پاسخ تمرین سری $\underline{7}$

سوال اول) $x = 2avu\cos\varphi \quad , \quad y = 2avu\sin\varphi \quad , \quad z = a(u^2 - v^2)$

$$\frac{\partial r}{\partial q_i} = h_i\,\hat{q}_i \qquad \frac{\partial r}{\partial q_i}\cdot\frac{\partial r}{\partial q_i} = h_i^2\,\hat{q}_i^2 = h_i^2$$

$$r = (2avu\cos\varphi)\hat{i} + (2avu\sin\varphi)\hat{j} + a(u^2 - v^2)\hat{k}$$

$$\frac{\partial r}{\partial v} = (2au\cos\varphi)\hat{i} + (2au\sin\varphi)\hat{j} + (-2av)\hat{k}$$

$$\frac{\partial r}{\partial v}\cdot\frac{\partial r}{\partial v} = h_v^2 = 4a^2u^2\cos^2\varphi + 4a^2u^2\sin^2\varphi + 4a^2v^2 = 4a^2u^2(\cos^2\varphi + \sin^2\varphi) + 4a^2v^2$$

$$= 4a^2(u^2 + v^2) \qquad \Rightarrow \boxed{h_v = 2a\sqrt{(u^2+v^2)}}$$

$$\frac{\partial r}{\partial u} = (2av\cos\varphi)\hat{i} + (2av\sin\varphi)\hat{j} + (2au)\hat{k}$$

$$\frac{\partial r}{\partial u}\cdot\frac{\partial r}{\partial u} = h_u^2 = 4a^2v^2\cos^2\varphi + 4a^2v^2\sin^2\varphi + 4a^2u^2 = 4a^2(v^2 + u^2)$$

$$\Rightarrow \boxed{h_u = 2a\sqrt{v^2+u^2}}$$

$$\frac{\partial r}{\partial \varphi} = (-2avu\sin\varphi)\hat{i} + (2avu\cos\varphi)\hat{j}$$

$$\frac{\partial r}{\partial \varphi}\cdot\frac{\partial r}{\partial \varphi} = h_\varphi^2 = 4a^2v^2u^2\sin^2\varphi + 4a^2v^2u^2\cos^2\varphi = 4a^2v^2u^2$$

$$\Rightarrow \boxed{h_\varphi = 2avu}$$

الف) $\nabla f = \sum_i \hat{q}_i \frac{1}{h_i} \frac{\partial f}{\partial q_i}$

$$\nabla f = \frac{1}{2a\sqrt{u^2+v^2}} \frac{\partial f}{\partial v} \hat{v} + \frac{1}{2a\sqrt{u^2+v^2}} \frac{\partial f}{\partial u} \hat{u} + \frac{1}{2avu} \frac{\partial f}{\partial \varphi} \hat{\varphi}$$

ب) $\nabla \cdot A = \left[\frac{\partial}{\partial v}(A_v h_u h_\varphi) + \frac{\partial}{\partial u}(A_u h_v h_\varphi) + \frac{\partial}{\partial \varphi}(A_\varphi h_v h_u) \right] \frac{1}{h_v h_u h_\varphi}$

$$= \frac{1}{8a^3 vu(u^2+v^2)} \left[4a^2 u \frac{\partial}{\partial v}\left(A_v v\sqrt{v^2+u^2}\right) + 4a^2 v \frac{\partial}{\partial u}\left(A_u u\sqrt{v^2+u^2}\right) + 4a^2(v^2+u^2) \frac{\partial A_\varphi}{\partial \varphi} \right]$$

ج) $\nabla \times A = \frac{1}{h_v h_u h_\varphi} \begin{vmatrix} \hat{v}(2a(v^2+u^2)) & \hat{u}(2a(v^2+u^2)) & \hat{\varphi}\, 2avu \\ \frac{\partial}{\partial v} & \frac{\partial}{\partial u} & \frac{\partial}{\partial \varphi} \\ 2a(v^2+u^2)A_v & 2a(v^2+u^2)A_u & 2avu A_\varphi \end{vmatrix}$

د) $\nabla \cdot \nabla f = \frac{1}{h_v h_u h_\varphi} \left[\frac{\partial}{\partial v}\left(\frac{h_u h_\varphi}{h_v} \frac{\partial f}{\partial v} \right) + \frac{\partial}{\partial u}\left(\frac{h_v h_\varphi}{h_u} \frac{\partial f}{\partial u} \right) + \frac{\partial}{\partial \varphi}\left(\frac{h_v h_u}{h_\varphi} \frac{\partial f}{\partial \varphi} \right) \right]$

سوال دوم)

$$\oint v\cdot da = \int (\nabla\cdot v)\, d\tau$$

$$\nabla\cdot v = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\left(\rho(2+\sin^2\theta)\right)\right) + \frac{1}{\rho}\frac{\partial}{\partial\theta}\left(\rho\sin\theta\cos\theta\right) + \frac{\partial}{\partial z}(3z^2)$$

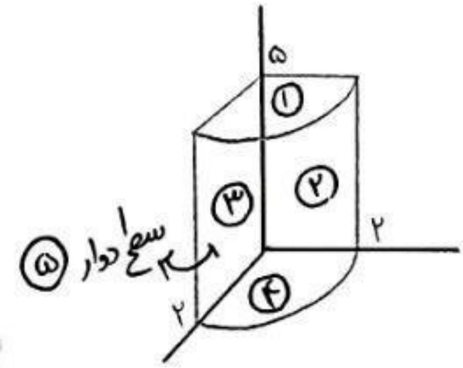
$$= \frac{1}{\rho}\left(2\rho(2+\sin^2\theta)\right) + \frac{1}{\rho}\left(\rho(\cos^2\theta - \sin^2\theta)\right) + 6z$$

$$= 5 + 6z$$

$$\int(\nabla\cdot v)\, d\tau = \int (5+6z)\rho\, d\rho\, d\theta\, dz = 5\int_0^2 \rho\, d\rho \int_0^{\frac{\pi}{2}} d\theta \int_0^5 dz \; +$$

$$6\int_0^2 \rho\, d\rho \int_0^{\frac{\pi}{2}} d\theta \int_0^5 z\, dz = 5(2)\left(\frac{\pi}{2}\right)(5) + 6(2)\left(\frac{\pi}{2}\right)\left(\frac{25}{2}\right)$$

$$= 25\pi + 75\pi = \underline{100\pi}$$



① سطح بالا : $z=5$, $da = \rho\, d\rho\, d\theta\, \hat{z}$

$$v\cdot da = 3z^2 \rho\, d\rho\, d\theta = 75\rho\, d\rho\, d\theta$$

$$\int v\cdot da = 75\int_0^2 \rho\, d\rho \int_0^{\frac{\pi}{2}} d\theta = 75(2)\left(\frac{\pi}{2}\right) = \underline{75\pi}$$

② سطح پشت : $\theta = \frac{\pi}{2}$, $da = d\rho\, dz\, \hat{\theta}$ $\qquad v\cdot da = \rho\sin\theta\cos\theta\, d\rho\, dz = 0$

$$\int v\cdot da = 0$$

③ سطح سمت چپ : $\theta = 0$ $\qquad \int v\cdot da = 0$

④ سطح پایین : $z = 0$ $\qquad \int v\cdot da = 0$

⑤ سطح دوار : $\rho = 2$, $da = \rho\, d\theta\, dz\, \hat{\rho}$

$$\int V\cdot da = \int \rho(2+\sin^2\theta)\,\rho\, d\theta\, dz = 4\int (2+\sin^2\theta)\, d\theta\, dz$$

$$= 4\int_0^{\frac{\pi}{2}} (2+\sin^2\theta)\, d\theta \int_0^5 dz = 20\left[\int_0^{\frac{\pi}{2}} 2\, d\theta + \int_0^{\frac{\pi}{2}} \sin^2\theta\, d\theta\right]$$

$$* \ \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$= 20\left[2\left(\frac{\pi}{2}\right) + \int_0^{\frac{\pi}{2}} \left(\frac{1-\cos 2\theta}{2}\right) d\theta\right] =$$

$$20\left[\pi + \frac{1}{2}\theta\Big|_0^{\frac{\pi}{2}} - \frac{1}{2}\sin 2\theta\Big|_0^{\frac{\pi}{2}}\right] = 20\left[\pi + \frac{\pi}{4}\right] = 20\left(\frac{5\pi}{4}\right) = 25\pi$$

$$\Rightarrow \int V\cdot da = 75\pi + 0 + 0 + 0 + 25\pi = 100\pi \quad \checkmark$$

سوال سوم)

$$\vec{A} = \frac{ky}{x^2+y^2}\,\hat{i} - \frac{kx}{x^2+y^2}\,\hat{j}$$

شکل نشانگر مسیر بسته‌ی دایره ای به معادله‌ی $x^2+y^2=a^2$ است.

$$\oint \vec{A}\cdot d\vec{l} = \int \vec{\nabla}\times\vec{A}\cdot d\vec{a}$$

سمت چپ: $\oint\left(\left(\frac{ky}{x^2+y^2}\right)\hat{i} - \left(\frac{kx}{x^2+y^2}\right)\hat{j}\right)\cdot\left(dx\,\hat{i} + dy\,\hat{j}\right)$

$$x^2+y^2=a^2 \ ; \quad x = a\cos\theta,\ dx = -a\sin\theta\, d\theta;\ y = a\sin\theta,\ dy = a\cos\theta\, d\theta$$

$$\oint A\cdot dl = \oint \left(\frac{k(a\sin\theta)}{a^2}\hat{i} - \frac{k(a\cos\theta)}{a^2}\hat{j} \right)\cdot\left(-a\sin\theta\, d\theta\,\hat{i} + a\cos\theta\, d\theta\,\hat{j} \right)$$

$$= k\oint \left(-\sin^2\theta - \cos^2\theta\right) d\theta = -k\int_0^{2\pi} d\theta = -2k\pi$$

سمت راست:

$$\vec{\nabla}\times\vec{A} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{ky}{x^2+y^2} & \frac{-kx}{x^2+y^2} & 0 \end{vmatrix} = \Big(0 - \underbrace{\frac{\partial}{\partial z}\frac{kx}{x^2+y^2}}_{0}\Big)\hat{i} - \Big(0 - \underbrace{\frac{\partial}{\partial z}\frac{ky}{x^2+y^2}}_{0}\Big)\hat{j}$$

$$+ \left(\frac{\partial}{\partial x}\left(\frac{-kx}{x^2+y^2}\right) - \frac{\partial}{\partial y}\left(\frac{ky}{x^2+y^2}\right)\right)\hat{k}$$

$$= \left(\frac{-k(x^2+y^2) - (-kx)(2x)}{(x^2+y^2)^2} - \frac{k(x^2+y^2) - (ky)(2y)}{(x^2+y^2)^2} \right)\hat{k}$$

$$= \frac{kx^2 - ky^2}{(x^2+y^2)^2} - \frac{kx^2 - ky^2}{(x^2+y^2)^2} = 0$$

$$\Rightarrow \int \vec{\nabla}\times\vec{A}\cdot d\vec{a} = 0$$

$$0 \neq -2k\pi$$

قضیه برقرار نیست. چرا؟ اگر به شکلی که بردار $\vec{A}$ تعریف شده است دقت کنیم در جایی که

به ازای x, y هردو صفر داشته باشیم تعریف نداریم.

سوال چهارم)

از قضیه دیورژانس استفاده می‌کنیم:

الف)

$$\nabla . E = \frac{\rho}{\varepsilon_0}$$

$$\int \nabla . E \, d\tau = \frac{1}{\varepsilon_0} \int \rho \, d\tau$$

$$\Rightarrow \int E . da = \frac{q}{\varepsilon_0}$$

$$\int \nabla . E \, d\tau = \int E . da$$

داخل: $E \int da = \frac{1}{\varepsilon_0} \int \rho(r) \, d\tau \quad ; \; d\tau = r^2 \sin\theta \, dr \, d\theta \, d\varphi$

$$4\pi r^2 E = \frac{1}{\varepsilon_0} \int_0^r \alpha r \left(r^2 \sin\theta \, dr \, d\theta \, d\varphi \right)$$

$* \; \sin\theta \, d\theta \, d\varphi = d\Omega$

$** \; \int d\Omega = 4\pi$

$$= \frac{1}{\varepsilon_0} \int_0^r \alpha r \; r^2 dr \, d\Omega = \frac{4\pi\alpha}{\varepsilon_0} \int_0^r r^3 dr$$

$$= \frac{4\pi\alpha}{\varepsilon_0} \frac{r^4}{4}$$

$$\Rightarrow \quad 4\pi r^2 E = \frac{4\pi\alpha}{\varepsilon_0} \frac{r^4}{4} \Rightarrow E = \frac{\alpha}{4\varepsilon_0} r^2$$

خارج: $E \int da = \frac{1}{\varepsilon_0} \int_0^R \rho(r) \, d\tau$

$$4\pi r^2 E = \frac{1}{\varepsilon_0} \int_0^R \alpha r \; r^2 dr \, d\Omega = \frac{\alpha}{\varepsilon_0} (4\pi) \frac{R^4}{4}$$

$$\Rightarrow E = \frac{\alpha R^4}{\varepsilon_0 4 r^2}$$

ب) $\nabla \times B = \mu_0 J$

با استفاده از استوکس

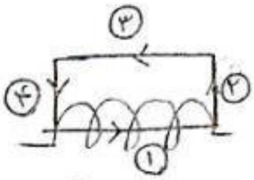
$$\int \nabla \times B \cdot da = \oint B \cdot dl$$

$$\int \nabla \times B \cdot da = \mu_0 \int J \cdot da = \mu_0 I$$

$$\oint B \cdot dl = \mu_0 I$$

یه مسیر بسته دلخواه انتخاب کنیم و انتگرال مسیر بسته را درنظر بگیریم:

سیم لوله با N دور



$$B_1 l + 0 + \underbrace{\cancel{B_3}}_{\to 0} l + 0 = Bl = \mu N I$$

$$\Rightarrow B = \frac{\mu N I}{l}$$

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$$\vec{G} = \frac{\sqrt{x^2+y^2+z^2}}{\sqrt{x^2+y^2}}\left[(x-y)\hat{i}+(x+y)\hat{j}\right]$$

ابتدا قبل از حل مسئله چون شکل رویه ای که انتگرال میگیریم یک محدوده عرقچین است پس بهترین کار آن است که $\vec{G}$ به دستگاه مختصات کروی ببریم.

$$\begin{cases} x = r\sin\theta\cos\varphi \\ y = r\sin\theta\sin\varphi \\ z = r\cos\theta \end{cases}$$

با استفاده از این روابط $\vec{G}$ را بدست می آوریم

$$\begin{cases} \hat{i} = \sin\theta\cos\varphi\,\hat{r} + \cos\theta\cos\varphi\,\hat{\theta} - \sin\varphi\,\hat{\varphi} \\ \hat{j} = \sin\theta\sin\varphi\,\hat{r} + \cos\theta\sin\varphi\,\hat{\theta} + \cos\varphi\,\hat{\varphi} \\ \hat{z} = \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta} \end{cases}$$

$$\vec{G} = \frac{\sqrt{(r\sin\theta\cos\varphi)^2+(r\sin\theta\sin\varphi)^2+(r\cos\theta)^2}}{\sqrt{(r\sin\theta\cos\varphi)^2+(r\sin\theta\sin\varphi)^2}}\left[(x-y)\hat{i}+(x+y)\hat{j}\right] = \frac{\cancel{r}}{\cancel{r}\sin\theta}\left[(x-y)\hat{i}+(x+y)\hat{j}\right]$$

$$= \frac{1}{\cancel{\sin\theta}}\left[(r\cancel{\sin\theta}\cos\varphi - r\cancel{\sin\theta}\sin\varphi)\hat{i} + (r\cancel{\sin\theta}\cos\varphi + r\cancel{\sin\theta}\sin\varphi)\hat{j}\right]$$

$$= r\left[(\cos\varphi - \sin\varphi)\hat{i} + (\cos\varphi + \sin\varphi)\hat{j}\right]$$

$$= r\left[(\cos\varphi - \sin\varphi)(\sin\theta\cos\varphi\,\hat{r} + \cos\theta\cos\varphi\,\hat{\theta} - \sin\varphi\,\hat{\varphi}) + (\cos\varphi + \sin\varphi)(\sin\theta\sin\varphi\,\hat{r} + \cos\theta\sin\varphi\,\hat{\theta} + \cos\varphi\,\hat{\varphi})\right]$$

$$= r\left[\sin\theta\cos^2\varphi\,\hat{r} - \sin\theta\cancel{\cos\varphi\sin\varphi}\,\hat{r} + \cos\theta\cos^2\varphi\,\hat{\theta} - \cos\theta\cancel{\sin\varphi\cos\varphi}\,\hat{\theta} - \cos\varphi\cancel{\sin\varphi}\,\hat{\varphi} + \sin^2\varphi\,\hat{\varphi}\right.$$

$$\left. + \sin\theta\cancel{\cos\varphi\sin\varphi}\,\hat{r} + \sin\theta\sin^2\varphi\,\hat{r} + \cos\theta\cancel{\cos\varphi\sin\varphi}\,\hat{\theta} + \cos\theta\sin^2\varphi\,\hat{\theta} + \cos^2\varphi\,\hat{\varphi} + \sin\varphi\cancel{\cos\varphi}\,\hat{\varphi}\right]$$

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$$\Rightarrow \quad \vec{G}(r,\theta,\varphi) = r\sin\theta\,\hat{r} + r\cos\theta\,\hat{\theta} + r\,\hat{\varphi}$$

$\oint \vec{G}\cdot d\vec{l} = ?$ $\qquad d\vec{l} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\varphi\,\hat{\varphi}$ $\qquad$ (الف

$$\vec{G}\cdot d\vec{l} = r\sin\theta\,dr + r^2\cos\theta\,d\theta + r^2\sin\theta\,d\varphi$$

$r = cte = 2 \rightarrow dr = 0 \qquad \theta = cte = \frac{\pi}{6} \Rightarrow d\theta = 0$

$$\oint \vec{G}\cdot d\vec{l} = \int r^2\sin\theta\,d\varphi = (2)^2\sin\pi/6\int_0^{2\pi} d\varphi = 4\times\frac{1}{2}\times 2\pi = \underline{\underline{4\pi}}$$

$\int_{S_1} \vec{\nabla}\times\vec{G}\cdot d\vec{a}$ $\qquad$ (ب

$$\vec{\nabla}\times\vec{G} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\,\hat{\varphi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\varphi} \\ r\sin\theta & r(r\cos\theta) & r\sin\theta(r) \end{vmatrix}$$

$$\Rightarrow \vec{\nabla}\times\vec{G} = \frac{1}{r^2\sin\theta}\left[r^2\cos\theta\,\hat{r} - 2r^2\sin\theta\,\hat{\theta} + r^2\sin\theta\cos\theta\,\hat{\varphi}\right] = \cot\theta\,\hat{r} - 2\hat{\theta} + \cos\theta\,\hat{\varphi}$$

$$d\vec{a} = r^2\sin\theta\,d\varphi\,d\theta\,\hat{r} + r\,dr\,d\theta\,\hat{\varphi} + r\sin\theta\,dr\,d\varphi\,\hat{\theta}$$

$$(\vec{\nabla}\times\vec{G})\cdot d\vec{a} = r^2\cos\theta\,d\varphi\,d\theta - 2r\sin\theta\,dr\,d\varphi + r\cos\theta\,dr\,d\theta$$

at S_1: $\quad r = cte = 2 \Rightarrow dr = 0$

$$\int_{S_1}(\vec{\nabla}\times\vec{G})\cdot d\vec{a} = \int r^2\sin\theta\,d\varphi\,d\theta = 4\int_0^{\pi/6}\sin\theta\,d\theta\int_0^{2\pi}d\varphi = \underline{\underline{4\pi}}$$

از قسمت قبل : $(\vec{\nabla}\times\vec{G})\cdot d\vec{a} = r^2\cos\theta\, d\varphi \underbrace{d\theta}_{0} - 2r\sin\theta\, dr\, d\varphi + r\cos\theta\, dr\underbrace{d\theta}_{0}$ ج)

$$\theta = cte = \pi/6 \rightarrow d\theta = 0$$

$$\int_{S_2} (\vec{\nabla}\times\vec{G})\cdot d\vec{a} = \int (-2r\sin\theta\, dr\, d\varphi) = -2\sin\frac{\pi}{6}\int_0^2 r\,dr \int_0^{2\pi} d\varphi = -4\pi$$

د)

$$\int_{S_1} \vec{G}\cdot d\vec{a} \qquad \vec{G} = r\sin\theta\,\hat{r} + r\cos\theta\,\hat{\theta} + r\hat{\varphi}$$

$$d\vec{a} = r^2\sin\theta\, d\varphi\, d\theta\,\hat{r} + r\sin\theta\, dr\, d\varphi\,\hat{\theta} + r\, dr\, d\theta\,\hat{\varphi}$$

$$\vec{G}\cdot d\vec{a} = r^3\sin^2\theta\, d\varphi\, d\theta + r^2\sin\theta\cos\theta\, \underbrace{dr}_{0}\, d\varphi + r^2\, \underbrace{dr}_{0}\, d\theta$$

$$\text{at } S_1: \quad r = cte = 2 \quad dr = 0$$

$$\int_{S_1} \vec{G}\cdot d\vec{a} = \int r^3\sin^2\theta\, d\varphi\, d\theta = 8\int_0^{\pi/6}\sin^2\theta \int_0^{2\pi} d\varphi = 2\pi\left(-\sqrt{3} + \frac{2\pi}{3}\right)$$

$$\vec{G}\cdot d\vec{a} = r^3\sin^2\theta\, \underbrace{d\varphi\, d\theta}_{0} + r^2\sin\theta\cos\theta\, dr\, d\varphi + r^2\, dr\, \underbrace{d\theta}_{0}$$ ه)

$$\text{at } S_2: \quad \theta = cte = \pi/6 \Rightarrow d\theta = 0$$

$$\int_{S_2} \vec{G}\cdot d\vec{a} = \int r^2\sin\theta\cos\theta\, dr\, d\varphi = \sin\pi/6\,\cos\pi/6 \int_0^2 r^2 dr \int_0^{2\pi} d\varphi = 4\sqrt{3}\,\frac{\pi}{3}$$

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$$\int_V \vec{\nabla}\cdot\vec{G}\, d\tau \qquad \vec{\nabla}\cdot\vec{G} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 G_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, G_\theta) + \frac{1}{r\sin\theta}\frac{\partial G_\varphi}{\partial\varphi} \quad \text{(ی}$$

$$\Rightarrow \vec{\nabla}\cdot\vec{G} = \frac{1}{r^2}\frac{\partial}{\partial r}\Big(\underbrace{r^2 (r\sin\theta)}_{r^3\sin\theta}\Big) + \frac{1}{\cancel{r}\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, \cancel{r}\cos\theta) + \cancel{\frac{1}{r\sin\theta}\frac{\partial r}{\partial\varphi}}^{\,0}$$

$$\Rightarrow \vec{\nabla}\cdot\vec{G} = 3\sin\theta + \frac{\cos 2\theta}{\sin\theta}$$

$$d\tau = r^2\sin\theta\, dr\, d\theta\, d\varphi$$

$$\int_V \vec{\nabla}\cdot\vec{G}\, d\tau = \int \Big(3\sin\theta + \frac{\cos 2\theta}{\sin\theta}\Big) r^2 \sin\theta\, dr\, d\theta\, d\varphi$$

$$= 3\int_0^2 r^2 dr \int_0^{\pi/6} \sin^2\theta\, d\theta \int_0^{2\pi} d\varphi + \int_0^2 r^2 dr \int_0^{\pi/6} \frac{\cos 2\theta}{\cancel{\sin\theta}} \cancel{\sin\theta}\, d\theta \int_0^{2\pi} d\varphi = \underline{\underline{2\pi\Big(-\frac{1}{\sqrt3} + \frac{2\pi}{3}\Big)}}$$

ز) در واقع این سوال برای بررسی قضیه‌ی استوکس و دیورژانس است

قضیه استوکس بیان می‌کند: $\int \vec{\nabla}\times\vec{V}\cdot d\vec{a} = \oint \vec{V}\cdot d\vec{l}$ "الف" این سوال سمت راست معادله

استوکس است قسمت ب و ج هرکدام سطح متفاوت برای سمت چپ معادله هستند.

قضیه‌ی استوکس برقرار است $\checkmark \leftarrow 4\pi =$ قسمت ب برای S_1 و $4\pi =$ قسمت الف

قضیه استوکس برقرار است چون سطح اختیاری $\checkmark \leftarrow -4\pi =$ قسمت ج برای S_2 و $4\pi =$ قسمت الف

بوده و برای حلقه پادساعتگرد چرخیدیم پس منفی ظاهر شده است.

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پس دیدیم برای محاسبه‌ی میدان برداری تحلیلی مانند $\vec{G}$ که روی سطح باشیم می‌توانیم برویم و آن را طبق قضیه‌ی استوکس روی مرز سطح حساب کنیم و برابر می‌کنند.

در قسمت "د" و "و" و "ه" قضیه‌ی دیورژانس را می‌بینیم:

$$\int_V \vec{\nabla}\cdot\vec{v}\, d\tau = \oint_{\partial V} \vec{v}\cdot d\vec{a}$$

قسمت د و قسمت ه با هم سمت راست معادله را می‌دهند:

$$\oint_S \vec{G}\cdot d\vec{a} = \int_{S_1} \vec{G}\cdot d\vec{a} + \int_{S_2} \vec{G}\cdot d\vec{a}$$

$$= 2\pi\left(-\sqrt{3} + \frac{2\pi}{3}\right) + 4\sqrt{3}\,\frac{\pi}{3}$$

$$= 2\pi\left(-\frac{1}{\sqrt{3}} + \frac{2\pi}{3}\right)$$

<= قضیه‌ی دیورژانس برقرار است.

قسمت "و" هم سمت چپ معادله است $= 2\pi\left(\frac{-1}{\sqrt{3}} + \frac{2\pi}{3}\right)$

پس دیدیم برای محاسبه‌ی میدان برداری تحلیلی مانند $\vec{G}$ که داخل حجم داریم G را محاسبه کنیم می‌توانیم برویم روی سطح و آن را محاسبه کنیم.

P4PCO