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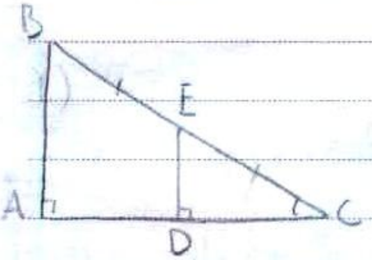
بنام خداوند یکتای بی همتا

۱۳۹۲/۱/۳۰

جواب دوره تمرین بهمن (هندسه)

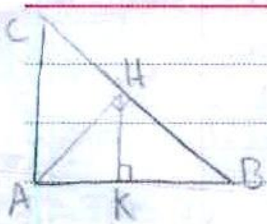
امیرحسین گرزین کلاس ۲۰۲



۱) $\left.\begin{matrix}\hat{A}=\hat{D}=90^\circ \\ \hat{C}=\hat{C} \text{ مشترک}\end{matrix}\right\} \xRightarrow{(زز)} \overset{\triangle}{ABC} \sim \overset{\triangle}{DEC} \Rightarrow \frac{CD}{AC}=\frac{DE}{AB}=\frac{CE}{BC}=\frac{1}{2}$ ①

① $\Longrightarrow \begin{cases} 2CD=AC & ② \\ 2DE=AB & ③ \end{cases}$

$$\frac{S_{DEC}}{S_{ABC}}=\frac{CD\times DE\times \frac{1}{2}}{AB\times AC\times \frac{1}{2}} \overset{②③}{=} \frac{CD\times DE\times \frac{1}{2}}{2CD\times 2DE\times \frac{1}{2}}=\frac{1}{4}$$



۲)

① $A\hat{C}H \sim ABC$ (زز) $\Rightarrow \frac{AC}{BC}=\frac{AH}{AB}=\frac{CH}{AC}$

② $A\hat{H}B \sim A\hat{B}C$ (زز) $\Rightarrow \frac{AH}{AC}=\frac{BH}{AB}=\frac{AB}{BC}$

③ $B\hat{H}K \sim \overset{\triangle}{A\hat{B}C}$ (زز) $\Rightarrow \frac{BK}{AB}=\frac{BH}{BC}=\frac{HK}{AC}$

④ $A\hat{H}B \sim H\hat{K}B$ (زز) $\Rightarrow \frac{HK}{AH}=\frac{BH}{AB}=\frac{BK}{BH}$

⑤ $A\hat{H}B \sim A\hat{K}H$ (زز) $\Rightarrow \frac{HK}{AH}=\frac{BH}{BC}=\frac{HK}{AC}$

⑥ $A\hat{H}K \sim H\hat{B}K$ (زز) $\Rightarrow \frac{AK}{HK}=\frac{AH}{BH}=\frac{HK}{BK}$

⑦ $A\hat{C}H \sim A\hat{H}B$ (زز) $\Rightarrow \frac{AC}{AB}=\frac{CH}{AH}=\frac{AH}{BH}$

⑧ $A\hat{C}H \sim B\hat{H}K$ (زز) $\Rightarrow$ —

بقیه‌ی تشابهات از بالایی‌ها بدست می آید

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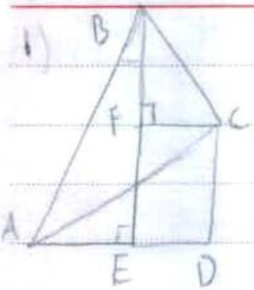
۳) از آنجایی که $5<y<x$ است پس ۵ ضلع کوچکتر در مثلث در نتیجه با ضلع کوچکتر مثلث دیگر متشابه

$$\frac{3}{5}=\frac{5}{y}=\frac{7}{x} \Rightarrow y=\frac{25}{3}, \quad x=\frac{35}{3} \Rightarrow x+y=\frac{25}{3}+\frac{35}{3}=\frac{60}{3}=20$$

۴)



$$BC^2+AC^2=AB^2 \Rightarrow 4^2+AC^2=8^2 \Rightarrow AC^2=48 \ (1) \Rightarrow AC=4\sqrt{3}$$

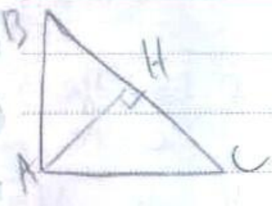
$$\Rightarrow AC^2=AD^2+CD^2 \overset{(1)}{\Longrightarrow} 48=6^2+CD^2 \Rightarrow CD^2=12 \Rightarrow CD=2\sqrt{3}$$

$$\frac{BC}{AB}=\frac{4}{8}=\frac{1}{2}, \quad \frac{CD}{AC}=\frac{2\sqrt{3}}{4\sqrt{3}}=\frac{1}{2} \xrightarrow{\text{ضلع مقابل } 30-60-90} B\hat{A}C=C\hat{A}D=30 \ (2) \Rightarrow \boxed{D\hat{A}B=60}$$

$$\Rightarrow ABE=180-90-60=30 \xrightarrow{\text{ضلع } 30-60-90} \frac{AB}{2}=AE \Rightarrow AE=\frac{8}{2}=4$$

$$AB^2=AE^2+BE^2 \quad 8^2=4^2+BE^2 \Rightarrow BE^2=48 \Rightarrow BE=4\sqrt{3} \Rightarrow \boxed{\frac{BE}{AB}=\frac{4\sqrt{3}}{8}=\frac{\sqrt{3}}{2}}$$

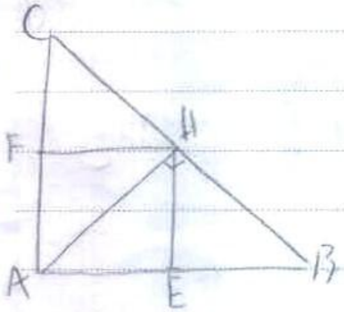
۵)



$\hat{B}=\hat{B}$ مشترک، $\hat{A}=\hat{H}=90$ $\xrightarrow{(ز ز)}$ $A\hat{B}C \sim H\hat{B}A \Rightarrow \frac{AB}{AC}=\frac{BH}{AB} \Rightarrow AB^2=BH\times BC$

$\hat{C}=\hat{C}$ مشترک، $\hat{A}=\hat{H}$ $\xrightarrow{(ز ز)}$ $A\hat{B}C \sim H\hat{C}A \Rightarrow \frac{AC}{BC}=\frac{CH}{AC} \Rightarrow AC^2=CH\times BC$

۶)



الف) $HE^2=AE\times BE$

ب) $AH^2=AE\times AB$

ج) $AB^2=BH\times BC$

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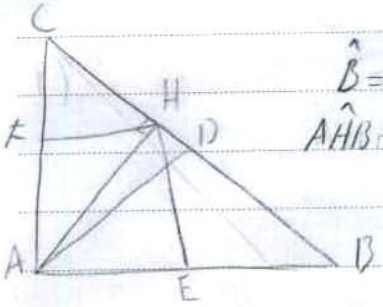

$\hat{B}=\hat{B}$, $A\hat{H}B=B\hat{A}C$ $\Big\}$ (ii) $\Rightarrow H\hat{B}A \sim A\hat{B}C \Rightarrow \frac{HB}{AB}=\frac{HA}{AC}=\frac{AB}{BC}$

① اگر دو مثلث با هم متشابه باشند نسبت‌های آن دو مثلث متناسب با

② $\frac{HE}{AD}=\frac{AB}{BC}$: نسبت اضلاع است در نتیجه داریم

$\hat{C}=\hat{C}$, $A\hat{H}C=B\hat{A}C=90$ $\Big\}$ (ii) $\Rightarrow H\hat{A}C \sim A\hat{B}C \Rightarrow \frac{HA}{AB}=\frac{HC}{AC}=\frac{AC}{BC}$

طبق ① داریم: ③ $\frac{HF}{AD}=\frac{AC}{BC}$

فیثاغورس: $AB^2+AC^2=BC^2 \Rightarrow \frac{AB^2+AC^2}{BC^2}=1 \Rightarrow (\frac{AB}{BC})^2+(\frac{AC}{BC})^2=1$

②③ $\Rightarrow (\frac{HE}{AD})^2+(\frac{HF}{AD})^2=1 \Rightarrow \frac{HE^2+HF^2}{AD^2}=1 \Rightarrow HE^2+HF^2=AD^2$

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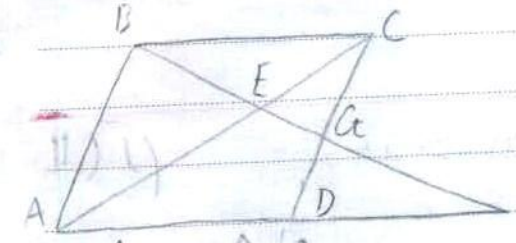

$AD \parallel BC \Rightarrow D\hat{A}C=A\hat{C}B$, $A\hat{E}F=B\hat{E}C$ $\Big\}$ (i) $\Rightarrow A\hat{F}E \equiv B\hat{E}C$

ب)

$A\hat{E}B=C\hat{E}G$, $B\hat{A}E=A\hat{C}G$ $\Big\}$ (ii) $\Rightarrow A\hat{B}E \sim G\hat{C}E \Rightarrow \frac{EG}{BE}=\frac{CE}{AE}$

① $\Rightarrow \frac{BE}{EF}=\frac{CE}{AE}$ $\Big\} \Rightarrow \frac{BE}{EF}=\frac{EG}{BE} \Rightarrow BE^2=EF \times EG$

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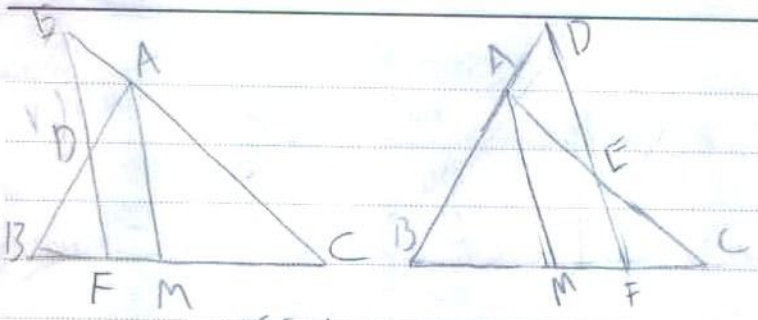

$AM \parallel EF \xrightarrow{\text{طبق تالس}} \frac{CE}{AC} = \frac{CF}{CM} \xrightarrow{\text{ترکیب صورت و مخرج}} \frac{AE}{AC} = \frac{MF}{CM}$

$AM \parallel FD \xrightarrow{\text{طبق تالس}} \frac{AD}{AB} = \frac{MF}{BM} \xrightarrow{BM = CM\ (=y)} \frac{AD}{AB} = \frac{MF}{CM}$

$\Rightarrow \frac{AD}{AB} = \frac{AE}{AC} \Rightarrow \frac{AD}{AE} = \frac{AB}{AC}$

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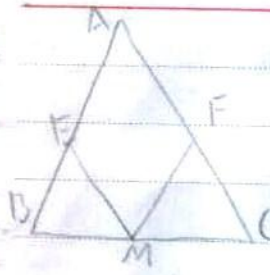

$EM \parallel AC \xrightarrow{\text{طبق تالس}} \frac{BE}{AE} = \frac{BM}{CM}$

$FM \parallel AB \xrightarrow{\text{طبق تالس}} \frac{AF}{CF} = \frac{BM}{CM}$

$\Rightarrow \frac{BE}{AE} = \frac{AF}{CF}$

$\Rightarrow BE \times CF = AE \times AF \Rightarrow \frac{AE \times AF}{BE \times CF} = 1$

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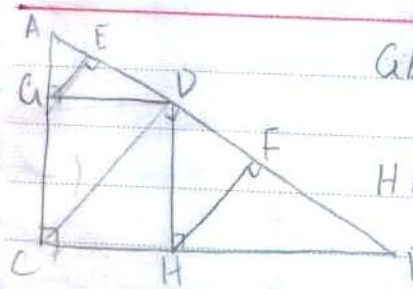

$GE^2 = AE \times ED \Rightarrow GE^2 = 4 \times 1 \Rightarrow GE = 2$

$HF^2 = DF \times FB \Rightarrow HF^2 = 4 \times 16 \Rightarrow HF = 8$

(ب

$GE^2 + ED^2 = GD^2 \Rightarrow 2^2 + 4^2 = GD^2 \Rightarrow GD = 2\sqrt{5}$

$HF^2 + DF^2 = DH^2 \Rightarrow 8^2 + 4^2 = DH^2 \Rightarrow DH = 4\sqrt{5}$

$S_{GHDG} = 2\sqrt{5} \times 4\sqrt{5} = 40$

(ج

$HF^2 + BF^2 = BH^2 \Rightarrow 8^2 + 16^2 = BH^2 \Rightarrow BH = 8\sqrt{5} \Rightarrow S_{BHF} = \frac{1}{2} \times 8\sqrt{5} \times 16 = 64\sqrt{5}$

$AE^2 + GE^2 = AG^2 \Rightarrow 1^2 + 2^2 = AG^2 \Rightarrow AG = \sqrt{5} \Rightarrow S_{AGE} = \frac{1}{2} \times 1 \times \sqrt{5} = \sqrt{5}$

$\Rightarrow \frac{S_{BHF}}{S_{AGE}} = 64$

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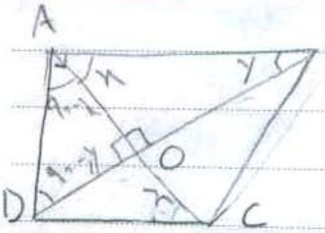

۱۳) ابتدا برای آسانی در نوشتار $B\hat{A}C = x$ و $A\hat{B}D = y$ درنتیجه داریم:

$AB \parallel CD \Longrightarrow B\hat{A}C = A\hat{C}D = x$

$C\hat{A}D = 90 - x \quad A\hat{D}B = 90 - y$

$A\hat{B}O : x + y = 90 \Longrightarrow x = 90 - y \ , \ y = 90 - x$

$\Rightarrow A\hat{D}B = A\hat{C}D$

$\left.\begin{array}{l} A\hat{D}B = A\hat{C}D \\ A\hat{D}C = B\hat{A}D \end{array}\right\} \Longrightarrow \overset{\triangle}{ACD} \sim \overset{\triangle}{ABD} \Longrightarrow \frac{AD}{AB} = \frac{CD}{AD} \Rightarrow AD^2 = AB \times CD$

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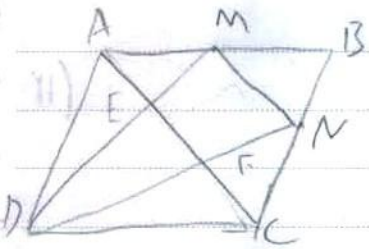

$AM \parallel BC \Rightarrow \begin{cases} M\hat{A}E = A\hat{C}D \\ A\hat{M}E = C\hat{D}M \end{cases} \overset{''}{\Rightarrow} A\hat{E}M \sim C\hat{D}E$ (۱) ۱۵

$CN \parallel AD \Rightarrow \begin{cases} A\hat{C}B = C\hat{A}D \\ D\hat{N}C = A\hat{D}N \end{cases} \overset{''}{\Rightarrow} N\hat{C}F \sim A\hat{D}F$ (۲)

(۱) $\Rightarrow \frac{AE}{CE} = \frac{AM}{CD} = \frac{AM}{AB} = \frac{1}{2}$ (۴) $\Rightarrow \frac{CF}{AF} = \frac{NC}{AD} = \frac{NC}{BC} = \frac{1}{2}$ (۱) $\Rightarrow \frac{ME}{DE} = \frac{1}{2}$ (۳)

طبق قضیه خط وسط $\Rightarrow MN \parallel AC \Rightarrow MN \parallel EF \xrightarrow{\text{طبق تالس}} \frac{DE}{DM} = \frac{DF}{DN} = \frac{EF}{MN}$

طبق قضیه خط وسط $\Rightarrow MN = \frac{1}{2}AC$ (۴) $\frac{EF}{MN} = \frac{DE}{Dm} = \frac{DE}{DE+ME} \overset{(۳)}{=} \frac{2ME}{2ME+ME} = \frac{2}{3}$

(۴) $\Rightarrow \frac{EF}{\frac{1}{2}AC} = \frac{2}{3} \Rightarrow \frac{EF}{AC} = \frac{1}{3}$ (۱) $\frac{AE}{CE} = \frac{1}{2} \Rightarrow \frac{AE}{AC} = \frac{1}{3} \overset{(۵)}{\Rightarrow} \frac{AE}{2EF} = \frac{1}{2} \Rightarrow AE = EF$ (۶)

$\frac{CF}{AF} = \frac{1}{2} \Rightarrow \frac{CF}{AC} = \frac{1}{3} \overset{(۵)}{\Rightarrow} \frac{CF}{2EF} = \frac{1}{2} \Rightarrow CF = EF \overset{(۶)}{\Rightarrow} AE = EF = CF$

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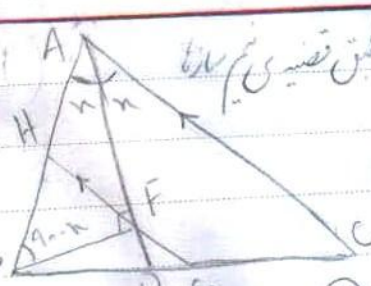

طبق قضیه نیمساز داخلی $\Rightarrow \frac{AB}{AC} = \frac{BD}{CD} \Rightarrow \frac{AB}{AC+AB} = \frac{BD}{BD+CD} \Rightarrow \frac{24}{24} = \frac{BD}{28}$

$BD = 12$ $B\hat{A}D = x \Rightarrow C\hat{A}D = x \Rightarrow A\hat{B}F = 90 - x$

(۱) $HG \parallel AC \Rightarrow D\hat{F}G = C\hat{A}D = x \Rightarrow A\hat{F}H = x$ $B\hat{A}D = x$ $\xrightarrow{\text{عکس تالس}} AH = HF$

$B\hat{H}F = B\hat{A}D + A\hat{F}H = x + x = 2x$ $B\hat{F}H = 180 - (B\hat{H}F + A\hat{B}F) = 180 - (2x + 90 - x) = 90 - x$

$\Rightarrow B\hat{F}H = A\hat{B}F \xrightarrow{\text{عکس مثلث متساوی الساقین}} BH = HF$, $AH = HF \Rightarrow AH = BH$, (۱) $\xrightarrow{\text{عکس تالس}} BG = CG$

$\Rightarrow \frac{BG}{CG} = \frac{1}{1} \Rightarrow \frac{BG}{BG+CG} = \frac{1}{2} \Rightarrow \frac{BG}{BC} = \frac{1}{2} \Rightarrow \frac{BD+DG}{BC} = \frac{1}{2} \Rightarrow \frac{12+DG}{28} = \frac{1}{2} \Rightarrow DG = 1$

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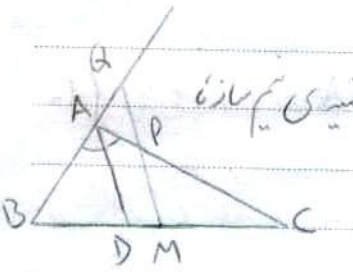

طبق قضیه‌ی تالس $\Rightarrow \frac{AB}{BD} = \frac{AC}{CD}$ ① BC وسط M $\Rightarrow BM = CM$ ②

$AD \parallel QM \xrightarrow{\text{طبق تالس}} \frac{AB}{BQ} = \frac{BD}{BM} \Rightarrow \frac{AB}{BD} = \frac{BQ}{BM}$ ③

$AD \parallel PM \xrightarrow{\text{طبق تالس}} \frac{CP}{AC} = \frac{CM}{CD} \Rightarrow \frac{CP}{CM} = \frac{AC}{CD}$ ④

① ③ $\Rightarrow \frac{CP}{CM} = \frac{BQ}{BM}$

② ④ $\Rightarrow CP = BQ$

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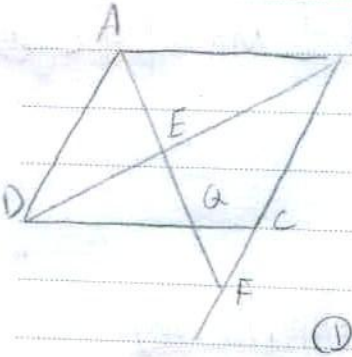
۱۸) این دقیقا مشابه سوال ۸ است



$AD \parallel BF \Rightarrow A\hat{D}E = F\hat{B}E$ ①

$A\hat{E}D = B\hat{E}F$ متقابل به رأس

$\Rightarrow \hat{ADE} \sim \hat{FBE}$ ①

$AB \parallel DQ \Rightarrow A\hat{B}E = Q\hat{D}E$ ②

$A\hat{E}B = D\hat{E}Q$

$\Rightarrow A\hat{E}B \sim Q\hat{E}D$ ②

① $\Rightarrow \frac{EA}{EF} = \frac{DE}{BE}$

② $\Rightarrow \frac{EQ}{EA} = \frac{DE}{DE}$

$\Rightarrow \frac{EA}{EF} = \frac{EQ}{EA} \Rightarrow EA^2 = EF \times EQ$

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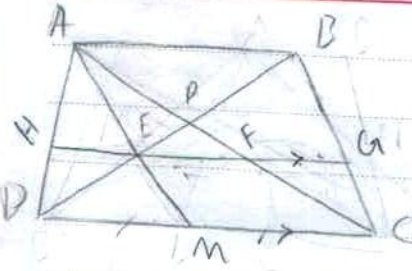

$CD \parallel HG \Rightarrow \frac{DH}{AD} = \frac{CG}{BC}$ ③

$HE \parallel AB \Rightarrow \frac{DH}{AD} = \frac{HE}{AB}$ ① $FG \parallel AB \Rightarrow \frac{CG}{BC} = \frac{FG}{AB}$ ②

①، ②، ③ $\Rightarrow HE = FG$ $DM = CM$ (زیرا AM میانه است) ④

$HE \parallel DM \Rightarrow \frac{HE}{DM} = \frac{AE}{AM}$ ⑤ $EF \parallel CM \Rightarrow \frac{EF}{CM} = \frac{AE}{AM}$ ⑥

③ ④ $\Rightarrow \frac{HE}{DM} = \frac{EF}{CM}$ ⑦ $\Rightarrow HE = EF$, $HE = FG \Rightarrow EF = FG = HE$

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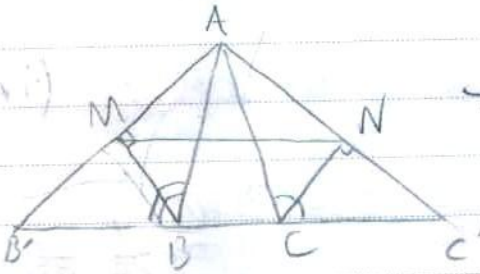
۲۰) ابتدا AM و AN را امتداد می‌دهیم تا ضلع BC را به ترتیب در B' و C' قطع کند. حال داریم:



$$\left.\begin{array}{l} A\hat{B}M = B'\hat{B}M \\ MB = MB \\ A\hat{M}B = B'\hat{M}B \end{array}\right\} \xrightarrow{(ض ز ض)} A\hat{B}M \cong B'\hat{B}M \Longrightarrow \left\{\begin{array}{l} AB = BB' \ (1) \\ AM = MB' \end{array}\right. \xrightarrow{\text{قضیه میانگین}} MN = \frac{1}{2}B'C' \ (3)$$

$$\left.\begin{array}{l} A\hat{C}N = C'\hat{C}N \\ CN = CN \\ A\hat{N}C = C'NC \end{array}\right\} \xrightarrow{(ز ض ز)} A\hat{C}N \cong C'\hat{C}N \Longrightarrow \left\{\begin{array}{l} AN = C'N \\ AC = CC' \ (2) \end{array}\right.$$

$$(3) \Longrightarrow MN = \frac{1}{2}(BB' + BC + CC') \xrightarrow{(1)(2)} MN = \frac{1}{2}(AB + BC + AC)$$

یا علی مدد