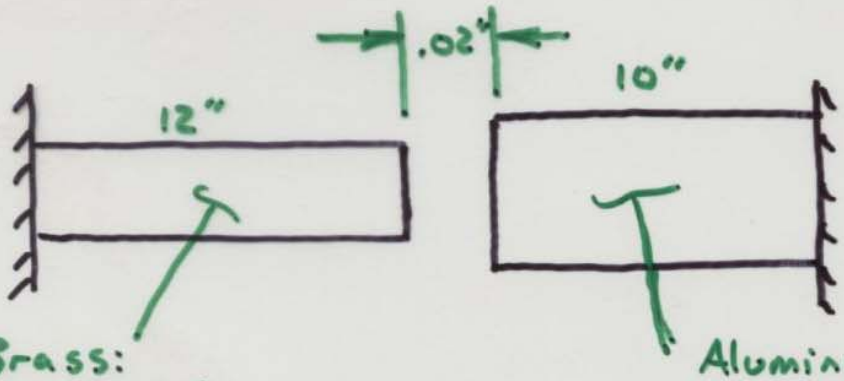
Example:

Given: Structure shown, initial temperature, $T_i = 60°F$
initial gap = .02", final temp, $T_f = 180°F$



$T_i = 60°F$

$T_f = 180°F$

$\Delta T = 120°F$

Brass:
$A = 3.14 in^2$
$E = 15 \times 10^6$ psi
$\alpha = 11.6 \times 10^{-6} °F^{-1}$

Aluminum:
$A = 7.07 in^2$
$E = 10.1 \times 10^6$ psi
$\alpha = 13.1 \times 10^{-6} °F^{-1}$

Find: Stress @ T_f. (will they touch? If No then $\sigma = 0$!!)

Total expansion without Constraint:

$$\delta_{Tot} = (\alpha L \Delta T)_B + (\alpha L \Delta T)_A$$

$$= 11.6 \times 10^{-6} F^{-1} (12")(120F) + 13.1 \times 10^{-6} F^{-1} (10")(120F)$$

$$= .0324" \quad (> .02" \therefore \text{will touch!})$$

Stress will be developed for:

$$.0324" - .02" = \underline{\underline{.0124"}} = \delta_{contact}$$

Force Required to squeeze the bars

.0124" = $\delta_{contact}$ => stress is developed in last .0124" of expansion.

$$\delta = \left(\frac{PL}{AE}\right)_B + \left(\frac{PL}{AE}\right)_A = P\left[\left(\frac{L}{AE}\right)_B + \left(\frac{L}{AE}\right)_A\right]$$

$$.0124" = P\left[\frac{12"}{3.14in^2(15\times10^6 psi)} + \frac{10"}{7.07in^2(10.1\times10^6 psi)}\right]$$

$\boxed{P = 31,400lb}$ => Experienced by both bars ∴ will produce σ in both

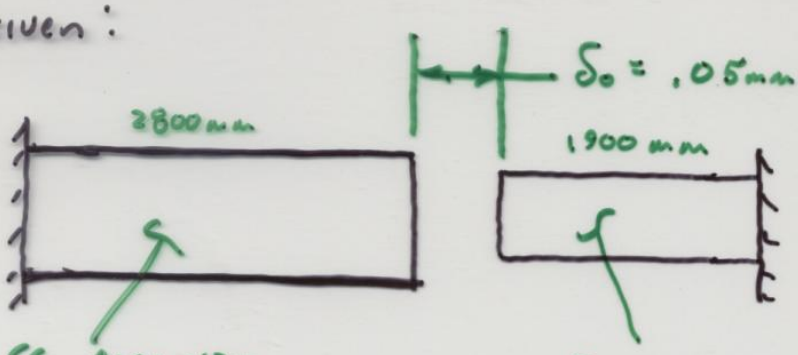
$$\sigma_{Brass} = \frac{P}{A} = \frac{31,400lb}{3.14in^2} \qquad \sigma_{Al} = \frac{31,400lb}{7.07in^2}$$

$\boxed{\sigma_{Brass} = 10Ksi \qquad \sigma_{Al} = 4.44\,Ksi}$ ✱

Summary:
1. Determine unrestrained, δ_{Tot}
2. Determine contact, $\delta_{contact}$
3. Solve for Load
4. Solve for Stress

Problem 4.42

Given:



T_o = 20°C

SS AISI 430
$\alpha_{SS} = 10.4 \times 10^{-6} °C^{-1}$
12mm square

Brass C3600
$\alpha_{Brass} = 20.5 \times 10^{-6} °C^{-1}$
8mm square

Find: T_f so just touching

Sol'n:

$$\delta_{Tot} = \delta_{SS} + \delta_B = \alpha_{SS} L_{SS} \Delta T + \alpha_B L_B \Delta T$$

$$\delta_{Tot} = \Delta T \left[\alpha_{SS} L_{SS} + \alpha_B L_B\right]$$

$$\Delta T = \frac{\delta_{Tot}}{\left[\alpha_{SS} L_{SS} + \alpha_B L_B\right]}$$

$$\Delta T = \frac{.05mm}{\left[10.4 \times 10^{-6} °C^{-1} (2800mm) + 20.5 \times 10^{-6} °C^{-1} (1900mm)\right]}$$

$\Delta T = .735°$ => $T_f = T_o + \Delta T$

$$\boxed{T_f = 20.735°C}$$

So @ $T_F = 20.735°C$ just touching

$T_F >$ will get Stress due to Contact!