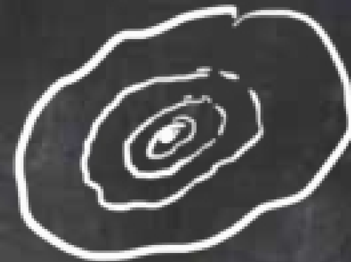
$$\rho = \lim_{\Delta\tau \to 0} \frac{\Delta m}{\Delta\tau} \qquad \bar{\rho} = \frac{\Delta m}{\Delta\tau}$$



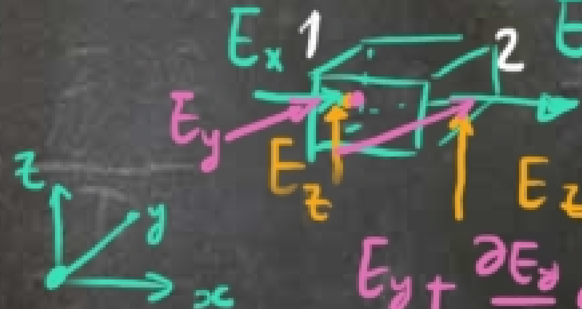
$$\rho = \lim_{\Delta\tau \to \epsilon} \frac{\Delta m}{\Delta\tau}$$

$$\oint \vec{E}\cdot d\vec{a}$$

$$\frac{1}{\Delta\tau}\oint \vec{E}\cdot d\vec{a} =$$ شار از سطح حجم

$$\operatorname{div}(\vec{E}) = \lim_{\Delta\tau \to 0} \frac{\oint \vec{E}\cdot d\vec{a}}{\Delta\tau}$$

دیورژانس



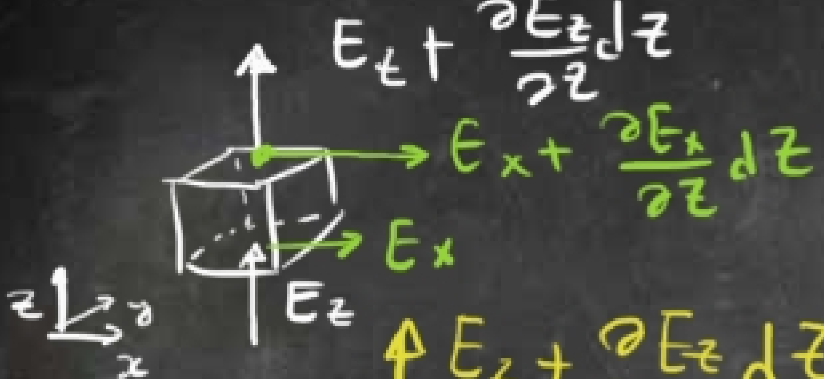
$$E_x + \frac{\partial E_x}{\partial x}dx$$

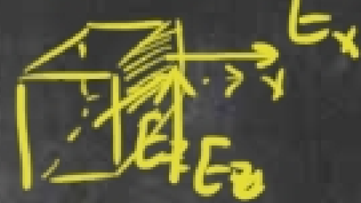
$$E_z + \frac{\partial E_z}{\partial x}dx$$

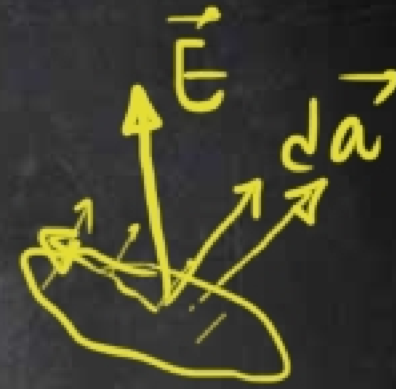
$$E_y + \frac{\partial E_y}{\partial x}dx$$

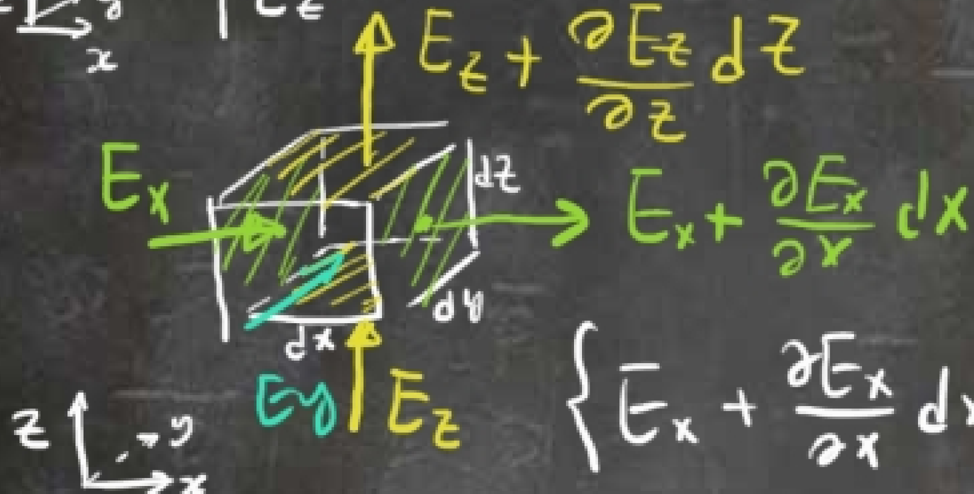
$$\vec{E} \to E_x,\ E_y,\ E_z$$

$$dE_x = \frac{\partial E_x}{\partial x}dx + \frac{\partial E_x}{\partial y}dy + \frac{\partial E_x}{\partial z}dz$$









$$\left\{E_x + \frac{\partial E_x}{\partial x} dx\right\} dy\,dz - E_x\,dy\,dz$$

$$\oint \vec{E} \cdot d\vec{a} = \frac{\partial E_x}{\partial x} dx\,dy\,dz + \frac{\partial E_z}{\partial z} dx\,dy\,dz + \frac{\partial E_y}{\partial y} dx\,dy\,dz$$

$$\mathrm{div}(\vec{E}) = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

$$\operatorname{div}(\vec{E}) = \lim_{\Delta\tau \to 0} \frac{\oint \vec{E}\cdot d\vec{a}}{\Delta\tau}$$

$$= \lim_{\Delta\tau \to 0} \frac{\left\{ \rho\,\Delta\tau / \epsilon_0 \right\}}{\Delta\tau} = \frac{\rho}{\epsilon_0}$$

$$\operatorname{div}(\vec{E}) = \frac{\rho}{\epsilon_0}$$

فرم دیفرانسیلی قانون گاوس

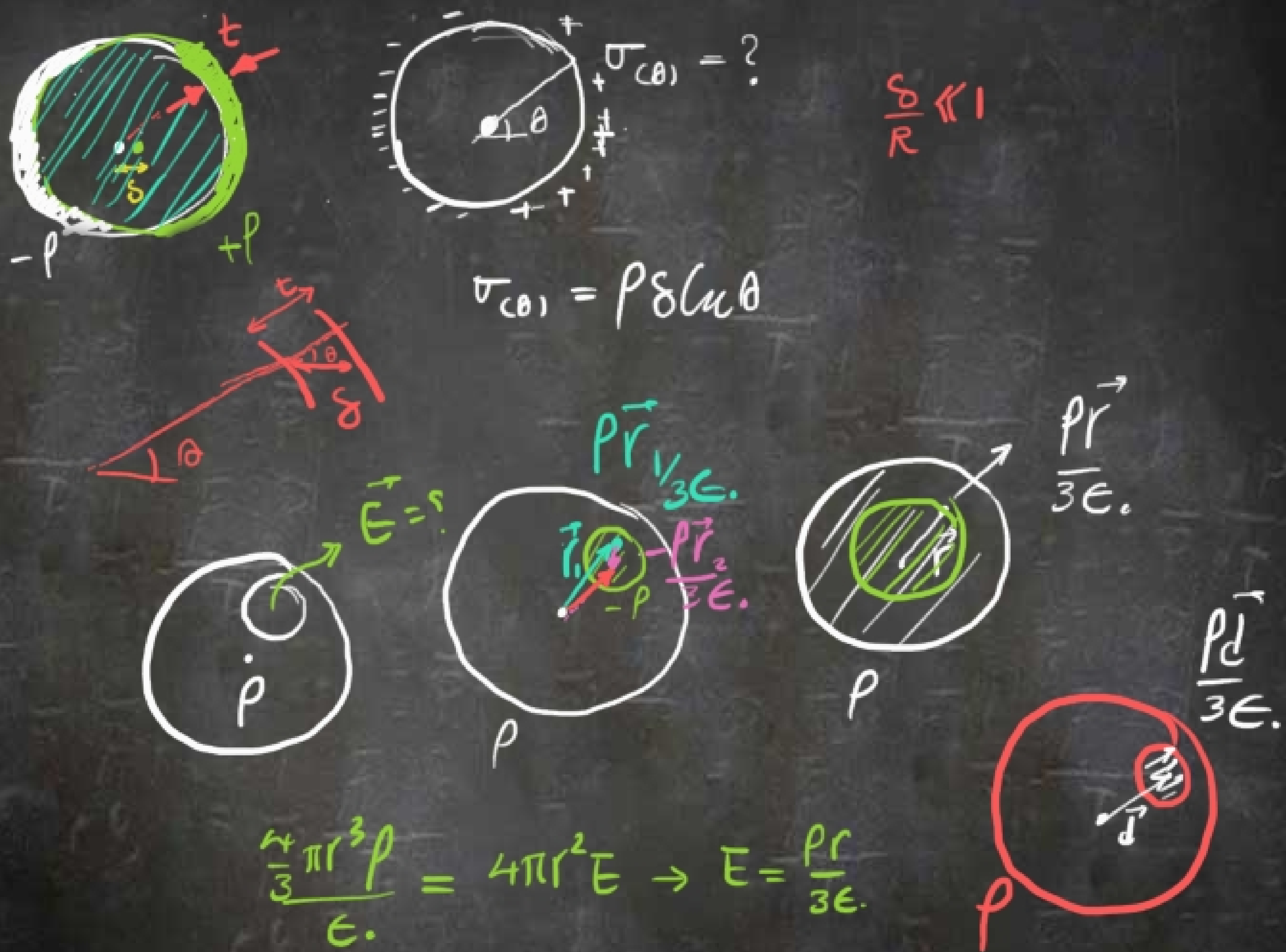
$$\sigma, \lambda, q \rightsquigarrow \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \Rightarrow \infty$$

ناپیوستگی در میدان الکتریکی

$$\sigma(\rho, t) = ?$$

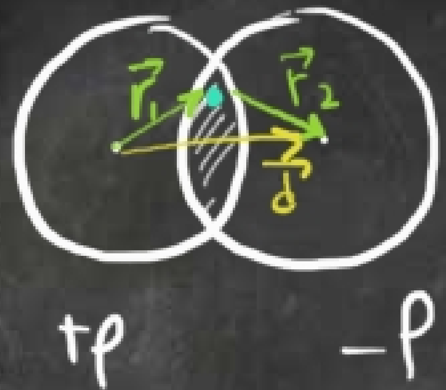
$$\Delta q = \rho\, t\, \Delta s = \sigma\, \Delta s$$

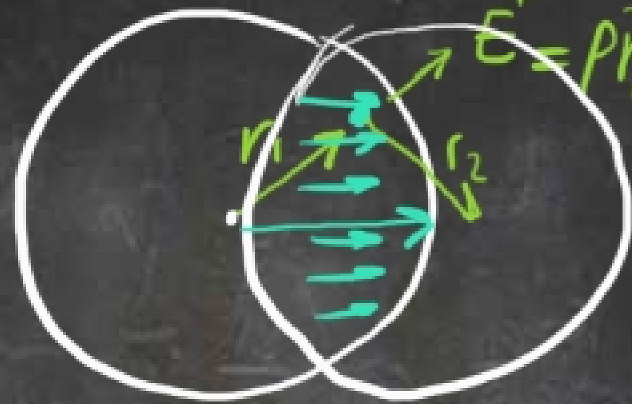
$$\rightarrow \sigma = \rho t$$



$$\sigma(\theta) = \rho\delta\cos\theta$$

$$\frac{\frac{4}{3}\pi r^3 \rho}{\epsilon_0} = 4\pi r^2 E \rightarrow E = \frac{\rho r}{3\epsilon_0}$$





$\vec{E} = \rho\vec{r}_1/3\epsilon_0$

$\vec{E}_2 = \rho\vec{r}_2/3\epsilon_0$

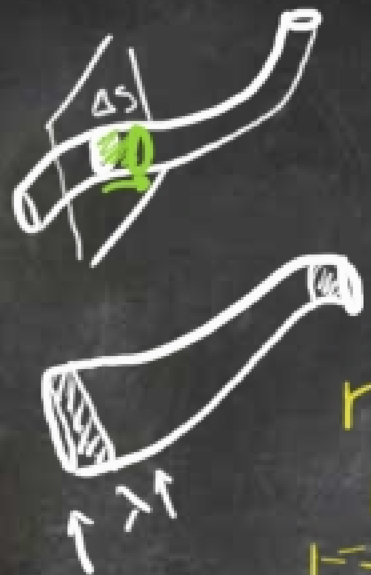
$$\vec{E} = \frac{\rho\vec{r}_1}{3\epsilon_0} + \frac{\rho\vec{r}_2}{3\epsilon_0} = \rho\vec{d}/3\epsilon_0$$

$$\vec{E} = \frac{\rho\delta}{3\epsilon_0}$$

$\sigma = \rho\delta\cos\theta$ ✓

$$E_0 = \frac{\rho\delta}{3\epsilon_0} = \frac{\sigma}{3\epsilon_0\cos\theta}$$

$\sigma = 3\epsilon_0 E_0 \cos\theta$ ✓



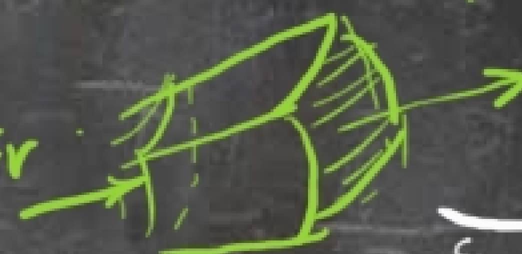
$\Delta S, \rho \rightsquigarrow \lambda \rightarrow \lambda = \rho \Delta S$

$\lambda dx = \rho \Delta S dx$

$r \sin\theta \, d\varphi$

$\rho \, d\varphi$

$E_r + \frac{\partial E_r}{\partial r} dr$



دیورژانس در مختصات کروی

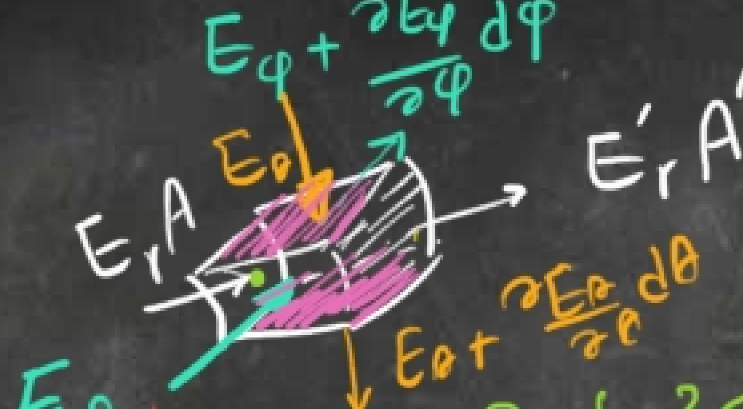
r^2

$2rdr \; r^2$

$$\left\{ E_r + \frac{\partial E_r}{\partial r} dr \right\} \left\{ (r+dr) d\theta \sin\theta \, d\varphi \right\}$$

$$- \left\{ E_r \right\} \left\{ r^2 \sin\theta \, d\theta \, d\varphi \right\}$$

$$= E_r \left(2r\,dr\,d\theta \sin\theta \, d\varphi \right) + \frac{\partial E_r}{\partial r} dr \, r^2 d\theta \sin\theta \, d\varphi$$



$$\frac{\partial}{\partial r}\{E_r A\}\,dr$$

$$d\Phi_r = \frac{\partial}{\partial r}\{r^2 \sin\theta\, d\theta\, d\varphi\, E_r\}\,dr = \sin\theta\, d\theta\, d\varphi\, \frac{\partial}{\partial r}[r^2 E_r]\,dr$$

$$d(xy) = dx\, y + dy\, x = (x+dx)(y+dy) - xy$$

$$\frac{\partial}{\partial r}(E_r A) = \frac{\partial E_r}{\partial r} A + E_r \frac{\partial A}{\partial r}$$

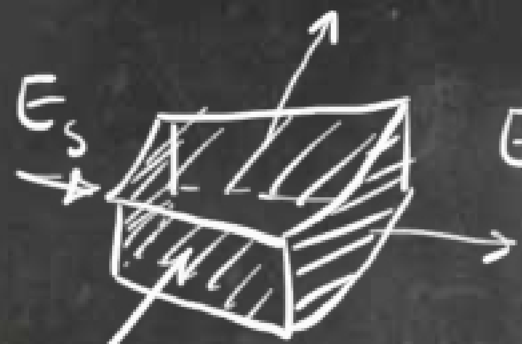
$$d\tau = r^2 \sin\theta\, d\theta\, d\varphi\, dr$$

$$d\Phi_\varphi = \frac{\partial E_\varphi}{\partial \varphi} d\varphi\, dr\, r\, d\theta$$

$$d\Phi_\theta = \frac{\partial}{\partial \theta}\{E_\theta\, r \sin\theta\, d\varphi\, dr\}\,d\theta$$

$$\mathrm{div}(\vec{E}) = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 E_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}(E_\varphi) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(E_\theta \sin\theta)$$

دیورژانس در مختصات استوانه‌ای



$$d\Phi = \frac{\partial}{\partial s}\{E_s\, dz\, s\, d\varphi\} ds + \frac{\partial}{\partial \varphi}\{E_\varphi\, dz\, ds\} \overset{d\varphi}{} + \frac{\partial}{\partial z}\{E_z\, s\, d\varphi\, ds\} \overset{dz}{}$$

$$d\Omega = ds\; s\, d\varphi\; dz$$

$$\operatorname{div}(\vec{E}) = \frac{1}{s}\frac{\partial}{\partial s}(s E_s) + \frac{1}{s}\frac{\partial}{\partial \varphi}\{E_\varphi\} + \frac{\partial}{\partial z} E_z$$

$$x \rightsquigarrow f_{(x)} \to f'_{(x)}$$

$$\frac{d^2}{dx^2}\{f_{(x)}\} = f''_{(x)}$$

خطی

$$L[x+y] = Lx + Ly$$

$$\frac{d}{dx}(f+g) = \frac{d}{dx} f + \frac{d}{dx} g$$

$$\int (f+g)\, dx = \int f\, dx + \int g\, dx$$

$$(A+B)^2 \neq A^2 + B^2 \qquad \sqrt{A+B} \neq \sqrt{A} + \sqrt{B}$$

عملگر غیرخطی

$$\frac{d}{dx} f_{(x)} = \checkmark$$

عملگر دل

$$\vec{\nabla} = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z} \right)$$

$$\vec{\nabla} f_{(x,y,z)} = \hat{x}\frac{\partial f}{\partial x} + \hat{y}\frac{\partial f}{\partial y} + \hat{z}\frac{\partial f}{\partial z}$$

$$\vec{\nabla} \cdot \vec{E} = \mathrm{div}(\vec{E}) \quad !!$$

$$\begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \cdot \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

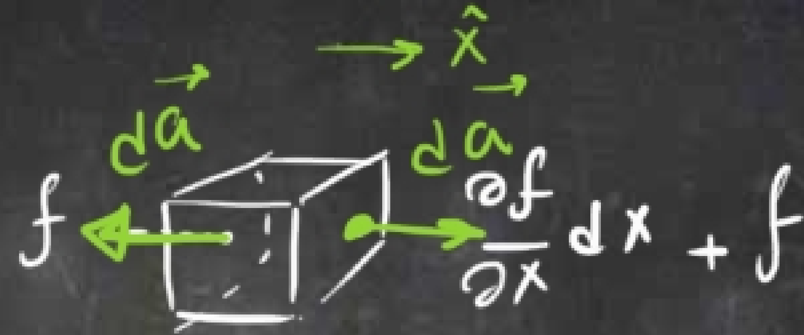
$$\vec{\nabla} f = \mathrm{grad}(f)$$

بردار

$$\underline{\mathrm{grad}(f)} \cdot d\vec{r} = df$$

$$\mathrm{grad}(f) = \lim_{\Delta\tau \to 0} \frac{\oint f\, d\vec{a}}{\Delta\tau}$$

$$\lim_{\Delta\Pi \to 0} \frac{\oint f \, d\vec{a}}{\Delta\Pi}$$



$$\frac{\partial f}{\partial x} dx\,dy\,dz\,\hat{x} + \frac{\partial f}{\partial y} dx\,dy\,dz\,\hat{y} + \frac{\partial f}{\partial z} \cdots$$

$$\rightarrow \operatorname{grad}(f) = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z} \quad !$$

$$\int \vec{E} \cdot d\vec{a} \qquad \int f \, d\vec{a}$$

$$\operatorname{grad}(f) = \lim_{\Delta\Pi \to 0} \frac{\int d\vec{a}\, f}{\Delta\Pi} \rightarrow \vec{\nabla} T \cdot d\vec{r} = dT$$

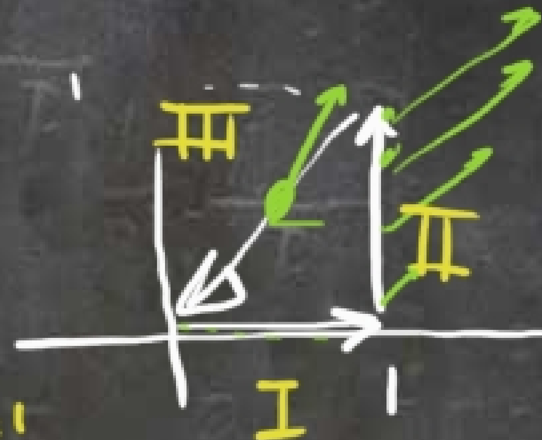
$$\operatorname{div}(\vec{E}) = \lim_{\Delta\Pi \to 0} \frac{\int d\vec{a} \cdot \vec{E}}{\Delta\Pi} \rightarrow \checkmark$$

$$\operatorname{Curl}(\vec{E}) = \lim_{\Delta\Pi \to 0} \frac{\int d\vec{a} \times \vec{E}}{\Delta\Pi} \rightarrow$$ جدید

چرخش یا سرکولاسیون

$$\Gamma = \oint_{c} \vec{E}\cdot d\vec{l}$$

$$\vec{E}_{(x,y)} = xy\,\hat{x} + y\,\hat{y}$$



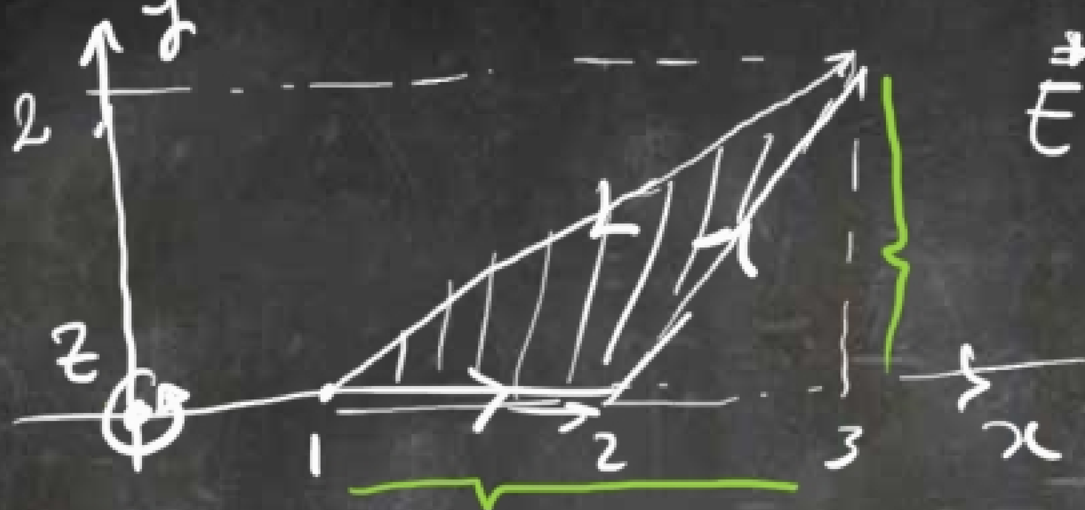
$y\,\hat{x} + y\,\hat{y}$

$$\int_{I} \vec{E}\cdot d\vec{l} = 0 \ , \quad \int_{II} \vec{E}\cdot d\vec{l} = \int_{y=0}^{y=1} \underbrace{\begin{pmatrix} y \\ y \end{pmatrix}\cdot\begin{pmatrix} 0 \\ dy \end{pmatrix}}_{y\,dy} = \frac{1}{2}$$

$$\int_{III} \begin{pmatrix} xy \\ y \end{pmatrix}\cdot\begin{pmatrix} dx \\ dy \end{pmatrix} = \int \begin{pmatrix} x^2 \\ x \end{pmatrix}\cdot\begin{pmatrix} dx \\ dx \end{pmatrix} = -\frac{1}{3} - \frac{1}{2} = \frac{-5}{6}$$

$x = y$!!!

$$\rightarrow \Gamma = -\frac{1}{3}\ !$$



$\Gamma_{c,\vec{E}}$

$$\vec{E} = \begin{pmatrix} x+y \\ xy \end{pmatrix}$$

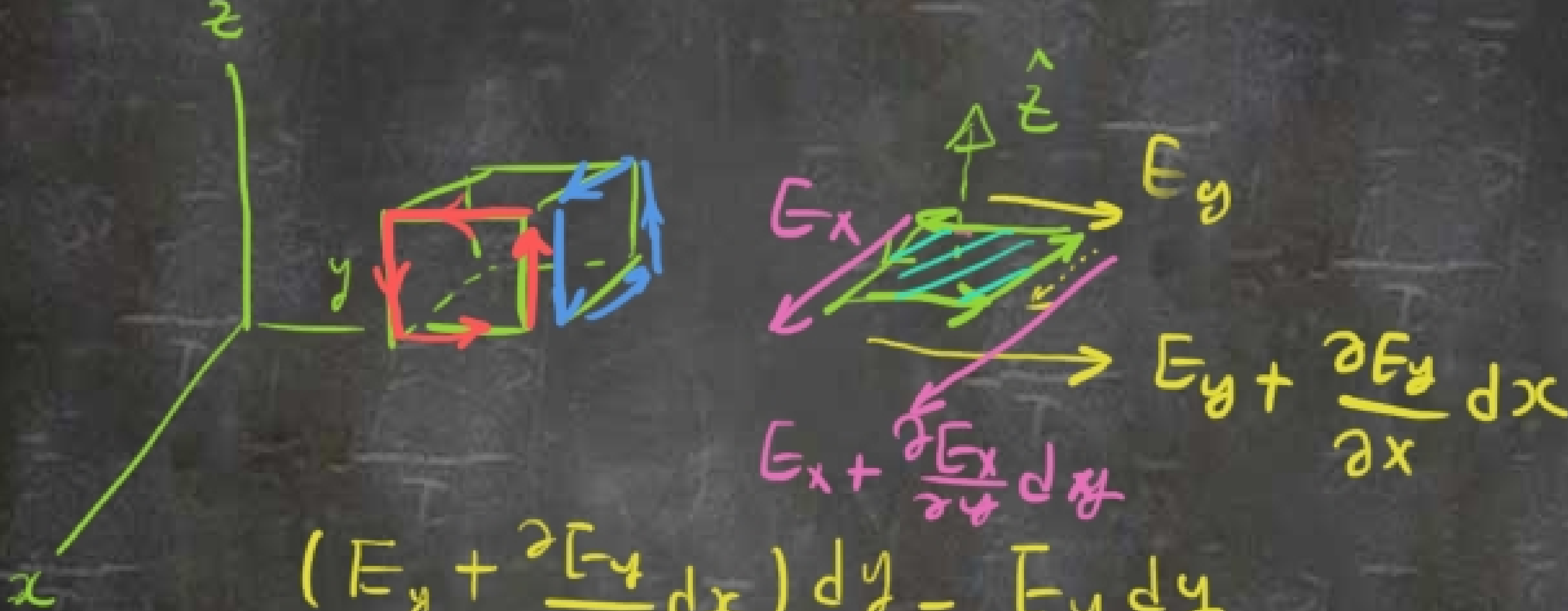
$$C_{I} = \int_{x=1}^{x=2} \begin{pmatrix} x \\ 0 \end{pmatrix} \cdot \begin{pmatrix} dx \\ 0 \end{pmatrix} = \frac{2}{2}$$

$$\Gamma = \checkmark$$

$$C_{II} = \int_{x=2}^{x=3} \begin{pmatrix} x+2x-4 \\ x(2x-4) \end{pmatrix} \cdot \begin{pmatrix} dx \\ 2dx \end{pmatrix} = \checkmark$$

$$C_{III} \int_{x=3}^{x=1} \begin{pmatrix} x+(x-1) \\ x(x-1) \end{pmatrix} \begin{pmatrix} dx \\ dy \end{pmatrix} = \checkmark$$

$$\mathrm{Curl}(\vec{E})_{\hat{n}} = \lim_{\Delta A \to 0} \frac{\oint_{\hat{n}} \vec{E} \cdot d\vec{l}}{\Delta A} = \lim_{\Delta A \to 0} \frac{\Gamma_{\hat{n}}}{\Delta A}$$



$$\left(E_y + \frac{\partial E_y}{\partial x} dx\right) dy - E_y\, dy$$

$$+ E_x\, dx - \left(E_x + \frac{\partial E_x}{\partial y} dy\right) dx$$

$$= \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right) dx\, dy$$

$$\mathrm{Curl}(\vec{E})_z = \frac{\partial}{\partial x} E_x - \frac{\partial}{\partial y} E_x$$

$$\mathrm{Curl}(\vec{E}) = \begin{pmatrix} \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \\ \\ \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \\ \\ \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \end{pmatrix}$$

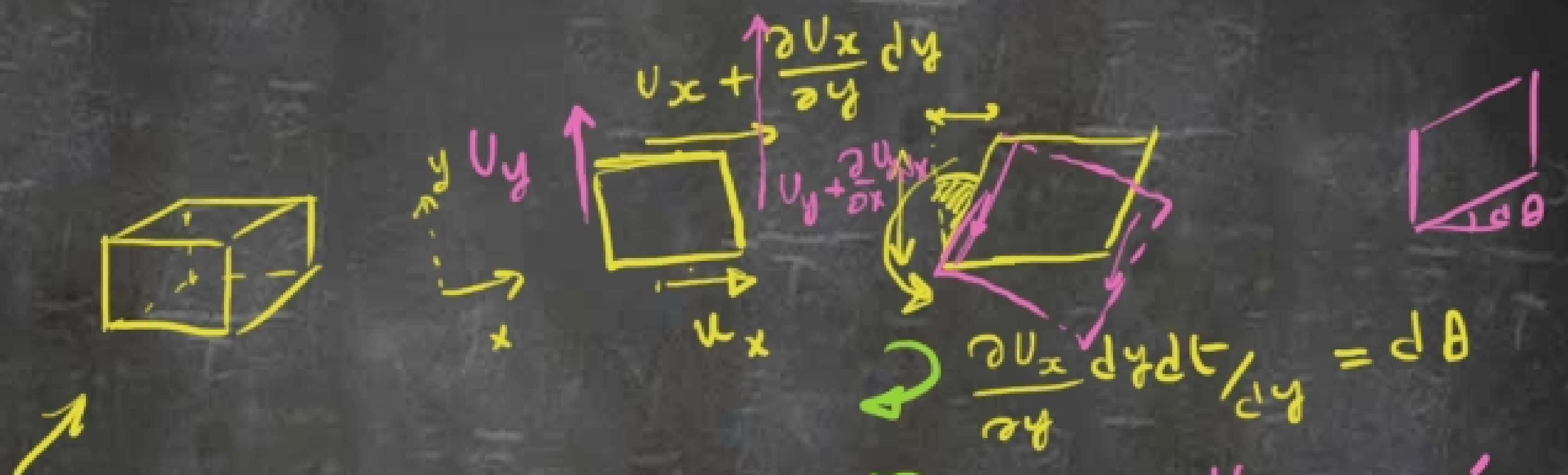
$$\underbrace{\vec{\nabla}}_{\begin{pmatrix} \partial/\partial x \\ \partial/\partial y \\ \partial/\partial z \end{pmatrix}} \times \underbrace{\vec{E}}_{\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}} = \det \begin{pmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{pmatrix}$$

grad → $\vec{\nabla} f$

div → $\vec{\nabla} \cdot \vec{v}$

curl → $\vec{\nabla} \times \vec{v}$

$$\vec{\nabla} = \lim_{\Delta\square \to 0} \frac{\oint d\vec{a}}{\Delta\square}$$

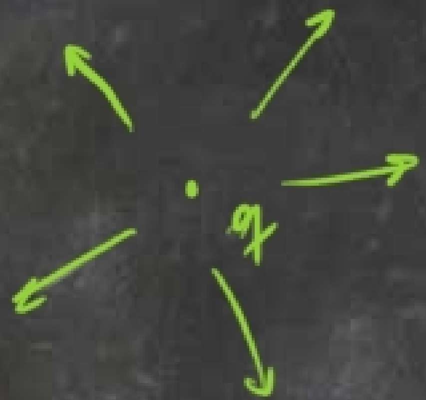


$$\frac{\partial u_x}{\partial y} dy\,dt / dy = d\theta$$

$$\frac{\partial u_y}{\partial x} dx\,dt / dx = d\theta'$$

$$\frac{\partial u_y}{\partial x} dt - \frac{\partial u_x}{\partial y} dt = \Delta\theta$$

$$! \; \omega_z = \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \rightarrow \vec{\nabla} \times \vec{V} = \vec{\omega}$$

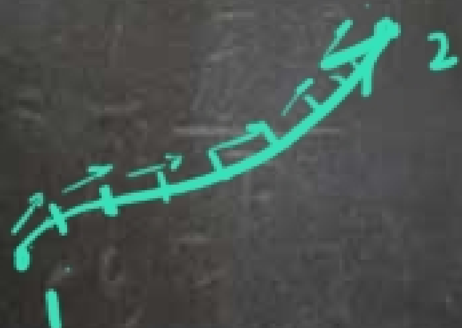


$$\vec{E} = \frac{kq\begin{pmatrix}x\\y\\z\end{pmatrix}}{(x^2+y^2+z^2)^{\frac{3}{2}}}$$

$$\vec{E} = \frac{kq\vec{r}}{r^3} \rightarrow \text{curl } \vec{E} = 0$$

✓ $\vec{\nabla}T \cdot d\vec{l} = dT$, ✓ $\lim\limits_{\Delta\tau \to 0} \dfrac{\oint \vec{E} \cdot d\vec{a}}{\Delta\tau}$

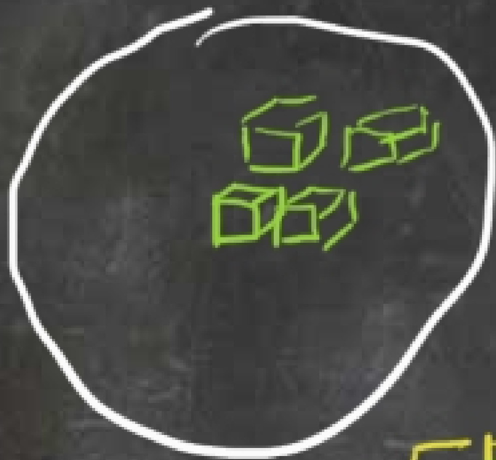
✓ $\text{curl}(\vec{E})_{\hat{n}} = \lim\limits_{\Delta A \to 0} \dfrac{\sqcap_{\hat{n}}}{\Delta A}$



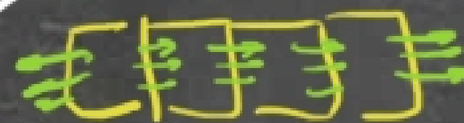
$$\int_1^2 \vec{\nabla}T \cdot d\vec{l} = T(\bar{r}_2) - T(\bar{r}_1)$$

تفسیر انتگرالی دیورژانس

$$(\vec{\nabla}\cdot\vec{E})\,d\tau$$

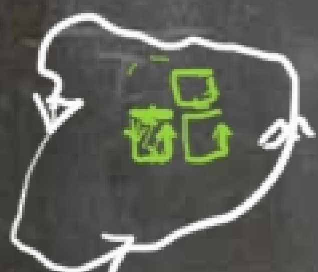


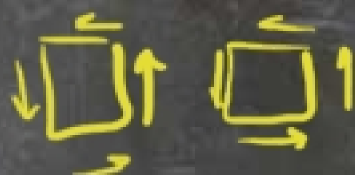
$$\oint_S \vec{E}\cdot d\vec{a} = \int_v (\vec{\nabla}\cdot\vec{E})\,d\tau$$



$$(\vec{\nabla}\times\vec{E})_z\,dA$$

تفسیر انتگرالی چرخش

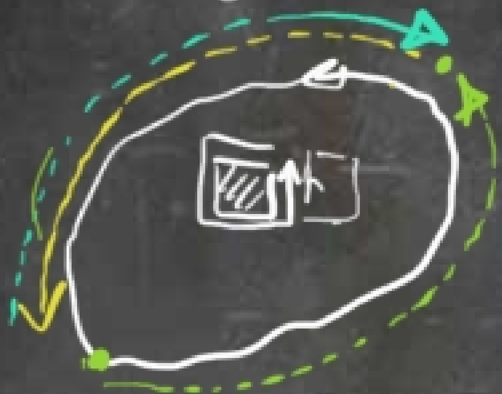




$$\oint_S (\vec{\nabla}\times\vec{E})_{\hat{n}}\,da = \int_c \vec{E}\cdot d\vec{l}$$

$$\vec{\nabla} \times \vec{E} = 0 \rightarrow \oint \vec{E} \cdot d\vec{l} = 0$$

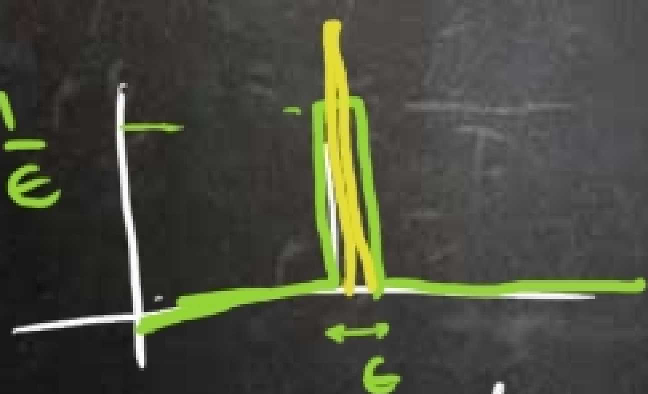
$$\int \vec{E} \cdot d\vec{l} = -\int \vec{E} \cdot d\vec{l}$$



$$\int \vec{E} \cdot d\vec{l} = \int \vec{E} \cdot d\vec{l}$$

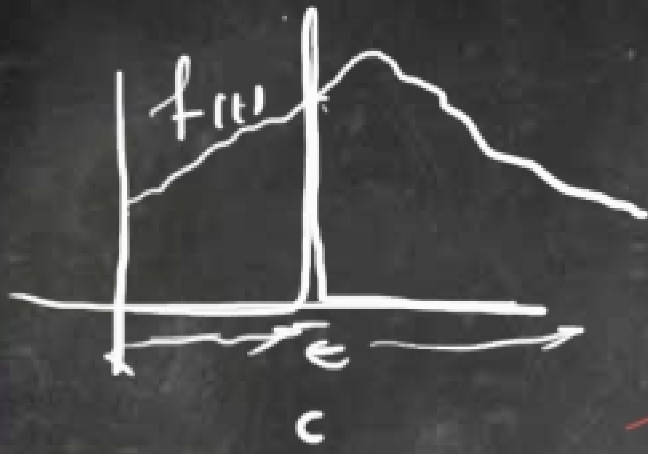
تابع دلتای دیراک

$$\vec{\nabla} \cdot \vec{E} = \lim_{\Delta\tau \to 0} \frac{\oint \vec{E} \cdot d\vec{a}}{\Delta\tau} = \frac{\Delta q}{\epsilon_0 \Delta\tau} = \frac{\rho}{\epsilon_0}$$



$$f_{(x)} = \delta_c(x)$$

$x = c$

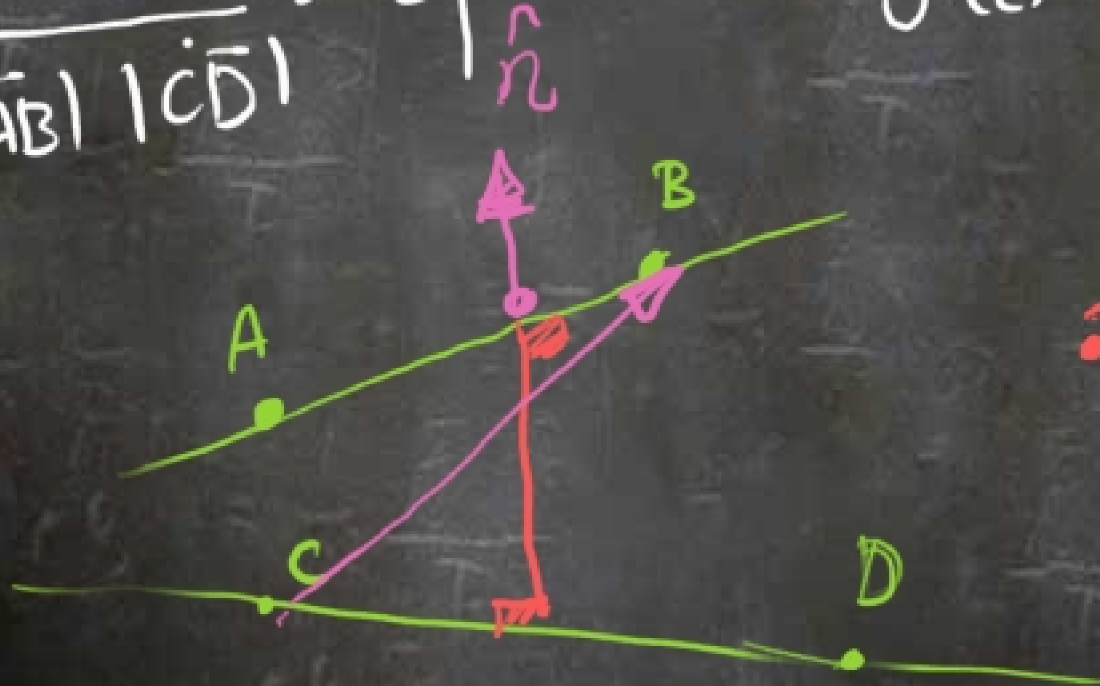


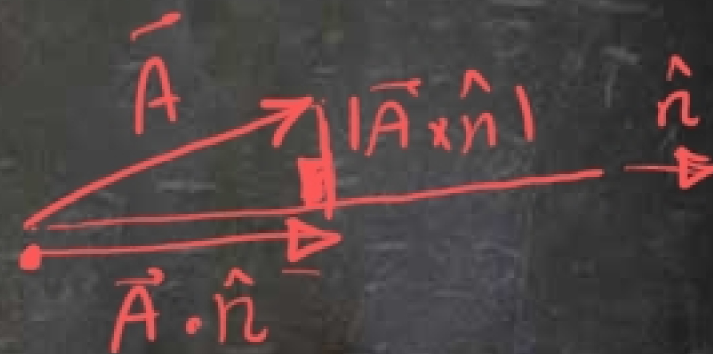
$$\int_0^\infty \delta_c(t)\,dt = 1$$

$$\int_0^\infty f(t)\,\delta_c(t)\,dt = f(c)$$

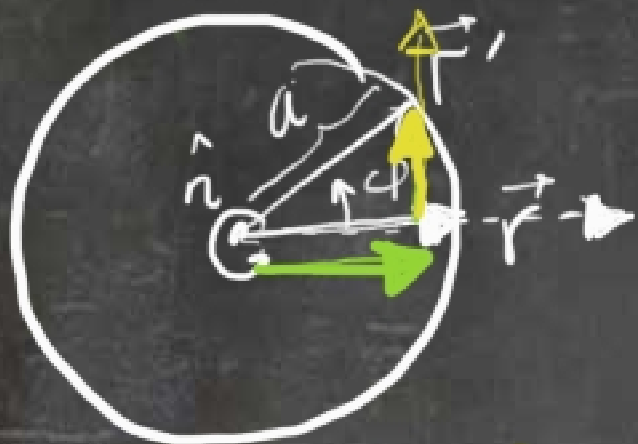
$$f(c)\,\frac{1}{\epsilon}\,\epsilon$$

$$\left|\frac{\vec{AB}\times\vec{CD}}{|\vec{AB}|\,|\vec{CD}|}\cdot\vec{CB}\right| = h \checkmark$$





$$\vec{r}' = (\vec{r}\cdot\hat{n})\hat{n} + (\vec{r} - [\vec{r}\cdot\hat{n}]\hat{n})\cos\varphi + \hat{n}\times\vec{r}\,\sin\varphi$$



$$a = |\vec{r}\times\hat{n}|$$

$$\{\vec{r} - (\vec{r}\cdot\hat{n})\hat{n}\}\cos\varphi$$

$\sin\varphi$

$$\hat{n}\times[\vec{r} - (\vec{r}\cdot\hat{n})\hat{n}] = \vec{n}\times\vec{r}$$

$$(\vec{n}\times\vec{r})\sin\varphi$$

The End