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Renewable Energy II

OPTIMAL SIZING OF SOLAR ELERIC (PHOTOVOLTAIC) AND THERMAL SYSTEMS

Abstract

Aside from their zero fuel costs, the solar systems can reduce the cost of electrical and thermal consumers. In residential regions, use of the photovoltaic (PV) systems causes to reduction in electricity cost. Also, the solar-thermal (ST) system used for water heating can reduce the natural gas consumption and cost. Beside the benefits of the solar systems, their high initial costs are considerable. The objectives of this study are: (1) find an optimal size of the PV system that the minimum electrical cost is reached, (2) find an optimal size of the ST system to minimize the thermal cost. To do this, initial data such as solar radiation, electrical and thermal load, ambient temperature, and the solar systems characteristics are gathered. Then, a mathematical model of the solar systems is presented to define the objective functions and constraints. A search algorithm (genetic algorithm) with limited search domain is used to optimal sizing. Also, a heuristic algorithm is applied to find the optimal sizes. The solar system is applied for a residential house and the results are presented. The results show that the optimal sizes of PV and ST systems are 13 m^2 and 12m^2 when total costs are 1891$ and 587$, respectively.

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Nomenclature

A_c	Collector area (m^2)
A_{pv}	PV area (m^2)
$A_{c-\max}$	Maximum possible collector area (m^2)
$A_{pv-\max}$	Maximum possible PV area (m^2)
C	Area independent cost of collector (\$)
C_{a-b}	Capacity dependent cost of battery ($\frac{\$}{\text{kWh}}$)
C_{a-c}	Area dependent cost of collector ($\frac{\$}{\text{m}^2}$)
C_{ann-b}	Annual cost of battery (\$)
C_{ann-c}	Annual cost of collector (\$)
C_{ann-pv}	Annual cost of PV (\$)
C_{a-pv}	Area dependent cost of PV ($\frac{\$}{\text{m}^2}$)
C_b	Total cost of battery (\$)
C_c	Total cost of collector (\$)
C_{I-b}	Initial cost of battery (\$)
C_{I-c}	Initial cost of collector (\$)
C_{I-pv}	Initial cost of PV (\$)
$C_{M\&O-b}$	Maintenance and operation costs of battery (\$)
$C_{M\&O-c}$	Maintenance and operation costs of collector (\$)
$C_{M\&O-pv}$	Maintenance and operation costs of PV (\$)
C_g	Natural gas cost (\$)
C_{g-ann}	Annual natural gas cost (\$)
C_{net}	Cost of electrical energy met by network (\$)
$C_{net-ann}$	Annual cost of electrical energy met by network (\$)
C_{pv}	Total cost of PV (\$)
C_{R-b}	Replacement cost of battery (\$)

C_{R-c}	Replacement cost of collector ($)
C_{R-pv}	Replacement cost of PV ($)
CS_b	Salvage cost of battery ($)
CS_c	Salvage cost of collector ($)
CS_{pv}	Salvage cost of PV ($)
$C_{total-pv}$	Total cost of PV system ($)
$C_{total-th}$	Total cost of ST system ($)
E	Total annual radiation $(\frac{\mathrm{J}}{\mathrm{m}^2})$
E_b	Battery electrical energy (kWh)
$E_{b-\max}$	Maximum battery electrical energy (kWh)
E_c	Collector output energy (kWh)
$E_c(t)$	Collector output energy at time period t (kWh)
E_g	Water heating energy met by natural gas (kWh)
$E_g(t)$	Water heating energy met by natural gas at time period t (kWh)
$E_l(t)$	Electrical load at time period t (kWh)
E_{net}	Total electrical load met by network (kWh)
$E_{net}(t)$	Total electrical load met by network at time period t (kWh)
E_{pv}	Total electrical energy from PV (kWh)
$E_{pv}(t)$	Total electrical energy from PV at time period t (kWh)
E_{th}	Total water heating load (kWh)
$E_{th}(t)$	Total water heating load at time period t (kWh)
E_{total}	Total electrical load (kWh)
f	Function
F_{c-g}	Radiation view factor between ground and collector
F_{c-s}	Radiation view factor between sky and collector
F_R	Collector removal factor

G	Intensity of solar radiation $(\frac{\text{W}}{\text{m}^2})$
G_{on}	Extraterrestrial normal radiation $(\frac{\text{W}}{\text{m}^2})$
G_{sc}	Solar constant $(\frac{\text{W}}{\text{m}^2})$
H_T	The total radiation on a tilted surface during a day $(\frac{\text{J}}{\text{m}^2})$
$\overline{H_T}$	Monthly average daily radiation on a tilted surface $(\frac{\text{J}}{\text{m}^2})$
I	Total radiation for an hour on a horizontal surface $(\frac{\text{J}}{\text{m}^2})$
i	Interest rate
I_b	Total beam radiation for an hour on a horizontal surface $(\frac{\text{J}}{\text{m}^2})$
I_d	Total diffuse radiation for an hour on a horizontal surface $(\frac{\text{J}}{\text{m}^2})$
I_o	The total solar radiation incident on an extraterrestrial horizontal surface during an hour $(\frac{\text{J}}{\text{m}^2})$
I_T	Total radiation for an hour on a tilted surface $(\frac{\text{J}}{\text{m}^2})$
$\overline{k_T}$	Monthly average clearness index
m	Number of month (=1 to 12)
N	Number of daylight hours
n	Day number of year
n_{month}	The number of month's day
n_y	Life time (year)
Pr_e	Electricity price $(\frac{\$}{\text{kWh}})$
Pr_g	Natural gas price $(\frac{\$}{\text{m}^3})$ or $(\frac{\$}{\text{kWh}})$
q_c	Actual useful thermal gain of collector (W)
R_b	Geometric factor

Re	Resolution of heuristic algorlthm
t	Time period index (month)
t'	Time period index of summer day (hour)
T_a	Ambient temperature (°C)
t_c	Total operation time of collector (hour)
T_i	Collector initial temperature (°C)
t_1, t_2	First and end hour of a time period (minute)
U_l	Overall heat transfer coefficient of collector $(\frac{\mathrm{W}}{\mathrm{m}^{2\circ}\mathrm{C}})$
$\alpha\tau$	Transmittance absorptance product of collector
β	Surface slope (degree)
η_{pv}	Efficiency of PV
γ	Surface azimuth angle (degree)
δ	Declination (degree)
θ	Angle of incidence (degree)
θ_z	Zenith angle (degree)
ρ_g	Ground reflectance
ϕ	Latitude (degree)
ω	Hour angle (degree)
ω_1, ω_2	First and end hour angles of a time period (degree)

1. Introduction

The objectives of this study are: (1) find an optimal size of the photovoltaic (PV) system that the minimum electrical cost is reached, (2) find an optimal size of the solar-thermal (ST) system to minimize the thermal cost for a residential house.

In recent years, some optimization methods are applied to various applications of solar systems to find the optimal size of PV and ST systems. PV and battery are used to meet electrical load in [1]-[8]. PV are used to water pumping application in some studies [9]-[10]. Also, water heating solar systems are used for optimal sizing problems [11]-[12].

In this study, in the PV system, the electrical demand is provided by network, PV system, and battery system. The ST system is only used for water heating. So, the water heating demand is met by natural gas and the ST system.

The procedure of this study is:

1- Gathering required information (solar radiation, electrical and thermal load, ambient temperature, and the solar systems characteristics)
2- Modelling of the systems (PV and ST)
3- Determination of the objective functions and constraints
4- Optimization algorithm (limited search domain algorithm)
5- Simulation and results
6- Conclusion

Next, systems description is presented in section 2. Section 3 formulates the problems. Section 4 explains the objective functions, constraints, and optimization algorithm. The simulation results are presented in Section 5. Finally, in Section 6, conclusion is given.

2- Description of the systems

2-1- The PV system

Figure (1) shows the PV system [1]. This system includes PV array, battery, and electrical consumers. The electrical load is met by PV array, battery, and electrical network. The battery is used for increasing reliability of the system and storing electrical energy when an excess solar energy is available.

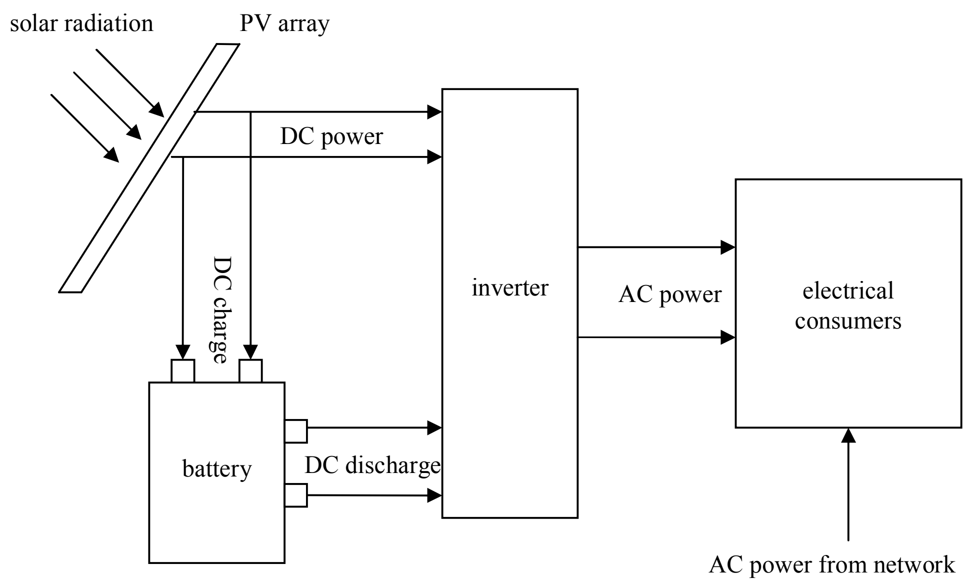

Figure 1- schematic of the PV system [1]

2-2- The ST system

Figure (2) shows the ST system used for water heating [10]. This system is used for a residential house and includes a solar collector, a conventional water heater, and hot water storage. Therefore, the hot water load is met by solar collector and conventional water heater.

Other thermal loads such as cooking and human heating could not support by such ST system because they are directly consume natural gas with no hot water need.

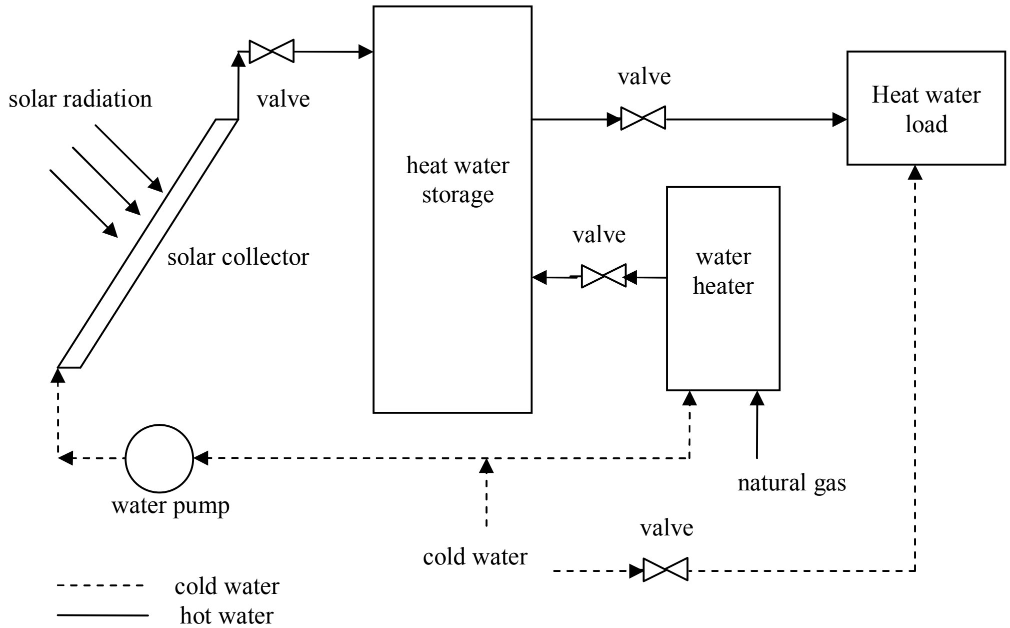

Figure 2- schematic of the ST system used for water heating [10]

3- Problem formulation

In this section, the formulation of the problem is presented.

3-1- Solar irradiation

The total radiation for an hour on a tilted surface (I_T) is [13]

$$I_T = I_b R_b + I_d F_{c-s} + I \rho_g F_{c-g} \tag{1}$$

Also, the total radiation on a tilted surface during a day (H_T) and total annual radiation (E) are as below, respectively [13]:

$$H_T = \sum_N I_T \tag{2}$$

$$E = \sum_m \sum_{n_{month}} \sum_N \{I_T\} \tag{3}$$

where [13],

$$I_b = I - I_d \tag{4}$$

$$I_d = \begin{cases} I(1 - 0.09\overline{K_T}) & \overline{K_T} \le 0.22 \\ I\left(0.95 - 0.16\overline{K_T} + 4.3(\overline{K_T})^2 - 16.6(\overline{K_T})^3 + 12.3(\overline{K_T})^4\right) & 0.22 \le \overline{K_T} < 0.8 \\ 0.165I & \overline{K_T} \ge 0.8 \end{cases} \tag{5}$$

$$R_b = \frac{\cos\theta}{\cos\theta_z} \tag{6}$$

$$\begin{aligned} \cos\theta = {} & \sin\phi\sin\delta\cos\beta + \cos\omega\cos\delta\cos\phi\cos\beta - \sin\delta\cos\phi\sin\beta\cos\gamma \\ & + \cos\gamma\cos\omega\cos\delta\sin\phi\sin\beta + \cos\delta\sin\beta\sin\gamma\sin\omega \end{aligned} \tag{7}$$

$$\cos\theta_z = \cos\omega\cos\delta\cos\phi + \sin\phi\sin\delta \tag{8}$$

$$I = \overline{K_T} I_o \tag{9}$$

$$I_o = \frac{12 \times 3600}{\pi} G_{on} \left(\cos\phi\cos\delta(\sin\omega_1 - \sin\omega_2) + \pi\left(\frac{\omega_1 - \omega_2}{180}\right)\sin\phi\sin\delta \right) \tag{10}$$

$$\omega_2 = 15^\circ (t_2 - 12:00') \tag{11}$$

$$\omega_1 = 15^\circ (t_1 - 12:00') \tag{12}$$

$$\omega = \frac{\omega_2 + \omega_1}{2} \tag{13}$$

$$F_{c-s} = \frac{1 + \cos\beta}{2} \tag{14}$$

$$F_{c-g} = \frac{1-\cos\beta}{2} \tag{15}$$

$$\delta = 23.45\sin(360\frac{284+n}{365}) \tag{16}$$

$$G_{on} = G_{sc}\left(1+0.033\cos(\frac{360n}{365})\right) \tag{17}$$

and constant parameters are:

$$G_{sc} = 1367 \ \frac{\mathrm{W}}{\mathrm{m}^2} \tag{18}$$

$$\phi = 34^\circ \tag{19}$$

$$\gamma = 0 \tag{20}$$

$$\rho_g = 0.6 \qquad \text{If } n < 60 \ \text{(winter)} \tag{21}$$

$$\rho_g = 0.2 \qquad \text{If } n > 59 \ \text{(other seasons)} \tag{22}$$

$$\beta = 34^\circ \tag{23}$$

The surface slope is considered equal to latitude to reach maximum annual solar energy [13]. The surface azimuth angle is considered zero to reach maximum annual solar energy [13].

3-2- PV array

The total energy from PV array can be achieved from [7]

$$E_{pv} = E\eta_{pv}A_{pv} \tag{24}$$

The total cost of PV array is [10]

$$C_{pv} = C_{I-pv} + C_{M\&O-pv} + C_{R-pv} - CS_{pv} \tag{25}$$

$$C_{I-pv} = C_{a-pv}A_{pv} \tag{26}$$

$$C_{M\&O-pv} = 0.02C_{I-pv} \tag{27}$$

$$CS_{pv} = 0.2C_{I-pv} \tag{28}$$

To calculate annual cost, the total cost is changed as [8]

$$C_{ann-pv} = C_{pv}\left(\frac{i(1+i)^{n_y}}{(1+i)^{n_y}-1}\right) \tag{29}$$

3-3- Solar collector

The useful thermal gain of a solar collector is [10]

$$q_c = F_R A_c \left[\alpha\tau G - U_l (T_i - T_a)\right]^+ \tag{30}$$

where + sign indicates that only positive values of q_c is considered in the study.

The total cost of PV system is

$$C_c = C_{I-c} + C_{M\&O-c} + C_{R-c} - CS_c \tag{31}$$

$$C_{I-c} = C + C_{a-c} A_c \tag{32}$$

$$C_{M\&O-c} = 0.02 C_{I-c} \tag{33}$$

$$CS_c = 0.2 C_{I-c} \tag{34}$$

To calculate annual cost, the total cost is changed as

$$C_{ann-c} = C_c \left(\frac{i(1+i)^{n_y}}{(1+i)^{n_y} - 1} \right) \tag{35}$$

3-4- Battery system

The total cost of battery system is [10

$$C_b = C_{I-b} + C_{M\&O-b} + C_{R-b} - CS_b \tag{36}$$

$$C_{I-b} = C_{a-b} E_{b-\max} \tag{37}$$

$$C_{M\&O-b} = 0.02 C_{I-b} \tag{38}$$

$$CS_b = 0.2 C_{I-b} \tag{39}$$

$$E_{b-\max} = \sum_{t'} \left(E_{pv}(t') - E_l(t') \right)^+ \tag{40}$$

t' indicates the summer day hours.

To calculate annual cost, the total cost is changed as [8]

$$C_{ann-b} = C_b \left(\frac{i(1+i)^{n_y}}{(1+i)^{n_y} - 1} \right) \tag{41}$$

4- Optimization algorithm

In this section, the objective functions, constraints, and optimization algorithm are presented.

4-1- The objective functions and constraints

4-1-1- The PV system

The objective function of PV system is

$$\mathrm{Min}\{C_{total-pv}\} \tag{42}$$

$$C_{total-pv} = C_{ann-pv} + C_{ann-b} + C_{net-ann} \tag{43}$$

$$C_{net-ann} = C_{net}(1+i)^{n_y} \tag{44}$$

$$C_{net} = E_{net}\,\mathrm{Pr}_e \tag{45}$$

$$E_{net} = E_{total} - E_b - E_{pv} \tag{46}$$

$$E_b = \sum_t \left(E_{pv}(t) - E_l(t)\right)^+ \tag{47}$$

where + sign indicates that the battery charges during hours that $E_{net} - E_{pv} < 0$ and discharges in other hours (peak-load hours)

Constraints

$$E_l(t) = E_{net}(t) + E_b(t) + E_{pv}(t) \tag{48}$$

$$E_{pv}(t) \geq 0 \tag{49}$$

$$E_{net} \geq 0$$

4-1-2- The ST system

The objective function of ST system is

$$\mathrm{Min}\{C_{total-th}\} \tag{50}$$

$$C_{total-th} = C_{ann-c} + C_{g-ann} \tag{51}$$

$$C_{g-ann} = C_g(1+i)^{n_y} \tag{52}$$

$$C_g = E_g \Pr_g \tag{53}$$

$$E_g = E_{th} - E_c \tag{54}$$

$$E_c = q_c t_c \tag{55}$$

Constraints

$$E_{th}(t) = E_g(t) + E_c(t) \tag{56}$$

$$E_c(t) \geq 0 \tag{57}$$

$$E_g \geq 0$$

4-2- optimization algorithm

4-2-1- optimization algorithm of PV system

In this section, the optimization algorithm of PV system is described.

To find the optimal size of PV array, hourly electrical load of a summer day is needed. In the summer, the electrical consumption is high. So, if the PV system size is selected according to a summer day, it could be perform more reliable for other days.

The procedure of finding the optimal size of PV system is:

1- Select a summer day
2- Gather required information (hourly electrical load, hourly solar radiation on a tilted surface, PV and battery system price and efficiency, and electricity price)
3- Find maximum and minimum possible values of A_{pv}

Note: the minimum A_{pv} is zero and the maximum A_{pv} is happened when E_{net} is zero. It means that the PV and battery system provide all of E_{total} of the selected summer day.

Obviously, if $A_{pv} > A_{pv-\max}$, total cost of PV system increases.

Note: The objective function of search method is only a function of A_{pv}.

According to Equations (25)-(29), (36)-(41)

$$C_{ann-pv} = \left(C_{a-pv} A_{pv}(1+0.02-0.2) + C_{R-pv}\right)\left(\frac{i(1+i)^{n_y}}{(1+i)^{n_y}-1}\right) = f(A_{pv}) \tag{58}$$

$$C_{ann-b} = \left(C_{a-b}E_{b-\max}(1+0.02-0.2)+C_{R-b}\right)\left(\frac{i(1+i)^{n_y}}{(1+i)^{n_y}-1}\right) \tag{59}$$

$$E_{b-\max} = f(E_{pv}) = f(A_{pv}) \Rightarrow C_{ann-b} = f(A_{pv}) \tag{60}$$

$$E_{net} = f(E_b, E_{pv}) = f(A_{pv}) \tag{61}$$

therefore,

$$C_{total-pv} = f(A_{pv}) \tag{62}$$

The objective function is subjected to some following constraints:

Equations (48)-(49)

In each time period, if $E_{net} - E_{pv} \geq 0$, then $E_b = 0$ and the battery is in idle state. If $E_{net} - E_{pv} < 0$, then $E_b = \left|E_{net} - E_{pv}\right|$. The battery is then discharges at peak-load hours.

$$0 < A_{pv} \leq A_{pv-\max} \tag{63}$$

4- Find the optimal A_{pv} when the $C_{total-pv}$ as a function of A_{pv} is minimized using a search method such as genetic algorithm (GA) when A_{pv} is limited between 0 and $A_{pv-\max}$.

The flowchart of optimization algorithm of PV system is presented in figure (3).

A heuristic algorithm can be used for optimal sizing as shown in figure (4). The Re is the resolution of search. For example, if $\mathrm{Re} = 0.5$, the heuristic algorithm find A_{pv} with the resolution of 0.5.

4-2-2- optimization algorithm of ST system

A same optimization algorithm with same procedure is used to optimal sizing of ST system. Similarly, he total cost of ST system is only a function of A_c and the minimum and maximum A_c are calculated. In this section, no battery is available.

The flowchart of optimization algorithm for ST system is similar to figures (3) and (4).

$$C_{total-th} = f(A_c) \tag{64}$$

$$0 < A_c \leq A_{c-\max} \tag{65}$$

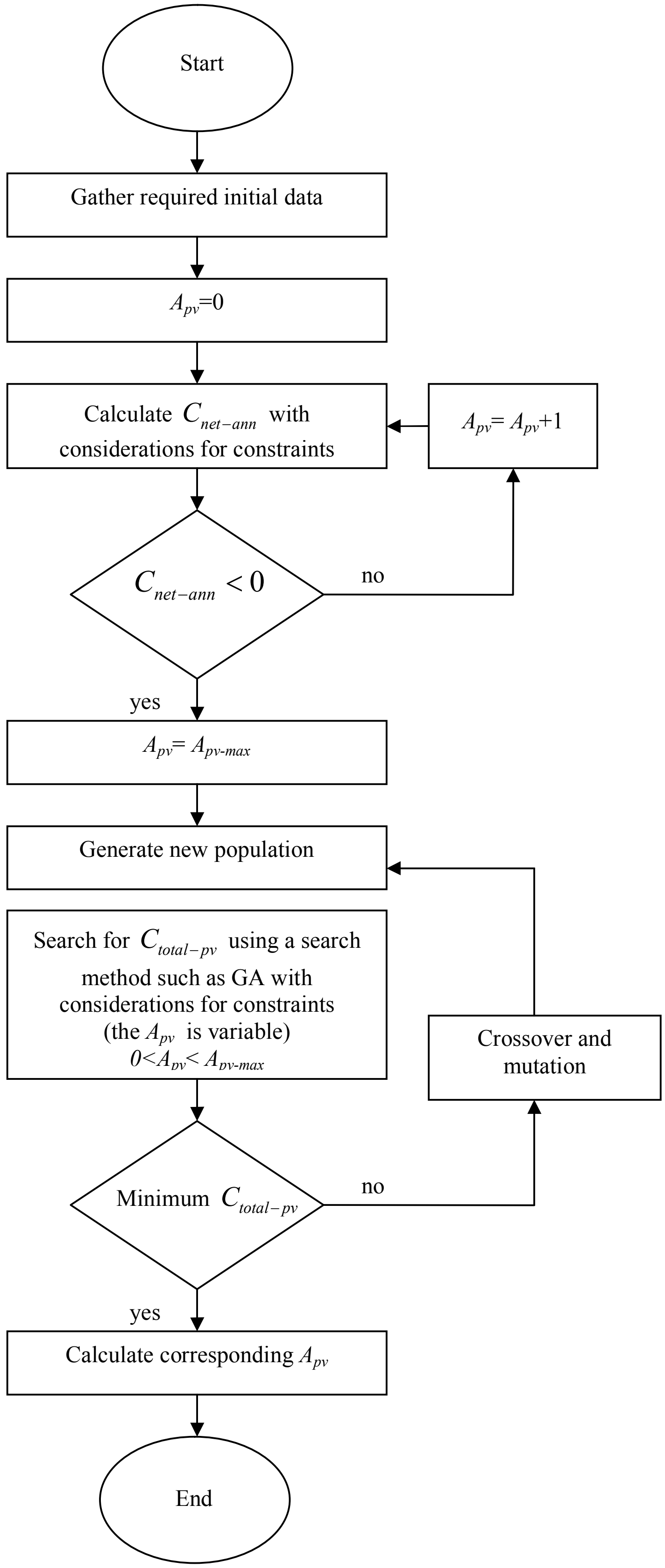
Start
Gather required initial data
A_{pv}=0
Calculate $C_{net-ann}$ with considerations for constraints
A_{pv}= A_{pv}+1
$C_{net-ann} < 0$
no
yes
A_{pv}= A_{pv-max}
Generate new population
Search for $C_{total-pv}$ using a search method such as GA with considerations for constraints (the A_{pv} is variable) $0<A_{pv}< A_{pv-max}$
Crossover and mutation
Minimum $C_{total-pv}$
no
yes
Calculate corresponding A_{pv}
End

Figure 3- the flowchart of optimization algorithm of PV system

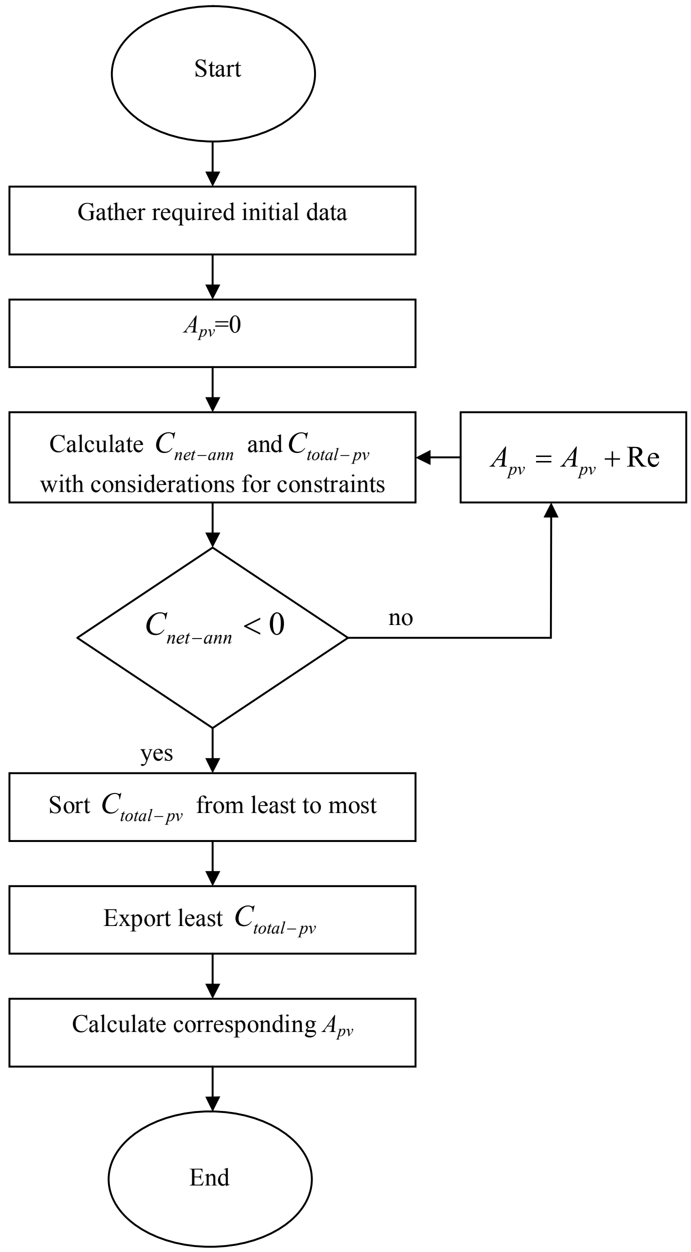

Figure 4- the flowchart of heuristic algorithm of PV system

5- Simulation results

In this section, the simulation results of PV and ST systems are presented.

5-1- simulation results of PV system

In this section, the simulation results of PV system are presented.

The necessary data are available as below:

Figures (5)-(6) show the selected summer day (Tuesday, 7 June, 2011) solar radiation and electrical load. The solar radiation and electrical load data are shown in table (1). Table (2) shows the electrical consumers characteristics.

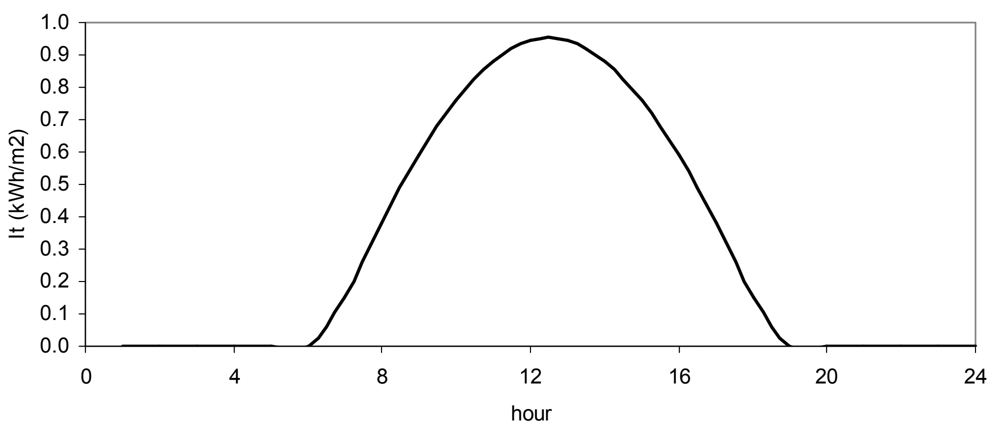

Figure 5- solar radiation of the selected summer day

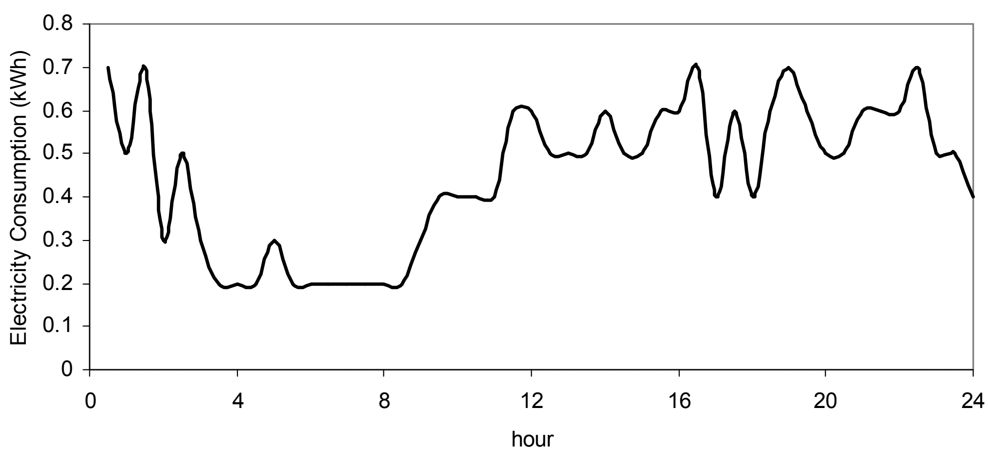

Figure 6- electrical load of the selected summer day

Table 1- solar radiation and electrical load data of the selected summer day

Hour	solar radiation (kWh/m^2)	electrical load (kWh)
1	0	1.2
2	0	1
3	0	0.8
4	0	0.4
5	0	0.5
6	0.001654	0.4
7	0.149002	0.4
8	0.382118	0.4
9	0.591125	0.5
10	0.761778	0.8
11	0.882449	0.8
12	0.944912	1.2
13	0.944912	1
14	0.882449	1.1
15	0.761778	1
16	0.591125	1.2
17	0.382118	1.1
18	0.149002	1
19	0.001654	1.3
20	0	1.1
21	0	1.1
22	0	1.2
23	0	1.2
24	0	0.9

Other data are:

City: Khomein- Iran

$\beta = 34^\circ$, $\phi = 34^\circ$, $\gamma = 0$, $\overline{k_T} = 0.77$

$C_{a-pv} = 450 \dfrac{\$}{\text{m}^2}$, $C_{R-pv} = C_{R-b} = 0$, $\eta_{pv} = 0.16$, $C_{a-b} = 1370 \dfrac{\$}{\text{kWh}}$ [7].

$i = 0.1$, $n_y = 10$, $\Pr_e = 0.0343 \dfrac{\$}{\text{kWh}}$.

The simulation is done fore 10-years with $i = 0.1$. The annual electrical load and solar radiation are shown in figures (7)-(8).

Table 2- electrical consumers characteristics

Consumer	Demand (W)	**amount**	Daily operation hours
refrigerator	700	1	8
	1000	1	7
washing machine	150	1	0.25
TV	110	1	10
computer	200	1	10
illumination (summer)	100	4	2
	200	2	1
	40	16	4
illumination (winter)	100	4	3
	200	2	2
	40	16	7
cooler	500	1	16
vacuum cleaner	1200	1	0.25

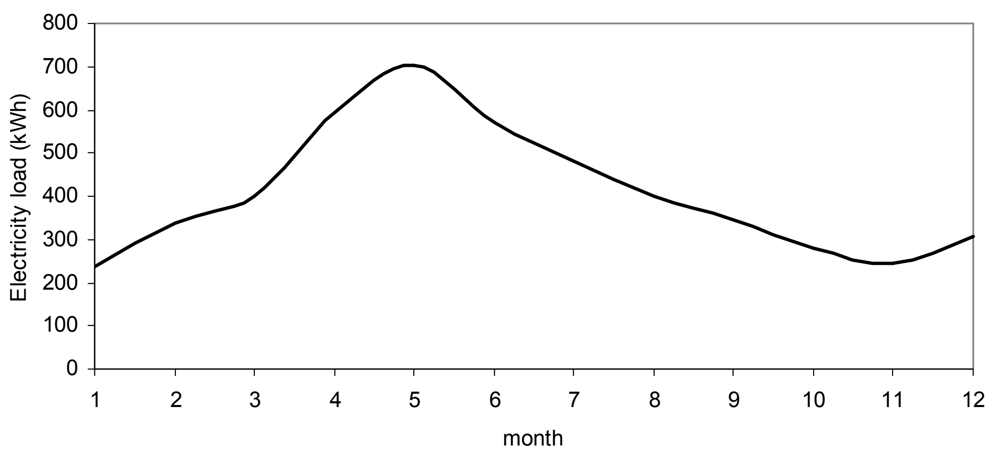

Figure 7- annual electrical load

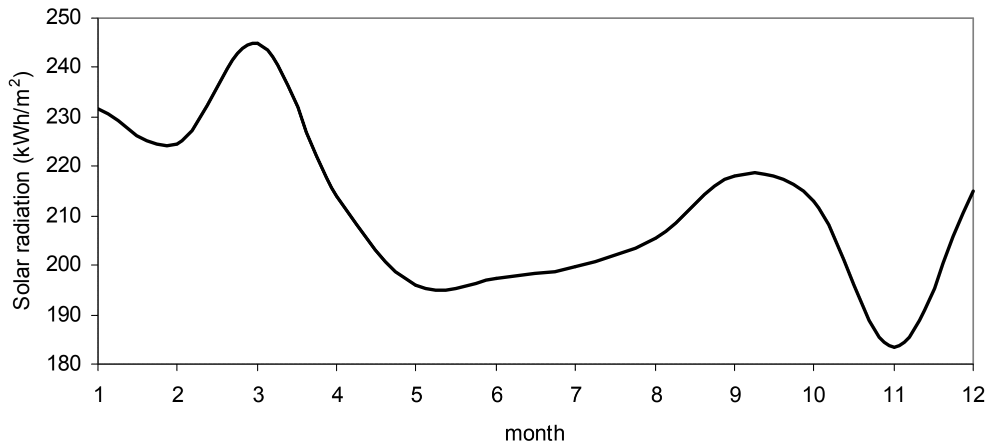

Figure 8- annual solar radiation

The $A_{pv-\max} = 13\text{m}^2$ and therefore, the search algorithm (GA) finds the optimal size of PV between 0 and 13m^2. The optimal size of PV is 12.98m^2 with $C_{total-pv} = 1886.3\$$ and then, a 13m^2 PV is selected. Figure (9) and table (3) show the $C_{total-pv}, C_{ann-pv}, C_{ann-b}, C_{net}$ variation versus A_{pv}. It is shown that the minimum $C_{total-pv}$ is corresponding to $A_{pv} = 13\text{m}^2$

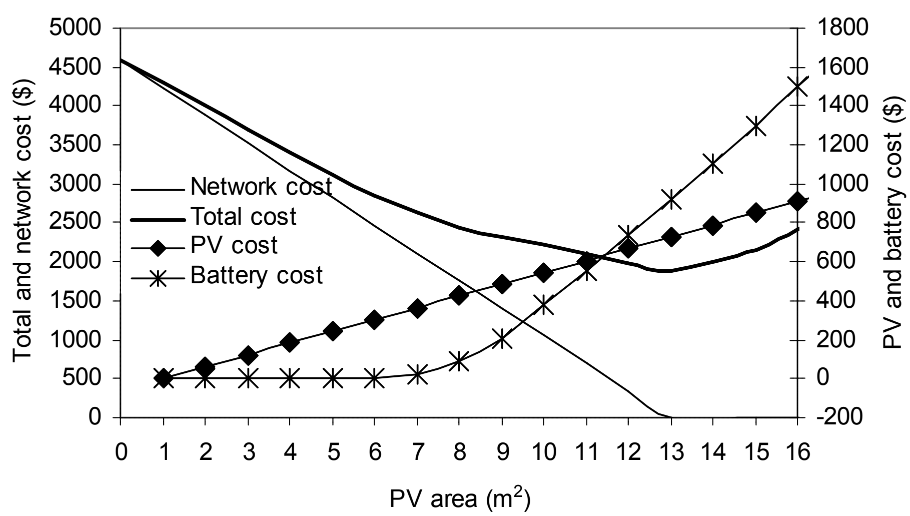

Figure 9- the $C_{total-pv}, C_{ann-pv}, C_{ann-b}, C_{net}$ variation versus A_{pv}

Table 3- the $C_{total-pv}, C_{ann-pv}, C_{ann-b}, C_{net}$ variation versus A_{pv}

A_{pv} (m^2)	C_{net} (\$)	$C_{total-pv}$ (\$)	C_{ann-pv} (\$)	C_{ann-b} (\$)
0	4586	4586	0	0
1	4232	4293	61	0
2	3879	4000	121	0
3	3526	3708	182	0
4	3173	3415	243	0
5	2819	3123	303	0
6	2466	2851	364	21
7	2113	2627	425	90
8	1759	2446	485	201
9	1406	2329	546	377
10	1053	2216	607	557
11	700	2104	667	737
12	346	1992	728	917
13	0	1891	789	1103
14	0	2149	849	1300
15	0	2408	910	1498
16	0	2666	971	1695

5-2- simulation results of ST system

In this section, the simulation results of ST system are presented.

The necessary data are available as below:

Figures (10)-(11) show the selected summer day (Tuesday, 7 June, 2011) solar radiation and water-heating load. As shown in figure (11), water-heating load is thermal load minus cooking load. The solar radiation and water-heating load data are shown in table (4). All thermal consumers characteristics are listed in table (5)

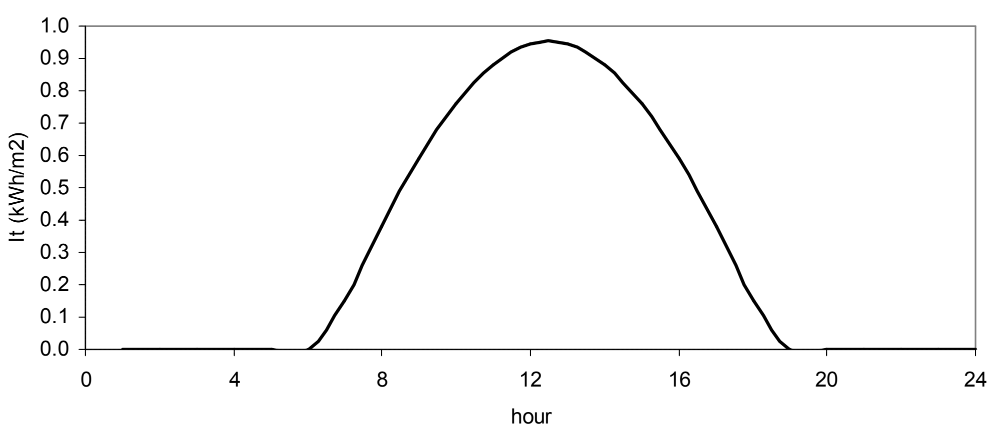

Figure 10- solar radiation of the selected summer day

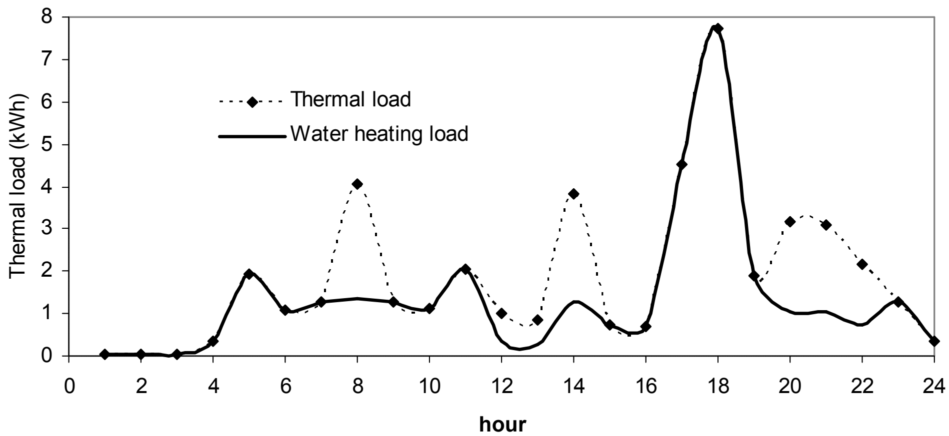

Figure 11- water-heating load of the selected summer day

Table 4- solar radiation and water-heating load data of the selected summer day

Hour	solar radiation (kWh/m^2)	water-heating load (kWh)
1	0	0.02134
2	0	0.03201
3	0	0.03201
4	0	0.34144
5	0	1.9206
6	0.001654	1.067
7	0.149002	1.2804
8	0.382118	1.347977
9	0.591125	1.29107
10	0.761778	1.10968
11	0.882449	2.04864
12	0.944912	0.33077
13	0.944912	0.27742
14	0.882449	1.26973
15	0.761778	0.72556
16	0.591125	0.70422
17	0.382118	4.53475
18	0.149002	7.73575
19	0.001654	1.88859
20	0	1.059887
21	0	1.027877
22	0	0.718447
23	0	1.29107
24	0	0.35211

Other data are:

$\beta = 34^\circ$, $\phi = 34^\circ$, $\gamma = 0$, $\overline{k_T} = 0.77$

$C_{a-c} = 333 \dfrac{\$}{\text{m}^2}$, $C = 400\$$, $C_{R-c} = 0$.

$i = 0.1$, $n_y = 10$, $\text{Pr}_g = 0.0837 \dfrac{\$}{\text{m}^3} = 0.0078 \dfrac{\$}{\text{kWh}}$, $F_R = 0.8, \alpha\tau = 0.9, U_l = 10 \dfrac{\text{W}}{\text{m}^{2\text{o}}\text{C}}$

[10], $T_i = 40^\text{o}\text{C}$.

The annual ambient temperature is shown in figure (12).

Table 5- thermal consumers characteristics

Consumer	Demand (kcal/hour)	amount	Daily operation hours
heater	1500-8500	4	10-24
water heater	9500	1	10
oven	big:3100	1	2
	medium:1550	2	2
	small: 1400	3	2

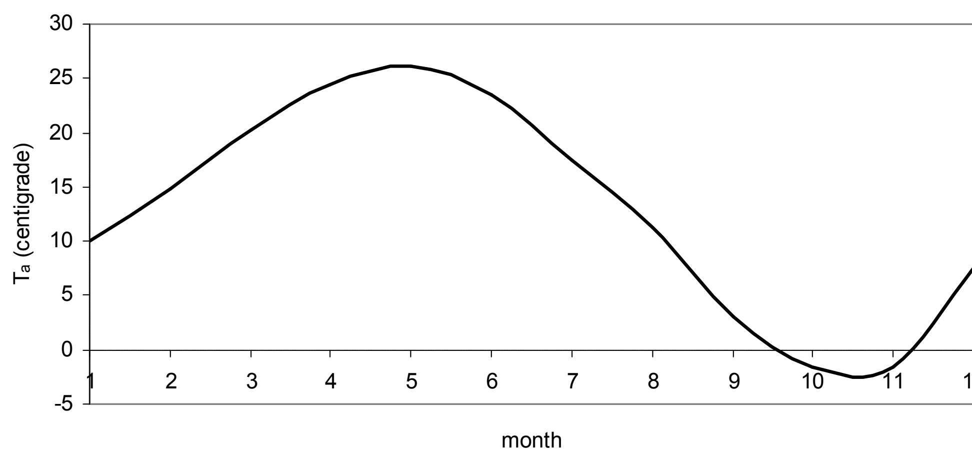

Figure 12- annual ambient temperature

The simulation is done fore 10-years with $i = 0.1$. The annual water-heating load is assumed to be fixed and solar radiation is shown in figures (13).

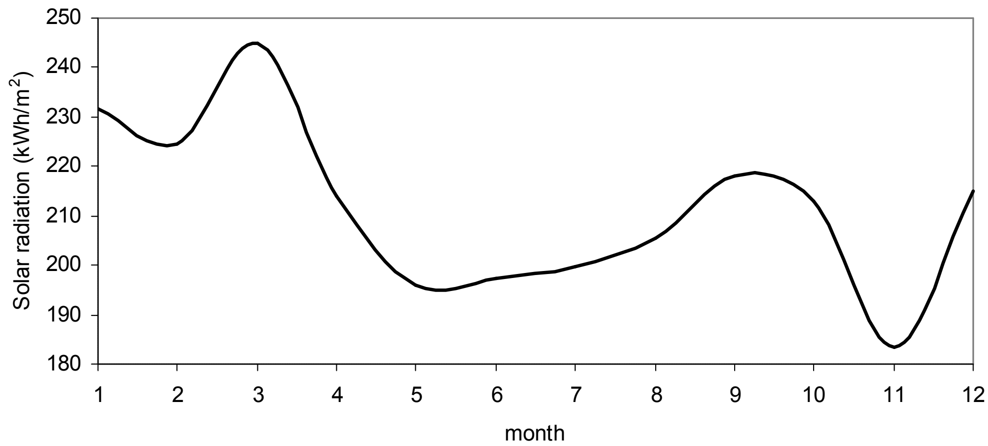

Figure 13- annual solar radiation

The $A_{c-\max} = 12\text{m}^2$ and therefore, the search algorithm (GA) finds the optimal size of collector between 0 and 12m^2. The optimal size of collector is 11.62m^2 with $C_{total-th} = 569.9\$$ and then, a 12m^2 PV is selected. Figure (14) and table (6) show the $C_{total-th}, C_{ann-c}, C_g$ variation versus A_c. It is shown that the minimum $C_{total-th}$ is corresponding to $A_c = 12\text{m}^2$

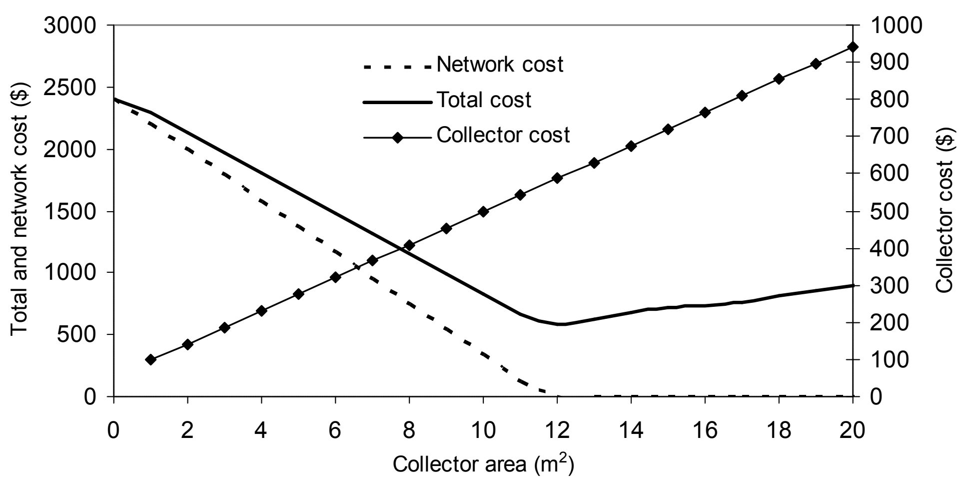

Figure 14- the $C_{total-th}, C_{ann-c}, C_g$ variation versus A_c

Table 6- the $C_{total-th}, C_{ann-c}, C_g$ variation versus A_c

$A_c(m^2)$	C_g (\$)	$C_{total-th}$ (\$)	C_{ann-c} (\$)
0	2406	2406	0
1	2199	2297	98
2	1992	2134	142
3	1785	1972	187
4	1578	1809	231
5	1371	1647	276
6	1164	1484	320
7	957	1321	364
8	750	1159	409
9	543	996	453
10	336	834	498
11	129	671	542
12	0	587	587
13	0	631	631
14	0	676	676
15	0	720	720
16	0	764	764
17	0	809	809
18	0	853	853
19	0	898	898
20	0	942	942

6- Conclusion

The optimal sizing of PV and ST systems is presented in this study.

In the PV system, the electrical demand is provided by network, PV system, and battery system.

The ST system is only used for water heating. So, the water heating demand is met by natural gas and the ST system.

The procedure of this study is:

1- Gathering required information (solar radiation, electrical and thermal load, ambient temperature, and the solar systems characteristics)

2- Modelling of the systems (PV and ST)

3- Determination of the objective functions and constraints

4- Optimization algorithm (limited search domain algorithm)

5- Simulation and results

The results show that the optimal sizes of PV and ST systems are 13 m^2 and 12m^2 when total costs are 1891$ and 587$, respectively.

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