تکلیف پایانی درس ریاضیات برای اقتصاد (ارشد دانشگاه مفید): (به صورت کار گروهی دو نفره)

قسمت اول: (سوالات 20 و21) دانشجویانی که رقم سمت راست شماره دانشجویی آنها زوج می باشد.

قسمت دوم: (سوالات 16 و 17) دانشجویانی که رقم سمت راست شماره دانشجویی آنها فرد می باشد

1- با توجه به رقم سمت راست شماره دانشجویی، یک گروه 2 نفره تشکیل دهید.
2- متن هر قسمت از سوالات مربوطه را ترجمه کنید. (برای اصطلاحات خاص معادل لاتین آنرا در پاورقی بیاورید (مثلا جواب تعادلی[1])
3- مطابق صورت مسأله مراحل را با نرم افزار maple حل کنید
4- جوابهای بدست آمده از هر مرحله را با توضیحات کامل را در فایل word وارد کنید.
5- برای متن خود یک چکیده در ابتدا و یک نتیجه گیری در انتها بیاورید. در متن خود مسأله و نتایج مسأله را تحلیل کنید.
6- درباره مدل های ذکر شده در متن سوالات تحقیق کنید و در حد یک پاراگراف توضیح دهید
7- مراجعی که استفاده کرده اید را ذکر کنید.
8- در تاریخ چهارشنبه 7 بهمن 1394 ساعت 9 الی 11 کلاس 355 گزارش خود را تحویل دهید. (حدود 4 صفحه A4)
9- گزارش باید بصورت پرینت شده و شامل همه مطالب فوق الذکر باشد.

راهنمایی: فصل دوم –بخش 5 (Sec. 2.5) از کتاب معادلا دیفرانسیل بویس.+ جزوه تدریس شده در کلاس درس + فایلهای آموزشی maple که در تلگرام قرار داده شده است)

نکته: تکلیف پایانی بخشی از درس می باشد و کسانی که تکلیف خود را تحویل نداده باشند به منزله ناتمام بودن درس می باشد.

[1] Equilibrium solution

Harvesting a Renewable Resource. Suppose that the population y of a certain species of fish (for example, tuna or halibut) in a given area of the ocean is described by the logistic equation

$$dy/dt = r(1 - y/K)y.$$

Although it is desirable to utilize this source of food, it is intuitively clear that if too many fish are caught, then the fish population may be reduced below a useful level and possibly even driven to extinction. Problems 20 and 21 explore some of the questions involved in formulating a rational strategy for managing the fishery.[15]

20. At a given level of effort, it is reasonable to assume that the rate at which fish are caught depends on the population y: the more fish there are, the easier it is to catch them. Thus we assume that the rate at which fish are caught is given by Ey, where E is a positive constant, with units of 1/time, that measures the total effort made to harvest the given species of fish. To include this effect, the logistic equation is replaced by

$$dy/dt = r(1 - y/K)y - Ey. \tag{i}$$

This equation is known as the **Schaefer model** after the biologist M. B. Schaefer, who applied it to fish populations.

(a) Show that if $E < r$, then there are two equilibrium points, $y_1 = 0$ and $y_2 = K(1 - E/r) > 0$.

(b) Show that $y = y_1$ is unstable and $y = y_2$ is asymptotically stable.

(c) A sustainable yield Y of the fishery is a rate at which fish can be caught indefinitely. It is the product of the effort E and the asymptotically stable population y_2. Find Y as a function of the effort E; the graph of this function is known as the yield–effort curve.

(d) Determine E so as to maximize Y and thereby find the **maximum sustainable yield** Y_m.

21. In this problem we assume that fish are caught at a constant rate h independent of the size of the fish population. Then y satisfies

$$dy/dt = r(1 - y/K)y - h. \tag{i}$$

The assumption of a constant catch rate h may be reasonable when y is large but becomes less so when y is small.

(a) If $h < rK/4$, show that Eq. (i) has two equilibrium points y_1 and y_2 with $y_1 < y_2$; determine these points.

(b) Show that y_1 is unstable and y_2 is asymptotically stable.

(c) From a plot of $f(y)$ versus y, show that if the initial population $y_0 > y_1$, then $y \to y_2$ as $t \to \infty$, but that if $y_0 < y_1$, then y decreases as t increases. Note that $y = 0$ is not an equilibrium point, so if $y_0 < y_1$, then extinction will be reached in a finite time.

(d) If $h > rK/4$, show that y decreases to zero as t increases, regardless of the value of y_0.

(e) If $h = rK/4$, show that there is a single equilibrium point $y = K/2$ and that this point is semistable (see Problem 7). Thus the maximum sustainable yield is $h_m = rK/4$, corresponding to the equilibrium value $y = K/2$. Observe that h_m has the same value as Y_m in Problem 20(d). The fishery is considered to be overexploited if y is reduced to a level below $K/2$.

[15] An excellent treatment of this kind of problem, which goes far beyond what is outlined here, may be found in the book by Clark mentioned previously, especially in the first two chapters. Numerous additional references are given there.

16. Another equation that has been used to model population growth is the Gompertz[14] equation

$$dy/dt = ry \ln(K/y),$$

where r and K are positive constants.

(a) Sketch the graph of $f(y)$ versus y, find the critical points, and determine whether each is asymptotically stable or unstable.

(b) For $0 \leq y \leq K$, determine where the graph of y versus t is concave up and where it is concave down.

(c) For each y in $0 < y \leq K$, show that dy/dt as given by the Gompertz equation is never less than dy/dt as given by the logistic equation.

17. (a) Solve the Gompertz equation

$$dy/dt = ry \ln(K/y),$$

subject to the initial condition $y(0) = y_0$.
Hint: You may wish to let $u = \ln(y/K)$.

(b) For the data given in Example 1 in the text ($r = 0.71$ per year, $K = 80.5 \times 10^6$ kg, $y_0/K = 0.25$), use the Gompertz model to find the predicted value of $y(2)$.

(c) For the same data as in part (b), use the Gompertz model to find the time τ at which $y(\tau) = 0.75K$.