates

As we already saw, a bid for euros in terms of the dollar can be generated by multiplying the $(\mathrm{EUR{-}JPY})_{\mathrm{bid}}$ by the $(\mathrm{JPY{-}USD})_{\mathrm{bid}}$. Similarly, the ask for euros in terms of the dollar can be generated by multiplying the $(\mathrm{EUR{-}JPY})_{\mathrm{ask}}$ by the $(\mathrm{JPY{-}USD})_{\mathrm{ask}}$. The bid and ask rates for a pair of currencies that are arrived using rates with respect to a third currency are referred to as *cross rates*. In our illustration, the common currency was the Japanese yen, which was the base currency with respect to the USD and the variable currency with respect to the euro. Thus, to compute cross rates from two given quotes in which the common currency is the base currency in one quotation and the variable currency in the other, we need to multiply the two quoted bid rates to arrive at the *cross bid* and multiply the two quoted ask rates to arrive at the *cross ask*.

However, sometimes the common currency may be either the base currency with respect to both the other currencies or the variable currency with respect to both. In such cases, the cross rate may be generated using the following logic.

Assume that a bank in NYC is quoting the following rates:

1.2225–1.2295 EUR–USD

0.6225–0.6350 AUD–USD

The USD is the variable currency in both cases. Consider the synthetic $(\mathrm{EUR{-}AUD})_{\mathrm{bid}}$. It requires the trader to sell euros and buy USD and then sell USD to buy AUD. If the dealer sells a euro, he will get US\$1.2225. If he were to sell US\$1, he will get 1/0.6350 AUD. Thus, the synthetic $(\mathrm{EUR{-}AUD})_{\mathrm{bid}} = 1.2225/0.6350 = 1.9252 = (\mathrm{EUR{-}USD})_{\mathrm{bid}} \div (\mathrm{AUD{-}USD})_{\mathrm{ask}}$.

Consider the synthetic ask. To buy euros, the trader can buy U.S. dollars by selling Australian dollars and then sell the U.S. dollars to acquire a euro. If the trader were to sell an Australian dollar, he will get US\$0.6225.

The USD price for buying a euro is 1.2295. Thus, to buy a euro using Australian dollars, he requires 1.2295/0.6225 AUD or 1.9751 EUR−AUD. So the synthetic $(\text{EUR}-\text{AUD})_{\text{ask}} = (\text{EUR}-\text{USD})_{\text{ask}} \div (\text{AUD}-\text{USD})_{\text{bid}}$. To arrive at a spot rate from two other rates that have a common variable currency, we need to divide opposite sides of the quotes. In this case, the synthetic EUR−AUD bid rate is obtained by dividing the EUR−USD bid rate by the AUD−USD ask rate. Similarly, the EUR−AUD ask rate is obtained by dividing the EUR−USD ask rate by the AUD−USD bid rate.

We consider a situation in which the common currency is the base currency in both cases. Assume that the following quotes are available:

0.8225−0.8375 USD−EUR

1.5225−1.5350 USD−AUD

Consider the synthetic EUR−AUD bid. If the trader sells a euro, she will get 1/0.8375 USD. She can sell this to get $1.5225 \times 1/0.8375 =$ AU\$1.8179, so the EUR−AUD bid is equal to the USD−AUD bid divided by the USD−EUR ask. Consider the EUR−AUD ask rate. To buy a U.S. dollar, the trader requires AU\$1.5350. With US\$1 she can acquire €0.8225, so to acquire €1 she requires 1.5350/0.8225 = AU\$1.8663. Thus, the EUR−AUD ask rate is equal to the USD−AUD ask divided by the USD−EUR bid. Hence, once again, to arrive at a spot rate from two other rates that have a common base currency, we need to divide opposite sides of the quotes.

Value Dates

Spot-market transactions have a value date of two business days after the trade date. If a spot transaction were to occur on a Monday, the value date would be Wednesday. The market for the purchase and sale of foreign currencies with a delivery date that is more than two business days after the trade date is referred to as the *forward market*. The delivery dates for forward contracts, as is the case for money market transactions, are based on the modified following business day convention and the end-to-end rule.

Assume that we are on March 5, 20XX, and that the spot value date is March 7. A one-month forward contract will have a value date of April 7, and a three-month forward contract will have a value date of June 7. If the value date were to be a holiday, then the delivery would be moved to the next business day. As in the case of the money market, if moving the value date forward because of a market holiday were to cause it to fall in the subsequent calendar month, then it would be moved back to the last business day of the same calendar month. Assume that we are on July 29, 20XX. The spot value date is July 31, and the value date for a one-month forward contract is August 31. If August 31 were a Sunday, the value date for the contract would be August 29.

According to the end-to-end rule, if the spot value date is the last business day of a month, then the forward value date will be the last business day of the corresponding month. If July 31 is the spot value date, then a two-month forward contract will have a value date of September 30.

The Forward Market

In most foreign-exchange markets, the standard maturities for forward contracts are 1 and 2 weeks and 1, 2, 3, 6, 9, and 12 months. Countries such as the United States have an active currency futures market. However, foreign currencies are one product where the volume of forward contract transactions is much higher than the trading volumes in futures markets.

If the forward rate for a maturity exceeds the spot rate, then we say that the foreign currency is *trading at a premium*. However, if the forward rate is less than the spot rate, then the foreign currency is said to *be trading at a discount*. If the two rates are equal, then the currency is said to be trading flat. Consider Example 8.5.

Example 8.5

Consider the following rates that are observed in NYC on a given day:

Spot: 1.2225−1.2275 EUR−USD

One-month forward: 1.2325—1.2405 EUR−USD

Spot: 0.6250−0.6305 AUD−USD

One-month forward: 0.6165−0.6235 AUD−USD

Spot: 0.9815−0.9855 CAD−USD

One-month forward: 0.9815−0.9855 CAD−USD.

The euro is trading at a forward premium, the Australian dollar at a forward discount, and the Canadian dollar flat.

Outright Forward Rates

Forward contracts that are undertaken in isolation are referred to as *outright forward contracts*, and the corresponding quotes are referred to as *outright forward rates*. However, whenever a bank in the interbank market undertakes a forward contract, it will be accompanied by a spot transaction in what is termed a *swap deal*. We will examine swap deals in detail shortly.

Outright forward rates have the same properties as spot rates. If the one-month rate for the euro is given as 1.2250−1.2325 EUR−USD, then the corresponding rates in terms of USD−EUR are 0.8114−0.8163 USD−EUR, where the USD−EUR bid is the reciprocal of the EUR−USD ask and the USD−EUR ask is the reciprocal of the EUR−USD bid.

The rules for calculating cross rates are also the same. Assume that on a given day the following rates are observed in the London market:

Three-month forward: 0.5000−0.5075 USD−GBP

Three-month forward: 0.7500−0.7560 EUR−GBP

The GBP is the variable currency in both cases.

The bid for a 3-month forward contract for euros in terms of the USD is given by 0.7500 ÷ 0.5075 = 1.4778 EUR−USD, while the ask rate may be computed as 0.7560 ÷ 0.5000 = 1.5120 EUR−USD.

Swap Points

The forward rates are not quoted directly in the interbank market. Instead, the difference between the forward and spot rates, which is termed the *forward margin* or *swap points*, is given, and the corresponding outright rates must be deduced from the data.

Consider the following data:

Spot: 0.5000−0.5075 USD−GBP

One-month forward points: 45/75

The numbers 45 and 75 represent the last two decimal places or what we have termed as *points*. The swap points represent the difference between the forward rate and the corresponding spot rate. However, it has not been specified whether the points should be added or subtracted in order to arrive at the outright forward rates. So the question is, do we add or subtract the numbers?

Remember that the spot market will have the lowest bid–ask spread and that the spread will steadily widen as we negotiate contracts for future points in time. This is because the relative liquidity in the spot market is the highest and the liquidity steadily declines as we go forward in time.

In this case, if we add 45 points on the left side and 75 points on the right side, the spread will widen; if we were to subtract the numbers, the spread will narrow. Thus, a quote such as 45/75 signifies that the foreign currency is at a forward premium and that the swap points need to be added. Hence, the corresponding outright forward rates in this case are 0.5045–0.5150 USD–GBP.

So swap points given as small number/large number connotes that the foreign currency is at a forward premium and the points must be added.

Consider the following situation. On a given day in Frankfurt, the rate for the U.S. dollar is given as follows:

Spot: 0.8000–0.8075 USD–EUR

Three-month swap points: 95/55

In this case, it is that if the forward market has a higher spread than the spot market, then we need to subtract the corresponding points from the respective sides. The equivalent outright forward rates in this case are as follows:

Three-month forward: 0.7905–0.8020

So the rule is that if the swap points are given as large number/small number, the foreign currency is at a forward discount and the points need to be subtracted.

If the quote states *par*, then the spot rate and the outright forward rates are identical. Sometimes, the swap points may be specified as $x–y$ A/P. A/P stands for *around par*. It signifies that the swap points on the left-hand side should be subtracted whereas that on the RHS should be added. Consider the following quote:

Spot: 0.8000–0.8005 USD–EUR

One-month forward: 9/5 A/P

The corresponding outright rates for a one-month forward contract are 0.7991−0.8010.

The logic used to derive the outright forward rates is valid in the case of direct quotes. In the case of indirect quotes, even though the swap points have the same meaning, the treatment is different. Small number/large number implies a foreign currency that is at a forward premium irrespective of whether rates are quoted directly or indirectly. However, if the rates were to be quoted indirectly, then the swap points must be subtracted from the respective sides. Similarly, if the swap points are given as large number/small number, it indicates a foreign currency that is at a forward discount. However, if rates are quoted indirectly, then the numbers need to be added. Consider the following illustration in Example 8.6.

Example 8.6

The following rates are observed in NYC on a given day:

Spot: 123.15−122.50 USD−JPY
One-month forward: 20/30

The Japanese yen is at a forward premium. However, the points must be subtracted, so the corresponding outright rates are

122.95−122.20 USD−JPY.

However, if the swap points had been given as 30/20, it would imply that the Japanese yen is at a forward discount. The points will have to be added to arrive at the outright rates. The corresponding quote is therefore

123.45−122.70 USD−JPY.

Broken-Dated Contracts

Most forward contracts are for standard time intervals such as one month or three months. At times, however, a client may approach a dealer seeking a contract with a nonstandard maturity date or a date that falls between two standard intervals. Such contracts are referred to as *broken-dated contracts*; to compute the applicable rate for such odd periods, a method of linear interpolation is used.

Example 8.7 illustrates the computation of the forward rates for a broken-dated contract.

Example 8.7

Consider the following rates that are quoted by BNP Paribas in Paris:

Spot: 0.8000−0.8125 USD−EUR

One month: 40/65

Two month: 60/95

The U.S. dollar is at a forward premium. Assume that today is August 16, 20XX, which is a Wednesday. The spot value date is August 18. The value date for a one-month contract is September 18, and that for a two-month contract is October 18.

Assume that a client approaches the bank, seeking to sell dollars on October 9, 2010, and that none of the three dates involved—that is, September 18 and October 9 and 18—are market holidays.

The number of days between September 18 and October 18 is 30. The number of days between September 18 and October 9 is 21 days, which represents 0.70 months. The premium for a 1-month forward purchase is 40 points, and that for a 2-month forward purchase is 60 points. The swap points for a contract maturing on October 9 may be calculated as

$$40 + 0.70 \times (60 - 40) = 54 \text{ points.}$$

The outright forward rate for a purchase transaction scheduled for October 9 is:

$$0.8000 + 0.0054 = 0.8054 \text{ USD}-\text{EUR.}$$

Covered Interest Arbitrage

We will derive the relationship between the spot rate and the forward rate for a given maturity using arbitrage arguments.

We will denote the spot rates, forward rates, and interest rates using the following symbols. Subsequently we will illustrate our arguments with the help of a numerical example.

Spot: S_b–S_a USD–EUR

Three-month forward: F_b–F_a USD–EUR

Borrowing and lending rates in Frankfurt (for euros): r_{db}–r_{dl}

Borrowing and lending rates in NYC (for dollars): r_{fb}–r_{fl}

Note: This is a direct quote for the U.S. dollar in Frankfurt. The rate for borrowing and lending euros is the domestic rate, whereas that for borrowing and lending dollars is the foreign rate.

We will first consider cash-and-carry arbitrage. It entails the acquisition of one unit of the foreign currency and the simultaneous assumption of a short position in a forward contract to sell the foreign currency after three months. To buy a U.S. dollar, the arbitrageur will have to borrow S_a EUR. This will be financed at a rate of r_{db}. He can invest the dollar at the rate r_{fl}. This will yield $(1 + r_{fl})$ USD at maturity, which can be sold forward at the outset at a rate F_b. At maturity, the amount borrowed in euros will have to be repaid with interest, which will entail an outflow of $S_a(1 + r_{db})$. To rule out arbitrage we require that

$$S_a(1 + r_{db}) \geq F_b(1 + r_{fl}) \Rightarrow F_b \leq S_a \times \frac{(1 + r_{db})}{(1 + r_{fl})}.$$

Consider a reverse cash-and-carry strategy. It will require the arbitrageur to borrow one U.S. dollar to acquire S_b euros. The amount that is borrowed in dollars will have to be financed at the rate r_{fb}. The euros can be invested at the rate r_{dl}. At the outset, the arbitrageur will have to go long in a forward contract to buy $(1 + r_{fb})$ dollars at the rate of F_a. To rule out arbitrage, we require that

$$F_a(1 + r_{fb}) \geq S_b(1 + r_{dl}) \Rightarrow F_a \geq S_b \times \frac{(1 + r_{dl})}{(1 + r_{fb})}.$$

A Perfect Market

In a perfect market, there will be no bid–ask spreads in either the spot or forward market and the borrowing rate for both currencies will be equal to the corresponding lending rates. In such a scenario, $S_a = S_b = S$

and $F_a = F_b = F$. The borrowing and lending rate in Frankfurt will be r_d, whereas that in NYC will be r_f. The no-arbitrage condition in such circumstances may be stated as

$$F = S \times \frac{(1 + r_d)}{(1 + r_f)}.$$

This is termed the *interest-rate parity condition*.

Example 8.8 illustrates these principles.

Example 8.8

The following rates are available in Frankfurt and NYC:

Spot: 0.8000−0.8125 USD−EUR

Three-month forward: 45−95

The outright forward rates are: 0.8045−0.8220 USD−EUR

Borrowing and lending rates in the United States (per annum): 4.25%, 4.00%

Borrowing and lending rates in the EU (per annum): 4.75%, 4.50%.

Let us first check out the condition for cash-and-carry arbitrage.

$$S_a \times \frac{(1 + r_{db})}{(1 + r_{fl})} = 0.8125 \times \frac{(1 + 0.25 \times 0.0475)}{(1 + 0.25 \times 0.04)} = 0.8140.$$

The forward bid of 0.8045 is less than this. Cash-and-carry arbitrage is ruled out.

Let us check out the condition for reverse cash-and-carry arbitrage.

$$S_b \times \frac{(1 + r_{dl})}{(1 + r_{fb})} = 0.8000 \times \frac{(1 + 0.25 \times 0.045)}{(1 + 0.25 \times 0.0425)} = 0.8005.$$

The forward ask of 0.8220 is greater than this. As a result, reverse cash-and-carry arbitrage is also ruled out.

Foreign-Exchange Swaps

It is a common practice for a bank to enter into a foreign-exchange swap with a counterparty. Such a transaction entails the purchase or sale of a currency on the spot value date with a simultaneous agreement to sell or purchase the same currency at a future date. In other words, it may involve a spot sale accompanied with a forward purchase or a spot purchase accompanied with a forward sale. In some cases, the transaction may entail a purchase or sale for a future date accompanied with a sale or purchase for a longer maturity. These transactions are referred to as *forward-to-forward swaps*.

In such transactions, the key variable is the forward margin or swap points for the foreign currency. The rate for the spot leg per se is irrelevant, and the spot purchase or sale may be done at the prevailing bid or the prevailing ask or at a rate that is close. Example 8.9 illustrates the mechanics of such transactions.

Example 8.9

BNP Paribas seeks to sell €1 million spot and acquire U.S. dollars for a period of six months. It approaches JPMorgan Chase (JPMC) for a swap transaction. The rates in the interbank market are as follows:

Spot: 1.2500−1.2625 EUR−USD

Six-month forward: 40/90

The two banks will usually agree on a spot rate between the bid and the ask rates. In this case, we assume that they agree on a rate of 1.2575. BNP will therefore deliver €1 million to JPMC in exchange for US$1,257,500. In the second leg, JPMC will be selling euros to BNP. The applicable forward margin will be 90 points. So BNP will take delivery of €1 million after six months at a rate of 1.2665 EUR−USD. This translates to a payable of US$1,266,500 for the bank.

The Cost

The cost of a foreign-exchange swap is a function of the interest-rate differential between the two currencies, and it is defined as the foreign-exchange value of the interest-rate differential between the currencies for the maturity of the swap. The party who holds the currency that pays a higher interest rate will effectively pay the counterparty, thereby neutralizing the rate differential and equalizing the returns on the two currencies.

Assume that covered interest arbitrage holds and for ease of exposition ignore bid–ask spreads and differential borrowing and lending rates. From the interest-rate parity condition, we know that

$$F = S\frac{(1+r_d)}{(1+r_f)} \Rightarrow \frac{F-S}{S} = \frac{r_d - r_f}{(1+r_f)}.$$

Thus,

$$F - S = S \times \frac{r_d - r_f}{(1+r_f)} \approx S \times (r_d - r_f)$$

$$\Rightarrow S - F = S \times \frac{r_f - r_d}{(1+r_f)} \approx S \times (r_f - r_d).$$

Example 8.10 illustrates the cost of/gain from a swap transaction.

Example 8.10

Assume that the spot rate for the euro is 1.25 EUR–USD. The interest rate in the U.S. market is 4 percent per annum, and that in the EU is 4.80 percent per annum. Consider a forward contract with six months to maturity. The forward rate, assuming that interest-rate parity holds, will be

$$1.2500 \times 1.02/1.024 = 1.2451 \text{ EUR–USD}.$$

Consider the following swap transaction. BNP Paribas agrees to sell €1 million to Bank of America at the spot rate and simultaneously agrees to buy back the euros after six months at the forward rate. The near-date transaction requires BNP to deliver €1 million in return for US$1.25 million, whereas the far-date transaction requires it to buy back €1 million by paying US$1.245 million. The gain for BNP is

$$1{,}250{,}000 - 1{,}245{,}100 = \$4{,}900.$$

This can be interpreted as follows. BNP is holding U.S. dollars, the currency that has a relatively lower interest rate, and Bank of America is holding euros, the currency that is paying a relatively higher interest rate. Bank of America has to make a payment of

$$1{,}000{,}000 \times 1.2500 \times (0.048 - 0.04) \times 0.5 \times 1/(1.024) = \$4{,}882.8125.$$

The two amounts are equal except for rounding errors.

The Motive

What could be the motivation for a swap such as the one undertaken by BNP Paribas with Bank of America? Even though banks regularly execute outright forward contracts with their nonbank clients, most interbank transactions that entail the use of forward contracts are done in the form of a foreign-exchange swap.

Assume that Bank of America has executed a forward contract with a client to buy €1 million six months later. It will have a long position in euros six months forward. If the bank desires to square-off this position, it will have to sell €1 million six months forward. But it may not always be easy to locate a counterparty with matching opposite needs and with whom an offsetting outright forward contract can be executed. It is easier to do a swap. In this case, the swap will require Bank of America to buy euros spot and sell the currency six months forward. This creates the required offsetting short forward positions in euros. In the process, however, the bank has created a long spot position in euros. This can be easily offset by selling the euros in the interbank market.[1]

A second reason why banks prefer to do foreign-exchange swaps rather than an outright forward is that, even though an outright forward is influenced by both the spot rate and the interest rates for the two currencies, a foreign-exchange swap is primarily influenced by the interest rates. An outright forward transaction has the effect of combining two related but different markets in a single transaction, which may not be acceptable to the bank, as compared to a swap that is primarily an interest-rate instrument.[2]

Example 8.11 demonstrates that a swap is primarily impacted by changing interest rates.

Example 8.11

Consider the following data:

Spot: 1.2500 EUR–USD
Interest rate for dollars = 4.00% per annum
Interest rate for euros = 2.80% per annum

A six-month outright forward would be priced at

$$1.2500 \times 1.02/1.014 = 1.2574 \text{ EUR}-\text{USD}.$$

If the spot rate were to increase to 1.2625, the forward rate would be 1.2700.

Assume that the rate for dollars remains at 4 percent per annum and that for the euro declines to 2 percent per annum. If the exchange-rate differential were to increase to 2 percent per annum, keeping the spot rate unchanged, the forward rate would be 1.2624.

A change of 125 points in the spot rate has changed the forward rate by 126 points, whereas a change of 0.8 percent in the interest-rate differential has changed the rate by 50 points.

Consider a foreign-exchange swap:

$$\begin{aligned} F - S &= S \times \frac{(r_d - r_f)}{(1 + r_f)} \\ &= 1.2500 \times \frac{(0.02 - 0.014)}{1.014} = 0.0074. \end{aligned}$$

If the spot rate increases to 1.2625, then F–S = 0.0075. So the swap price changes by one point. However, if the interest rate for euros were to decline to 2 percent per annum, then

$$F - S = 1.2500 \times \frac{(0.02 - 0.01)}{1.01} = 0.0124$$

which corresponds to a change of 50 points. The swap price is sensitive to interest-rate changes but is relatively insensitive to changes in the spot rate. An outright forward contract, on the other hand, is sensitive to both parameters.

Interpretation of the Swap Points

Consider the following quote in the foreign-exchange market in NYC on a given day:

Spot: 1.2500−1.2625 EUR−USD

One-month forward: 50/90

The first figure (the swap points on the left) represents a transaction in which the quoting bank is selling the base currency spot and buying it back one month forward. In this illustration, the quoting bank is prepared to offer a premium of 50 points. The second figure (the swap points on the right) represents a transaction in which the quoting bank is buying the base currency spot and selling the same one month forward. In this illustration, the quoting bank is charging a premium of 90 points. If we term the spot transaction as the *near-date transaction* and the forward contract as the *far-date transaction*, then the swap points on the left are for a transaction that entails the sale of the base currency on the near-date and its acquisition on the far-date, and the swap points on the right are for a transaction that entails the purchase of the base currency on the near-date and its subsequent sale on the far-date.[3] This statement should, of course, be perceived from the standpoint of the quoting bank.

A Clarification

There is a derivative product known as a *currency swap*. It requires two parties to swap interest-rate payments in the two chosen currencies periodically, based on prespecified benchmarks. The principal amounts on the basis of which the interest flows are calculated are always exchanged at the end of the swap. However, there may or may not be an exchange of principal at inception. Example 8.12 illustrates this.

Example 8.12

Commonwealth Bank and Barclays agree to on a swap wherein Commonwealth will deliver AU$1 million to Barclays at the outset in exchange for £500,000. The exchange is based on the prevailing spot rate of 0.5000 AUD−GBP. As per the agreement Commonwealth agrees to make payments in British pounds every six months at the LIBOR prevailing at the start of the interest-computation period, while Barclays agrees to make payments in Australian dollars at the same frequency at a fixed rate of 8 percent per annum. The swap has a maturity of five years at the end of which the two banks will swap back the currencies based on the original spot rate.

Short-Date Contracts

A short-date transaction may be defined as one for which the value date is before the swap value date. There are two possibilities—*value today* and *value tomorrow*—where *tomorrow* refers to the next business day. In the foreign-exchange markets, three types of one-day swaps are quoted:

1. Between today and tomorrow, which is referred to as *overnight* or *O/N*;
2. Between tomorrow and the next day, which is referred to as *tom/next* or *T/N*; and
3. Between the spot date and the next day, which is referred to as *spot/next* or *S/N*.

Let us first consider a value tomorrow transaction. An outright sale for a bank, with value tomorrow, may be viewed as a combination of the following transactions: spot sale of euros accompanied by a swap that comprises of sale of euros for tomorrow accompanied by a spot purchase of euros.

Let us assume that the tom/next swap points are given as 25/15. The question is, how do we deal with these points? Do we deal with them in the same way as we do for normal forward contracts or is there a difference? Before we proceed, let us reconsider a one-month forward contract from the standpoint of the quoting bank.

An outright sale one month forward may be viewed as a spot sale accompanied by a swap that entails the spot purchase and forward sale of the base currency. Assume that the swap points are given as a/b. The first number represents a transaction for selling spot and buying forward, and the second number represents a transaction for buying spot and selling forward. Thus, $a < b$ implies that the foreign currency is at a forward premium, and we will add the numbers to the respective sides with the rationale that if the currency is at a forward premium, then the premium the bank will offer when it buys forward will be less than what it will charge when it sells forward. On the other hand, $a > b$ implies that the foreign currency is a forward discount and that we should subtract the numbers from the respective sides with the rationale that if the currency is at a forward discount, then the discount the bank will apply when it buys will be larger than what it will offer when it sells. As explained earlier, the swap points on the left are for selling the base currency on the near date and buying it on the far date, whereas the points on the right are for buying the base currency on the near date and selling it on the far date.

Consider a value tomorrow sale transaction. As we have seen, it may be viewed as a combination of the spot sale of euros accompanied by a swap that entails the spot purchase of euros with the sale of euros for tomorrow. The difference between this swap and the swap corresponding to a one-month outright forward sale is that this transaction is for the purchase of euros on the far date, whereas in the earlier case it was for the sale of euros on the far date. In this case, if the swap points are given as a/b, then the points applicable for an outright sale with value tomorrow are the points on the left side, whereas those applicable for an outright purchase are those on the right side. Remember that the transaction scheduled for tomorrow is the near-date leg, and the spot transaction represents the far-date leg.

Consider a number like 25/15. The implications are that when the bank is buying for value tomorrow it will give a premium of 15 points, whereas when it is selling for value tomorrow it will charge a premium of 25 points. The numbers cannot represent a discount because the bank will not offer a larger discount when it sells as compared to what it levies when it buys. On the other hand, what if the points had been given as 15/20? It implies that the bank will apply a discount of 20 points when it buys euros for value tomorrow; for sale transactions with value tomorrow, it will offer a discount of only 15 points.

As we can surmise from the preceding arguments, the way to deal in this case with the swap points is to reverse them and then proceed as before. In other words, for a value tomorrow transaction, if the tom/next swap points are given as a/b, we first invert them to obtain b/a. Then if it connotes a smaller number followed by a larger number, we will add the points to the respective sides. However, if it is a larger number followed by a smaller number, we subtract the points from the respective sides. As Steiner points out, the inversion of the swap points is purely from the standpoint of computation. The swap points themselves will not be quoted after inversion.

Example 8.13 demonstrates the computation of rates for a short-date contract.

Example 8.13

The following quotes are observed in NYC on a given day:

Spot: 1.2500–1.2600 EUR–USD
T/N: 15/8

If we reverse the swap points, it represents a small number/large number. The base currency is at a forward premium. The rates for a transaction with value tomorrow are

$$1.2508 - 1.2615 \text{ EUR} - \text{USD}.$$

Consider the procedure for arriving at a quote for a value today sale transaction. This can be viewed as a combination of three transactions:

1. Sale of euros spot;
2. A swap entailing a sale of euros for value tomorrow accompanied with a spot purchase of euros; and
3. A swap entailing a sale of euros for value today accompanied with a purchase of euros for value tomorrow.

The computation of the outright value today rates may be best illustrated with the help Example 8.14.

Example 8.14

Consider the following quotes:

Spot: 1.2500–1.2625 EUR–USD
O/N: 15/8
T/N: 12/5

The rates for value today would be computed as follows:

Bid: 1.2500 + 0.0008 + 0.0005 = 1.2513 EUR–USD
Ask: 1.2625 + 0.0015 + 0.0012 = 1.2652 EUR–USD

Swap points for short-duration transactions such as O/N and T/N may be very small. Often they may be less than a point and will be expressed either in fractions or as decimals. Consider the following quote for an O/N swap: $2\frac{1}{4}$ /1 $\frac{3}{4}$.

It implies a quote of 2.25/1.75 points. At times, the quote may be expressed directly in decimals—that is, as 2.25/1.75. In either case, care should be exercised when they are being added to or subtracted from the spot rates.

Option Forwards

A client who is negotiating a forward contract may be uncertain at times about the exact date on which she will have to pay or receive the foreign currency. In such cases, she has the freedom to negotiate a forward contract with the flexibility to transact within a specified time period. An exporter based in Philadelphia may be of the opinion that she will receive a payment in euros some time between two to three months from the date of negotiation of the forward contract. In such a situation, she can negotiate a forward contract with an option to complete the contract on any date during the stated period. The option writer in such cases is the foreign-exchange dealer, who is typically a bank. The dealer will quote a rate for the option forward in such cases under the assumption that the contract will be completed on the worst possible day from her point of view. The implications of this would depend on whether the dealer is buying or selling the foreign currency and whether the currency is quoting at a premium or at a discount. We will illustrate various situations with the help of Examples 8.15, 8.16, 8.17, 8.18, 8.19, and 8.20.

Example 8.15

Raytheon is importing goods from Zurich and is required to make a payment in euros. The company is of the opinion that the payment will have to be made sometime between one to two months from today, although it is unsure about the exact date. So it seeks to negotiate a forward contract with an option to take delivery of the euros at any point in time after one month but before two months.

Assume that the following rates are prevailing in the interbank market:

Spot: 1.2225–1.2350 EUR–USD

One-month forward: 30/65

Two-month forward: 45/95

Three-month forward: 60/120

The euro is at a forward premium. The relevant exchange rate for the bank is the ask rate, because it is selling foreign currency to Raytheon. If the bank assumes that the contract will be completed after one month, it should charge a premium of 65 points; if it assumes that the currency will have to be sold after two months, it should charge a premium of 95 points. Because the bank is charging a premium, it will charge the higher of the two values. The rate quoted by the bank will be $1.2350 + 0.0095 = 1.2445$ EUR–USD.

Example 8.16

Let us continue with the previous example but with a different set of data. Assume that the spot rates are the same but that the euro is at a forward discount and that the swap points are as follows:

One-month forward: 65/30

Two-month forward: 95/45

Three-month forward: 120/60

In this case, if the bank assumes that the deal will be completed after one month, it should offer a discount of 30 points to the client. If it assumes that the currency will have to be sold after two months, it should offer a discount of 45 points. Because the bank is offering a discount, it will offer the lower of the two values. So, in this case, the rate quoted by the bank will be $1.2350 - 0.0030 = 1.2320$ EUR–USD.

Example 8.17

Again take the case of Raytheon and the bank. We will assume that the spot rate remains the same but that the swap points for the various maturities are as follows:

One-month forward: 30/65

Two-month forward: 95/45

Three-month forward: 120/60

The euro is trading at a premium for a one-month transaction but at a discount for a two-month transaction. If the transaction is assumed to be completed after one month, then the dealer will have to charge a premium of 65 points. If he assumes instead that the transaction will take place two months after, then he has to offer a discount of 45 points. Because he is selling foreign currency, he will charge the premium of 65 points. The rate quoted by the bank will be 1.2350 + 0.0065 + 1.2415 EUR−USD.

Example 8.18

Purovita has exported pet food to the United Kingdom and expects to be paid in pound sterling sometime between one and two months from today. The current rates in the interbank market are as follows:

Spot: 1.7500−1.7625 GBP−USD

One-month forward: 35/75

Two-month forward: 55/105

Three-month forward: 80/150

The pound is trading at a forward premium. In this case, the bank will be acquiring the foreign currency from the client, so the applicable rate is the spot bid of 1.7500. If the bank assumes that the transaction will be completed after one month, it should offer a premium of 35 points; if it assumes that the transaction will be completed after two months, it should offer 55 points. Because it is acquiring the foreign currency, the bank will offer the lower of the two values. The rate at which the bank will buy will be $1.7500 + 0.0035 = 1.7535$ GBP−USD.

Example 8.19

Take the case of Purovita, which is expecting to be paid in pounds in between one and two months. Assume that the spot rates are the same as earlier but that the swap points are as follows:

One-month forward: 75/35

Two-month forward: 105/55

Three-month forward: 150/80

The pound is trading at a forward discount. If the bank assumes that the trade will take place after one month, it should offer a discount of 75 points; and if it assumes that the deal will be completed after two months, it should offer 105 points. Because the bank is buying the foreign currency, it will offer the higher discount of 105 points. The applicable rate will be $1.7500 - 0.0105 = 1.7395$ GBP−USD.

Example 8.20

Let us once again take the case of Purovita and the bank. We will assume that the spot rate remains the same but that the swap points for the various maturities are as follows:

One-month forward: 65/30

Two-month forward: 45/95

Three-month forward: 60/120

The pound is at a discount for a one-month contract but at a premium for a two-month contract. If the bank assumes that the transaction will take place after one month, it has to offer a discount of 65 points. If it assumes that the deal will be consummated after two months, it has to offer a premium of 45 points. Because the bank is buying the foreign currency, it will offer a discount of 65 points. The applicable rate will be $1.7500 - 0.0065 = 1.7435$ GBP−USD.

Nondeliverable Forwards

A traditional forward contract entails the acquisition of a foreign currency if a long position is taken or the delivery of a foreign currency if a short position is taken. In the case of a nondeliverable forward contract, there is no need to acquire or deliver a foreign currency. Instead, the two parties simply settle the profit or loss with respect to the prevailing spot rate two business days before the maturity date of the forward contract.

Examples 8.21 and 8.22 illustrate the mechanics of a non-deliverable forward contract.

Example 8.21

On March 15, 20XX, Axis Bank and HSBC agree to a nondeliverable forward contract wherein Axis Bank will acquire US$1 million three months forward at a rate of 42.2500 USD−INR. On June 15, the two banks agree to settle the difference at a rate of 43.4000 USD−INR. In this case, the dollar has appreciated with respect to the Indian rupee. Axis Bank will be entitled to receive

$$1{,}000{,}000(43.4000 - 42.2500) = 1{,}150{,}000 \text{ INR.}$$

Nondeliverable forwards can be offset before maturity. In this case the profit or loss may be received/paid either at the original date of maturity of the contract or on the day of offsetting.

Futures Markets

FOREX futures trading is active on the CME Group. The exchange currently offers futures contracts on the currencies listed in Table 8.2. The contract details are given in Table 8.3.

Example 8.22

On March 15, 20XX, Axis Bank and HSBC agree to a nondeliverable forward contract wherein Axis Bank agrees to acquire US$1 million three months forward at a rate of 42.2500 USD−INR. On May 21, Axis Bank decides to offset the contract with another nondeliverable forward contract with the same maturity date. Assume that the rate on May 21 is 43.1250 USD−INR.

The original contract would have been settled on June 17. So one possibility is that the profit or loss, which in this case is $1{,}000{,}000 \times (43.1250 - 42.2500) = 875{,}000$ INR, is paid to Axis Bank by HSBC on June 17.

The other possibility is that the present value of this amount is paid to Axis Bank on May 21 itself. Between May 21 and June 17 there are 27 days. Assume that the interest rate in the Indian market is 8 percent per annum. If so, the present value of this amount as computed on May 21 is

$$\frac{875{,}000}{\left(1 + 0.08 \times \dfrac{27}{360}\right)} = 869{,}781.31 \text{ INR.}$$

This amount can be paid by HSBC if the deal were to be settled on May 21 itself.

The basic unit of a price is a point. The point description denotes the U.S. dollar equivalent of one point, whereas the tick size connotes the minimum observable fluctuation in the value of a contract.

E-Mini and E-Micro Contracts

The CME Group offers contracts on certain assets with a smaller contract size. These contracts, called *E-mini contracts*, are currently available for the euro and the Japanese yen. The contract size is €62,500 for the futures contract on the euro and ¥6,250,000 for the contract on the Japanese yen.

Contracts with very small sizes are referred to as *E-micro contracts*. These are currently available for six currency pairs, details of which are given in Table 8.4.

Table 8.2
FOREX Futures on the CME: The Underlying Currencies

Australian Dollar	Brazilian Real
British Pound	Canadian Dollar
Chinese Renminbi	Czech Koruna
Euro	Hungarian Forint
Israeli Shekel	Japanese Yen
Korean Won	Mexican Peso
New Zealand Dollar	Norwegian Krone
Polish Zloty	Russian Ruble
South African Rand	Swedish Krona
Swiss Franc	Turkish Lira

Table 8.3
FOREX Futures Contract Details

Currency	Contract Size	Symbol	Point Description	Tick Size
Australian Dollar	100,000	AUD	$0.0001	$10.00
Brazilian Real	100,000	BRL	$0.0001	$ 5.00
British Pound	62,500	GBP	$0.0001	$ 6.25
Canadian Dollar	100,000	CAD	$0.0001	$10.00
Chinese Renminbi	1,000,000	CNY	$0.00001	$10.00
Czech Koruna	4,000,000	CZK	$0.000001	$ 8.00
Euro	125,000	EUR	$0.0001	$12.50
Hungarian Forint	30,000,000	HUF	$0.0000001	$ 6.00
Israeli Shekel	1,000,000	ILS	$0.00001	$10
Japanese Yen	12,500,000	JPY	$0.000001	$12.50
Korean Won	125,000,000	KRW	$0.0000001	$12.50
Mexican Peso	500,000	MXN	$0.00001	$12.50
New Zealand Dollar	100,000	NZD	$0.0001	$10.00
Norwegian Krone	2,000,000	NOK	$0.00001	$20
Polish Zloty	500,000	PLN	$0.00001	$10.00
Russian Ruble	2,500,000	RUB	$0.00001	$25
South African Rand	500,000	ZAR	$0.00001	$12.50
Swedish Krona	2,000,000	SEK	$0.00001	$20
Swiss Franc	125,000	CHF	$0.0001	$12.50
Turkish Lira	200,000 USD	TRY	0.0001 TRY	20 TRY

Table 8.4
E-Micro Futures Contract Details

Currency-Pair	Contract Size	Tick Size
AUD/USD	10,000 AUD	1.00 USD
EUR/USD	12,500 EUR	1.25 USD
GBP/USD	6,250 GBP	0.625 USD
USD/CAD	10,000 USD	1.00 CAD
USD/CHF	10,000 USD	1.00 CHF
USD/JPY	10,000 USD	100 JPY

Cross Rate Futures

In addition to contracts on foreign currencies with respect to the U.S. dollar, the CME Group offers futures contracts based on the exchange rates of two foreign or non-American currencies with respect to each other. These are known as cross-rate futures contracts. The contract specifications are given in Table 8.5.

Hedging Using Currency Futures

Futures contracts on foreign currencies may be used by traders to hedge their risks in the spot market. A party who knows it will be selling a foreign currency in the future will be worried that the currency may depreciate or, in other words, that the price of the currency in terms of the domestic currency may fall. It will hedge by going short in futures contracts.

On the other hand, a party that knows it will be acquiring a foreign currency in the future will be concerned that the currency may appreciate, which would mean that the price of that currency in terms of the domestic currency may rise. Such parties will seek to hedge by going long in futures contracts.

Examples 8.23 and 8.24 illustrate buying and selling hedges.

Table 8.5
Cross-Rate Contract Details

Cross Rate	Contract Size	Point Description	Tick Size
Australian Dollar/ Canadian Dollar	200,000 AUD	0.0001 CAD/AUD	20.00 CAD
Australian Dollar/ Japanese Yen	200,000 AUD	0.01 JPY/AUD	2,000 JPY
Australian Dollar/New Zealand Dollar	200,000 AUD	0.0001 NZD/AUD	20.00 NZD
British Pound/Japanese Yen	125,000 GBP	0.01 JPY/GBP	1,250.00 JPY
British Pound/Swiss Franc	125,000 GBP	0.0001 CHF/GBP	12.50 CHF
Canadian Dollar/ Japanese Yen	200,000 CAD	0.01 JPY/CAD	2,000 JPY
Chinese Renminbi/ Euro	1,000,000 CNY	0.00001 EUR/CNY	10.00 EUR
Chinese Renminbi/ Japanese Yen	1,000,000 CNY	0.001 JPY/CNY	1,000 JPY
Euro/Australian Dollar	125,000 EUR	0.0001 AUD/EUR	12.50 AUD
Euro/British Pound	125,000 EUR	0.0001 GBP/EUR	6.25 GBP
Euro/Canadian Dollar	125,000 EUR	0.0001 CAD/EUR	12.50 CAD
Euro/Czech Koruna	4,000,000 CZK	0.000001 EUR/CZK	8.00 EUR
Euro/Hungarian Forint	30,000,000 HUF	0.0000001 EUR/HUF	6.00 EUR
Euro/Japanese Yen	125,000 EUR	0.01 JPY/EUR	1,250.00 JPY
Euro/Norwegian Krone	125,000 EUR	0.001 NOK/EUR	62.50 NOK
Euro/Polish Zloty	500,000 PLN	0.00001 EUR/PLN	10.00 EUR
Euro/Swedish Krona	125,000 EUR	0.001 SEK/EUR	62.50 SEK
Euro/Swiss Franc	125,000 EUR	0.0001 CHF/EUR	12.50 CHF
Euro/Turkish Lira	125,000 EUR	0.0001 TRY/EUR	12.50 TRY
Swiss Franc/Japanese Yen	250,000 CHF	0.01 JPY/CHF	1,250.00 JPY

Example 8.23 (A Selling Hedge)

Purovita has exported pet food to South Africa and is scheduled to receive 7.5 million South African rand (ZAR) after 75 days. It decides to hedge the dollar value of its receivable by going short in ZAR futures. On the CME Group, each futures contract on the rand is for 500,000 ZAR. The company needs to go short in 15 contracts.

Assume that the rates on January 15, the day on which the hedge is initiated, are as follows:

Spot: 0.1335–0.1415 ZAR–USD

Three-month futures: 0.1370–0.1490 USD

Because the company needs to go short in futures, the applicable rate is the bid rate of 0.1370. The futures contract is scheduled to expire on April 15.

Assume that on April 1 the rates in the market are as follows:

Spot: 0.1220–0.1275 ZAR–USD

April futures: 0.1235–0.1310 ZAR–USD

If the company had not hedged, it would have had to sell the 7.5 million rand for

$$7{,}500{,}000 \times 0.1220 = \$915{,}000.$$

However, because it has hedged, the proceeds from the spot market will be augmented by the profit or loss from the futures market. In this case, the April contracts will have to be offset at the ask rate of 0.1310 ZAR–USD. The cash flow from the market is

$$15 \times 500{,}000 \times (0.1370 - 0.1310) = \$45{,}000.$$

The total cash inflow in dollars is $915{,}000 + 45{,}000 = \ 960{,}000$. The effective exchange rate is therefore

$$\frac{960{,}000}{7{,}500{,}000} = 0.1280 \text{ ZAR–USD}.$$

Example 8.24 (A Buying Hedge)

Edgar Royce has imported books from the United Kingdom and is due to pay £1.25 million after 75 days. Each futures contract on the pound is for £62,500. Because the company will be buying the foreign currency, it needs to go long in 20 futures contracts.

Assume that the rates on January 15, the day on which the hedge is initiated, are as follows:

Spot: 1.7500–1.7525 GBP–USD

Three-month futures: 1.7620–1.7665 GBP–USD

Because the company needs to go long in futures, the applicable rate is the ask rate of 1.7665. The futures contract is scheduled to expire on April 15.

Assume that on April 1 the rates in the market are as follows:

Spot: 1.7725–1.7750 GBP–USD

April futures: 1.7775–1.7820 GBP–USD

If the company had not hedged, it would have had to buy £1.25 million for

$$1{,}250{,}000 \times 1.7750 = \$2{,}218{,}750.$$

However because it has hedged, the cash flow in the spot market will be accompanied by the profit or loss from the futures market, which will have implications for the effective rate of exchange. In this case the April contracts will have to be offset at the bid rate of 1.7775 GBP–USD. The cash flow from the futures market is

$$20 \times 62{,}500 \times (1.7775 - 1.7665) = \$13{,}750.$$

The total cash outflow in dollars is 2,218,750 − 13,750 = \$2,205,000. The effective exchange rate is therefore

$$\frac{2{,}205{,}000}{1{,}250{,}000} = 1.7640 \text{ GBP–USD}.$$

Exchange-Traded Foreign Currency Options

NASDAQ OMX PHLX offers options contracts on a number of foreign currencies, with settlement in U.S. dollars. The currencies on which contracts are available are given in Table 8.6 and the contract sizes are given in Table 8.7.

Table 8.6
Underlying Assets for FOREX Options Contracts

Australian Dollar (AUD)
British Pound (GBP)
Canadian Dollar (CAD)
Euro (EUR)10
Japanese Yen (JPY)
Mexican Peso (MXN)
New Zealand Dollar (NZD)
Norwegian Krone (NOK)
South African Rand (ZAR)
Swedish Krona (SEK)
Swiss Francs (CHF)

Table 8.7
Contract Sizes for FOREX Options Contracts

Currency	**Contract Size**
AUD	10,000
GBP	10,000
CAD	10,000
EUR	10,000
JPY	1,000,000
MXN	100,000
NZD	10,000
NOK	100,000
ZAR	100,000
SEK	100,000
CHF	10,000

The options are European in nature, and contracts expire on the Saturday following the third Friday of the expiration month. At any point in time, four months from the March cycle plus two near-term months are available—that is, there is a choice of six contract months at any point in time. Assume that we are on September 1, 20XX. Contracts expiring in September, October, November, and December of the year will be available, as will contracts expiring in March and June of the following year.

Option premiums are quoted in points, with one point equivalent to \$100. On September 15, 2010, the Canadian dollar (CAD) was quoting at 0.9700 CAD−USD. A call options contract with a strike price of 80 cents was quoting at 17.25. The premium for the contract was US\$1,725.

Examples 8.25 and 8.26 illustrate the use of options from a speculative standpoint.

Example 8.25 (Speculating with FOREX Options)

The Canadian dollar is quoting at 0.9700 CAD–USD in the spot market. A call option with $X = 90$ and 3 months to expiration is quoting at 8.50. So the premium per contract is US\$850. Mitch decides to go long in 100 call options contracts because he expects the Canadian dollar to appreciate.

Assume that three months after the Canadian dollar is quoting at 0.9975 CAD–USD. Because the contract is in the money, it will be exercised and the payoff will be

$$(0.9975 - 0.9000) \times 10{,}000 \times 100 = \text{US\$97,500}.$$

Because the premium for 100 contracts was US\$85,000, the profit for the option holder is US\$12,500.

Example 8.26

The Canadian dollar is quoting at 0.9700 CAD–USD. A put option with $X = 102.50$ and 3 months to expiration is quoting at 6.25. The premium per contract is US\$625. Janice decides to go short in 100 put-option contracts because she expects the Canadian dollar to depreciate.

Assume that three months after the Canadian dollar is quoting at 0.8750 CAD–USD. The contract is in the money and will be exercised. The payoff will be

$$(1.0250 - 0.8750) \times 10{,}000 \times 100 = \text{US\$150,000}.$$

Because the premium for 100 contracts was US\$62,500, the profit is US\$87,500.

The Garman-Kohlhagen Model

The Garman-Kohlhagen model is a variation of the Black-Scholes model that was developed to price European options on foreign currencies. In addition to the usual parameters required to calculate an option premium using the Black-Scholes model, the Garman-Kohlhagen model requires us to specify the riskless rate of interest in the foreign country, r_f. Per the model, the price of a European call option is given by

$$C_E = S_t e^{-r_f(T-t)} N(d_1) - Xe^{-r(T-t)} N(d_2)$$

where

$$d_1 = \frac{ln\left(\frac{S_t}{X}\right) + \left(r - r_f + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

and $d_2 = d_1 - \sigma\sqrt{T-t}$.

The price of a European put is given by

$$P_E = Xe^{-r(T-t)} N(-d_2) - S_t e^{-r_f(T-t)} N(-d_1).$$

Example 8.27 illustrates the application of the Garman-Kohlhagen model.

Example 8.27

The spot rate for euros in New York is 1.3000 EUR–USD. The riskless rate in the United States is 3 percent per annum, and the rate in the EU is 3.75 percent per annum. The volatility of the exchange rate is 20 percent. Consider European call options with an exercise price of 120 and four months to expiration.

$$\begin{aligned} d_1 &= \frac{ln\left(\frac{130}{120}\right) + \left(0.03 - 0.0375 + \frac{(0.20)^2}{2}\right) \times \frac{1}{3}}{0.20\sqrt{\frac{1}{3}}} \\ &= 0.7293 \\ d_2 &= 0.6138 \\ N(d_1) &= 0.7671 \text{ and } N(d_2) = 0.7303 \\ C_E &= 130e^{-\frac{0.0375}{3}} \times 0.7671 - 120e^{-\frac{0.03}{3}} \times 0.7303 = 11.7202. \end{aligned}$$

The premium per contract is US$1,172.

Put-Call Parity

The put-call parity relationship for foreign currency options may be expressed as

$$C_{E,t} - P_{E,t} = S_t e^{-r_f(T-t)} - Xe^{-r(T-t)}.$$

The Binomial Model

The binomial model can be used to value options on foreign currencies. The only difference is with respect to the definition of the risk-neutral probability p. For currency options, the probability is defined as

$$p = \frac{\frac{r}{r_f} - d}{u - d}.$$

Example 8.28 demonstrates the use of the binomial model to value a currency option.

Example 8.28

Consider a call option on the euro. The current exchange rate is 1.3500 EUR–USD. Every period, the exchange rate may go up by 20 percent or down by 20 percent. The riskless rate in the United States is 4 percent, and the corresponding rate in the EU is 5 percent. The exercise price of the option is 125, and there are two periods to expiration.

The tree for the evolution of the exchange rate is depicted in Figure 8.1.

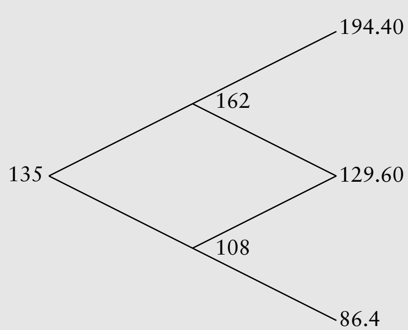


Figure 8.1 The Evolution of the Exchange Rate

$$C_{uu,T} = 69.40,\ C_{ud,T} = 4.60,\ C_{dd,T} = 0$$

$$p = \frac{\frac{1.04}{1.05} - 0.8}{0.4} = 0.4762 \text{ and } (1-p) = 0.5238$$

$$C_{u,T-1} = \frac{0.4762 \times 69.4 + 0.5238 \times 4.60}{1.04} = 34.094.$$

The intrinsic value is 37, so we will use the intrinsic value for proceeding backward:

$$C_{d,T-1} = \frac{0.4762 \times 4.60 + 0.5238 \times 0}{1.04} = 2.1063.$$

The intrinsic value is zero. The model value at time $T-2$ is given by

$$C_{T-2} = \frac{0.4762 \times 37 + 0.5238 \times 2.1063}{1.04} = 18.0026.$$

The intrinsic value is 135 – 125 = 10. The option premium is 18.0026, or US$1,800.26, per contract.

Endnotes

1. See Apte (2006).
2. See Steiner (2002).
3. You will understand the import of this better when we study short-date contracts.

CHAPTER 9

Mortgages and Mortgage-Backed Securities

Introduction

In most developed countries, the right to own a home is largely considered a fundamental right. However, few people are in a position to pay the cost of buying a home from their own personal funds, so most of the money required to buy a home must be borrowed. A loan that is collateralized by real estate property is called a *mortgage loan*. The lender is called the *mortgagee*, and the borrower is called the *mortgagor*. In the event that the mortgagor defaults on the loan, the lender can seize the property and sell it to recover what is due him. This is called the *right of foreclosure*.

Market Participants

There are three categories of players in the mortgage market: (1) mortgage originators; (2) mortgage servicers; and (3) mortgage insurers.

Mortgage Originator

Who is a mortgage originator? The original lender or the party who first extends a loan to the acquirer of the property is called the *mortgage originator*. Originators include:

- Thrifts or savings-and-loan associations;
- Commercial banks;

- Mortgage bankers;
- Life insurance companies; and
- Pension funds.

Income for the Originator

The originator gets income from various sources. First, when a loan is granted, the originator will levy an origination fee. The fee is expressed in terms of points, with each point representing 1 percent of the borrowed funds. For example, if a lender charges 1.5 points on a loan of $200,000, the origination fee will amount to $3,000. Most originators will also levy an application fee and certain processing fees.

The second source of income is the profit that can be earned if and when the mortgage loan is sold by the originator to another party. A mortgage loan is a type of debt, so it is vulnerable to interest-rate fluctuations. If the interest rate declines, such a sale leads to a gain for the originator. This is called a *secondary marketing profit*. Of course, if interest rates rise after the loan is disbursed, the originator will incur a loss when the loan is sold.

If the originator decides to hold the loan as an asset rather than sell it, periodic income will be earned in the form of interest.

Mortgage Servicers

Every mortgage loan must be serviced. Mortgage loan servicing includes the following:

- Collecting monthly payments and forwarding the proceeds to the owner of the loan;
- Sending payment notices to mortgagors;
- Reminding mortgagors whose payments are overdue;
- Maintaining records of principal balances;
- Administering escrow balances for real estate taxes and insurance purposes;
- Initiating foreclosure proceedings if necessary; and
- Furnishing tax information to mortgagors when applicable.

Mortgage servicers include bank-related entities, thrift-related entities, and mortgage bankers.

Escrow Accounts

An escrow account is a trust account held in the name of the home owner to pay statutory levies such as property taxes, as well as insurance premiums. The maintenance of such an account helps to ensure that payments are made when due, because it becomes the lender's responsibility to do so.

In most cases, the borrower makes deposits on a monthly basis, along with the loan payment, and the payments accrue at the lender. The amount required to be deposited every month is a function of the cost of insurance and the tax assessment of the property concerned. It fluctuates from year to year.

Income for the Servicer

The primary source of income is the servicing fee, which is a fixed percentage of the outstanding mortgage balance. Because the mortgage loan is an amortized loan, where the outstanding principal declines with each payment, the revenue from servicing, in dollar terms, will decline over time. The second source of income arises on account of the interest that can be earned by the servicer from the escrow account that the borrower often maintains with the servicer. The third source of revenue is the float earned on the monthly mortgage payment. This opportunity arises because there is a delay that is permitted between the time the servicer receives the monthly payment and the time that the payment has to be sent to the investor.

Mortgage Insurers

There are two types of mortgage-related insurance. The first type, *mortgage insurance* or *private mortgage insurance*, is originated by the lender to insure against default by the borrower. It is usually required by lenders on loans with a *loan-to-value* (LTV) *ratio* that is greater than 80 percent. The amount insured will be some percentage of the loan, and it may decline as the LTV declines.

What is the LTV ratio? Every borrower will have to make a down payment, which is the difference between the price of the property and the loan amount. The LTV ratio is obtained by dividing the loan amount by the market value of the property. The lower the LTV ratio, the greater the protection for the lender in the event of default by the borrower.

The second type of mortgage-related insurance—typically called *credit life*—is acquired by the borrower, usually through a life insurance company. This is not required by the lender. Such policies provide for the continuation of mortgage payments after the death of the insured person so that survivors can continue to live in the house.

Government Insurance and Private Mortgage Insurance

As we have seen, lenders insist on insurance if the LTV ratio is more than 80 percent. However, low- to middle-income borrowers may not be in a position to deposit 20 percent of the value of the property. At the same time, their credit rating makes them ineligible to obtain insurance from private parties. To facilitate the acquisition of properties by this strata of society, government agencies are available that provide loan guarantees in lieu of insurance. These agencies include:

- The Federal Housing Administration (FHA);
- The Department of Veterans' Affairs (VA); and
- The Department of Agriculture's Rural Housing Service (RHS).

Borrowers who are not eligible for loans from these agencies must obtain *private mortgage insurance* (PMI).

Secondary Sales

Once a loan is granted, the originator can either hold it as an asset or sell it to an investor who may hold it as an investment or pool it with many other individual mortgage loans and use them as collateral for the issuance of securities. Of course, the issuance of securities backed by a pool of underlying mortgages may be undertaken by the originator. When a mortgage pool is used as collateral for the issuance of a security, it is said to be *securitized*.

Two federally sponsored credit agencies and many private agencies buy mortgages, pool them, and issue securities backed by the pool to investors. Such agencies are referred to as *conduits*. The two federally sponsored agencies are the Federal National Mortgage Association (Fannie Mae) and the Federal Home Loan Mortgage Corporation (Freddie Mac). Such agencies buy what they call a *conforming mortgage*, or one that meets the underwriting criteria established by the agencies from the standpoint of being eligible to be included in a pool for the purpose of being securitized. A conforming mortgage must satisfy three criteria: (1) PTI ratio, (2) maximum LTV ratio, and (3) maximum loan amount.

The *payment-to-income* (PTI) ratio is the ratio of monthly payments (the loan payment plus any real estate tax payments) to the borrower's monthly income. The lower the ratio, the greater is the likelihood of the borrower being able to pay as scheduled.[1]

Mortgages that are nonconforming because they are for amounts in excess of the purchasing limit set by these agencies are termed *jumbo mortgages*. Those that do not meet the underwriting guidelines such as credit quality or LTV ratio are termed *subprime mortgages*.

Risks in Mortgage Lending

Investors who invest in mortgage loans are exposed to four main risks: (1) default risk; (2) liquidity risk; (3) interest-rate risk; and (4) prepayment risk.

Default Risk

Default or credit risk is the risk that the borrower will default. For government-insured mortgages, the risk is minimal because the insuring agencies are government sponsored. For privately insured mortgages, the risk can be gauged by the credit rating of the insurance company. For uninsured mortgages, the risk will depend on the credit quality of the borrower.

Liquidity Risk

Mortgage loans tend to be rather illiquid because they are large and indivisible. Although an active secondary market exists for such loans, bid–ask spreads are large relative to other debt instruments.

Interest-Rate Risk

The price of a mortgage loan in the secondary market moves inversely with interest rates, just like any other debt security. Moreover, because the loans are for long terms to maturity, the impact of an interest-rate change on the price of the mortgage can be significant.

Prepayment Risk

Most home owners pay off all or a part of their mortgage balance before the maturity date. Payments made in excess of the scheduled principal repayments are called *prepayments*. Prepayments occur for one of several reasons. First, borrowers tend to prepay the entire mortgage when they sell their home. The sale of the house may be the result of a change of employment that necessitates moving, the purchase of a more expensive home, or a divorce in which the settlement requires sale of the marital residence.

Second, if market interest rates decline below the loan rate, the borrower may prepay the loan because of the ability to refinance at a lower rate. Third, in the case of home owners who cannot meet their mortgage obligations, the property will be repossessed and sold. The proceeds from the sale will be used to pay off the mortgage in the case of uninsured mortgages. For an insured mortgage, the insurance company will pay off the balance. Finally, if the property is destroyed by a so-called act of God, the insurance proceeds will be used to pay off the mortgage. The effect of prepayments, regardless of the reason, is that the cash flows from the mortgage become unpredictable.

Example 9.1 presents a detailed illustration of the refinancing of a mortgage loan.

Example 9.1

Cindy Hopkins has taken a mortgage loan for \$800,000. It is a 10-year mortgage with a nominal interest rate of 12 percent per annum. Installments are due every 6 months, and interest is compounded semiannually.

At the end of the first year, just after she has paid the second installment, the interest on housing loans drops to 10.8 percent per annum. The refinancing fee is 1.75 percent of the amount being refinanced. If Cindy's opportunity cost of funds is 9.6 percent per annum, is refinancing an attractive option?

The quantum of the semiannual interest payment is given by

$$\frac{800{,}000 \times 0.06}{\left[1 - \dfrac{1}{(1+0.06)^{20}}\right]} = 69{,}747.65.$$

From the discussion of amortized loans in Chapter 2, we know that the amount outstanding after the second installment is

$$\frac{69{,}747.65}{0.06} \times \left[1 - \frac{1}{(1+0.06)^{18}}\right] = \$755{,}199.90.$$

The cost of refinancing is

$$755{,}199.80 \times 0.0175 = \$13{,}216.$$

The installment amount if the loan is refinanced is

$$\frac{755{,}199.90 \times 0.054}{\left[1 - \frac{1}{(1+0.054)^{18}}\right]} = 66{,}638.91.$$

So the saving every 6 months for 18 periods is

$$69{,}747.65 - 66{,}638.91 = \$3{,}108.74.$$

The present value of the saving is

$$\frac{3{,}108.74}{0.048} \times \left[1 - \frac{1}{(1+0.048)^{18}}\right] = \$36{,}914.46.$$

The net saving is

$$36{,}914.46 - 13{,}216 = \$23{,}698.46.$$

Because the net saving is positive, refinancing is an attractive proposition.

Other Mortgage Structures

The level-payment mortgage, which entails a constant monthly payment for the life of the mortgage, is the simplest of mortgage structures. Other mortgage loans with more complex features are possible. We will discuss some of them.

Adjustable-Rate Mortgage

In an *adjustable-rate mortgage* (ARM), the interest rate is not fixed for the life of the loan but is reset periodically. The monthly payments will rise if the interest rate at the time of resetting is higher than previously, and it will fall if the interest rate declines. The rate on such mortgages is linked to a benchmark such as the LIBOR or the rate on Treasury securities. Example 9.2 illustrates such a mortgage loan, assuming that rates are reset at the end of every year.

Example 9.2

Consider a four-year loan with an initial principal of \$800,000. Payments are to be made monthly, with the interest rate being reset every year. Assume that the benchmark is LIBOR and that the mortgage rate is LIBOR + 150 basis points (bp). Assume that the values of LIBOR observed over the next three years are as shown in Table 9.1.

Table 9.1
Observed Values for LIBOR

Time in Years	LIBOR
0	4.20
1	4.56
2	4.14
3	4.74

The monthly installment for the first year is

$$\frac{800{,}000 \times 0.00475}{\left[1 - \dfrac{1}{(1+0.00475)^{48}}\right]} = 18{,}678.19.$$

The outstanding balance at the end of the first year is

$$\frac{18{,}678.19}{0.00475} \times \left[1 - \frac{1}{(1+0.00475)^{36}}\right] = \$616{,}722.81.$$

The monthly installment for the second year is

$$\frac{616{,}722.81 \times 0.00505}{\left[1 - \dfrac{1}{(1 + 0.00505)^{36}}\right]} = 18{,}778.67.$$

The outstanding balance at the end of the second year is

$$\frac{18{,}778.67}{0.00505} \times \left[1 - \frac{1}{(1 + 0.00505)^{24}}\right] = \$423{,}442.35.$$

The monthly installment for the third year is

$$\frac{423{,}442.35 \times 0.0047}{\left[1 - \dfrac{1}{(1 + 0.0047)^{24}}\right]} = 18{,}698.61.$$

The outstanding balance at the end of the third year is

$$\frac{18{,}698.61}{0.0047} \times \left[1 - \frac{1}{(1 + 0.0047)^{12}}\right] = \$217{,}676.16.$$

The monthly installment for the fourth year is

$$\frac{217{,}676.16 \times 0.0052}{\left[1 - \dfrac{1}{(1 + 0.0052)^{12}}\right]} = 18{,}758.63.$$

The outstanding balance at the end of the fourth year is zero.

Option to Change the Maturity

In the case of certain ARMs, the borrower may be given the option to keep the monthly installment at the initial level and have the maturity date altered every time the rate is reset. Let us see the implications of this for the mortgage discussed in the preceding example.

The outstanding balance after one year is \$616,722.81. If we keep the equated monthly payment at \$18,678.19, the life of the loan if the rate is reset at 9 percent is given by

$$\frac{616{,}722.81 \times 0.0075}{18{,}678.19} = \left[1 - \frac{1}{(1 + 0.0075)^T}\right] \Rightarrow T = 38.08 \text{ months.}$$

Rate Caps

In the case of ARMs, there may be a cap on how much the interest rate may increase or decrease per period. The cap may be a periodic cap or a lifetime cap. Consider a four-year mortgage with a principal of \$800,000. Assume that payments are made monthly and that the interest rate every period is the prevailing LIBOR at the start of the period plus 150 bp. The values of LIBOR observed over the next three years are as shown in Table 9.2.

Assume that there is a periodic cap of 1.5 percent and a lifetime cap of 9 percent. If so, the applicable mortgage rates for each period would be as shown in Table 9.3.

Table 9.2
Observed Values for LIBOR

Time in Years	LIBOR	Mortgage Rate
0	4.20	5.70
1	6.00	7.50
2	7.00	8.50
3	8.00	9.50

Table 9.3
Mortgage Rates in the Presence of a Cap

Period	Mortgage Rate
0	5.70
1	7.20
2	8.50
3	9.00

Let us analyze the entries in Table 9.3. The starting mortgage rate is 5.70 percent, which is less than the lifetime cap of 9 percent. The rate at the end of the first year should be 7.50 percent in the absence of a periodic cap. However, because there is a periodic cap of 1.5 percent, the rate cannot increase or decrease by more than 1.5 percent. The rate for the second year is 7.20 percent, which is the rate for the first year plus 1.5 percent. The uncapped rate for the third year is 8.50 percent, which is the same as the capped rate, because 8.50 percent is only 1.3 percent more than the rate for the previous year. Finally, the rate for the fourth year in the absence of a cap should be 9.50 percent. This does not violate the requirement for a periodic cap, because the change is only 1 percent compared to the previous year. However, because there is a lifetime cap of 9 percent, the rate cannot exceed 9 percent in any period. The rate for the fourth year will be 9 percent.

Payment Caps

If the adjustable-rate mortgage has a payment cap, there will be a limit on how much the monthly payments can increase or decrease at the end of every period. Let us take the case of the mortgage that we studied in the preceding example. The mortgage rate for each year is as shown in Table 9.4.

Assume there is a payment cap of 2 percent. In the absence of the cap, the monthly payments in every year and the outstanding balance at the beginning of each year will be as given in Table 9.5.

However, if there is a payment cap of 2 percent, then the monthly payment in the second year cannot exceed

$$18{,}678.19 \times 1.02 = 19{,}051.75.$$

The interest component in the thirteenth month will be

$$616{,}722.81 \times \frac{0.075}{12} = 616{,}722.81 \times 0.00625 = \$3{,}854.52.$$

Table 9.4
Observed Values for LIBOR

Time in Years	LIBOR	Mortgage Rate
0	4.20	5.70
1	6.00	7.50
2	7.00	8.50
3	8.00	9.50

Table 9.5
Monthly Payments & Outstanding Balances

Time in Years	Monthly Payments	Outstanding Balance
0	18,678.19	616,722.81
1	19,183.91	426,313.07
2	19,378.35	222,178.34
3	19,481.38	0

The principal repayment will therefore be only

$$19{,}051.75 - 3{,}854.52 = \$15{,}197.23$$

as compared to

$$19{,}183.91 - 3{,}854.52 = \$15{,}329.39$$

in the absence of the cap. The cap leads to a slower repayment of principal. The outstanding balance at the end of the twenty-fourth month is given by

$$616{,}722.81 = \frac{19{,}051.75}{0.00625} \times \left[1 - \frac{1}{(1.00625)^{12}}\right] + \frac{X}{(1.00625)^{12}}$$
$$\Rightarrow X = 427{,}954.71$$

as compared to \$426,313.07 in the absence of the cap. The monthly payment in year three will therefore be

$$\frac{427{,}954.71 \times \dfrac{0.085}{12}}{\left[1 - \dfrac{1}{\left(1 + \dfrac{0.085}{12}\right)^{24}}\right]} = \$19{,}452.97.$$

The cap for the period is

$$19{,}051.75 \times 1.02 = \$19{,}432.79.$$

The monthly payment in year three will be \$19,432.79.

The outstanding balance at the end of the thirty-sixth month is given by

$$427{,}954.71 = \frac{19{,}432.79}{\frac{0.085}{12}} \times \left[1 - \frac{1}{\left(1 + \frac{0.085}{12}\right)^{12}}\right] + \frac{X}{\left(1 + \frac{0.085}{12}\right)^{12}}$$

$$\Rightarrow X = \$223{,}285.74.$$

The monthly payment for year four will therefore be

$$\frac{223{,}285.74 \times \frac{0.095}{12}}{\left[1 - \frac{1}{\left(1 + \frac{0.095}{12}\right)^{12}}\right]} = \$19{,}578.48.$$

Negative Amortization

There are times when a situation could arise in which the capped monthly payment is less than the monthly interest that is due for a subsequent month. If so, instead of a reduction in the outstanding balance, the deficit in the interest payment will be added to the outstanding balance, causing the principal to increase in value. This is referred to as *negative amortization*. We will illustrate it with the help of Example 9.3.

Example 9.3

Consider a 30-year ARM with an initial loan amount of \$800,000. The initial mortgage rate is 7.5 percent, and payments are to be made on a monthly basis. There is a payment cap of 5 percent.

The monthly installment for the first year is

$$\frac{800{,}000 \times 0.00625}{\left[1 - \frac{1}{(1 + 0.00625)^{360}}\right]} = 5{,}593.72.$$

The outstanding balance at the end of the first year is

$$\frac{5{,}593.72}{0.00625} \times \left[1 - \frac{1}{(1 + 0.00625)^{348}}\right] = \$792{,}625.88.$$

Assume that the mortgage rate at the end of the year is 9.75 percent. The monthly payments in the second year in the absence of a cap would be

$$\frac{792{,}625.88 \times 0.008125}{\left[1 - \frac{1}{(1 + 0.008125)^{348}}\right]} = 6{,}849.99.$$

However, because of the cap, the payment has to be

$$5{,}593.72 \times 1.05 = 5{,}873.41.$$

The interest due for the thirteenth month is

$$792{,}625.88 \times 0.008125 = 6{,}440.09.$$

There is a deficit of

$$6{,}440.09 - 5{,}873.41 = 566.68.$$

This will be added to the principal. The principal at the end of the thirteenth month will be

$$792{,}625.88 + 566.68 = \$793{,}192.56.$$

The amortization schedule for the first two years is given in Table 9.6. The evidence of negative amortization is obvious.

Table 9.6
Illustration of Negative Amortization

Month	Monthly Installment	Interest Component	Principal Component	Outstanding Balance
0				800000.00
1	5593.72	5000.00	593.72	799406.28
2	5593.72	4996.29	597.43	798808.85
3	5593.72	4992.56	601.16	798207.68
4	5593.72	4988.80	604.92	797602.76
5	5593.72	4985.02	608.70	796994.06

Table 9.6
Continued

Month	Monthly Installment	Interest Component	Principal Component	Outstanding Balance
6	5593.72	4981.21	612.51	796381.55
7	5593.72	4977.38	616.34	795765.22
8	5593.72	4973.53	620.19	795145.03
9	5593.72	4969.66	624.06	794520.97
10	5593.72	4965.76	627.96	793893.00
11	5593.72	4961.83	631.89	793261.11
12	5593.72	4957.88	635.84	792625.28
13	5873.41	6440.08	−566.67	793191.95
14	5873.41	6444.68	−571.27	793763.22
15	5873.41	6449.33	−575.92	794339.14
16	5873.41	6454.01	−580.60	794919.73
17	5873.41	6458.72	−585.31	795505.05
18	5873.41	6463.48	−590.07	796095.11
19	5873.41	6468.27	−594.86	796689.98
20	5873.41	6473.11	−599.70	797289.67
21	5873.41	6477.98	−604.57	797894.24
22	5873.41	6482.89	−609.48	798503.72
23	5873.41	6487.84	−614.43	799118.15
24	5873.41	6492.84	−619.43	799737.58

Graduated-Payment Mortgage

Young home buyers may not have the disposable income required to avail a conventional fixed-rate mortgage. However, many people in this age bracket have the potential to earn substantially more in the coming years. To encourage such parties to take loans, the graduated payment mortgage was designed. Such a mortgage starts with a level of payment that steadily increases every year up to a point, and then remains steady thereafter. We will illustrate it with the help of an Example 9.4.

Example 9.4

Mathew Thomas is being offered a mortgage loan by First National Housing Bank with the following terms. The monthly payment will increase by 5 percent every year for five years and remain constant thereafter. The mortgage rate is 9.75 percent per annum for a loan amount of $800,000. What will the payment schedule look like?

Let us denote the monthly payment for the first year by A. Then

$$800{,}000 = \frac{A}{0.008125}\left[1-\frac{1}{(1.008125)^{12}}\right] + \frac{\frac{A\times 1.05}{0.008125}\left[1-\frac{1}{(1.008125)^{12}}\right]}{(1.008125)^{12}}$$

$$+\frac{\frac{A\times (1.05)^2}{0.008125}\left[1-\frac{1}{(1.008125)^{12}}\right]}{(1.008125)^{24}} + \frac{\frac{A\times (1.05)^3}{0.008125}\left[1-\frac{1}{(1.008125)^{12}}\right]}{(1.008125)^{36}}$$

$$+\frac{\frac{A\times (1.05)^4}{0.008125}\left[1-\frac{1}{(1.008125)^{12}}\right]}{(1.008125)^{48}} + \frac{\frac{A\times (1.05)^5}{0.008125}\left[1-\frac{1}{(1.008125)^{300}}\right]}{(1.008125)^{60}}$$

$$= 11.3896A + 10.8524A + 10.3405A + 9.8527A + 9.3880A + 88.1330A$$
$$= 139.9562A$$
$$\Rightarrow A = 5{,}716.07.$$

The monthly payment values for each year are given in Table 9.7.

Table 9.7
Monthly Payments for a GPM

Year	Monthly Payment
1	5,716.07
2	6,001.87
3	6,301.97
4	6,617.07
5	6,947.92
6-30	7,295.31

The monthly payments for the first two years are given in Table 9.8. In this case, too, there is negative amortization.

Table 9.8
Amortization Schedule for a GPM

Month	Monthly Installment	Interest Component	Principal Component	Outstanding Balance
0				800000
1	5716.07	6500.00	−783.93	800783.93
2	5716.07	6506.37	−790.30	801574.23
3	5716.07	6512.79	−796.72	802370.95
4	5716.07	6519.26	−803.19	803174.14
5	5716.07	6525.79	−809.72	803983.86
6	5716.07	6532.37	−816.30	804800.16
7	5716.07	6539.00	−822.93	805623.09
8	5716.07	6545.69	−829.62	806452.71
9	5716.07	6552.43	−836.36	807289.07
10	5716.07	6559.22	−843.15	808132.22
11	5716.07	6566.07	−850.00	808982.23
12	5716.07	6572.98	−856.91	809839.14
13	6001.87	6579.94	−578.07	810417.21
14	6001.87	6584.64	−582.77	810999.98
15	6001.87	6589.37	−587.50	811587.49
16	6001.87	6594.15	−592.28	812179.76
17	6001.87	6598.96	−597.09	812776.86
18	6001.87	6603.81	−601.94	813378.80
19	6001.87	6608.70	−606.83	813985.63
20	6001.87	6613.63	−611.76	814597.39
21	6001.87	6618.60	−616.73	815214.13
22	6001.87	6623.61	−621.74	815835.87
23	6001.87	6628.67	−626.80	816462.67
24	6001.87	6633.76	−631.89	817094.56

WAC and WAM

All the mortgages that constitute the pool being securitized will not have the same mortgage rate or time to maturity. It is a common practice in the market for securitized assets to take cognizance of the *weighted-average coupon* (WAC) rate and the weighted-average time to maturity.

The WAC of a mortgage pool is determined by weighting the mortgage rate of each loan in the pool by the principal amount outstanding. In other words, the weight for each mortgage rate is the outstanding amount of that mortgage divided by the cumulative outstanding amount of all the mortgages in the pool. Similarly, the *weighted-average maturity* (WAM) is found by weighting the remaining number of months to maturity of each of the loans in the pool by the principal amount outstanding. We illustrate the calculations as follows for a hypothetical pool.

Calculation of WAC and WAM

Assume that a pool consists of 10 mortgage loans. The mortgage rate for each loan and its remaining term to maturity are given in Table 9.9.

The weight to be attached to each of the mortgage rates and terms to maturity, in order to compute the WAC and the WAM, are given in Table 9.10, as are the calculated values of the two variables.

The WAC for the pool is 6.9265 percent, and the WAM is 301.07 months.

Pass-Through Securities

Mortgage loans are extremely illiquid. Besides, the lender faces significant exposure to credit risk as well as prepayment risk, so making a secondary market for whole loans is an extremely difficult proposition. However, it

Table 9.9
Mortgage Loans, Their Rates, and Times to Maturity

S. No.	Principal Outstanding	Mortgage Rate (r_s)	Term to Maturity (T_s)
1	975,000	6.25%	336
2	925,000	6.50%	324
3	900,000	7.25%	300
4	875,000	7.50%	325
5	825,000	7.00%	350
6	800,000	6.75%	280
7	850,000	6.60%	275
8	910,000	6.95%	250
9	810,000	7.15%	265
10	880,000	7.40%	300
Total	8,750,000		

Table 9.10
Calculation of WAC and WAM

S. No.	Weight (w_s)	$r_s \times w_s$	$T_s \times w_s$
1	0.1114	0.6964	37.44
2	0.1057	0.6871	34.25
3	0.1029	0.7457	30.86
4	0.1000	0.7500	32.50
5	0.0943	0.6600	33.00
6	0.0914	0.6171	25.60
7	0.0971	0.6411	26.71
8	0.1040	0.7228	26.00
9	0.0926	0.6619	24.53
10	0.1006	0.7442	30.17
Total	1.0000	6.9265%	301.07

is possible to issue liquid debt securities that are backed by an underlying pool of mortgage loans: the cash flows stemming from the underlying loans are *passed through* to the holders of these debt securities, hence the name. Each holder of a pass-through security is entitled to a pro rata undivided share of each cash flow that emanates from the underlying pool, as when the home owners make monthly payments. Each monthly payment will consist of an interest component, a principal component, and potentially an additional amount on account of prepayment. Any amount that constitutes a prepayment is termed as an *unscheduled principal*, as opposed to the scheduled or expected principal repayment. Example 9.5 illustrates the mechanics of a pass-through.

Example 9.5

Four home loans with a principal of \$100,000 each and 30 years to maturity have been pooled. The mortgage rate for two loans is 7.50 percent per annum, and the rate for the remaining two is 9.00 percent per annum. One hundred securities backed by these loans have been created. These securities have been distributed as follows: 25 to Alex, 50 to Morena, 15 to Mark, and 10 to Davena. These securities are identical in all respects, and the holder of a security is entitled to 1 percent of any cash flow that arises from the underlying loans.

The meaning of the term *undivided* may be understood as follows. Alex owns 25 percent of the securities, which is equivalent to 25 percent of the underlying collateral and amounts to one loan of $100,000. However, by virtue of the fact that he has 25 percent of the securities, it cannot be construed that Alex owns one underlying loan in its entirety. All that we can say is that Alex is entitled to 25 percent of each cash flow that emanates from the underlying pool.

The first month's payment for a loan with a rate of 7.50 percent is $699.21. This consists of $625 of interest and $74.21 of scheduled principal. For the loan with a rate of 9.00 percent, the first month's payment will be $804.62, which is composed of $750 of interest and $54.62 of principal.

The total cash flow from the underlying pool in the first month will be

$$2 \times 699.21 + 2 \times 804.62 = \$3{,}007.66.$$

Each of the 100 mortgage-backed securities that have been issued is entitled to $30.08 in the first month. Because Alex owns 25 securities, he will receive

$$25 \times 30.08 = \$752.00.$$

By the same logic, Morena will receive $1,504.00, Mark will receive $451.20, and Davena will receive $300.80.

Assume that each of the four home owners makes an additional principal payment of $100 at end of the first month. This was not scheduled and consequently is a prepayment. The total cash flow from the underlying loans on account of this prepayment is $400. Each mortgage-backed security will receive $4 more. Because Alex has 25 securities, he will receive $100 extra, which means that his total inflow will be $852.00. By the same logic, Morena will receive $1,704.00, Mark will receive $511.20, and Davena will receive $340.80.

Cash Flows for a Pass-Through

Assume that the underlying pool consists of four loans of $100,000 each with a rate of 9 percent and 360 months to maturity. The WAC will remain constant at 9 percent, and the WAM at any point of time will be equal to the time remaining in the life of the loans. Before we can project the cash flows, we need to consider servicing and guaranteeing fees and consider the important issue of prepayments.

The cash flow that is passed through to investors in a given period will not be equal to the payment received from the underlying pool. The monthly cash flow for a pass-through is less than the monthly cash flow from the underlying loans by an amount equal to the servicing and guaranteeing fees. The coupon rate on the pass-through will be less than the mortgage rate of the underlying pool by an amount equal to the servicing and guaranteeing fees. We will assume a servicing fee of 0.5 percent per annum and a guaranteeing fee of 0.5 percent per annum. In our case, the coupon rate of the pass-through will be 8 percent.

Prepayment Conventions

It is impossible to predict the monthly cash flow stream that a pass-through holder receives with absolute certainty because the timing and amount of principal that is prepaid is subject to considerable uncertainty. Because a pass-through is a debt security, we need to project the cash flows from it in order to determine its value. Such a projection entails an assumption about the prepayment behavior of the mortgage owners over the life of the pool. The prepayment rate that is assumed is called the *prepayment speed*.

Single-Month Mortality Rate

The *single-month mortality rate for a given month t* (SMM_t) is defined as

$$SMM_t = \frac{SP_t - L_t}{SP_t}$$

where SP_t is the scheduled principal outstanding at the end of month t and L_t is the actual principal outstanding at the end of month t.

The actual principal at the end of month t is given by

$$L_t = SP_t[1 - SMM_t].$$

Consider a mortgage with N months to maturity and a monthly interest rate of r. The original loan balance is given by

$$L_0 = \frac{A_1}{r} \times \left[1 - \frac{1}{(1+r)^N}\right]$$

where A_1 is the scheduled monthly installment for month 1. The scheduled principal at the end of the first month is given by

$$\begin{aligned} \mathrm{SP}_1 &= \mathrm{L}_0 - [\mathrm{A}_1 - r \times \mathrm{L}_0] \\ &= \mathrm{L}_0 \times (1 + r) - \mathrm{A}_1 \\ &= \frac{\mathrm{A}_1}{r}\left[1 - \frac{1}{(1+r)^{(N-1)}}\right]. \end{aligned}$$

The actual balance at time 1 is given by

$$\mathrm{L}_1 = \mathrm{SP}_1[1 - \mathrm{SMM}_1]$$

$$\mathrm{L}_1 = \frac{\mathrm{A}_2}{r} \times \left[1 - \frac{1}{(1+r)^{(N-1)}}\right]$$

where A_2 is the scheduled monthly installment for month 2.

$$\Rightarrow \mathrm{SP}_1[1 - \mathrm{SMM}_1] = \frac{\mathrm{A}_2}{r} \times \left[1 - \frac{1}{(1+r)^{(N-1)}}\right]$$

$$\Rightarrow \frac{\mathrm{A}_1}{r}\left[1 - \frac{1}{(1+r)^{(N-1)}}\right][1 - SMM_1] = \frac{\mathrm{A}_2}{r} \times \left[1 - \frac{1}{(1+r)^{(N-1)}}\right]$$

$$\Rightarrow \mathrm{A}_2 = \mathrm{A}_1 \times [1 - \mathrm{SMM}_1].$$

Similarly, it can be shown that

$$\begin{aligned} \mathrm{A}_3 &= \mathrm{A}_2 \times [1 - \mathrm{SMM}_2] \\ &= \mathrm{A}_1 \times [1 - \mathrm{SMM}_1] \times [1 - \mathrm{SMM}_2]. \end{aligned}$$

In general, the scheduled monthly installment in month t is given by

$$\mathrm{A}_t = \mathrm{A}_1 \prod_{i=1}^{t-1} [1 - \mathrm{SMM}_i].$$

If $\mathrm{SMM}_i = \mathrm{SMM}\ \forall\ i$, then

$$\mathrm{A}_t = \mathrm{A}_1 \times [1 - \mathrm{SMM}]^{t-1}.$$

Let us denote the monthly payment on the original principal in the absence of prepayments by A. Then we can state

$$A_t = A \prod_{i=1}^{t-1} [1 - \mathrm{SMM}_i]$$
$$= A \times [1 - \mathrm{SMM}]^{t-1} \text{ if } \mathrm{SMM}_i = \mathrm{SMM} \forall i.$$

In the absence of prepayments, the interest and principal components of the tth payment and the outstanding principal at the end of t periods are given by

$$I_t^* = A \times \left[1 - \frac{1}{(1+\mathrm{r})^{N-t+1}}\right]$$
$$P_t^* = \frac{A}{(1+r)^{N-t+1}}$$
$$SP_t^* = \frac{A}{r}\left[1 - \frac{1}{(1+r)^{N-t}}\right].$$

In the presence of prepayments, the actual principal outstanding at the end of $t-1$ is given by

$$\mathrm{L}_{t-1} = \frac{A_t}{r} \times \left[1 - \frac{1}{(1+r)^{N-t+1}}\right].$$

In the presence of prepayments, the interest and principal components of the tth payment are given by

$$I_t = A_t \times \left[1 - \frac{1}{(1+r)^{N-t+1}}\right]$$

and

$$P_t = \frac{A_t}{(1+r)^{N-t+1}}.$$

The scheduled principal at the end of the tth period is given by

$$SP_t = \frac{A_t}{r}\left[1 - \frac{1}{(1+r)^{N-t}}\right].$$

We know that

$$A_t = A \prod_{i=1}^{t-1} [1 - \mathrm{SMM}_i].$$

Thus,

$$I_t = I_t^* \prod_{i=1}^{t-1} [1 - \text{SMM}_i]$$
$$P_t = P_t^* \prod_{i=1}^{t-1} [1 - \text{SMM}_i]$$

and

$$\text{SP}_t = SP_t^* \prod_{i=1}^{t-1} [1 - \text{SMM}_i].$$

The actual principal at the end of t periods is given by

$$\text{SP}_t \text{ x } [1 - \text{SMM}_t].$$

Therefore,

$$L_t = SP_t^* \prod_{i=1}^{t} [1 - \text{SMM}_i].$$

Example 9.6 gives a detailed illustration of cash-flow calculations for a pass-through security.

Example 9.6

Consider a pool of four mortgage loans. Each loan has an outstanding principal of $100,000 and 24 months to maturity. The mortgage rate is 9 percent for each loan. The servicing fee is 0.5 percent of the outstanding balance, and the guarantor's fee is also 0.5 percent of the outstanding balance. The SMM is 2.5 percent for all the months.

The cash flows in the absence of prepayments are given in Table 9.11.

Table 9.11
Amortization Schedule in the Absence of Prepayments

Month	Mthly. Pmt.	Serv. Fee	Grnt. Fee	Net Int.	Prin. Pmt.	Out. Prin.	Cash Flow
0						400,000	
1	18273.90	166.67	166.67	2666.67	15273.90	384726.10	17940.56
2	18273.90	160.30	160.30	2564.84	15388.45	369337.65	17953.29

Table 9.11
Continued

Month	Mthly. Pmt.	Serv. Fee	Grnt. Fee	Net Int.	Prin. Pmt.	Out. Prin.	Cash Flow
3	18273.90	153.89	153.89	2462.25	15503.86	353833.79	17966.12
4	18273.90	147.43	147.43	2358.89	15620.14	338213.64	17979.04
5	18273.90	140.92	140.92	2254.76	15737.29	322476.35	17992.05
6	18273.90	134.37	134.37	2149.84	15855.32	306621.03	18005.17
7	18273.90	127.76	127.76	2044.14	15974.24	290646.79	18018.38
8	18273.90	121.10	121.10	1937.65	16094.05	274552.74	18031.69
9	18273.90	114.40	114.40	1830.35	16214.75	258337.99	18045.10
10	18273.90	107.64	107.64	1722.25	16336.36	242001.63	18058.62
11	18273.90	100.83	100.83	1613.34	16458.88	225542.74	18072.23
12	18273.90	93.98	93.98	1503.62	16582.33	208960.42	18085.94
13	18273.90	87.07	87.07	1393.07	16706.69	192253.72	18099.76
14	18273.90	80.11	80.11	1281.69	16831.99	175421.73	18113.69
15	18273.90	73.09	73.09	1169.48	16958.23	158463.49	18127.71
16	18273.90	66.03	66.03	1056.42	17085.42	141378.07	18141.84
17	18273.90	58.91	58.91	942.52	17213.56	124164.51	18156.08
18	18273.90	51.74	51.74	827.76	17342.66	106821.85	18170.43
19	18273.90	44.51	44.51	712.15	17472.73	89349.12	18184.88
20	18273.90	37.23	37.23	595.66	17603.78	71745.34	18199.44
21	18273.90	29.89	29.89	478.30	17735.81	54009.53	18214.11
22	18273.90	22.50	22.50	360.06	17868.83	36140.70	18228.89
23	18273.90	15.06	15.06	240.94	18002.84	18137.86	18243.78
24	18273.90	7.56	7.56	120.92	18137.86	0.00	18258.78

Mthly. Pmt. = monthly payment; Serv. Fee = servicing fee; Grnt. Fee = guaranteeing fee; Net. Int. = net interest; Prin. Pmt. = principal payment; Out. Prin. = outstanding principal

Analysis of Amortization in the Absence of Prepayments

Consider the entries for the first month in Table 9.11. The principal outstanding at the beginning of the month is \$400,000. The monthly payment due is given by

$$400{,}000 = \frac{A}{0.0075}\left[1 - \frac{1}{(1.0075)^{24}}\right]$$

$$\Rightarrow A = \$18{,}273.90.$$

The servicing fee is

$$\frac{0.005}{12} \times 400{,}000 = \$166.67.$$

The guarantee fee for every month is the same as the servicing fee. The net interest to be passed through for the first month is

$$\frac{0.08}{12} \times 400{,}000 = \$2{,}666.67.$$

The principal payment for the month is

$$\$18{,}273.90 - \$166.67 - \$166.67 - \$2{,}666.67 = \$15{,}273.90.$$

The total cash flow for the holders of the pass-through securities is

$$\$2{,}666.67 + \$15{,}273.90 = \$17{,}940.57.$$

Let us consider the repayment schedule assuming that the SMM is 2.5 percent for all months.

A Revised Analysis (Assuming Prepayments)

Let us analyze the entries in Table 9.12. The payment for the first month is the same as in the preceding case that is \$18,273.90. The servicing and guaranteeing fees are also the same as those obtained previously because they are based on the same outstanding principal: \$400,000. The net interest to be passed through is \$2,666.67, which is the same as the value obtained in the absence of prepayments. The scheduled outstanding balance at the end of the first month is

$$\$400{,}000 - \$15{,}273.90 = \$384{,}726.10$$

We have assumed a constant SMM of 2.50 percent, so the prepayment at the end of the first month is $0.025 \times \$384{,}726.10 = \$9{,}618.15$. The actual outstanding principal at the end of the first month is then

$$\$384{,}726.10 - \$9{,}618.15 = \$375{,}107.95$$

The total cash flow for the month for the pass-through holders is the sum of the net interest plus the scheduled principal payment plus the prepayment, which in this case is

$$\$2{,}666.67 + \$15{,}273.90 + \$9{,}618.15 = \$27{,}558.72.$$

Let us consider the entries for the second month. The total payment for the month is given by

$$375{,}107.95 = \frac{A}{0.0075}\left[1 - \frac{1}{(1.0075)^{23}}\right]$$

$$\Rightarrow A = \$17{,}817.05.$$

The servicing and guaranteeing fees for the second month are

$$\frac{0.005}{12} \times 375{,}107.95 = \$156.29.$$

The net interest for the second month is

$$\frac{0.08}{12} \times 375{,}107.95 = \$2{,}500.72.$$

The scheduled principal payment for the month is

$$\$17{,}817.05 - \$156.29 - \$156.29 - \$2{,}500.72 = \$15{,}003.74.$$

The scheduled outstanding principal at the end of the month is

$$\$375{,}107.95 - \$15{,}003.74 = \$360{,}104.21.$$

The prepayment at the end of the month is $0.025 \times \$360{,}104.21 = \$9{,}002.61$. The actual outstanding principal at the end of the second month is therefore

$$\$360{,}104.21 - \$9{,}002.61 = \$351{,}101.60.$$

The cash flow for the month that is directed toward the pass-through holders is

$$\$2{,}500.72 + \$15{,}003.74 + \$9{,}002.61 = \$26{,}507.06.$$

The analysis can be extended along similar lines to the remaining months.

Table 9.12
Amortization Schedule in the Presence of Prepayments

Mt. Pmt.	Mly. Fee	Ser. Fee	Grn. Fee	Net Int.	Sch. Princ. Pmt.	Sch. Out. Prin.	Pre Pmt.	Act. Out. Prin.	CF
0						400,000		400,000	
1	18273.90	166.67	166.67	2666.67	15273.90	384726.10	9618.15	375107.95	27558.72
2	17817.05	156.29	156.29	2500.72	15003.74	360104.21	9002.61	351101.60	26507.06
3	17371.62	146.29	146.29	2340.68	14738.36	336363.24	8409.08	327954.16	25488.12
4	16937.33	136.65	136.65	2186.36	14477.68	313476.48	7836.91	305639.57	24500.95
5	16513.90	127.35	127.35	2037.60	14221.60	291417.97	7285.45	284132.52	23544.65
6	16101.05	118.39	118.39	1894.22	13970.06	270162.46	6754.06	263408.40	22618.34
7	15698.53	109.75	109.75	1756.06	13722.96	249685.44	6242.14	243443.30	21721.15
8	15306.06	101.43	101.43	1622.96	13480.24	229963.06	5749.08	224213.99	20852.27
9	14923.41	93.42	93.42	1494.76	13241.81	210972.18	5274.30	205697.88	20010.87
10	14550.33	85.71	85.71	1371.32	13007.59	192690.29	4817.26	187873.03	19196.17
11	14186.57	78.28	78.28	1252.49	12777.52	175095.51	4377.39	170718.12	18407.39
12	13831.90	71.13	71.13	1138.12	12551.52	158166.60	3954.17	154212.44	17643.80
13	13486.11	64.26	64.26	1028.08	12329.51	141882.93	3547.07	138335.85	16904.67
14	13148.95	57.64	57.64	922.24	12111.43	126224.42	3155.61	123068.81	16189.28
15	12820.23	51.28	51.28	820.46	11897.21	111171.60	2779.29	108392.31	15496.96
16	12499.72	45.16	45.16	722.62	11686.78	96705.53	2417.64	94287.89	14827.03
17	12187.23	39.29	39.29	628.59	11480.07	82807.82	2070.20	80737.62	14178.85
18	11882.55	33.64	33.64	538.25	11277.02	69460.60	1736.52	67724.09	13551.78
19	11585.49	28.22	28.22	451.49	11077.56	56646.53	1416.16	55230.37	12945.21
20	11295.85	23.01	23.01	368.20	10881.62	44348.75	1108.72	43240.03	12358.54
21	11013.45	18.02	18.02	288.27	10689.15	32550.88	813.77	31737.11	11791.19
22	10738.12	13.22	13.22	211.58	10500.09	21237.02	530.93	20706.09	11242.59
23	10469.66	8.63	8.63	138.04	10314.37	10391.73	259.79	10131.93	10712.20
24	10207.92	4.22	4.22	67.55	10131.93	0.00	0.00	0.00	10199.48

Mly. Pmt. = monthly payment; Ser. Fee = servicing fee; Grn. Fee = guaranteeing fee; Net. Int. = net interest; Sch. Prin. Pmt. = scheduled principal payment; Sch. Out. Prin. = scheduled outstanding principal; Pre Pmt. = prepayment; Act. Out. Prin. = actual outstanding principal; CF = cash flow

Average Life

Unlike a bond that repays its principal in the form of a bullet payment at maturity, a pass-through security returns a part of its principal every month. It is a common practice to compute the average life of a pass-through security. Let PCF_t be the total principal received at time t. PCF_t is the sum of the scheduled principal received at time t and the prepayment received at the same point in time. The average life may be defined as

$$\frac{\sum_{t=1}^{T} t \times \mathrm{PCF}_t}{L_0}.$$

In the case of the pass-through previously discussed, the average life in the absence of prepayments is 12.8578 months, or 1.0715 years. However, when we assume an SMM of 2.50 percent per month, the average life declines to 10.6677 months, which is equivalent to 0.8890 years. The higher the prepayment speed that is assumed, the shorter the average life of the security.

Cash-Flow Yield

The cash-flow yield for a pass-through is the internal rate of return (IRR) computed using the projected cash-flow stream. The result is a monthly rate, which is usually converted to a *bond equivalent yield* (BEY) to facilitate comparisons with conventional debt securities. If we denote the IRR as i_m, where i_m denotes a monthly rate, then the bond equivalent yield may be expressed as

$$2[(1 + i_m)^6 - 1].$$

Example 9.7 presents the cash flow yield and the bond equivalent yield for a mortgage-backed security under various pre-payment assumptions.

Example 9.7

Assume that the pool of four mortgages with a principal of $100,000 each and with a coupon of 9 percent is securitized, and 400 securities are issued. The par value of each security is $1,000, and each security will be eligible for 1/400 of each cash flow emanating from the underlying pool. The monthly cash flows for each security, assuming an SMM of 2.50 percent, are given in Table 9.13.

Table 9.13
Cash Flows for a Security with a Par Value of $1,000

Month	Total Cash Flow from the Underlying Mortgage Pool	Cash Flow for the Pass-through Security
1	27,558.72	68.90
2	26,507.06	66.27
3	25,488.12	63.72
4	24,500.95	61.25
5	23,544.65	58.86
6	22,618.34	56.55
7	21,721.15	54.30
8	20,852.27	52.13
9	20,010.87	50.03
10	19,196.17	47.99
11	18,407.39	46.02
12	17,643.80	44.11
13	16,904.67	42.26
14	16,189.28	40.47
15	15,496.96	38.74
16	14,827.03	37.07
17	14,178.85	35.45
18	13,551.78	33.88
19	12,945.21	32.36
20	12,358.54	30.90
21	11,791.19	29.48
22	12,242.59	28.11
23	10,712.20	26.78
24	10,199.48	25.50

The cash-flow yield and the corresponding bond equivalent yield for this security are given as follows for three different price scenarios, and three assumed prepayment speeds in each case.

From Table 9.14, the BEY for a pass-through is inversely related to the price of the security, as is the case for a conventional debt security. If the security is priced at par, the cash-flow yield is equal to the coupon rate, irrespective of the rate that is assumed for prepayment.

Table 9.14
Bond Equivalent Yields

Price	SMM	Cash Flow Yield	BEY
1,000	2.5%	0.6667%	8.1349%
1,000	4.0%	0.6667%	8.1349%
1,000	1.0%	0.6667%	8.1349%
975	2.5%	0.9184%	11.2770%
975	4.0%	0.9455%	11.6176%
975	1.0%	0.8927%	10.9543%
1,025	2.5%	0.4243%	5.1459%
1,025	4.0%	0.3987%	4.8323%
1,025	1.0%	0.4487%	5.4452%

Conditional Prepayment Rate

To project the cash flows from a pass-through, we need to make an assumption about the prepayment behavior. The prepayment rate that is assumed for a pool of mortgages is called the *conditional prepayment rate* (CPR). The term *conditional* connotes that the rate is conditional on the remaining balance in the pool.

The CPR corresponding to the assumed prepayment speed has to be converted to the equivalent SMM. The formula for conversion is the following:

$$[1 - \text{SMM}]^{12} = 1 - \text{CPR}$$
$$\Rightarrow \text{SMM} = 1 - [1 - \text{CPR}]^{\frac{1}{12}}.$$

The rationale for this formula is the following. Assume that the SMM is constant for a year. Let $SP^*_{12\times N}$ be the scheduled principal at the end of year N in the absence of prepayments, and let $L_{12\times N}$ be the actual principal at the end of the Nth year. We know that

$$L_{12\times N} = SP^*_{12\times N} \times [1 - \text{SMM}]^{12\times N}.$$

If we denote the corresponding annual prepayment rate as CPR, then

$$L_{12\times N} = SP^*_{12\times N} \times [1 - \text{CPR}]^{N}.$$

Thus,

$$1 - \text{CPR} = [1 - SMM]^{12}$$
$$\Rightarrow \text{SMM} = 1 - [1 - \text{CPR}]^{\frac{1}{12}}.$$

PSA Prepayment Benchmark

The Public Securities Association (PSA) prepayment benchmark is expressed as a monthly series of annual prepayment rates.[2]

The PSA benchmark assumes that prepayment rates are lower for newly originated mortgages and will then speed up as the mortgages become seasoned. It has been observed that home owners are less likely to move in the initial years of the loan. They are also less likely to refinance or make large unscheduled principal repayments. The PSA benchmark assumes that the prepayment rate will steadily increase for the first 30 months of the loan and then stabilize thereafter.

The benchmark assumes the following CPR for 30-year mortgages:

- CPR of 0.2 percent for the first month;
- An increase of 0.2 percent per month for the next 30 months until it reaches 6 percent; and
- 6 percent for the remaining years.

This benchmark is called *100 percent PSA* or *100 PSA*.

Let t be the number of months after the mortgage was originated.

$$\text{If } t \leq 30; \text{CPR} = 6 \times \frac{t}{30}\%, \text{whereas}$$
$$\text{If } t > 30; \text{CPR} = 6\%$$

Sample calculations for the 100 PSA benchmark are given as follows:

$$\text{Month} - 10: \ \text{CPR} = 6 \times \frac{10}{30} = 2\%$$
$$\text{SMM} = 1 - (1 - .02)^{\frac{1}{12}} = 0.001682 \equiv 0.1682\%$$
$$\text{Months } 31\text{–}360: \ \text{CPR} = 6\%$$
$$\text{SMM} = 1 - (1 - .06)^{\frac{1}{12}} = 0.005143 \equiv 0.5143\%.$$

Slower or faster prepayment speeds are referred to as a *percentage* of the standard PSA benchmark. For example, 50 PSA means one-half of the

CPR of the PSA benchmark, and 150 PSA means 1.5 times the CPR of the PSA benchmark.

The case of 150 PSA, the SMM may be calculated as follows:

$$\text{Months} - 10: \ \text{CPR} = 1.50 \times 6 \times \frac{10}{30} = 3\%$$

$$\text{SMM} = 1 - (1 - .03)^{\frac{1}{12}} = 0.002535 \equiv 0.2535\%$$

$$\text{Months } 30{-}360: \ \text{CPR} = 1.50 \times 6\% = 9\%$$

$$\text{SMM} = 1 - (1 - .09)^{\frac{1}{12}} = 0.007828 \equiv 0.7828\%.$$

It should be noted that 150 PSA does not mean that the SMM for a month is 1.5 times the SMA for the same month under the 100 PSA assumption. It is the CPR for the month that is 1.5 times the CPR under the 100 PSA assumption.

Example 9.8 illustrates the cash flows from a pass-through under the assumption of 100 PSA.

Example 9.8 (100 PSA)

Consider a pass-through backed by a single loan of $100,000 with 360 months to maturity and a mortgage rate of 9 percent per annum. Assume there are no servicing or guaranteeing fees. The projected cash flows assuming 100 PSA are given in Table 9.15.

The average life of the pass-through is 12.08 years.

Table 9.15
Cash Flows Assuming 100 PSA

Mth.	Mthly. Pmt.	Int. Pmt.	Sch. Prin. Pmt.	Sch. Out. Prin.	Pre Pmt.	Act. Out. Prin.	Cash Flow
1	804.62	750.00	54.62	99945.38	16.67	99928.70	821.30
12	795.75	737.10	58.65	98221.92	198.64	98023.29	994.39
24	767.88	705.98	61.90	94068.57	384.82	93683.75	1152.70
36	724.49	660.61	63.88	88016.81	452.67	87564.14	1177.16
48	681.02	615.34	65.68	81979.09	421.62	81557.47	1102.64
60	640.16	572.62	67.53	76282.30	392.32	75889.98	1032.48
72	601.75	532.31	69.44	70905.39	364.67	70540.72	966.42
84	565.64	494.25	71.39	65828.56	338.56	65490.00	904.20

Table 9.15
Continued

Mth.	Mthly. Pmt.	Int. Pmt.	Sch. Prin. Pmt.	Sch. Out. Prin.	Pre Pmt.	Act. Out. Prin.	Cash Flow
96	531.71	458.30	73.41	61033.16	313.89	60719.26	845.60
108	499.80	424.33	75.47	56501.65	290.59	56211.06	790.39
120	469.81	392.21	77.60	52217.54	268.56	51948.98	738.37
132	441.63	361.84	79.79	48165.27	247.71	47917.56	689.34
144	415.13	333.09	82.04	44330.24	227.99	44102.25	643.12
156	390.22	305.87	84.35	40698.69	209.31	40489.37	599.53
168	366.81	280.08	86.72	37257.64	191.62	37066.02	558.42
180	344.80	255.63	89.17	33994.90	174.84	33820.06	519.64
192	324.11	232.43	91.68	30898.98	158.91	30740.07	483.02
204	304.66	210.40	94.26	27959.05	143.79	27815.26	448.46
216	286.38	189.46	96.92	25164.92	129.42	25035.49	415.81
228	269.20	169.55	99.65	22506.96	115.75	22391.21	384.96
240	253.05	150.59	102.46	19976.14	102.74	19873.40	355.79
252	237.87	132.52	105.35	17563.90	90.33	17473.57	328.20
264	223.59	115.28	108.32	15262.20	78.49	15183.71	302.09
276	210.18	98.81	111.37	13063.44	67.19	12996.25	277.36
288	197.57	83.06	114.51	10960.45	56.37	10904.08	253.94
300	185.71	67.98	117.73	8946.47	46.01	8900.45	231.73
312	174.57	53.52	121.05	7015.10	36.08	6979.02	210.65
324	164.10	39.64	124.46	5160.32	26.54	5133.78	190.64
336	154.25	26.28	127.97	3376.42	17.36	3359.06	171.62
348	145.00	13.42	131.57	1658.02	8.53	1649.49	153.52
360	136.30	1.01	135.28	0.00	0.00	0.00	136.30

Mth. = month; Mthly. Pmt. = monthly payment; Int. Pmt. = interest payment; Sch. Prin. Pmt. = scheduled principal payment; Sch. Out. Prin. = scheduled outstanding principal; Pre. Pmt. = prepayment; Act. Out. Prin. = actual outstanding principal

Analysis of Cash Flows Assuming 100 PSA

Consider the data for the first month. The monthly payment is given by

$$100{,}000 = \frac{A}{0.0075} \times \left[1 - \frac{1}{(1.0075)^{360}}\right]$$

$$\Rightarrow A = \$804.62.$$

The interest component for the first month is

$$100{,}000 \times \frac{0.09}{12} = \$750.$$

The scheduled outstanding principal at the end of the month is

$$\$100{,}000 - (\$804.62 - \$750) = \$99{,}945.38.$$

The CPR for the first month is 0.2 percent. So the SMM is

$$1 - (1 - .002)^{\frac{1}{12}} = 0.0001668 \equiv 0.01668\%.$$

The prepayment for the month is

$$\$99{,}945.38 \times 0.0001668 = \$16.67.$$

The actual outstanding principal at the end of the first month is

$$\$99{,}945.38 - \$16.67 = \$99{,}928.71.$$

The cash flow for the pass-through is

$$\$750 + \$54.62 + \$16.67 = \$821.29.$$

Example 9.9 illustrates the cash flows from a pass-through under an assumption of 200 PSA.

Example 9.9 (200 PSA)

Consider a pass-through backed by a single loan of \$100,000 with 360 months to maturity and a mortgage rate of 9 percent per annum. Assume that there are no servicing or guaranteeing fees. The projected cash flows assuming 200 PSA are given in Table 9.16.

The average life of the pass-through is eight years. As can be seen, the more rapid prepayments lead to a reduction in the average life of the security.

Table 9.16
Cash Flows Assuming 200 PSA

Mth.	Mthly. Pmt.	Int. Pmt.	Sch. Prin. Pmt.	Sch. Out. Prin.	Pre Pmt.	Act. Out. Prin.	Cash Flow
1	804.62	750.00	54.62	99,945.38	33.38	99,912.00	838.00
12	786.84	728.85	57.99	97,122.15	397.31	96,724.84	1,184.15
24	731.68	672.70	58.98	89,634.10	750.71	88,883.39	1,482.39
36	648.97	591.75	57.22	78,842.54	835.43	78,007.11	1,484.41
48	571.10	516.01	55.08	68,746.86	728.46	68,018.40	1,299.55
60	502.56	449.55	53.02	59,886.42	634.57	59,251.85	1,137.14
120	265.22	221.41	43.81	29,477.80	312.35	29,165.45	577.57
180	139.96	103.77	36.20	13,799.60	146.22	13,653.38	286.19
240	73.86	43.96	29.91	5,830.94	61.79	5,769.15	135.65
300	38.98	14.27	24.71	1,877.81	19.90	1,857.92	58.88
360	20.57	0.15	20.42	0.00	0.00	0.00	20.57

Mth. = month; Mthly.Pmt. = monthly payment; Int. Pmt. = interest payment; Sch. Prin. Pmt. = scheduled principal payment; Sch. Out. Prin. = scheduled outstanding principal; Pre. Pmt. = prepayment; Act. Out. Prin. = actual outstanding principal

Collateralized Mortgage Obligations

In the case of a pass-through, there is only one class of security that is issued, and every security is entitled to the same undivided share of the cash flows from the underlying pool. The feature of such securities is that prepayments have the same implications for all the security holders.

In real life, however, investors differ with respect to the prepayment exposure and the average security life sought by them. The securitization process can take these preferences into account by creating multiple classes of securities or what are referred to in the parlance of mortgage-backed securities as *tranches*. There are clearly specified rules as to how the cash flows from the underlying pool are to be directed to the holders of the various tranches. Such securities, with multiple tranches, are referred to as *collateralized mortgage obligations* (CMOs). We will introduce the concept by studying a very simple CMO structure known as a *sequential pay CMO*.

Table 9.17
Amortization for Tranche A

Month	Interest Received	Principal Received	Cash Flow	Outstanding Balance
1	750.00	285.18	1035.18	99714.82
2	747.86	353.50	1101.36	99361.32
10	712.36	895.96	1608.31	94085.02
20	622.99	1543.95	2166.94	81521.67
30	487.00	2127.57	2614.58	62806.32
40	330.88	2026.39	2357.27	42091.53
50	182.18	1930.42	2112.60	22359.64
60	40.49	1839.42	1879.92	3559.90
61	26.70	1830.58	1857.28	1729.32
62	12.97	1729.32	1742.29	0.00

Sequential Pay CMO

Assume that a pool of mortgages consists of four loans with a principal of \$100,000 each. Each loan has a maturity of 360 months and a mortgage rate of 9 percent. The average life of a pass-through, backed by such loans, under the assumption of 100 PSA is 12.08 years, as we have just seen.

Assume that instead of creating a single type of security like a pass-through, four types of securities known as tranches A to D are created. All principal payments from the underlying pool, both scheduled and unscheduled, will first go to tranche A. During this time, the remaining tranches will continue to earn interest on the outstanding principal. Once tranche A is fully paid off, all subsequent principal payments, scheduled and unscheduled, will be directed to the holders of tranche B. Once again, while tranche B is being paid off, holders of tranches C and D will get interest on their outstanding principal. Extending the logic, the redemption of tranche B will see further principal payments being directed to tranche C, with holders of tranche D getting only the interest due. Finally, once tranche C securities are retired, all further principal payments will be directed to the holders of tranche D.

Let us assume that one security is issued for each tranche, with a face value of \$100,000, and that there are no servicing or guaranteeing fees. The cash-flow schedules for the four tranches are given in Tables 9.17 to 9.20.

Table 9.18
Amortization for Tranche B

Month	Interest Received	Principal Received	Cash Flow	Outstanding Balance
1	750.00	0.00	750.00	100000.00
10	750.00	0.00	750.00	100000.00
20	750.00	0.00	750.00	100000.00
30	750.00	0.00	750.00	100000.00
40	750.00	0.00	750.00	100000.00
50	750.00	0.00	750.00	100000.00
60	750.00	0.00	750.00	100000.00
61	750.00	0.00	750.00	100000.00
62	750.00	92.47	842.47	99907.53
70	655.48	1753.13	2408.61	85644.07
80	526.78	1671.32	2198.11	68566.30
90	404.08	1593.77	1997.85	52283.01
100	287.05	1520.27	1807.32	36752.77
110	175.40	1450.62	1626.02	21936.22
120	68.85	1384.63	1453.48	7795.93
125	17.41	1352.94	1370.35	968.18
126	7.26	968.18	975.44	0.00

Table 9.19
Amortization for Tranche C

Month	Interest Received	Principal Received	Cash Flow	Outstanding Balance
1	750.00	0.00	750.00	100000.00
20	750.00	0.00	750.00	100000.00
40	750.00	0.00	750.00	100000.00
60	750.00	0.00	750.00	100000.00
80	750.00	0.00	750.00	100000.00
100	750.00	0.00	750.00	100000.00
120	750.00	0.00	750.00	100000.00
126	750.00	378.53	1128.53	99621.47
140	620.00	1262.90	1882.90	81403.49
160	438.49	1153.77	1592.25	57311.25
180	272.52	1056.02	1328.54	35280.26
200	120.47	968.57	1089.05	15094.36
210	49.20	928.41	977.60	5631.42
215	14.67	909.16	923.83	1047.35
216	7.86	905.38	913.23	141.97
217	1.06	141.97	143.03	0.00

Table 9.20
Amortization for Tranche D

Month	Interest Received	Principal Received	Cash Flow	Outstanding Balance
1	750.00	0.00	750.00	100000.00
30	750.00	0.00	750.00	100000.00
60	750.00	0.00	750.00	100000.00
90	750.00	0.00	750.00	100000.00
120	750.00	0.00	750.00	100000.00
150	750.00	0.00	750.00	100000.00
180	750.00	0.00	750.00	100000.00
210	750.00	0.00	750.00	100000.00
217	750.00	759.64	1509.64	99240.36
240	602.36	820.79	1423.15	79493.62
270	427.81	730.43	1158.23	56310.55
300	271.93	654.98	926.90	35601.82
330	131.58	592.44	724.03	16951.79
340	87.77	574.17	661.94	11128.88
350	45.29	557.08	602.37	5482.15
360	4.06	541.13	545.18	0.01

Analysis of Amortization for Tranche A of a Sequential Pay CMO

Consider the first month. The scheduled principal that is received from the underlying pool is

$$400{,}000 = \frac{A}{0.0075} \times \left[1 - \frac{1}{(1.0075)^{360}}\right]$$

$$\Rightarrow A = \$3{,}218.49.$$

The interest component for the month is

$$\$400{,}000 \times 0.0075 = \$3{,}000.$$

The scheduled principal for the month is \$3,218.49 − \$3,000 = \$218.49. This will be directed toward tranche A.

The prepayment for the month is

$$[\$400{,}000 - \$218.49] \times 0.0001668 = \$66.68.$$

This will also be directed to tranche A. The total principal received by tranche A is \$218.49 + \$66.68 = \$285.17.

The interest receivable by this security for the month is

$$\$100{,}000 \times 0.0075 = \$750.$$

The total cash flow received by tranche A in the first month is

$$\$750 + \$285.17 = \$1{,}035.17.$$

Consider the second month. The monthly payment received from the underlying pool is

$$399{,}714.82 = \frac{A}{0.0075} \times \left[1 - \frac{1}{(1.0075)^{359}}\right]$$
$$\Rightarrow A = \$3{,}217.95.$$

The interest component for the month is

$$\$399{,}714.82 \times 0.0075 = \$2{,}997.86.$$

The scheduled principal for the month is

$$\$3{,}217.95 - \$2{,}997.86 = \$220.09.$$

The prepayment for the month is

$$[\$399{,}714.82 - \$220.09] \times 0.0003339 = \$133.41.$$

So the total principal for the month is \$220.09 + \$133.41 = \$353.50.

The entire amount will be directed toward tranche A. The interest receivable by this tranche for the second month is given by

$$[\$100{,}000 - \$285.17] \times 0.0075 = \$747.86.$$

The total cash flow that is directed toward tranche A at the end of the second month is \$747.86 + \$353.50 = \$1,101.36.

At the end of the sixty-first month, tranche A has an outstanding principal of \$1,729.32. The total principal from the underlying pool at the end of the sixty-second month is \$1,821.79. Of this, \$1,729.32 will be directed to tranche A. The balance of \$92.47 will be paid to tranche B. The total cash flow received by tranche A in the sixty-second month is

$$(\$1{,}729.32 \times 0.0075) + \$1{,}729.32 = \$1{,}742.29.$$

Analysis of Amortization for Tranche B

Let us turn our attention to tranche B. For the first sixty-one months, tranche B will receive only interest on the outstanding principal of \$100,000, which amounts to \$750 per month. In the sixty-second month, this tranche receives \$92.47 by way of principal payment as previously explained. The total cash flow for this month is \$750 + \$92.47 = \$842.47. The outstanding principal at the end of the month is \$100,000 − \$92.47 = \$99,907.53. In the 125th month, the outstanding principal for this tranche is \$968.18. In the 126th month, the underlying pool makes a total principal payment of \$1,346.71. In this month, tranche B receives \$7.26 worth of interest and \$968.18 of principal, which amounts to a total cash flow of \$975.44. The excess principal—that is, \$1,346.71 − \$968.18 = \$378.53—is directed to tranche C.

Analysis of Amortization for Tranche C

Let us consider tranche C. Until the 125th month, this tranche receives a monthly cash flow of \$750, which represents the interest on an outstanding principal of \$100,000. In the 126th month, there is a principal repayment of \$378.53, as previously explained. The total cash flow for this month is therefore \$1,128.53. At the end of 216 months, there is an outstanding principal of \$141.97. So in the 217th month, this tranche receives an interest payment of \$1.06. In this month, the total principal received from the underlying pool is \$901.61. After paying \$141.97 to tranche C, the excess of \$759.64 is directed toward tranche D.

Analysis of Amortization for Tranche D

Finally, let us analyze the last tranche. Tranche D receives interest on the initial outstanding principal until month 216 that amounts to \$750 per month. In month 217, it receives a principal payment of \$759.64 as previously explained. The tranche is fully paid off in month 360.

The average life of the collateral as calculated for a pass-through security based on the pool is 12.08 years. However, each tranche of the CME has a different maturity and average life. The details are given in Table 9.21.

Table 9.21
Maturity and Average Life of the Securities

Security	Maturity in Months	Average Life in Months
Collateral	360	144.92
Tranche A	62	36.13
Tranche B	126	92.74
Tranche C	217	168.43
Tranche D	360	282.39

Extension Risk and Contraction Risk

Pass-through securities and CMOs expose investors to prepayment risk. In a declining interest-rate environment, prepayments impact security holders in two ways. First, since they are debt securities, we would expect the price of such securities to rise in a scenario where rates are declining. However, in the case of mortgage-backed securities, holders of underlying bonds are likely to prepay and refinance at a lower rate when market rates go down. The overall result is price compression: the rise in price will not be as large as in the case of plain vanilla bonds. The second adverse consequence for holders of such securities is, of course, reinvestment risk, as is the case for all debt holders. In other words, cash flows that are received in a declining interest-rate environment have to be reinvested at lower rates of interest. The risk from these two adverse consequences is termed *contraction risk*.

Let us consider a rising interest-rate environment. We would expect the prices of mortgage-backed securities to fall. However, unlike in the case of plain vanilla bonds, principal payments are likely to slow down because holders of underlying mortgage loans are less likely to refinance when mortgage rates are rising. This will exacerbate the anticipated price decline. In such a situation, the security holders would like prepayments to be high because it would afford them an opportunity to reinvest larger amounts at high rates of interest. The adverse impact for mortgage-backed securities in a rising interest-rate environment is termed *extension risk*.

Accrual Bonds

In the case of the sequential pay CMO that we just studied, every tranche receives interest every month, based on the principal outstanding for that particular tranche at the beginning of the month. However, many sequential pay CMO structures do not require that every tranche be paid interest every month based on the outstanding principal. In the case of a sequential pay CMO with an *accrual bond* (also termed a *Z bond*[3]), however, the Z bond will not receive any monthly interest until the previous tranches are fully retired. Therefore, the interest for the Z bond will accrue and be added to its original principal (negative amortization) until the other classes of the CMO have been paid off. In the months before the retirement of the penultimate tranche, the interest that would otherwise have been paid to the Z bond is directed to the tranche that is receiving principal at that point in time, and it helps speed up the repayment of principal.

Let us consider a CMO in which tranches A, B, and C are as specified in the preceding illustration. However, the last tranche is an accrual bond. The inclusion of the Z bond will reduce the maturity as well as the average life of each of the three earlier tranches. Tables 9.22 to 9.25 illustrate the amortization schedules of the various tranches of a CMO that includes a Z-bond.

Table 9.22
Amortization for Tranche A in the Presence of a Z-bond

Month	Interest Received	Principal Received	Cash Flow	Outstanding Balance
1	750.00	1035.18	1785.18	98964.82
2	742.24	1109.13	1851.36	97855.69
10	660.19	1698.13	2358.31	86326.76
20	508.59	2408.36	2916.94	65403.26
30	305.54	3059.04	3364.58	37679.15
40	77.15	3030.12	3107.27	7256.67
42	31.72	3025.63	3057.35	1203.21
43	9.02	1203.21	1212.23	0.00

Table 9.23
Amortization for Tranche B in the Presence of a Z-bond

Month	Interest Received	Principal Received	Cash Flow	Outstanding Balance
1	750.00	0.00	750.00	100000.00
10	750.00	0.00	750.00	100000.00
20	750.00	0.00	750.00	100000.00
30	750.00	0.00	750.00	100000.00
40	750.00	0.00	750.00	100000.00
43	750.00	1820.34	2570.34	98179.66
50	600.57	3012.03	3612.60	77063.95
60	374.98	3004.94	3379.92	46991.80
70	149.54	3009.08	3158.61	16929.01
75	36.61	3015.44	3052.04	1865.71
76	13.99	1865.71	1879.70	0.00

Table 9.24
Amortization for Tranche C in the Presence of a Z-bond

Month	Interest Received	Principal Received	Cash Flow	Outstanding Balance
1	750.00	0.00	750.00	100000.00
10	750.00	0.00	750.00	100000.00
20	750.00	0.00	750.00	100000.00
30	750.00	0.00	750.00	100000.00
40	750.00	0.00	750.00	100000.00
50	750.00	0.00	750.00	100000.00
60	750.00	0.00	750.00	100000.00
70	750.00	0.00	750.00	100000.00
76	750	1151.35	1901.35	98848.65
80	673.40	3024.71	3698.11	86761.89
90	445.69	3052.16	3497.85	56373.74
100	215.52	3091.80	3307.32	25644.37
109	5.49	731.83	737.32	0.00

Analysis of Cash Flows for a CMO with an Accrual Bond

The cash flows from the underlying pool at the end of the first month are as follows.

Interest = \$3,000, scheduled principal \$218.49, prepayment = \$66.69.

The outstanding principal for each of the four tranches is \$100,000. Tranches A, B, and C will therefore receive an interest payment of \$750 for the month. The accrual bond will not receive any interest. Tranche A will receive the following amount by way of principal payment

$$\$3{,}000 + \$218.49 + \$66.69 - (750 \times 3) = \$1{,}035.18.$$

The total cash flow for this tranche is \$750 + \$1,035.18 = \$1,785.18. The outstanding principal at the end of the first month is \$100,000 − \$1,035.18 = \$98,964.82.

Because the Z bond does not receive any interest during the month, there will be negative amortization. The outstanding principal for this bond at the end of the first month is \$100,750.

Let us consider the second month. The cash flows from the underlying pool are as follows: interest = \$2,997.86, scheduled

Table 9.25
Amortization for the Z-Bond

Month	Interest Due	Interest Received	Principal Received	Cash Flow	Outstanding Balance
1	750	0.00	0.00	(750.00)	100750.00
2	755.63	0.00	0.00	(755.63)	101505.63
10	802.17	0.00	0.00	(802.17)	107758.25
20	864.41	0.00	0.00	(864.41)	116118.41
30	931.47	0.00	0.00	(931.47)	125127.18
50	1081.61	0.00	0.00	(1081.61)	145295.69
60	1165.52	0.00	0.00	(1165.52)	156568.10
70	1255.94	0.00	0.00	(1255.94)	168715.05
90	1458.38	0.00	0.00	(1458.38)	195909.25
100	1571.53	0.00	0.00	(1571.53)	211108.38
108	1668.33	0.00	0.00	(1668.33)	224112.42
109	1680.84	1680.84	725.59	2406.43	223386.83
120	1568.85	1568.85	1384.63	2953.48	207795.91
140	1370.00	1370.00	1262.90	2632.90	181403.48
160	1188.49	1188.49	1153.77	2342.25	157311.23
180	1022.52	1022.52	1056.02	2078.54	135280.24
200	870.47	870.47	968.57	1839.05	115094.34
220	730.86	730.86	890.46	1621.32	96557.85
240	602.36	602.36	820.79	1423.15	79493.59
260	483.75	483.75	758.78	1242.53	63740.99
280	373.94	373.94	703.73	1077.66	49154.44
300	271.93	271.93	654.98	926.90	35601.78
320	176.81	176.81	611.96	788.78	22962.97
340	87.77	87.77	574.17	661.94	11128.83
360	4.06	4.06	541.13	545.18	(0.05)

principal = \$220.09, prepayment = \$133.41. Tranche A has an outstanding principal of \$98,964.82. The interest for the month for this tranche is

$$\$98{,}964.82 \times 0.0075 = \$742.24.$$

Tranches B and C will each receive \$750 for the month by way of interest because there has been no principal repayment. The principal repayment that is directed at tranche A for this month is

$$\$2{,}997.86 + \$220.09 + \$133.41 - \$742.24 - (750 \times 2) = \$1{,}109.12.$$

The outstanding principal at the end of the second month is therefore \$98,964.82 − \$1,109.12 = \$97,855.70.

Tranche Z will not receive any cash flow during the month. The interest due is

$$\$100{,}750 \times 0.0075 = \$755.63.$$

The outstanding principal at the end of the second month is \$100,750 + \$755.63 = \$101,505.63.

Let us turn to the forty-third month. The cash flows from the underlying collateral are as follows: interest = \$2,535.51, scheduled principal = \$259.71, prepayment = \$1,737.35.

The outstanding principal for tranche A at the end of the forty-second month is \$1,203.21. This tranche will receive a cash flow of \$1,203.21 × (1.0075) = \$1,212.23. With this, tranche A is retired.

Tranches B and C are entitled to an interest payment of \$750 each. The principal payment that is received by tranche B during this month is

$$\$2{,}535.51 + \$259.71 + \$1{,}737.35 - \$1{,}212.23 - (750 \times 2) = \$1{,}820.34.$$

The total cash flow that is received by tranche B at the end of this month is \$2,570.34. The outstanding principal is \$100,000 − \$1,820.34 = \$98,179.66.

Similarly, tranche B is retired at the end of 76 months and tranche C at the end of 109 months. From the 109th month onward, the Z bond starts receiving repayments of principal. The outstanding principal for the bond at the end of month 108 is \$224,112.42. In the 109th month, the tranche receives an interest payment of \$224,112.42 × 0.0075 = \$1,680.84. It also receives a principal payment of \$725.59 that reduces the outstanding principal to \$223,386.83. The tranche is fully paid off at the end of the 360th month.

From Table 9.26, the average life of each of the three tranches has reduced because of the presence of the Z bond, which appeals to investors

Table 9.26
Maturity and Average Life of the Securities in the Presence of a Z Bond

Security	Maturity in Months	Average Life in Months
Collateral	360	144.92
A	43	25.06
B	76	59.50
C	109	92.53
Z	360	

who are primarily concerned with the specter of reinvestment risk. Because such bonds do not entail the receipt of any cash flows until all the other classes are fully retired, reinvestment risk is totally eliminated until the Z bond starts receiving its first cash flow.

Floating-Rate Tranches

Floating-rate bonds can be created from a fixed-rate tranche. We will illustrate this using tranche C in the preceding illustration, which has a par value of \$100,000 and a 9 percent coupon. To create a floater, we have to simultaneously create an *inverse* floater.[4] The cumulative principal of the two tranches will be \$100,000 in this case. There is no rule as to what the principal values of the respective tranches should be as long as their combined par value is \$100,000.

Let us assume that the benchmark for the two tranches is the three-month LIBOR and that we wish to create a floater with a par value of \$60,000 and an inverse floater with a par value of \$40,000. The coupon on the floater is LIBOR + m basis points, whereas that for the inverse floater is $C - L \times$ LIBOR.

It is a common practice to specify the coupon for an inverse floater as

$$C - L \times \text{benchmark}.$$

C is the cap or maximum interest on the inverse floater, and L is referred to as the *coupon leverage*. The concept of a cap can be easily understood. The lower limit for LIBOR is zero. The upper limit for the coupon on the inverse floater is C. The coupon leverage measures the rate of change of the coupon of the inverse floater with respect to the benchmark.

We know that the total annual interest available is 9 percent of 100,000, or 9,000. This is also the maximum interest available for the floating tranche. The cap on the floater is $9{,}000 \div 60{,}000 = 15$ percent. The ratio of floaters to inverses is 3:2, which implies that the coupon leverage for the inverse floater is 1.50. If we assume that the coupon on the floater is LIBOR + 150 bp, then the minimum coupon on the floater is 1.50 percent, which would be the case if LIBOR was zero. The minimum interest on the floater is $\$60{,}000 \times 0.015 = \$900$. So the maximum interest for the inverse floater is $\$9{,}000 - \$900 = \$8{,}100$. The cap on the inverse is therefore $8{,}100 \div 40{,}000 = 0.2025 = 20.25$ percent. We can therefore express the coupons of the two tranches as

$$\text{Floater coupon: LIBOR } + 1.50\%$$
$$\text{Inverse floater coupon: } 20.25\% - 1.50 \times \text{LIBOR}$$

The weighted-average coupon is

$$0.60 \times (\text{LIBOR} + 1.50) + 0.40 \times (20.25 - 1.50 \times \text{LIBOR}).$$

If LIBOR is 7.50 percent, the weighted average will be

$$0.60 \times 9 + 0.40 \times 9 = 9\%.$$

Notional Interest Only Tranche

Let us go back to the sequential pay CMO we studied earlier. We assumed that the underlying mortgages had a coupon rate of 9 percent and that each of the four tranches that we created also had a coupon of 9 percent. Each tranche of a CMO will have a different coupon rate, and there will be an excess representing the difference between the interest on the underlying collateral and the coupons on the various tranches. Therefore, it is a common practice to create a tranche that is entitled to receive only the excess coupon interest.[5] Such tranches are referred to as *notional interest-only securities*.

We will assume that the underlying collateral has a coupon of 9 percent and that the four tranches created from it have coupon rates as shown in Table 9.27.

Consider tranche A. It pays a coupon of 7.50 percent, whereas the coupon on the collateral is 9 percent. There is excess interest of 1.50 percent. Let us assume that we wish to create a notional interest-only security with a coupon of 8.40 percent. To find the notional amount on which the interest payable to this security will be calculated, we proceed

Table 9.27
Coupon Rates for a Sequential Pay CMO

Tranche	Initial Principal ($)	Coupon Rate (%)
A	140,000	7.50
B	84,000	7.75
C	84,000	8.00
D	92,000	8.25

as follows. An excess interest of 1.50 percent on \$140,000 worth of principal can be used to pay an interest of 8.40 percent on a principal amount of \$25,000.

Similarly, in the case of tranche B, an excess interest of 1.25 percent on a principal amount of \$84,000 can be used to pay an interest of 8.40 percent on a principal of \$12,500. Tranche C, using the same logic, generates excess interest to support \$10,000 worth of principal, and the last tranche is capable of servicing a principal of \$8,214.2850. The excess interest generated can service a bond with a principal of 25,000 + 12,500 + 10,000 + 8,214.285 = \$55,714.285. This is the notional principal for the tranche that is scheduled to receive the excess interest. The term *notional* connotes that this principal amount is used merely to compute the interest payable to this class for the month and that the principal per se is not repaid.

The contribution of a tranche to the notional principal is in general given by the following formula.

$$\text{Notional Amount for an x \% IO Class} = \frac{\text{Par Value of the tranche} \times \text{Excess Interest}}{\frac{x}{100}}.$$

In the case of tranche A in the preceding illustration,

$$\text{Notional Amount for an 8.40\% IO Class} = \frac{140{,}000 \times 0.015}{0.0840} = \$25{,}000.$$

For the first month, the IO class will receive a cash flow of

$$0.0840 \times 55{,}714.285 \times \frac{1}{12} = \$390.$$

Let us assume a prepayment speed of 100 PSA. The principal paid to tranche A at the end of the first month will be \$285.18. This will reduce the outstanding principal for this tranche to 140,000 − 285.18 = \$139,714.82. The contribution of this tranche to the notional principal for the second month is

$$\frac{139{,}714.82 \times 0.0150}{0.0840} = \$24{,}949.08.$$

The contributions of the other tranches will not be affected. For the second month, the notional IO class will receive interest on a principal of

$$24{,}949.08 + 12{,}500 + 10{,}000 + 8{,}214.285 = \$55{,}663.36.$$

The interest for this tranche for the second month will be

$$\frac{55{,}663.36 \times 0.084}{12} = \$389.64.$$

Interest-Only and Principal-Only Strips

Stripped mortgage-backed securities can be created by either directing all the principal or all the interest to a particular class. The security that is scheduled to receive only the principal is referred to as the *principal-only* (PO) *class*. The other security is termed the *interest-only* (IO) *class*.

Let us first consider the PO class. The yield obtained by the security holder will depend on the speed with which the underlying pool generates prepayments. The more quickly that prepayments are received, the greater the rate of return for holders of such securities. In the case of IO class holders, however, the desire is to have slow prepayments because such securities do not receive any principal payments, and the income (which is exclusively by way of interest payments) will be higher, the larger the outstanding balance on the underlying pool (which is tantamount to a slower prepayment rate).

PAC Bonds

PAC is an acronym for *planned amortization class*. In the case of such bonds, if the prepayment pattern falls within a prespecified range, then the cash flows received by the security are known.

PAC bonds are not created in isolation. They are accompanied by a category of bonds that are termed *support bonds*. It is this category that absorbs the prepayment risk, thereby protecting the PAC bonds against extension risk as well as contraction risk.

To create a PAC bond, we need to specify a range of prepayment speeds. These are referred to as *PAC collars*. The PAC schedule lists the principal amount payable to the PAC bondholders if the prepayment speed is within the range provided by the lower and upper PAC collars. The schedule is obtained by taking the lower of the principal cash flows generated by the lower collar and the upper collar. Consider the Example 9.10.

Example 9.10

A pool of four mortgage loans of $100,000 each is used to create a PAC bond and a support bond. The lower PAC collar is 100 PSA, and the upper collar is 250 PSA. Table 9.28 lists the principal payments from the underlying pool, assuming four different prepayment speeds between the lower and upper collars.

Table 9.28
Principal Payments for a 100 PSA to 250 PSA Range

Month	At 100 PSA	At 150 PSA	At 200 PSA	At 250 PSA	Minimum Principal Payment
1	285.18	318.57	352.00	385.45	285.18
2	353.50	420.36	487.32	554.40	353.50
3	421.79	522.13	622.71	723.51	421.79
4	490.01	623.83	758.02	892.57	490.01
5	558.13	725.37	893.11	1061.35	558.13
10	895.96	1,227.95	1,560.54	1,893.75	895.96
50	1,930.42	2,553.91	3,066.72	3,477.57	1,930.42
100	1,520.27	1,720.23	1,775.68	1,729.92	1,520.27
101	1,513.14	1,706.67	1,756.27	1,705.78	1,513.14
102	1,506.04	1,693.22	1,737.08	1,681.96	1,506.04
114	1,423.79	1,539.79	1,522.14	1,420.35	1,420.35
150	1,206.84	1,157.52	1,021.39	851.68	851.68
200	968.57	777.74	581.68	412.60	412.60
250	788.87	521.43	326.18	194.79	194.79
300	654.98	348.48	178.44	88.02	88.02
350	557.08	231.79	93.62	36.63	36.63
360	541.13	213.48	81.67	30.24	30.24

The sum total of the principal payments per the PAC schedule is $278,088.22. This represents the maximum principal for the PAC bonds. For our illustration, we have assumed that the PAC bond has a principal of $277,500, whereas the support bond has a principal of $122,500

Analysis of Principal Payments for Speeds between the Lower and Upper PAC Collars

Until the 113th month, the lowest principal payment is obtained under the assumption of 100 PSA. From the 114th month onward, the lowest principal payment is observed under the assumption of 250 PSA.

Table 9.29
Principal Payments at 150 PSA

Month	Total Principal	Payment to PAC Bond	Payment to Support Bond
1	318.57	285.18	33.39
2	420.36	353.50	66.85
3	522.13	421.79	100.34
4	623.83	490.01	133.82
5	725.37	558.13	167.24
50	2,553.91	1,930.42	623.48
100	1,720.23	1,520.27	199.96
150	1,157.52	851.68	305.85
200	777.74	412.60	365.13
250	521.43	194.79	326.64
300	348.48	88.02	260.46
344	243.49	13.53	229.96
345	241.51	0.00	241.51
350	231.79	0.00	231.79
355	222.46	0.00	222.46
360	213.48	0.00	213.48

We will first consider the prepayment scenario corresponding to 150 PSA as shown in Table 9.29.

Analysis (The Case of 150 PSA)

Until the 344th month, the total principal received is in excess of the amount required to satisfy the PAC schedule. The difference is therefore directed to the support bond. In the 344th month, the outstanding amount on the PAC bond is $13.53. The total principal received is $243.49. The excess of $229.96 is directed to the support bond. In subsequent months, because the PAC bond has been fully paid off, the entire principal received is directed to the support bond. The average life of the PAC bond is 98.13 months, and that for the support bond is 156.69 months.

Let us consider the prepayment scenario corresponding to 250 PSA as shown in Table 9.30.

Analysis (The Case of 250 PSA)

Until the 113th month, the principal payment, scheduled as well as unscheduled, from the underlying collateral and based on the assumption of 250 PSA, is greater than the amount required to be directed toward the PAC bond on the basis of the PAC schedule. Until the 113th month

Table 9.30
Principal Payments at 250 PSA

Month	Total Principal	Payment to PAC Bond	Payment to Support Bond
1	385.45	285.18	100.27
2	554.40	353.50	200.90
3	723.51	421.79	301.72
4	892.57	490.01	402.56
5	1,061.35	558.13	503.23
50	3,477.57	1,930.42	1,547.15
100	1,729.92	1,520.27	209.65
113	1,440.54	1,430.45	10.09
150	851.68	851.68	0.00
200	412.60	412.60	0.00
250	194.79	194.79	0.00
300	88.02	88.02	0.00
344	40.97	13.53	27.44
345	40.22	0.00	40.22
350	36.63	0.00	36.63
355	33.31	0.00	33.31
360	30.24	0.00	30.24

there is a cash inflow for the support bond. From the 114th until the 343rd month, the total principal payment is just adequate to satisfy the PAC schedule. The payment directed toward the support bond during these months is nil. In the 344th month, the total principal received is \$40.97. The outstanding balance for the PAC bond is \$13.53. The payment for the support bond is \$27.44. Because the PAC bonds are fully paid off at this stage, in subsequent months all principal received is directed to the support bond.

As to be expected, the average life of the PAC bond continues to remain at 98.13 months. However, the average life of the support bond falls to 44.07 months. This highlights the protection provided by the support bond. In this, case the entire impact of contraction resulting from faster prepayment is absorbed by the support bond.

Let us consider the prepayments at 0 PSA or, in other words, a situation corresponding to no prepayments as shown in Table 9.31.

Analysis (The Case of 0 PSA)

Until the 161st month, the total principal received is inadequate to satisfy the PAC schedule. The entire principal received is directed

Table 9.31
Principal Payments at 0 PSA

Month	Total Princiapl	Scheduled Payment to PAC Bond	Actual Payment to PAC Bond	Deficit	Cumulative Deficit	Payment to Support Bond
1	218.49	285.18	218.49	(66.69)	(66.69)	0.00
2	220.13	353.50	220.13	(133.37)	(200.06)	0.00
3	221.78	421.79	221.78	(200.01)	(400.07)	0.00
4	223.44	490.01	223.44	(266.57)	(666.64)	0.00
5	225.12	558.13	225.12	(333.01)	(999.65)	0.00
50	315.09	1,930.42	315.09	(1,615.33)	(64,444.79)	0.00
100	457.82	1,520.27	457.82	(1,062.46)	(130,879.07)	0.00
150	665.19	851.68	665.19	(186.49)	(163,337.65)	0.00
161	722.17	727.35	722.17	(5.18)	(164,288.15)	0.00
162	727.59	716.96	727.59	10.63	(164,277.53)	0.00
163	733.05	706.72	733.05	26.33	(164,251.20)	0.00
200	966.49	412.60	966.49	553.89	(152,986.07)	0.00
250	1,404.27	194.79	1,404.27	1,209.48	(108,618.11)	0.00
300	2,040.35	88.02	2,040.35	1,952.33	(29,870.86)	0.00
314	2,265.35	69.61	2,265.35	2,195.74	(735.60)	0.00
315	2,282.34	68.43	804.03	2,213.90	0.00	1,478.30
316	2,299.45	67.28	67.28	2,232.18	0.00	2,232.18
343	2,813.46	41.73	41.73	2,771.73	0.00	2,771.73
344	2,834.56	13.53	13.53	2,821.03	0.00	2,821.03
345	2,855.82	0.00	0.00	2,855.82	0.00	2,855.82
350	2,964.54	0.00	0.00	2,964.54	0.00	2,964.54
355	3,077.39	0.00	0.00	3,077.39	0.00	3,077.39
360	3,194.53	0.00	0.00	3,194.53	0.00	3,194.53

toward the PAC bond, and the support bond does not receive any payment. The deficit with respect to the PAC schedule is accumulated until the end of the 161st month when it reaches a value of $164,228.15. From the 162nd until the 314th month, the entire principal is directed to the PAC bond, and the deficit is steadily reduced. At the end of the 314th month, the cumulative deficit is $735.60. The scheduled payment for the PAC bond for the 315th month is $68.43. A cash flow of $804.03 is directed to the PAC bond. The total cash flow for the month is $2,282.34. Thus, $1,478.31 is directed to the support bond. From here on until the 344th month, the principal received is in excess of what is required to satisfy the PAC schedule, and the excess is directed toward the support bond.

Table 9.32
Principal Payments at 400 PSA

Month	Total Princiapl	Scheduled Payment to PAC Bond	Actual Payment to PAC Bond	Payment to Support Bond	Outstanding for Support Bond
1	485.99	285.18	285.18	200.81	122,299.19
2	756.32	353.50	353.50	402.82	121,896.37
3	1,027.31	421.79	421.79	605.52	121,290.85
4	1,298.44	490.01	490.01	808.43	120,482.42
5	1,569.15	558.13	558.13	1,011.02	119,471.40
40	5,282.47	2,026.39	2,026.39	3,256.08	1,426.02
41	5,160.49	2,016.56	3,734.47	1,426.02	0.00
42	5,041.32	2,006.79	5,041.32		
50	4,181.24	1,930.42	4,181.24		
100	1,289.82	1,520.27	1,289.82		
150	391.27	851.68	391.27		
200	115.53	412.60	115.53		
250	32.57	194.79	32.57		
300	8.42	88.02	8.42		
350	1.77	36.63	1.77		
360	1.23	30.24	1.23		

The principal outstanding for the PAC bond as of the end of month 343 is $13.53. The principal received in month 344 is $2,834.56. Thus, $2,821.03 is directed to the support bond at the end of this month. For all subsequent months, the entire principal received is directed to the support bond. The average life of the PAC bond is 214.83 months, and that of the support bond is 339.10 months. Although slower prepayment increases the average life of the bonds, the impact on the support bond, as is to be expected, is greater than the impact on the PAC bond.

Finally, let us consider a scenario corresponding to 400 PSA as shown in Table 9.32.

Analysis (The Case of 400 PSA)

Until the fortieth month, the total principal received is in excess of what is required by the PAC schedule, so the excess is directed to the support bond. As of the end of the fortieth month, the outstanding principal on the support bond is $1,426.02. In the forty-first month, this amount is directed to the support bond, and the balance is directed to the PAC bond. With this, the support bond is fully paid off. In subsequent months the entire principal received is directed to the PAC bond. The average life of the PAC bond is 70.43 months, and that of the support bond is 24.78 months.

Agency Pass-Throughs

The U.S. federal government has set up three agencies to guarantee mortgage-backed securities. Pass-throughs guaranteed by such agencies are referred to as *agency pass-throughs* as compared to *private-label pass-throughs*, which are issued by nongovernment entities such as commercial banks and savings-and-loan associations. The three federal agencies are the Government National Mortgage Association (Ginnie Mae), the Federal National Mortgage Association (Fannie Mae), and the Federal Home Loan Mortgage Corporation (Freddie Mac).

Ginnie Mae

Ginnie Mae was established in 1968. Unlike the other two federal agencies, Ginnie Mae is fully owned by the U.S. government. Ginnie Mae securities, like other Treasury securities such as T-bonds, are backed by the full faith and credit of the federal government.

Note that Ginnie Mae as an organization neither originates nor acquires and pools mortgage loans and nor does it issue securities. What happens is that private institutions that have been approved by Ginnie Mae pool home loans and issue mortgage-backed securities that have the guarantee of Ginnie Mae. The guaranty from Ginne Mae ensures that holders of the pass-throughs receive timely payment of principal and interest. If a borrower fails to make the required payment on a mortgage on time, then the issuer is expected to use its own funds to ensure that the security holders receive timely payment. If an issuer is unable to ensure that payments are made per schedule, then Ginnie Mae will step in to make the required payment to the security holders. This is the essence of the guaranty provided by Ginnie Mae.

Ginnie Mae guarantees securities backed by single-family and multifamily loans that are insured by U.S. government agencies. These are the Federal Housing Administration (FHA), the Department of Agriculture, the Department of Veterans Affairs, and the Office of Public and Indian Housing.

Ginnie Mae has two programs: the Ginnie Mae I MBS Program, for the issuance of securities backed by single-family or multifamily loans, and the Ginnie Mae II MBS program for the issuance of securities backed by single-family loans. In the case of Ginnie Mae I, all loans must bear the same rate of interest, with certain exceptions. However, in the case of Ginnie Mae II, the interest rates for a pool of mortgages may vary across loans. All Ginnie Mae I securities must bear a fixed rate of interest, whereas some Ginnie Mae II securities may bear an adjustable interest rate. The minimum pool size for Ginnie Mae I is US$1 million. Payment to pass-through holders is made on the fifteenth of each month. For Ginnie Mae II securities, the minimum pool size is US$250,000 for multilender pools and US$1 million for single-lender pools. Payment to holders is made on the twentieth of each month.

Fannie Mae

Fannie Mae was founded in 1938 at a time when the U.S. economy was coming to terms with the Great Depression. In 1968, it was set up as a shareholder-owned corporation by the U.S. Congress. Fannie Mae and Freddie Mac are referred to as *government-sponsored enterprises* (GSEs). They are referred to as *quasi-government organizations* because, although they are essentially private, they were set up by Congress. Most people believe that securities issued by Fannie Mae and Freddie Mac carry an implicit federal government guarantee; that is, although there is no explicit guarantee from the government from the standpoint of the credit risk of securities issued by the GSEs, it is believed that the government will inevitably bail them out in adverse circumstances, because they are considered economically too important to be allowed to fail. This belief has been reinforced by the bailout of these organizations during the recent subprime crisis.

Fannie Mae buys mortgage loans and securitizes them for sale to investors in the secondary mortgage market. To reiterate, unlike the case of Ginnie Maes, the federal government does not ensure timely payment of principal and interest on Fannie Mae securities. The guarantee on such securities is provided by Fannie Mae itself.

Freddie Mac

Freddie Mac is a GSE set up by Congress to compete with Fannie Mae. Like the former, it also pools loans and securitizes them. As in the case of Fannie Mae securities, holders of Freddie Mac securities are provided an assurance from Freddie Mac from the standpoint of timely payment of principal and interest. There is no explicit guarantee from the federal government.

Endnotes

1. The monthly income should be taken as the take-home salary and not as the cost to company (CTC).
2. This model was developed by the Public Securities Association, which was subsequently renamed the Bond Market Association (BMA). In 2006, the BMA merged with the Securities Industry Association to form the Securities Industry and Financial Markets Association (SIFMA). However, the prepayment benchmark is still referred to as the *PSA benchmark*.
3. Because of its similarities with a zero-coupon bond.
4. Although the coupon of a floating-rate bond moves up and down with the corresponding changes in the benchmark, the coupon of an inverse floater moves inversely with the benchmark. An inverse floater may have its coupon specified as 10% − LIBOR. If the LIBOR rises, the coupon falls and vice versa.
5. Note that this tranche is not entitled to any principal payments.

CHAPTER 10

Swaps

Introduction

What exactly is a swap transaction? As the name suggests, it entails the swapping of cash flows between two counterparties. There are two broad categories of swaps: interest-rate swaps and currency swaps.

In the case of an interest-rate swap (IRS), all payments are denominated in the same currency. The two cash flows being exchanged will be calculated using different interest rates. One party may compute its payable using a fixed rate of interest and the other may calculate what it owes based on a market benchmark such as LIBOR. Such interest-rate swaps are referred to as *fixed−floating swaps*. A second possibility is that both payments may be based on variable or floating rates. The first party may compute its payable based on LIBOR, whereas the counterparty may calculate what it owes based on the rate for a Treasury security. Such a swap is referred to as a *floating−floating swap*. It should be obvious to the reader that we cannot have a fixed−fixed swap. Consider a deal in which Bank ABC agrees to make a payment to the counterparty every six months on a given principal at the rate of 5.25 percent per annum in return for a counterpayment based on the same principal computed at a rate of 6 percent per annum. This is an arbitrage opportunity for Bank ABC, because what it owes every period will always be less than what is owed to it. No rational counterparty will therefore agree to be a party to such a contract. In the case of an interest-rate swap, before the exchange of interest on a scheduled payment date, there must be a positive probability of a net payment being received for both parties to the deal.

In the case of an interest-rate swap, a principal amount needs to be specified to facilitate the computation of interest. But there is no need to physically exchange this principal at the outset. Therefore, the underlying principal amount in such transactions is referred to as a *notional principal*.

Let us consider a currency swap. Such a contract also entails the payment of interest by two counterparties to each other, the difference being that the payments are denominated in two different currencies. Because two currencies are involved, there are three interest-computation methods possible: fixed–fixed, fixed–floating, and floating–floating. Examples 10.1 and 10.2 illustrate this.

Example 10.1

Two banks that are parties to a swap deal. Bank Exotica agrees to pay interest to Bank Halifax at the rate of 6.40 percent per annum on a principal of $2,500,000. In return, the counterparty agrees to pay Bank Exotica an amount that is computed on the same principal but that is based on the LIBOR prevailing at the outset of the payment period. This is a fixed–floating swap. We will assume that interest is payable at the end of every six months for a period of two years.

Assume that the observed values of LIBOR over a two-year horizon are as depicted in Table 10.1.

Table 10.1
Six-Month LIBOR as Observed at Six-Month Intervals

Time (Months)	LIBOR (per Annum) (%)
0	6.20
6	6.55
12	6.75
18	6.10

We will assume that the LIBOR used for computing the interest payable for a period will be the value observed at the start of the period, although the interest per se is payable at the end of the period. This is the common practice in financial markets and is referred to as a system of *determined in advance and paid in arrears*. Rarely are we likely to observe a case of *determined in arrears and paid in arrears*. The second method entails payment based on the LIBOR prevailing at the end of the period for which the interest is due.

The periodic interest payable by Bank Exotica is

$$\$2{,}500{,}000 \times 0.064 \times 0.5 = \$80{,}000.$$

This amount will be invariant for the life of the swap for two reasons. First, the bank is paying interest based on a fixed rate. Second, we are assuming that every six-month period corresponds to exactly one-half of a year. The gap between successive interest-payment dates may vary and depends on the day-count convention that is assumed.

The periodic interest that is payable by Bank Halifax will vary because the benchmark rate is variable. The interest payable by the bank for the first period is

$$\$2{,}500{,}000 \times 0.0620 \times 0.50 = \$77{,}500.$$

The net transfer at the end of the first six-month period is a cash flow of $2,500 from Bank Exotica to Bank Halifax.

The amounts payable by the two banks and the net payment at the end of every period are given in Table 10.2.

Table 10.2
Amounts Payable by the Two Counterparties

Time (Months)	Payment to Be Made by Bank Exotica ($)	Payment to Be Made by Bank Halifax ($)	Net Payment (to be made by Bank Exotica) ($)
6	80,000	77,500	2,500
12	80,000	81,875	(1,875)
18	80,000	84,375	(4,375)
24	80,000	76,250	3,750

Consider the last column of Table 10.2. It describes the net payment to be made or received by Bank Exotica. Positive amounts indicate that Bank Exotica will experience a *cash outflow*: what it owes is more than what it is owed. Amounts in parentheses indicate a net *cash inflow*: what the bank owes is less than what is owed to it.

Example 10.2

We consider a currency swap. Bank SudAfrique enters into a swap with Bank Oriental with the following terms. The two banks will first exchange South African rands (ZAR) for U.S. dollars (USD) at the rate of 7.50 USD−ZAR. The principal amount in USD is $2,400,000 and is payable by Bank Orient in exchange for the South African currency. Every six months thereafter, Bank Sud Afrique will pay interest at the rate of 4.75 percent per annum on the principal amount of $2,400,000. Bank Orient, on the other hand, will pay interest at the rate of 7.25 percent per annum on the equivalent amount in ZAR. The terms of the deal specify that the two banks will exchange back the currencies at the end of two years at the original spot rate.

Table 10.3 shows the cash flows payable and receivable by the two banks.

Table 10.3
Cash Flows for a Fixed−Fixed Currency Swap

Time (Months)	Payment by Bank Sud Afrique	Payment by Bank Oriental
0	18,000,000 ZAR	2,400,000 USD
6	57,000 USD	652,500 ZAR
12	57,000 USD	652,500 ZAR
18	57,000 USD	652,500 ZAR
24	57,000 USD	652,500 ZAR
24	2,400,000 USD	18,000,000 ZAR

Contract Terms

In swap contracts such as those discussed previously, certain terms and conditions need to be specified at the very outset to avoid ambiguities and potential future conflicts.

Every swap contract must spell out the identities of the two counterparties to the deal. In our first illustration, the two counterparties are Bank Exotica and Bank Halifax; in the second, they are Bank Sud Afrique and Bank Oriental.

The tenor or maturity of the swap refers to the date on which the last exchange of cash flows between the two parties will take place. In both preceding illustrations, the tenor was two years. Unlike exchange-traded products such as futures contracts, in which the exchange specifies a maximum maturity for contracts on an asset, swaps being over-the-counter (OTC) products can have any maturity that is agreed on in bilateral discussions.

The interest rates on the basis of which the two parties have to make payments should be spelled out. To avoid ambiguities, the basis on which both cash inflow and cash outflow are arrived at for both counterparties should be explicitly stated.

In the case of the preceding interest-rate swap, Bank Exotica was a fixed-rate payer with the interest rate fixed at 6.40 percent per annum. Bank Halifax was a floating-rate payer with the amount payable based on the 6-month LIBOR prevalent at the start of the interest-computation period.

For currency swaps, we also need to specify the currencies in which the two parties will make payments to each other, in addition to the specification of interest rates to be adopted by them. In the preceding illustration of a currency swap, Bank Sud Afrique was a fixed-rate payer with the interest being payable in U.S. dollars at the rate of 4.75 percent per annum. Bank Oriental also was a fixed-rate payer, the difference being that it was required to pay interest in South African rands at the rate of 7.25 percent per annum.

The frequency with which the cash flows are to be exchanged has to be defined. In our illustrations, we assumed that cash-flow exchanges would take place at six-month intervals. In the market, such swaps are referred to as *semi–semi swaps*. Other contracts may entail payments on a quarterly basis or annual basis. The benchmark that is chosen for the floating-rate payment is usually based on the frequency of the exchange. A swap that entails the exchange of cash flows at six-month intervals will specify six-month LIBOR as the benchmark, whereas a swap that entails the exchange of payments at three-month intervals will specify the three-month LIBOR as the benchmark. The most popular benchmark in the market is the six-month LIBOR.

The day-count convention that is used to compute the interest must be explicitly stated. In our illustration, we assumed that every six-month period amounted to exactly one half of a year. The underlying convention is referred to as 30/360: every month is assumed to consist of 30 days, and the year as a whole is assumed to consist of 360 days. Other possibilities include actual/360 or actual/365.

The principal amounts that serve as the basis for each party to figure out payments to the counterparty have to be stated. In the case of interest-rate swaps, there is only one currency involved. Nonetheless, the magnitude of the principal has to be specified to facilitate the computation of interest. In the case of currency swaps, the two currencies involved and the principal amounts in each have to be specified. In other words, the rate of exchange for conversion between them should be stated.

Market Terminology

We first consider interest-rate swaps. As we discussed, there are two possible counterpayments: fixed–floating and floating–floating. A swap in which one of the rates is fixed is referred to as a *coupon swap*. On the other hand, a swap in which both the parties are required to make payments based on varying rates is referred to as a *basis swap*. The IRS that we considered previously is a coupon swap: Bank Exotica was required to pay a fixed rate of 6.40 percent per annum in exchange for a payment from Bank Halifax that was based on LIBOR.

In a coupon swap, the party that agrees to make payments based on a fixed rate is the *payer*, and the counterparty that is committed to making payments on a floating-rate basis is the *receiver*.[1] However, these terms cannot be used in the case of basis swaps, because both cash-flow streams are based on floating rates. It is important to be explicit and to avoid ambiguities. For each counterparty, both rates—for making and receiving payments—should be explicitly stated. In the case of the interest-rate swap that we previously considered, the terms would be stated somewhat as follows.

Counterparties: Bank Exotica and Bank Halifax

- Interest rate 1: A fixed rate of 6.40 percent to be paid by Bank Exotica to Bank Halifax.
- Interest rate 2: A variable rate based on the six-month LIBOR prevailing at the onset of the corresponding six-month period to be paid by Bank Halifax to Bank Exotica.

Key Dates

There are four important dates for a swap. Consider the two-year swap between Bank Exotica and Bank Halifax. Assume that the swap was negotiated on June 10, 2010, with a specification that the first payments would be for a six-month period commencing on June 15, 2010. June 10 will be referred to as the *transaction date*. The date from which the interest counterpayments start to accrue is termed the *effective date*. In our illustration, the effective date is June 15.

By assumption, our swap has a tenor of two years; the last exchange of payments will take place on June 15, 2012. This date will be referred to as the *maturity date* of the swap. We will assume that the four exchanges of cash flows will occur on December 15, 2010; June 15, 2011; December 15, 2011; and June 15, 2012. The first three dates on which the floating rate will be reset for the next six-month period are referred to as *reset* or *refixing dates*.

Inherent Risk

An interest swap exposes both the parties to interest-rate risk. Consider the fixed–floating swap between Bank Exotica and Bank Halifax. For the fixed-rate payer, the risk is that the LIBOR may decline after the swap is entered into. If so, although the payments to be made by it would remain unchanged, its receipts would decline in magnitude. From the standpoint of the floating-rate payer, the risk is that the benchmark—in this case, the LIBOR—may increase after the swap is agreed on. If so, even though its receipts would remain unchanged, its payables would increase. A priori we cannot be sure whether LIBOR will increase or decline. *Ex ante*, both the parties to the contract are exposed to interest-rate risk.

The Swap Rate

What exactly is the swap rate? The term refers to the fixed rate of interest that is applicable in a coupon swap. There are two conventions for quoting the interest rate. The first practice is to quote the full rate in percentage terms. This is referred to as an *all-in price*. In certain interbank markets, however, the practice is to quote the fixed rate as a difference or spread in basis points between the fixed rate and a benchmark interest rate. The benchmark that is used is the yield for a government security with a time to maturity equal to the tenor of the swap. Example 10.3 best describes the two methods.

Illustrative Swap Rates

Consider the hypothetical quotes for euro-denominated interest-rate swaps on a given day as shown in Table 10.4. Assume that the

Example 10.3

Let us consider the interest-rate swap between Bank Exotica and Bank Halifax in which the former agreed to pay a fixed rate of 6.40 percent per annum. Such a specification is a manifestation of an all-in price because the fixed rate is quoted in full.

Consider an alternative scenario in which the yield to maturity for a 2-year T-note is 5.95 percent per annum. If so, the spread will be quoted as 45 basis points. The implication is that the fixed rate is 45 basis points greater than the yield on a security having the same maturity.

corresponding floating rate is the six-month EURIBOR (Euro Interbank Offered Rate).

Such a rate schedule, which is typically provided by a professional swap dealer, may be interpreted as follows. A swap dealer will give a two-way quote in which the bid is the fixed rate at which he is willing to do a swap that requires him to pay the fixed rate. The ask represents the rate at which he will do a swap that requires him to receive the fixed rate. The spread must be positive.

Determining the Swap Rate

Let us reconsider the coupon swap between Bank Exotica and Bank Halifax. The swap rate was arbitrarily assumed to be 6.40 percent. We will demonstrate how this rate will be set.

Table 10.4
Bid–Ask Quotes for Euro-Denominated IRS

Tenor (Year)	Bid (%)	Ask (%)
1	1.50	1.55
2	1.70	1.75
3	1.90	1.95
5	2.25	2.30
10	2.90	2.95
15	3.25	3.30
25	3.20	3.25

To understand the pricing of swaps, consider an alternative financial arrangement in which Bank Exotica issues a fixed-rate bond with a principal of US$2,500,000 and uses the proceeds to acquire a floating-rate bond with the same principal. Assume that the benchmark for the floating-rate bond is the six-month LIBOR and that both bonds pay coupons on a semiannual basis. The initial cash flow is zero because the proceeds of the fixed-rate issue will be just adequate to purchase the floating-rate security. Every six months the bank will receive a floating rate of interest based on the six-month LIBOR on a principal amount of US$2,500,000 and will have to make an interest payment for the same principal, based on a fixed rate of interest. When the two bonds mature, the amount received when the bank redeems the principal on the floating-rate bond held by it will be just adequate for it to repay the principal on the bond it issued. The net cash flow at maturity is therefore zero. Structurally this financial arrangement corresponds to an interest-rate swap with a notional principal of US$2.5 million in which the bank pays a fixed rate of interest every six months in return for an interest stream that is based on the six-month LIBOR. To ensure that the combination of the two bonds exactly matches the structure of the swap, we need to assume that the bonds also pay coupons based on the same day-count convention, which in this case has been assumed to be 30/360.

We will demonstrate how to determine the fixed rate of a coupon swap. Let us assume that the term structure for LIBOR is as shown in Table 10.5.

Table 10.5
Observed Term Structure

Time to Maturity (Months)	**Interest Rate Per Annum (%)**
6	5.25
12	5.40
18	5.75
24	6.00

These interest rates need to be converted to discount factors, which can be done as follows.[2] The discount factor for the first payment (after six months) is

$$\frac{1}{\left(1 + \frac{0.0525}{2}\right)} = 0.9744.$$

The remaining discount factors may be computed as follows:

$$\frac{1}{\left(1+\frac{0.0540}{2}\right)^2} = 0.9481$$

$$\frac{1}{\left(1+\frac{0.0575}{2}\right)^3} = 0.9185$$

$$\frac{1}{\left(1+\frac{0.0600}{2}\right)^4} = 0.8885.$$

Because we are at the start of a coupon period, the price of the floating-rate bond should be equal to the principal value of US\$2,500,000. Therefore, the question is, what is the coupon rate that will make the fixed-rate bond also have the same value at the outset? Let us denote the annual coupon in dollars by C.

We require that

$$\frac{C}{2} \times [0.9744 + 0.9481 + 0.9185 + 0.8885] + 2{,}500{,}000 \times 0.8885$$
$$= 2{,}500{,}000$$
$$\Rightarrow \frac{C}{2} = \$74{,}741.92$$
$$\Rightarrow C = \$149{,}483.85.$$

This implies that the coupon rate is

$$c = \frac{149483.85}{2{,}500{,}000} = 0.059794 \equiv 5.9794\%.$$

The Market Method

The convention in the LIBOR market is that if the number of days for which the rate is quoted is N, then the corresponding discount factor is given by

Table 10.6
Discount Factors per Market Convention

Time to Maturity (Months)	Discount Factor
6	0.9744
12	0.9488
18	0.9206
24	0.8929

$$\frac{1}{\left(1+i\times\frac{N}{360}\right)}.$$

Consider the 18-month rate of 5.75 percent. The corresponding discount factor is

$$\frac{1}{\left(1+i\times\frac{540}{360}\right)}=\frac{1}{\left(1+0.0575\times\frac{540}{360}\right)}=0.9206.$$

The vector of discount factors for our example is shown in Table 10.6. The corresponding swap rate is 5.7323 percent.

Valuation of a Swap During Its Life

Let us consider the swap between Bank Exotica and Bank Halifax. Assume that two months have elapsed since the swap was entered into and that the current term structure of interest rates is as given in Table 10.7. We have used the market method to compute the discount factors.

Table 10.7
The Term Structure of Interest Rate after Two Months

Time to Maturity (Months)	Interest Rate per Annum (%)	Discount Factor
4	5.50	0.9820
10	5.80	0.9539
16	6.05	0.9254
22	6.25	0.8972

The value of the fixed-rate bond can be obtained using the following equation:

$$\begin{aligned}&\frac{C}{2} \times [0.9820 + 0.9539 + 0.9254 + 0.8972] + 2{,}500{,}000 \times 0.8972\\&= 74{,}741.92 \times 3.7584 + 2{,}500{,}000 \times 0.8972\\&= \$\ 2{,}523{,}910.\end{aligned}$$

The value of the floating-rate bond may be computed as follows. The next coupon known for it would have been set at time zero—that is two months before the date of valuation. The magnitude of this coupon is

$$0.0525 \times 2{,}500{,}000 \times \frac{1}{2} = \$65{,}625.$$

Once this coupon is paid, the value of the bond will revert back to its face value of 2,500,000. The value of this bond after four months will be $2,565,625. Its value today is:

$$2{,}565{,}625 \times 0.9820 = \$2{,}519{,}444.$$

From the standpoint of the fixed-rate payer, the swap is equivalent to a long position in a floating-rate bond that is combined with a short position in a fixed-rate bond. The value of the swap for the fixed-rate payer is

$$2{,}519{,}444 - 2{,}523{,}910 = (\$4{,}466).$$

In other words, because the value of the fixed-rate liability is higher than that of the floating-rate asset, the value of the swap for Bank Exotica, the fixed-rate payer, is negative. This implies that if the bank seeks a cancellation of the original swap, then it will have to pay $4,466 to the counterparty. This transaction can only take place with the consent of the counterparty, of course.

From the standpoint of the counterparty, the value of the swap is $4,466. In the event of cancellation of the contract, it will get a cash flow of this magnitude from Bank Exotica.

Terminating a Swap

As we have just seen, one way to exit from an existing swap contract is to have it cancelled with the approval of the counterparty. This will entail an inflow or outflow of the value of the swap, depending on how interest rates have moved, after the last periodic cash flow was exchanged. In market parlance, this is known as a *buyback* or *closeout*. In our illustration, Bank Exotica would have to pay $4,466 to Bank Halifax to close out the contract. Bank Exotica will have the option of selling the swap to a party other than Bank Halifax, the original counterparty. This would, of course, require the approval of Bank Halifax. In this case as well, the party who buys the swap from the bank will expect to be paid the value of the swap, which is $4,466 in this case.

Another way for Bank Exotica to exit from its commitment would be to do an opposite swap with a third party. It would have to do a 22-month swap with a party in which it pays LIBOR and receives the fixed rate. This is known as a *reversal*.

Remember: if it were to do so, two swaps would exist. So Bank Exotica would be exposed to credit risk from the standpoint of both counterparties.

The Role of Banks in the Swap Market

In the early years, when the swap market was evolving, it was a standard practice for banks to play the role of an intermediary. They would bring together two counterparties in return for what was termed an *arrangement fee*. As the market has evolved, such arrangement fees have become extremely rare except perhaps for contracts that are very exotic or unusual.

These days, most banks will don the mantle of a principal. The reasons why they are required to do so are twofold. First, nonbank counterparties are reluctant to reveal their identity when entering into such deals, so they are more comfortable dealing with a bank while negotiating. Second, for most parties to such transactions, it is easier to evaluate the credit risk while dealing with a bank than while negotiating with a nonbank counterparty. And, as we have seen, swaps being OTC transactions always carry an element of counterparty risk.

In the days when the market was in its infancy, banks would primarily do reversals. They would do a fixed−floating deal with a party only if they were hopeful of immediately concluding a floating−fixed deal for the same tenor with a third party. Parties that carry equal and offsetting swaps in their books are said to be running a *matched book*. As we saw earlier, such parties are exposed to default risk from both counterparties. These days, banks are less finicky about maintaining such a matched position, and in most cases they are willing to take on the inherent exposure for the period until they can eventually locate a party for an offsetting transaction.

Motivation for the Swap

A party to a swap may enter into the contract with a speculative motive or with an incentive to hedge. In addition, such transactions may also be used to undertake what is known as *credit arbitrage* arising from the comparative advantage enjoyed by the participating institutions. We will analyze each potential use.

Speculation

Morgan Bank and Brown Brothers Bank are both players in the U.S. capital market, but they have different expectations as to where interest rates are headed. Morgan speculates that rates are likely to steadily decline over the next two years, whereas Brown Brothers expects rates to rise steadily over the same period. Assume they enter into a coupon swap with a tenor of two years in which Morgan agrees to pay interest at LIBOR every three months on a notional principal of $100 million, and Brown Brothers agrees to pay interest on the same notional principal and with the same frequency but at a fixed rate of z percent per annum.

Both parties are speculating on the interest rate. If Morgan is right and rates do decline as it anticipates, then its floating-rate payments are likely to be lower than its fixed-rate receipts, thereby leading to net cash inflows for it. But if Brown Brothers is correct and rates rise steadily as it anticipates, its floating-rate receipts are likely to be in excess of its fixed-rate payments, thereby leading to net cash inflows for it. Because both parties are speculating, *ex post* one will stand vindicated and the other will have to countenance a loss.

Hedging

Swaps can be used as a hedge against anticipated interest-rate movements. Donutz a company based in Detroit, has taken a loan from First National Bank at a rate of LIBOR + 75 basis points (bp). The company is worried that rates are going to increase and seeks to hedge by converting its liability into an effective fixed-rate loan. One way to do so would be to renegotiate the loan and have it converted to a loan carrying a fixed rate of interest. This may not be easy in real life, however. There will be a lot of administrative and legal issues and related costs. It may be easier for the company to negotiate a coupon swap in which it pays fixed and receives LIBOR.

Assume that Morgan Bank agrees to enter into a swap with Donutz, paying LIBOR in return for a fixed interest stream based on a rate of 5.75 percent per annum.

The net result from the standpoint of Donutz may be analyzed as follows:

- Outflow 1 (interest on the original loan): LIBOR + 75 bp
- Inflow 1 (receipt from Morgan Bank): LIBOR
- Outflow 2 (payment to Morgan Bank): 5.75 percent
- Net outflow: 5.75% + 0.75% = 6.50% per annum.

The company has converted its loan to an effective fixed-rate liability carrying interest at the rate of 6.50 percent per annum.

Comparative Advantage and Credit Arbitrage

At times there are situations in which a party, despite being at a disadvantage from the standpoint of interest payments with respect to another party in both fixed-rate and variable-rate debt markets, may enjoy a comparative advantage in one of the two.

Assume that Infosys, a software company based in San Jose, can borrow at a fixed rate of 7.50 percent per annum and at a variable rate of LIBOR + 125 bp. IBM is in a position to borrow at a fixed rate of 6 percent per annum and a variable rate of LIBOR + 60 bp. Infosys has to pay 150 bp more as compared to IBM if it borrows at a fixed rate but only 65 basis points more if it borrows at a floating rate. We say that although IBM enjoys an absolute advantage from the standpoint of borrowing in terms of both fixed-rate and floating-rate debt, Infosys has a comparative advantage if it borrows on a floating-rate basis.

Assume that Infosys wants to borrow at a fixed rate while IBM would like to borrow at a floating rate. It can be demonstrated that an interest-rate swap can be used to lower the effective borrowing costs for both parties compared to what they would have had to pay in its absence.

Let us assume that IBM borrows $10 million at a fixed rate of 6 percent per annum, and Infosys borrows the same amount at LIBOR + 125 bp. The two parties can then enter into a swap in which Infosys agrees to pay interest on a notional principal of $10 million at the rate of 5.75 percent per annum in exchange for a payment based on LIBOR from IBM. The effective interest rate for the two parties may be computed as follows:

- IBM: 6% + LIBOR − 5.75% = LIBOR + 25 bp
- Infosys: LIBOR + 125 bp + 5.75% − LIBOR = 7.00%

IBM has a saving of 35 basis points on the floating-rate debt, and Infosys has a saving of 50 bp on the fixed-rate debt. What exactly does this cumulative saving of 85 basis points represent? IBM has an advantage of 1.50 percent in the market for fixed-rate debt and 65 bp in the market for floating-rate debt. The difference of 85 basis points manifests itself as the savings for both parties considered together.

Let us introduce a bank into the picture. Assume that IBM borrows at a rate of 6 percent per annum and enters into a swap with First National Bank in which it must pay LIBOR in return for a fixed-rate stream based on a rate of 5.65 percent. Infosys, on the other hand, borrows at LIBOR + 125 bp and enters into a swap with the same bank, receiving LIBOR in return for payment of 5.85 percent.

The net result of the transaction may be summarized as follows:

- IBM: Effective interest paid = 6% + LIBOR − 5.65% = LIBOR + 35 bp
- Infosys: Effective interest paid = LIBOR + 125 bp + 5.85% − LIBOR = 7.10%
- First National Bank: Profit from the transaction = LIBOR − 5.65% − LIBOR + 5.85% = 20 bp.

The difference in this case is that the comparative advantage of 85 basis points has been split three ways. IBM saves 25 basis points, Infosys saves 40 basis points, and the bank makes a profit of 20 basis points.

We must point out that the transaction that entails a role for the bank is more realistic than a deal in which the two companies identify and directly enter into a swap with each other.

Swap Quotations

The swap rate in the market can be quoted in four different ways.[3] First, there are two possibilities with respect to the interest payments: they may be on either an annual basis or a semiannual basis. Second, payments may be settled either on a bond market or on a money market basis, the difference being that the first convention is based on a 365-day year and an actual/365 day-count convention, whereas the second is based on a 360-day year and an actual/360 day count convention.

Quotes on a semiannual basis can be converted to equivalent values on an annual basis and vice versa. Example 10.4 illustrates the required procedures.

Example 10.4

The fixed rate for a swap is given as 7.25 percent per annum payable semiannually. To convert this to the corresponding value for an annual pay basis, we need to find the effective annual rate. This is given by

$$\left(1+\frac{0.0725}{2}\right)^2-1$$
$$=0.0738 \equiv 7.38\%.$$

Consider a situation in which the fixed rate is quoted as 7.25 percent per annum payable annually. To convert this to an equivalent rate for a semiannual pay basis, we need to find the nominal annual rate that will yield 7.25 percent on an effective annual basis if compounding is undertaken semiannually.

This may be calculated as

$$\left(1+\frac{r}{2}\right)^2=1.0725$$
$$\Rightarrow \frac{r}{2}=(1.0725)^{0.50}-1$$
$$r=2\times 0.0356=0.0712 \equiv 7.12\%.$$

The fixed rates payable are quoted on a bond basis, whereas floating rates are quoted on a money market basis. To convert from a bond basis to a money market basis, we have to multiply the quote by 360/365; if we seek to do the reverse, we multiply by 365/360.

Matched Payments

Numero Uno Corporation has issued bonds carrying a coupon of 6.25 percent per annum. It is of the opinion that market rates could decline, so it wishes to undertake a swap in which it has to pay floating and receive fixed. Assume that the bonds pay interest on a semiannual basis and that the swap also entails a six-month exchange of cash flows. The principal of the bonds is assumed to be \$10 million, as is the notional principal of the swap.

We will assume that Bank Deux is willing to arrange a swap that is suitable for Numero Uno but with a swap rate of 5.80 percent per annum. However, the company wants a fixed payment of \$312,500 to match the cash outflow on account of the bonds it issued. Bank Deux would accommodate the request as follows. Because Numero Uno wants to receive a higher fixed payment compared to what it would ordinarily have received, it must compensate it in the form of a positive spread with respect to the floating-rate payment that it is required to make. In this case, the difference in the fixed-rate payments is \$45,000 per annum, which corresponds to 45 basis points. So the bank will require Numero Uno to make a payment based on a rate of LIBOR + 45 basis points while making the floating counterpayment every six months.

Currency Swaps

A currency swap is like an interest-rate swap in the sense that it requires two counterparties to commit themselves to the exchange of cash flows at prespecified intervals. The difference in this case is that the two cash-flow streams are denominated in two different currencies. The two counterparties also agree to exchange, at the end of the stated time period or the maturity date of the swap, the corresponding principal amounts computed at an exchange rate that is fixed right at the outset.[4]

As mentioned earlier, because the counterpayments are denominated in two different currencies, there are three possibilities from the standpoint of interest computation. In other words, these swaps may be on any of the following bases:

- Fixed rate−fixed rate;
- Fixed rate−floating rate;
- Floating rate−floating rate.

A currency swap always requires an exchange of principal at the time of maturity. However, there do exist contracts that entail an exchange of principal twice: at inception as well as at maturity. Such swaps are referred to as *cash swaps*.

We will assume that the swap described in the preceding illustration is a cash swap. Such a contract will typically entail the following transactions:

- Bank Atlantic will borrow US$7.5 million in New York.
- Bank Europeana will borrow €6 million in Frankfurt.
- Bank Atlantic will transfer the dollars to Bank Europeana in exchange for the equivalent payment denominated in euros.
- On the maturity date of the swap, the principal amounts will be swapped back. In most cases, the terminal exchange is based on the exchange rate that was prevailing at the outset—in other words, the same exchange rate that was used to compute the initial exchange of principal. In our illustration, therefore, we will assume that Bank Atlantic will make a payment of €6 million to Bank Europeana after two years in exchange for a cash flow of US$7.5 million. Such a swap in which the same amount of principal is exchanged is known as a *par swap*.[5]
- At the end of two years, Bank Atlantic would use the dollars received by it to repay the U.S. dollar−denominated loan that it had taken two years prior.
- Bank Europeana would use the euros received by it to repay the loan that it had taken in Frankfurt two years earlier.

As given in Example 10.5, a swap transaction such as this enables each party to service the debt of the counterparty. Bank Atlantic has taken a dollar-denominated loan. It will service it using the dollar-denominated payments that it receives every six months from Bank Europeana. Similarly, Bank Europeana has taken a euro-denominated loan that it will service using the euro-denominated payments it periodically receives from Bank Atlantic.

Example 10.5

Bank Atlantic and Bank Europeana agree to execute a contract in which they will exchange cash flows denominated in U.S. dollars and the euro over two years. Per the terms of the agreement, Bank Atlantic will make a payment at the end of every six months denominated in euros and at an interest rate of 4.25 percent per annum. The payments will be based on a principal amount of €6 million. Bank Europeana, on the other hand, will make payments at the same frequency but denominated in dollars and with an interest rate of 5.40 percent per annum. These payments will be based on a principal of US$7.5 million.

At the end of the two-year period, Bank Atlantic will pay €6 million to Bank Europeana, which in turn will make a counterpayment of US$7.5 million. The implicit exchange rate of 0.8000 USD−EUR is fixed right at the outset and is typically the spot rate of exchange prevailing in the currency market at that point in time.

Cross-Currency Swaps

Technically speaking, the term *currency swap* is applicable only for transactions that entail the exchange of cash flows computed on a fixed rate−fixed rate basis, such as the deal that we have just studied. Currency swaps in which one or both payments are based on a floating rate of interest should, strictly speaking, be termed *cross-currency* swaps. Within cross-currency swaps we make a distinction between coupon swaps, which are on a fixed rate−floating rate basis, and basis swaps, which are on a floating rate−floating rate basis.

Valuation

We will explore the mechanics of currency-swap valuation by focusing on the swap between Bank Atlantic and Bank Europeana. The swap may be viewed as a combination of the following transactions. Assume that Bank Atlantic has issued a fixed-rate bond in euros with a principal of €6 million and converted the proceeds to dollars at the spot rate of 0.8000 USD−EUR. The proceeds in dollars can be perceived as having been invested in fixed-rate bonds denominated in dollars. From the perspective of the counterparty, the transaction may be perceived as follows. Assume that Bank Europeana has issued fixed-rate, dollar-denominated bonds with a face value of US$7.5 million, converted the proceeds to euros at the prevailing spot rate, and invested the equivalent in euros in fixed-rate bonds in that currency. Every six months Bank Atlantic will receive interest in dollars and pay interest in euros, and Bank Europeana will pay interest in dollars and receive interest in euros. So a currency swap between two parties is equivalent to a combination of transactions in which each party issues a bond in one currency to the other, and uses the proceeds to acquire a bond issued by the counterparty. The fixed rate that is applicable in either currency is the coupon rate associated with a par bond in that currency, as we have seen in the case of fixed−floating

Table 10.8
Term Structure for U.S. Dollars and the Euro

Time to Maturity (Months)	USD−LIBOR (%)	Discount Factor	Euro−LIBOR (%)	Discount Factor
6	4.25	0.9792	3.80	0.9814
12	4.40	0.9579	3.60	0.9653
18	4.75	0.9335	3.25	0.9535
24	4.60	0.9158	3.50	0.9346

interest-rate swaps. However, because two currencies are involved, we need the term structure of interest rates for both currencies.

Consider the data given in Table 10.8.

The value of the fixed rate for U.S. dollar may be determined as follows. Consider a bond with a face value of US$2.5 million. The coupon for a par bond denominated in U.S. dollar is given by

$$\begin{aligned}
&\frac{C}{2} \times [0.9792 + 0.9579 + 0.9335 + 0.9158] + 2{,}500{,}000 \times 0.9158 \\
&\quad = 2{,}500{,}000 \\
&\Rightarrow \frac{C}{2} = \$55{,}593.70 \\
&\Rightarrow C = \$111{,}187.40.
\end{aligned}$$

This corresponds to a rate of 4.4475 percent per annum.

Similarly we can compute the coupon for a bond denominated in euros

$$\begin{aligned}
&\frac{C}{2} \times [0.9814 + 0.9653 + 0.9535 + 0.9346] + 2{,}500{,}000 \times 0.9346 \\
&\quad = 2{,}500{,}000 \\
&\Rightarrow \frac{C}{2} = \$42{,}635.86 \\
&\Rightarrow C = \$85{,}271.72.
\end{aligned}$$

This corresponds to a coupon of 3.4109 percent per annum.

If the swap had been such that the rate for dollars was fixed while that for the payment in euros was variable, then the applicable rate would be 4.4475 percent for the dollar-denominated payments and LIBOR for the counterpayments. On the other hand, if the rate is fixed for the payments in euros and is variable for the dollar-denominated payments, then the applicable rate is LIBOR for the payment in dollars and 3.4109 for the payments in euros. Finally, if payments in both currencies are on a floating-rate basis, then it is LIBOR for LIBOR.

Currency Risks

A currency swap, as we would expect, exposes both parties to currency risk. Let us consider the swap in which Bank Atlantic borrows and makes a payment of US$7.5 million to Bank Europeana in return for a counterpayment of €6 million. The exchange rate for the cash-flow swap was 0.8000 USD−EUR.

At maturity, Bank Europeana would pay back US$7.5 million to the U.S. bank and receive €6 million in return. Assume that in the intervening three years the dollar has appreciated to 0.8125 USD−EUR. Bank Atlantic would benefit by the fact that the terminal exchange of principal is based on the original exchange rate of 0.8000 USD−EUR. Because, if €6 million are converted at the prevailing rate of 0.8125 USD−EUR, it stands to receive only US$7,384,615. Therefore, the counterparty that is slated to receive the currency that has appreciated during the life of the swap stands to gain from the fact that the terminal exchange of principal is based on the exchange rate prevailing at the outset, whereas the other party stands to lose. In our illustration, Bank Atlantic avoids a loss of US$115,385, and Bank Europeana forgos an opportunity to save an identical amount.

Hedging with Currency Swaps

A swap can be used as a mechanism for hedging foreign-currency exposure. Telekurs, a telecom company based in Frankfurt, has issued a Yankee bond for US$25 million, carrying interest at the rate of 6 percent per annum payable semiannually. The bonds have four years to maturity, and the company seeks to hedge its exposure to the U.S. dollar−euro exchange rate, because all its income is primarily denominated in euros. Assume that the current exchange rate is 0.8000 USD−EUR.

The company can use a swap to hedge its currency exposure. Because it has a payable in U.S. dollars, it needs a contract in which it will receive cash flows in dollars and make payments in euros. If the company is of the opinion that rates in the euro zone are likely to rise, then it can negotiate a swap in which it makes a fixed rate−based payment in euros to a counterparty in exchange for a fixed rate−based income stream in dollars. The dollar inflows should be structured to match the company's projected outflows in that currency. This will lead to a situation in which its exposure to the dollar is perfectly hedged.

Endnotes

1. In some swap markets, the fixed-rate payer is termed the *buyer* and the counterparty is termed the *receiver*.
2. The discount factor for a given maturity is the present value of a dollar to be received at the end of the stated period.
3. See Coyle (2001).
4. See Geroulanos (1998).
5. See Geroulanos (1998).

APPENDIX 1

Partial Derivative of the Price with Respect to the Yield

From the bond-pricing equation, we know that

$$P = \frac{\frac{C}{2}}{\frac{y}{2}} \times \left[1 - \frac{1}{\left(1 + \frac{y}{2}\right)^N}\right] + \frac{M}{\left(1 + \frac{y}{2}\right)^N}.$$

The derivative of the second term with respect to the yield is

$$-\frac{NM}{2\left(1 + \frac{y}{2}\right)^{N+1}}.$$

which is obviously negative.

We will now demonstrate that the derivative of the first term is also negative by considering the present value of an annuity that pays \$$A$ per period.

$$\mathrm{PV} = \frac{A}{r}\left[1 - \frac{1}{(1+r)^N}\right]$$

$$\frac{\partial \mathrm{PV}}{\partial r} = -\frac{A}{r^2}\left[1 - \frac{1 + r(N+1)}{(1+r)^{N+1}}\right].$$

We need to show that the term in the bracket is positive or, in other words, that

$$1 + r(N+1) < (1+r)^{N+1}.$$

From the expression for a Maclaurin series, we know that

$$(1+r)^{N+1} = 1 + r(N+1) + \frac{N(N+1)r^2}{2} + h.o.t.$$

Thus, it is indeed the case that

$$1 + r(N+1) < (1+r)^{N+1}.$$

Hence, the partial derivative of the present value of the annuity with respect to the discount rate is negative. Because the bond price consists of an annuity and a terminal cash flow, the derivative of whose present value has already been shown to be negative, we conclude that the partial derivative of the price of a plain vanilla bond with respect to the YTM is negative.

APPENDIX 2

We will derive the expression for y given that

$$P\left(1+\frac{y}{2}\right)\left(1+\frac{y}{2}\left\{\frac{T_m-\frac{365}{2}}{\frac{365}{2}}\right\}\right)=V.$$

From the above expression, when we expand the terms we get

$$P+\frac{Py}{2}+\frac{Py}{2}\times\frac{2T_m-365}{365}+\frac{Py^2}{4}\times\frac{2T_m-365}{365}=V$$
$$\Rightarrow y^2\times\frac{P}{4}\times\frac{2T_m-365}{365}+y\times\frac{P}{2}\times\left(1+\frac{2T_m-365}{365}\right)+(P-V)=0.$$

This is a quadratic equation of the form

$$ax^2+bx+c=0.$$

The roots are given by

$$\frac{-b\pm\sqrt{b^2-4ac}}{2a}.$$

Substituting the appropriate values for a, b, and c, and simplifying, we get,

$$\frac{-\frac{2T_m}{365}\pm 2\sqrt{\left(\frac{T_m}{365}\right)^2-\left(\frac{2T_m}{365}-1\right)\left(1-\frac{V}{P}\right)}}{\frac{2T_m}{365}-1}.$$

Bibliography

Apte, P. G. *International Financial Management*. New Delhi: Tata McGraw-Hill, 2006.

Apte, P. G. *International Financial Management*. New Delhi: Tata McGraw-Hill, 2008.

Bose, R. *An Introduction to Documentary Credits*. New Delhi: Macmillan, 2006.

Chance, D. M. *An Introduction to Options and Futures*. New York: The Dryden Press, 1991.

Chance, D. M. *An Introduction to Derivatives and Risk Management*. Mason, OH: South-Western, 2004.

Chicago Board Options Exchange. *Margin Manual*. Chicago: Author, 2000.

Chicago Board Options Exchange. *Understanding Index Options*. Chicago: Author, 2001.

Choudhry, M. *An Introduction to Repo Markets*. London: Securities Institute, 2001.

Choudhry, M. *An Introduction to Bond Markets*. London: Securities Institute, 2001.

Choudhry, M. *Analyzing and Interpreting the Yield Curve*. New York: John Wiley, 2004.

Comotto, R. *The Money Market*. London: Euromoney Publications, 1998.

Common Stocks, Preferred Stocks—Basic Concepts: http://thismatter.com

Copeland, T. L. "Liquidity Changes Following Stock Splits". *Journal of Finance*, March 1979.

Coyle, B. *Currency Swaps*. London: Financial World Publishing, 2000.

Coyle, B. *Equity Finance*. London: Financial World Publishing, 2002.

Coyle, B. *Interest-Rate Swaps*. London: Financial World Publishing, 2001.

Coyle, B. *Money Markets*. London: Financial World Publishing, 2001.

Coyle, B. *Overview of the Markets*. London: Financial World Publishing, 2002.

Curley, M. T. *Margin Trading from A to Z*. New York: John Wiley, 2008.

Damodaran, A. *Investment Valuation*. New York: John Wiley, 1996.

Davidson, A., Sanders, A., Wolff, L., and Ching, A. *Securitization*. New York: John Wiley, 2003.

Dembroski, S. *The Globalization of Financial Markets*. London, Ontario: University of Western Ontario, 1990.
Dodd, R. *Primer on Repurchase Agreements Market*. Online at www.financialpolicy.org.
Dubofsky, D. A. *Options and Financial Futures: Valuation and Uses*. New York: McGraw-Hill, 1992.
Edwards, F. R., and Ma, C. W. *Futures and Options*. New York: McGraw-Hill, 1992.
Fabozzi, F. J., and Modigliani, F. *Mortgage and Mortgage-Backed Securities Markets*. Cambridge, MA: Harvard Business School Press, 1992.
Fabozzi, F. J. *Bond Markets: Analysis and Strategies*. Englewood Cliffs, NJ: Prentice Hall, 1996.
Fabozzi, F. J., Bhattacharya, A. K., and Berliner, W.S. *Mortgage-Backed Securities*. New York: John Wiley, 2007.
Ferri, M. G., and Kidwell, D. S. *Overview of the Financial System in Selected Topics in Investment Management for Financial Planning*, F. J. Fabozzi and S. Kole, eds. New York: Dow Jones-Irwin, 1985.
Figlewski, S., Silber, W. L., and Subrahmanyam, M. G. *Financial Options: From Theory to Practice*. New York: Irwin, 1990.
Garbade, K. D. *Fixed Income Analytics*. Cambridge: MIT Press, 1996.
Geddes, H. R. *An Introduction to Corporate Finance: Transactions and Techniques*. London: Securities Institute, 2001.
Geisst, C. R. *A Guide to Financial Institutions*. New York: Macmillan, 1993.
Geroulanos P. *An Introduction to Swaps*. London: The Securities Institute, 1998.
Gibson, N. *Essential Finance*. London: The Economist/Profile Books, 2004.
Gray, S. *Repo of Government Securities: Handbooks in Central Banking No. 16*. London: Bank of England, 1998.
Guide to the Markets—Investor's Business Daily
Harris, L. *Trading and Exchanges: Market Microstructure for Practitioners*. London: Oxford University Press, 2003.
Hegde, S. P., and Parameswaran, S. K. *Futures Markets: 200 Questions and Answers*. New York: John Wiley, 2007.
Hull, J. *Fundamentals of Futures and Options Markets*. Englewood Cliffs, NJ: Prentice Hall, 2004.
Hull, J. *Fundamentals of Futures and Options Markets*. Upper Saddle River, NJ: Prentice Hall, 2004.
Hull, J. *Options, Futures, and Other Derivatives*. Upper Saddle River, NJ: Prentice Hall, 2006.
Kellison, S. G. *The Theory of Interest*. New York: Irwin McGraw-Hill, 1991.
Kolb, R. W. *Futures, Options, and Swaps*. New York: Blackwell, 2003.

Kolev, I. *Mortgage-Backed Securities*. Washington, DC: Financial Policy Forum, 2004.

Koontz, S. R., and Purcell, W. D. *Agricultural Futures and Options: Principles and Strategies*. Englewood Cliffs, NJ: Prentice Hall, 1999.

Liaw, K. T. *Capital Markets*. Mason, OH: Thomson South-Western, 2004.

Liaw, K. T. *Capital Markets*. Mason, OH: Thomson/South-Western, 2004.

Liaw, K. T. *The Business of Investment Banking*. New York: John Wiley, 1999.

Livingston, M. *Bonds and Bond Derivatives*. New York: Blackwell Publishing, 2004.

Lofton, T. *Getting Started in Futures*. New York: John Wiley, 2005.

Logue, D. E. *The WG&L Handbook of International Finance*. Mason, OH: South-Western, 1995.

McDonald, R. L. *Derivatives Markets*. New York: Pearson Education, 2006.

McInish, T. H. *Capital Markets: A Global Perspective*. New York: Blackwell, 2000.

Melvin, M. *International Money and Finance*. New York: Addison Wesley, 1997.

Mobius, M. *Foreign Exchange: An Introduction to the Core Concepts*. New York: John Wiley, 2009.

Options Clearing Corporation. *Long-Term Equity Anticipation Securities*. Chicago: Author, 2004.

Options Clearing Corporation. *Options: Essential Concepts and Trading Strategies*. New York: McGraw-Hill, 1999.

Options Clearing Corporation. *Understanding Stock Options*. Chicago: Author, 1994.

Palat, R. *Retail Banking*. India: Executive Excellence Books, 2006.

Parameswaran, S. K. *Futures and Options: Concepts and Applications*. New Delhi: Tata McGraw-Hill, 2010.

Parameswaran, S. K. *Futures and Options: Concepts and Applications*. New Delhi: McGraw-Hill, 2010.

Reilly, F. K., and Brown, K. C. *Investment Analysis and Portfolio Management*. New York: The Dryden Press, 2000.

Resnick, B. G. *The Globalization of World Financial Markets in Advances in Business Financial Management: A Collection of Readings*, P. L. Cooley, ed. New York: The Dryden Press, 1996.

Rose, P. S. *Money and Capital Markets*. New York: McGraw-Hill, 2000.

Rose, P. S. *Commercial Bank Management*. New York: McGraw-Hill Irwin, 2002.

Rose, P. S. *Money and Capital Markets*. New York: McGraw-Hill, 2000.

Ross, S. A., Westerfield, R. W., and Jaffe, J. *Corporate Finance*. New York: Irwin McGraw-Hill.

Saunders, A., and Cornett, M. M. *Financial Institutions Management*. New York: McGraw-Hill, 2003.

Scott, D. L. *How Wall Street Works*. New York: McGraw-Hill, 1999.

Siegel, D. R., and Siegel, D. F. *Futures Markets.* New York: The Dryden Press, 1990.

Siegel, D. R., and Siegel, D. F. *Futures Markets*. New York: The Dryden Press, 1990.

Siegel, D. R., and Siegel, D. F. *Futures Markets*. New York: The Dryden Press, 1990.

Simmons, M., and Dalgleish, E. *Corporate Actions: A Guide to Securities Event Management*. New York: John Wiley, 2006.

Smith, S. D., and Spudeck, R. E. *Interest Rates: Principles and Applications*. Belmont, CA: Thomson Learning, 1993.

Steiner, B. *Foreign Exchange and Money Markets*. Oxford, UK: Butterworth-Heinemann, 2002.

Steiner, B. *Foreign Exchange and Money Markets*. London: Butterworth-Heinemann, 2002.

Steiner, R. *Mastering Financial Calculations*. London: Financial Times Prentice Hall, 1998.

Stigum, M., and Robinson, F. L. *Money Market and Bond Calculations*. New York: Irwin, 1996.

Stigum, M., and Crescenzi, A. *Stigum's Money Market*. New York: McGraw-Hill, 2007.

Stoll, H. R., and Whaley, R. E. *Futures and Options: Theory and Applications*. Mason, OH: South-Western, 1993.

Strumeyer, G. *Investing in Fixed Income Securities*. New York: John Wiley, 2005.

Subject: Stocks—Tracking Stock: http://invest-faq.com

Sundaresan, S. *Fixed Income Markets and Their Derivatives*. San Diego: Academic Press, 2009.

Taff, L. G. *Investing in Mortgage Securities*. St. Lucie Press, 2003.

Taggart, R. A. *Quantitative Analysis For Investment Management.* Englewood Cliffs, NJ: Prentice Hall, 1996.

Taggart, R. A. *Quantitative Analysis for Investment Management*. Englewood Cliffs, Prentice Hall, 2002.

Teweles, R. J., and Bradley, E. S. *The Stock Market*. New York: John Wiley, 1998.

Viney, C. *Financial Institutions, Instruments, and Markets*. New York: McGraw-Hill, 2000.

Weiss, D. M. *After The Trade is Made.* New York: New York Institute of Finance, 1993.

Web Sites

British Bankers' Association (http://www.bba.org.uk)
Canada Business (www.canadabusiness.ca)
Chicago Board Options Exchange (http://www.cboe.com)
Citi Depositary Receipt Services (http://www.citiadr.idmanagedsolutions.com/www/front_page.idms)
Commercial Loan Advice (http://www.loanuniverse.com)
Credit and Collections (http://www.creditmanagementworld.com)
Dow Jones Indexes (http://www.djindexes.com)
Euromoney (www.euromoney.com)
Fannie Mae (http://www.fanniemae.com)
Federal Financial Institutions Examination Council (http://www.ffiec.gov)
Federal Reserve Bank of New York (www.newyorkfed.org)
Federal Reserve of New York (http://www.newyorkfed.org)
Financial Careers (http://www.financecareers.about.com)
Ginnie Mae (http://www.ginniemae.com)
Import-Export Trade (http://malaysiaexports.com)
Incorporation Service (http://www.activefilings.com)
Investing for Beginners (http://beginnersinvest.about.com)
J.P. Morgan Depositary Receipts (http://adr.com)
Money Blue Book (www.moneybluebook.com)
NASDAQ (http://www.nasdaq.com)
Poznak Law Firm (www.poznaklaw.com)
Real Estate Investment (http://www.mreic.com)
Reuters (www.reuters.com)
SEC (http://www.sec.gov)
Stock Investing Advice (http://www.fool.com)
Stocks (http://stocks.about.com)
The CME Group (http://www.cmegroup.com)
The Investment FAQ (http://invest-faq.com)

Tokyo Stock Exchange (http://tse.or.jp)
Trade and Business Knowledge (http://www.resources.alibaba.com)
Trading Glossary (http://www.trading-glossary.com)
Treasury Securities (http://www.treasurydirect.gov)
Wiki Q&A (http://www.answers.com)
Wikipedia (http://en.wikipedia.org)

Index

G

Z