

Answer questions in the space provided, or on attached sheets with carefully labeled graphs. A notepad report using the Architect is OK, too.

Name/Date _____

Course/Section _____

Exploration 9.2. Harvesting a Natural Resource

Human societies use resources from their environments. We harvest animals and plants for food, construction, fuel, and many other uses. The harvesting of a biological resource must be done carefully, because overexploitation of the population can cause severe harm, or even extinction, to the resource. As a society we have become much more sensitive about the need to balance the benefits of resource consumption against the impact of that consumption on the exploited populations and their environment.

Resource management is an important tool for minimizing the negative effects of harvesting. Mathematical models are tools for understanding the impact of harvesting on a population, so that we can then design management policies, such as quotas on the annual harvest.

In this exploration, you will analyze a simple model for harvesting a single species. To be specific, suppose that the habitat is a forest and the resource is a species of pine tree. The number of trees grows logistically with an intrinsic growth rate r , and the forest will support at most K trees (measured in millions of board feet). You are a consulting ecologist, asked to model the effect of a lumber company's harvesting strategy on the pine forest. The company harvests the trees *proportionally*: in a unit of time (a year, for example), the company removes a fixed fraction h of the trees. Harvesting reduces the net rate of growth of the forest; this leads you to propose the following model for the effect of harvesting:

$$N' = rN\left(1 - \frac{N}{K}\right) - hN, \quad N(0) = N_0 \quad (6)$$

The last term, $-hN$, is the *harvesting term*. Notice that when $h = 0$ (i.e., no trees are harvested), the model reduces to the logistic equation.

1. Open ODE Architect and enter the ODE for the harvesting model given by equation (6). Set the growth rate to $r = 0.1 \text{ year}^{-1}$, the carrying capacity to $K = 1000$ million board feet, and the population size IC to $N_0 = 100$ at $t = 0$. Describe the growth of the forest when there is no harvesting ($h = 0$). You'll have to choose a good time interval to best display your results.

2. Keep r and K fixed and plot solution curves for various (positive) values of the harvesting coefficient h . You can do this exploration most efficiently by sweeping the parameter h . After you have studied a variety of harvest rates, explain how harvesting affects the pine population. Your explanation should address the following questions: How does the growth of the pine population with harvesting compare to its growth without harvesting? What is the long-term effect of harvesting? How are the time dynamics of the forest growth affected by harvesting?

3. The annual yield Y of the harvest is the amount of lumber removed per year. This is just $Y = hN$ when there are N units of lumber in the forest. The yield will vary through time as the amount of lumber (trees) in the forest varies in time. If the harvest rate is too high, the long-term yield will tend to zero ($Y \rightarrow 0$) and the forest will become overexploited. If the harvest rate is very low, the yield will also be very low. As the consultant to the company, you are asked: What should the harvest rate be to give the largest *sustainable yield* of lumber? That is to say, what optimal harvest rate will maximize $\lim_{t \rightarrow \infty} Y(t)$? Attack the problem graphically using ODE Architect to plot graphs of the yield function for various values of h . Assume that $r = 0.1$, $K = 1000$, and $N_0 = 100$. If you can, provide an analytic solution to the question, and check your results using the Architect. Suppose that the company follows your recommendation and harvests pine at the optimal rate. When the size of the forest reaches equilibrium, how much lumber (trees) will there be, and how does this amount compare to the size of the forest without harvesting?