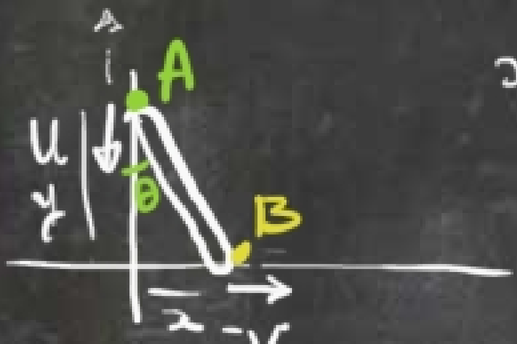
پایستگی انرژی

قید هندسی → هندسه → $\frac{d}{dt}$(هندسه)

سرعت نزدیک شدن و دور شدن = سرعت در راستای خط واصل

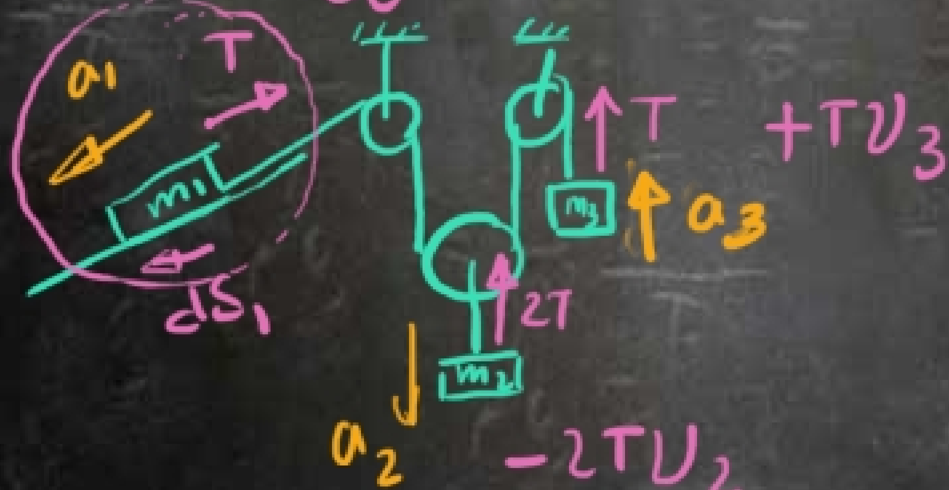


$$x^2+y^2=l^2 \rightarrow 2x\dot{x}+2y\dot{y}=0$$

$$\dot{x}=v\,,\ \dot{y}=-u \rightarrow \frac{x}{y}=\frac{u}{v}$$

$$u\cos\theta = v\sin\theta \rightarrow \tan\theta = y/x = \frac{v}{u}$$

$$-T\,dS_1/dt = -Tv_1$$

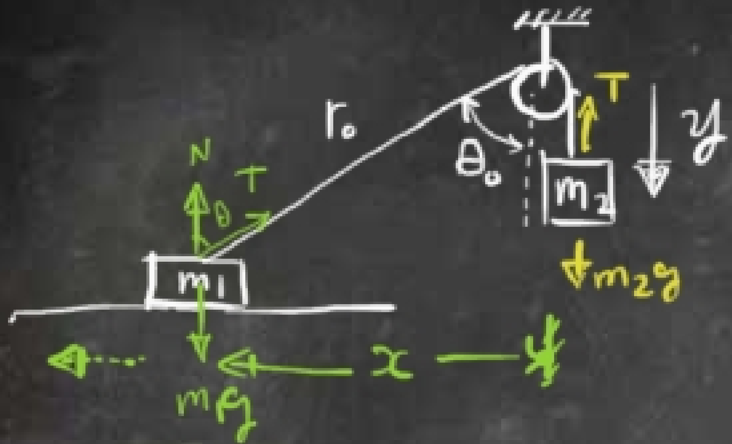


$$-Tv_1-2Tv_2+Tv_3=0$$

$$v_1+2v_2=v_3$$

$$\rightarrow a_1+2a_2=a_3 \checkmark$$

روش انرژی



$$y_{(t)}\,,\ r_{(t)}\,,\ \theta_{(t)} = ?$$

$$\left\{\begin{array}{l} T\cos\theta + N = mg \\ T\sin\theta = -m\ddot{x} \\ m_2 g - T = m_2\ddot{y} \end{array}\right\} \xleftrightarrow{T,\,r,\,\theta} \left\{\begin{array}{l} \left.\begin{array}{l} x \\ y \end{array}\right\} \rightarrow r,\theta \\ T \\ \theta \end{array}\right.$$

r,θ

معادله‌ی دیگر؟

$$r\sin\theta = x$$

$$\frac{d}{dt}\{r\sin\theta\} = \dot{r}\sin\theta + r\cos\theta\,\dot{\theta} = \dot{x}$$

$$r + y = l \rightarrow \frac{d}{dt}(r+y) = 0 \rightarrow \dot{r} = -\dot{y} \rightarrow \ddot{y} = -\ddot{r}$$

$$\rightarrow \ddot{r}\sin\theta + \dot{r}\cos\theta\,\dot{\theta} + \dot{r}\cos\theta\,\dot{\theta} - r\sin\theta\,\dot{\theta}^2 + r\cos\theta\,\ddot{\theta} = \ddot{x}$$

$$\left\{\begin{array}{l} T\sin\theta = -m(\ddot{r}\sin\theta + 2\dot{r}\dot{\theta}\cos\theta - r\dot{\theta}^2\sin\theta + r\cos\theta\,\ddot{\theta}) \\ m_2 g - T = -m_2\ddot{r} \end{array}\right\}$$

$$r\cos\theta = c \rightarrow \dot{r}\cos\theta - r\sin\theta\,\dot{\theta} = 0$$

$$\left\{\begin{aligned} T\sin\theta &= -m_1\left(\ddot{r}\sin\theta + 2\dot{r}\dot{\theta}\cos\theta - r\dot{\theta}^2\sin\theta + r\cos\theta\,\ddot{\theta}\right) \\ m_2 g - T &= -m_2\ddot{r} \end{aligned}\right\}$$

$$r\cos\theta = H \rightarrow \dot{r}\cos\theta - r\sin\theta\,\dot{\theta} = 0$$

$$m_2 g + m_1\left(\ddot{r} - r\dot{\theta}^2 + 2\dot{r}\dot{\theta}\cot\theta + r\ddot{\theta}\cot\theta\right) = -m_2\ddot{r}$$

→ جایگزین

$$r = \frac{H}{\cos\theta} \rightarrow \dot{r} = H\sec\theta\tan\theta\,\dot{\theta} \rightarrow \ddot{r} = H\sec\theta\tan^2\theta\,\dot{\theta}^2 + H\sec^3\theta\,\dot{\theta}^2 + H\sec\theta\tan\theta\,\ddot{\theta}$$

$$d(\sec\theta) = \sec\theta\tan\theta\,d\theta$$

$$d(\csc\theta) = -\csc\theta\cot\theta\,d\theta$$

$$\rightarrow \boxed{f(\theta)\,\ddot{\theta} + g(\theta)\,\dot{\theta}^2 + h(\theta) = 0}$$ معادله دیفرانسیل

معادله ی ساده $\rightarrow$ دنبال عدد ثابت

معادله دیفرانسیل $\rightarrow$ دنبال تابع

معادله ی دیفرانسیل $\rightarrow$ $\dot{x} = \alpha x^2$

$x(t) = ?$

$$\frac{dx}{dt} = \alpha x^2 \rightarrow \frac{dx}{x^2} = \alpha dt \rightarrow -\frac{1}{x(t)} = \alpha t + c$$

$$d\left(\frac{-1}{x}\right) = d(\alpha t)$$

$$\rightarrow x(t) = \frac{-1}{\alpha t + c} \rightsquigarrow \text{شرایط اولیه}$$

$x(t=0) = 1$

$\hookrightarrow c = -1$

$$\frac{d^8}{dt^8}x = x^{(8)}$$

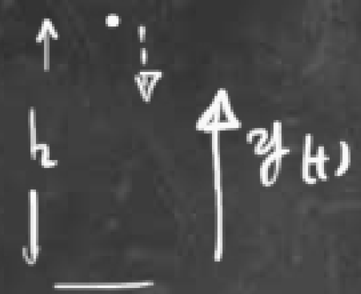
مرتبه ۳ $\;\dddot{x} + 3\dot{x}^8 - \sin x = 0$

$$\frac{d^4}{dt^4}x + \dot{x}^7 - \sin x\,\dot{x} = 2 + 33x^2 \rightsquigarrow \text{مرتبه ۴}$$

متغیر مستقل ، متغیر وابسته !؟

$x(t)$

$t(x)$



$$y(t) = h - \frac{gt^2}{2}$$

$$t(y) = \sqrt{\frac{2}{g}(h-y)}$$

$\frac{dx(t)}{dt}$

$$\ddot{x} + x = 0$$

وابسته x $x(t)$

متغیر مستقل t

خطی (یعنی توابع وابسته خطی باشند)

معادله دیفرانسیل

غیرخطی

$$\ddot{x} + 2x - 3 = 0$$ خطی

$$\ddot{x}^2 + 2x - 3 = 0$$ غیرخطی

$$x^2 + \ddot{x} = 0$$ غیرخطی

$$\ddot{x} + 2\dot{x} - 3x = 2$$

$$\ddot{x} + 2x\dot{x} - 3x = 2$$

غیرخطی

$$\sin x + \dot{x} = 0$$

پس غیرخطی

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \cdots$$

$$\ddot{x} + \underbrace{\sin t}_{\text{متغیر}}\, x = \cos t \cdot t^2$$ خطی

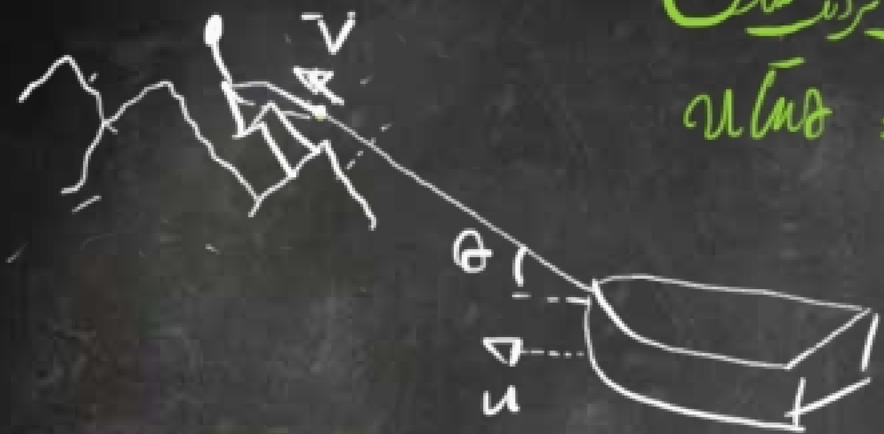
$$y(x) \rightsquigarrow \quad y'' + 2y'^2 + \sin x = 0$$

غیرخطی مرتبه ۲

$$y'' + \sin x\, y' + x y = 0$$

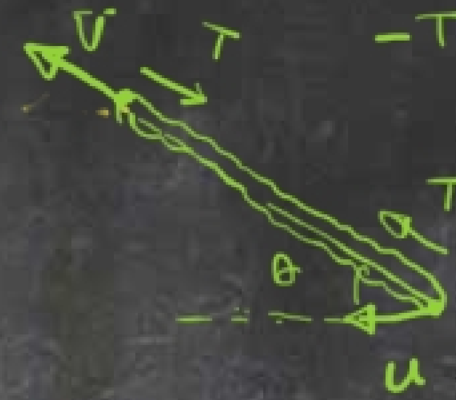
خطی مرتبه ۲

$$x_t = \dot{x} = \frac{dx}{dt}$$



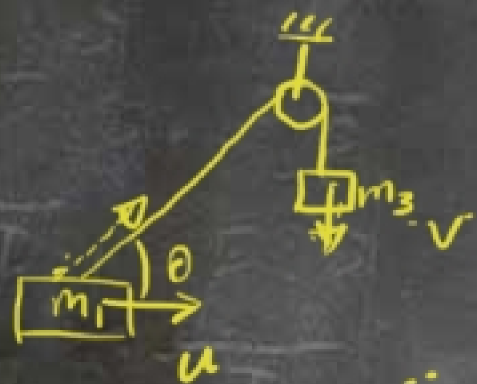
سرعت نزدیک شدن

$$u\cos\theta = v$$



$$-Tv + Tu\cos\theta = 0$$

$$v = u\cos\theta$$



$$u = \dot{x} \quad , \quad v = \dot{y}$$

$$u\cos\theta = v$$

$$\dot{u}\cos\theta - u\sin\theta\,\dot{\theta} = \dot{v}$$

$$\ddot{x} \qquad\qquad \ddot{y}$$

اصطکاک

ایستایی (حرکت نسبی نداریم)

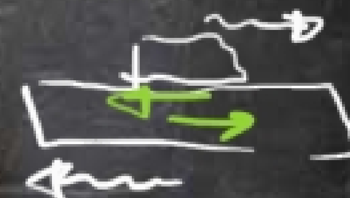
جنبشی (حرکت نسبی داریم)

جهت؟! اندازه چقدر است؟! کی به هم تبدیل می‌شوند؟!!

جهت؟ خلاف حرکت نسبی

ایستایی

off



جهت ⇐ حرکت نسبی

ایستایی

۳۵N

جهت خلاف

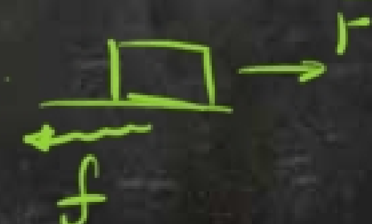
استفاده از عقل!!

۲۴kN (جنبشی در سطح)

جنبشی

ایستایی

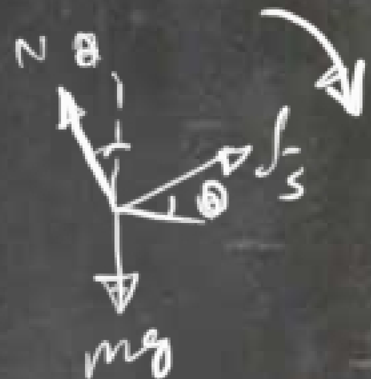
نوع دوم

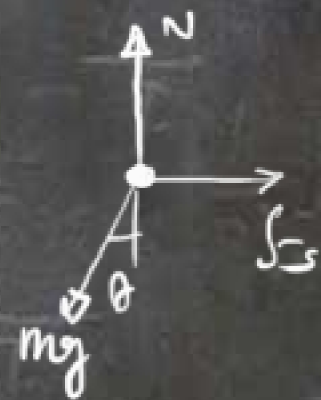


ه شتاب m حساب کنید

μ_s ، μ_k بر حسب

$$\left\{\begin{aligned} f_s\cos\theta &= N\sin\theta \\ f_s\sin\theta + N\cos\theta &= mg \end{aligned}\right\}$$





$$f_s = mg\sin\theta$$
$$N = mg\cos\theta$$

f_s ✓

N ✓

$$f_s \leq \mu_s N$$

$$mg\sin\theta \leq \mu_s mg\cos\theta \rightarrow \boxed{\tan\theta \leq \mu_s}$$

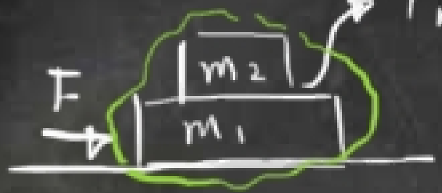
$$f_k = \mu_k N$$
$$f_{s,max} = \mu_s N$$

if $\tan\theta \leq \mu_s \rightarrow a = 0$

if $\tan\theta > \mu_s \rightarrow mg\sin\theta - \mu_k mg\cos\theta = ma$ ✓

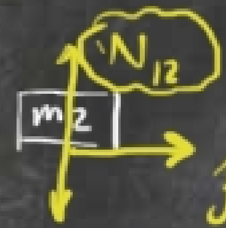
حساب a_1 و a_2 = ؟

μ_k , μ_s



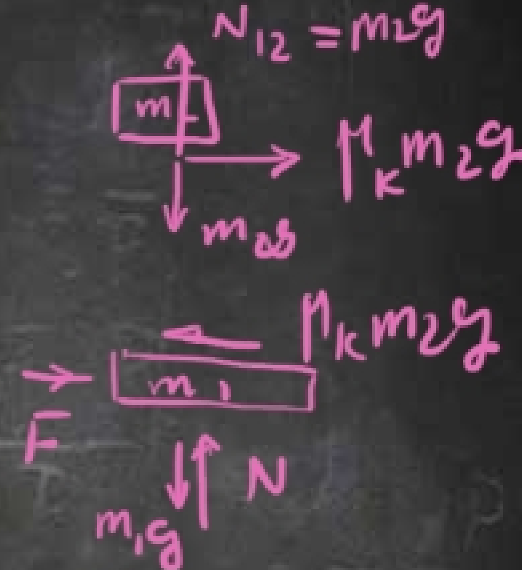
m_1+m_2

$$a = \frac{F}{m_1+m_2}$$



$$f_s \begin{cases} f_s = m_2 \dfrac{F}{m_1+m_2} \\ N = m_2 g \end{cases}$$

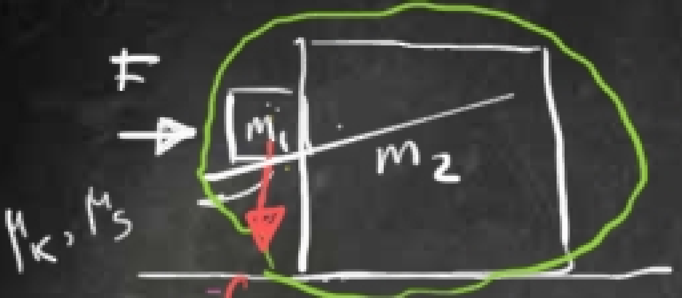
[illegible] استی



$$m_2 \frac{F}{m_1+m_2} \leqslant \mu_s m_2 g \rightarrow$$

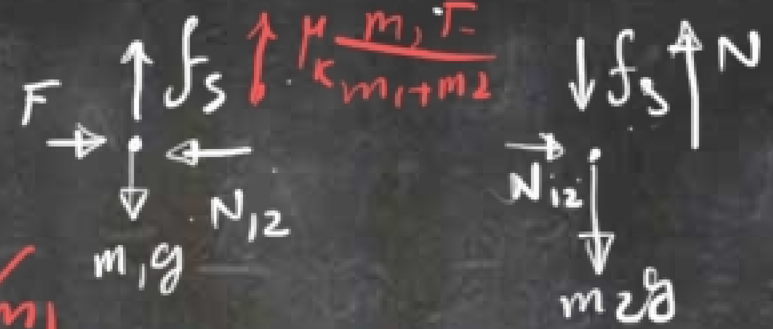
$$\text{if} \left\{ \frac{F}{(m_1+m_2)g} \leqslant \mu_s \right\} \xrightarrow{\text{استی}} a_1 = a_2 = \frac{F}{m_1+m_2}$$

$$\text{if} \left\{ \frac{F}{(m_1+m_2)g} > \mu_s \right\} \xrightarrow{\text{جنبشی}} \begin{aligned} a_2 &= \mu_k g \\ a_1 &= \frac{F - \mu_k m_2 g}{m_1} \end{aligned}$$



$$\left(m_1 g - \mu_k m_2 \frac{F}{m_1+m_2}\right) / m_1$$

$m_1 , m_2 = ؟!$



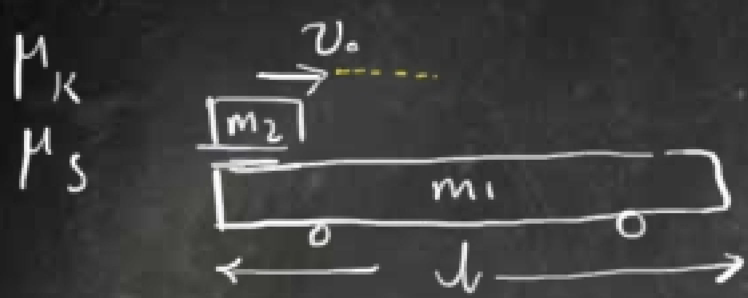
$$\begin{cases} F - N_{12} = m_1 a \\ f_s = m_1 g \end{cases} \qquad \begin{cases} N = m_2 g + f_s \\ N_{12} = m_2 a \end{cases}$$

$$a = \frac{F}{m_1+m_2} \rightarrow N_{12} = m_2 \frac{F}{m_1+m_2}$$

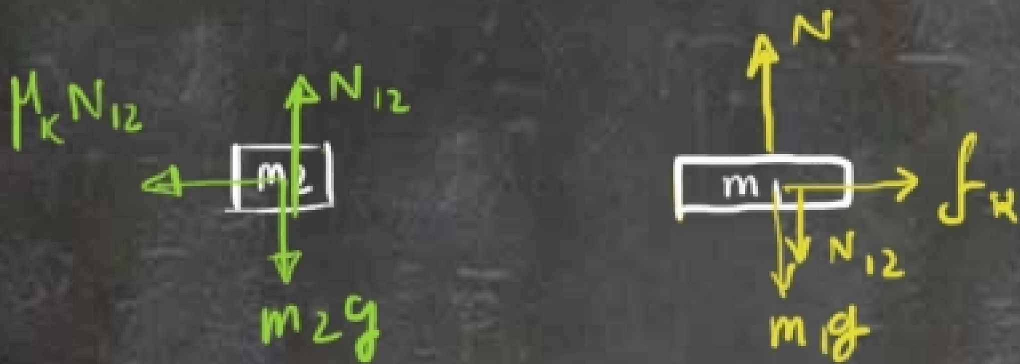
$$\text{if } \left\{ m_1 g \leq \frac{\mu_s m_2 F}{m_1+m_2} \right\} \rightarrow \vec{a}_1 = \vec{a}_2 = \frac{F}{m_1+m_2} \hat{x}$$

$$\text{if } \left\{ m_1 g > \frac{\mu_s m_2 F}{m_1+m_2} \right\} \rightarrow \checkmark$$

$$\vec{a}_1 = \left(\frac{F}{m_1+m_2} \right) - \left(g - \mu_k \frac{m_2}{m_1} \frac{F}{m_1+m_2} \right)$$



شرطی برای μ_k بیابید تا جسم m_2 سقوط نکند

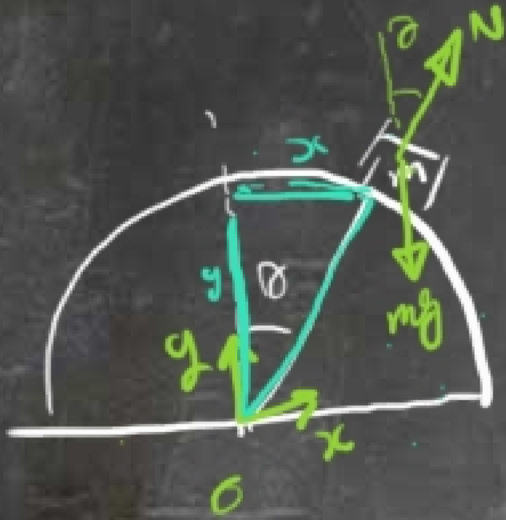


$$-\mu_k m_2 g = m_2 a_2 \rightarrow a_2 = -\mu_k g$$

$$\mu_k m_2 g = m_1 a_1 \rightarrow a_1 = \mu_k \frac{m_2}{m_1} g$$

$$v_0 - \mu_k g t = \mu_k \frac{m_2}{m_1} g t \rightarrow t = \frac{v_0}{\mu_k g \left(1 + \frac{m_2}{m_1}\right)}$$

$N_{(\theta)},\ N_{(t)},\ \dot{\theta}_{(\theta)},\ \theta(t)$



$$\begin{cases} N\cos\theta - mg = m\ddot{y} \\ N\sin\theta = m\ddot{x} \end{cases}$$

θ

x

y

N

$$x^2+y^2=R^2$$

$$2x\dot{x}+2y\dot{y}=0 \rightarrow x\dot{x}+y\dot{y}=0$$

$$\boxed{\dot{y}^2+y\ddot{y}+\dot{x}^2+x\ddot{x}=0}$$

$$\boxed{\tan\theta=\frac{x}{y}}$$

$$\boxed{\frac{x}{y}=\frac{\ddot{x}}{\ddot{y}+g}}$$

$$y = R\cos\theta \rightarrow \dot{y} = -R\sin\theta\,\dot{\theta} \rightarrow \ddot{y} = -R\cos\theta\,\dot{\theta}^2 - R\ddot{\theta}\sin\theta$$

$$x = R\sin\theta \rightarrow \dot{x} = R\cos\theta\,\dot{\theta} \rightarrow \ddot{x} = -R\sin\theta\,\dot{\theta}^2 + R\cos\theta\,\ddot{\theta}$$

$\cos\theta\ \sin\theta \rightsquigarrow$ $$N\cos\theta = mg + m(-R\cos\theta\,\dot{\theta}^2 - R\ddot{\theta}\sin\theta)$$

$\sin\ \cos\theta \rightsquigarrow$ $$N\sin\theta = m(-R\dot{\theta}^2\sin\theta + R\ddot{\theta}\cos\theta)$$

$$0 = mg\sin\theta - mR\ddot{\theta}(\sin^2\theta + \cos^2\theta) \rightarrow \boxed{g\sin\theta = R\ddot{\theta}}$$

$$\boxed{N = mg\cos\theta - mR\dot{\theta}^2}$$

$$dt = \frac{d\theta}{\dot{\theta}}$$

Golden RelationShip $\rightsquigarrow$

$$\ddot{\theta} = \frac{d\dot{\theta}}{dt} = \frac{d\dot{\theta}}{(d\theta/\dot{\theta})} = \boxed{\frac{\dot{\theta}\,d(\dot{\theta})}{d\theta}} = \frac{d\left(\frac{\dot{\theta}^2}{2}\right)}{d\theta}$$

$$\ddot{\theta} = \frac{\dot{\theta}\, d\dot{\theta}}{d\theta} = \frac{d\left(\frac{\dot{\theta}^2}{2}\right)}{d\theta}$$

Golden !!!

$$g\sin\theta = R\ddot{\theta} \Rightarrow \ddot{\theta} = \frac{g}{R}\sin\theta$$

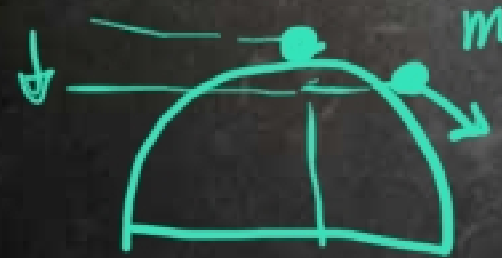
$$\rightsquigarrow \frac{d\left(\frac{\dot{\theta}^2}{2}\right)}{d\theta} = \frac{d(\dot{\theta}^2)}{2d\theta} = \frac{g}{R}\sin\theta$$

$$\rightarrow \int_0^{\dot{\theta}^2} d(\dot{\theta}^2) = \int_0^{\theta} \frac{2g}{R}\sin\theta\, d\theta = d\left(-\frac{2g}{R}\cos\theta\right)$$

شرط اولیه در تعیین
انتگرال معین

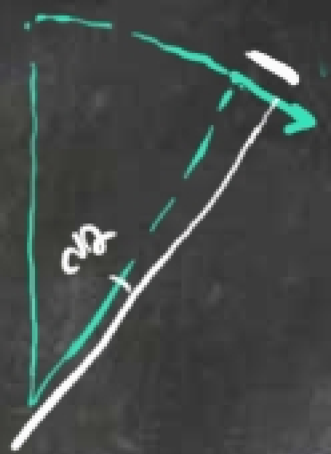
$$\boxed{\dot{\theta}^2 = -\frac{2g}{R}\cos\theta + \frac{2g}{R} = \frac{2g}{R}(1-\cos\theta)}$$

$$\boxed{N = mg(3\cos\theta - 2)}$$



$$mgR(1-\cos\theta) = \frac{1}{2}mR^2\dot{\theta}^2$$

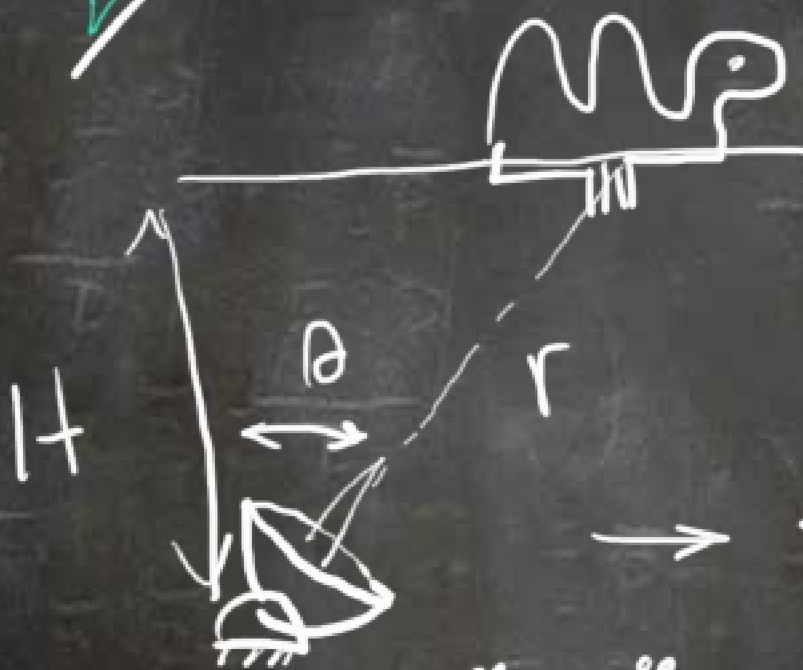
$$\dot{\theta} = \frac{2g}{R}(1-\cos\theta)$$



$$\frac{R\,d\theta}{dt} = v \rightarrow R\dot{\theta} = v$$

$\dot{r}$ $\ddot{r}$

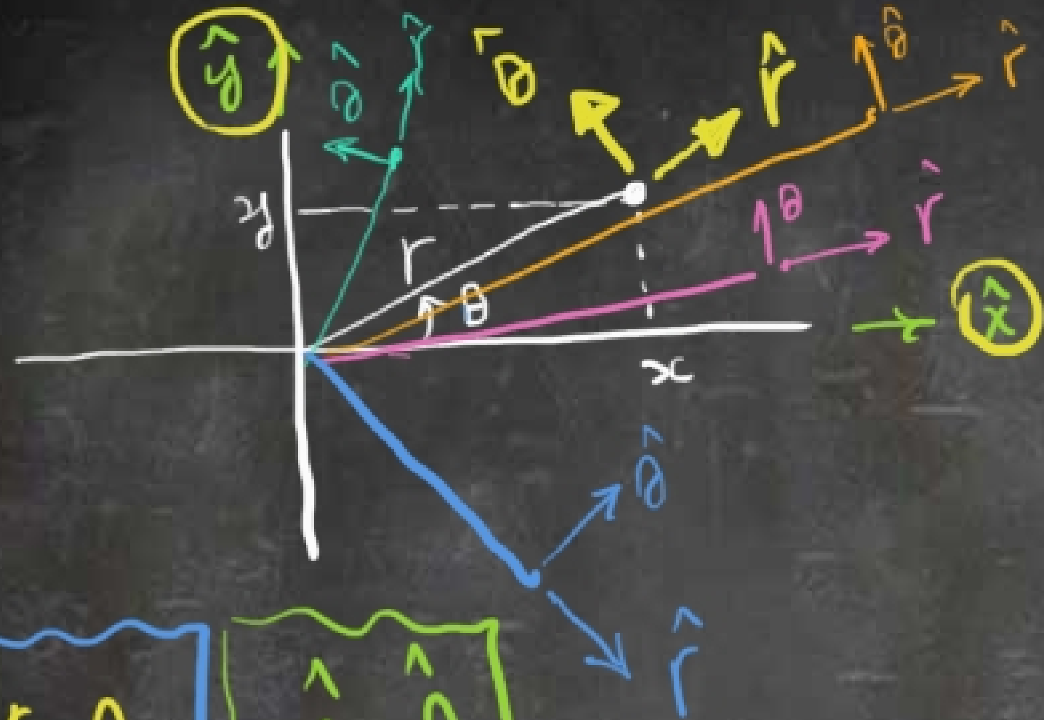
$\dot{\theta}$ $\ddot{\theta}$



$$\begin{Bmatrix} r\cos\theta = H \\ r\sin\theta = ut \end{Bmatrix}$$

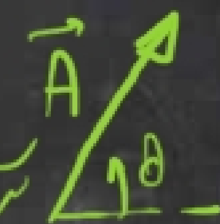
$$\rightarrow \begin{Bmatrix} \dot{r}\sin\theta + r\cos\theta\,\dot{\theta} = u \\ \dot{r}\cos\theta - r\sin\theta\,\dot{\theta} = 0 \end{Bmatrix}$$

$\ddot{r}$ ✓, $\ddot{\theta}$ ✓



r, θ

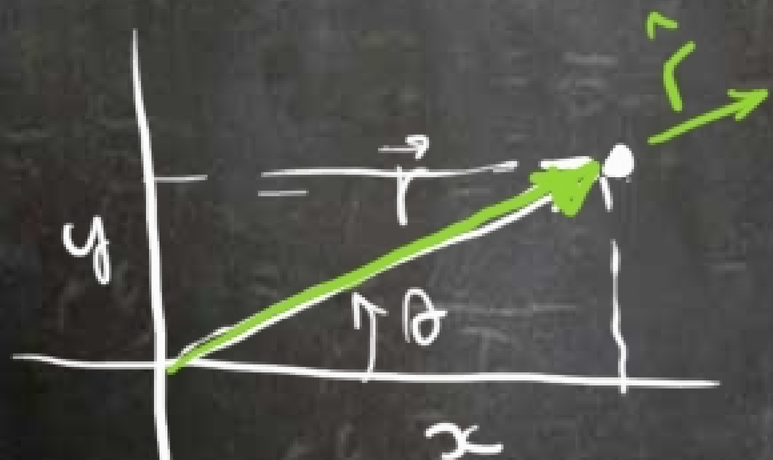
$\hat{r}, \hat{\theta}$



$$A\cos\theta\,\hat{x} + A\sin\theta\,\hat{y}$$

$$\begin{pmatrix} A\cos\theta \\ A\sin\theta \end{pmatrix}$$

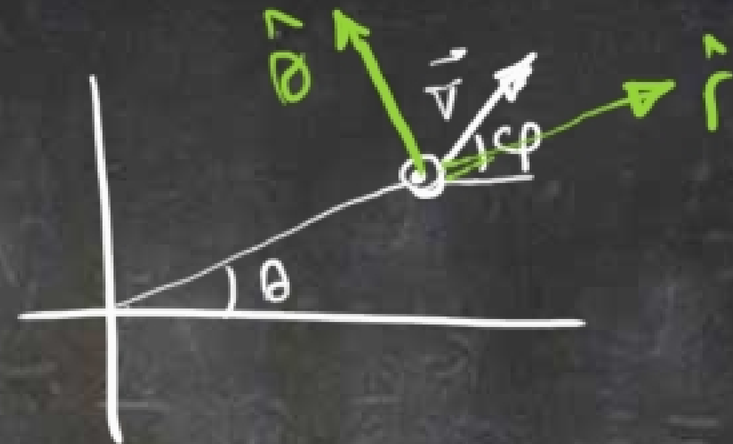
$$\frac{\vec{r}}{|\vec{r}|} = \hat{r}$$



$$\vec{r} = r\cos\theta\,\hat{x} + r\sin\theta\,\hat{y}$$
$$= x\hat{x} + y\hat{y}$$

$$\boxed{\vec{r} = r\hat{r}}$$

$$\begin{cases} \vec{r} = x\hat{x} + y\hat{y} \\ \vec{r} = r\hat{r}_{(\theta)} \end{cases}$$



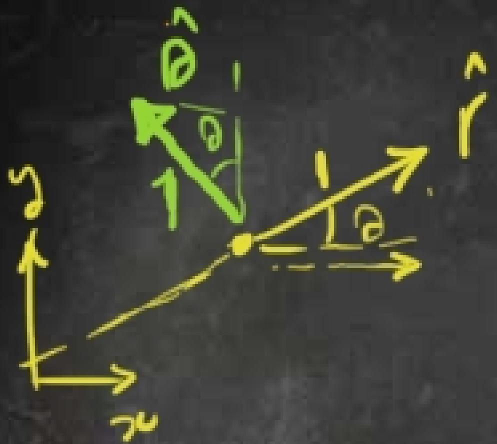
$$\vec{v} = v\cos\varphi\,\hat{x} + v\sin\varphi\,\hat{y} = \dot{x}\hat{x} + \dot{y}\hat{y}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{x} + \frac{dy}{dt}\hat{y}$$

$$\vec{v} = \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix}$$

$$\vec{v} = v\cos(\varphi-\theta)\,\hat{r} + v\sin(\varphi-\theta)\,\hat{\theta}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r\frac{d\hat{r}}{dt}$$

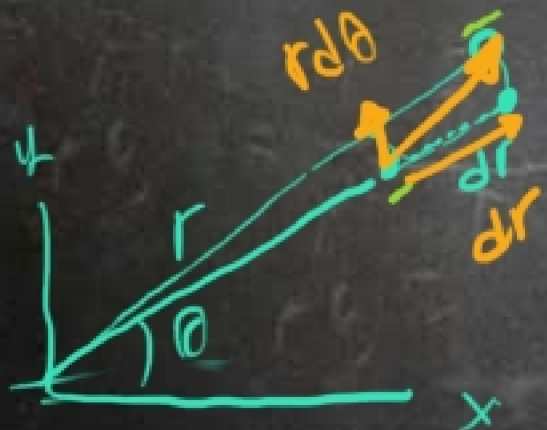


$$\hat{r} = \cos\theta\,\hat{x} + \sin\theta\,\hat{y}$$

$$\frac{d}{dt}\hat{r} = -\sin\theta\,\dot{\theta}\,\hat{x} + \cos\theta\,\dot{\theta}\,\hat{y}$$

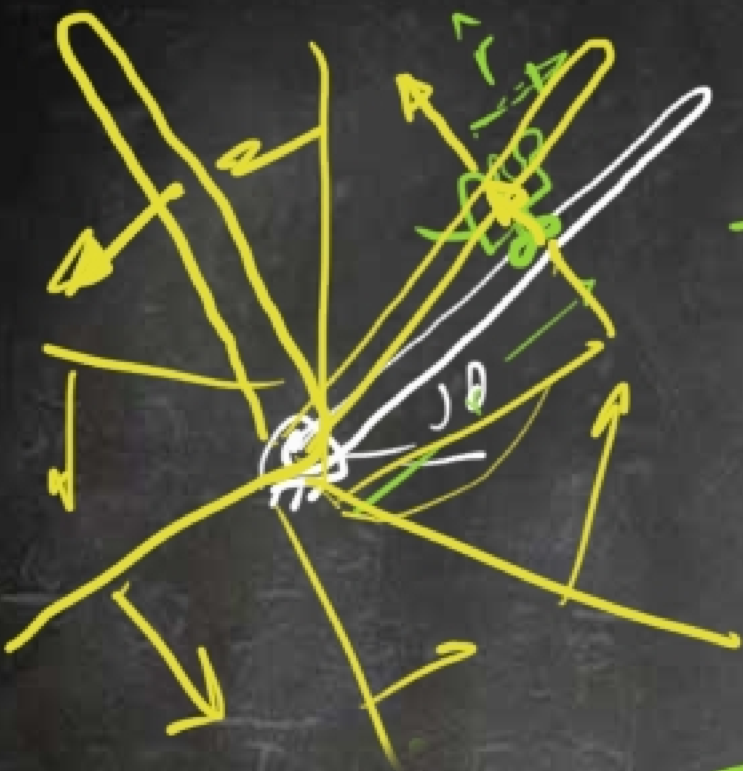
$$\vec{v} = \dot{r}\hat{r} + r\left(-\sin\theta\,\dot{\theta}\,\hat{x} + \cos\theta\,\dot{\theta}\,\hat{y}\right)$$

$$= \dot{r}\hat{r} + r\dot{\theta}\underbrace{\left(-\sin\theta\,\hat{x} + \cos\theta\,\hat{y}\right)}_{\hat{\theta}} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$



$$d\vec{r} = dr\,\hat{r} + r\,d\theta\,\hat{\theta}$$

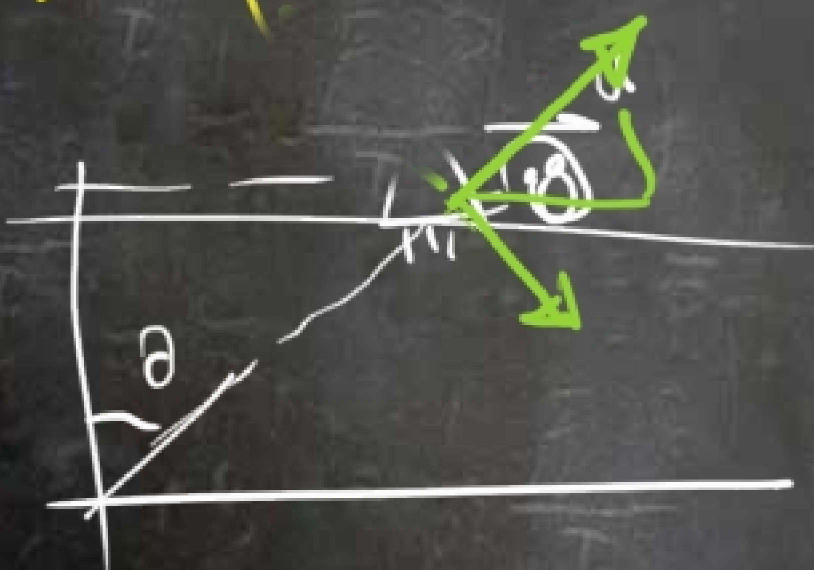
$$\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} = \dot{x}\hat{x} + \dot{y}\hat{y}$$



θ $\quad d r\hat{r} \quad \Rightarrow \dot{r}\hat{r}$

$$\vec{r} = x\hat{x} + y\hat{y} = r\hat{r}$$

$$\vec{v} = \dot{x}\hat{x} + \dot{y}\hat{y} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$



$$r\sin\theta = ut$$

$$r\cos\theta = H$$

$$u\cos\theta = \dot{r} \quad \checkmark$$

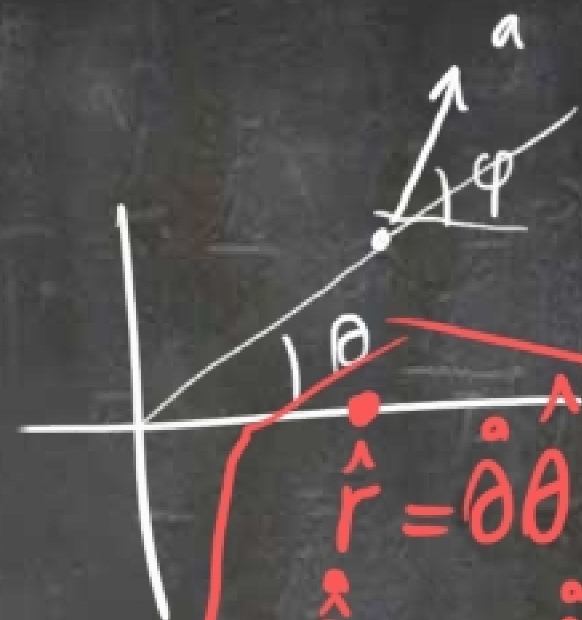
$$u\sin\theta = r\dot{\theta} \quad \checkmark$$

$$\vec{v} = \dot{x}\hat{x} + \dot{y}\hat{y}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \ddot{x}\hat{x} + \ddot{y}\hat{y}$$

$$\vec{a} = a\cos\varphi\,\hat{x} + a\sin\varphi\,\hat{y}$$

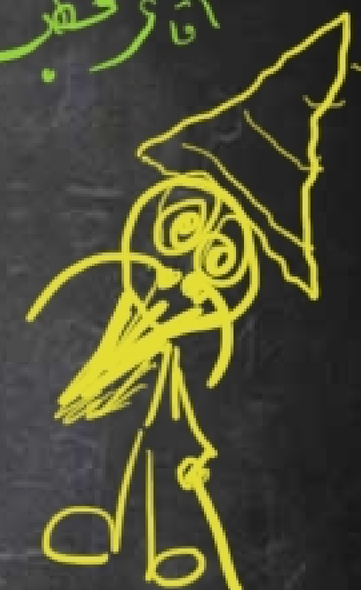
شتاب قطبی



$$\dot{\hat{r}} = \dot{\theta}\hat{\theta}$$

$$\dot{\hat{\theta}} = -\dot{\theta}\hat{r}$$

آقای قطبی

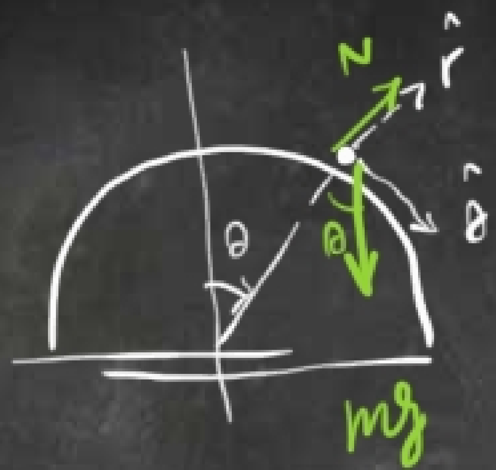


$$\vec{a} = a\cos(\varphi-\theta)\hat{r} + a\sin(\varphi-\theta)\hat{\theta}$$

$$\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

$$\frac{d\vec{v}}{dt} = \ddot{r}\hat{r} + \dot{r}\dot{\theta}\hat{\theta} + \dot{r}\dot{\theta}\hat{\theta} + r\ddot{\theta}\hat{\theta} + r\dot{\theta}(-\dot{\theta}\hat{r})$$

$$= (\ddot{r} - r\dot{\theta}^2)\hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta}$$

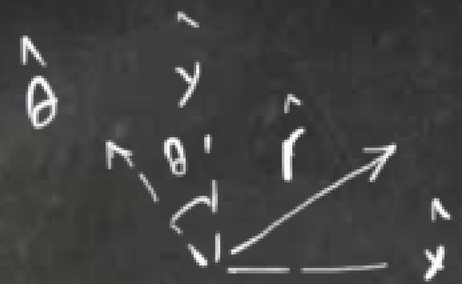


$$mg\sin\theta = m(2\dot{r}\dot{\theta} + r\ddot{\theta})$$

$$\boxed{g\sin\theta = R\ddot{\theta}} \longrightarrow \text{Golden} \checkmark$$

$$N - mg\cos\theta = m(\ddot{r} - r\dot{\theta}^2)$$
$$= -mR\dot{\theta}^2$$

$$\left\{\begin{aligned} \dot{\hat{r}} &= \dot{\theta}\hat{\theta} \\ \dot{\hat{\theta}} &= \frac{d\hat{\theta}}{dt} = -\dot{\theta}\hat{r} \end{aligned}\right\}$$



$$\hat{\theta} = -\sin\theta\,\hat{x} + \cos\theta\,\hat{y}$$

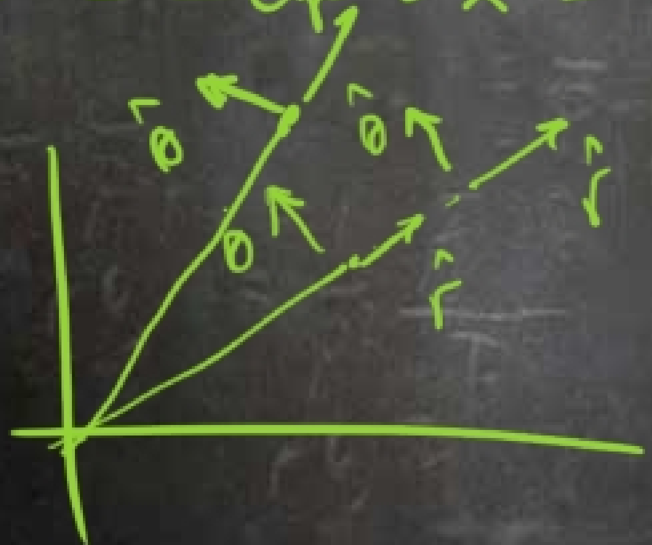
$$\hat{r} = \cos\theta\,\hat{x} + \sin\theta\,\hat{y}$$

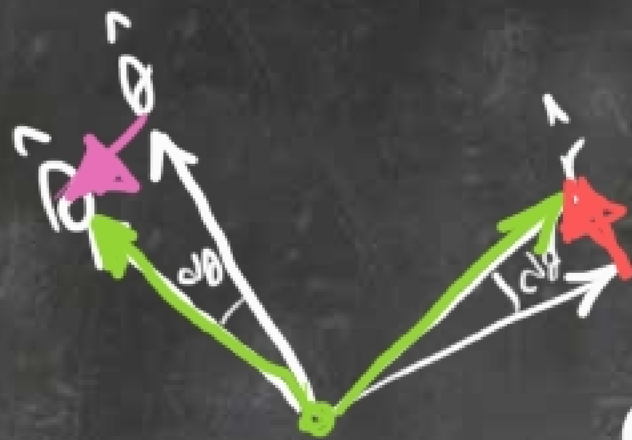
$$\dot{\hat{r}} = -\sin\theta\,\dot{\theta}\,\hat{x} + \cos\theta\,\dot{\theta}\,\hat{y}$$

$$= \dot{\theta}\,\underbrace{(\cos\theta\,\hat{y} - \sin\theta\,\hat{x})}_{\hat{\theta}} = \dot{\theta}\,\hat{\theta}$$

$$\dot{\hat{\theta}} = -\cos\theta\,\dot{\theta}\,\hat{x} - \sin\theta\,\dot{\theta}\,\hat{y} = -\dot{\theta}(\hat{x}\cos\theta + \hat{y}\sin\theta)$$

$$= -\dot{\theta}\,\hat{r}$$





$$\frac{d\theta\,\hat{\theta}}{dt} = \dot{\theta}\hat{\theta}$$

$$\hat{r}\, d\hat{\theta} = -d\theta\, \hat{r}$$

$$\underline{\underline{\frac{d\hat{\theta}}{dt} = -\dot{\theta}\hat{r}}}$$

$$r_{(0)} = r_0$$

$$\dot{r}_{(0)} = 0$$

معادله در جهت $\hat{r}$:

$$0 = m(\ddot{r} - r\dot{\theta}^2)$$

$$\text{(I)} \quad \boxed{\ddot{r} = r\omega^2}$$

معادله در جهت $\hat{\theta}$:

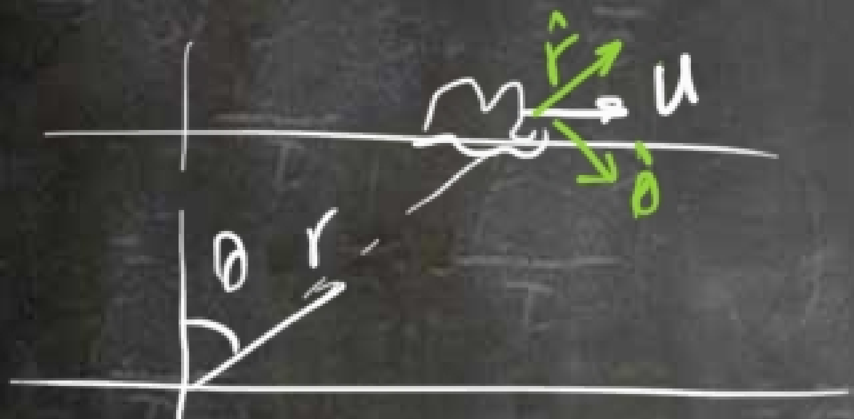
$$\text{(II)} \quad N = m(2\dot{r}\dot{\theta} + r\ddot{\theta})$$

$$\boxed{\dot{r}^2 = \omega^2(r^2 - r_0^2)}$$

$$\boxed{\frac{d(\dot{r}^2)}{2dr} = r\omega^2} \rightarrow d(\dot{r}^2) = 2\omega^2 r\,dr = d(\omega^2 r^2)$$

$$\dot{r}^2 = r^2\omega^2 + C \quad -\omega^2 r_0^2$$

$$
\left\{
\begin{aligned}
\vec{r} &= x\hat{x} + y\hat{y} = r\hat{r}_{(\theta)} \\
\vec{v} &= \dot{x}\hat{x} + \dot{y}\hat{y} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} \\
\vec{a} &= \ddot{x}\hat{x} + \ddot{y}\hat{y} = (\underbrace{\ddot{r} - r\dot{\theta}^2}_{})\hat{r} + (\underbrace{2\dot{r}\dot{\theta} + r\ddot{\theta}}_{})\hat{\theta}
\end{aligned}
\right)
$$



$$\ddot{r} = r\dot{\theta}^2$$

$$2\dot{r}\dot{\theta} = -r\ddot{\theta}$$

فصل ۱: نورشناسی هندسی

فصل ۲ و ۳ کتاب هالیدی الکترومغناطیس

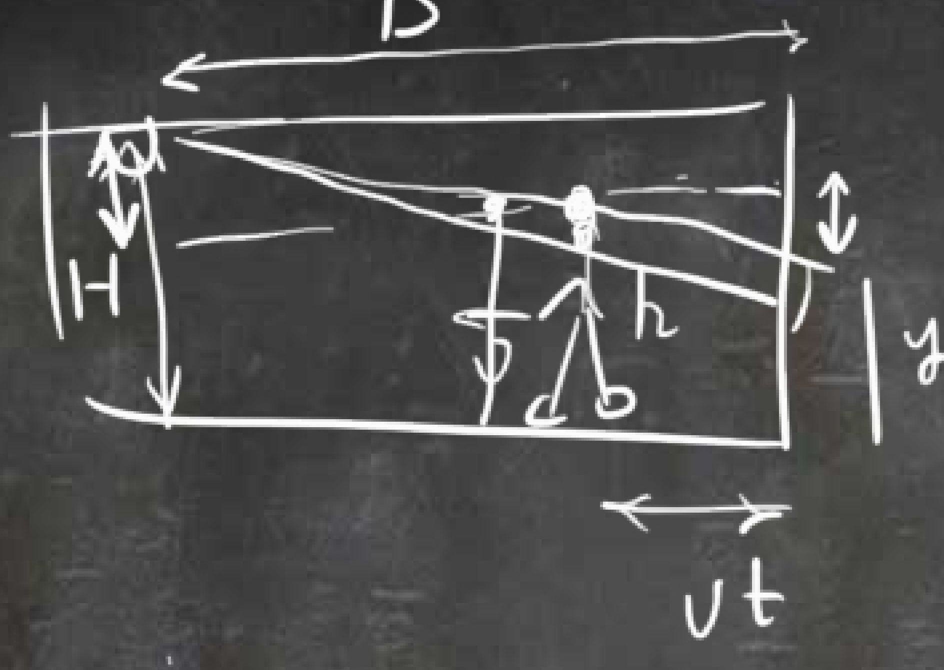
فصل ۴ کتاب ≡ فصل ۱ فاولز

سینماتیک یک بعد دو بعد ایرودوف و هالیدی

دینامیک ایرودوف + هالیدی

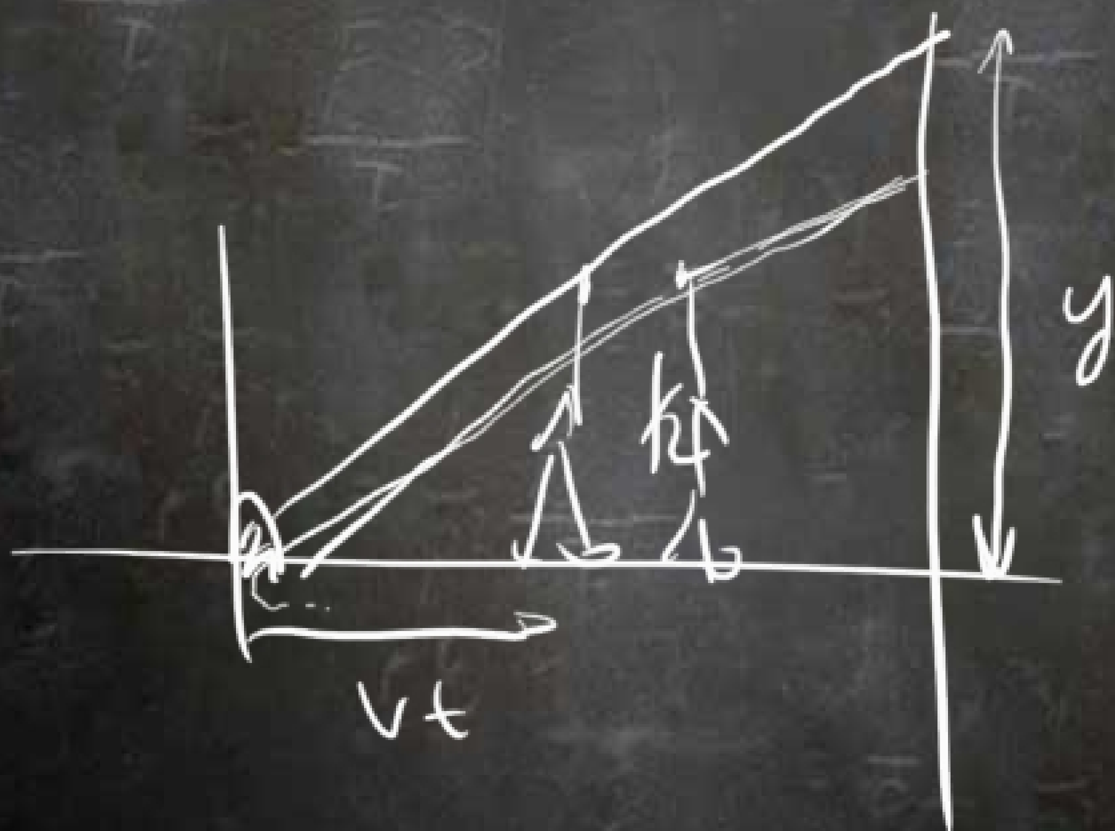
+

جزوات همکلاسی



$$y = h - (H-h)\frac{vt}{D-vt}$$

$$\dot{y} = (H-h)\frac{vt}{D-vt}\left\{\frac{1}{t} + \right.$$



$$y = h\frac{D}{vt} = \frac{hD}{v}t^{-1}$$

$$\boxed{\dot{y} = \frac{-hD}{vt^2}}$$

The End