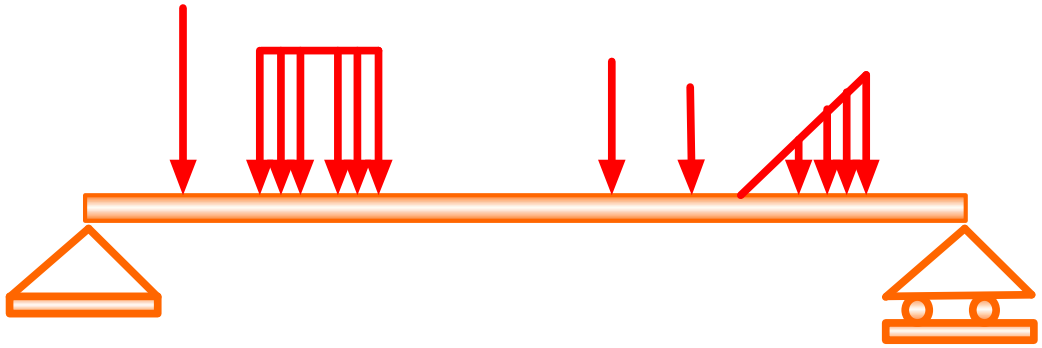
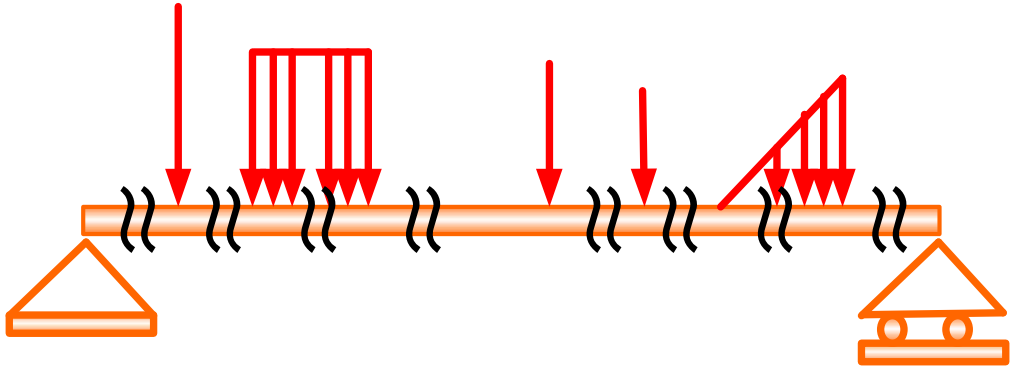
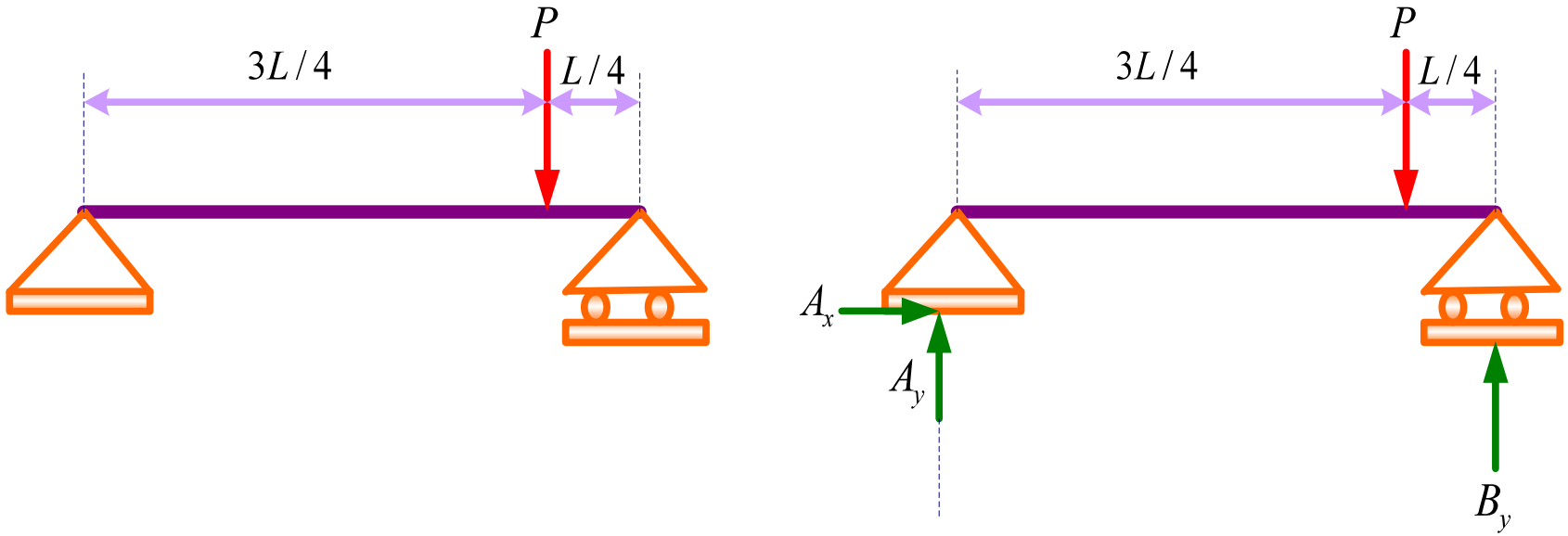
دیاگرام نیروی برشی و گشتاور خمشی



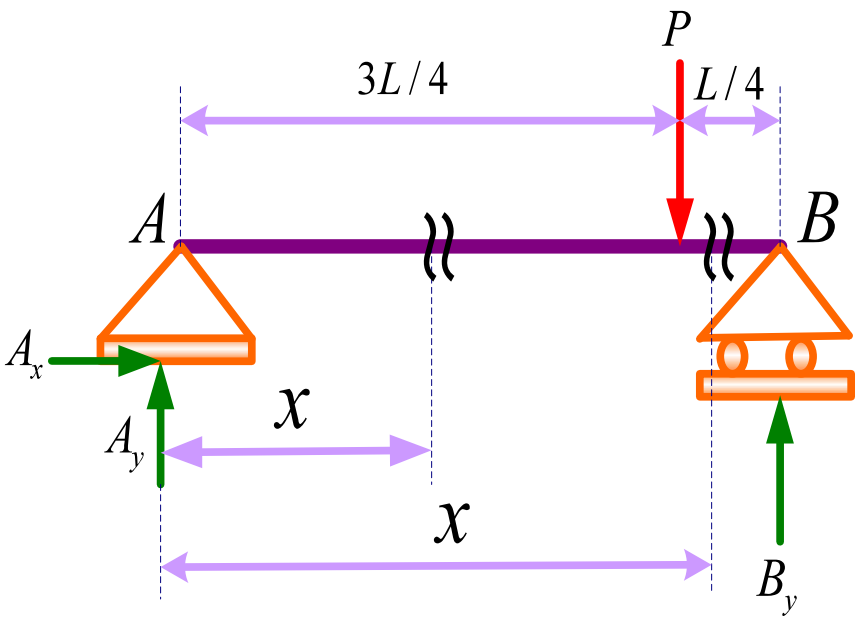
قبل از هر اتفاق جدید و بعد از آن یک برش لازمست. برای هر نیروی توزیعی هم یک برش لازمست.

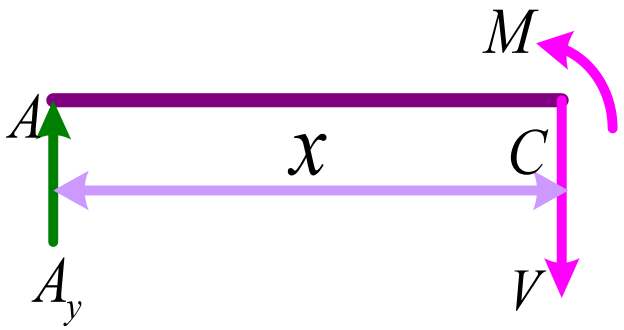


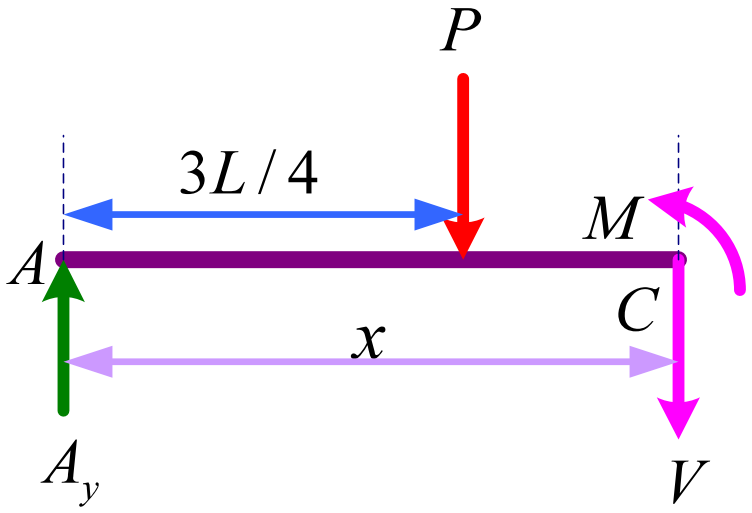


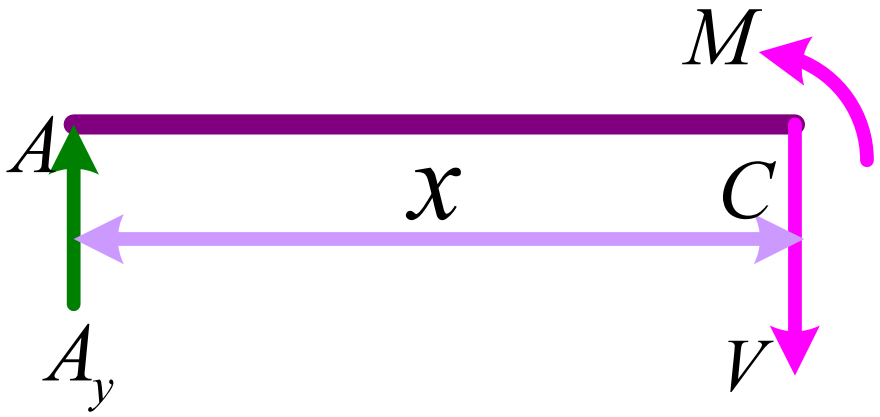
$$\sum M_A = 0 \Rightarrow \quad LB_y - P\left(\frac{3l}{4}\right) = 0 \Rightarrow B_y = \frac{3P}{4}$$

$$\sum F_y = 0 \Rightarrow A_y + B_y - P = 0 \Rightarrow \quad A_y = \frac{P}{4}$$

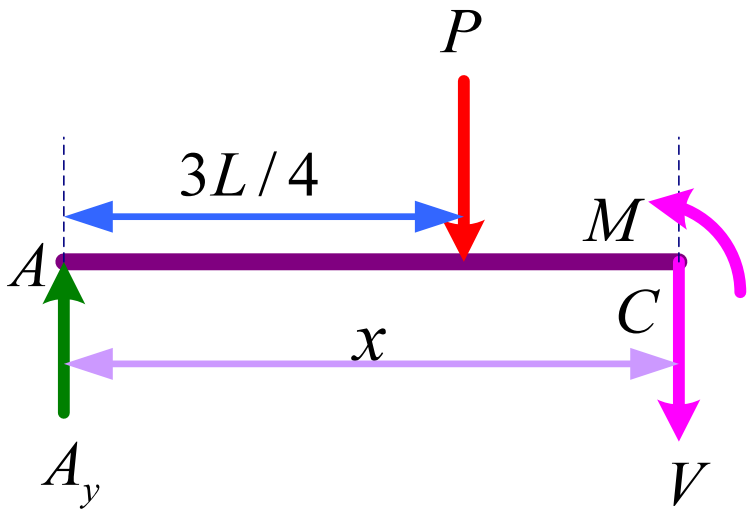

P
3L / 4
L / 4
A
B
A_x
A_y
x
x
B_y


M
A
x
C
A_y
V

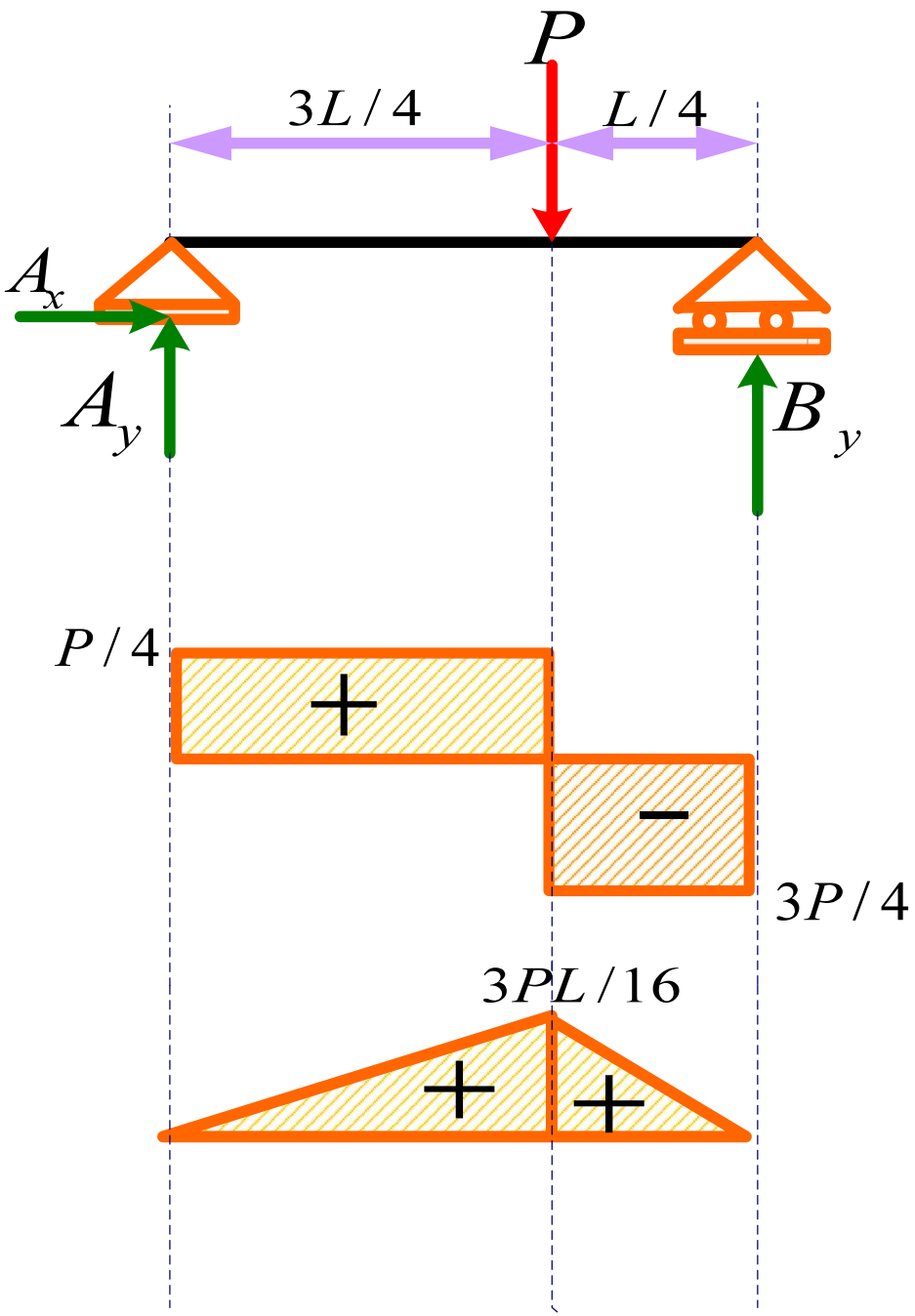

P
3L / 4
M
A
C
x
A_y
V



$$\left.\begin{aligned} \sum F_y = 0 \Rightarrow A_y - V = 0 \Rightarrow V = \frac{P}{4} \\ \sum M_C = 0 \Rightarrow A_y x - M = 0 \Rightarrow M = \frac{P}{4} x \end{aligned}\right\} \quad 0 \le x \le 3L/4$$



$$\left.\begin{aligned} \sum F_y = 0 \Rightarrow A_y - P - v = 0 \;\Rightarrow\; v = \frac{-3P}{4} \\ \sum M_C = 0 \Rightarrow A_y x - \left(x - \frac{3L}{4}\right)P - M = 0 \Rightarrow M = \frac{3P}{4}(l - x) \end{aligned}\right\} \quad 3L/4 \le x \le L$$



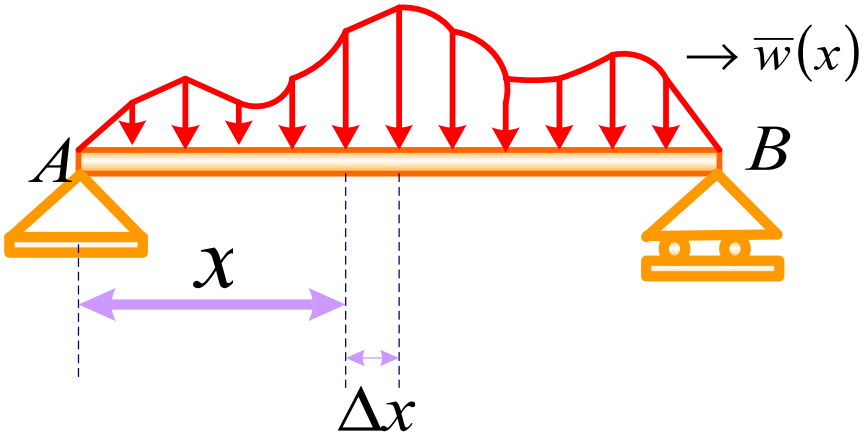
$$V = \frac{P}{4} \qquad M = \frac{P}{4}x$$

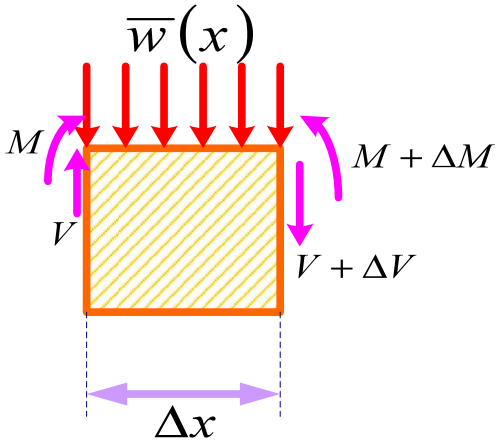
$$0 \le x \le 3L/4$$

$$v = \frac{-3P}{4} \qquad M = \frac{3P}{4}(l - x)$$

$$3L/4 \le x \le L$$

رابطه ما بین نیروی برشی و گشتاور خمشی



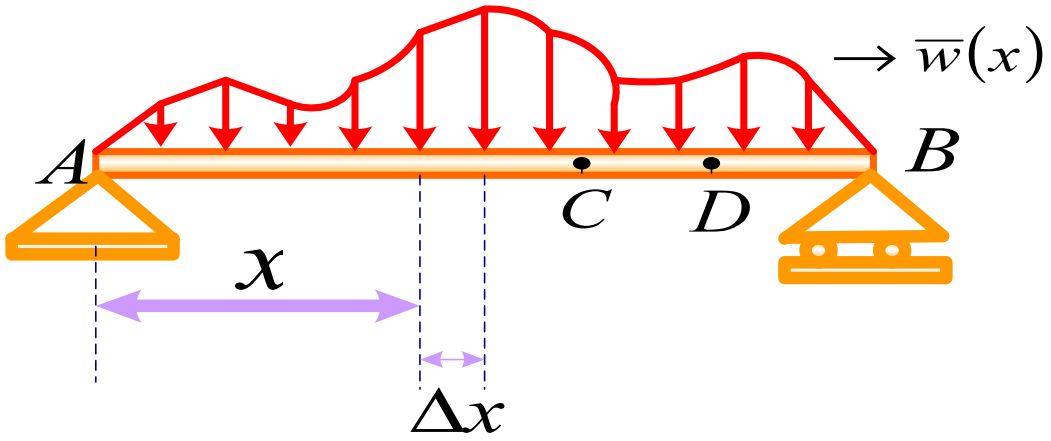


$$\sum F_y = 0 \Rightarrow V + \Delta V - V + \overline{W}\Delta x = 0 \qquad \frac{\Delta V}{\Delta x} = -\overline{W}(x)$$

$$\lim \frac{\Delta V}{\Delta x} = \frac{dv}{dx} = -\overline{W}(x) \qquad \frac{dv}{dx} = -\overline{W}(x)$$

$\Delta x \to 0$

منهای شدت نیرو = شیب منحنی تغییرات نیروی برشی برحسب طول تیر

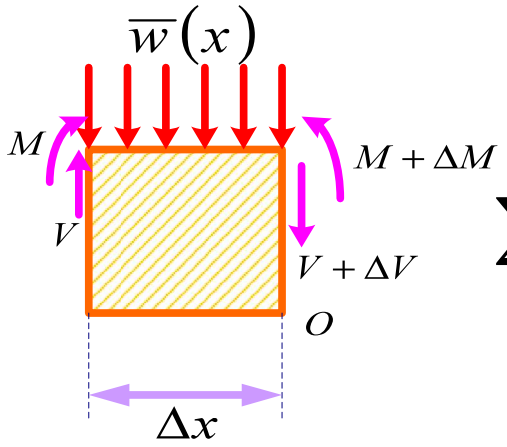


$$\frac{dv}{dx} = -\overline{W}(x)$$

$$dv = -\overline{W}(x)dx$$

$$\int_{V_c}^{V_d} dv = -\int_{x_c}^{x_d} \overline{W}(x)dx \Rightarrow V_d - V_c = -\int_{x_c}^{x_d} \overline{W}(x)dx$$

سطح زیر منحنی تغییرات شدت نیرو **بر**حسب طول تیر در فاصله x_c تا x_d



$$\sum M_O = 0 \Rightarrow V\Delta x + M - \overline{W}(x)\Delta x\left(\frac{\Delta x}{2}\right) - (M+\Delta M) = 0$$

$$V\Delta x - \overline{W}(x)\frac{\Delta x^2}{2} - \Delta M = 0$$

$$V - \frac{\overline{W}(x)}{2}\Delta x - \frac{\Delta M}{\Delta x} = 0 \Rightarrow \frac{\Delta M}{\Delta x} = -\frac{\overline{W}(x)}{2}\Delta x + V \qquad \lim_{\Delta x \to 0}\frac{\Delta M}{\Delta x} = \frac{dM}{dx} = V$$

$$V = \frac{dM}{dx}$$

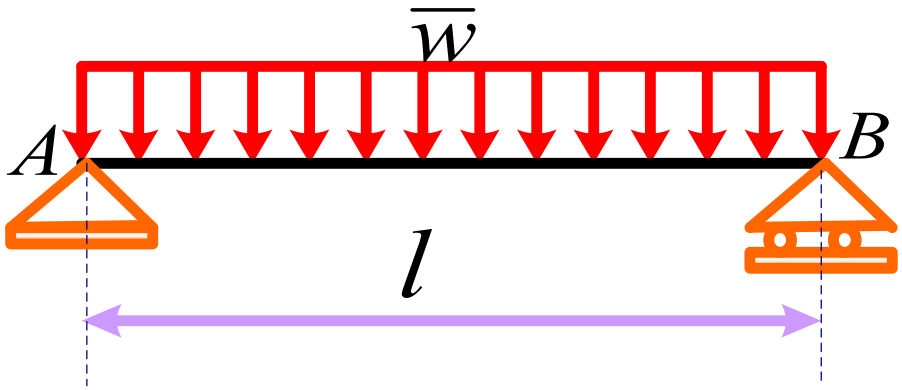
نیروی برشی = شیب منحنی تغییرات گشتاور خمشی بر حسب طول تیر

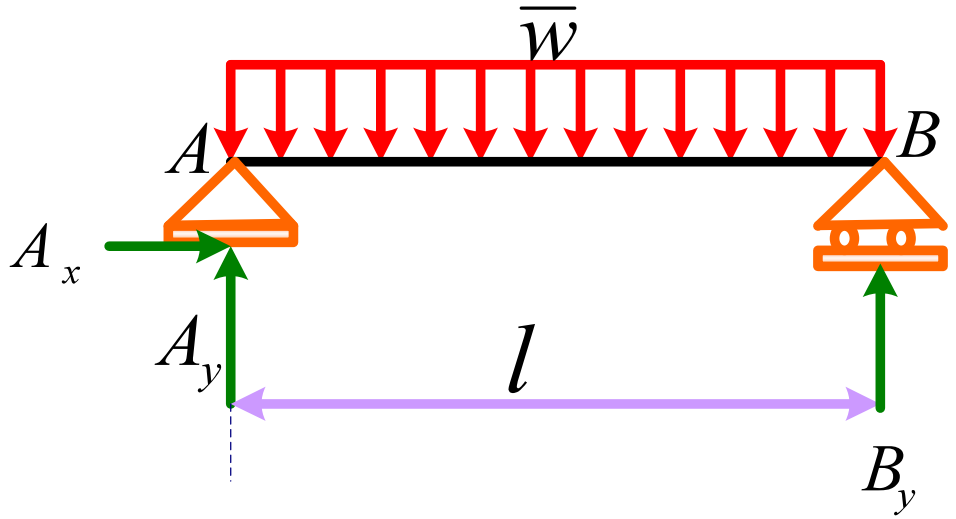
$$dM = Vdx \Rightarrow$$

$$\int_{M_c}^{M_d} dM = \int_{x_c}^{x_d} Vdx \Rightarrow M_d - M_c = \int_{x_c}^{x_d} Vdx$$

سطح زیر منحنی تغییرات نیروی برشی بر حسب طول تیر در فاصله x_c تا x_d

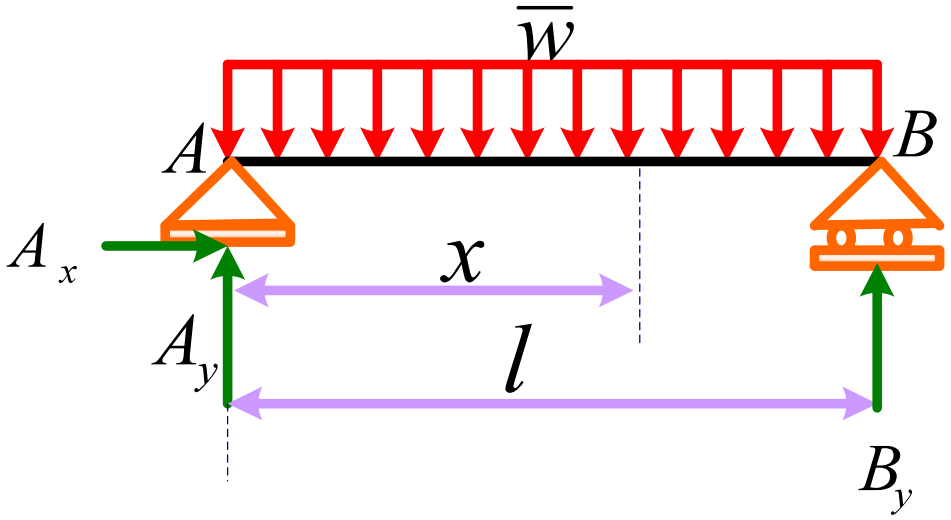
مثال: برای تیر نشانداده شده در شکل دیاگرام های نیروی برشی وگشتاور خمشی را رسم کنید.





$$\sum M_A = 0 \Rightarrow LB_y - \bar{W}(l)\left(\frac{l}{2}\right) = 0 \Rightarrow B_y = \bar{W}\frac{l}{2}$$

$$\sum F_y = 0 \Rightarrow A_y + B_y - \bar{W}(l)\left(\frac{l}{2}\right) = 0 \Rightarrow A_y = \overline{W}\frac{l}{2}$$



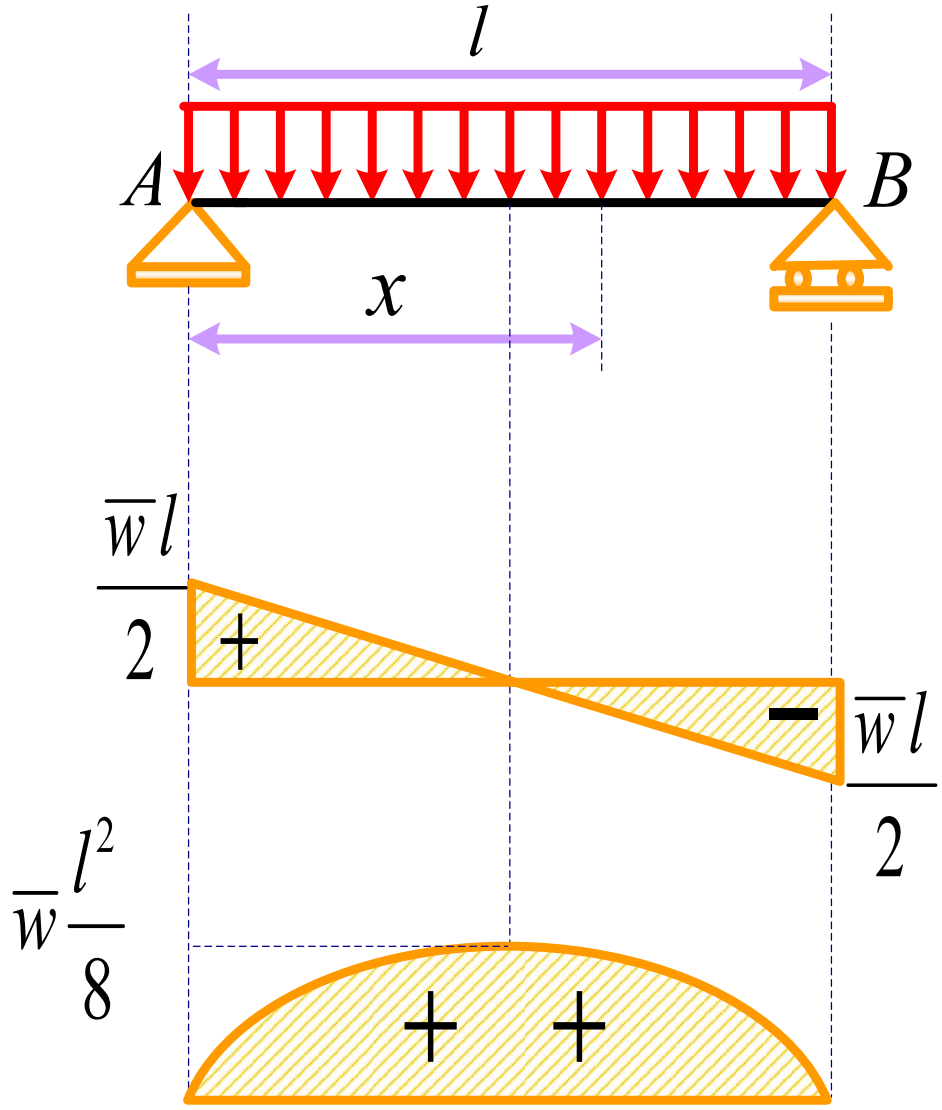
$$A_y = \overline{W}\frac{l}{2}$$

$$\frac{dV}{dx} = -\overline{W} \Rightarrow dV = -\overline{W}dx \Rightarrow \int_{A_y}^{V} dV = -\overline{W}\int_0^x dx$$

$$\Rightarrow V - A_y = -\overline{W}x \Rightarrow V = \overline{W}\left(\frac{l}{2} - x\right)$$

$$\frac{dM}{dx} = V = \overline{W}\left(\frac{l}{2} - x\right) \Rightarrow dM = \overline{W}\left(\frac{l}{2} - x\right)dx$$

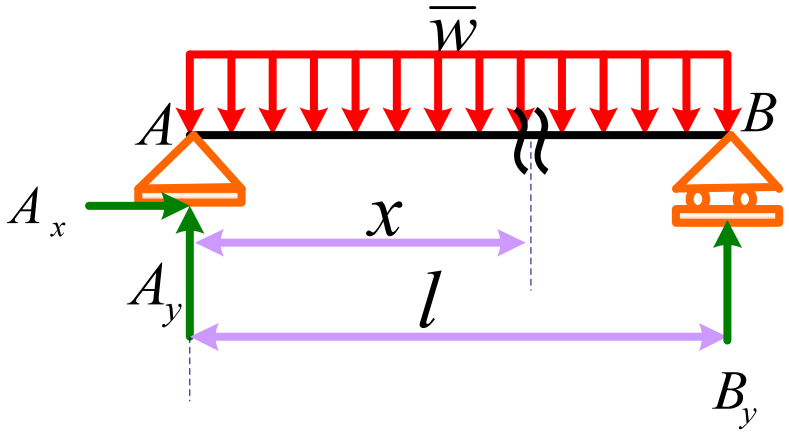
$$\int_0^M dM = \overline{W}\int_0^x \left(\frac{l}{2} - x\right)dx \Rightarrow M = \overline{W}\left(\frac{l}{2}x - \frac{x^2}{2}\right) \Rightarrow M = \frac{\overline{W}}{2}\left(lx - x^2\right)$$

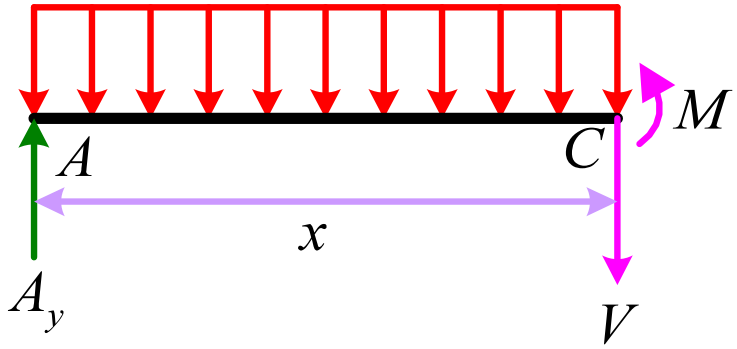


$$V = \overline{W}\left(\frac{l}{2} - x\right)$$

$$M = \frac{\overline{W}}{2}\left(lx - x^2\right)$$

حل مسئله قبل با روش مقطع:



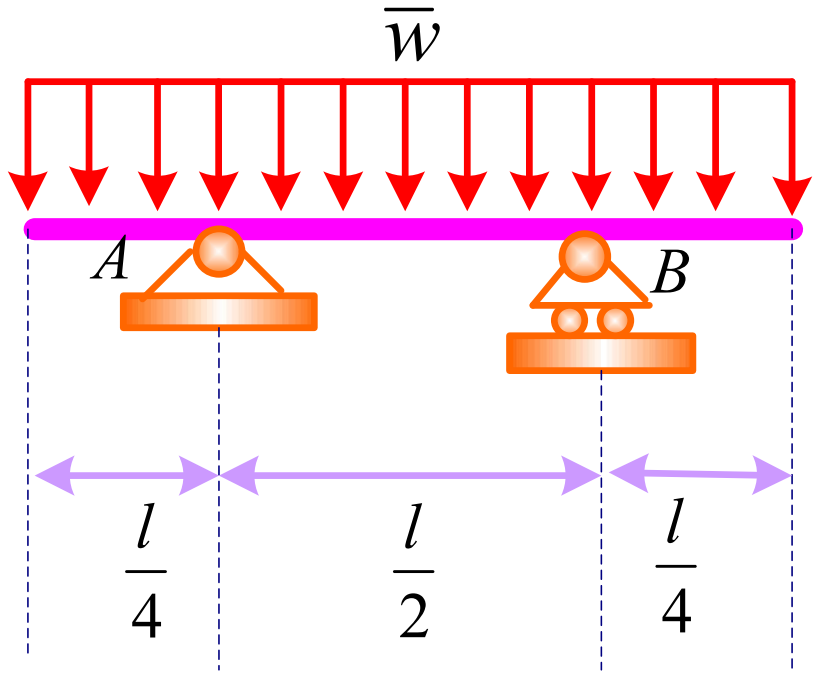


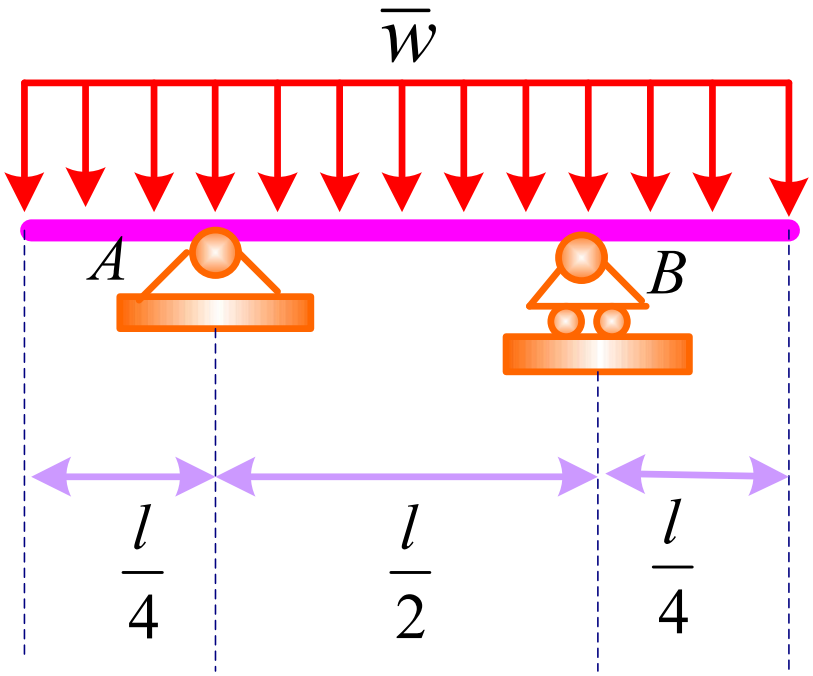
$$\sum F_y = 0 \Rightarrow A_y - \overline{W}x - V = 0 \qquad V = A_y - \overline{W}(x) = \overline{W}\frac{l}{2} - \overline{W}x$$

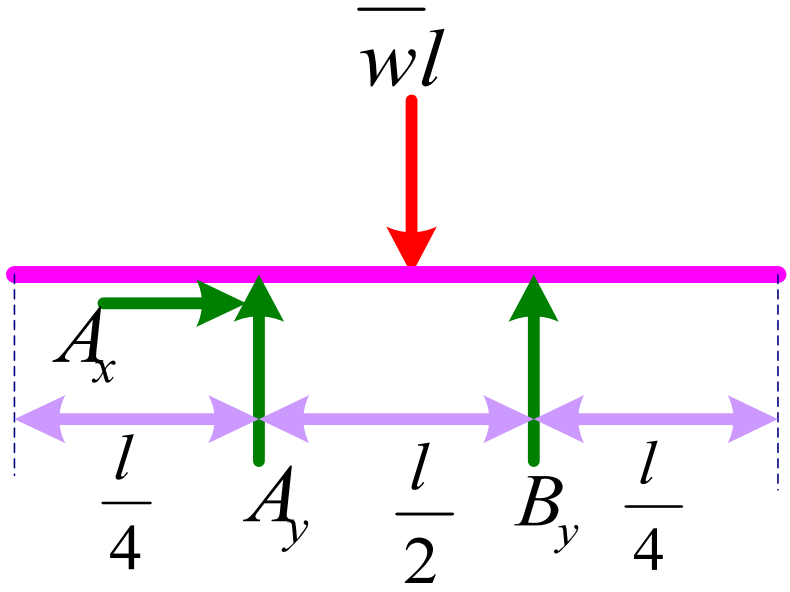
$$V = \overline{W}\left(\frac{l}{2} - x\right) \qquad \sum M_C = 0 \Rightarrow A_y x - \overline{W}x\left(\frac{x}{2}\right) - M = 0$$

$$M = \overline{W}\frac{l}{2}x - \overline{W}\frac{x^2}{2} = \frac{\overline{W}}{2}\left(lx - x^2\right)$$

مثال: دیاگرام های نیروی برشی و گشتاور خمشی را برای تیر نشان داده شـده در شکل که تحت تأثیر بار توزیعی با شدت ثابت قرار گرفته است رسم کنید.

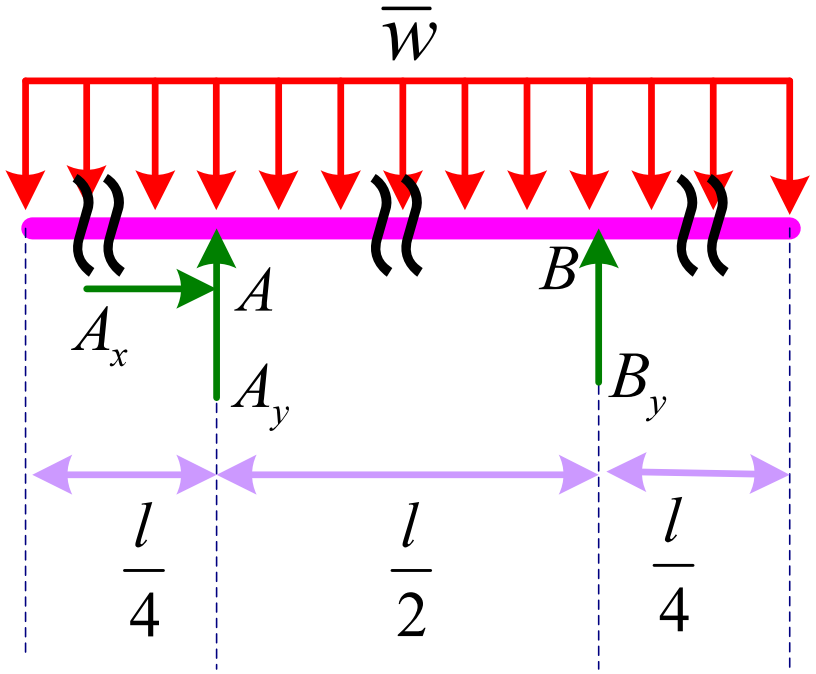


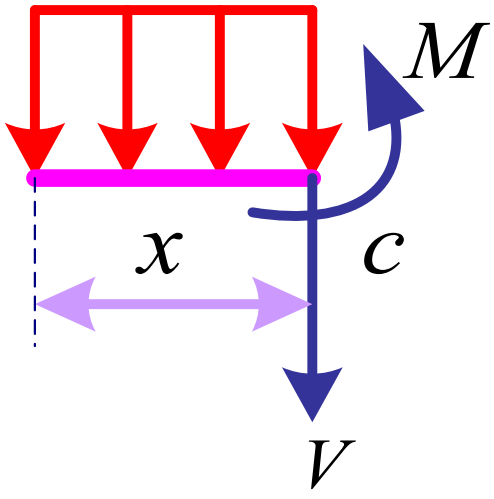




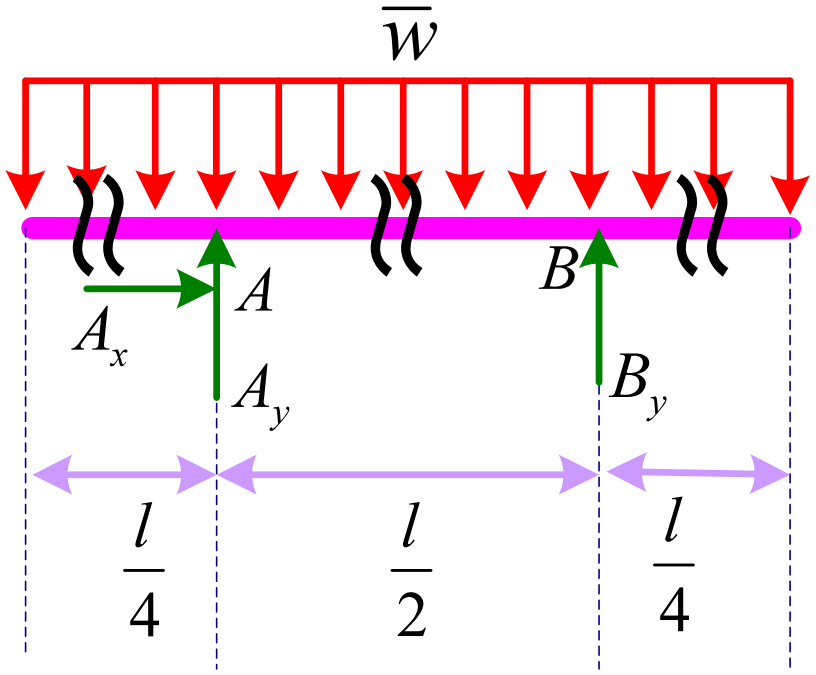
$$\sum M_A = 0 \Rightarrow B_y\left(\frac{l}{2}\right) - \overline{W}l\left(\frac{l}{4}\right) = 0 \Rightarrow B_y = \frac{\overline{W}l}{2}$$

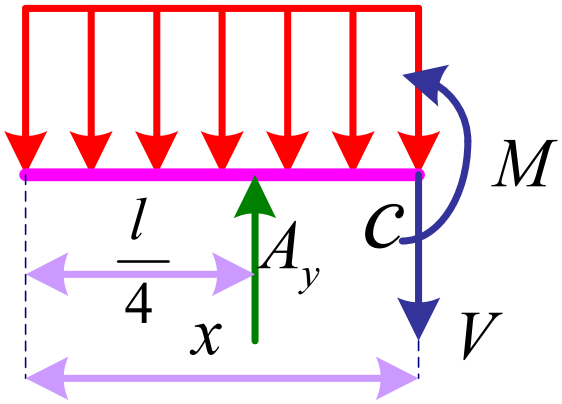
$$\sum F_y = 0 \Rightarrow A_y + B_y - \overline{W}l = 0 \Rightarrow A_y = \overline{W}\frac{l}{2}$$





$$\left.\begin{aligned} \sum F_y = 0 \Rightarrow \overline{W}x + v = 0 \Rightarrow v = -\overline{W}x \\ \sum M_C = 0 \Rightarrow \overline{W}x\left(\frac{x}{2}\right) + M = 0 \Rightarrow M = -\frac{Wx^2}{2} \end{aligned}\right\} 0 \le x \le l/4$$





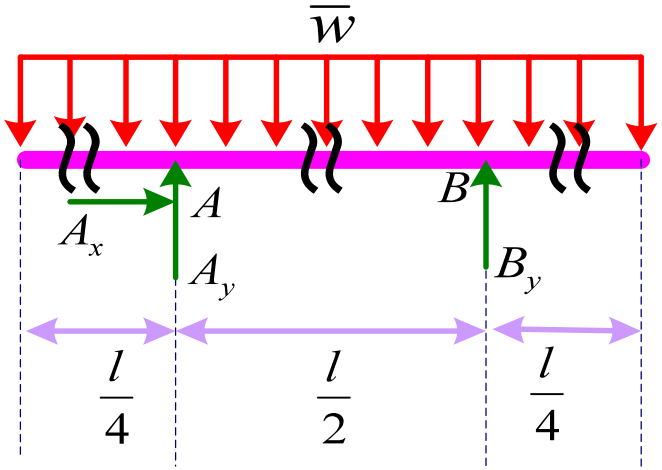
$$A_y = \overline{W}\frac{l}{2}$$

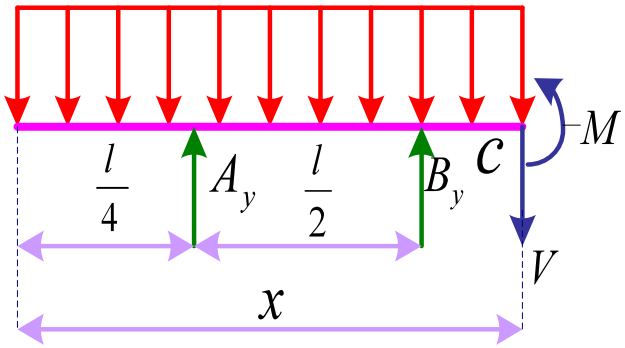
$$\sum F_y = 0 \Rightarrow A_y - \overline{W}x - v = 0 \Rightarrow v = \overline{W}\left(\frac{l}{2} - x\right)$$

$$\sum M_C = 0 \Rightarrow -A_y\left(x - \frac{l}{4}\right) + \overline{W}x\left(\frac{x}{2}\right) + M = 0$$

$$\Rightarrow M = -\frac{\overline{W}x^2}{2} + \frac{\overline{W}l}{2}\left(x - \frac{l}{4}\right)$$

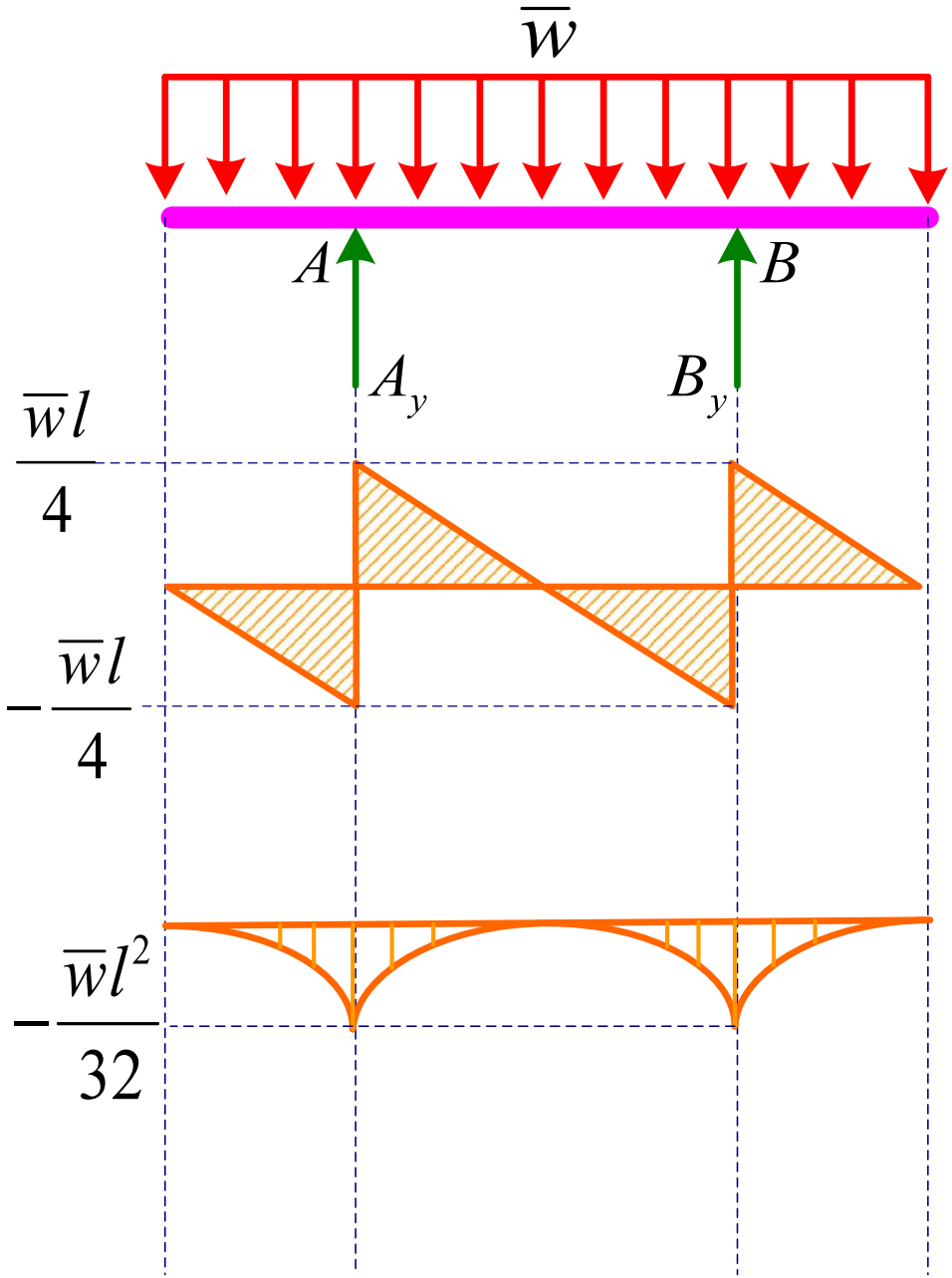
$$\frac{l}{4} \le x \le 3\frac{l}{4}$$





$$\sum F_y = 0 \Rightarrow A_y + B_y - \overline{W}x - v = 0 \Rightarrow v = \overline{W}(l - x)$$

$$\left.\begin{aligned} \sum M_C = 0 \Rightarrow -A_y\left(x - \frac{l}{4}\right) + \overline{W}x\left(\frac{x}{2}\right) - B_y\left(x - \frac{3l}{4}\right) + M = 0 \\ \Rightarrow M = \frac{\overline{W}l}{2}\left(x - \frac{l}{4}\right) + \frac{\overline{W}l}{2}\left(x - \frac{3l}{4}\right) - \frac{\overline{W}x^2}{2} \end{aligned}\right\} \quad \frac{3l}{4} \le x \le l$$



$$A_y = \overline{W}\frac{l}{2} \qquad B_y = \frac{\overline{W}l}{2}$$

$$v = -\overline{W}x \qquad M = -\frac{\overline{W}x^2}{2}$$

$$0 \le x \le l/4$$

$$v = \overline{W}\left(\frac{l}{2} - x\right) \qquad M = -\frac{\overline{W}x^2}{2} + \frac{\overline{W}l}{2}\left(x - \frac{l}{4}\right)$$

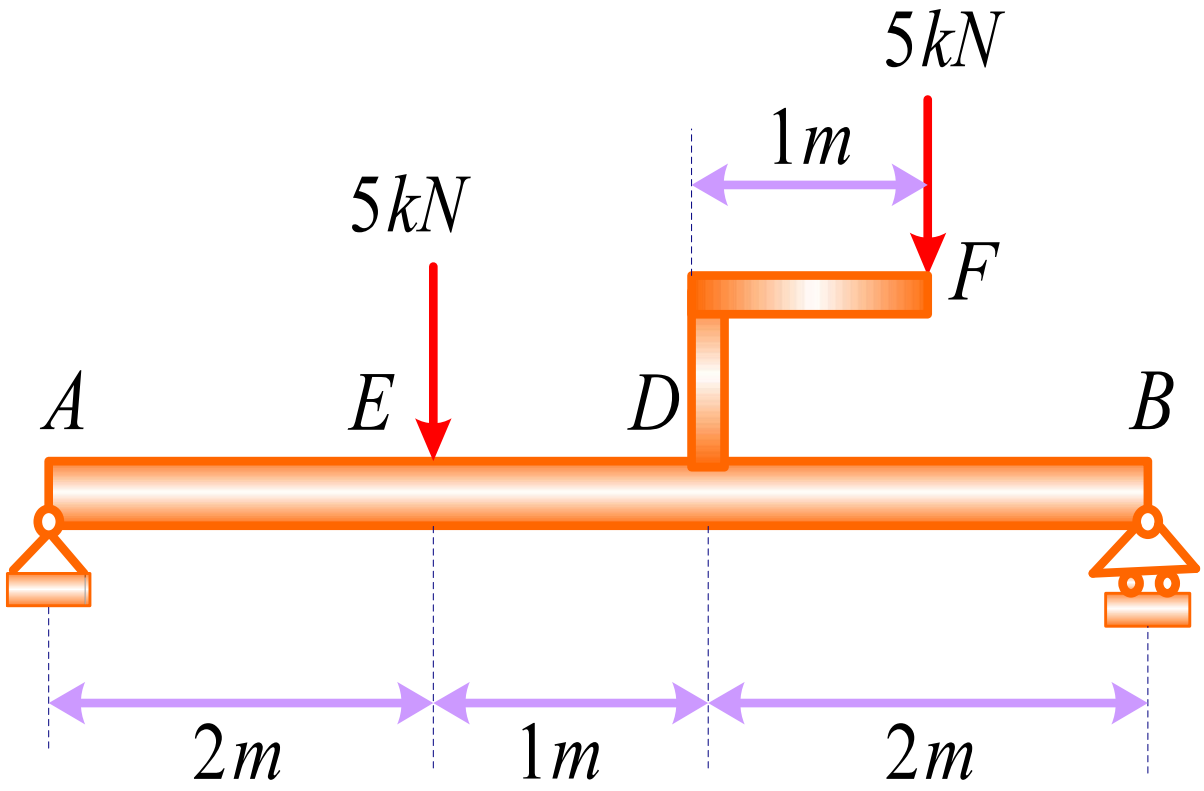
$$l/4 \le x \le 3l/4$$

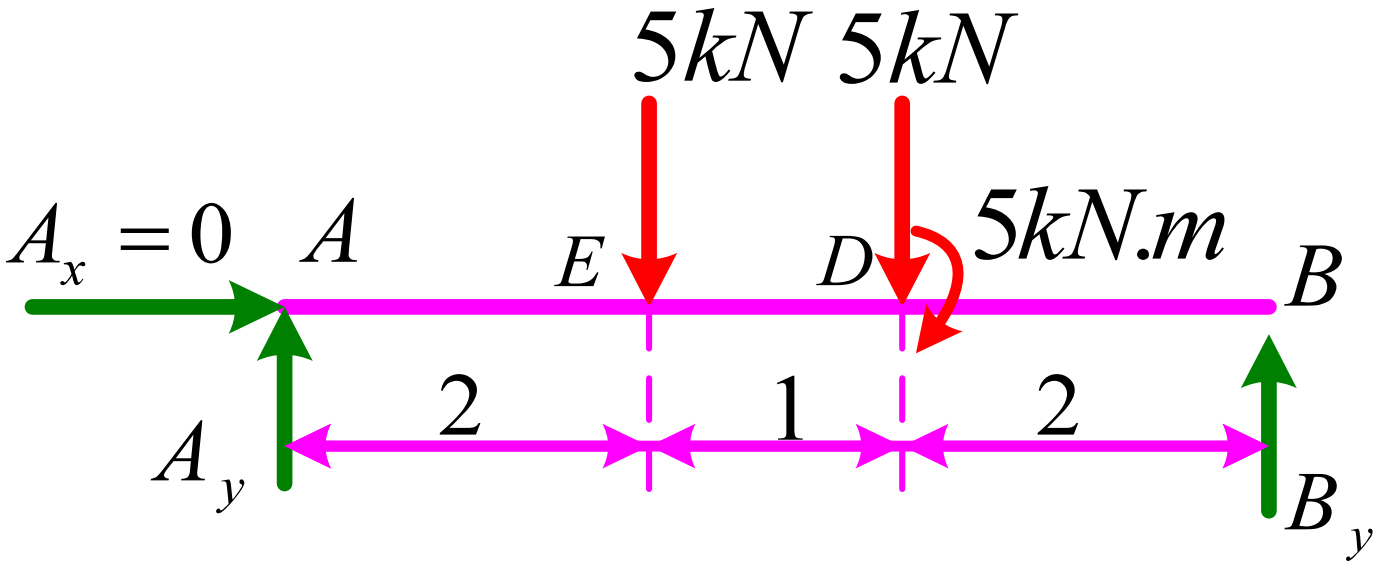
$$v = \overline{W}(l - x)$$

$$M = \frac{\overline{W}l}{2}\left(x - \frac{l}{4}\right) + \frac{\overline{W}l}{2}\left(x - \frac{3l}{4}\right) - \frac{\overline{W}x^2}{2}$$

$$3l/4 \le x \le l$$

مثال: دیاگرام های نیروی برشی و گشتاور خمشی را برای تیر نشان داده شده در شکل رسم کنید؟



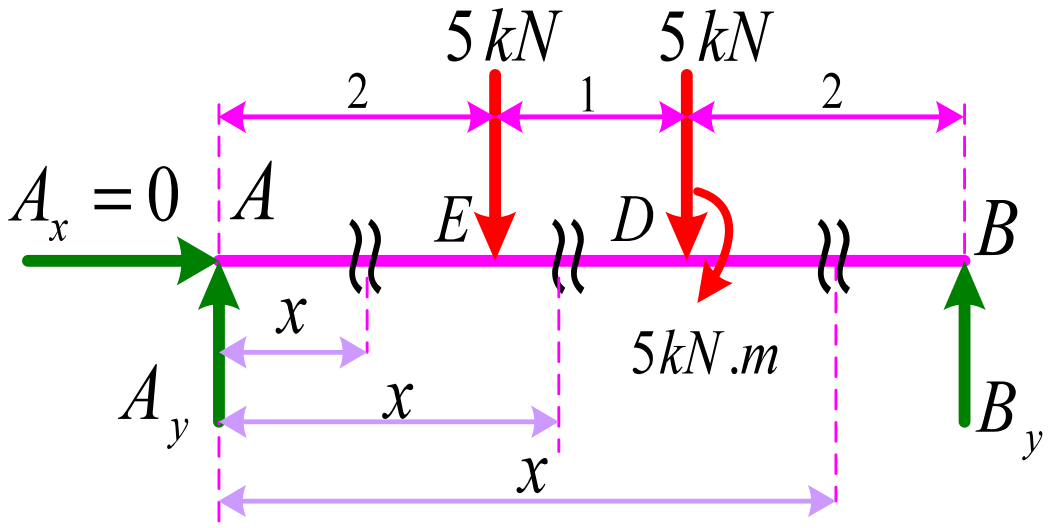


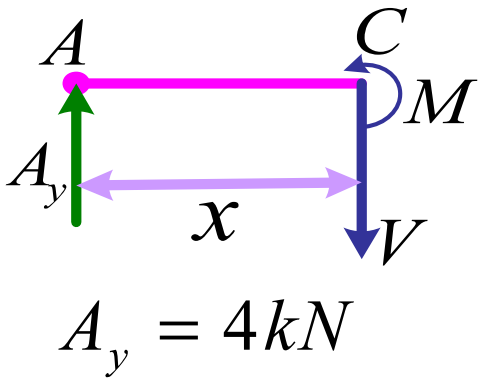
$$\sum M_A = 0 \Rightarrow 2\times 5 + 3\times 5 + M - 5B_y = 0$$

$$B_y = 6kN$$

$$\sum F_y = 0 \Rightarrow A_y + B_y - 10 = 0$$

$$A_y = 4kN$$

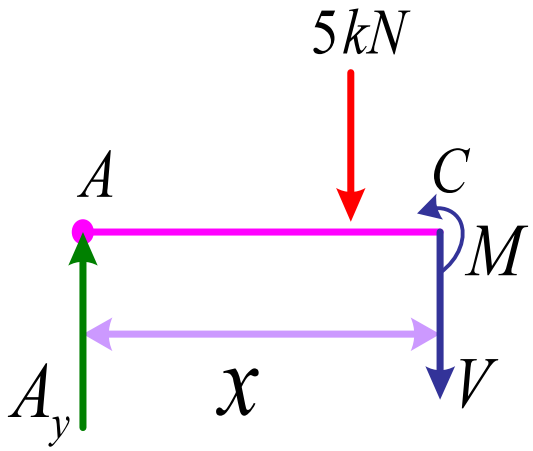




$A_y = 4kN$

$$\sum F_y = 0 \Rightarrow v = 4kN \qquad \sum M_C = 0 \Rightarrow M - A_y x = 0 \Rightarrow M = 4x$$

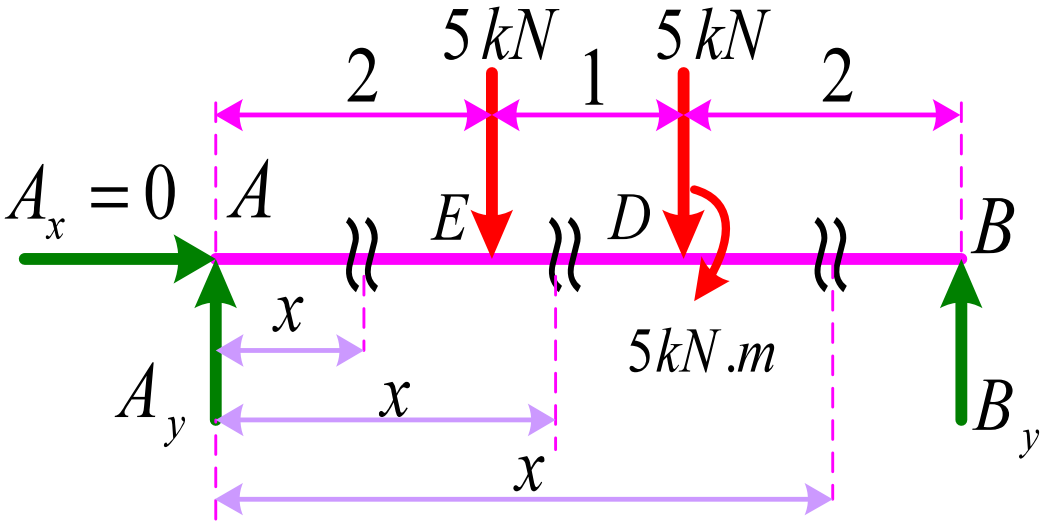
$$0 \le x \le 2$$

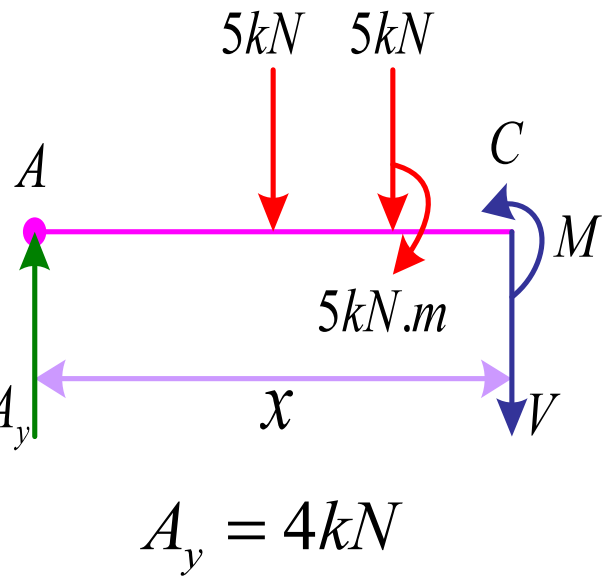


$$\sum F_y = 0 \Rightarrow v = -1kN$$

$$\sum M_C = 0 \Rightarrow M + 5(x-2) - 4x = 0$$

$$\Rightarrow M = -x + 10 \qquad 2 \le x \le 3$$



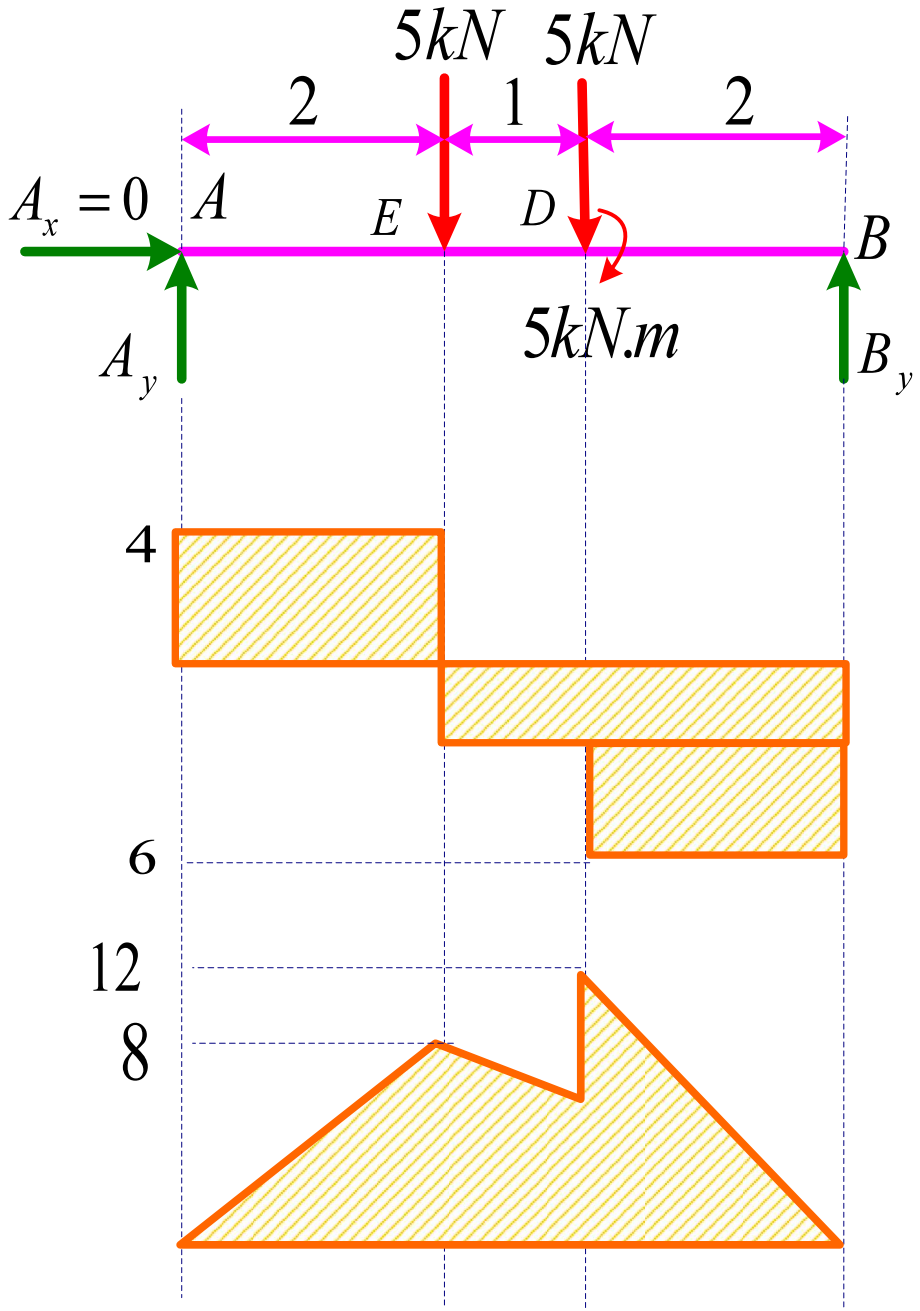


$A_y = 4kN$

$$\sum F_y = 0 \Rightarrow v = -6kN$$

$$\sum M_C = 0 \Rightarrow -M - 5(x-3) - 5(x-2) + 5 + 4x = 0$$

$$M = -6x + 30 \qquad 3 \le x \le 5$$



$$v = 4kN \qquad M = 4x$$

$$0 \le x \le 2$$

$$v = -1kN \qquad M = -x + 10$$

$$2 \le x \le 3$$

$$v = -6kN \quad M = -6x + 30$$

$$3 \le x \le 5$$

$$A_y = 4kN \qquad B_y = 6kN$$