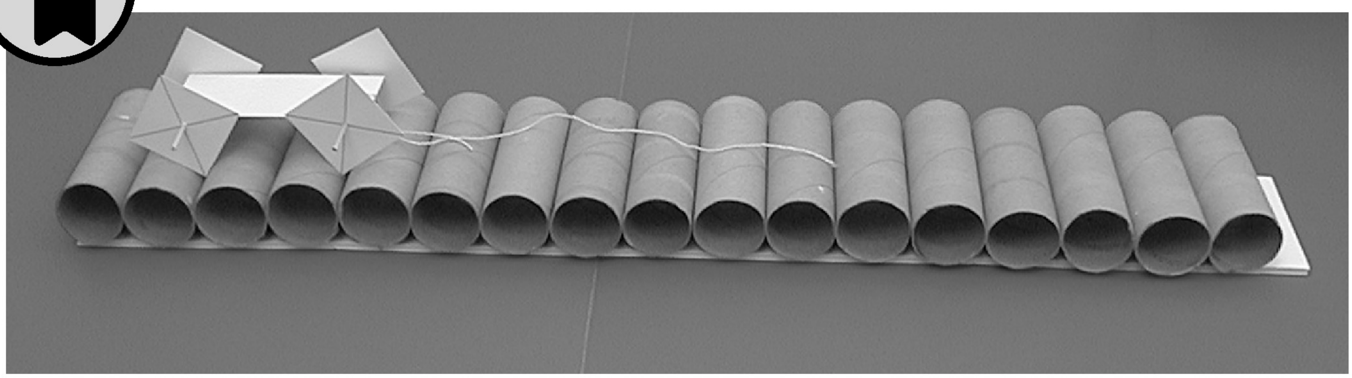
Square Wheels

You may not be able to put a square peg in a round hole, but you can make a square wheel roll on a round road.

A square wheel will roll smoothly, with its axle at a constant height, on a surface with properly spaced bumps of the right size and shape.



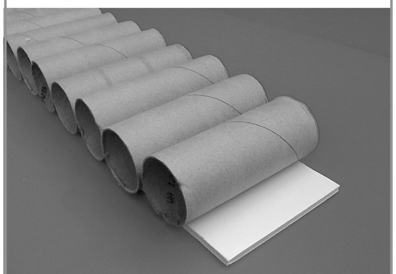
Materials

- hot glue gun and glue sticks
- about 20 cardboard toilet paper tubes (all approximately the same diameter)
- foam core, stiff cardboard, or mat board to serve as a base for the cardboard tubes, about 4 in × 30 in (10 cm × 75 cm)
- ruler
- poster board or mat board, approximately 8 in × 10 in (20 cm × 25 cm)
- pencil or pen
- pushpin
- scissors
- drinking straw
- 2 bamboo skewers
- paper clip
- string, about 12 in (30 cm)

ASSEMBLY

1 Use hot glue to attach a cardboard tube at one end of the base. The length of the tube should be placed across the base, as shown in figure 1.

Figure 1



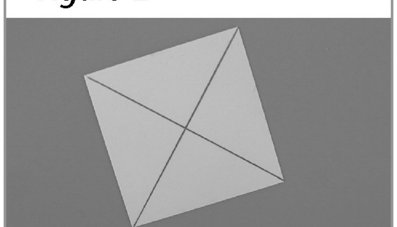
Toilet paper rolls glued to base

2 Continue gluing tubes to the base, with each tube just touching the one before it, until you reach the other end of the base.

3 Measure the diameter of three or four of the cardboard tubes. The diameters should be approximately $1\frac{11}{16}$ inches (4.3 cm). If this is the case, cut four square wheels from the poster board, with sides of 2 inches (5.1 cm). If the tubes you obtain have a significantly different diameter, then make the sides of the square wheels equal to 1.2 times the diameter (see Box o' Math at the end of this snack).

4 Locate the center of each square wheel by drawing two diagonals, as shown in figure 2.

Figure 2



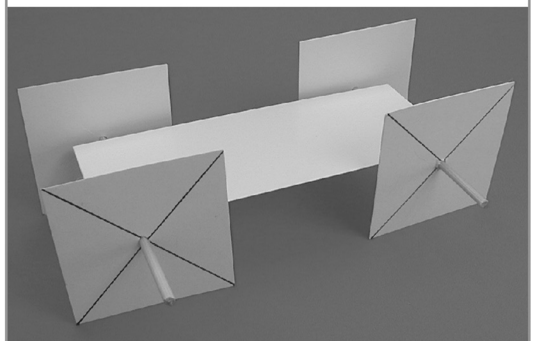
Using diagonals to locate the center of the square wheel

5 Poke a small hole in the center of each square wheel with a pushpin, taking care not to bend or crease the wheel.

6 From the poster board, cut out a 2- × 5-inch (5- × 12-cm) piece for the cart body.

7 Cut two sections of straw, each 2 inches (5 cm) long.

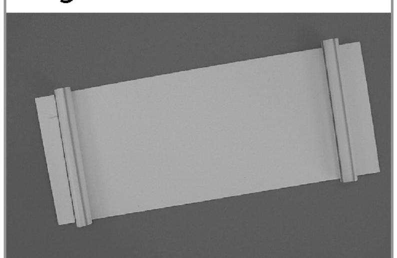
Figure 4



Cart with wheels

8 Hot glue the straw sections to the rectangular piece of poster board $\frac{3}{8}$ inch (1 cm) from each end, as shown in figure 3. This assembly will be the body of a small cart.

Figure 3



Drinking-straw axle holders

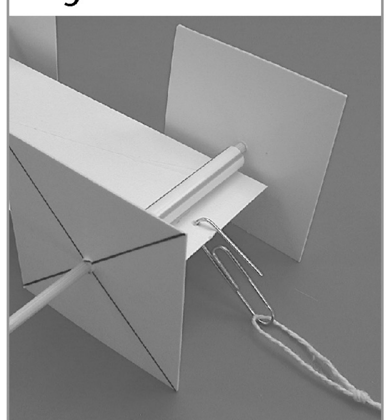
9 Cut the skewers into two 5-inch-long (12-cm) pieces, each with a point at one end. These will be axles. (If you are unable to cut the skewers with the scissors, just break the skewers or cut them with a utility knife.)

10 Slide one of the square wheels onto a skewer until it is about $\frac{3}{4}$ inch (2 cm) from the non-pointed end. Slide the pointed end through the straw, and then slide the other square wheel onto the skewer. Adjust the positions of the wheels so that they are aligned with each other and are fairly close to the edge of the cart. The wheel-and-axle assembly should turn freely in the straws. Assemble the other set of wheels the same way. When all the wheels are on, the cart should look like the one in figure 4.

11 Use the pushpin to poke a hole in the body of the cart between the straw and one end, equidistant from the edges. Put one end of the paper clip through the hole, and adjust until it is positioned as shown in figure 5.

12 Tie a loop in the end of the string, and place it on the paper clip as shown in figure 5.

Figure 5



Paper clip and string on cart

➥ Helpful Hint

When you place the cart on the road, the wheels on each axle should be aligned with each other, as shown in figure 6. Also, be sure that the wheels are reasonably perpendicular to the axles and are not excessively wobbly. If you have trouble keeping the wheels on the same axle aligned, or if they are too tilted or wobbly, use a small amount of hot glue to hold them in place on the axle.

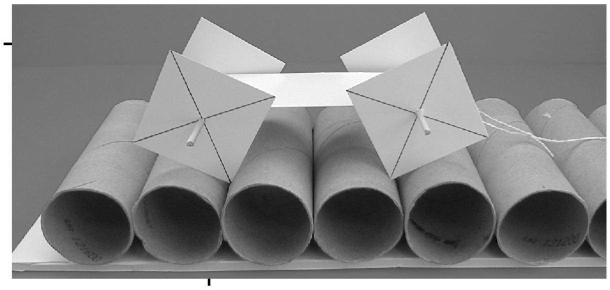


***Figure 6** Square wheels on the rounded road*

To Do and Notice

Place the cart at one end of the cardboard-tube "road" and pull gently on the string so that the cart travels along the road. Notice that the cart rolls along smoothly and that the axles stay at a reasonably constant height.

What's Going On?

The cart rolls smoothly along the bumpy road because the vertical distance from each axle to the horizontal base of the road is always about the same. Each axle moves from a point above a low spot between two tubes (see figure 7) to a point above a high spot on a tube (see figure 8). The increasing height of the point on the circular tube where the tube contacts the wheel is compensated for by the decreasing distance on the wheel between the axle and the edge of the square where it contacts the tube. The same thing happens in reverse as each axle moves from a position above a high spot to a position above a low spot.

A special shape called a *catenary curve* (see Did You Know?), not a circle, is the curve that will give an absolutely level ride with square wheels. A road made with circles is a reasonably close approximation, however, and is easier to build from commonly available materials.

On a flat road, this would be the ultimate exercise bike! Mathematician Stan Wagon was inspired to build his square-wheeled tricycle after seeing the Exploratorium's Square Wheels exhibit.

So What?

A key problem in designing automobile transmissions involves gear teeth. Gear teeth must mesh together without slipping, because slipping results in frictional wear. In order for gears to mesh smoothly, engineers must design teeth that have matching shapes—a problem that's quite similar to designing the particular bumpy road that will provide a smooth ride for square wheels.

Figure 7

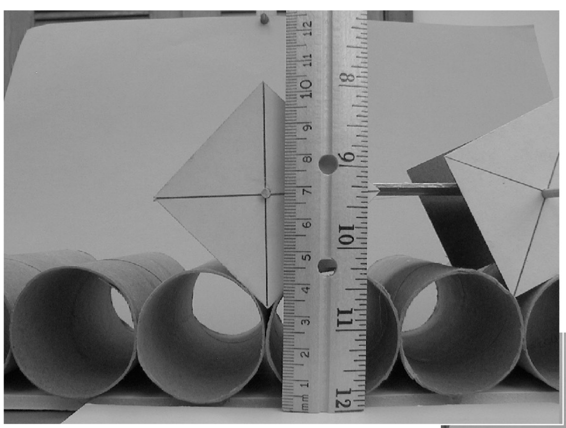
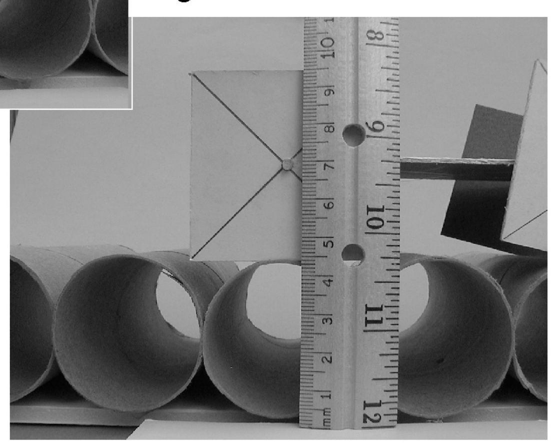


Figure 8



The height of the axle always remains about the same distance from the base (6.8 cm in figure 7 and 6.9 cm in figure 8). In figure 7, the vertical distance between the axle and the bottom of the wheel is maximum; because the wheel is in a depression, however, the remainder of the distance to the base is minimum. In figure 8, the situation is reversed. The vertical distance between the axle and the bottom of the wheel is minimum, but the remainder of the distance to the base is maximum.

Box o' Math

Calculating Wheel Size

To travel smoothly over the array of tubes, the sides of the square wheels have to be 1.2 times the diameter of the tubes. The equations below explain how this relationship is derived; the diagram shows you how the math applies to the square wheels and the "road." Note that l is the side of the square and d is the diameter of the circle (which represents the tube). The circumference of the tube = $2\pi r$.

$$\cos 45 = \frac{AC}{AB}, \text{ or } AB = \frac{AC}{\cos 45} = \frac{r}{\cos 45}$$

$$AD = r$$

$$DB = AB - AD = \frac{r}{\cos 45} - r = r\left(\frac{1}{\cos 45} - 1\right) = r\left(\frac{1}{.71} - 1\right) = r(1.41 - 1) = 0.41r$$

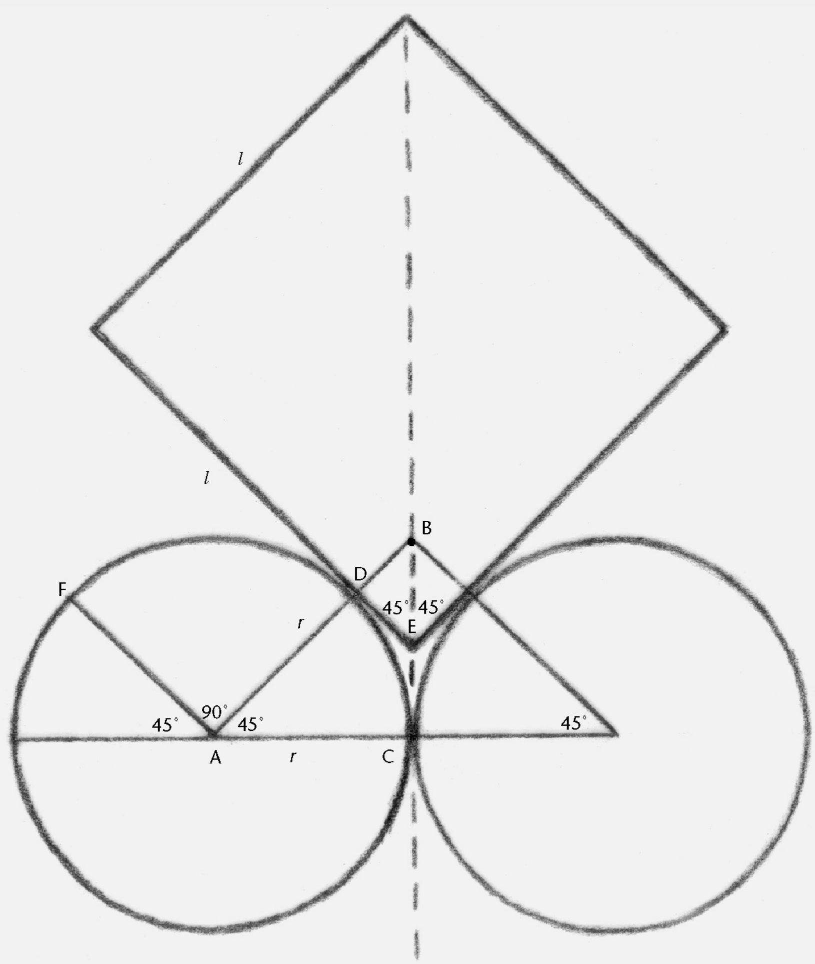
$$l = \widehat{DF} + 2DE$$

$$\widehat{DF} = \frac{2\pi r}{4}$$

$$DE = DB = 0.41r$$

$$l = \frac{2\pi r}{4} + 2 \times 0.41r = 0.5 \times 3.14r + 0.82r = 1.57r + 0.82r = 2.4r$$

$$r = \frac{d}{2}, \therefore\ l = 2.4 \times \frac{d}{2} = 1.2d$$



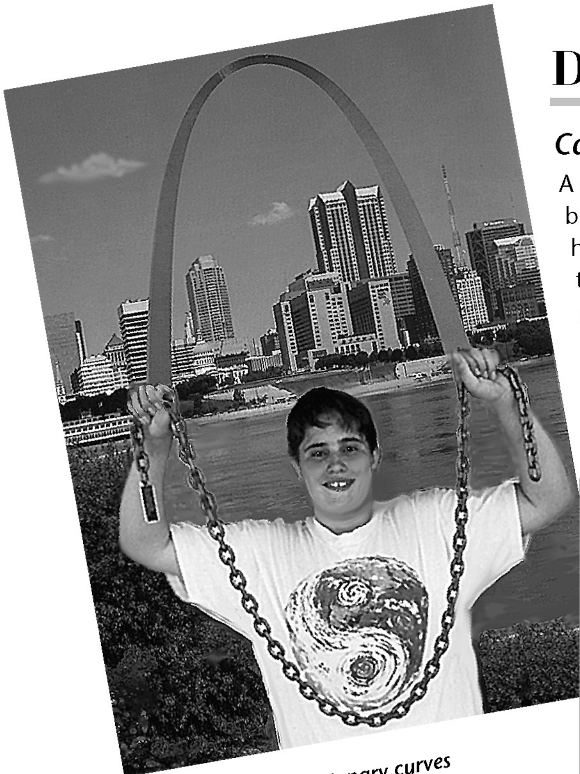


Figure 9 Two catenary curves

Did You Know?

Cat and Who?

A catenary curve is the shape a flexible rope or chain assumes when it hangs loosely and freely between two supports. Turned upside down, a catenary curve is the shape that will provide the greatest strength to an arch supporting only its own weight, such as the Gateway Arch in St. Louis.

Going Further

Deluxe Version

The article listed in the Credits & References section contains a template for a catenary curve and instructions for building a catenary road and a set of matching square wheels from plywood. If you have access to power tools, you might consider building this project.

Credits & References

This snack is based on the Exploratorium exhibit of the same name.

Regester, Jeffrey. "A Long and Bumpy Road." *The Physics Teacher,* April 1997. (Also reprinted in *Apparatus for Teaching Physics: A Collection of "Apparatus for Teaching Physics" Columns from The Physics Teacher,* 1987–1998, edited by Karl Mamola, American Association of Physics Teachers, 1998, pages 46–47.)