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Vector Calculus:

Coordinate Systems

EE3321

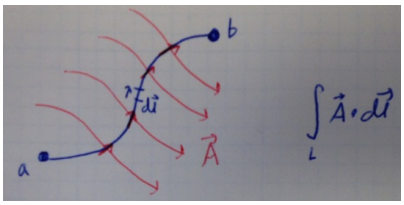
Electromagnetic Field Theory

Outline

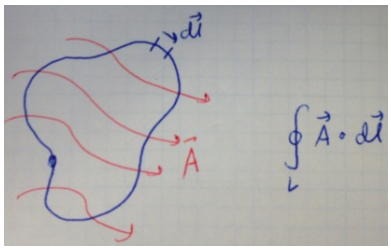
- Line, Surface, & Volume Integrals
- Cartesian Coordinates
- Cylindrical Coordinates
- Spherical Coordinates

Line Integrals

Ordinary Line Integral

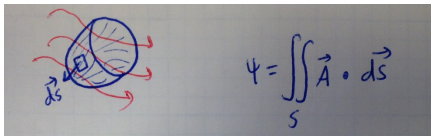


Closed-Contour Line Integral

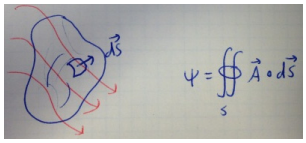


Surface Integrals

Ordinary Surface Integral



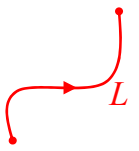
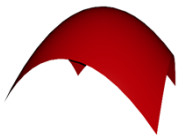
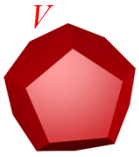
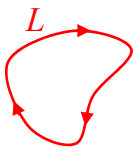
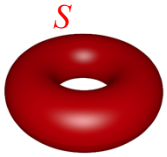
Closed-Surface Integral



Volume Integrals

Ordinary Volume Integral	Closed-Volume Integral
$\iiint_V dV$	?

Types of Integrations

Ordinary Line Integral	Ordinary Surface Integral	Ordinary Volume Integral
$\int_L d\ell$ 	$\iint_S ds$ 	$\iiint_V dv$ 
Closed-Contour Line Integral	**Closed-Contour Surface Integral**	**Closed-Contour Volume Integral**
$\oint_L d\ell$ 	$\oiint_S ds$ 	~~$\iiint_V dV$~~ ?

Cartesian Coordinates

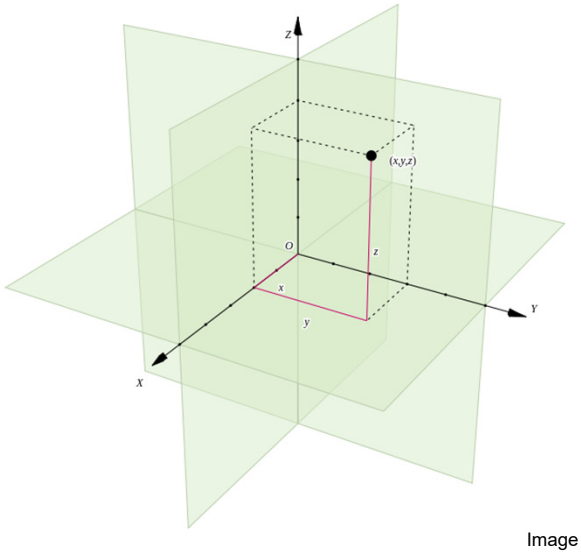
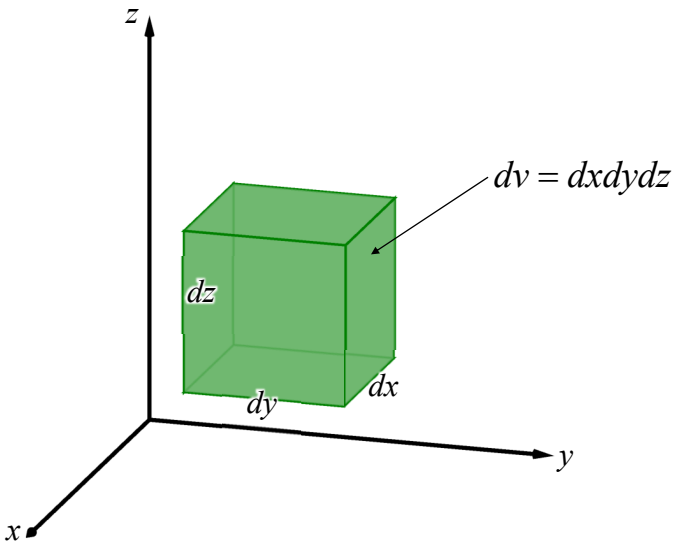
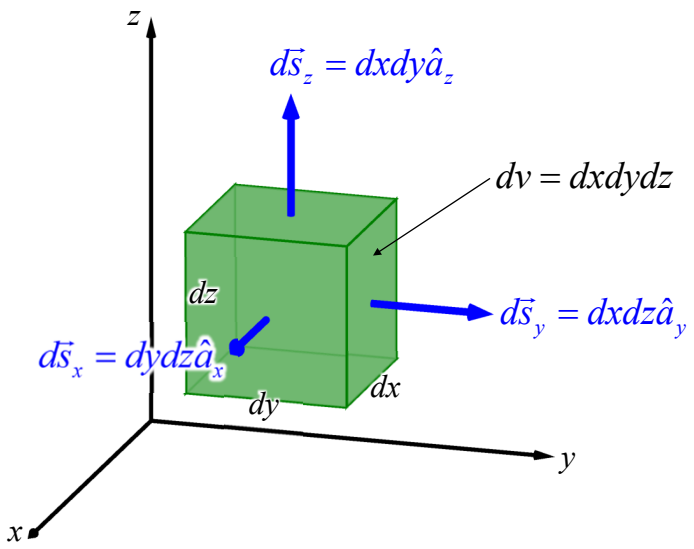


Image courtesy of Wikipedia.

Cartesian Differentials (1 of 2)



Cartesian Differentials (2 of 2)



Cylindrical Coordinates

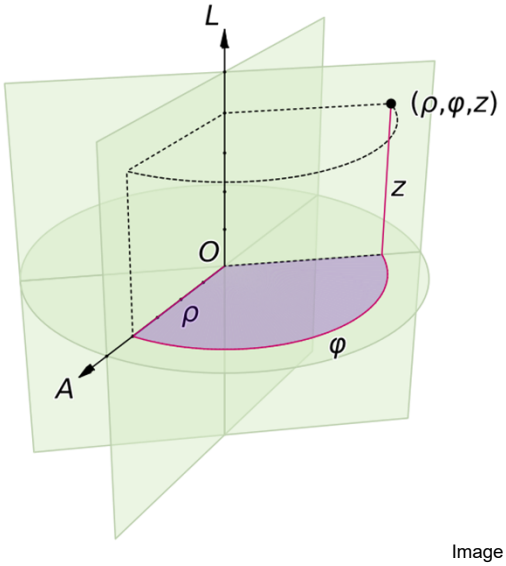
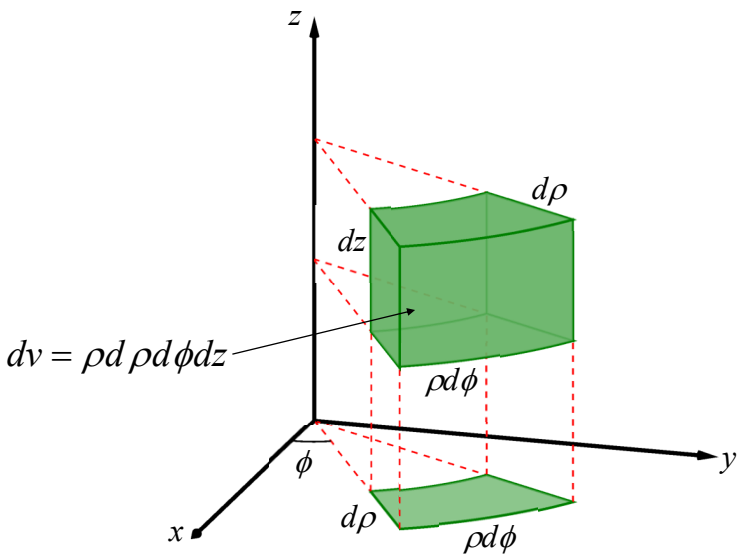
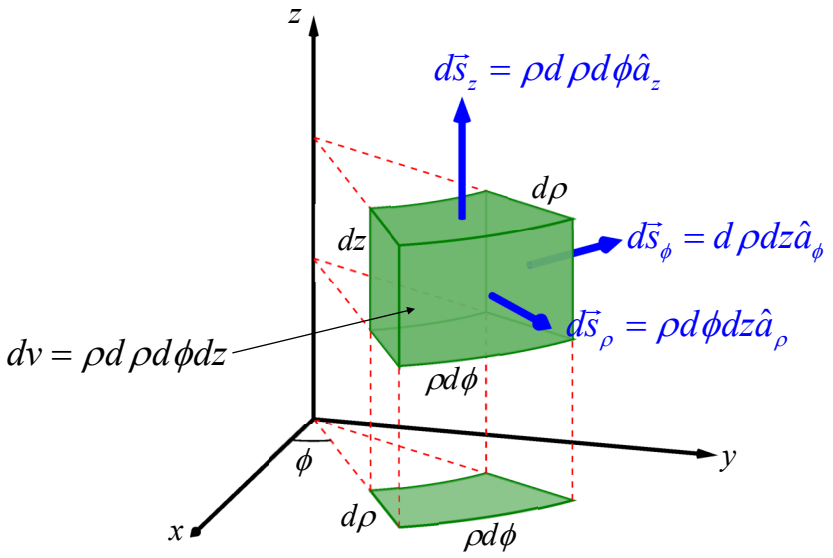


Image courtesy of Wikipedia.

Cylindrical Differentials (1 of 2)



Cylindrical Differentials (2 of 2)



Spherical Coordinates

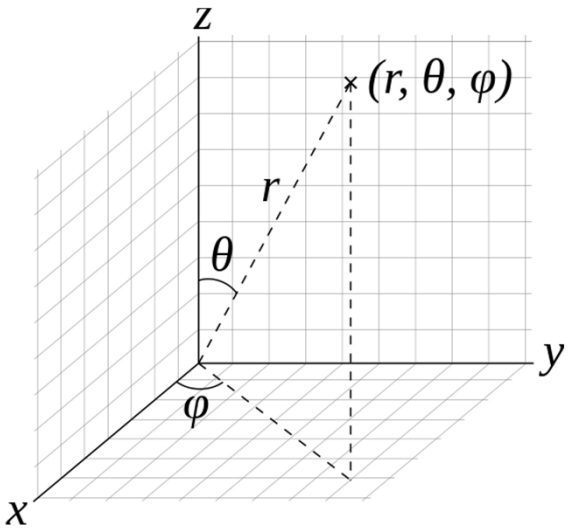
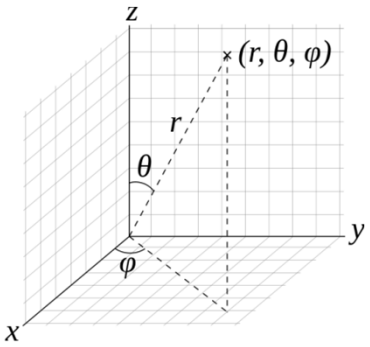
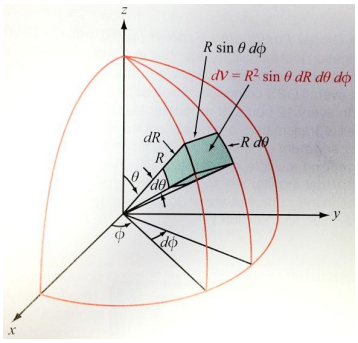


Image courtesy of Wikipedia.

Spherical Differentials





Sadiku, Matthew NO. *Elements of electromagnetics*. Oxford university press, 2014.

$$0 \le r \le \infty$$
$$0 \le \theta \le \pi$$
$$0 \le \phi < 2\pi$$

Summary of Differentials

Cartesian Coordinates

Differential Element	Formula	Notes
Length	$d\vec{\ell} = dx \cdot \hat{x} + dy \cdot \hat{y} + dz \cdot \hat{z}$	Vector is tangential to line
Normal Area	$d\vec{s} = dydz \cdot \hat{x} + dxdz \cdot \hat{y} + dxdy \cdot \hat{z}$	Vector is normal to surface
Volume	$dv = dxdydz$	Scalar

Cylindrical Coordinates

Differential Element	Formula	Notes
Length	$d\vec{\ell} = d\rho \cdot \hat{\rho} + \rho d\phi \cdot \hat{\phi} + dz \cdot \hat{z}$	Vector is tangential to line
Normal Area	$d\vec{s} = \rho d\phi dz \cdot \hat{\rho} + d\rho dz \cdot \hat{\phi} + \rho d\rho d\phi \cdot \hat{z}$	Vector is normal to surface
Volume	$dv = \rho d\rho d\phi dz$	Scalar

Spherical Coordinates

Differential Element	Formula	Notes
Length	$d\vec{\ell} = dr \cdot \hat{r} + rd\theta \cdot \hat{\theta} + r\sin\theta d\phi \cdot \hat{\phi}$	Vector is tangential to line
Normal Area	$d\vec{s} = r^2 \sin\theta d\theta d\phi \cdot \hat{r} + r\sin\theta dr d\phi \cdot \hat{\theta} + rdrd\theta \cdot \hat{\phi}$	Vector is normal to surface
Volume	$dv = r^2 \sin\theta dr d\theta d\phi$	Scalar