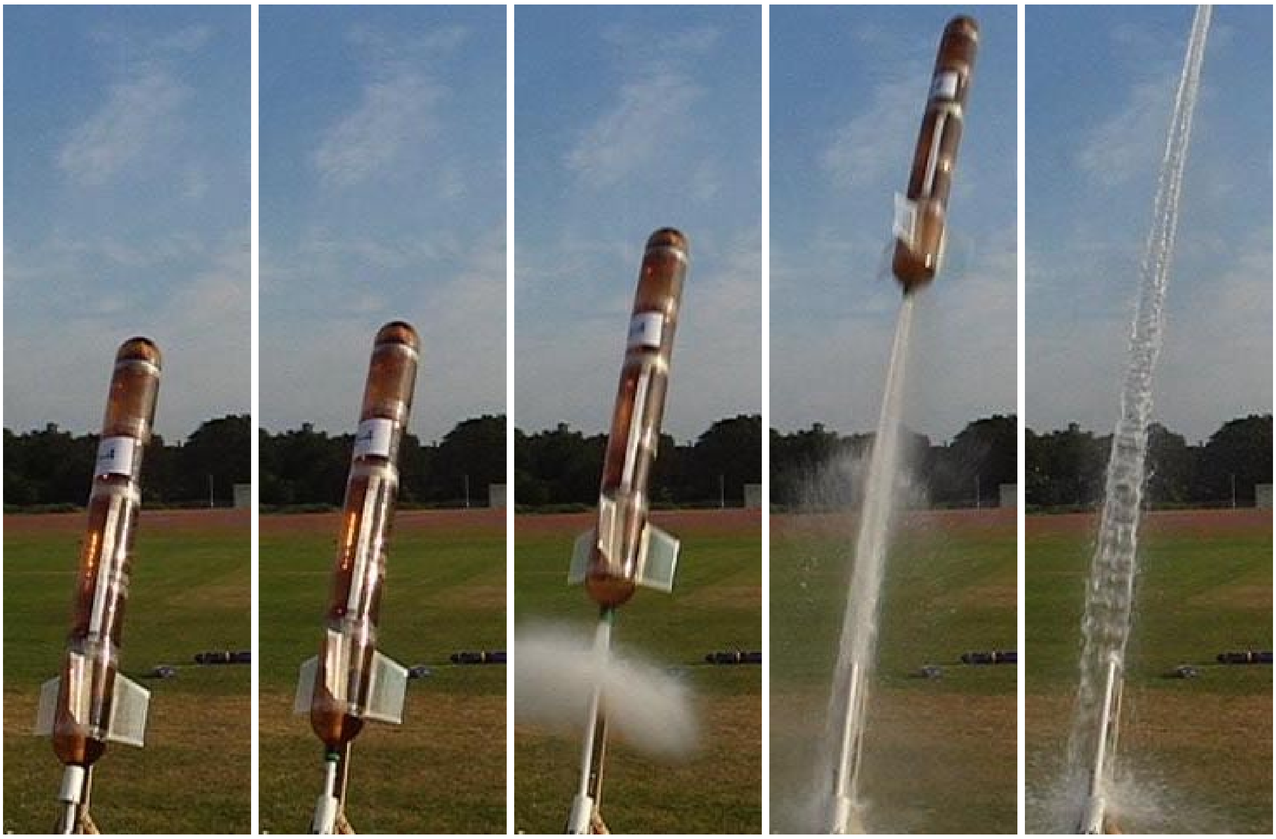


A guide to building and understanding the physics of

Water Rockets

Version 1.02
June 2007

Warning:
Water Rocketeering is a potentially dangerous activity and individuals following the instructions herein do so at their own risk.

Exclusion of liability:
Serco and NPL Management Limited cannot exclude the risk of accident and, for this reason, hereby exclude, to the maximum extent permissible by law, any and all liability for loss, damage, or harm, howsoever arising.

Contents

WATER ROCKETS

Acknowledgements

I began writing this guide to support people entering the *NPL Water Rocket Competition*. So the first acknowledgement has to be to Dr. Nick McCormick, who founded the competition many years ago and who is still the driving force behind the activity at NPL. Nick's instinct for physics and fun has brought pleasure to thousands.

The inspiration to actually begin writing this document instead of just saying that *someone* ought to do it, was provided by Andrew Hanson. Once I began writing, lots of people assisted me, many from the NPL Water Rocket Helpers Team, but I would particularly like to thank, Dave Lowe, Jaco Stander and Gergely Vargha for advice about building launchers, permission to use photographs of their equipment, and for generally putting me right on one or two finer points of rocket design.

Finally, the Water Rocket activity is supported by NPL's management, and by *Serco*, and I am grateful to both the organisations, and many individuals within them. Their support for this kind of activity is one of the reasons that NPL is such a great place to work.

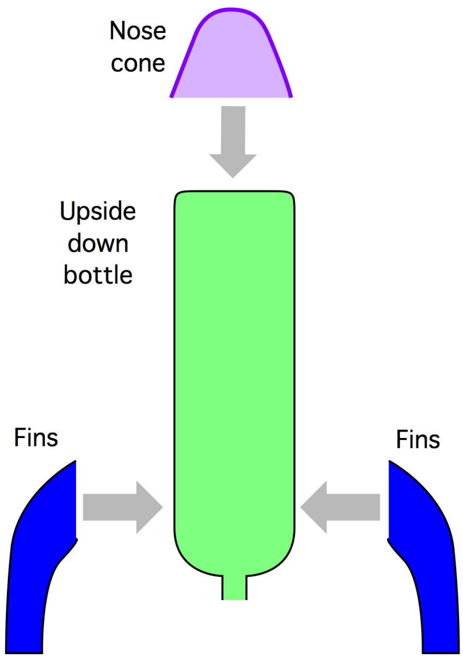
Thanks to all of you

Michael de Podesta

April 2006

Section 1: What is a water rocket?

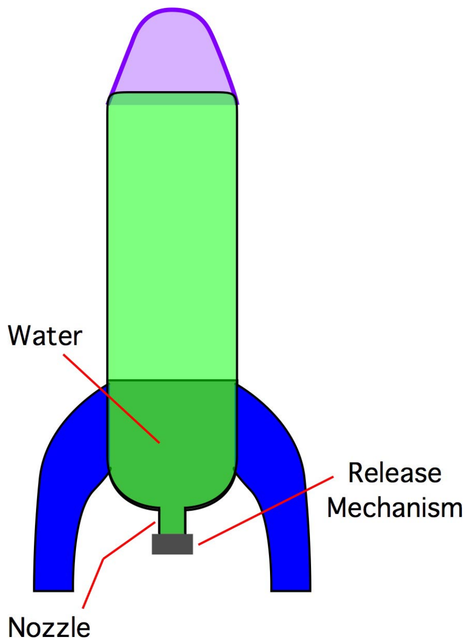
At its simplest, a water rocket is basically an upside down fizzy drinks bottle, which has had a 'nose' cone and some fins added.



The nose cone
The job of the nose cone is to make the rather snub-nosed end of the fizzy drinks bottle more aerodynamic. Also if you have 'payload' on your rocket, or a parachute mechanism, this is probably where it will be placed.

The fins
Others might disagree, but I think the fins are the parts of a rocket that really give a rocket its character. Technically, the fins are important for ensuring that the rocket flies smoothly

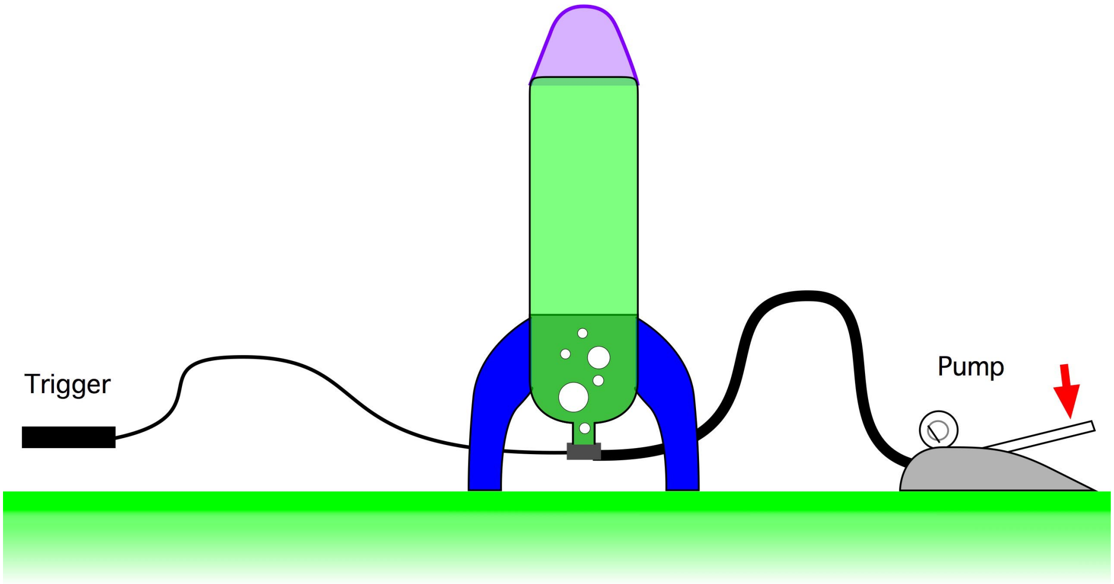
Once we have added the fins and the nose cone, we have something which *looks* like a rocket. But how do we make it *go* like a rocket?



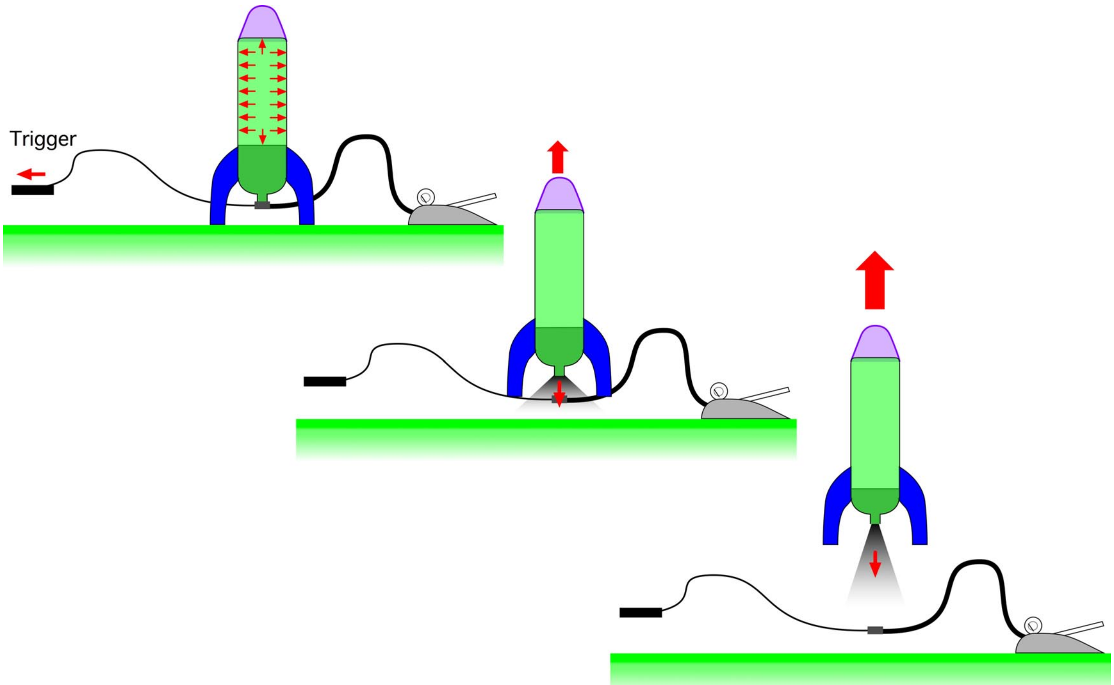
First we need to add some water, and some kind of release mechanism, that will keep the water in the bottle, until we choose to release it. The water will then leave the bottle through its nozzle.

Typically the bottle will be between about one quarter and one third filled with water.

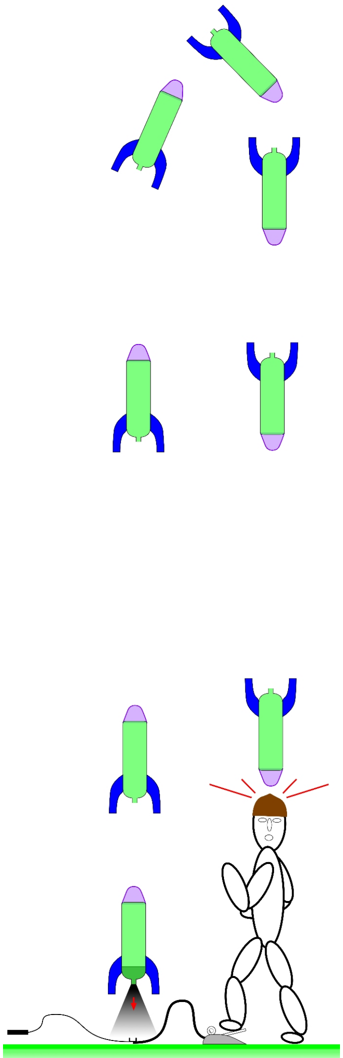
Launch
To launch the water rocket, we need to pump air into the rocket: this provides the energy for the launch. As the air enters, it bubbles up through the water and pressurises the 'empty' space above the water. You can see that the release mechanism has to be really quite clever, allowing air into the rocket, while not allowing the water to escape until we activate a trigger.



When the trigger activates the release mechanism, the pressurised air within the rocket pushes the water rapidly out through the nozzle, sending the rocket rapidly into the air.



Peak launch velocities can easily reach 30 metres per seconds (about 60 miles per hour), and without too much difficulty its possible for a rocket to reach heights in excess of 30 m. But launching a rocket straight up in the air can be dangerous…

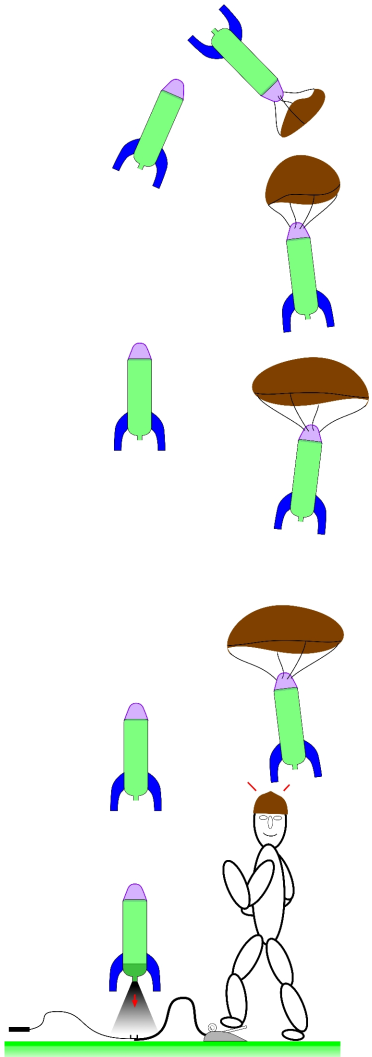


There are two ways to make your water rocket launch safe.

The first way is to use a parachute or other similar device to slow the descent of the rocket. Seeing a rocket launch, reach its peak, deploy a parachute and descend gracefully to Earth, is a really great sight.

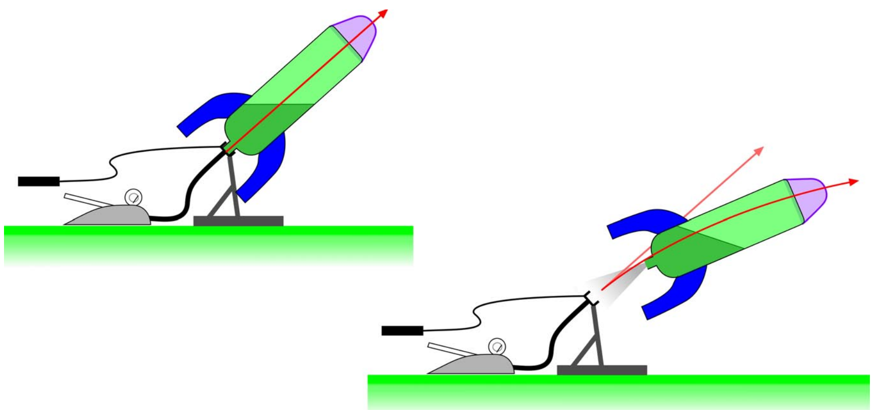
Unfortunately, its not a very common sight, because getting a parachute to open at just the right time is very tricky, and requires real ingenuity.

The second way to safely launch your water rocket is to launch it an angle. Of course, this makes it safe for you, but potentially dangerous for passers by!



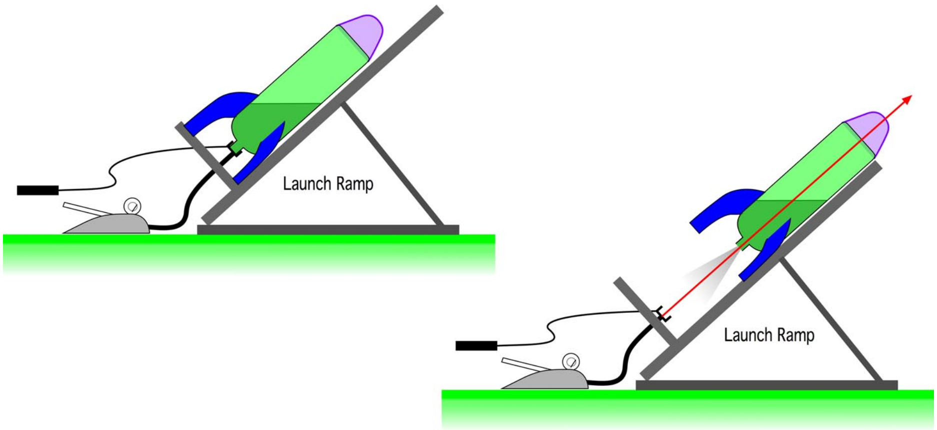
One of the best features about launching at angle is that water rockets can travel really impressive distances. Reaching 30 or 40 metres should be quite achievable, but distances beyond 100 m are possible with some careful design.

The main problem with launching the rocket at an angle is that the rocket can no longer stand on its own feet, and if it is supported entirely by its nozzle, then it tends to flop over. This happens before launch, and most importantly, it happens just after launch before the rocket has begun to move quickly.

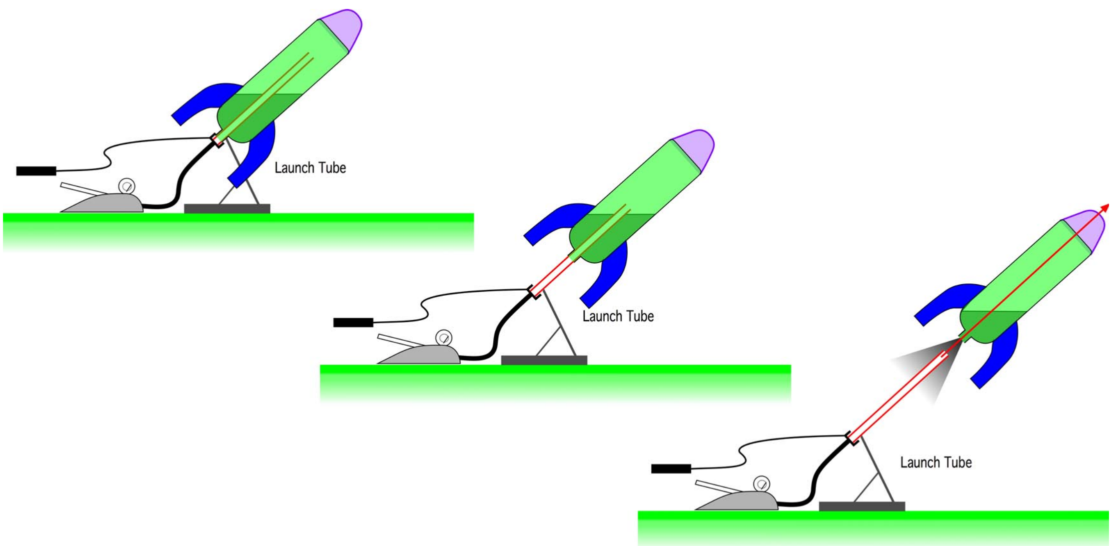


There are two standard ways to solve this problem: *launch ramp* and *launch tubes*.

Launch Ramps
A launch ramp supports the weight of the rocket before launch and just after launch, until its speed has built up.



Launch Tubes
A launch tube is a tube that runs through the nozzle of the rocket. When the trigger activates the release mechanism, the rocket slides along the launch tube before fully attaining 'free flight'. This has two advantages. The first and most obvious advantage is that the launch tube stops the rocket from 'flopping over' just after launch. In this respect it acts like a kind of 'internal launch ramp'. The second advantage is not quite so obvious. Once the trigger has been activated, the high pressure gas inside the rocket expands, and pushes (as the middle section of the figure below shows) the rocket along the launch tube. As it slides along the launch tube it accelerates, and it can be moving quite fast when it leaves the launch tube. However, while it is on the launch tube, it is not losing any water. This gives the rocket a kind of 'moving start' and allows it to use its charge of water more effectively. This can significantly improve its performance.



Why?

So now you know what a water rocket is. But perhaps the question still lingers: What's the point? The answer is very simple: building and launching rockets is just enormously enjoyable. It combines the simple pleasure of watching in awe at the power of a compressed gas, with the rather more subtle pleasure of mastering an engineering problem. In short, its fun for all ages.

The challenge: Some teams, designs and launches from NPL's Water Rocket Challenge

Photo Credits: Photos from the NPL Water Rocket Challenge Web Site. Thanks to Mike Parfitt, Steve Forrester, Clive Scoggins, Stuart Rogers

Section 2: How to make a basic water rocket

In Section 1, we saw what a water rocket was. In this section we'll see in detail how to make a basic water rocket that will fly pretty well in a wide range of conditions. We'll cover how to launch your rocket in Section 3.

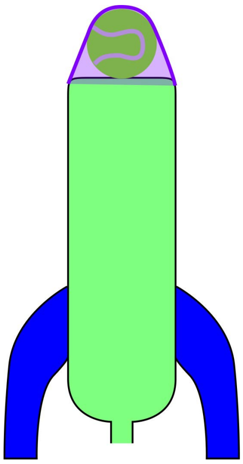
2.1 A Basic Rocket

What you will need

- A two-litre fizzy drinks bottle: this will form the main body of the rocket. Be sure only to use bottles that contained *fizzy* drinks: similar looking bottles which contained *still* drinks (cordial, milk drinks etc.) are not suitable. Fzzy drinks bottles are made from PET (short for Polyethylene Terephthalate), an enormously strong plastic.
- A tennis ball, or rubber ball weighing about 60 g. This will form the main part of the nose.
- Some corrugated cardboard, or better still, corrugated plastic. This will be used to make the fins.
- 'Duck' tape or equivalent strong, sticky tape.
- Scissors or a knife.
- Time: Between 30 and 40 minutes

When completed it will look like…

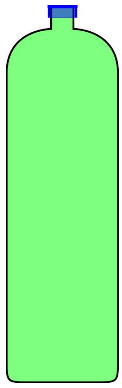
Schematic



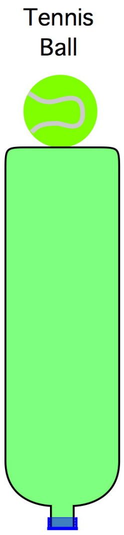
Actual

First of all…

…you start with a fizzy drinks bottle. You need to empty out the fizzy drink, get rid of the labels, and rinse it with water. Supermarkets sell 'value' ranges of lemonade and fizzy water that cost only perhaps 20 pence per bottle so this shouldn't cost too much.



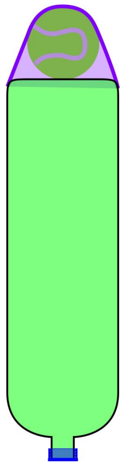
Now you need to add a *nose cone* and *some fins*

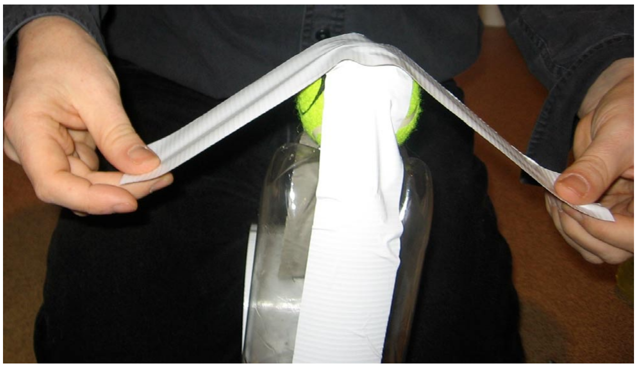


The nose cone

The nose cone needs to be slightly pointed, and as we'll see in Section 4, it's also important to have a little bit of weight towards the front of the rocket.

My favourite way of achieving both these aims is simply to tape a tennis ball to the end of the bottle.

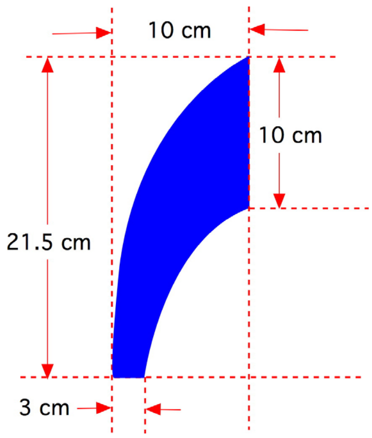




This might not look quite as aerodynamic as you were hoping for, but trust me, it will fly!

The fins

These fins were cut out of an old estate agent's 'For Sale' board. More technically, this corrugated plastic (known as *Corriflute*™) is waterproof, and has excellent rigidity for its weight. If you can't find any old 'For Sale' signs, a source of *Corriflute* is listed in Section 9. However, then there are many suitable alternatives.



Corrugated cardboard will do, but does tend to go soggy after a few launches. Also many packaging materials have the same design requirements as water rocket fins (high rigidity-to-weight ratio). One common choice is to cut up old CD's to use as fins. If you do this please then make sure you put tape over any sharp edges in case your rocket should hit someone.

I've used three fins rather than four, because three fins means one less fin to cut out! I've included a picture showing the actual dimensions I used on these fins, but this design is far from optimal. I like this design because the rocket can stand on the fins (which is actually quite handy), and they make the rocket look a little bit like the rockets from *Tintin* books.

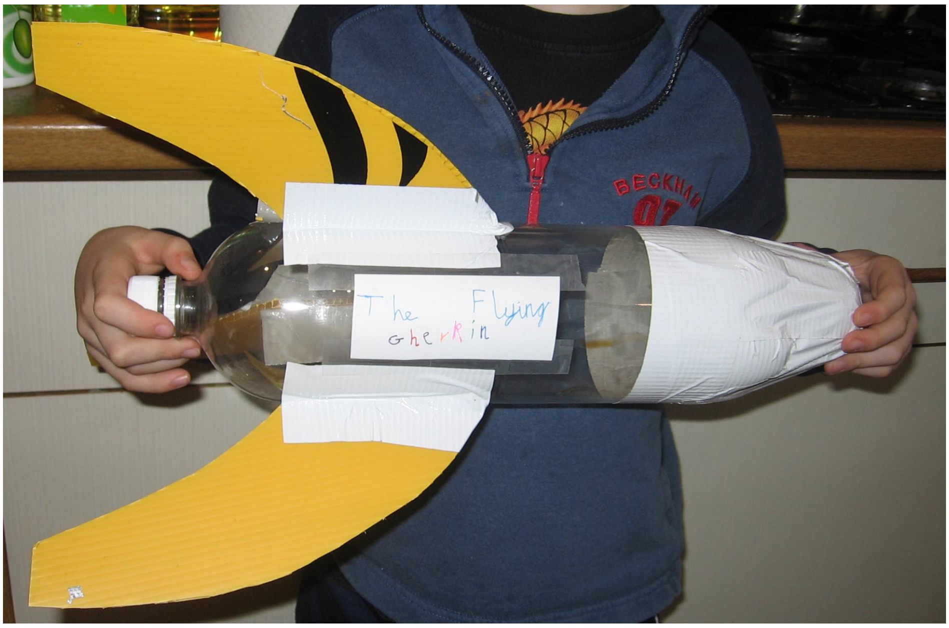
The fins are simply taped to the side of the rocket. They need to be reasonably firmly attached in order to stop them being ripped off during the launch. The fins will almost certainly be damaged on landing, but then they will not be too difficult to repair.

Whether you use this fin design or your own, the important things about the fins are that:

- All the fins should be the same as each other,
- They should be positioned towards the back of the rocket.
- They should arranged symmetrically around the rocket (every 120° if you have three fins or every 90° if you have four)
- They should be thin when viewed 'head on'

Decoration

Decorating and naming your rocket can give a disproportionate amount of pleasure for the time it takes: I give you: *The Flying Gherkin*!



Critique

This rocket is not optimised in many ways, and in Section 4 we'll see how to optimise each part of the rocket, and discuss the different design compromises that you will need to make. But you can probably already see that it could be made lighter, the nose cone could be made more aerodynamic, and the fins could be reduced in size. However, the aerodynamics are not *too* bad, and the weight and fin size are such as to keep the rocket stable in flight. In short: it's not a bad starting point.

Vital Statistics

Internal Volume	2 litres
Mass when empty	171 g (0.171 kg)
Length	45 cm (0.45 m)
Area of Fins	1200 cm^2
Frontal Cross Sectional area	62 cm^2

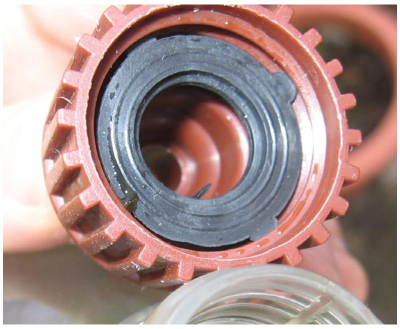
Section 3: Launchers

Launchers are more complicated to build than rockets, and it will take you much longer to build a launcher than it will to build a rocket. For this reason I think you might like to consider investing in a commercial launcher. I tested the rocket described in the previous section using a launch system available from *Maplin* (Section 9).

The *Maplin* system uses a special nozzle which screws onto the bottle in place of its cap. The nozzle is shaped to fit into normal garden hose fittings, and the system comes with a quick release based on normal garden hose connectors, and activated by a neat system based on a bicycle brake cable mechanism.

Right: details of the special nozzles used by the Maplin Launcher

Far Right: The nozzle screwed onto the rocket.

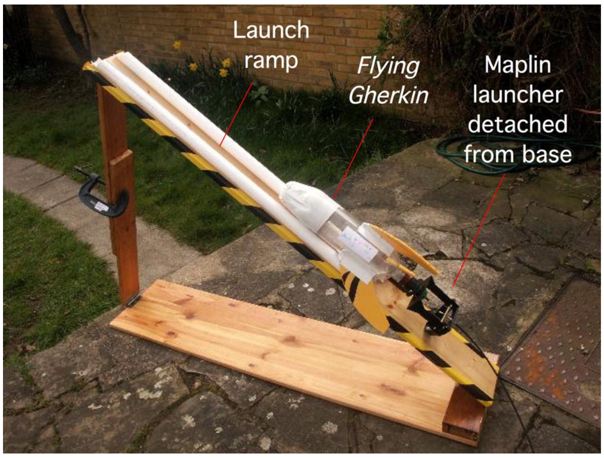


Right: The rocket *The Flying Gherkin* installed on the *Maplin* Launcher ready for launch

Far Right: Filling the rocket with a measured charge of water.

Personally, I have used parts of the *Maplin* launcher mechanism, but attached them to my own launch ramp. It's not the best design in the world, but it is *my* design.

And it may be that you too long to design and build your own launcher. In which case, the following pages may be of assistance.

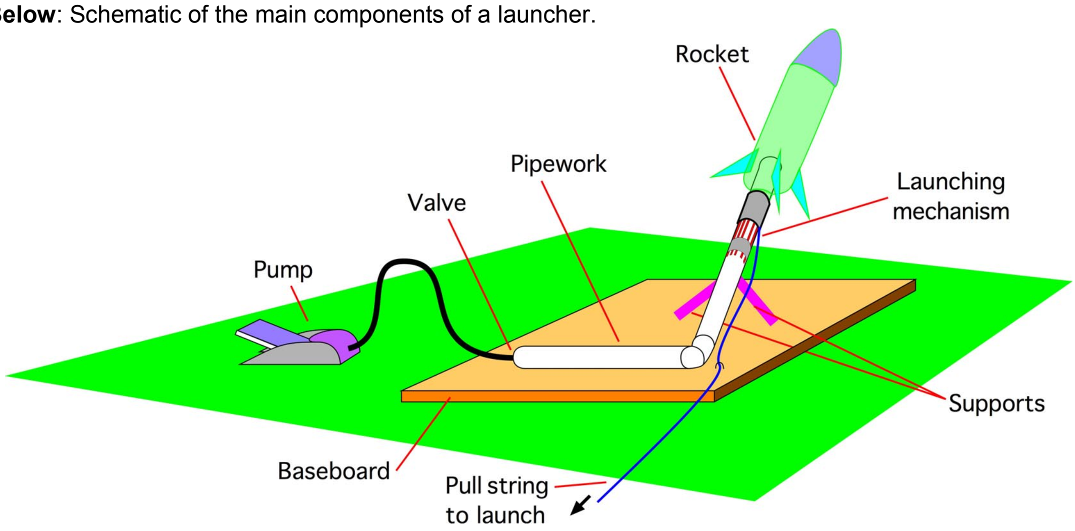


Making your own launcher

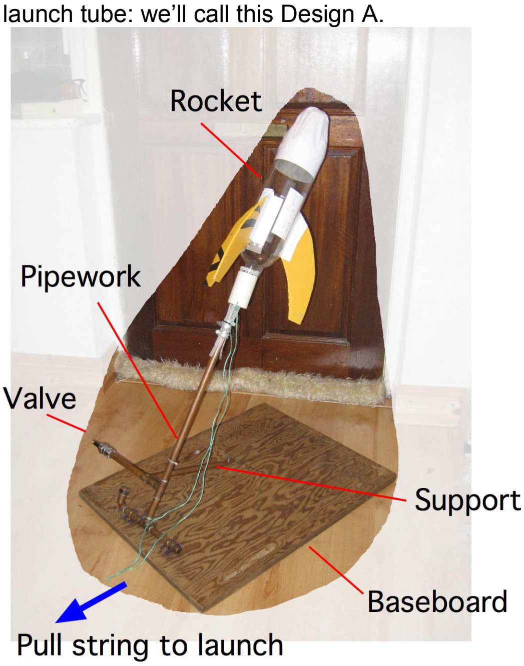
First of all, a confession: before I made the launcher shown on the previous page, I made a launcher similar to those featured below. However it didn't work very well. So these instructions are not based on what I *have* done, but rather on what I *would* have done if I had been clever enough to consult my colleagues before I started building. Launchers are more complicated than water rockets, and if you are attempting this, then you are probably quite good at DIY, and won't need complete instructions. So in this section I will only describe those parts that I think are not obvious. I will describe two designs, one without a launch tube (Design A) and one with a launch tube (Design B). If you attend any water rocket gathering, you will see that these designs are simply two stars in a galaxy of possible designs.

When completed your launcher *might* look something like…

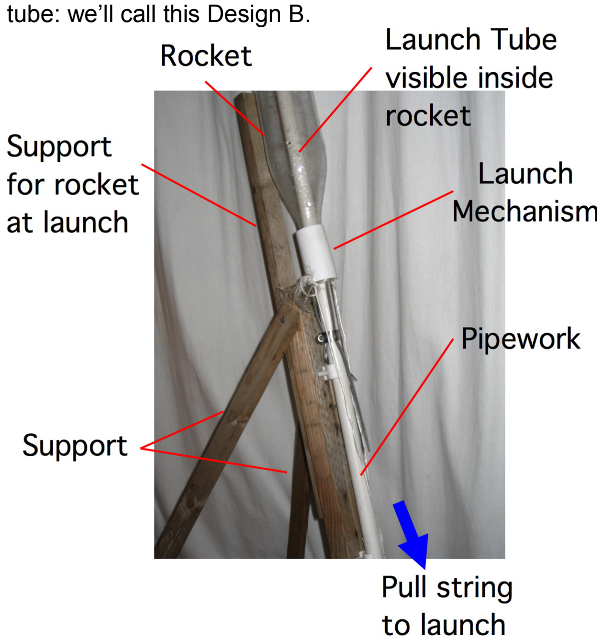
Below: Schematic of the main components of a launcher.



Below Left : A typical rocket launcher without a launch tube: we'll call this Design A.



Below Right : A typical rocket launcher with a launch tube: we'll call this Design B.



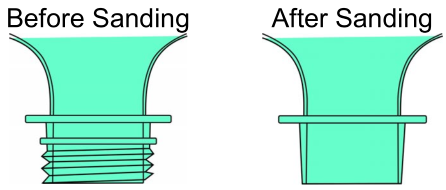
The launching Mechanism

Connecting the rocket to the launcher and designing a launching mechanism is probably the trickiest part of the construction. Let's look at how the two designs approach this.

Design A

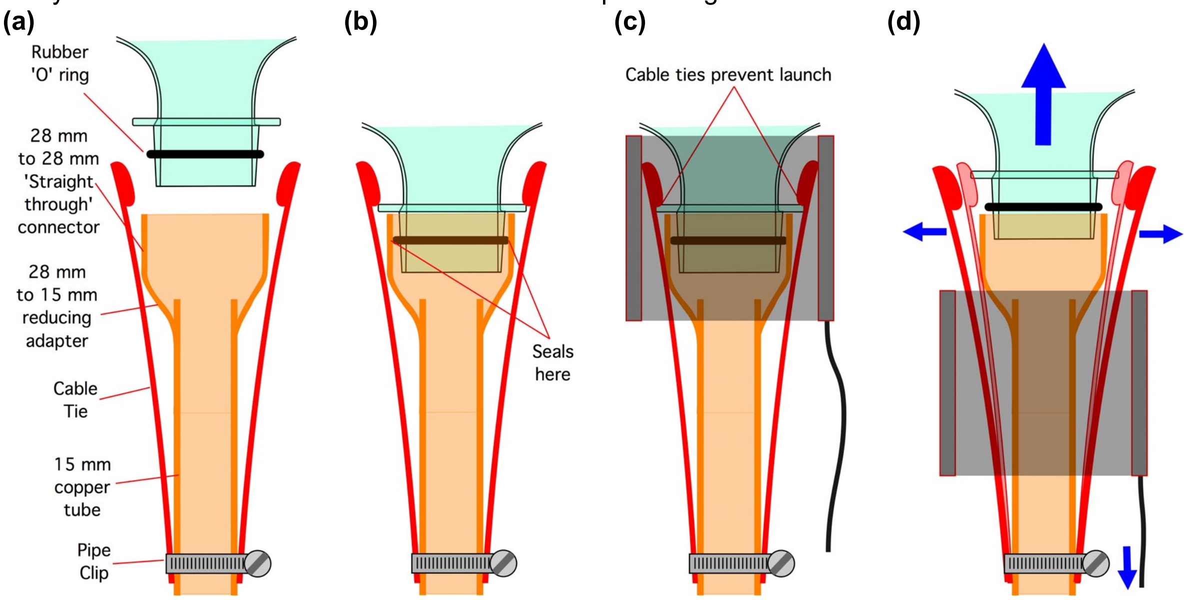
Design A is Jaco Stander's version of the Ian Clark's cable tie launcher (see Section 9 for a web link), and is constructed out of standard 15 mm copper plumbing tube soldered together. Soldering is probably quite hard for beginners, so as an alternative, the launcher could have been made using either compression fittings (which can be made pressure tight with spanners) or 'push fit' fittings (which require no additional tools). However, the design depends on small details of the fittings so you will need to check what will and won't work with the particular fittings you choose.

In this design, the screw thread on the outer part of the bottle is removed with sand paper to make a smooth surface. This can be done by hand, but a much better finish can be achieved by spinning the bottle and applying gentle pressure with sand paper.



The launcher and launching procedure is illustrated in the Figure below.

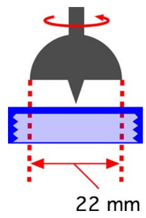
The launching technique for Design A (a) The bottle fits into a 28 mm to 28 mm 'straight through' connector, attached to a 28 mm to 15 mm reducing adapter, which in turn is attached to 15 mm copper pipe. (b) The modified bottle-end is fitted snugly into the 28 mm throat of the adapter. The pressurised seal is achieved by the use of a thin 28 mm diameter 'O' ring. Notice that at this point all the pipework visible in the illustration will be filled with water. (c) The bottle is then locked in place by cable ties, which in turn are held in place by large diameter plastic plumbing tube. At this point the rocket can be pressurised, and will not launch until (d) the large diameter plumbing tube is pulled away. This allows the cable ties to move outwards permitting the rocket to launch.



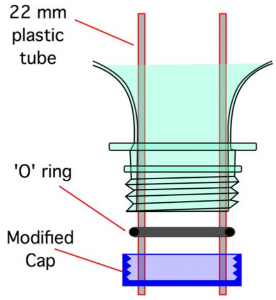
The major problem with Design A is that the rocket is only supported at the neck, and since a large rocket can weigh several kilograms when filled with water, this makes the rocket liable to 'sag' before launch. This could be overcome by the addition of either a launch ramp to support the rocket, or a launch tube. A photograph of the launcher in action can be seen in the collection on Page 5 (one down from the top right), where Jaco has used an improvised launch ramp.

Design B

Design B is by Dave Lowe, and uses a launch tube to add support and give extra momentum at launch. It is constructed out of two types of plumping pipe: standard 22 mm diameter plastic plumbing tube, and undersized 21.5 mm plastic 'overflow' pipe. It is assembled using simple push-fit plumbing connections.

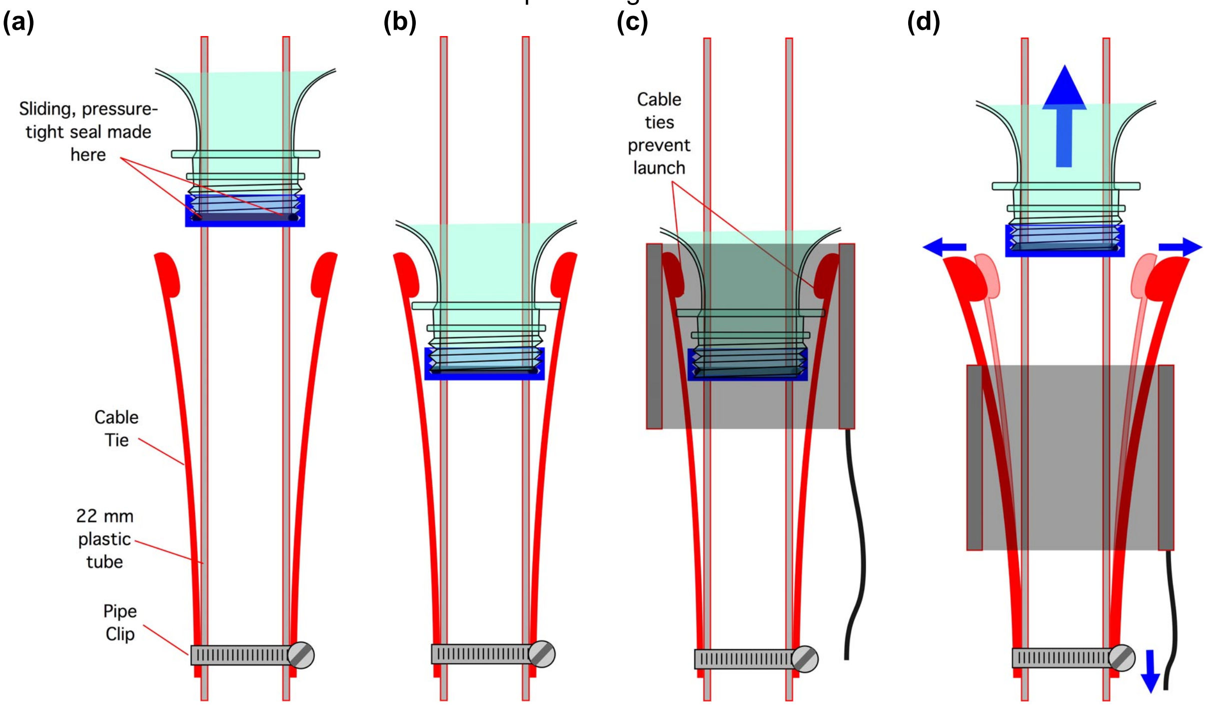


In this design, a 22 mm diameter hole is drilled through a standard bottle cap. Be careful when drilling because any drilling operation could be hazardous! The plastic of the cap is soft, and we recommend the use of wood bit rather than a conventional high-speed steel drill. The cap can now be slipped over the 21.5 mm overflow pipe trapping an 'O' ring between the bottle top and the tube. This should form a pressure-tight sliding seal against the wall of the tube.



- Standard 22 mm plumbing tube will *not* fit inside the neck of PET bottle. To make sure you get the correct type of pipe, take a bottle along to the shop to check the pipe will fit before you buy it!
- If the bottle is a tight fit on the tube, reduce its diameter slightly using sandpaper. One technique is to fit the tube into a drill and rotate it, and hold fine sand paper against the tube.
- You may need to add a small amount of lubricating oil or grease to allow the bottle to slide easily along the tube.

The launching technique for Design B (a) The bottle with its modified cap slips over a piece of polished 22 mm plastic pipe. In this design, the pipework visible in the illustration extends far enough into the bottle to prevent water overflowing into the pipes. (b) The bottle is then locked in place by cable ties, which in turn (c) are held in place by large diameter plastic plumbing tube. At this point the rocket can be pressurised, and will not launch until (d) the large diameter plumbing tube is pulled away. This allows the cable ties to move outwards permitting the rocket to launch.



Designs A and B both use cable ties as a key component in the launching mechanism, but as the photographs below show, they adopt a what I can only describe as different design philosophies. Dave has gone for the *minimum* of three cable ties, and Jaco has gone for the *maximum* number of cable ties that can be arranged around the rim of the bottle. Which is better? I don't know: they both work very reliably!

Photographs of Designs A and Design B

Details of the launch mechanism of Design A. **Below**. The cable ties loosely arranged around the 28 mm to 15 mm plumbing adapter. **Right**. The large diameter plumbing tube has been lifted up to clamp the cable ties over the neck of the rocket.

Details of the launch mechanism of Design B.

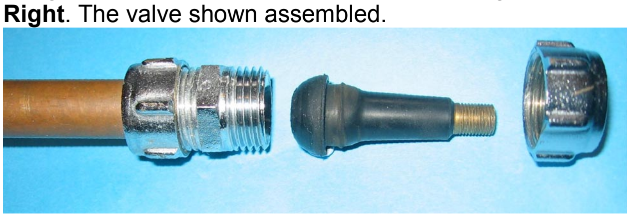
Right. The arrangement of the bottle, the sealing 'O' ring and the modified cap used to clamp the 'O' ring against the launch tube.

Far Right. The three cable ties held loosely arranged around the neck of a bottle. The large diameter plumbing tube has not been lifted up completely to clamp the cable ties over the neck of the rocket. Notice that launch tube continues inside the water rocket

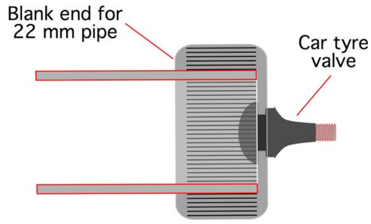
2. Pressurised connections & the pumping valve

Both designs A and B use pipework systems that are readily available from plumber's merchants and DIY stores. However one part of a rocket launcher which is not available from shops is a component to allow connection between these pipework systems and a bicycle pump. Both designs tackle this by installing either a bicycle tyre valve or a car tyre valve. The Figure over the page shows details of Design A's connector.

Details of the pumping valve in Design A. **Left**. The valve is shown disassembled, showing (from left to right) a standard 15 mm compression fitting (called a 'union'); a car tyre valve; the compression nut. **Right**. The valve shown assembled.



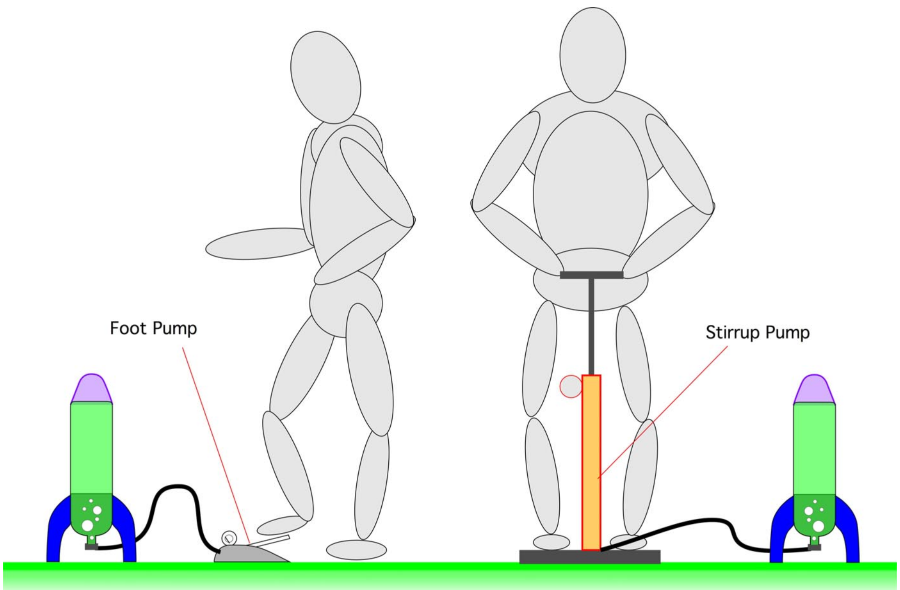
A similar arrangement can be made with 22 mm plumbing fittings, but there it will be necessary to drill a hole in an end-cap, and to fit the car tyre valve in place. Glue or sealant should not be necessary, because the tyre valve fitting is designed so that as the pressure increases, the seal will improve. A similar design can also be made using bicycle tyre valves



Pump types

One last feature of launcher design concerns the choice of pump used to pressurise the water rocket. There are broadly three types of pumps available: Hand pumps, foot pumps, and stirrup pumps. Any of these can be used, but the 'must have' feature for any pump you choose is a pressure gauge: if the pump doesn't have a pressure gauge then you will have no idea how your rocket will perform and be unable get it to behave reproducibly.

Having said that any type of pump can be used, I would definitely ***not*** recommend a normal bicycle hand pump. The amazing performance of water rockets comes from energy stored in the compressed air, and the source of the work required to compress the air is your arms and legs. Aside, from normally lacking a pressure gauge, hand-powered pumps are very hard work. Stirrup pumps (which allow the work to be shared across both arms), and foot pumps (especially dual-piston pumps) are both popular, but amongst the people who do a lot of rocketeering, the stirrup pump seems to be the preferred choice.



Section 4: Optimising Rocket Design

Aside from actually firing the rockets, designing the rocket itself is the part of rocketeering I enjoy most. In this section we'll look at some of the factors that you will need to consider if you want to optimise the design of your rocket.

Design considerations

Size (Volume)

The first consideration is the size of the rocket you want to construct. Looking around the shops, you will see a wide range of fizzy drinks bottles available, and any of them can be modified to make a water rocket. It's common to find 500 ml, 1 litre, 2 litre and even 3 litre bottles. Larger bottles tend make more spectacular launches, but if you want to go larger than three litres then you will need to construct a rocket by joining together more than one bottle. There's some tips on how to make multi-bottle rockets later on in this section

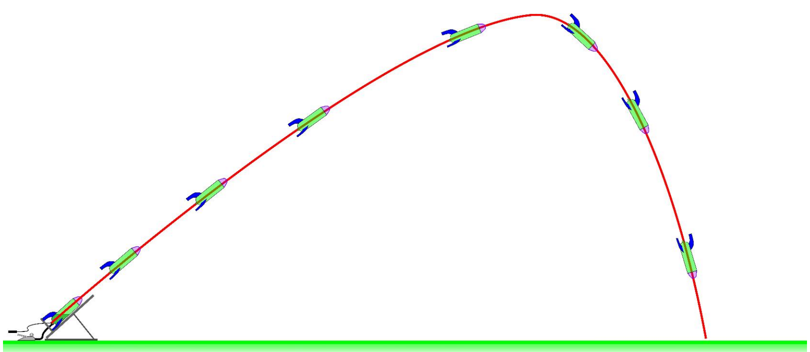
The volume of the rocket determines the maximum amount of energy that can be stored in the compressed gas. The energy is proportional to both the *pressure* and the *volume*. There are limits to the pressure that the rocket can sustain (5 atmospheres [or 75 psi] appears to be a safe working limit) and so in order to increase the total amount of energy available, it is necessary to use a larger rocket. With a little ingenuity it is possible to increase the volume with relatively little cost in terms of added weight.

Weight

The lower the weight of your water rocket, the better it will fly. Most of the work of designing a lightweight rigid structure has been done for you already by the manufacturers of the fantastically strong PET bottles. In order to capitalise on the strength-to-weight ratio of the bottles, you need to avoid adding too much weight as you improve the aerodynamics of the bottle. It is also important to add the weight in the correct places so that your rocket is aerodynamically stable. The distribution of weight along the length of the rocket is one of the factors which determines whether it will fly like rocket, or like a bottle. What's the difference?

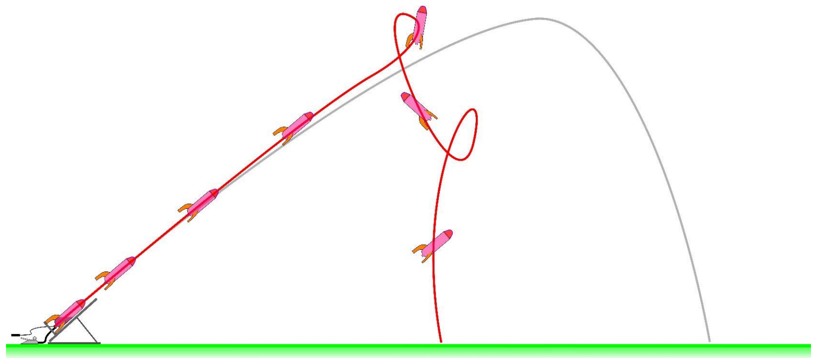
An aerodynamically stable rocket flies with its nose first, and should have a flight trajectory like a beautiful smooth arc.

Right: An aerodynamically stable rocket trajectory. Notice that air-resistance tends to make the trajectory asymmetric, with the rocket falling rather more steeply than it ascends.



An aerodynamically unstable rocket may start out with its nose first, but its flight will quickly become unstable and it will flap and tumble in the air, and then simply fall to Earth.

Right: An aerodynamically un-stable 'bottle' trajectory. Several commercially sold rocket systems have rockets that perform in this way.

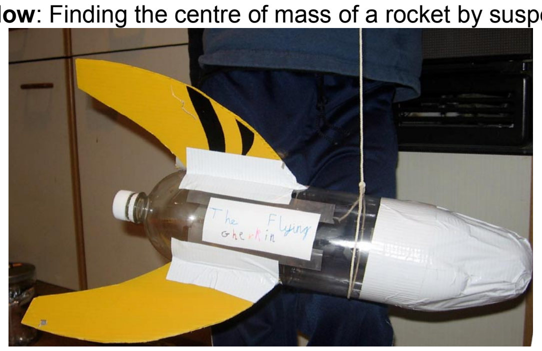


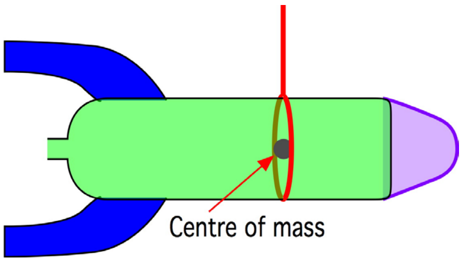
In order to make your rocket fly '*like a rocket*' rather than '*like a bottle*', the weight needs to be in the front half of the rocket. However depending on the design of your fins, this may or may not be enough to ensure aerodynamically stable flight. One of the most important properties of your rocket is the position of its *centre of mass*, sometimes called its *centre of gravity*.

Estimating the position of the Centre of Mass

Since your rocket will spend most of its flight without any water in it, this makes it easy to find its the centre of mass by simply tying a string around the rocket and moving the suspension point along the rocket until you find the balance point. The further forward this balance point, the more likely it is that your rocket will be stable in flight.

Below: Finding the centre of mass of a rocket by suspending it from a thread.



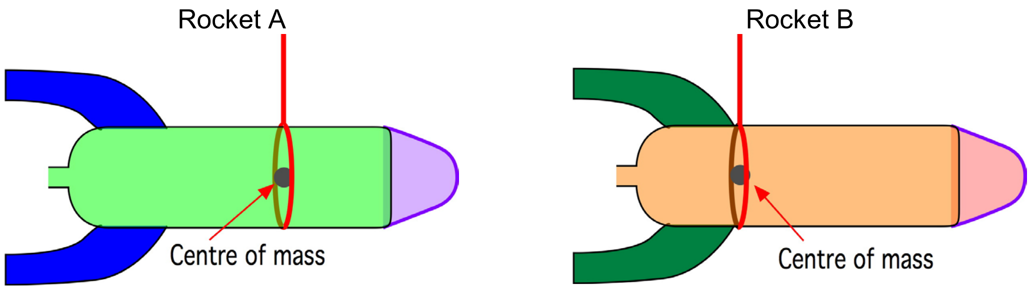


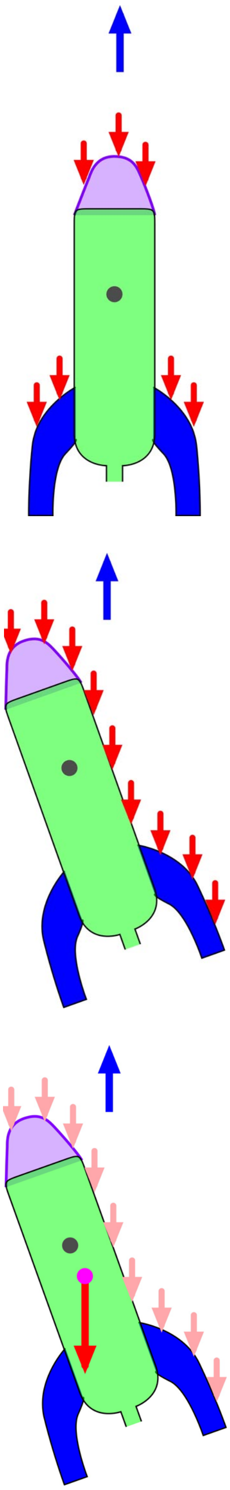
Fins

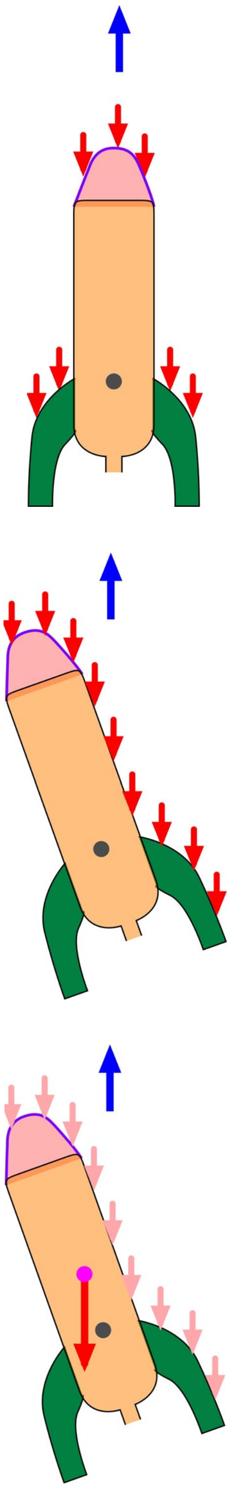
The fins on a rocket provide a mechanism by which aerodynamically stable flight can be ensured. To understand the role of the fins, it is necessary to consider the forces on a rocket when it becomes slightly misaligned in flight. If these forces act to increase the degree of misalignment, then the rocket will not fly well. If these forces act to decrease the degree of misalignment, then the rocket will fly… like a rocket! We'll see how the fins help to achieve stability in the next section.

Aerodynamic stability

To understand aerodynamic stability we need to consider the forces which act on the rocket both when it is flying correctly, and also when it is misaligned. Let's consider two different rockets (let's call them *Rocket A* and *Rocket B*) which are the same shape and have the same fins, but which have different weight distributions and so have their centres of mass positioned at different places. In particular let's assume that Rocket B has its centre of mass much further back than in Rocket A.







Now let's think about the forces when the rocket is travelling in the direction of the blue arrow.

The main drag forces act on all the surfaces exposed to air moving past the rocket. For a typical rocket oriented 'correctly', these forces act mainly on the nose cone, because the fins are usually very thin and expose very little cross section to the air through which they move.

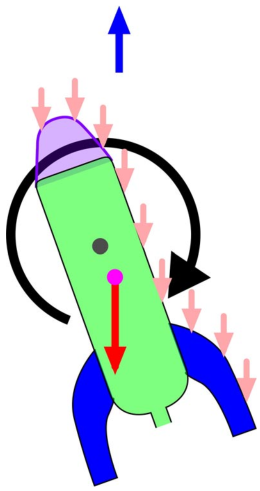
Now consider what would happen if the rocket became slightly misaligned. In this case much more of the rocket would be exposed, and the drag forces would increase significantly.

The forces would act:

- on the nose of the rocket,
- along the exposed side of the rocket,
- and on the fins.

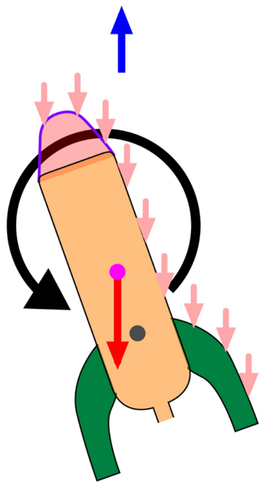
The forces along each portion of the rocket are difficult to calculate or measure precisely, but there will be some point on the rocket which is their effective point of action. This point is known as the *centre of pressure*, and is marked with a purple dot in the figures right and left.

Because the shapes of the two rockets are the same, the *centre of pressure* lies in the same place. But because the *centre of mass* occurs in different places on each rocket, the effect of the same drag forces on each rocket is quite different.



For *Rocket A* (left) the centre of mass lies further forwards along the rocket axis than the centre of pressure. The extra drag forces therefore act more on the back end of the rocket and tend to 'push it back into line'. Technically we say the drag forces exert a torque which acts about the centre of mass to restore optimal flight attitude.

For *Rocket B* (right) the centre of mass lies further backwards along the rocket axis than the centre of pressure. The extra drag forces therefore act more on the front end of the rocket and tend to 'push it even further out of line'.



So it is the relative positions of the *centre of mass* and the *centre of pressure* that determines whether a rocket is aerodynamically stable (like *Rocket A*) or unstable (like *Rocket B*). We saw in a previous section how to determine the position of the centre of mass, but how do we determine the position of the centre of pressure?

Estimating the position of the Centre of Pressure

Estimating the position of the centre of pressure turns out to be rather hard to do *accurately*, but there is a simple technique which you can use to make a rough estimate of its position. This involves making a flat 'silhouette' of your rocket. To understand why this is relevant look at the photographs below which show what the rocket would like if it became misaligned in flight.

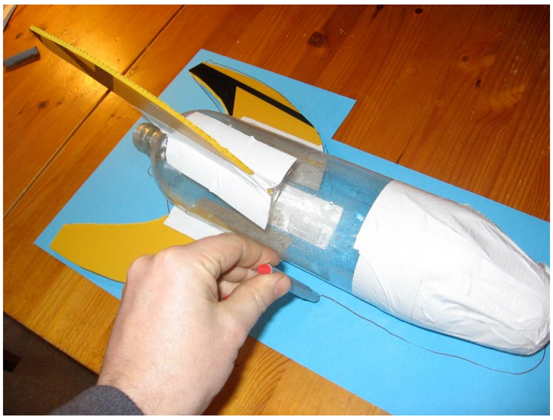
Left: A photograph of the 'Flying Gherkin' from directly above its nose cone: this is what you would see if the rocket were flying directly towards you.

Right: Exposed surfaces of the rocket: The circle shows the area of the rocket exposed to the oncoming air. The fins are rather thin and move easily through the air.

Left: This picture shows the 'Flying Gherkin' slightly misaligned: this is what you would see if the rocket were flying directly towards you, but its back end had swung around slightly.

Right: In this attitude, additional surfaces (outlined and shaded) are exposed to the oncoming air. Some of these surfaces are on the side of the rocket and some are on the fins.

The silhouette technique considers what would happen if for some reason your rocket were flying through the air sideways. This is obviously a more extreme scenario than the misalignments considered above, but let's follow the logic through. If this were happening then the surfaces of the rocket exposed to oncoming air would not form a circle (as when the rocket is correctly oriented) but rather would look like a silhouette of the entire rocket. The position of the centre of pressure of the rocket can be estimated making a silhouette (or cut out) of the rocket, and then estimating the centre of mass *of the cut-out*.



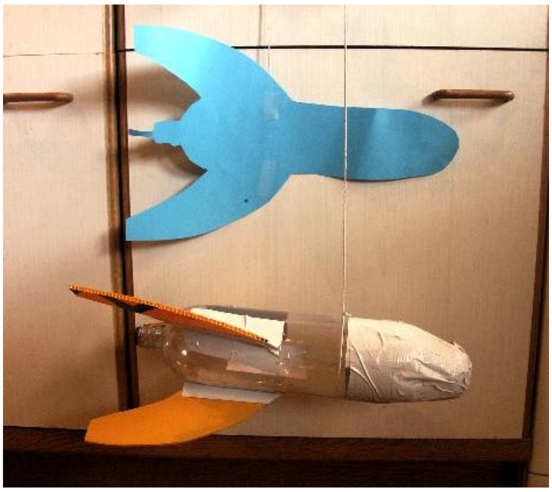


Illustration of the silhouette technique for estimating the centre of pressure.

Above Left: Drawing around the rocket.

Above Right: Silhouette (cut out)of the rocket .

Right. Assessing the centre of mass of the rocket and its silhouette together. We estimate the *centre of pressure of the rocket* to be in roughly the same relative position as the *centre of mass of the silhouette*. Notice that the rocket design with its large light, fins projecting back from the body of the rocket help to keep the centre of pressure towards the rear of the rocket. Also, extra weight in the nose of the rocket (a tennis ball) helps to keep centre of mass towards the front of the rocket.

As the photograph above shows, the centre of mass *of the silhouette* is much further back along the rocket body than the centre of mass *of the rocket itself*. To the extent that the centre of mass *of the silhouette* really is a good estimator for the centre of pressure *of the rocket*, we can see immediately that *The Flying Gherkin* is aerodynamically stable. If the *Flying Gherkin* were flying sideways, then the air pressure would cause an effective force to act at the centre of pressure. Since the centre of pressure lies further back along the rocket than the centre of mass, the air pressure causes the rear of the rocket to be pushed backwards, and the nose of the rocket to swing forward, restoring the correct flight attitude.

Tip: If your rocket is bigger than a sheet or two of A4 paper, then rather than drawing around your rocket, you may find it easier to make a scale drawing of your rocket.

Drag

As the water leaves the rocket's nozzle, it pushes the rocket forward. But this acceleration is decreased because the rocket needs to push air out of the way. The force required to push air out of the way is known as *aerodynamic drag*, and without specialist facilities, it is rather difficult to measure.

Travelling at just a few metres per second we are hardly aware of drag, but at higher speeds, drag dominates the motion of projectiles. For the rocket-shaped projectiles we are interested in, drag forces become significant above approximately 10 metres per second. Just after launch, a water rocket might reach a maximum speed of 20 metres per second, and a high-pressure rocket might reach 40 metres per second. At speeds such as this it is essential to create a design with low drag. Assuming your design is basically rocket shaped (pointy-nose, long body, fins) then you can minimise the drag by considering the following points.

Nose: This nose needs to be :
- Cone shaped, but there is no need to make it excessively pointy. In fact, from a safety point of view this is really quite undesirable.
- Weight may need to be placed in the nose. I am fond of using tape around a tennis ball, but other designs use plasticine stuffed into a cardboard or plastic nose cone.

Body: The body needs to be:
- As smooth as possible.
- For a given rocket volume, long thin rockets tend to have lower drag than short fat rockets.

Fins: The fins need to be:
- Thin and light
- Arranged symmetrically around the body of the rocket: usually there are three or four of them.
- Positioned as far back along the rocket as possible

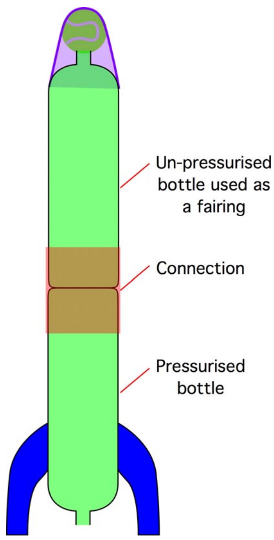
Fairings

A fairing is defined in my online dictionary as:

fair·ing1 n

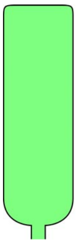
`a streamlined structure added to an aircraft, car, or other vehicle to reduce drag. See also` *`cowling`*

Fairings are used in two quite different ways. The first technique is used when joining two bottles together: a mid-section of a bottle is cut out and placed around the joint for strength and streamlining (see Figure on page 22). The second technique (see right) adds whole bottles or parts of bottles onto the water rocket 'engine' to produce a longer rocket. This is a useful way of moving the centre of mass forward along the rocket to improve stability.

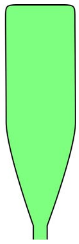


Nozzle

The nozzle is the 'transducer' which converts the energy of the expanding air into linear momentum of the water exiting the back of the rocket. The *efficiency* of the nozzle measures the extent to which energy is wasted in this process. Losses can occur due to friction, viscosity, and off-axis acceleration of the water. You have rather little control over the first two of these properties, but your design can affect the third process.



Consider the two bottles shown left and right: the left-hand figure represents the standard fizzy drinks bottle, and gives acceptable nozzle performance. However, the style of bottle shown right offers improved nozzle performance. Its gently sloping 'shoulders' guide water in just the right direction and little energy is wasted in pushing the water 'sideways' towards the nozzle. Unfortunately this style seems to be only available in one litre sizes.



One further parameter may be used to describe a nozzle: its *flow impedance*. This is mainly determined by the minimum cross-sectional area: the larger the area, the lower the impedance, and the quicker water comes out; the smaller the area, the higher the impedance, and the slower water comes out. In the limiting case of a very high impedance, water will simply trickle out the bottle, and it will not leave the ground.

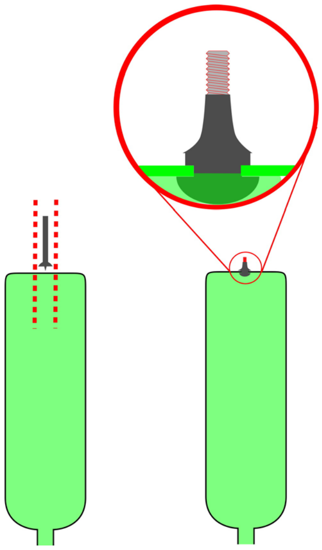
<u>Multibottle rockets</u>

Joining two or more bottles together to make a pressure-tight joint is simple in principle, but tricky in practice. The basic technique is to find a component (typically a plumbing connector or valve) which will mechanically join the bottles, and then to seal the component in place. Let's look at a couple of similar techniques.

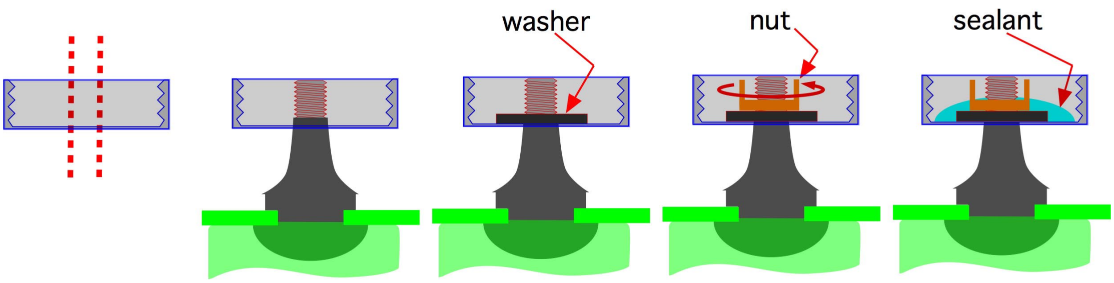
In the first technique one begins by drilling a hole in the centre of the bottom of a bottle. The hole should be around 12.5 mm in diameter (a half-inch drill will be fine). A wood drill will generally give a better result than a standard high-speed steel drill.

Now one inserts a car tyre valve into the bottom of the bottle. This is the same kind of valve that Jaco and Dave used to make their pumping valve and is illustrated on Page 14

Before inserting the valve into the bottle, the insides of the valve need to be removed with needle-nosed pliers or similar. This should result in a straight hole through the centre of valve with a diameter of roughly 3 mm.



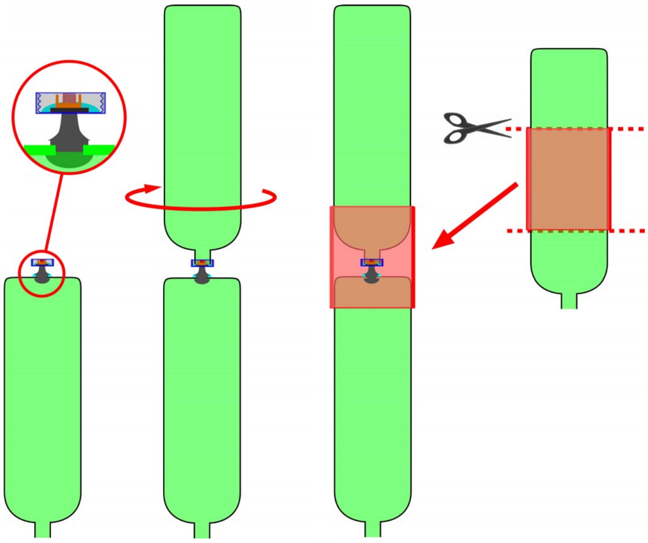
Now we need to attach another bottle to this valve. To do this we begin with a standard bottle cap and drill a hole which will allow the stem of the tyre valve to protrude as shown below. We now have to make the connection pressure tight.



To do this one first places a rubber washer over the valve stem, and squeezes it tightly into place with a nut made from a cut down cap for a tyre valve. One then seals the entire connection with silicone sealant and allows the assembly to set for 24 hours.

The arrangement now looks like the leftmost figure in the set right. One can now screw a second bottle into the first bottle to make a pressure tight seal. The resulting joint should be pressure tight, but it is not very aerodynamic, and will also be rather fragile and flexible.

The last step is to add a fairing to reduce drag. One clever way to do this is to cut out the central part of yet another bottle and slip it over the combined bottles.

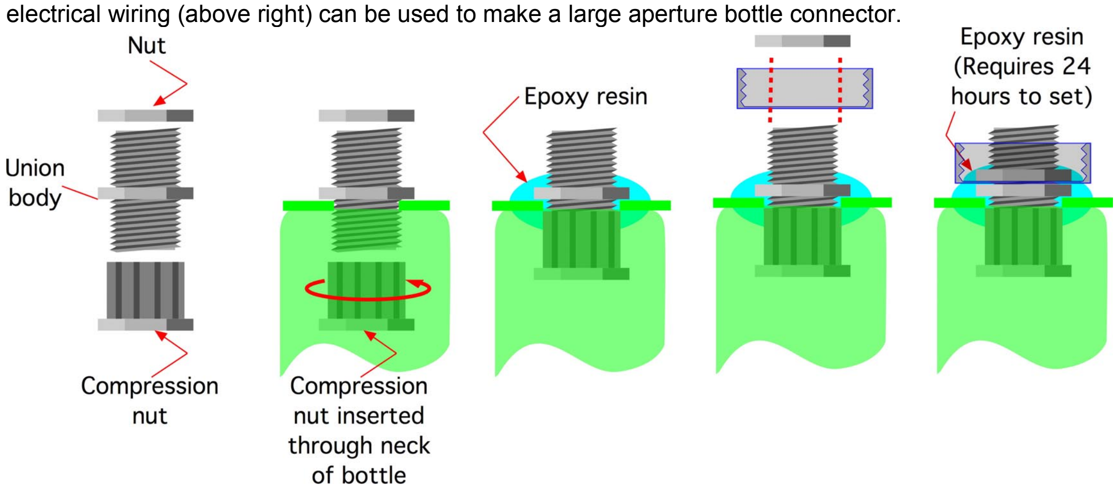


To do this you will find it convenient to slightly reduce the pressure in the combined bottles by sucking on them. The combined bottles will crumple a little, but this does not seem to affect their strength *as long as the plastic is not creased*. When re-inflated, the fairing will be held firmly in place.

This process can then be repeated to make rockets as large as you have the patience to create. Please note, however, that since the energy stored in a large rocket can be considerable when pressurised, you should be sure to observe the pressure safety precautions (Section 8).

Joining bottles together. **Left**: Bottle cap attached to the bottom of a bottle. **Centre**: Two bottles joined together (the fairing is not yet in place) **Far Right**: There are many other ways of joining bottles together using cable entry grommets or similar. Small hands are an advantage for this fiddly work.

Alternative technique: Using similar principles to those described above, a cable entry grommet for electrical wiring (above right) can be used to make a large aperture bottle connector.

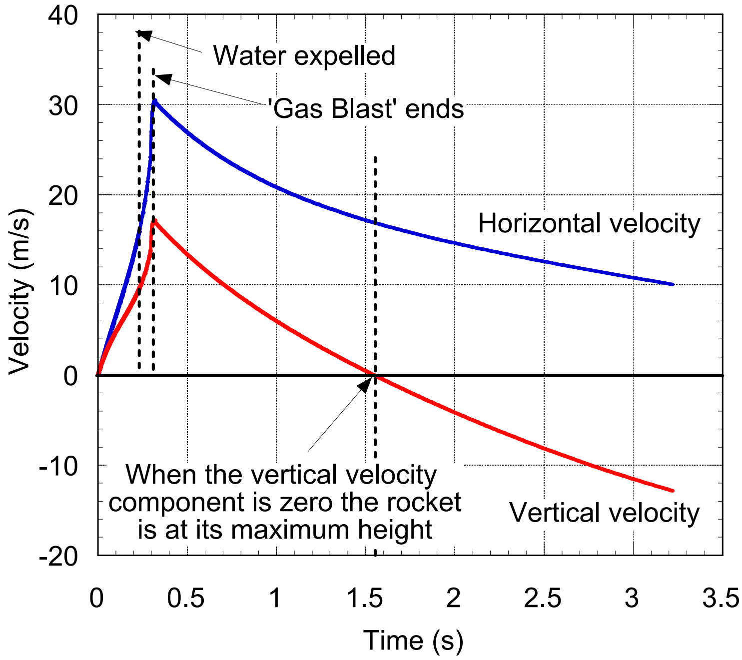


Parachutes

I am not in a position to tell you how to get a parachute to deploy correctly, because I have never managed to do it myself! Also, when I've asked people who have done it what the secret is, they have been somewhat… secretive! But I can tell you why it is so difficult, and I can pass on one or two hints given by some of the less secretive rocketeers.

Generally its considered that the ideal time for deployment is when the rocket reaches its maximum height (*apogee*). The problem is to tell when this occurs.

The Figure (right) shows the calculated speed of a standard rocket as a function of time. From this graph it is possible to see why detecting the apogee is a problem. After the acceleration phase, both the horizontal and vertical components of the rocket velocity gently decrease. At the apogee, there is no change in the forces acting on the rocket, and so it is hard to detect.



In order to arrange for the parachute to deploy correctly, I have seen only two generic types of mechanism. The first detects the height of the rocket, and the second activates at a pre-determined time. Another possibility would be to have a mechanism based on the orientation of the rocket, but I have never seen this implemented.

The first height-detecting mechanism I have seen consisted of a piece of thread attached to the ground: it broke during the rapid acceleration phase.

The second height-detecting mechanism consisted of a rocketeer with a radio-control. The parachute was stowed in the nose of the rocket beneath a cover which was held in place by nylon fishing line. The radio control activated a heater that melted the fishing line, causing elastic bands to pull off the cover, allowing the parachute to deploy. This ingenious mechanism worked perfectly. Most of the time. This type of deployment mechanism could also be used with a timer.

The mechanisms based on timing involved a tiny clockwork timer (See Section 9) which could be set to activate after periods of just a few seconds or so. Arranging for the timer to be started on launch, and then deploy correctly proved very tricky. When the mechanism worked, it worked perfectly, but frequently it deployed the parachute on launch, or failed to deploy it all.

Section 5: Testing your Rocket

The way you test your rocket is what distinguishes those who want to just have bit of fun (which is great in itself) and those who want to understand and improve their design (which the first step on the road to being a successful engineer). At the heart of this test process is *measurement*. You need to:

- *measure* the properties of rocket before launch, and then
- *measure* the performance of rocket.

You then need to use your understanding of the launch process and flight dynamics to try to work out which launch properties most significantly affect the performance of the rocket.

Rocket Properties

Weight of Empty Rocket: This is (pretty obviously) the weight of the rocket without the water in it. This will be the part of the rocket which makes the whole journey. Using electronic kitchen scales it is not too difficult to measure to the nearest gramme, which is more than accurate enough for our purposes.

Total Volume: If you have just a single bottle design, the total volume is likely to be very close to the volume stated on the label. If you have constructed a multi-bottle rocket, then you probably need to measure this. The easiest way is to weigh the rocket empty (see above) and then weigh it full of water. Each gramme of excess weight corresponds to 1 cubic centimetre of water. Or each kilogramme corresponds to 1 litre of water.

Water Volume: This is something you can easily customise and which makes a big difference to the performance. A good starting point is generally to fill with about one quarter water. The optimum filling depends on a number of factors, but is generally in the range from 20% to 30%. One thing you can do before you leave home, is to mark the side of the rocket with tape to show where (say) the 20% or 25% mark is: remember you will generally be filling the rocket when it is upside down so this mark will generally be in a non-obvious position.

Launch Angle: If the rocket were an un-powered projectile with no aerodynamic drag, then the angle to give the greatest range would be 45°. However, this is not the case for a water rocket, although the optimum angle is unlikely to be very far from 45°. My feeling is that launching slightly more vertically than this gives the best range, but you should check this for your rocket.

Launch Pressure: Increasing the pressure increases the stored energy at launch, which increases the maximum speed attained by the rocket, and this increases the launch range, flight time, and maximum height. However, you will find that increasing the launch pressure by a given amount (a) becomes harder to do and (b) makes less and less difference. The reason is aerodynamic drag which increase very rapidly with increasing launch speed, and 'steals' all the kinetic energy imparted to the rocket. If you have a launch pressure of 5 atmospheres (75 psi) and are still looking for improvements, then its better to try reducing drag rather than increasing the pressure further.

Other features: You need to note how you have set the fins, or whether you are trying any other interesting alterations either to the rocket or to the launcher.

Rocket Performance

Ground Range: This is the distance between the launch point and the point where the rocket hits the ground. When you begin rocketeering, increasing the range is simplest and most obvious measure of success, but as you get better, you will find that increasing the range is not so obviously a good thing. The first problem is that once the range becomes long (beyond 100 m or so) it takes a long time to measure the range and retrieve your rocket. Secondly, depending on the kind of space you have available to launch into, it becomes increasingly difficult to ensure that your rocket will not hit a passer by, or leave the field or park where you are practicing. At the extremes of the range — over 200 m — it simply becomes impossible to find anywhere to safely launch!

The best way to measure the range is probably with a wheel-based odometer.

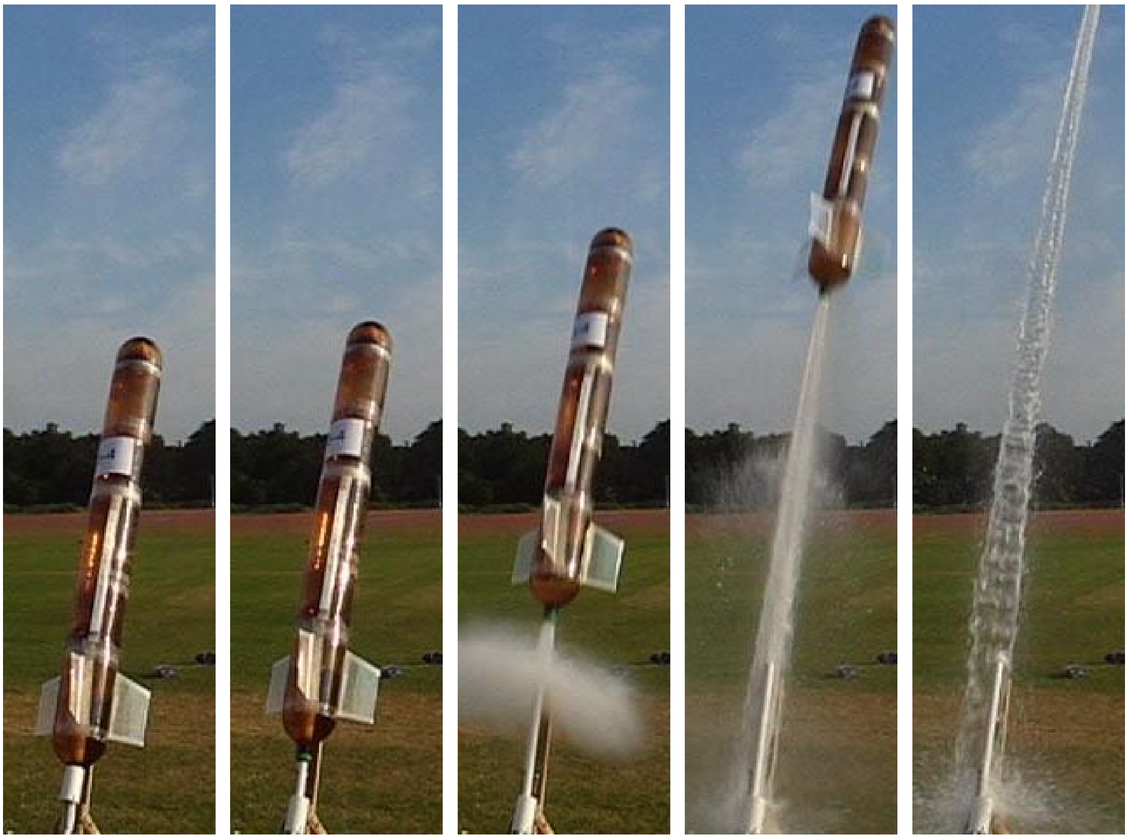
However, for most practical purposes, counting an adult's purposeful strides to the landing point will suffice for comparative measurement. If you want to go one step better than this (no pun intended), the adult can calibrate their stride by walking 10 purposeful paces, and measuring the distance travelled. From this you can work out the actual length of each stride in metres.

Height: This is a really interesting property of the rocket's trajectory, but unfortunately one that is very hard to measure. If you really want to know how high the rocket goes, then I would recommend a straight up launch (see Page 3 for warning!) with a long length of sewing thread attached to the tail of the rocket. The thread should be laid out on the ground, so that as the rocket increases in altitude it can lift the thread off the ground. If you want to measure the height for a non-vertical launch, then (aside from complicated triangulation from analysis of multiple video films) the only way I know of is to use an altitude data logger available from model aircraft shops (See Section 9).

Time in the air: The length of time spent in the air is a good measure of rocket performance, and is probably best measured with a sports stopwatch. With a little practice you should be able to measure this to the nearest tenth of a second or so. If your rocket has no parachute, then your flight times are likely to be under 10 seconds, but if you use a parachute, then flight times could be longer than a minute.

Launch Velocity: One very useful addition to your measurement armoury is the advent of affordable digital cameras which will take video clips. Typically the cameras can record at either 10 or 15 frames per second. By analysing video footage frame by frame it is possible to make estimates of previously un-measurable properties such as launch velocity. These measurements can be considerably improved by placing a metre rule or other object of known size in the field of view close to the rocket trajectory. It will also help to place the camera on a tripod so that there is no shake at the moment of launch.

To analyse a movie frame by frame, the files produced by the movie can be viewed with the *Quicktime*™ video player downloadable from the web (Section 9). The figure overleaf shows five frames extracted from a movie (*.avi* format) from a typical digital still camera operating in 'movie mode'.



$t = 0$ $t = 0.066$ s $t = 0.133$ s $t = 0.200$ s $t = 0.266$ s

Between the second and third frame, the rocket travels approximately 1 bottle length (roughly 0.4 metres) in $1/15^{th}$ of a second, which corresponds to a speed of approximately 6 metres per second. Between the third and fourth frame, the rocket travels approximately 3 bottle lengths (roughly 1.2 metres) in $1/15^{th}$ of a second, which corresponds to a speed of approximately 18 metres per second (40 miles per hour). The increase in speed from 6 metres per second to 18 metres per second takes only $1/15^{th}$ of a second, which corresponds to an acceleration of $12 \times 15 = 180$ metres per second per second. Colloquially, this is around 18*g*, where *g* is the acceleration due to gravity at the Earth's surface.

Aside from quantitative results, viewing the movies is also instructive in showing how the water leaves the rocket. In this case it is clear that the water really does move mainly backwards along the axis of the rocket indicating a satisfactory nozzle design.

Testing !	Before you begin testing, please read Section 8 on Safety. Water rockets are on the whole pretty safe, but the potential exists for a nasty accident that will take all the pleasure out of the endeavour. So for your own sake, and others, *follow the safety guidelines*.

Tips

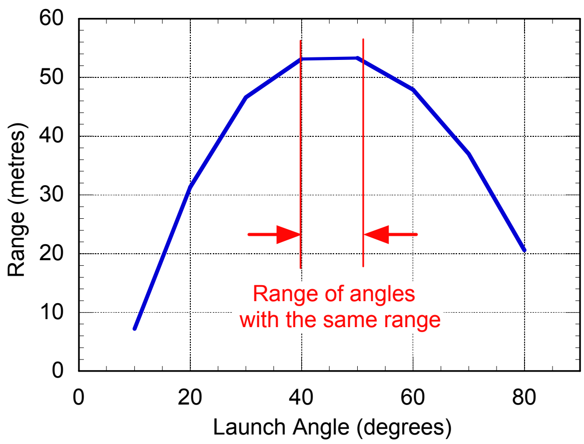
- Bring enough water: a barrel used for brewing beer is helpful, having a tap at the bottom. Plastic hose, funnels, and measuring cylinders are all likely to come in handy.
- Try to get into the habit of recording what you do as you do it. Its amazing how the process of simply writing down 'what I tried: what happened' can help to clarify what may seem confusing results.
- Enter results on a laptop, or use a sheet such as the one on Page 28.

- Try to use the computer model (Section 7) to estimate some of the relevant parameters, and to predict what you think will happen.
- Don't worry too much about precise measurements. Estimating most quantities to within 5% to 10% is generally sufficient to gain a good understanding.

Ideas for experiments to try

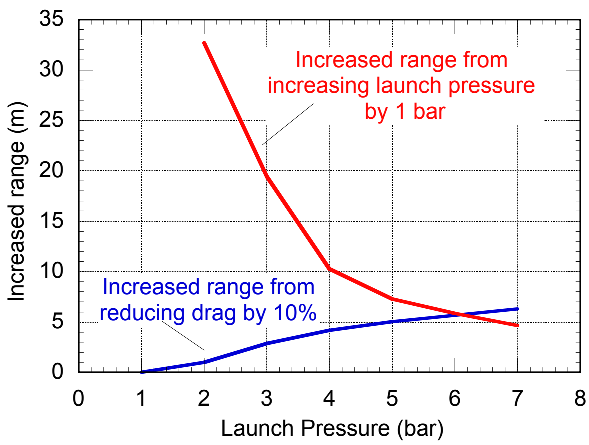
- **Try doing the same thing three times**. This will allow you to assess the reproducibility of your rocket's performance. If you can't get your rocket to do roughly the same thing when you launch it in the same circumstances, then you are not going to be able really optimise its performance in any meaningful way.
- **Try launching with no water**: This is a nice demonstration of the principle of rocket propulsion. The rocket will still fly, but if you then add even a small amount of water (perhaps just 5% filling of the rocket), you should see a dramatic effect on the rocket's performance.
- **Launch in teams of at least two**. Its good to talk about what's happening as you launch and to explain your ideas, and one person can act as Safety Marshal or timer as the other launches.
- **Try changing the launch angle and recording the range**. You should find a range of angles (probably close to 45°) where the range is insensitive to the precise launch angle.

  The graph right shows typical results from the water rocket simulator. It shows that changing the launch angle by ±5° around 45°, makes very little difference to the range. This makes this angle setting good for testing the dependence of range on other rocket parameters, such as launch pressure.



- **Try changing the launch pressure and recording the range**. You should find that increasing the launch pressure always increases the range, but by smaller and smaller amounts.

  The graph right shows typical results from the water rocket simulator. It shows that at high pressures, reducing the drag factor of the rocket has an increasing benefit rather than simply using the 'brute force' higher pressure approach.



Water Rocket Test Sheet

Date	Time	Rocket Identifier

Launch Parameters

Launch Number			
Rocket Mass			
Rocket Volume			
Filling Factor			
Launch Mass			
Launch Angle			
Launch Pressure			

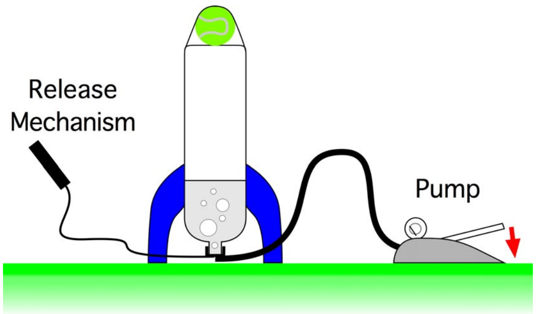
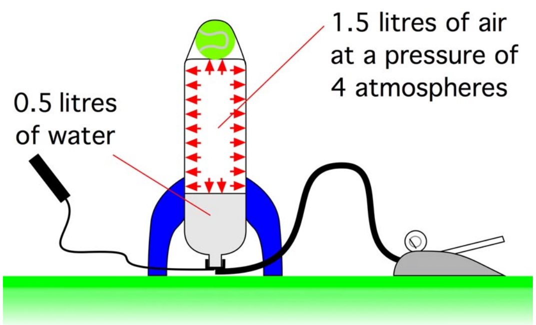
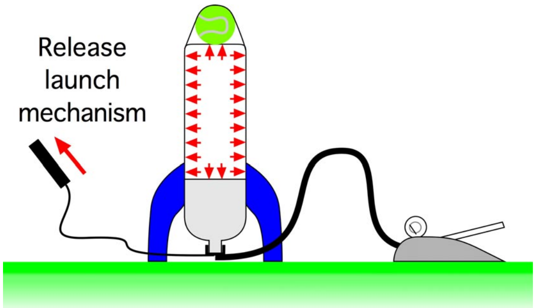
Launch Results

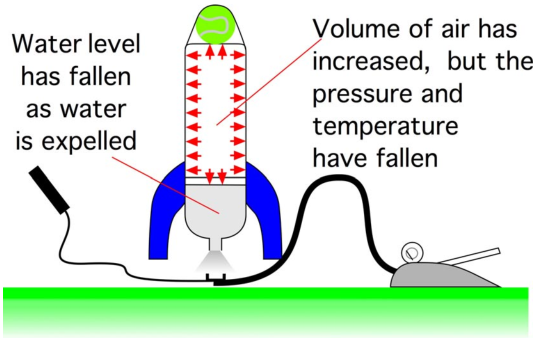
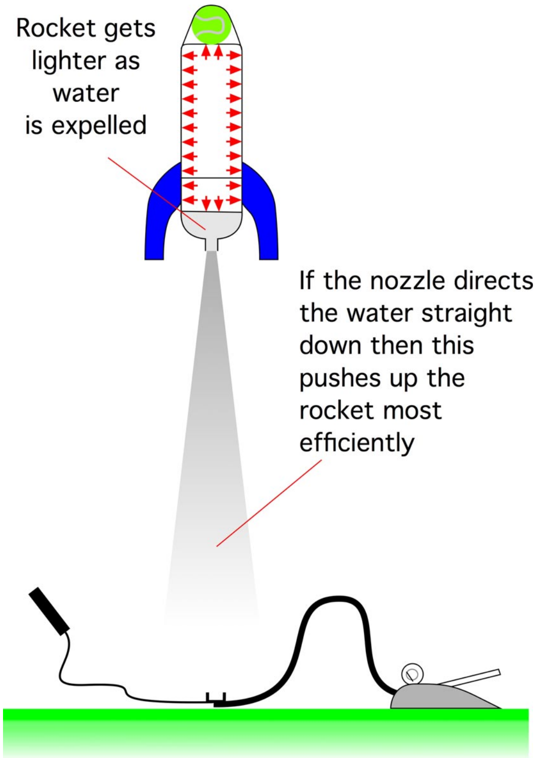
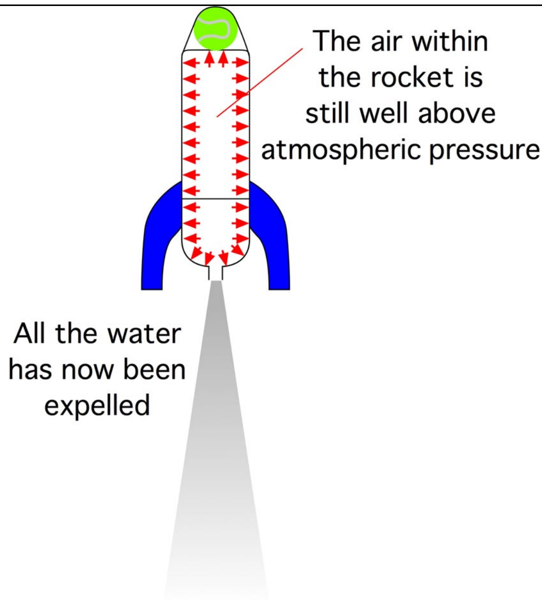
Range			
Time			
Height			
Video file identifier			
Other			

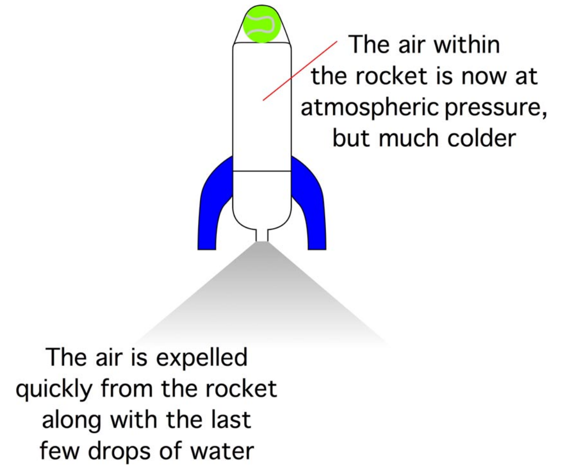
Other Notes

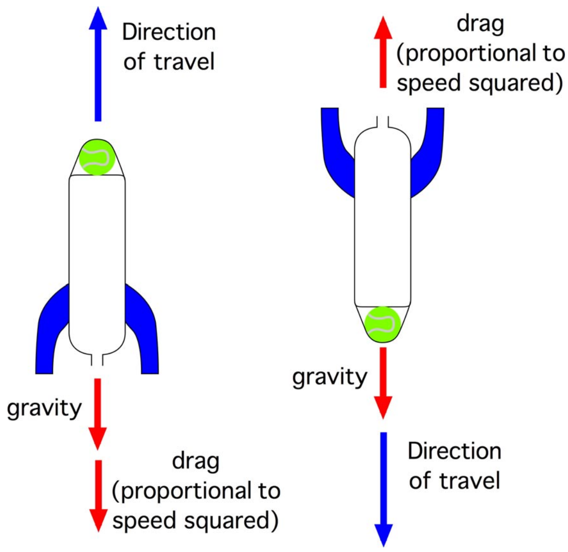
Section 6: Physics of a water rocket

This section is for people who want to understand in general terms what happens during a rocket launch. The left hand column has the time before or after launch in seconds; the middle column shows 'what's happening'; and the right hand column contains my commentary. We assume a vertical launch of a two-litre rocket weighing 100 grammes when empty, and quarter filled with water at launch. The speeds and heights quoted are those derived from the water rocket simulator software described in Section 7. Further details of the calculations can be found in Section 10.

Time	What's happening?	Comments
– 60 s		The rocket is filled, and then placed on its stand. Everyone nearby is warned that the rocket is about to be pressurised, and then pumping commences. As the air is compressed it gets hot, and you should be able to feel this near the exit of the pump. However, as the air bubbles through the water it cools down again, so the air in your water rocket should be close to the temperature of the water
– 30 s		Your chosen launch pressure is reached. To be specific, we'll assume that the gauge on the foot pump you have used reads 3 atmospheres. Since the pressure before you started was one atmosphere the actual pressure of the air in the bottle is now 4 atmospheres. 1 atmosphere – sometimes called 1 bar, is roughly equivalent to: • 15 pounds per square inch (psi) • 2 kilograms per square centimetre • 100000 pascal (Pa) The pressurised gas is the energy source for the rocket. A good rocket design will convert the maximum amount of stored energy into kinetic energy of the rocket.
– 5 s	5...4…3…2…1… 	The final launch warning is given and if everything is safe, the launch mechanism is released. Just before launch the force on the nozzle is very large. At this point the 500 ml of water weighing 500 g (0.5 kg) makes up most of the mass of the rocket.

Time	What's happening?	Comments
+ 0.01 s		As the catch is released, the gas pushes the water out through the nozzle, and the rocket begins to lift off. The pressure of the air begins to fall, and also its temperature drops as the air expands. After 10 milliseconds the speed is still quite slow (around 1 metre per second) because the rocket still has a heavy load of water on board. The rocket has moved less than a centimetre.
+0.1 s		Just under half the water has now left the rocket. The rocket has reached a height of around 0.5 metres and is travelling upwards at approximately 10 metres per second. This corresponds to very large acceleration of around 100 metres per second per second, or roughly 10*g*. If the nozzle causes water to be sprayed sideways, then this adds nothing to the lift forces. Frame by frame analysis of a video (Page 26) should show a 'tube of water' trailing behind the accelerating rocket.
+0.22 s		Amazingly all the water has now left the rocket. If you could just imagine emptying 500 ml of water out of a bottle it might easily take 10 seconds. It has now all gone in just under a quarter of a second! The rocket is now travelling at around 26 metres per second. The pressure has fallen to 2.4 bar from its initial value of 4 bar. As the air has expanded it cools and is now at approximately –19 °C. But it is still pressurised. However now that the exit is not blocked by water, the air finds it considerably easier to leave the rocket than the water did.

Time	What's happening?	Comments
+0.25 s		After another 30 milliseconds, the pressurised air has left the rocket giving the rocket a last boost. This boost can have quite a considerable effect because the rocket is now much lighter than it was on launch. At the end of this phase, the rocket is moving at its maximum speed of around 35 metres per second. The rocket now enters the 'cruise' or 'ballistic' phase of its flight.

+ 0.25 s to + 5.4 s		In this phase of its flight, the only forces acting on the rocket are gravity, and the aerodynamic drag force. Gravity always acts vertically downward on the rocket, and limits the maximum height to around 34 metres. The rocket then falls, striking the ground after around 5.4 seconds at a speed of approximately 20 metres per second If the rocket is well designed, the aerodynamic drag always acts to oppose the direction of motion, so the drag acts downwards as the rocket ascends, and upwards, as the rocket descends. If the rocket is not so well designed, the aerodynamic forces can act on the rocket in other directions and cause it to tumble. If your rocket does this, look at the Section 4 on aerodynamic stability.

Section 7: Computer Simulation

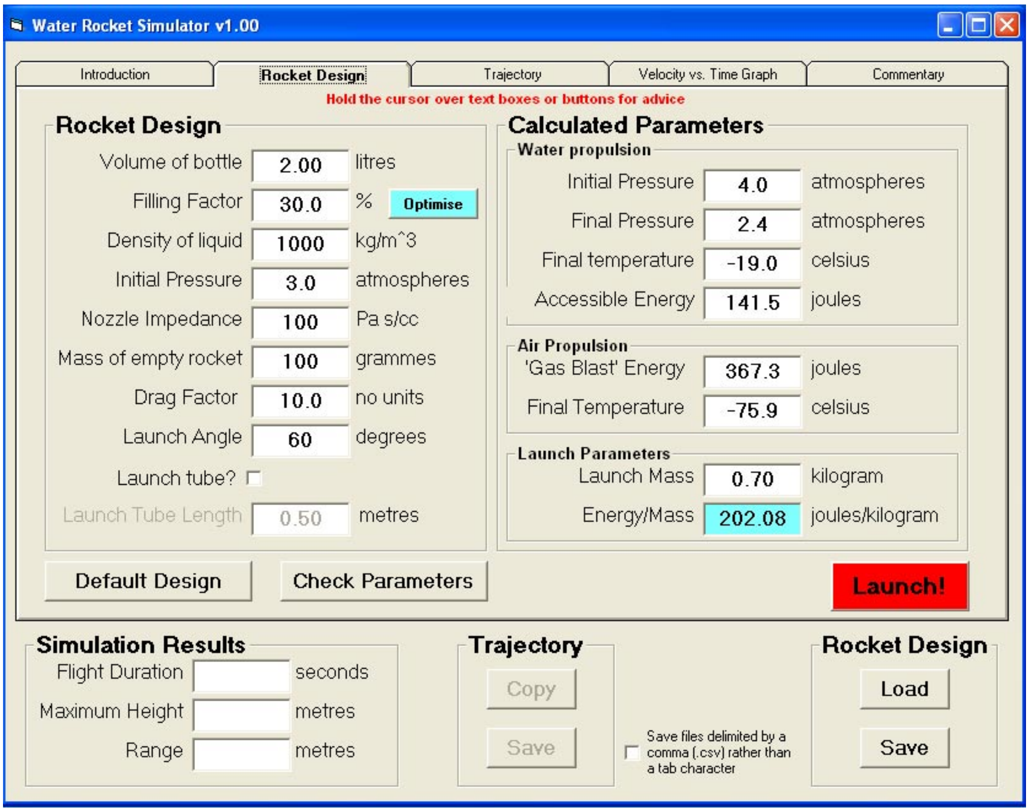
Water rocket simulation software has been written to operate under the *Windows*™ operating system. The software can be downloaded from the NPL water rocket site at:

www.npl.co.uk/waterrockets

Disclaimer. This software has ***not*** been developed under NPL quality procedures and is not warranted for any use whatsoever. Got that? I can't be clearer. The software comes with no guarantee that it will do anything at all. That said, we believe that it is pretty *Good for Nothing* ™

Introduction

First of all you need to install the software using the standard Windows installer. When you launch the software and you will be faced with a screen similar to the following.



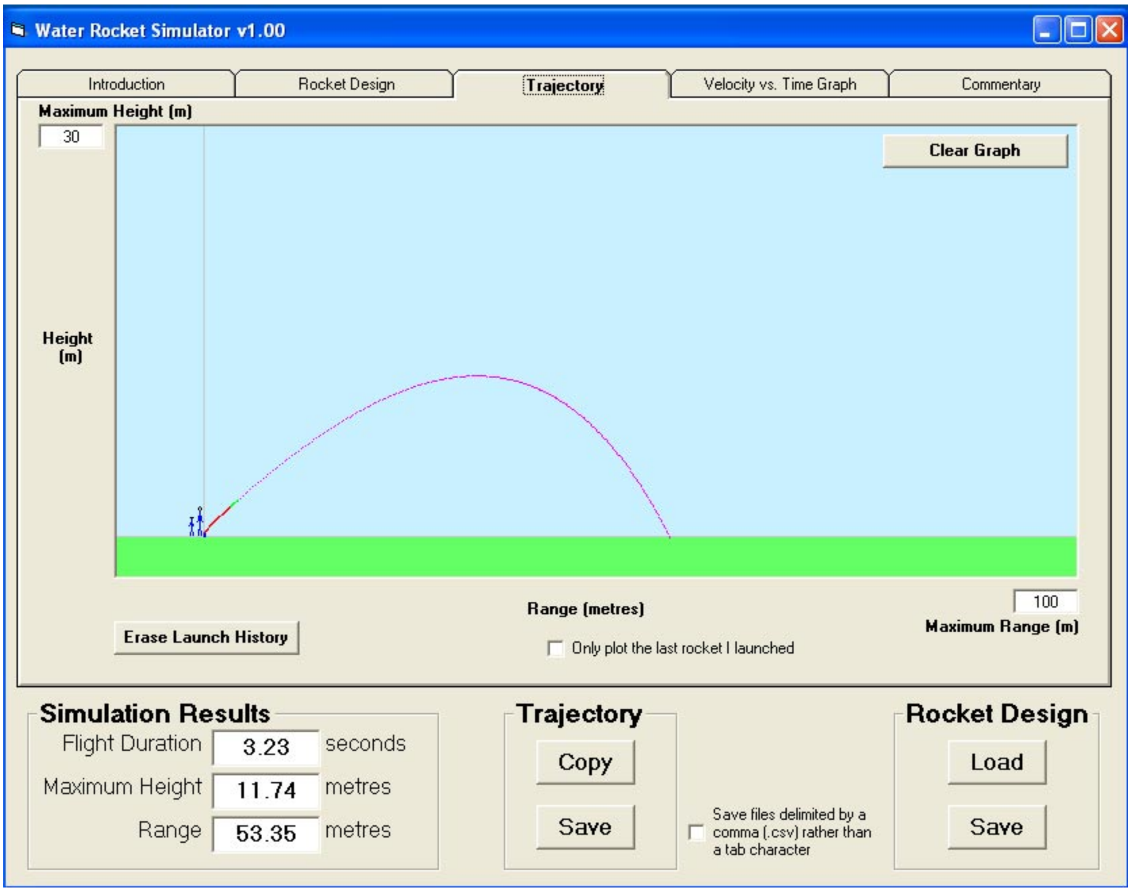
First of all…

You probably feel tempted to click the big red 'Launch' button. Well go ahead! The application will switch screens to show you the calculated trajectory of the rocket with the design parameters on the left-hand side of the window above. The key predictions for the flight duration, maximum height and range can be seen in the lower left of the screen.

The basic idea of the program is that you:

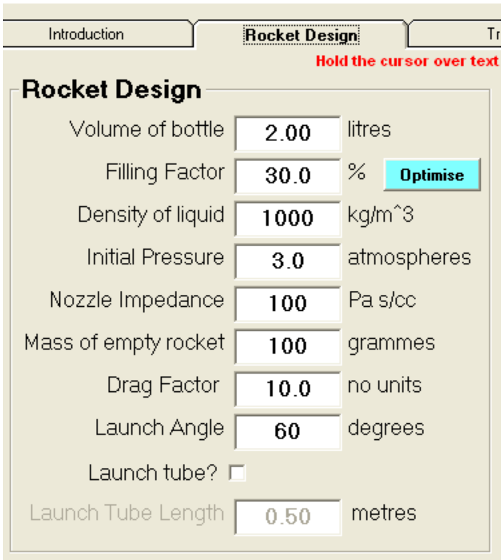
- design a 'virtual' rocket by choosing values for some key parameters;
- launch your 'virtual' rocket by clicking the **launch** button
- look at the results and revise your design.

This cycle of design/experiment/measurement of results/re-design is the fundamental process of engineering. The purpose of the software is to speed up the design cycle, allowing you to focus your real design efforts on those parameters likely to have the most effect on your real rocket.



Rocket Design Tab

The left-hand side of the screen features the parameters that you can control. They are all inside a **Rocket Design** control box. Holding the cursor over a text box will bring up a short help message.



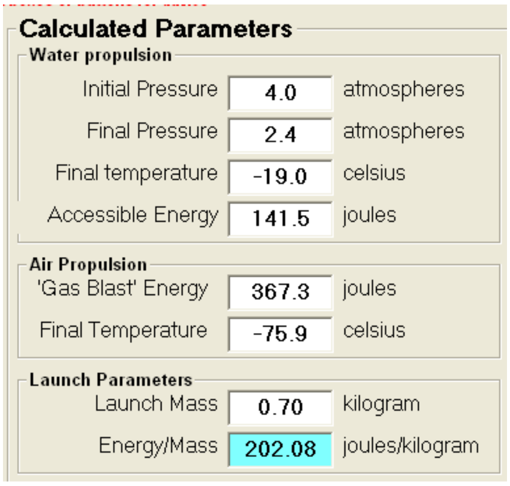
Most of the parameters are easy to estimate, but some are not so easy. In particular, the *Nozzle Impedance* and the *Drag Factor* can be problematic.

The *nozzle impedance* is a number which characterises how many cubic centimetres of water leave the nozzle per second per pascal of pressure difference between the inside and outside of the rocket.

Using typical figures, a pressure of 1 atmosphere (100000 Pa) causes 100 cubic centimetres of water to leave the rocket in about 0.1 seconds, a flow rate of 1000 cubic centimetres per second. So this corresponds to an impedance of 100000/1000 = 100 Pa s/cc. Increasing the nozzle impedance *reduces* the flow of water for a given pressure difference.

The *drag factor* is a number which characterises the magnitude of the drag force on the rocket. On this scale, a tennis ball has a value of 10, and a football has value close to 100. Most rockets will lie in the range between 10 and 100. If you take special care you can achieve lower values, and you will see that if everything else is optimally designed, it is the drag factor which will ultimately limit the performance of the rocket

When you click either the **launch** button or the **check parameters** button the application will calculate some basic parameters that describe the launch, such as the amount of energy stored in the compressed gas at launch

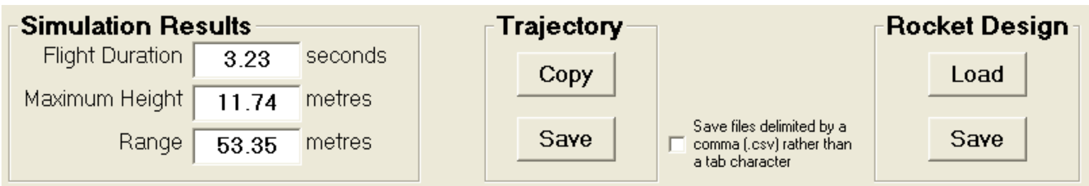


The 'gas blast' refers to the phase of the rocket flight after the water has been ejected during which (the rocket being rather light) can undergo extreme accelerations.

The energy per unit mass field shows the value of the initial energy stored in the gas at launch divided by the launch mass.

The optimise button in the rocket design panel, evaluates the energy per unit mass for all values of filling factor from 1% to 99% and fills in the value which maximises the energy per unit mass at launch

If you have a rocket design you like, you can save the key parameters in a text file (.txt), by clicking the appropriate SAVE button. Saved Rockets can similarly be reloaded for further study by using the LOAD button.



Trajectory

After the LAUNCH button is clicked, the application switches to the trajectory tab. The trajectory of the most recent launch is shown colour coded:

- The portion of the trajectory during which water is being ejected is shown in red
- The 'gas blast' phase of the trajectory during which air is being ejected is shown in green
- The 'ballistic' portion of the trajectory during which the rocket is in free un-propelled flight is shown in purple

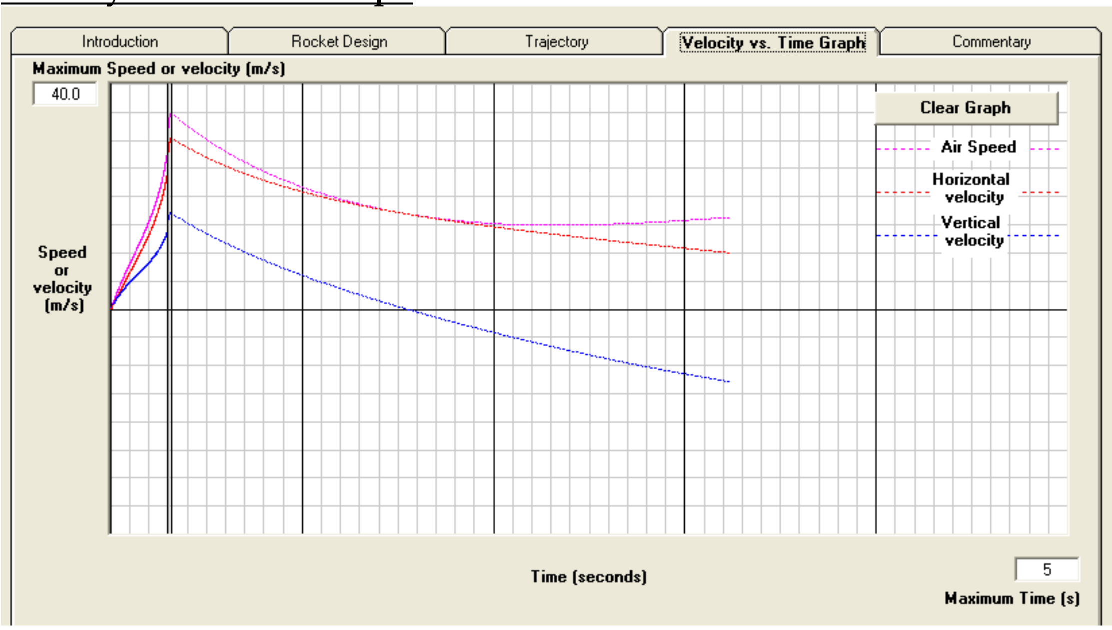
The ten previous launches are shown as blue lines. If you want to always see only on the last calculated trajectory, then check the 'Only plot the last rocket I launched' check box.

If the screen becomes too cluttered then clicking the 'Erase Launch History' will clear the screen.

The maximum values of the range and altitude are preset at values likely to be appropriate. You can change them as you see fit, but you will need to click the launch button again to see the effect.

The calculated trajectory can be saved as a tab delimited data file by clicking the 'SAVE' button in the Trajectory panel. This file can be opened in many applications including MS *Excel* ™ for further analysis. Alternatively, clicking the COPY button places a copy of the data on the clipboard which can then be simply pasted into an open *Excel* ™ spreadsheet.

Velocity versus Time Graph



The ***velocity vs. time*** tab shows a graph of three quantities as a function of time

- **Vertical velocity (blue)** is the speed in the vertical direction. This curve rises, reaches a peak and then declines, falls through zero and reaches negative values, which corresponds to travelling downwards. The point at which the vertical velocity reaches zero is *apogee*, or the point of maximum altitude
- **Horizontal velocity (red)** is the speed in the horizontal direction. This curve rises, reaches a peak and then declines gradually, but in general it does not reach zero
- Air speed is the raw speed of the rocket. It is calculated as:

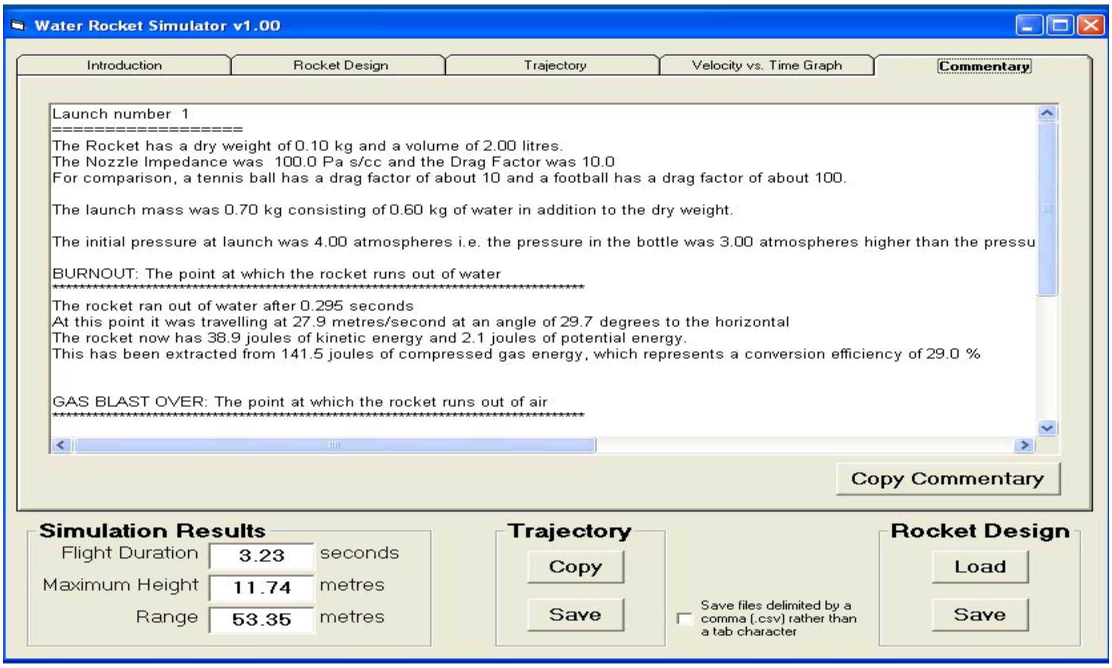
$$Air \;\; speed = \sqrt{(Vertical \;\; velocity)^2 + (Horizontal \;\; velocity)^2}$$

Notice that when the rocket is at apogee, the air speed is equal to the horizontal velocity.

Also marked as vertical lines on the graph are: the time at which the water runs out; the time at which the internal pressure is equal to atmospheric pressure; the passage of each tenth of a second.

The scale of the graph is pre-set to match most flights with a maximum speed of 40 m s^{-1} and a flight time of 5 seconds. If you should find that the curves leave the graph, then you can reset the maximum values of the graph with your own values. After re-scaling, you will need to re-launch your rocket in order to see the curves.

Commentary



The last tab panel summarises the story of your last launch. Clicking the [**Copy Commentary]** button places this text on the clipboard so that it can be pasted into a word processor.

Bugs reports, comments, and suggestions for improvements

There are several known bugs in the water rocket simulator. The effect of these bugs is to introduce inaccuracies in the results of some of the rocket calculations in certain regimes. If you find any bugs that you think I might not know about, please let me know by e-mail at:

michael.depodesta@npl.co.uk

Disclaimer. This software has ***not*** been developed under NPL quality procedures and is not warranted for any use whatsoever. Got that? I can't be clearer. The software comes with no guarantee that it will do anything at all. That said, we believe that it is pretty *Good for Nothing* ™.

Section 8: Safety

Building and launching water rockets is a pretty safe pastime, and I say that as someone who has had a water rocket land on their head more than once. But there are some hazards associated with both the launching of water rockets, and their construction and you should be aware of them. Taking some simple precautions should keep things safe

Sharp knives and blades: any sharp knife or blade presents a potential accident waiting to happen, especially with children present. So when using craft knives or blades:

- Always cut away from your fingers
- When not using the blade, always cover the sharp surface with either the manufactures cover, or failing that a cork, or piece of soft wood.

Rocket design: Do not use any sharp points on either the nose cone or the fins and never use metal fixtures or fittings external to the rocket body.

Pressurised objects and pipes: during launching and testing, pipework and connections will be pressurised, and large forces can be exerted on different parts of your system. Outright failure of component is extremely rare (see below for pressure limits) but it is common for connections to 'creep' while under pressure and then to pop out suddenly.

- When your launch system is pressurised, it should be treated like an unexploded firework. In particular you should keep small children away.
- We recommend that *safety spectacles* and *ear plugs* or *ear defenders* be worn while the launcher is pressurised.

Pressure limits: at times the desire may come upon you to increase the pressure just a little bit more. If you feel tempted, please note the following.

- Use only PET bottles designed for fizzy drinks or carbonated water. Do *not* use PET bottles used (for example) for fruit cordial or milk drinks. These are not safe.
- Aside from leaking connections, the most likely component to fail under pressure is the water rocket itself. The precise pressure at which bottles will explode depends on the bottle design, its history, as well as any of the strange things you may have done to it. Collectively, the rocketeers at NPL feel that if you are using relatively new undamaged bottles, then keeping the pressure below 5 bar (75 psi or 5 kg per square centimetre) will pretty much avoid the risk of explosion.
- If you feel the temptation to increase the pressure above this, then I recommend you re-design your rocket with less drag: this will have the same effect without any additional risk (See page 27 for a graph).

Launch procedure: when launching the rocket you should avoid any possibility that the rocket will hit any living thing. Since the rocket could land up to 100 metres away, this represents something of a challenge in any public access space!

- Please pick your spot carefully: most public parks are not suitable for any but the shortest flights.
- Launch in a team, with one person's job being to ensure safety. They should look out for people wandering into the firing range.
- Begin by firing at low pressures until you become familiar with your launch system. Remember that an accidental launch of the rocket is a real possibility, *whenever the rocket is pressurised*,
- Do not launch with children playing nearby.

Section 9: Useful Information

Where to buy

Please note that mentioning a shop in this section in no way constitutes an endorsement by NPL. This is a list of useful outlets which many people around NPL have used.

Shop	Description	Web SIte
Rokit	Simple water rocket kits	www.rokit.com
Maplin Electronics	Nationwide chain of electronics and hobby shops	www.maplin.co.uk
Free Flight Supplies	Source for small timers and sundry other wonders	www.freeflightsupplies.co.uk
Middlesex University Teaching Resources	Source for *Corriflute*™ corrugated plastic and a host of other materials.	www.mutr.co.uk *Corriflute*™ link: www.mutr.co.uk/prodDetail.aspx?prodID=771
Gordon Tarling	Source of altimeters, and flight data loggers.	www.gordontarling.co.uk
Real Raptors	Source of altimeters, and flight data loggers.	www.realraptors.co.uk
RS	Web & phone service for ordering an astonishingly wide variety of engineering and electronic components	rswww.com
Fun Learning	Educational toys: kind sponsor of the NPL water rocket event	www.brightminds.co.uk
Natural World	Educational toys: kind sponsor of the NPL water rocket event	www.thenaturalworld.com

Pressure Units

Pressure is measured in a number of different units, and even more irritatingly, from a number of different starting pressures(!) so it is worth a word or two about converting between units.

Pressure is the force per unit area exerted on the walls of a container.

The SI unit of pressure is the *pascal* (Pa) and 1 Pa is equal to 1 N m^{-2} (Newton per metre squared). One pascal is a very low pressure, and the pressure of atmospheric air is typically within a few percent of 100,000 Pa.

Pressure gauges found on foot pumps, generally indicate pressure in one of three units:

- pounds per square inch (psi): The pressure caused by the force of gravity acting on a mass of one pound (0.45 kg) spread over an area of 1 square inch (2.54 cm x 2.54 cm). A pressure of 1 psi is approximately 6912 Pa, so atmospheric pressure is roughly 14.5 psi.
- kilograms per centimetre squared ($kg\ cm^{-2}$): The pressure caused by the force of gravity acting on a mass of one kilogram spread over an area of 1 square centimetre. A pressure of 1 $kg\ cm^{-2}$ is approximately 98000 Pa, so atmospheric pressure is roughly 1.02 $kg\ cm^{-2}$.
- bar: standard atmospheric pressure equal to 101325 Pa

Gauge pressure is the pressure indicated on the pressure *gauge* of a typical foot pump, and reads zero when the system is unpressurised i.e. when its actual pressure is equal to one atmosphere. So in order to work out the actual pressure inside a water rocket we need to add one atmosphere's worth of pressure to the pressure indicated on the gauge. So for example:

- If a tyre gauge indicates zero pressure, the pressure is atmospheric pressure, which corresponds to around 100,000Pa.
- If a tyre gauge indicates 2.5 $kg\ cm^{-2}$, the pressure is 2.0 $kg\ cm^{-2}$ above atmospheric pressure, which corresponds to around $100{,}000 + 2.0 \times 98{,}000 = 296{,}000$ Pa
- Sometimes the designation *psig* is used to indicate that the pressure is the pressure is specified relative to atmospheric pressure rather than the absolute pressure

Weights of typical fizzy drinks bottles

Cap	3 g
1.0 litre bottle without cap	36 g
2.0 litre bottle without cap	48 g
3.0 litre bottle without cap	56 g

Downloadable Media application for analysing movies

The *Quicktime*™ media application can be downloaded for free for Windows and Mac computers from:

- www.apple.com/quicktime/download/

Water Rocket Web sites

Searching the web for water rocket related information will bring up a great many links and several excellent sites. Rather than overwhelm you with possibilities, we think the following sites will make good starting points for further investigations

- www.npl.co.uk/waterrockets
  (NPL Water Rockets Page)

- www.et.byu.edu/~wheeler/benchtop
  (Educational Water Rockets Page)

- www.smoke.com.au/~ic/water-rocket.html
  (Ian Clark's Water Rocket Page)

Section 10: Some interesting details

This section is for people who want to understand in more detail how water rockets work. Unfortunately, this involves mathematics and physics which generally isn't taught until university. Anyway, here at the end of the document, where most people don't bother to read, I have put down the calculations for anyone who is interested.

Calculation 1: The work done in expelling the water

The compressed gas above the water in the rocket is the *energy source* for the rocket. We need to understand how this internal energy of the gas is converted into kinetic energy of the rocket as a whole.

When a gas at pressure P expands from volume V to $V + \mathrm{d}V$ then it does work $\mathrm{d}W = P\mathrm{d}V$. In this case the work is done forcing water through the constricted neck of the water rocket. The work done on the water increases its momentum and kinetic energy, and decreases the energy stored in the gas. The rocket energy increases in reaction to the increase in momentum of the water.

To work out how much work the air does as it expels the water, we need to imagine repeating this process over and over and adding up the small amounts $P\mathrm{d}V$ from each incremental expansion. The work done by the gas in an incremental expansion is

$$\mathrm{d}W = P\mathrm{d}V \tag{1}$$

We can add up all the incremental expansions by integrating from the initial volume to the final volume

$$W = \int_{\substack{\text{initial}\\\text{volume}}}^{\substack{\text{final}\\\text{volume}}} P\mathrm{d}V \tag{2}$$

The expansion the gas is rapid enough to be closely *adiabatic*, and so conforms to the law:

$$PV^{\gamma} = K \tag{3}$$

where γ is the ratio of the principal specific heats, and K is a constant. So the work done in the expansion is:

$$W = \int_{\substack{\text{initial}\\\text{volume}}}^{\substack{\text{final}\\\text{volume}}} \frac{K}{V^{\gamma}}\mathrm{d}V \tag{4}$$

which integrates to:

$$W = K\left[\frac{V^{-\gamma+1}}{-\gamma+1}\right]_{\substack{\text{initial}\\\text{volume}}}^{\substack{\text{final}\\\text{volume}}} \tag{5}$$

We can simplify this as follows

$$\begin{aligned} W &= K\left[\frac{V_{\text{final}}^{-\gamma+1}}{-\gamma+1}\right] - K\left[\frac{V_{\text{initial}}^{-\gamma+1}}{-\gamma+1}\right] \\ &= \frac{K}{-\gamma+1}\left[V_{\text{final}}^{-\gamma+1} - V_{\text{initial}}^{-\gamma+1}\right] \end{aligned} \tag{6}$$

The final volume is the simply the full volume of the rocket, V. The initial volume is simply $(1-f)V$ where f is the filling fraction of the rocket. Substituting we find

$$\begin{aligned} W &= \frac{K}{-\gamma+1}\left[V^{-\gamma+1} - V^{-\gamma+1}(1-f)^{-\gamma+1}\right] \\ &= \frac{KV^{-\gamma+1}}{-\gamma+1}\left[1-(1-f)^{-\gamma+1}\right] \end{aligned} \tag{7}$$

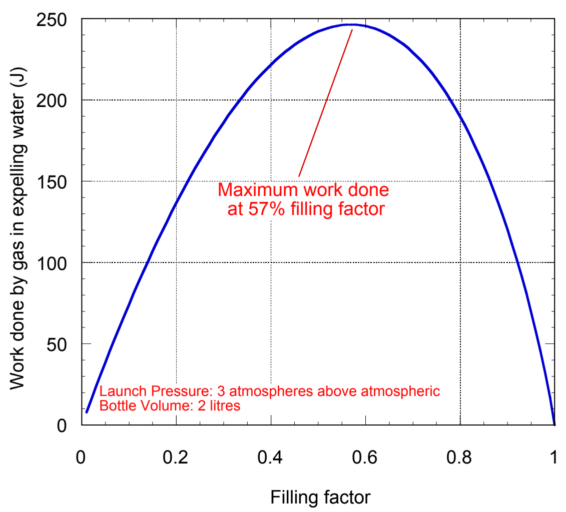
The value of K can be determined from the initial conditions where $K = P(1-f)^{\gamma}V^{\gamma}$

$$\begin{aligned} W &= \frac{\overbrace{P(1-f)^{\gamma}V^{\gamma}}^{K}V^{-\gamma+1}}{-\gamma+1}\left[1-(1-f)^{-\gamma+1}\right] \\ &= \frac{PV}{-\gamma+1}\left[(1-f)^{\gamma}-(1-f)\right] \end{aligned} \tag{8}$$

For air, gamma takes the value 1.4, and so for a two litre bottle pressurised to 3 atmospheres above atmospheric pressure (P = 4 x 10^5 Pa) with a filling fraction of 30% (0.3) this amounts to:

$$\begin{aligned} W &= \frac{4\times10^5\times2\times10^{-3}}{-0.4}\left[0.7^{1.4}-0.7\right] \\ &= 186.1 \text{ J} \end{aligned} \tag{9}$$

Using Equation 8 we can study how the work done expelling the water varies with filling fraction, f. This is plotted on the graph right for the specific example mentioned above.

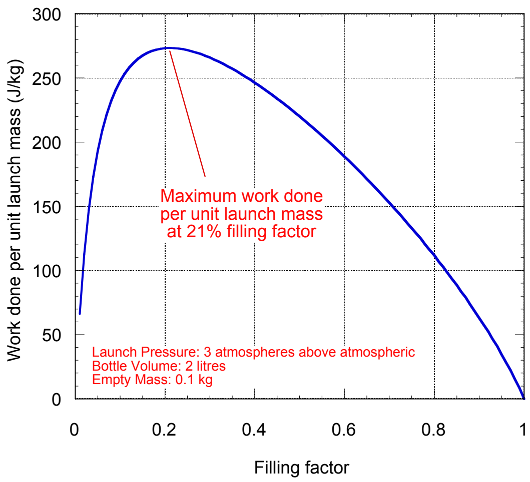


The graph shows a clear peak in the amount of energy extracted from the compressed air, and we can understand why quite easily. When the bottle is full of water, the volume of compressed gas is so small that the stored energy is small.

As the filling factor is reduced, i.e. as we reduce the amount of water in the bottle (and increase the volume of air), the stored energy increases because the volume of gas increases. The peak arises because the work that the gas does in pushing the water out of the bottle depends on the gas *expanding*. If the bottle is full of air, then in cannot do much work before all the water is expelled.

However, 57% filling of a bottle will not give the optimal launch. One can see this by observing that filling the rocket 57% full of water will make for a heavy rocket and most of the energy will be used in lifting water rather than rocket. We can estimate the optimal filling factor by dividing the result of Equation 8 by the mass of the rocket at launch (which increases with increasing f). The work done 'per unit mass at launch' is given by:

$$\frac{W}{rocket \;\; mass} = \frac{1}{m_o + density \times fV} \left(\frac{PV}{-\gamma + 1} \left[(1-f)^{\gamma} - (1-f) \right] \right) \qquad (10)$$



Where m_o is the mass of the rocket when empty, and *density* is the density of water. Using Equation 10 we can study how the work done per unit launch mass varies as a function of filling fraction. This is plotted on the graph right for the specific example mentioned above. This calculation yields a much smaller optimal filling fraction than Equation 8, and one which is much more likely to agree with your experience in the field.

__Calculation 2: The temperature of the air after the water has been expelled__

When the air expands during launch, it cools very significantly. The temperature at the end of the expansion can be calculated from the law:

$$TV^{\gamma-1} = \text{constant} \qquad (11)$$

during an adiabatic expansion. So we can write:

$$T_{\text{initial}} V_{\text{initial}}^{\gamma-1} = T_{\text{final}} V_{\text{final}}^{\gamma-1} \qquad (12)$$

Re-arranging this as an expression for T_{final} we find:

$$\begin{aligned} T_{\text{final}} &= \frac{T_{\text{initial}} V_{\text{initial}}^{\gamma-1}}{V_{\text{final}}^{\gamma-1}} \\ &= T_{\text{initial}} \left[\frac{V_{\text{initial}}}{V_{\text{final}}} \right]^{\gamma-1} \end{aligned} \qquad (13)$$

We can now notice that the final volume of the gas is just the volume of the bottle, V, and the initial volume is $V(1-f)$. Substituting these results into Equation 13 we find:

$$\begin{aligned} T_{\text{final}} &= T_{\text{initial}} \left[\frac{V(1-f)}{V} \right]^{\gamma-1} \\ T_{\text{final}} &= T_{\text{initial}} (1-f)^{\gamma-1} \end{aligned} \qquad (14)$$

It is important to realise the temperature in these equations is the *absolute* or *thermodynamic temperature* which is offset from the Celsius scale by 273.15 K. So if the initial temperature is 20 °C, then the initial temperature to use in Equation 14 is 273.15 + 20 = 293.15 K. So for a 30% filling factor, the final temperature is:

$$\begin{aligned} T_{\text{final}} &= 293.15 \times 0.7^{0.4} \\ &= 254.2 \;\; \text{K} \end{aligned} \qquad (15)$$

which corresponds to a temperature of around –19°C. This is cold, but the gas will cool even further more than this in just another few milliseconds!

Calculation 3: The work done by the expanding air: the 'gas blast'

Once the water has been expelled, the bottle is full of compressed air with nothing to keep it in the rocket except the flow impedance of the nozzle. Roughly speaking, the viscosity of air is 100 times less than that of water, and the air leaves the bottle in about one hundredth of the time it took the water to leave: i.e. extremely quickly. During this 'gas blast' phase of flight, the rocket shows a very noticeable acceleration.

Before we can proceed, we need to calculate the pressure at the end of water expulsion phase. Applying Equation 3 we arrive at:

$$\begin{aligned} P_{\text{final}} &= P_{\text{initial}}\left[\frac{V_{\text{initial}}}{V_{\text{final}}}\right]^{\gamma} \\ &= P_{\text{initial}}\left[\frac{V(1-f)}{V}\right]^{\gamma} \\ &= P_{\text{initial}}(1-f)^{\gamma} \end{aligned} \tag{16}$$

Where P_{initial} is the pressure to which the rocket was pressurised before launch. We now apply Equation 3 again to the expanding gas so that P_{final} from Equation 16 is now considered as P_{initial} for the new expansion.

For the next expansion, the initial volume for this expansion is the volume of the bottle. The initial pressure is the pressure at the point at which all the water has been expelled: the result of Equation 16. The final pressure is atmospheric pressure, and the final volume is unknown. So we need to arrange Equation 3 to find the final volume.

$$\begin{aligned} P_{\text{initial}}V_{\text{initial}}^{\gamma} &= P_{\text{final}}V_{\text{final}}^{\gamma} \\ V_{\text{final}}^{\gamma} &= V_{\text{initial}}^{\gamma}\frac{P_{\text{initial}}}{P_{\text{final}}} \\ &= V^{\gamma}\frac{\overbrace{P_{\text{initial}}(1-f)^{\gamma}}^{\textit{from Equation } 16}}{P_{\text{atmospheric}}} \end{aligned} \tag{17}$$

So for example, if the initial pressure before launch of the air is 3 atmospheres above atmospheric (i.e. 4 atmospheres), and the filling factor is 30%, then for a two litre bottle the final volume of the gas is:

$$V_{\text{final}} = V(1-f)\left[\frac{P_{\text{initial}}}{P_{\text{atmospheric}}}\right]^{1/\gamma} \tag{18}$$

$$\begin{aligned} V_{\text{final}} &= 2\times10^{-3}\times0.7[4]^{1/1.4} \\ &= 2\times10^{-3}\times1.88 \\ &= 3.77\times10^{-3}\text{m}^{3} = 3.77 \textit{ litres} \end{aligned} \tag{19}$$

So the gas expands by only a factor two or so. We now know the initial volume (V) and the final volume (Equation 18) of the gas as it expands. We can now essentially repeat the calculation from Equation 3 onwards to calculate the work done by the expanding gas in the gas blast phase. Since the expansion is adiabatic we know that:

$$PV^{\gamma} = K \tag{*3 and 20}$$

The work done in an infinitesimal expansion can be integrated to give the total work done:

$$W = \int_{\substack{\text{initial}\\\text{volume}}}^{\substack{\text{final}\\\text{volume}}} \frac{K}{V^{\gamma}} \mathrm{d}V \tag{*4 and 21}$$

As we saw previously, this integrates to:

$$W = \frac{K}{-\gamma + 1}\left[V_{\text{final}}^{-\gamma+1} - V_{\text{initial}}^{-\gamma+1}\right] \tag{*6 and 22}$$

In Equation 22 we can evaluate the terms as follows

- K is given by evaluating PV^{γ} at the start of the expansion, where P is given by Equation 16.
- V_{initial} is the volume of the rocket V
- V_{final} is given by Equation 18

Substituting, Equation 22 becomes

$$W = \frac{\overbrace{P_{\text{initial}}V^{\gamma}}^{\textit{from Equation 16}}}{-\gamma+1}\left[\overbrace{\left(V(1-f)\left[\frac{P_{\text{initial}}}{P_{\text{atmospheric}}}\right]^{1/\gamma}\right)^{-\gamma+1}}^{\textit{from Equation 18}} - V^{-\gamma+1}\right] \tag{23}$$

After some tedious simplification this becomes:

$$W = \frac{P_{\text{initial}}V}{-\gamma+1}\left[(1-f)^{-\gamma+1}\left[\frac{P_{\text{initial}}}{P_{\text{atmospheric}}}\right]^{(1/\gamma-1)} - 1\right] \tag{24}$$

We can evaluate this using our standard rocket values of a 2 litre rocket, 30% full, pressurised to 3 atmospheres above atmospheric pressure.

$$\begin{aligned} W &= \frac{4\times10^{5}\times2\times10^{-3}}{-0.4}\left[(0.7)^{-0.4}[4]^{-0.286} - 1\right] \\ &= \frac{8\times10^{2}}{-0.4}[1.153\times0.673-1] \\ &= \frac{8\times10^{2}}{-0.4}[-0.224] \\ &= 448\ \text{J} \end{aligned} \tag{25}$$

Comparing this with the figure (181.6 J) for the work done by the gas in expelling the water (Equation 9) we see that more than twice as much work is done by the gas in expanding after the water has left the rocket. However, while this ‘gas blast’ produces a notable acceleration, because the air density is roughly 1000 times less than the density of water, it imparts relatively little momentum to the rocket.

Calculation 4: The temperature of the gas after the 'gas blast'

When the air within the rocket expands after the water has been expelled — 'the gas blast' — it cools even further than we calculated in Calculation 2. We can calculate the final temperature by applying Equation 13 for an adiabatic (rapid) expansion

$$T_{\text{final}} = T_{\text{initial}}\left[\frac{V_{\text{initial}}}{V_{\text{final}}}\right]^{\gamma-1} \qquad (\text{*13 and 26})$$

We can calculate all the value on the right hand side of this equation.

- The initial temperature for this expansion is final temperature of the previous calculation:

$$T_{\text{initial}} = T_{\text{before launch}}(1-f)^{\gamma-1} \qquad (\text{*14 and 27})$$

- The initial volume of the expansion is just the rocket volume, V.
- The final volume is the result we calculated previously:

$$V_{\text{final}} = V(1-f)\left[\frac{P_{\text{before launch}}}{P_{\text{atmospheric}}}\right]^{1/\gamma} \qquad (\text{*18 and 28})$$

Substituting these values into Equation 26 we find:

$$\begin{aligned} T_{\text{final}} &= T_{\text{before launch}}(1-f)^{\gamma-1}\left[\frac{V}{V(1-f)\left[\frac{P_{\text{before launch}}}{P_{\text{atmospheric}}}\right]^{1/\gamma}}\right]^{\gamma-1} \\ &= T_{\text{before launch}}(1-f)^{\gamma-1}\left[(1-f)\left[\frac{P_{\text{before launch}}}{P_{\text{atmospheric}}}\right]^{1/\gamma}\right]^{1-\gamma} \end{aligned} \qquad (29)$$

Recalling that the temperature in these equations is the *absolute* or *thermodynamic temperature* which is offset from the Celsius scale by 273.15 K, we can estimate that for a launch pressure of 3 atmospheres above atmospheric pressure (i.e. 4 atmospheres) and for a 30% filling factor, the final temperature is expected to be

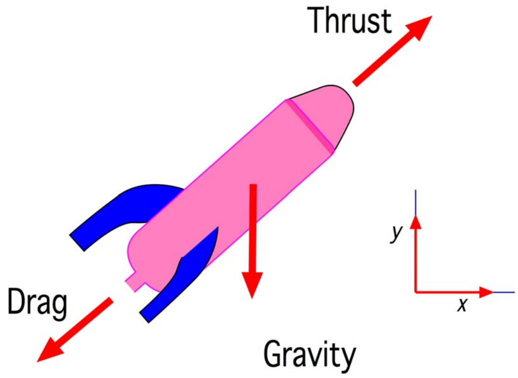
$$\begin{aligned} T_{\text{final}} &= 293.15\times 0.7^{0.4}\left[0.7[4]^{1/1.4}\right]^{-0.4} \\ &= 293.15\times 0.673 \\ &= 197.28\ \text{ K} \\ &= -75.9\ {}^{\circ}\text{C} \end{aligned} \qquad (30)$$

This is extremely cold, but obviously the gas will not stay so cool for long, and will have warmed up by the time the rocket lands. Also the figures calculated in Equations 15 and 30 probably overestimate the cooling because we have not taken account of the water content of the air in the bottle. For the two-litre bottle we have been considering, the air will contain roughly 17.4 mg of water vapour per litre of air (i.e. 24.4 mg). As the air cools, this vapour will condense, which absorbs around 2500 joules per gram of water vapour in the air, or roughly 60 J for the bottle we considered. This will slow down the cooling, and increase the minimum temperature reached by (very roughly) a few tens of degrees.

Calculation 5: The dynamics of the rocket

It is not possible to write down simple formulae for the behaviour of a water rocket as it flies: that is why I wrote the simulator program (Section 7). However, even though space is very short, I will try to indicate how the software works. The dynamics of the rocket are calculated by making estimates of the three forces acting on the rocket. These are illustrated in the Figure below.

The first force is *gravity*, and its magnitude is simply mg, where m is the mass of the rocket. However, remember that the rocket mass will change during the flight as the rocket expels its water. The next force is the *thrust*, and we will say some more about how this is calculated below. For now we note that we assume the thrust acts along the axis of the rocket. And the last force is the *aerodynamic drag* which has the form:



$$F_{\text{drag}} = kv^2 \tag{31}$$

where v is the air speed, and k is a constant which depends on how aerodynamic your rocket is. We assume that the rocket is aerodynamically stable and that the drag force always acts in the opposite direction to the velocity vector. To simulate the dynamics of the rocket we calculate the sum of the three forces mentioned above, and resolve the force into its x and y components. We then use Newton's third law ($\mathbf{F} = m\mathbf{a}$) to calculate acceleration due to the net force on the rocket.

At this point we make an approximation: we estimate the change in each component of velocity due to each component of the acceleration a as:

$$\Delta v_{\text{x}} \approx a_{\text{x}} \Delta t \tag{32}$$

which is strictly only accurate in the limit $\Delta t \to 0$. Having an estimate for the change in velocity during the time Δt, we can then estimate the change in each component of the position of the rocket as:

$$\Delta x \approx v_{\text{x}} \Delta t \tag{33}$$

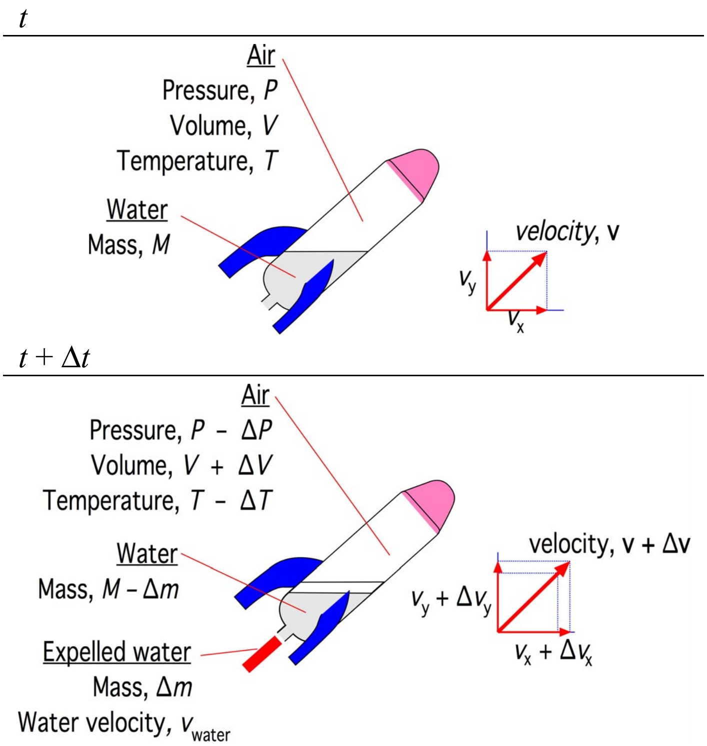
This process is repeated typically a few hundred or a few thousand times during the flight with short time steps, and seems to give a fair approximation to realistic flight dynamics.

Calculation 6: The thrust

The calculation of the thrust on the rocket is the trickiest part of the calculation, and draws on parts of all the previous calculations. The calculation is also different in the different stages of the rocket's flight. The figure below illustrates the general principle used in the simulation and shows the situation of the rocket as it is expelling water. The rocket is shown at two times separated by Δt.

To calculate the thrust we proceed as follows:

- First we calculate the work done by the expanding gas as it adiabatically expands, and expels a small mass Δm of water.
- Using the *principle of conservation of energy*, we assume that this work goes to increasing (a) the kinetic and potential energy of the rocket and its remaining water and (b) the kinetic energy of the expelled water. The implicit assumption here is that the nozzle is 100% efficient in converting between the two forms of energy.



- Using the *principle of conservation of momentum* in combination with the principle of conservation of energy, we arrive at some (really quite complicated) equations that yield the speed of the expelled water. From the speed of the expelled water v_{water}, its momentum $\Delta p = \Delta m\, v_{water}$ can be calculated, and this is just equal and opposite to the momentum imparted to the rocket.
- The thrust is then estimated as the rate of change of momentum from:

$$Thrust = \frac{\Delta p}{\Delta t} \tag{34}$$

The thrust is then input to the calculation of the rocket dynamics discussed previously.

Despite numerous checks of the software I am still not convinced that the software correctly predicts rocket behaviour in all circumstances, but I do think any errors are rather small for most plausible launch parameters. However, the principles outlined above should suffice to allow you to write your own software.