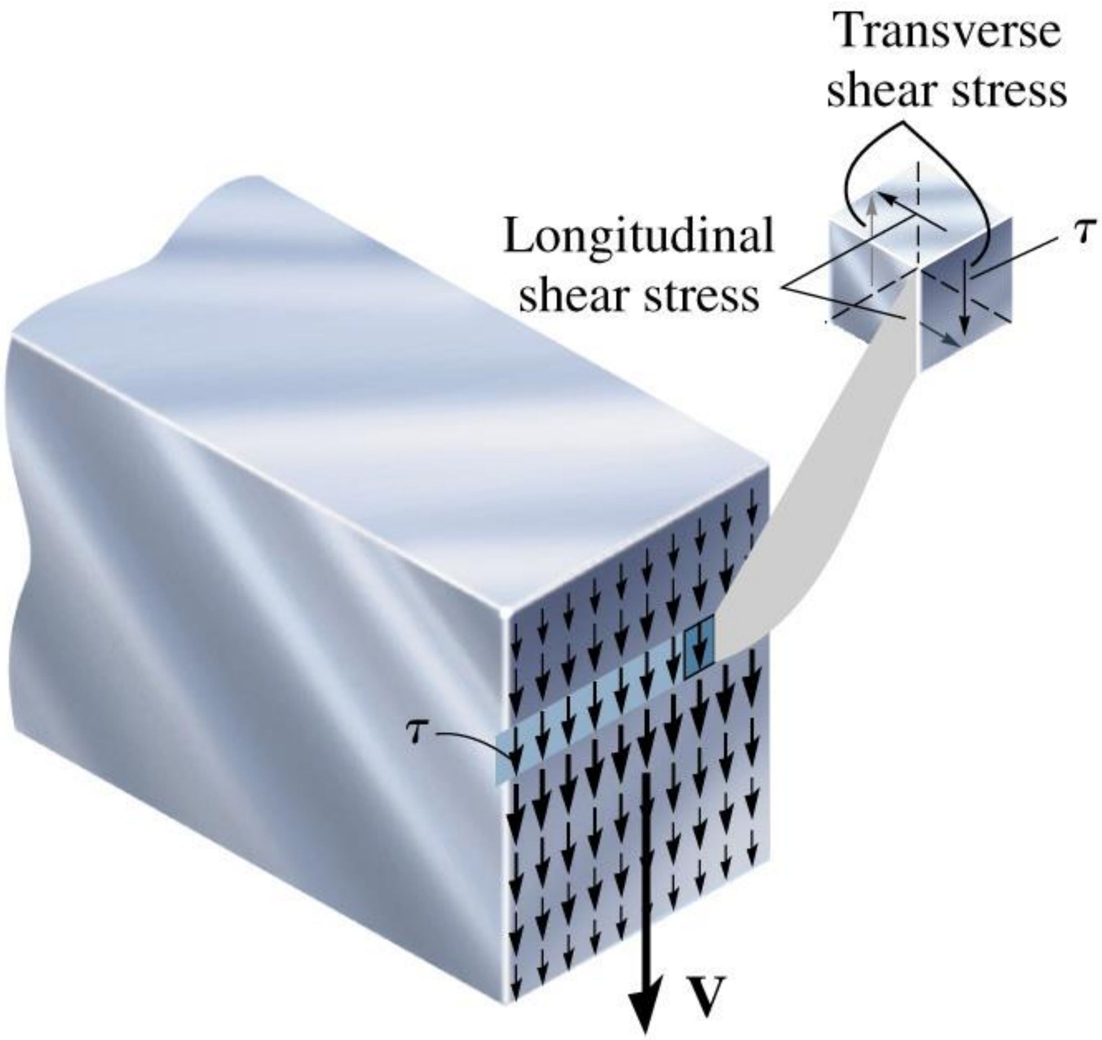
فصل ۷- برش عرضی (تنش برشی تیر)

۷-۱: برش در اعضای صاف و مستقیم

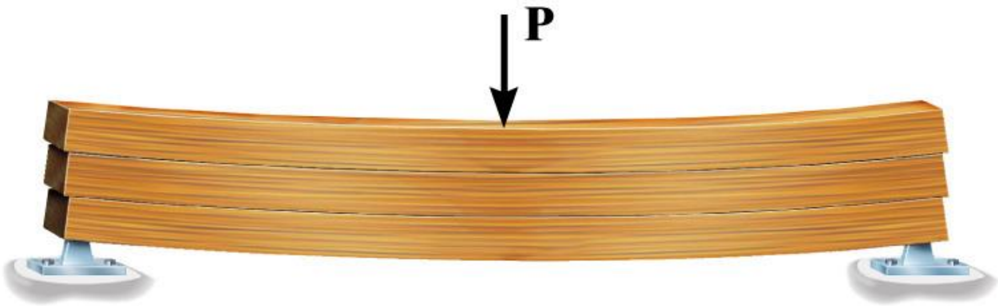


نیروی برش داخلی، تغییر شکل، کرنش و تنش برسی ایجاد میکند.

نکته: به دلیل طبیعت تنش برشی، سبب کرنش طولی و جانبی ایجاد می شود.

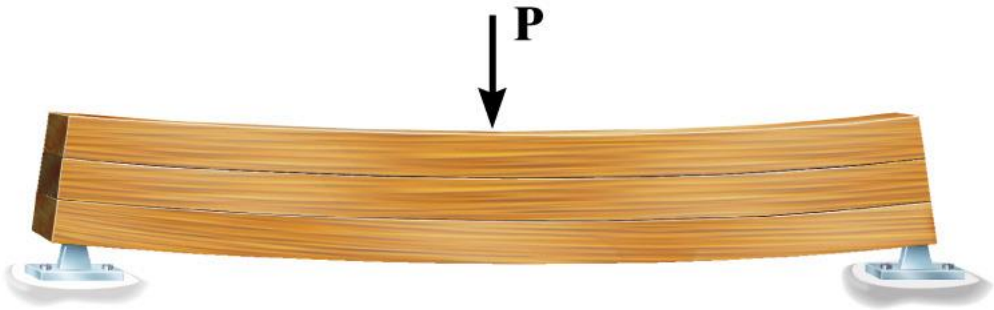
مثال فیزیکی:

وقتی که الوارها به همدیگر چسبانده میشوند، تنش برشی در سطوحی ایجاد میشود که از لغزش جلوگیری میکند.



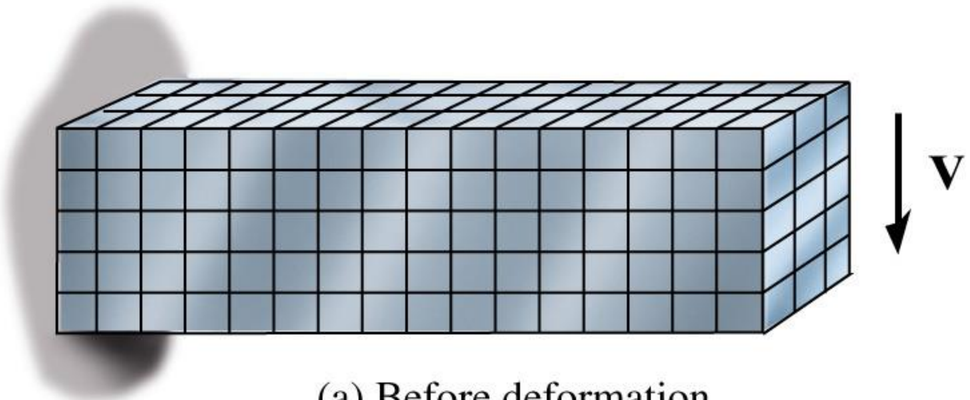
Boards not bonded together

(a)

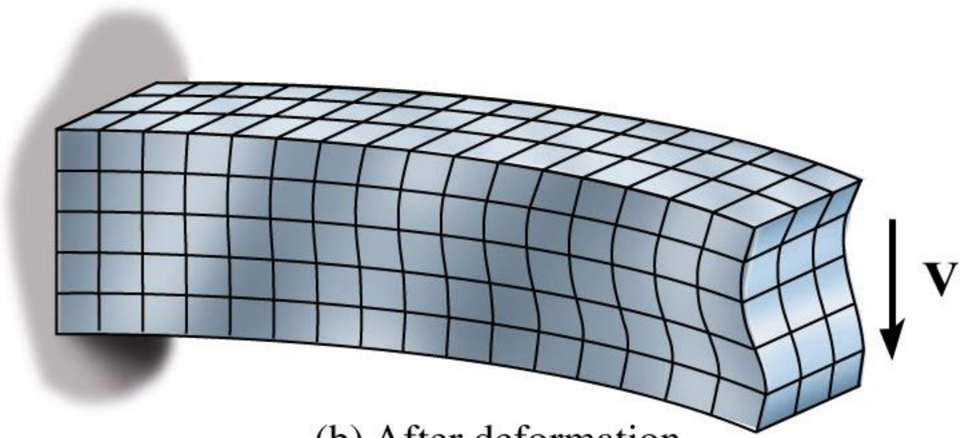


Boards bonded together

(b)



(a) Before deformation

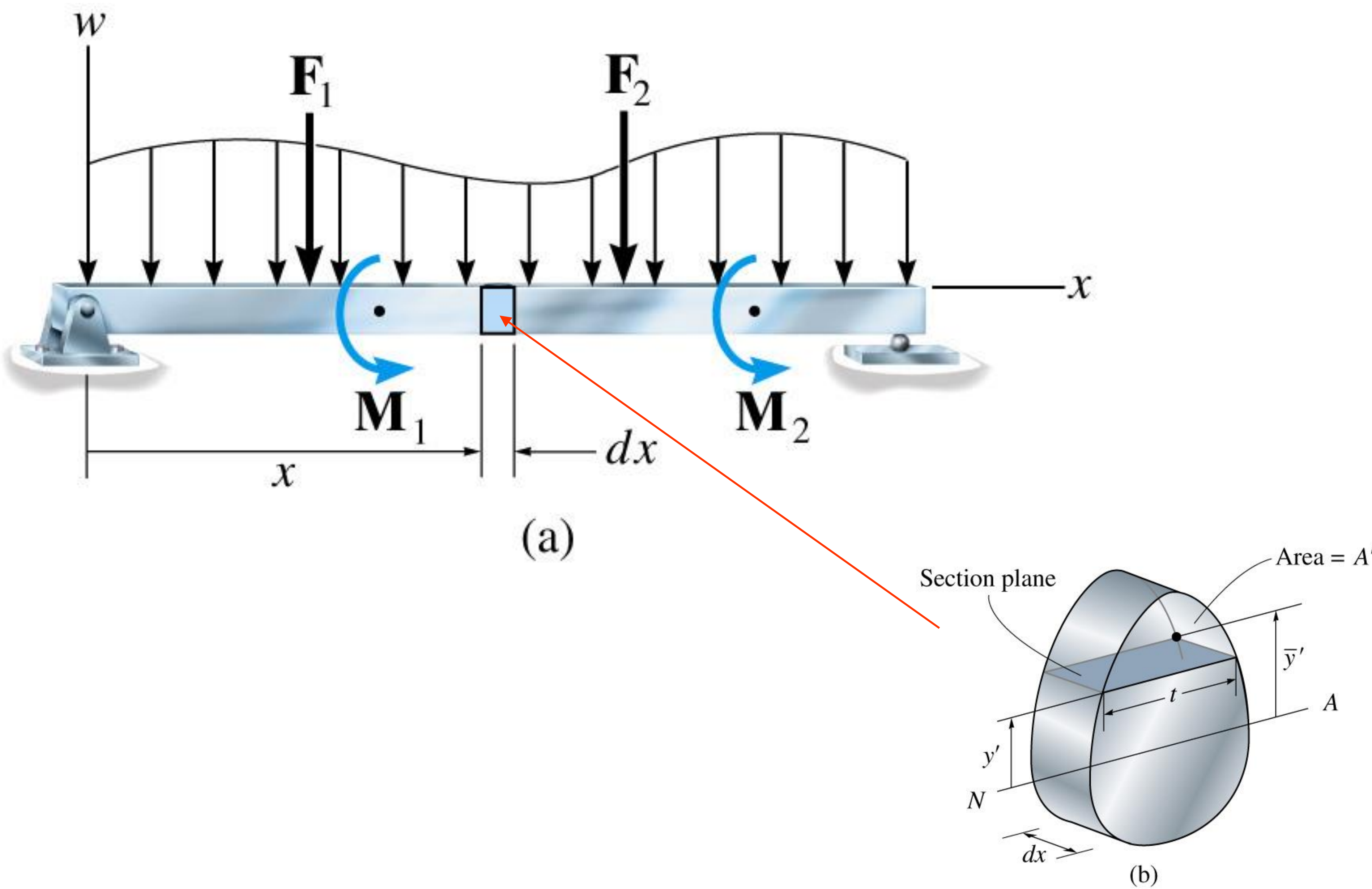


(b) After deformation

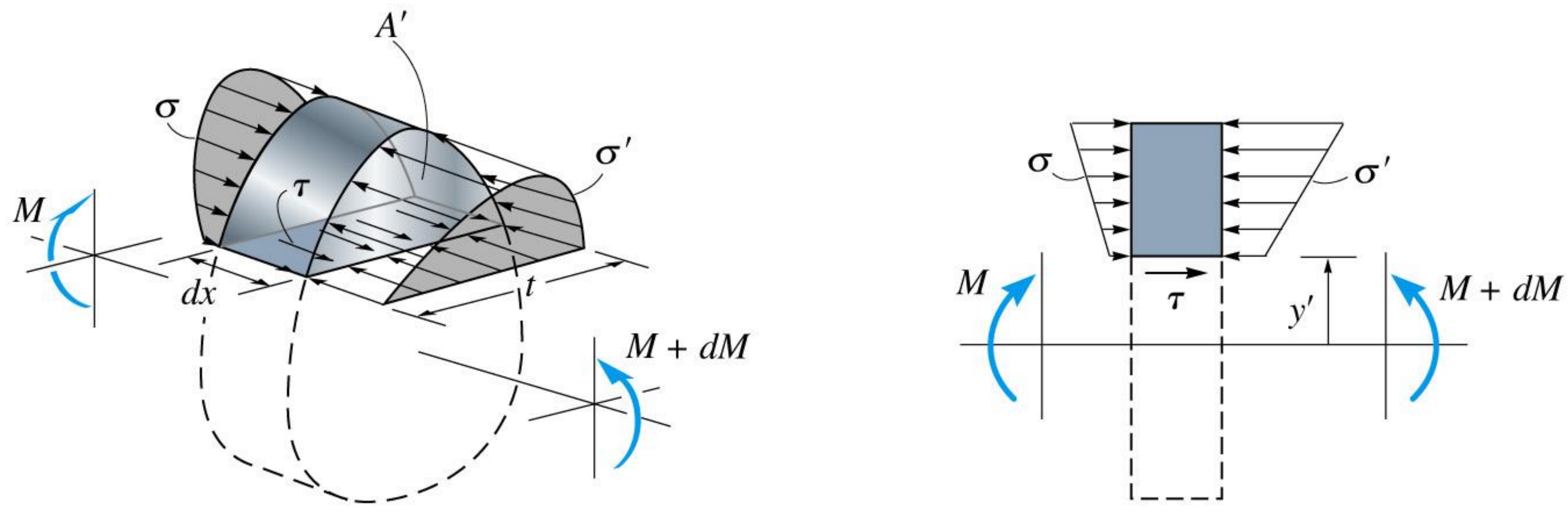
به تغییر شکل ها توجه کنید:

نکته کلیدی، تغییر شکل ها یکنواخت نیستند !!

۶-۲: فرمول تنش برشی:



استخراج معادله تنش برشی تیر:



Three-dimensional view (d) Profile view

$$\underset{\leftarrow}{+}\sum F_x = 0$$

$$\int_{A'} \sigma' dA - \int_{A'} \sigma dA - \tau(t dx) = 0$$

$$\int_{A'} \left(\frac{M + dM}{I}\right) y dA - \int_{A'} \left(\frac{M}{I}\right) y dA - \tau(t dx) = 0$$

ادامه استخراج معادله تنش برشی تیر:

$$\tau = \frac{1}{It}\left(\frac{dM}{dx}\right)\int_{A'} y\,dA$$

یادآوری

dM/dx = V

Q= ممان اول سطح

$$\tau = \frac{VQ}{It}$$

نیروی برشی داخلی

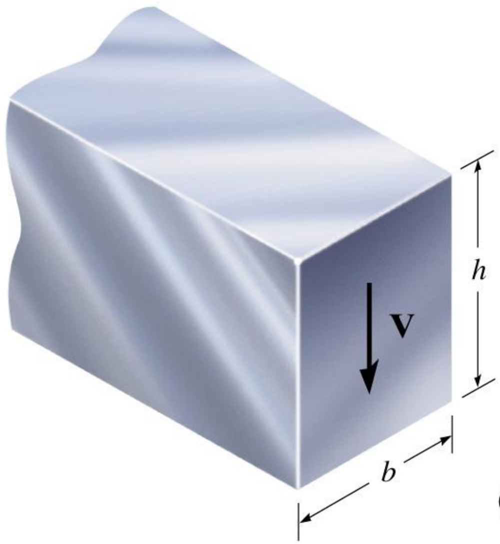
ممان اول سطح در نقطه مورد نظر

ضخامت سطح مقطع در نقطه مورد نظر

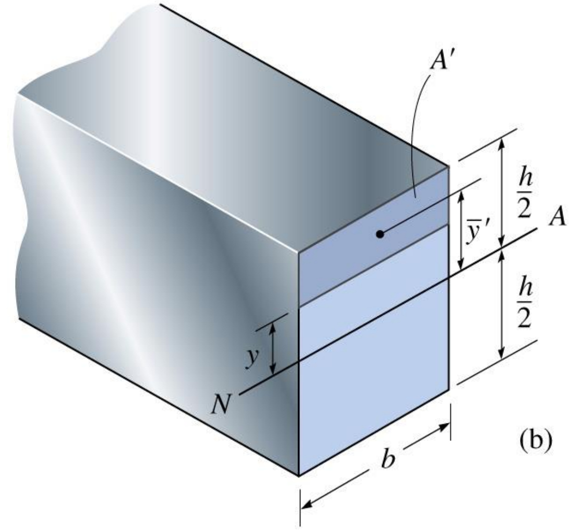
ممان اینرسی کل سطح مقطع

$$Q = \bar{y}' \cdot A'$$

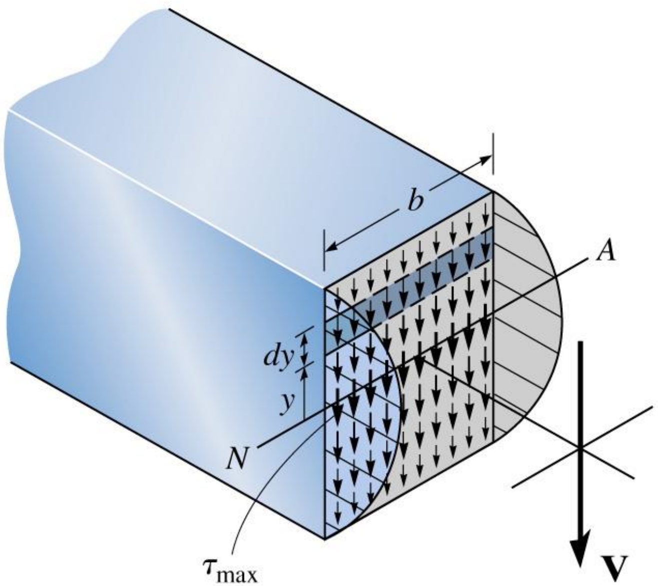
مثال: سطح مقطع مستطیلی



(a)

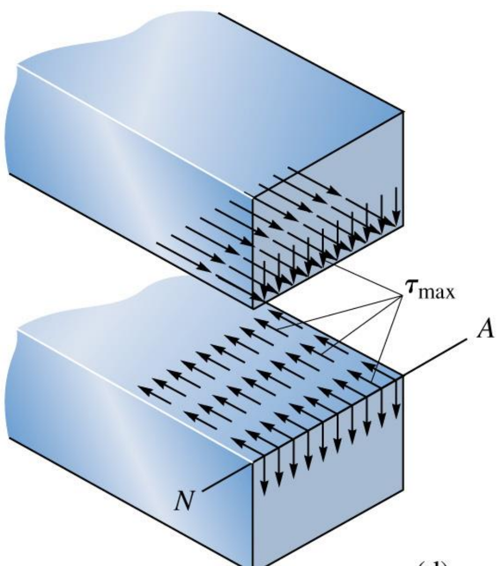


(b)



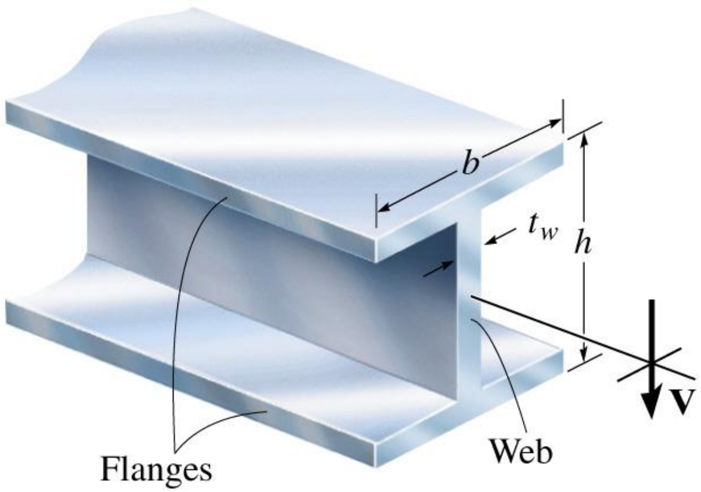
Shear–stress distribution

(c)

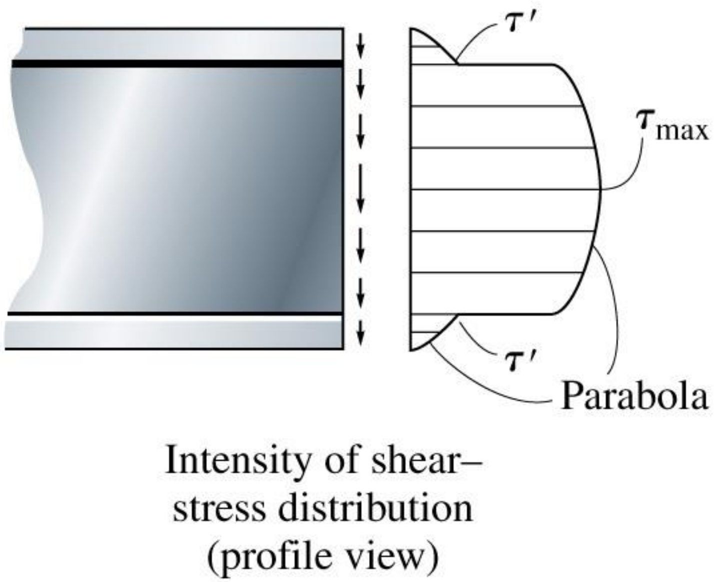


(d)

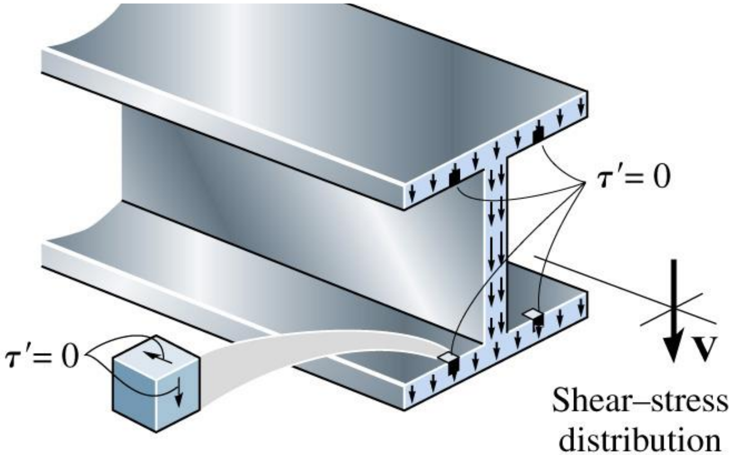
مثال: تیر I شکل



(a)



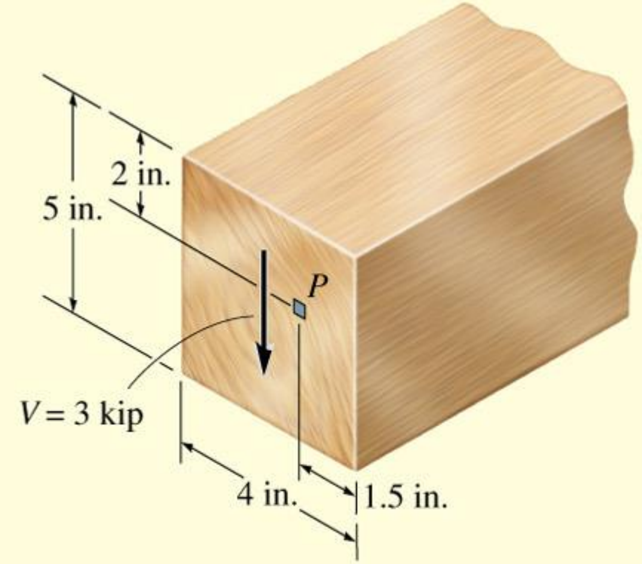
(c)



(b)

EXAMPLE 7.1

The beam shown in Fig. 7–10*a* is made of wood and is subjected to a resultant internal vertical shear force of $V = 3$ kip. (a) Determine the shear stress in the beam at point *P*, and (b) compute the maximum shear stress in the beam.



(a)

Solution

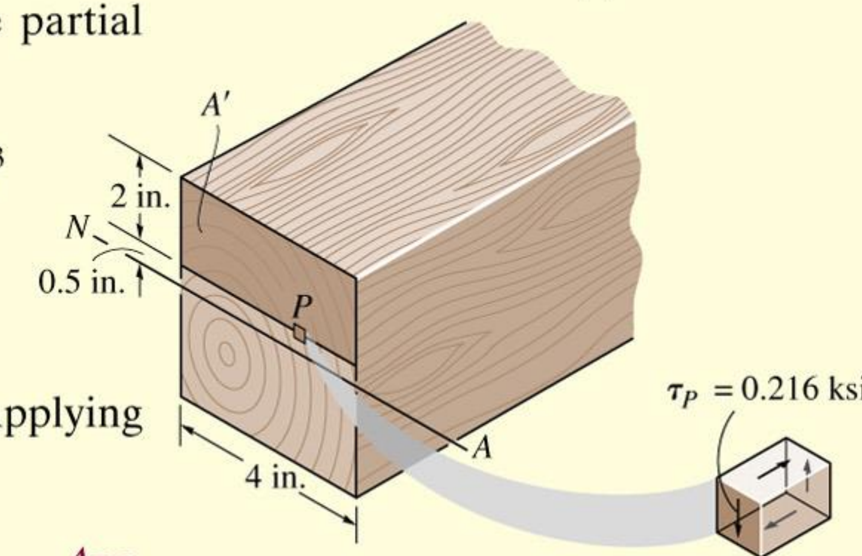
Part (a).

Section Properties. The moment of inertia of the cross-sectional area computed about the neutral axis is

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(4\text{ in.})(5\text{ in.})^3 = 41.7\text{ in}^4$$

A horizontal section line is drawn through point *P* and the partial area A' is shown shaded in Fig. 7–10*b*. Hence

$$Q = \bar{y}'A' = \left[0.5\text{ in.} + \frac{1}{2}(2\text{ in.})\right](2\text{ in.})(4\text{ in.}) = 12\text{ in}^3$$



(b) (c)

Shear Stress. The shear force at the section is $V = 3$ kip. Applying the shear formula, we have

$$\tau_P = \frac{VQ}{It} = \frac{(3\text{ kip})(12\text{ in}^3)}{(41.7\text{ in}^4)(4\text{ in.})} = 0.216\text{ ksi} \qquad \textit{Ans.}$$

Since τ_P contributes to V, it acts downward at P on the cross section. Consequently, a volume element of the material at this point would have shear stresses acting on it as shown in Fig. 7–10*c*.

Part (b).

Section Properties. Maximum shear stress occurs at the neutral axis, since t is constant throughout the cross section and Q is largest for this case. For the dark shaded area A' in Fig. 7–10*d*, we have

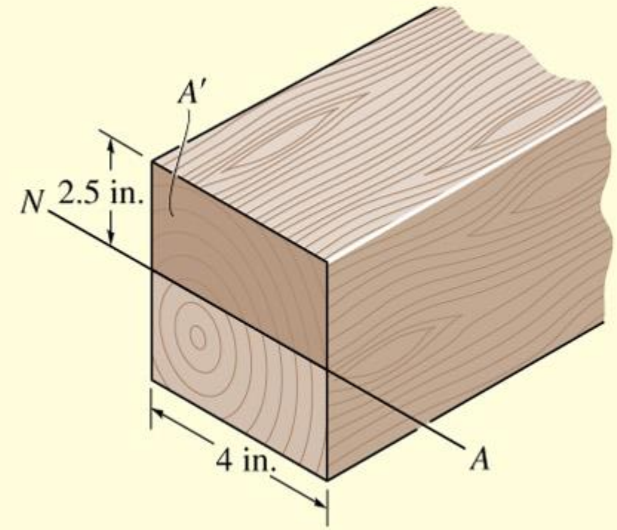
$$Q = \bar{y}'A' = \left[\frac{2.5\text{ in.}}{2}\right](4\text{ in.})(2.5\text{ in.}) = 12.5\text{ in}^3$$

Shear Stress. Applying the shear formula yields

$$\tau_{max} = \frac{VQ}{It} = \frac{(3\text{ kip})(12.5\text{ in}^3)}{(41.7\text{ in}^4)(4\text{ in.})} = 0.225\text{ ksi} \qquad \textit{Ans.}$$

Note that this is equivalent to

$$\tau_{max} = 1.5\frac{V}{A} = 1.5\frac{3\text{ kip}}{(4\text{ in.})(5\text{ in.})} = 0.225\text{ ksi} \qquad \textit{Ans.}$$

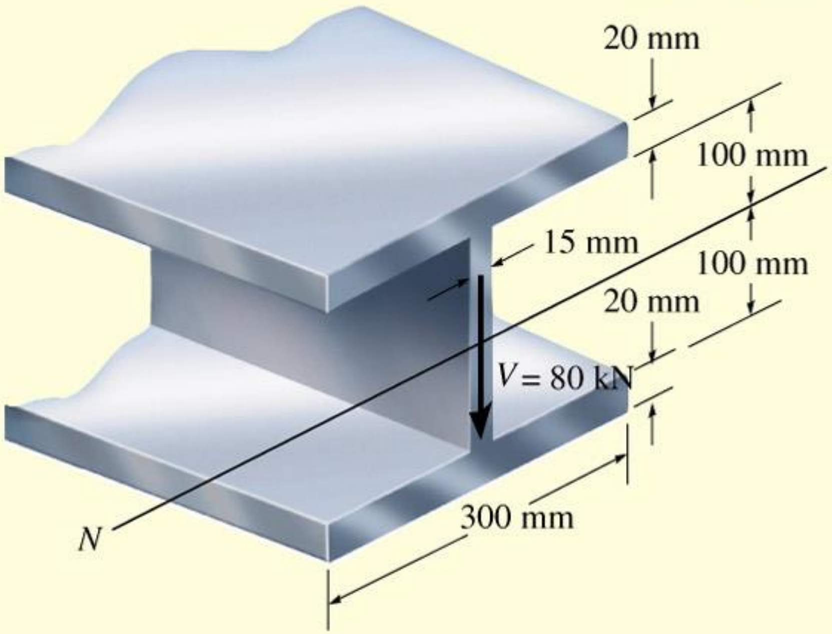


(d)

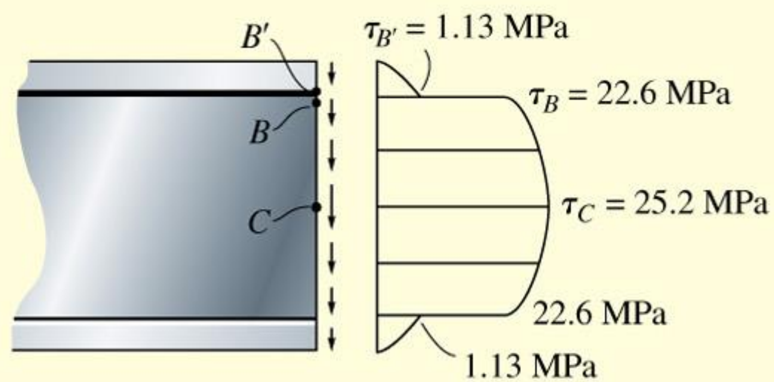
Fig. 7–10

EXAMPLE 7.2

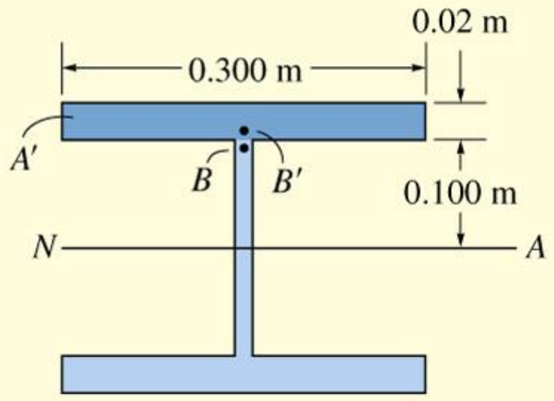
A steel wide-flange beam has the dimensions shown in Fig. 7–11*a*. If it is subjected to a shear of $V = 80$ kN, (a) plot the shear-stress distribution acting over the beam's cross-sectional area, and (b) determine the shear force resisted by the web.



(a)



(b)



(c)

Fig. 7–11

Solution

Part (a). The shear-stress distribution will be parabolic and varies in the manner shown in Fig. 7–11*b*. Due to symmetry, only the shear stresses at points B', B, and C have to be computed. To show how these values are obtained, we must first determine the moment of inertia of the cross-sectional area about the neutral axis. Working in meters, we have

$$I = \left[\frac{1}{12}(0.015\text{ m})(0.200\text{ m})^3\right]$$

$$+ 2\left[\frac{1}{12}(0.300\text{ m})(0.02\text{ m})^3 + (0.300\text{ m})(0.02\text{ m})(0.110\text{ m})^2\right]$$

$$= 155.6(10^{-6})\text{ m}^4$$

For point B', $t_B{}' = 0.300$ m, and A' is the dark shaded area shown in Fig. 7–11*c*. Thus,

$$Q_{B'} = \bar{y}'A' = [0.110\text{ m}](0.300\text{ m})(0.02\text{ m}) = 0.660(10^{-3})\text{ m}^3$$

so that

$$\tau_{B'} = \frac{VQ_{B'}}{It_{B'}} = \frac{80\text{ kN}(0.660(10^{-3})\text{ m}^3)}{155.6(10^{-6})\text{ m}^4(0.300\text{ m})} = 1.13\text{ MPa}$$

For point B, $t_B = 0.015$ m and $Q_B = Q_{B'}$, Fig. 7–11*c*. Hence

$$\tau_B = \frac{VQ_B}{It_B} = \frac{80\text{ kN}(0.660(10^{-3})\text{ m}^3)}{155.6(10^{-6})\text{ m}^4(0.015\text{ m})} = 22.6\text{ MPa}$$

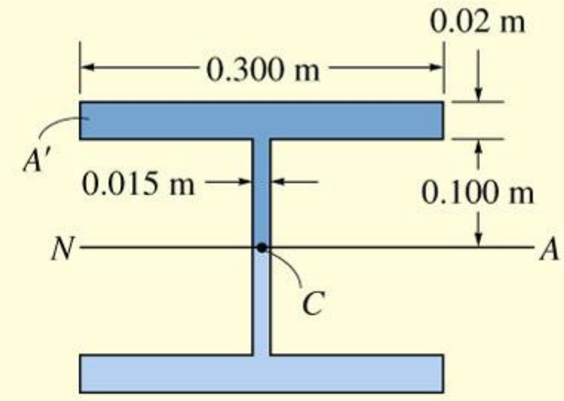
Note from the discussion of "Limitations on the Use of the Shear Formula" that the calculated value for both $\tau_{B'}$ and τ_B will actually be very misleading. Why?

For point C, $t_C = 0.015$ m and A' is the dark shaded area shown in Fig. 7–11*d*. Considering this area to be composed of two rectangles, we have

$$Q_C = \Sigma y'A' = [0.110\text{ m}](0.300\text{ m})(0.02\text{ m}) + [0.05\text{ m}](0.015\text{ m})(0.100\text{ m}) = 0.735(10^{-3})\text{ m}^3$$

Thus,

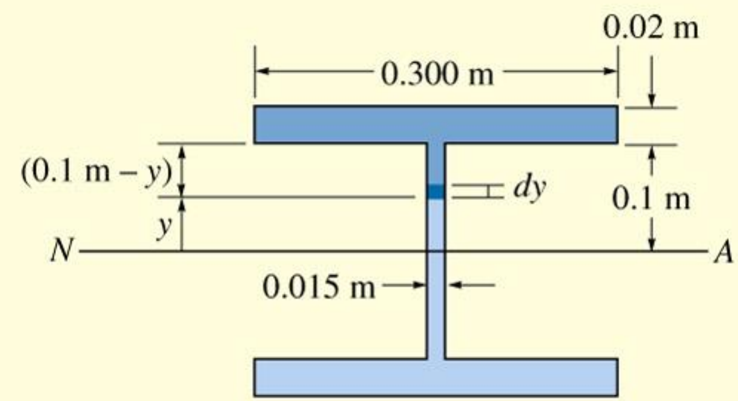
$$\tau_C = \tau_{\max} = \frac{VQ_C}{It_C} = \frac{80\text{ kN}[0.735(10^{-3})\text{ m}^3]}{155.6(10^{-6})\text{ m}^4(0.015\text{ m})} = 25.2\text{ MPa}$$



(d)

Part (b). The shear force in the web will be determined by first formulating the shear stress at the *arbitrary* location y within the web, Fig. 7–11*e*. Using units of meters, we have

$$I = 155.6(10^{-6})\ \text{m}^4$$
$$t = 0.015\ \text{m}$$
$$A' = (0.300\ \text{m})(0.02\ \text{m}) + (0.015\ \text{m})(0.1\ \text{m} - y)$$
$$Q = \Sigma \bar{y}' A' = (0.11\ \text{m})(0.300\ \text{m})(0.02\ \text{m}) + \left[y + \tfrac{1}{2}(0.1\ \text{m} - y)\right](0.015\ \text{m})(0.1\ \text{m} - y)$$
$$= (0.735 - 7.50\, y^2)(10^{-3})\ \text{m}^3$$



(e)

Fig. 7–11

so that

$$\tau = \frac{VQ}{It} = \frac{80\ \text{kN}(0.735 - 7.50\, y^2)(10^{-3})\ \text{m}^3}{(155.6(10^{-6})\ \text{m}^4)(0.015\ \text{m})}$$
$$= (25.192 - 257.07\, y^2)\ \text{MPa}$$

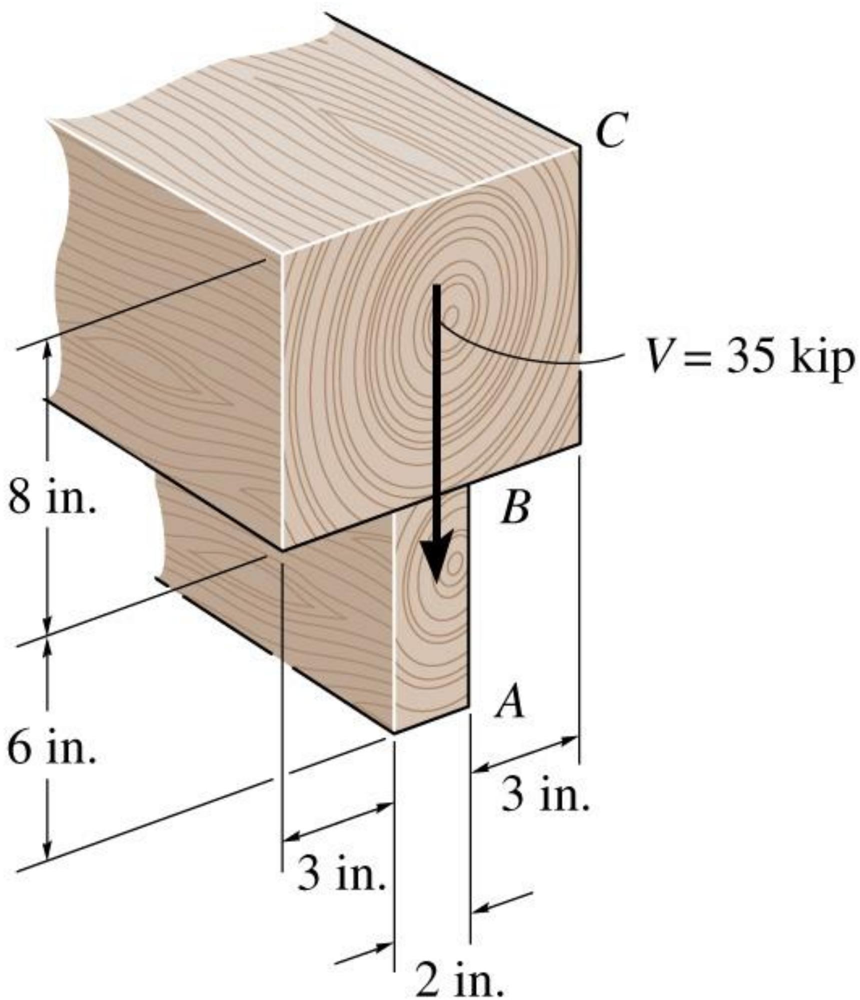
This stress acts on the area strip $dA = 0.015\,dy$ shown in Fig. 7–11*e*, and therefore the shear force resisted by the web is

$$V_w = \int_{A_w} \tau\, dA = \int_{-0.1\ \text{m}}^{0.1\ \text{m}} (25.192 - 257.07\, y^2)(10^6)(0.015\ \text{m})\, dy$$

$$V_w = 73.0\ \text{kN} \qquad \textit{Ans.}$$

Note that by comparison, the web supports 91% of the total shear (80 kN), whereas the flanges support the remaining 9%. Try solving this problem by finding the force in one of the flanges (3.496 kN) using the same method. Then $V_w = V - 2V_f = 80\ \text{kN} - 2(3.496\ \text{kN}) = 73.0\ \text{kN}$.

تیری تحت نیروی برشی ۳۵ کیلوپاسکال قرار دارد. توزیع تنش برشی روی سطح مقطع را ترسیم کنید. همچنین نیروی برشی برآیند روی قسمت AB را محاسبه کنید.



سوال: انتظار دارید تنش برشی کجا ماکزیمم باشد؟ چرا تنش برشی در نقطه اتصال مهم است؟