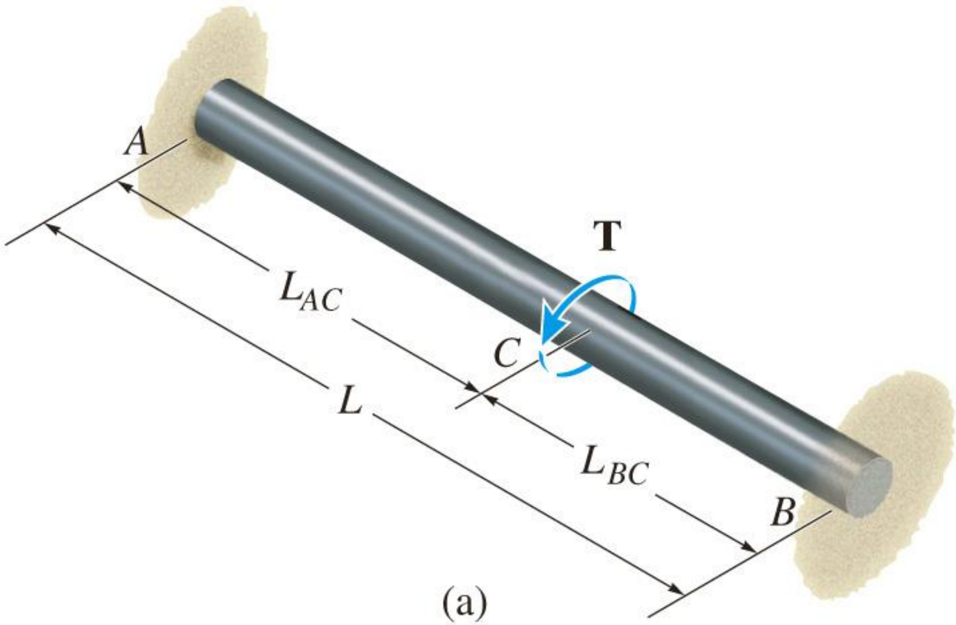
۵-۵ مسائل پیچش اعضای نامعین استاتیکی



(a)

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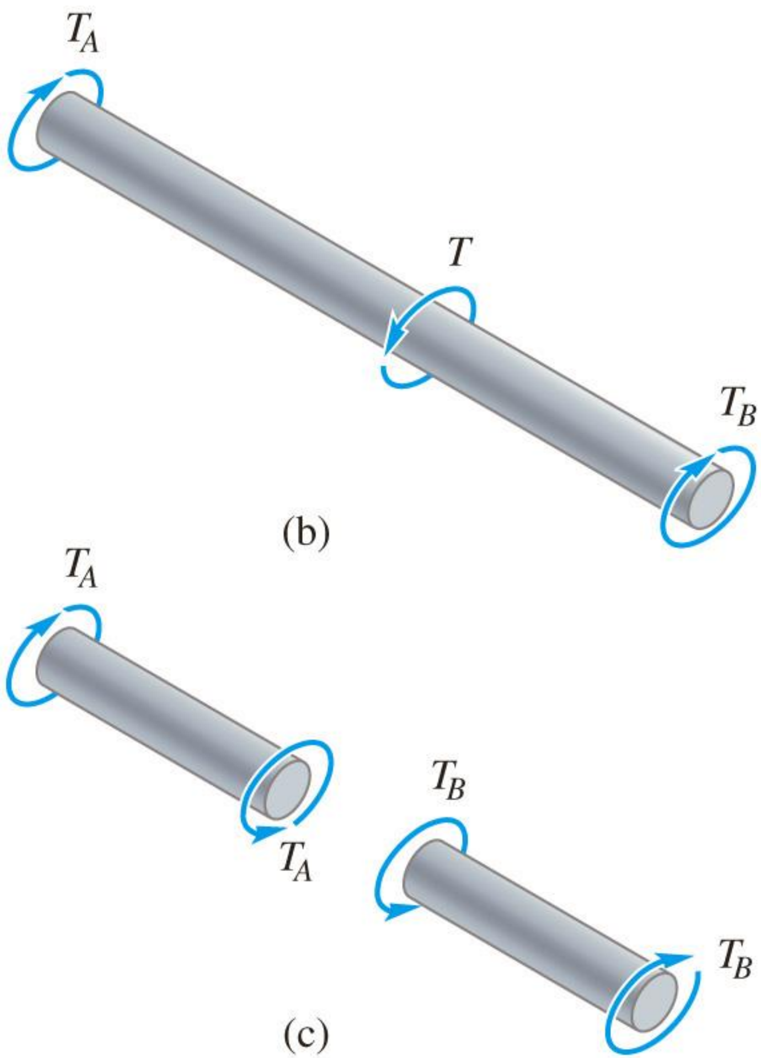
آیا این عضو تحت پیچش از نظر استاتیکی نامعین است؟

۱- تعادل:

$$\Sigma M_x = 0 \Rightarrow T - T_A - T_B = 0$$

۱ معادله، ۲ مجهول

از نظر استاتیکی نامعین است



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۲- معادله سازگاری:

$$\phi_{A/B} = 0$$

$$\frac{T_A L_{AC}}{JG} - \frac{T_B L_{BC}}{JG} = 0$$

اکنون ۲ معادله و ۲ مجهول است.
می توانیم TA و TB را بیابیم.

اگر مسئله از نظر استاتیکی نامعین باشد:

۱. از معادله استاتیکی استفاده میکنیم تا یک معادله بدست آوریم
۲. از معادله سازگاری استفاده می کنیم (اطلاعاتی که در مورد تغییر شکل ها می دانیم) تا معادله دیگری بدست آوریم.
۳. معادلات ۱ و ۲ را حل میکنیم تا گشتاورهای مجهول محاسبه شود
۴. تنش برشی (τ) را بدست می آوریم.

EXAMPLE 5.11

The solid steel shaft shown in Fig. 5–25*a* has a diameter of 20 mm. If it is subjected to the two torques, determine the reactions at the fixed supports *A* and *B*.

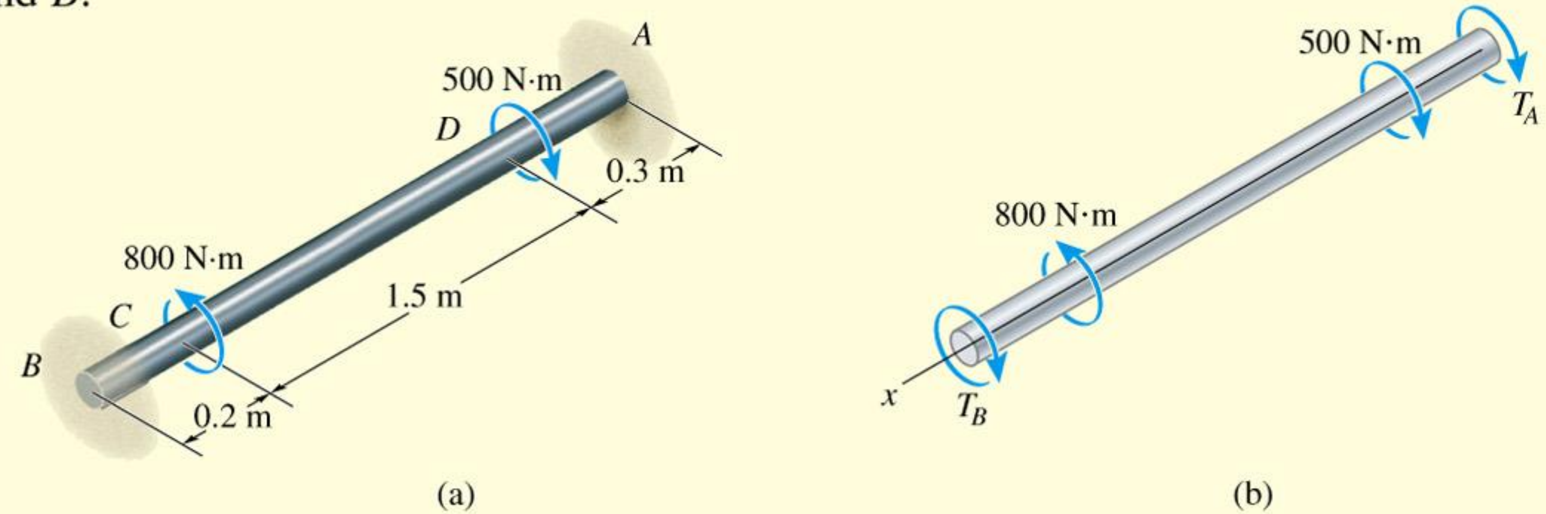


Fig. 5–25

Solution

Equilibrium. By inspection of the free-body diagram, Fig. 5–25*b*, it is seen that the problem is statically indeterminate since there is only *one* available equation of equilibrium, whereas $\mathbf{T}_A$ and $\mathbf{T}_B$ are unknown. We require

$$\Sigma M_x = 0; \qquad -T_B + 800\ \text{N}\cdot\text{m} - 500\ \text{N}\cdot\text{m} - T_A = 0 \qquad (1)$$

Compatibility. Since the ends of the shaft are fixed, the angle of twist of one end of the shaft with respect to the other must be zero. Hence, the compatibility equation can be written as

$$\phi_{A/B} = 0$$

This condition can be expressed in terms of the unknown torques by using the load–displacement relationship, $\phi = TL/JG$. Here there are three regions of the shaft where the internal torque is constant, BC, CD, and DA. On the free-body diagrams in Fig. 5–25*c* we have shown the internal torques acting on segments of the shaft which are sectioned in each of these regions. Using the sign convention established in Sec. 5.4, we have

$$\frac{-T_B(0.2\text{ m})}{JG} + \frac{(T_A + 500\text{ N}\cdot\text{m})(1.5\text{ m})}{JG} + \frac{T_A(0.3\text{ m})}{JG} = 0$$

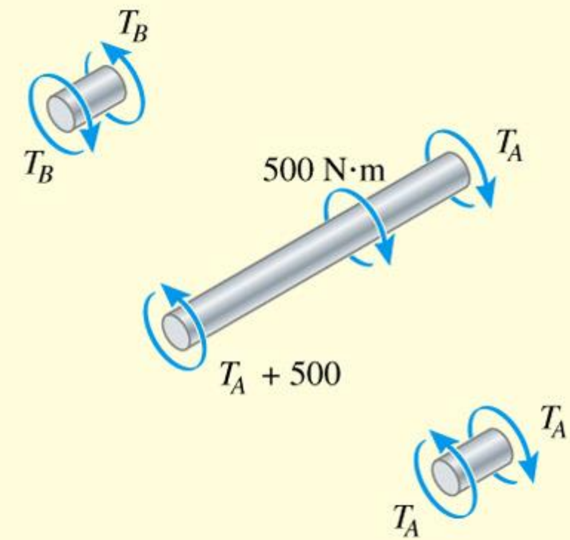
or

$$1.8T_A - 0.2T_B = -750 \qquad (2)$$

Solving Eqs. 1 and 2 yields

$$T_A = -345\text{ N}\cdot\text{m} \qquad T_B = 645\text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

The negative sign indicates that $\mathbf{T}_A$ acts in the opposite direction of that shown in Fig. 5–25*b*.



(c)

Fig. 5–25

EXAMPLE 5.12

The shaft shown in Fig. 5–26*a* is made from a steel tube, which is bonded to a brass core. If a torque of $T = 250\ \text{lb}\cdot\text{ft}$ is applied at its end, plot the shear-stress distribution along a radial line of its cross-sectional area. Take $G_{st} = 11.4(10^3)$ ksi, $G_{br} = 5.20(10^3)$ ksi.

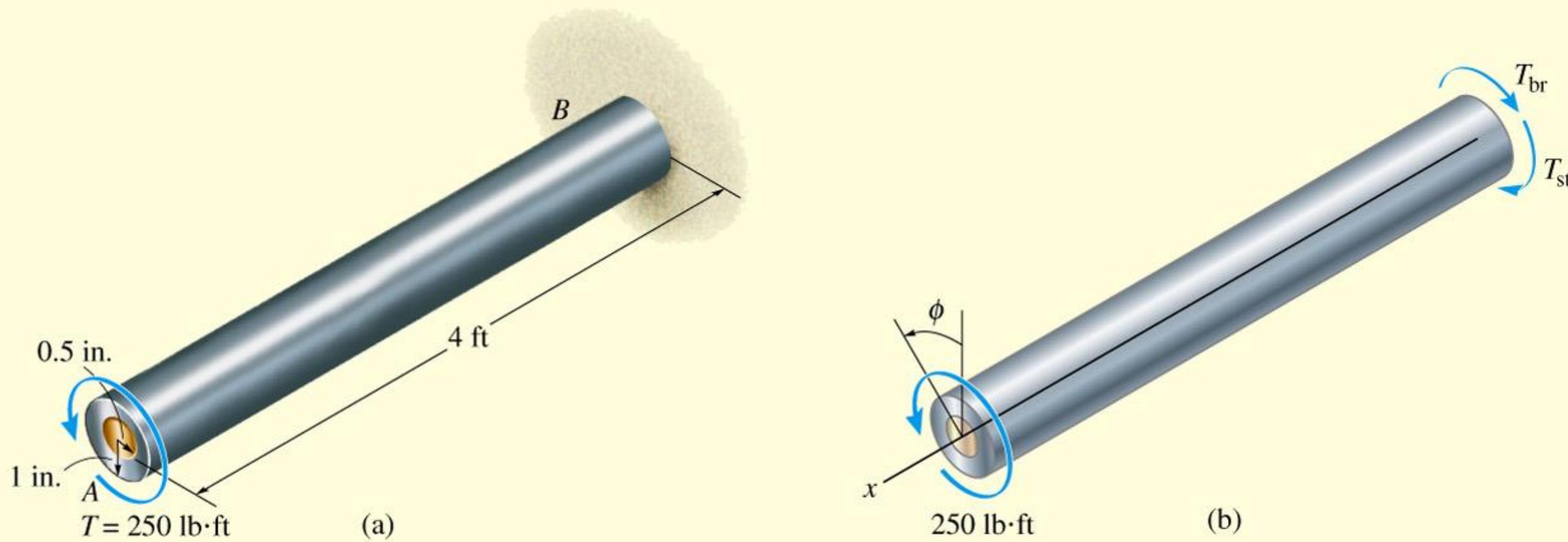


Fig. 5–26

Solution

Equilibrium. A free-body diagram of the shaft is shown in Fig. 5–26*b*. The reaction at the wall has been represented by the unknown amount of torque resisted by the steel, T_{st}, and by the brass, T_{br}. Working in units of pounds and inches, equilibrium requires

$$-T_{st} - T_{br} + 250\ \text{lb} \cdot \text{ft}(12\ \text{in./ft}) = 0 \qquad (1)$$

Compatibility. We require the angle of twist of end A to be the same for both the steel and brass since they are bonded together. Thus,

$$\phi = \phi_{st} = \phi_{br}$$

Applying the load–displacement relationship, $\phi = TL/JG$, we have

$$\frac{T_{st}L}{(\pi/2)[(1\ \text{in.})^4 - (0.5\ \text{in.})^4]11.4(10^3)\ \text{kip/in}^2} = \frac{T_{br}L}{(\pi/2)(0.5\ \text{in.})^4 5.20(10^3)\ \text{kip/in}^2}$$

$$T_{st} = 32.88T_{br} \qquad (2)$$

Solving Eqs. 1 and 2, we get

$$T_{st} = 2911.0 \text{ lb}\cdot\text{in.} = 242.6 \text{ lb}\cdot\text{ft}$$
$$T_{br} = 88.5 \text{ lb}\cdot\text{in.} = 7.38 \text{ lb}\cdot\text{ft}$$

These torques act throughout the entire length of the shaft, since no external torques act at intermediate points along the shaft's axis. The shear stress in the brass core varies from zero at its center to a maximum at the interface where it contacts the steel tube. Using the torsion formula,

$$(\tau_{br})_{max} = \frac{(88.5 \text{ lb}\cdot\text{in.})(0.5 \text{ in.})}{(\pi/2)(0.5 \text{ in.})^4} = 451 \text{ psi}$$

For the steel, the minimum shear stress is also at this interface,

$$(\tau_{st})_{min} = \frac{(2911.0 \text{ lb}\cdot\text{in.})(0.5 \text{ in.})}{(\pi/2)[(1 \text{ in.})^4 - (0.5 \text{ in.})^4]} = 988 \text{ psi}$$

and the maximum shear stress is at the outer surface,

$$(\tau_{st})_{max} = \frac{(2911.0 \text{ lb}\cdot\text{in.})(1 \text{ in.})}{(\pi/2)[(1 \text{ in.})^4 - (0.5 \text{ in.})^4]} = 1977 \text{ psi}$$

The results are plotted in Fig. 5–26*c*. Note the discontinuity of *shear stress* at the brass and steel interface. This is to be expected, since the materials have different moduli of rigidity; i.e., steel is stiffer than brass ($G_{st} > G_{br}$) and thus it carries more shear stress at the interface. Although the shear stress is discontinuous here, the *shear strain* is not. Rather, the shear strain is the *same* for both the brass and the steel. This can be shown by using Hooke's law, $\gamma = \tau/G$. At the interface, Fig. 5–26*d*, the shear strain is

$$\gamma = \frac{\tau}{G} = \frac{451 \text{ psi}}{5.2(10^6) \text{ psi}} = \frac{988 \text{ psi}}{11.4(10^6) \text{ psi}} = 0.0867(10^{-3}) \text{ rad}$$

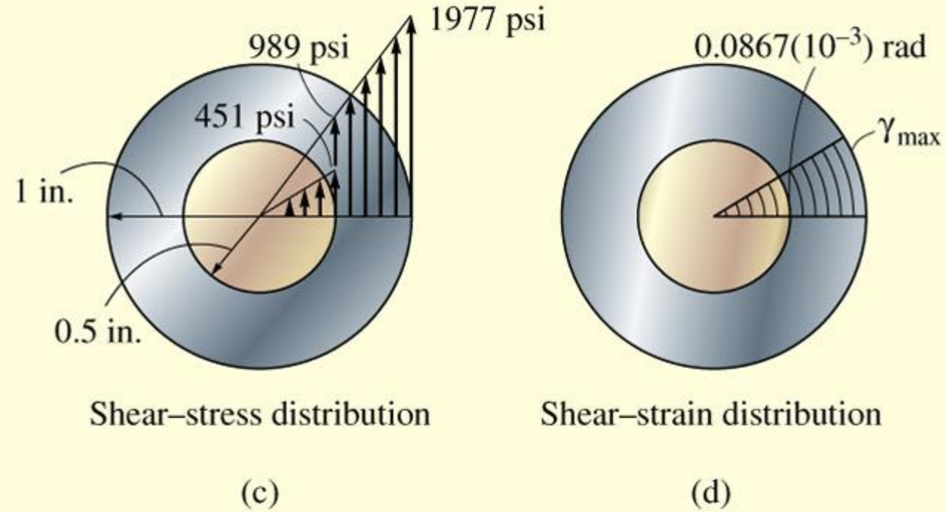
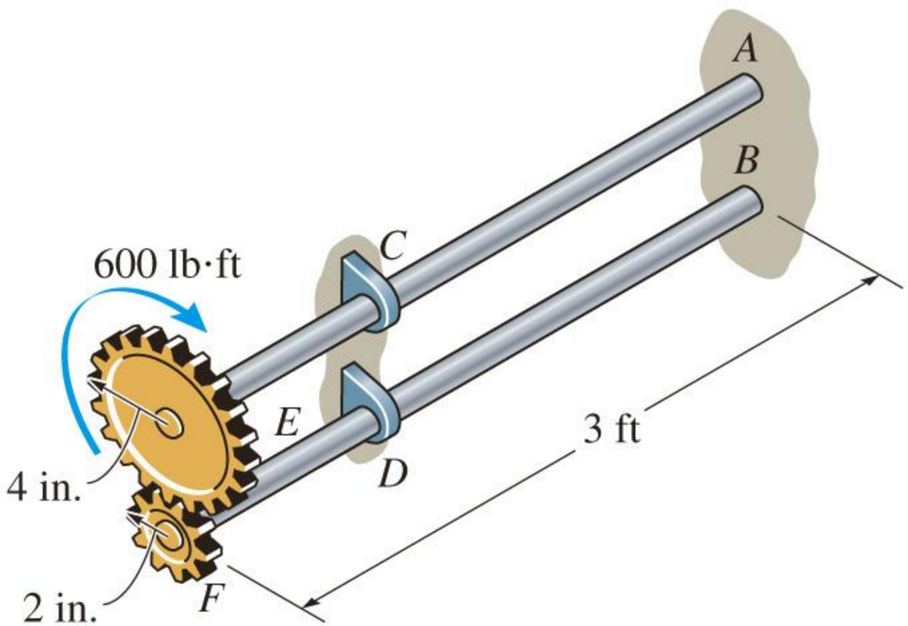


Fig. 5–26

تمرین: تنش برشی ماکزیمم را در هر شافت بیابید.



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