۱۲-۵: روش برهم نهی (Super position)

یادآوری:

$$EI\frac{d^4 v}{dx^4} = w(x)$$

این معادله دیفرانسیل، دو شرط اصلی برای برهم نهی را ارضا می کند:

۱) بار w(x) به صورت خطی با تغییر شکل v(x) مرتبط است.

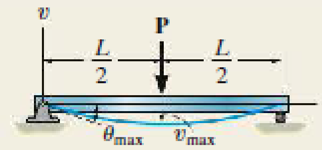
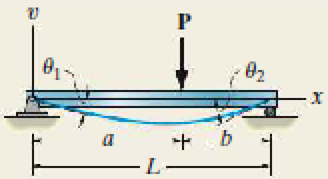
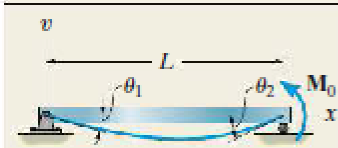
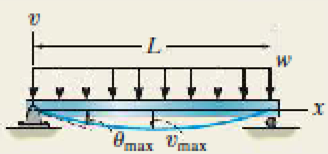
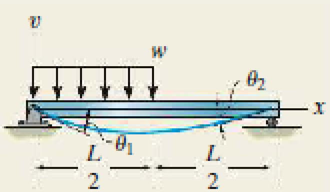
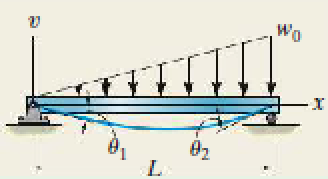
۲) بار به صورت قابل توجهی هندسه اصلی را تغییر ندهد.

برهم نهی چگونه عمل می کند؟؟

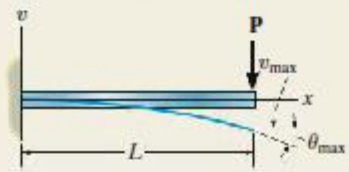
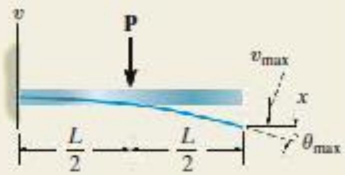
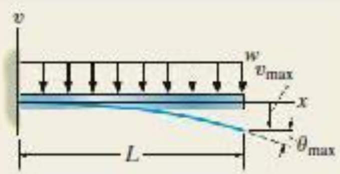
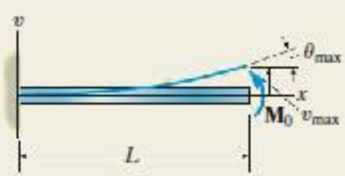
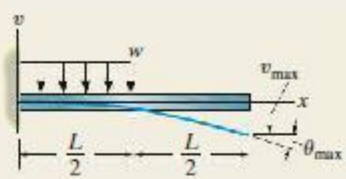
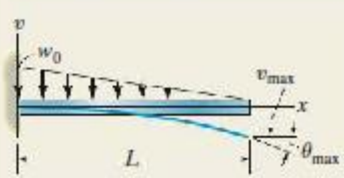
۱) تیر را به یک سری موارد شناخته شده تقسیم می کنیم (ضمیمه C کتاب)

۲) سپس $v_{tot} = v_1 + v_2 + v_3 + \ldots\ldots$

ضمیمه C :

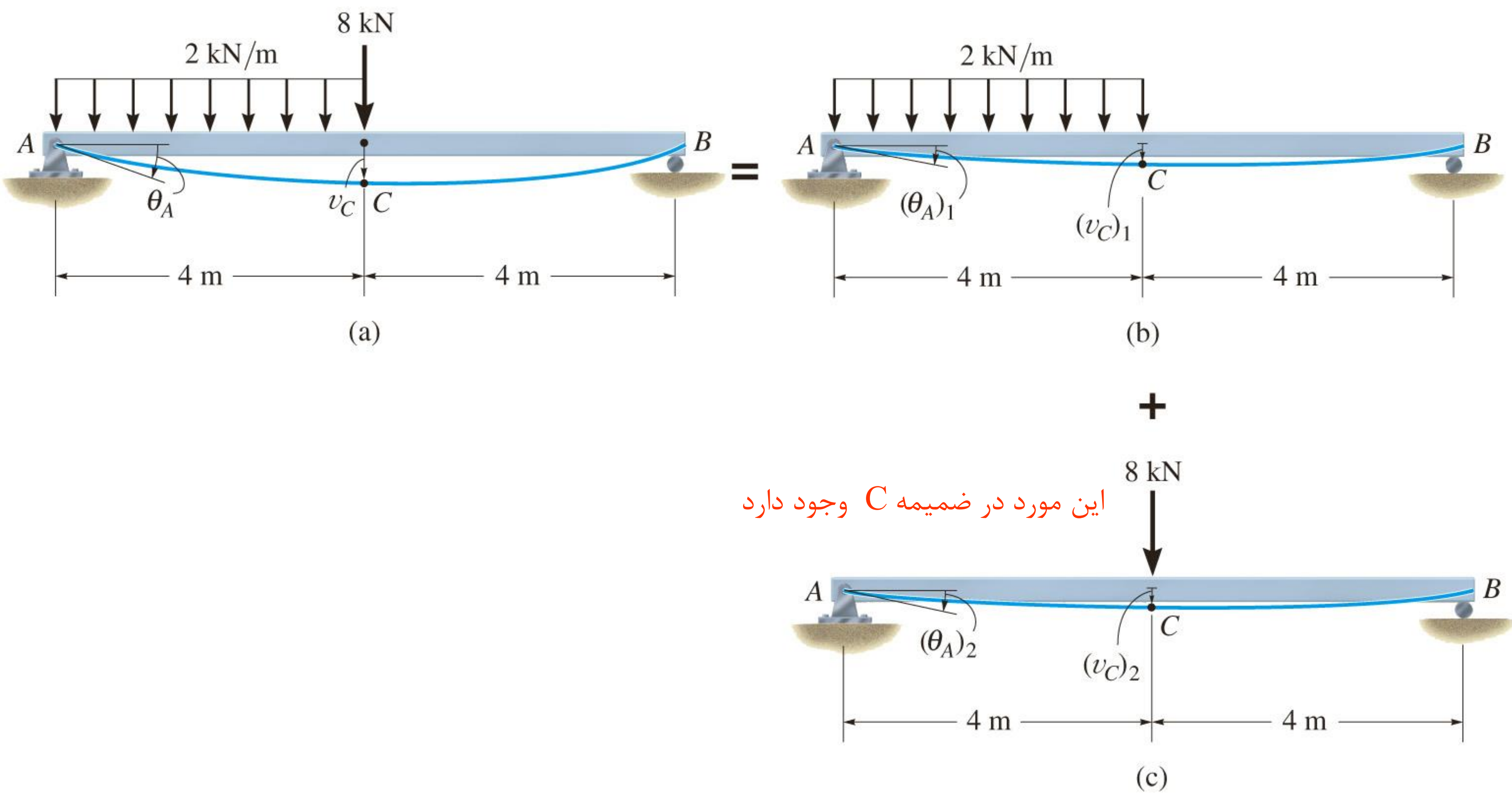
Beam	Slope	Deflection	Elastic Curve
	$\theta_{max} = \dfrac{-PL^2}{16EI}$	$v_{max} = \dfrac{-PL^3}{48EI}$	$v = \dfrac{-Px}{48EI}(3L^2 - 4x^2)$ $0 \le x \le L/2$
	$\theta_1 = \dfrac{-Pab(L + b)}{6EIL}$ $\theta_2 = \dfrac{Pab(L + a)}{6EIL}$	$v\Big\rvert_{x=a} = \dfrac{-Pba}{6EIL}(L^2 - b^2 - a^2)$	$v = \dfrac{-Pbx}{6EIL}(L^2 - b^2 - x^2)$ $0 \le x \le a$
	$\theta_1 = \dfrac{-M_0L}{6EI}$ $\theta_2 = \dfrac{M_0L}{3EI}$	$v_{max} = \dfrac{-M_0L^2}{9\sqrt{3}EI}$ at $x = 0.5774L$	$v = \dfrac{-M_0x}{6EIL}(L^2 - x^2)$
	$\theta_{max} = \dfrac{-wL^3}{24EI}$	$v_{max} = \dfrac{-5wL^4}{384EI}$	$v = \dfrac{-wx}{24EI}(x^3 - 2Lx^2 + L^3)$
	$\theta_1 = \dfrac{-3wL^3}{128EI}$ $\theta_2 = \dfrac{7wL^3}{384EI}$	$v\Big\rvert_{x=L/2} = \dfrac{-5wL^4}{768EI}$ $v_{max} = -0.006563\dfrac{wL^4}{EI}$ at $x = 0.4598L$	$v = \dfrac{-wx}{384EI}(16x^3 - 24Lx^2 + 9L^3)$ $0 \le x \le L/2$ $v = \dfrac{-wL}{384EI}(8x^3 - 24Lx^2$ $+ 17L^2x - L^3)$ $L/2 \le x < L$
	$\theta_1 = \dfrac{-7w_0L^3}{360EI}$ $\theta_2 = \dfrac{w_0L^3}{45EI}$	$v_{max} = -0.00652\dfrac{w_0L^4}{EI}$ at $x = 0.5193L$	$v = \dfrac{-w_0x}{360EIL}(3x^4 - 10L^2x^2 + 7L^4)$

ضمیمه C :

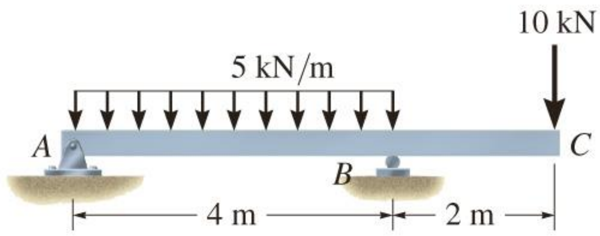
Beam	Slope	Deflection	Elastic Curve
	$\theta_{max} = \frac{-PL^2}{2EI}$	$v_{max} = \frac{-PL^3}{3EI}$	$v = \frac{-Px^2}{6EI}(3L - x)$
	$\theta_{max} = \frac{-PL^2}{8EI}$	$v_{max} = \frac{-5PL^3}{48EI}$	$v = \frac{-Px^2}{12EI}(3L - 2x) \quad 0 \le x \le L/2$ $v = \frac{-PL^2}{48EI}(6x - L) \quad L/2 \le x \le L$
	$\theta_{max} = \frac{-wL^3}{6EI}$	$v_{max} = \frac{-wL^4}{8EI}$	$v = \frac{-wx^2}{24EI}(x^2 - 4Lx + 6L^2)$
	$\theta_{max} = \frac{M_0 L}{EI}$	$v_{max} = \frac{M_0 L^2}{2EI}$	$v = \frac{M_0 x^2}{2EI}$
	$\theta_{max} = \frac{-wL^3}{48EI}$	$v_{max} = \frac{-7wL^4}{384EI}$	$v = \frac{-wx^2}{24EI}\left(x^2 - 2Lx + \frac{3}{2}L^2\right)$ $0 \le x \le L/2$ $v = \frac{-wL^3}{384EI}(8x - L)$ $L/2 \le x \le L$
	$\theta_{max} = \frac{-w_0 L^3}{24EI}$	$v_{max} = \frac{-w_0 L^4}{30EI}$	$v = \frac{-w_0 x^2}{120EIL}(10L^3 - 10L^2 x + 5Lx^2 - x^3)$

تغییر شکل ها مجهول است

این مورد در ضمیمه C وجود دارد



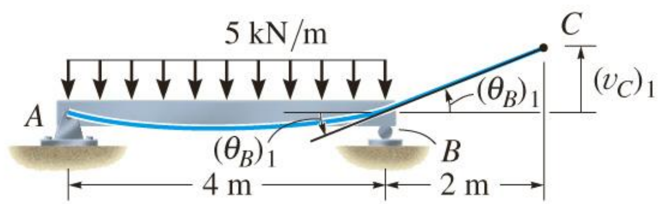
این مورد در ضمیمه C وجود دارد



10 kN
5 kN/m
A
B
C
4 m
2 m

(a)

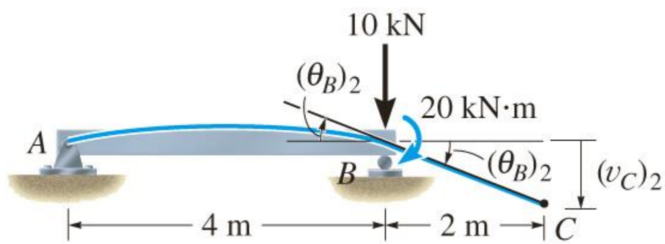
=



5 kN/m
C
A
$(\theta_B)_1$
$(v_C)_1$
$(\theta_B)_1$
B
4 m
2 m

(b)

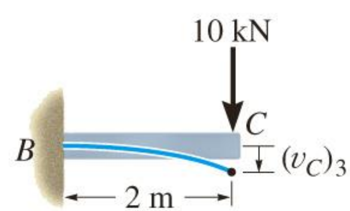
+



10 kN
$(\theta_B)_2$
20 kN·m
A
B
$(\theta_B)_2$
$(v_C)_2$
4 m
2 m
C

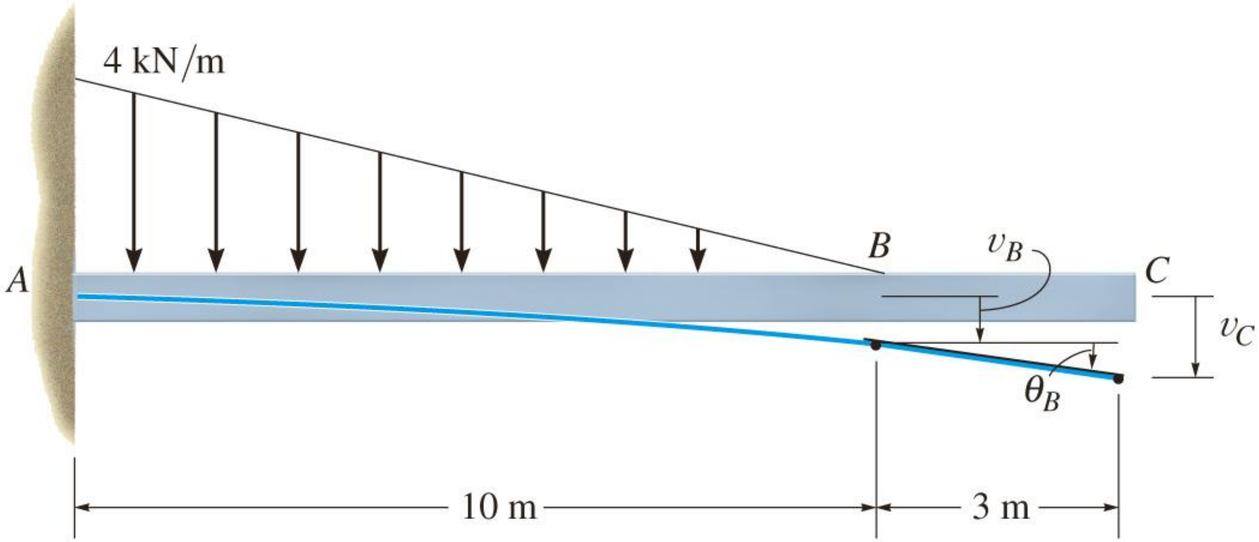
(c)

+



10 kN
C
B
$(v_C)_3$
2 m

(d)


4 kN/m
A
B
v_B
C
v_C
θ_B
10 m
3 m

EXAMPLE 12.13

Determine the displacement at point C and the slope at the support A of the beam shown in Fig. 12–29a. EI is constant.

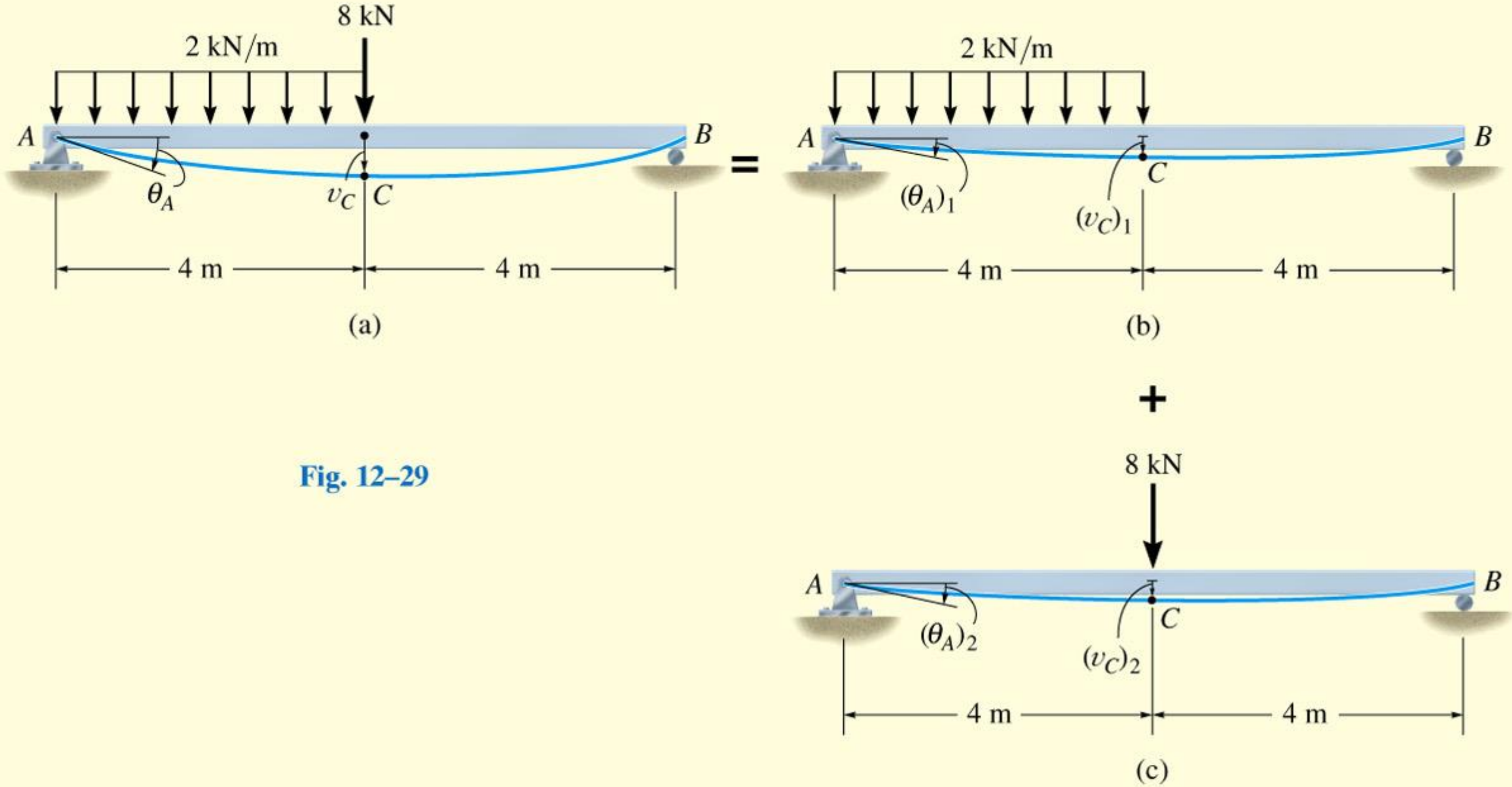


Fig. 12–29

Solution

The loading can be separated into two component parts as shown in Figs. 12–29*b* and 12–29*c*. The displacement at *C* and slope at *A* are found using the table in Appendix C for each part.

For the distributed loading,

$$(\theta_A)_1 = \frac{3wL^3}{128EI} = \frac{3(2\text{ kN/m})(8\text{ m})^3}{128EI} = \frac{24\text{ kN}\cdot\text{m}^2}{EI}\circlearrowright$$

$$(v_C)_1 = \frac{5wL^4}{768EI} = \frac{5(2\text{ kN/m})(8\text{ m})^4}{768EI} = \frac{53.33\text{ kN}\cdot\text{m}^3}{EI}\downarrow$$

For the 8-kN concentrated force,

$$(\theta_A)_2 = \frac{PL^2}{16EI} = \frac{8\text{ kN}(8\text{ m})^2}{16EI} = \frac{32\text{ kN}\cdot\text{m}^2}{EI}\circlearrowright$$

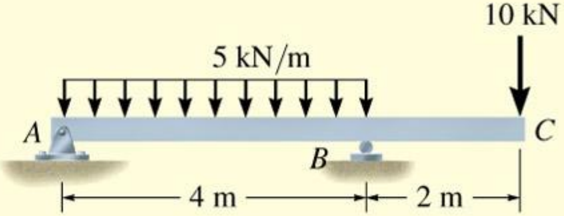
$$(v_C)_2 = \frac{PL^3}{48EI} = \frac{8\text{ kN}(8\text{ m})^3}{48EI} = \frac{85.33\text{ kN}\cdot\text{m}^3}{EI}\downarrow$$

The total displacement at *C* and the slope at *A* are the algebraic sums of these components. Hence

$$(+\circlearrowright) \qquad \theta_A = (\theta_A)_1 + (\theta_A)_2 = \frac{56\text{ kN}\cdot\text{m}^2}{EI}\circlearrowright \qquad \textit{Ans.}$$

$$(+\downarrow) \qquad v_C = (v_C)_1 + (v_C)_2 = \frac{139\text{ kN}\cdot\text{m}^3}{EI}\downarrow \qquad \textit{Ans.}$$

EXAMPLE 12.14



(a)

Determine the displacement at the end C of the overhanging beam shown in Fig. 12–30*a*. EI is constant.

Solution

Since the table in Appendix C *does not* include beams with overhangs, the beam will be separated into a simply supported and a cantilevered portion. First we will calculate the slope at B, as caused by the distributed load acting on the simply supported span, Fig. 12–30*b*.

$$(\theta_B)_1 = \frac{wL^3}{24EI} = \frac{5\ \text{kN/m}(4\ \text{m})^3}{24EI} = \frac{13.33\ \text{kN}\cdot\text{m}^2}{EI} \circlearrowright$$

Since this angle is *small*, $(\theta_B)_1 \approx \tan(\theta_B)_1$, and the vertical displacement at point C is

$$(v_C)_1 = (2\ \text{m})\left(\frac{13.33\ \text{kN}\cdot\text{m}^2}{EI}\right) = \frac{26.67\ \text{kN}\cdot\text{m}^3}{EI}\uparrow$$

Next, the 10-kN load on the overhang causes a statically equivalent force of 10 kN and couple moment of 20 kN · m at the support B of the simply supported span, Fig. 12-30*c*. The 10-kN force does not cause a displacement or slope at B; however, the 20-kN · m couple moment does cause a slope. The slope at B due to this moment is

$$(\theta_B)_2 = \frac{M_0 L}{3EI} = \frac{20\ \text{kN}\cdot\text{m}(4\ \text{m})}{3EI} = \frac{26.67\ \text{kN}\cdot\text{m}^2}{EI} \circlearrowright$$

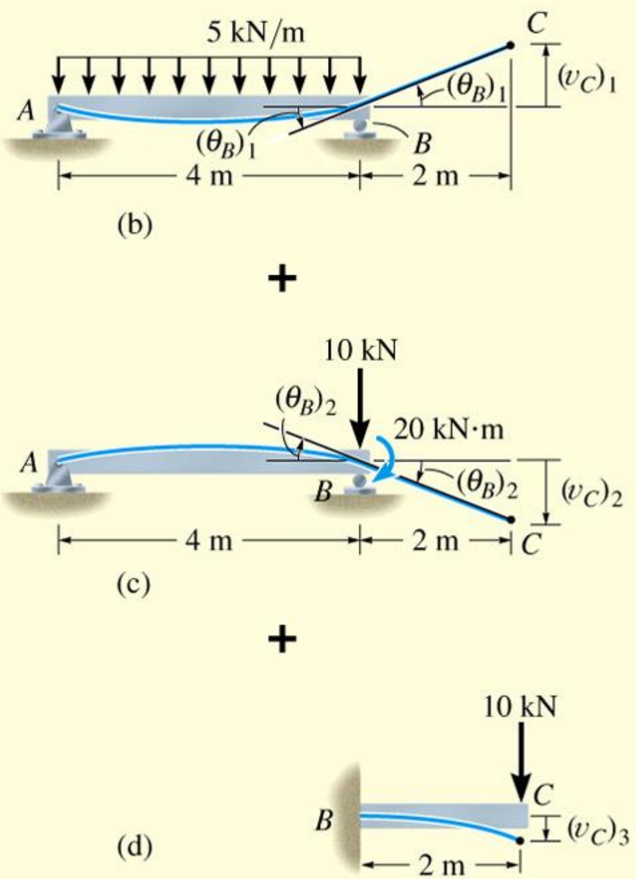


Fig. 12–30

So that the extended point C is displaced

$$(v_C)_2 = (2\text{ m})\left(\frac{26.7\text{ kN}\cdot\text{m}^2}{EI}\right) = \frac{53.33\text{ kN}\cdot\text{m}^3}{EI}\downarrow$$

Finally, the cantilevered portion BC is displaced by the 10-kN force, Fig. 12–30*d*. We have

$$(v_C)_3 = \frac{PL^3}{3EI} = \frac{10\text{ kN}(2\text{ m})^3}{3EI} = \frac{26.67\text{ kN}\cdot\text{m}^3}{EI}\downarrow$$

Summing these results algebraically, we obtain the final displacement of point C,

$$(+\downarrow) \qquad v_C = -\frac{26.7}{EI} + \frac{53.3}{EI} + \frac{26.7}{EI} = \frac{53.3\text{ kN}\cdot\text{m}^3}{EI}\downarrow \qquad \textit{Ans.}$$

EXAMPLE 12.15

Determine the displacement at the end C of the cantilever beam shown in Fig. 12–31. EI is constant.

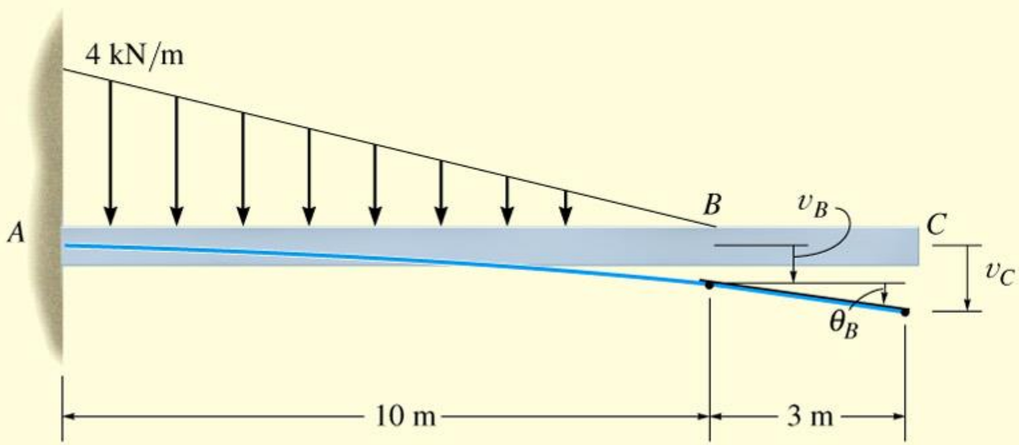


Fig. 12–31

Solution

Using the table in Appendix C for the triangular loading, the slope and displacement at point B are

$$\theta_B = \frac{w_0 L^3}{24EI} = \frac{4\ \text{kN/m}(10\ \text{m})^3}{24EI} = \frac{166.67\ \text{kN}\cdot\text{m}^2}{EI}$$

$$v_B = \frac{w_0 L^4}{30EI} = \frac{4\ \text{kN/m}(10\ \text{m})^4}{30EI} = \frac{1333.33\ \text{kN}\cdot\text{m}^3}{EI}$$

The unloaded region BC of the beam remains straight, as shown in Fig. 12–31. Since θ_B is small, the displacement at C becomes

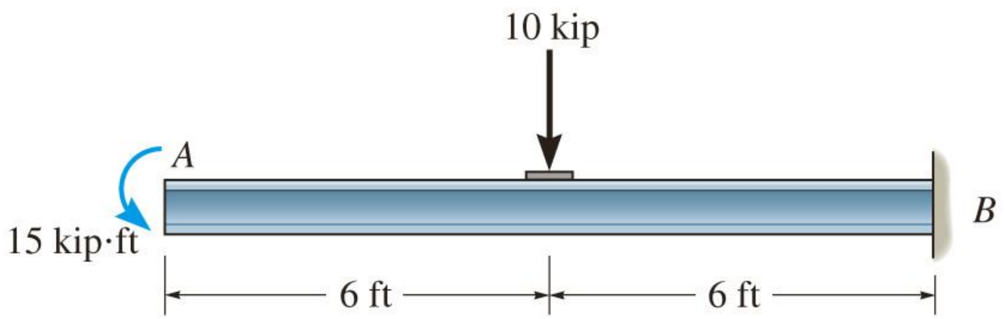
$$(+\downarrow) \qquad v_C = v_B + \theta_B\,(3\ \text{m})$$

$$= \frac{1333.33\ \text{kN}\cdot\text{m}^3}{EI} + \frac{166.67\ \text{kN}\cdot\text{m}^2}{EI}(3\ \text{m})$$

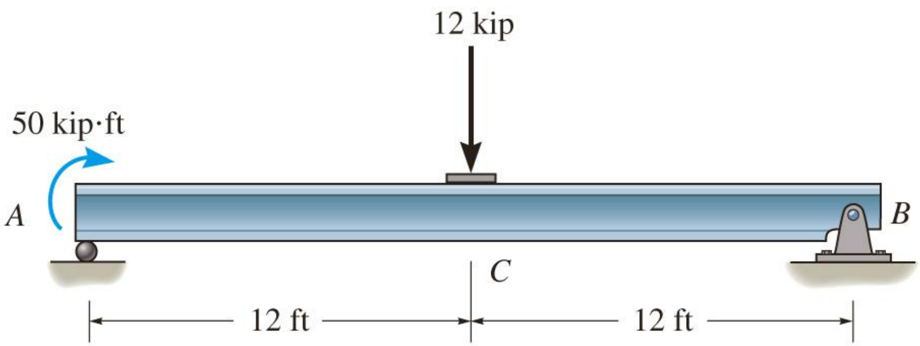
$$= \frac{1833\ \text{kN}\cdot\text{m}^3}{EI}\downarrow$$

Ans.

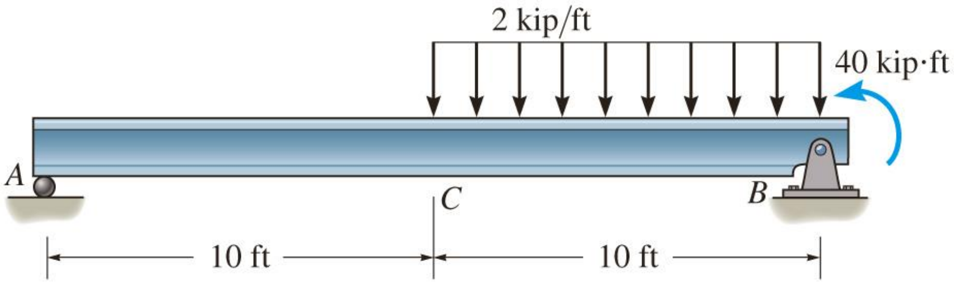
برای موارد چگونه زیر از برهم نهی استفاده می کنید؟



1.



2.



3.

تمرین: تیر با تکیه گاه های ساده نشان داده شده یک بار یکنواخت با شدت 2 kip/ft را تحمل می کند. بر اساس استاندادرد، بیشترین مقدار تغییر شکل نباید از 1/360 طول بخش تجاوز کند. از ضمیمه B کتاب فولاد سبک وزن -Z 36 را انتخاب کنید را به گونه ای که این شرایط را برآورده کند. همچنین بررسی کنید که تنش ها در قابل قبول باشند اگر σ_{allow} = 24 ksi و τ_{allow} = 14 ksi. .