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Example 1:

The ordinates of a rainfall mass curve of a particular storm are tabulated below.

Time	Cumulative rainfall in mm
7 00	0
7 15	9.5
7 30	17
7 45	27
8 00	40.5
8 15	49
8 30	63
8 45	84
9 00	95
9 15	102
9 30	110
9 45	112
10 00	112

- a) Construct the hyetograph of this storm using a uniform time interval of 15 minutes.
- b) Compute the maximum intensities of rainfall for durations of 15, 30, 45, 60, 90, 120 and 180 minutes and plot the intensity duration graph.
- c) For the intensity duration graph of part b fit an appropriate regression equation.

Solution:

a) The intensities of rainfall are calculated using uniform time interval of 15 minutes as given in the following table. Also The corresponding hyetograph is shown in fig.1.

Time	Rainfall in mm	Rainfall in successive 15 mt. intervals	Rainfall intensity in ^{mm/} hr
7 00	0		
7 15	9.5	9.5	38
7 30	17	7.5	30
7 45	27	10	40
8 00	40.5	13.5	54
8 15	49	8.5	34
8 30	63	14	56
8 45	84	21	82
9 00	95	11	44
9 15	102	7	28
9 30	110	8	32
9 45	112	2	8
10 00	112	0	0



Fig.1. Rainfall Hyetograph

Time	ime Cumulative Rainfall in any possible time interval equal to							
Time	Rainfall	15 mt	30 mt	45 mt	60 mt	90 mt	120 mt	180 mt
7 00	0							
7 15	9.5	9.5						
7 30	17	7.5	17					
7 45	27	10	17.5	27				
8 00	40.5	13.5	23.5	31	40.5			
8 15	49	8.5	22	32	39.5			
8 30	63	14	22.5	36	46	63		
8 4 5	84	21	35	43.5	57	74.5		
9 00	95	11	32	46	54.5	78	95	
9 15	102	7	18	39	53	75	92.5	
9 30	110	8	15	26	47	69.5	<i>93</i>	
9 45	112	2	10	17	28	63	85	
10 00	112	0	2	10	17	49	71.5	112

b) The necessary calculations to obtain the maximum intensities are shown in the following table.

Maximum intensity for 15 minute duration: $\Rightarrow = \frac{21}{0.25} = 84 \frac{mm}{hr}$ Maximum intensity for 30 minute duration: $\Rightarrow = \frac{35}{0.5} = 70 \frac{mm}{hr}$ Maximum intensity for 45 minute duration: $\Rightarrow = \frac{46}{0.75} = 61.33 \frac{mm}{hr}$ Maximum intensity for 60 minute duration: $\Rightarrow = \frac{57}{1} = 57 \frac{mm}{hr}$ Maximum intensity for 90 minute duration: $\Rightarrow = \frac{78}{1.5} = 52 \frac{mm}{hr}$ Maximum intensity for 120 minute duration: $\Rightarrow = \frac{95}{2} = 47.5 \frac{mm}{hr}$ Maximum intensity for 180 minute duration: $\Rightarrow = \frac{112}{3} = 37.33 \frac{mm}{hr}$

The intensity–duration graph for this storm is shown in fig.2.



Fig.2. Intensity– Duration Graph

c) The relationship between intensity and duration is assumed to be of the following form. $i = \frac{a}{(t+b)^c}$, the necessary computations to

arrive at the best values of a, b and c are shown in next table. The detailed analysis is given only for one trial value of "b" which is assumed to be 8. Similar analysis is carried out with other values of "b" equal to 10, 12 and 14 respectively. On substitution of the relevant summations in following equations $\sum \log i = \sum [\log a - c \log(t + b)]$ $\sum [\log i \log(t + b)] = \log a \sum \log(t + b) - c \sum [\log(t + b)]^2$ 12.2777=7log a -12.8709 c 22.3604=12.8709log a - 24.2585 c The solution of the above two simultaneous equations is c=0.362; log a =2.4196 \Rightarrow a=262.76 Therefore the best equation with b=8 is $i = \frac{262.76}{(t + 8)^{0.362}}$

Duration in minutes t	Max.intensity in mm/hr i	log i	log(t+b)	log i log (t+b)	(log(t+b))^2	\hat{i}	$\Delta i = i - \hat{i}$	Δi^2
15	84	1.9242	1.3617	2.6203	1.8543	84.4527	-0.4527	0.2049
30	70	1.8450	1.5797	2.9148	2.4957	70.4170	-0.4170	0.1738
45	61.33	1.7876	1.7242	3.0824	2.9731	62.4268	-1.0968	1.2030
60	57	1.7558	1.8325	3.2176	3.3580	57.0415	-0.0415	0.0017
90	52	1.7160	1.9912	3.4169	3.9649	49.973	2.0270	4.1087
120	47.5	1.6766	2.1072	3.5331	4.4403	45.3679	2.1320	4.5455
180	37.33	1.5720	2.2741	3.5751	5.1717	39.4742	-2.1442	4.5978
	Σ	12.2776	12.8708	22.3605	24.2583			14.8359

Using the above equation the intensities for various durations are determined and entered in column \hat{i} . The sum of the squared deviation for this case is 14.8356. The procedure is repeated for other value of "b" and results are as given under.

Trial No	b	а	С	$\sum \Delta i^2$
1	8	262.76	0.362	14.836
2	10	281.33	0.375	14.475
3	12	300	0.387	14.412
4	14	320	0.399	14.636

It is thus seen that the sum of the squared deviations is least when b=12. therefore the desired regression equation for the data under consideration is

 $i = \frac{300}{\left(t + 12\right)^{0.387}}$

Example 2:

The rainfall data obtained from 7 rain gauge stations located in and around the basin area shown in fig.3. The basin has an area of $2790 km^2$.

a) Compute the average depth of rainfall for the basin using the three methods discussed in the lesson.

b) Analyse this data and develop the depth – area – duration curves for the storm.

Time	Cumulative Rainfall in mm								
hrs	Α	В	С	D	E	F	G		
4	0	0	0	0	0	0	0		
6	12	0	0	0	0	0	0		
8	18	15	0	0	0	6	0		
10	27	24	0	0	9	15	6		
12	36	36	18	6	24	24	9		
14	42	45	36	18	36	33	15		
16	51	51	51	36	42	36	18		
18	51	63	66	51	60	39	18		
20	51	72	87	66	66	42	18		
22	51	72	96	81	66	42	18		
24	51	72	96	81	66	42	18		



Fig.3. Rain gauge stations in and around the basin area

Solution:

- a) The total rainfalls of 51,72,96,81,66,42 and 18 mm are indicated at the corresponding rain gauge stations A,B,C,D,E,F and G, respectively on the map. The average depth of precipitation is obtained by the three methods as given below.
 - 1) Arithmetic mean

$$P = \frac{P_1 + P_2 + \dots + P_n}{n} = \frac{51 + 72 + 81 + 66 + 42}{5} = 62.4mm$$

2) Theissen method

The construction of Theissen polygons for the given rain gauge network in the basin is shown in fig 4. the required computations to find the average rainfall depth are given in the following table.

Rain gauge	Observed rainfall	Theissen polygon	Rainfall volume
station	in mm	area in km ²	in km ² _mm
A	51	775	39525
В	72	463	33336
С	96	58	5568
D	81	294	23814
E	66	505	33330
F	42	455	19110
G	18	240	4320
	Σ	2790	159003

$$P = \frac{\sum P_i A_i}{\sum A_i} = \frac{159003}{2790} = 56.9903mm$$

3) Isohyets method

The isohyets of 30, 45, 60 and 75 mm of rainfall are drawn on the basin area map with linear interpolation and are shown in fig 4 too. The areas between successive isohyets are measured and tabulated below for necessary computations.



Fig.4. Theissen polygon and isohyetal map

Isohyet value in mm	Net area between isohyets in km ²	Average rainfall in mm	Rainfall volume in km ² _mm
75	330	80	26400
60	755	67.5	50962.5
45	1195	52.5	62737.5
30	435	37.5	16312.5
<30	75	27	2025
\sum	2790		158437.5

$$P = \frac{\sum P_i A_i}{\sum A_i} = \frac{158437.5}{2790} = 56.7876mm$$

 b) The isohyets divide the basin area into five zone (see above table and fig 4) From the Theissen polygons and the isohyets the following information is noted;

Zone I is influenced by 3 stations B, C and D with areas 58, 58, and $214 \text{ }km^2$, respectively Zone II by stations A, B, D and E with areas 45, 405, 80, and 225 $\text{ }km^2$ Zone III by A, E, and F with areas 730, 280, and 185 $\text{ }km^2$ Zone IV by G and F with areas 165 and 270 $\text{ }km^2$ Zone V by G with 75 $\text{ }km^2$

The cumulative average depth of rainfall for each zone can be calculated with the data at individual stations at A, B, C, D, E, F, and G by adopting the corresponding Theissen weights. For example in zone I, the average cumulative rainfall at any time, is computed from

$$P_I = \frac{58 \times P_B + 58 \times P_C + 214 \times P_D}{330}$$

where PB, PC, and PD are the cumulative rainfalls at stations B, C, and D at the same time. That is

$$P_I = 0.1758P_B + 0.1758P_C + 0.6484P_D$$

These results are shown in the following table

Time	Zone I	Zone II	Zone III	Zone IV	Zone V
4	0	0	0	0	0
6	0	0.72	7.33	0	0
8	2.64	9.12	11.93	3.73	0
10	4.22	17.16	20.94	11.59	6
12	13.38	22.25	31.34	18.31	9
14	25.91	39.28	39.2	26.18	15
16	41.27	46.74	46.57	29.18	18
18	55.75	60.12	51.25	31.04	18
20	70.75	68.33	53.11	32.90	18
22	82.06	69.92	53.11	32.90	18
24	82.06	69.92	53.11	32.90	18

Cumulative average rainfall in mm in zones

In the next step the average cumulative rainfalls for accumulated areas are worked out. Here again the weights are used in proportion to the areas of zone. For example the cumulative average rainfall over the first three zones is given as

$$P_{I+II+III} = \frac{330P_I + 755P_{II} + 1195P_{III}}{(330+755+1195)} = 0.145P_I + 0.331P_{II} + 0.524P_{III}$$

The results are as tabulated below

		0			
Time hrs	I 330 km ²	I+II 1085 km ²	I+II+III 2280 km ²	I+II+III+IV 2715 km^2	$I + II + III + IV + V$ $2790 \ km^2$
1115	000 km	1005 KM	2200 Km	2713 KM	2770 KM
4	0	0	0	0	0
6	0	0.5	4.08	3.53	3.34
8	2.64	7.15	9.65	8.7	8.47
10	4.22	15.28	17.26	16.35	16.08
12	13.38	24.43	28.04	26.49	26.02
14	25.91	35.22	37.3	35.52	34.97
16	41.27	45.08	45.85	43.18	42.51
18	55.75	58.79	54.84	51.03	50.14
20	70.75	69.07	60.71	56.26	55.22
22	82.06	73.61	62.87	58.08	56.98
24	82.06	73.61	62.87	58.08	56.98

Cumulative average rainfall in mm for accumulated areas

Now for any zone the maximum average depth of rainfall for various durations of 4, 8, 12, 16 and 20 hrs. can be obtained from the above table using the procedure explained in previous example. The results are given in the next table.

Duration	Maximum average rainfall in mm						
hrs.	$330 \ km^2 \qquad 1085 \ km^2 \qquad 2280 \ km^2 \qquad 2715 \ km^2 \qquad 2790 \ k$						
4	29.84	23.99	20.04	19.17	18.89		
8	57.37	44.64	37.58	34.68	34.06		
12	77.84	61.92	51.06	47.56	46.8		
16	82.06	73.11	60.71	56.26	55.22		
20	82.06	73.61	62.87	58.08	56.78		

For each duration the maximum depths of rainfall are plotted against the area on logarithmic scale as shown in fig.5.



Fig.5. Dept – Area – Duration Curves