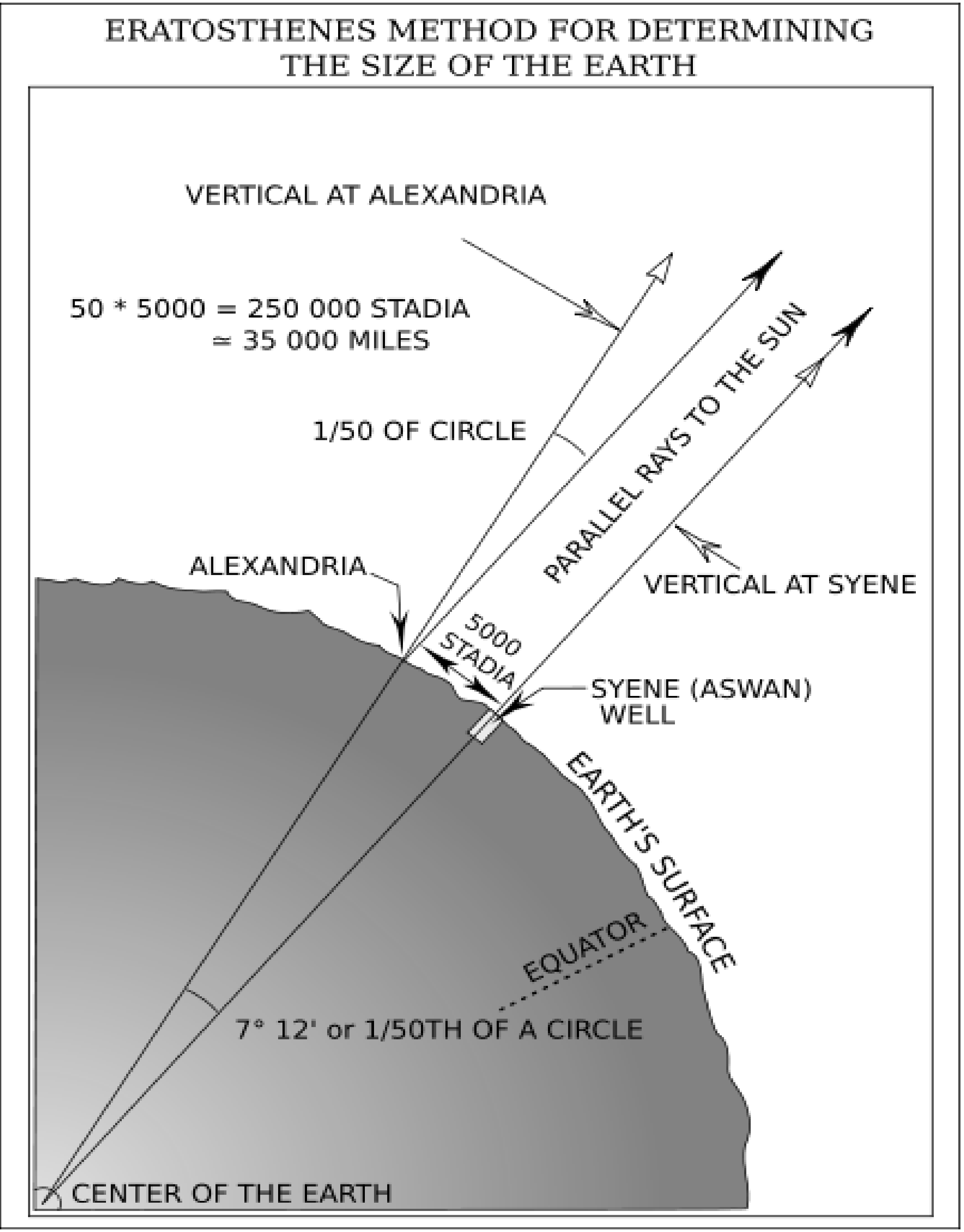
EARTH AND ITS SHAPE

- Approximately 72% of the planet's surface (~$3.6x10^8$ km^2) is covered by saline water that is customarily divided into several principal oceans and smaller seas, with the ocean covering approximately 71% of the Earth's surface.

Eratosthenes (276 - 195 B.C.)



The Earth’s Shape

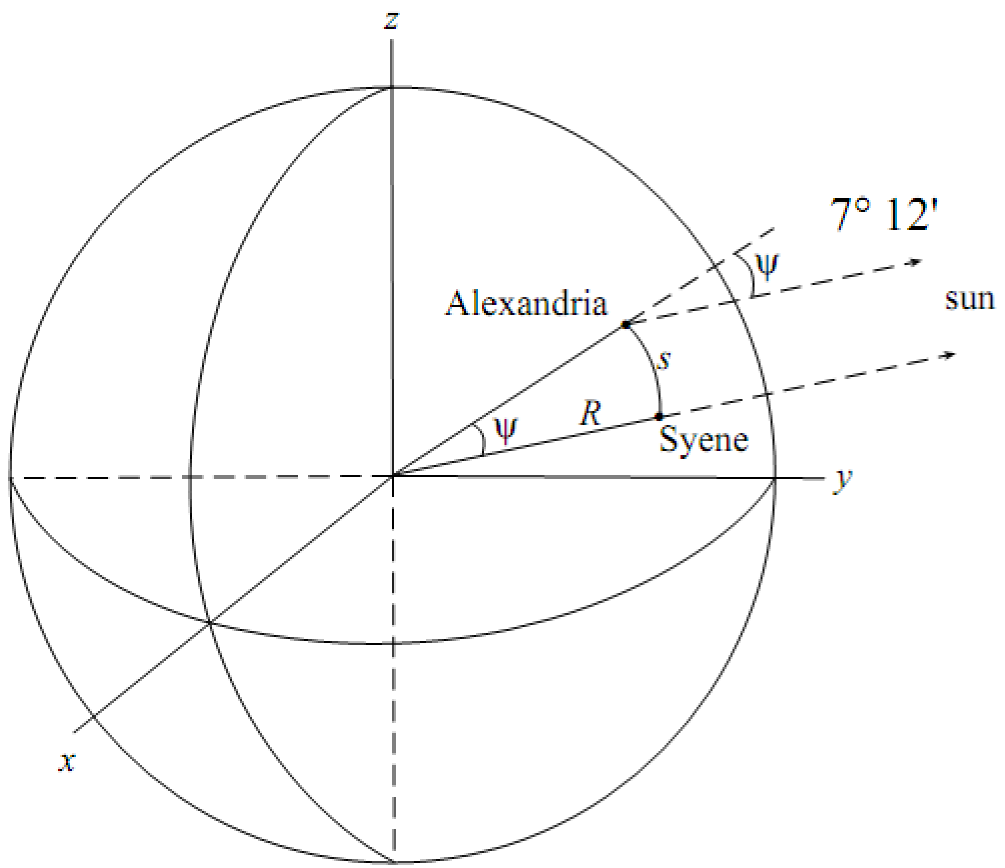
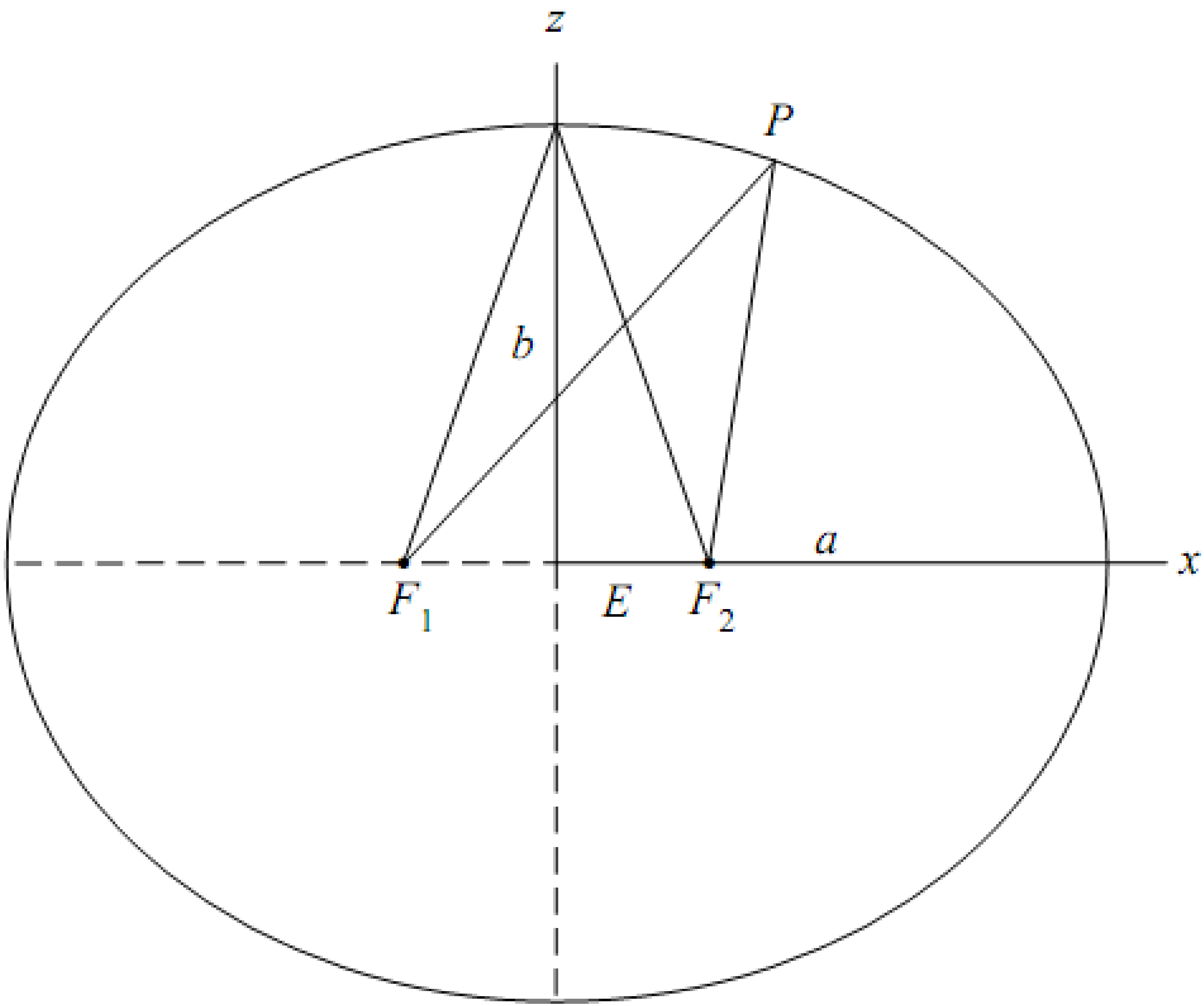


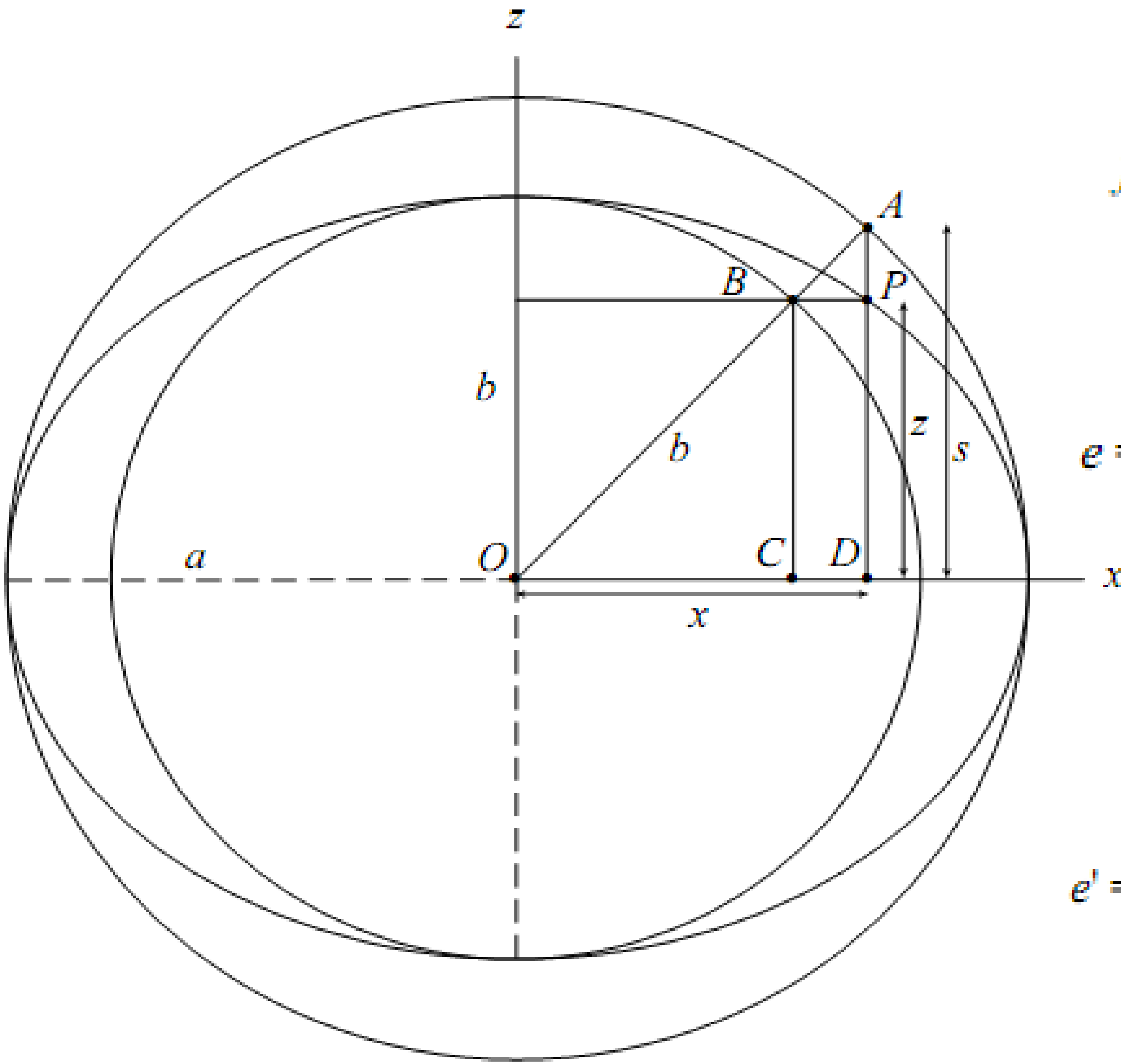
Figure 1.4: Eratosthenes’ determination of Earth’s radius.

to arrive at a radius of $R = 6267$ km , which differs from the actual mean Earth radius by only 104 km (1.6%).

- **In 1687 Isaac Newton published the that a rotating self-gravitating fluid body in equilibrium takes the form of an oblate ellipsoid of revolution which he termed an oblate spheroid. A great many ellipsoids have been used with various sizes and centres but modern (post GPS) ellipsoids are centred at the actual center of mass of the Earth or body being modeled.**



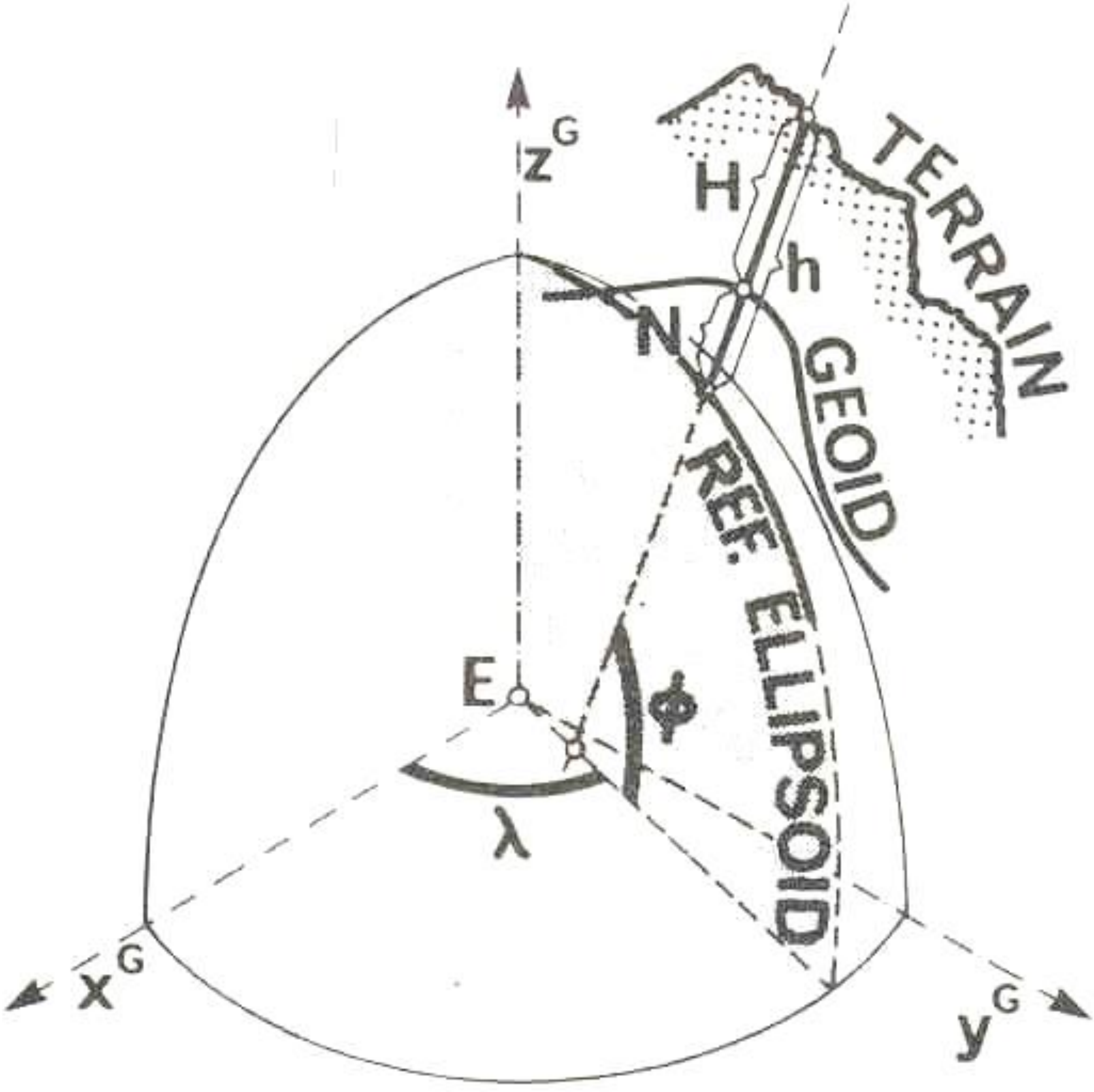
z
P
b
a
x
F1
E
F2



$$f = \frac{a-b}{a} \;;$$

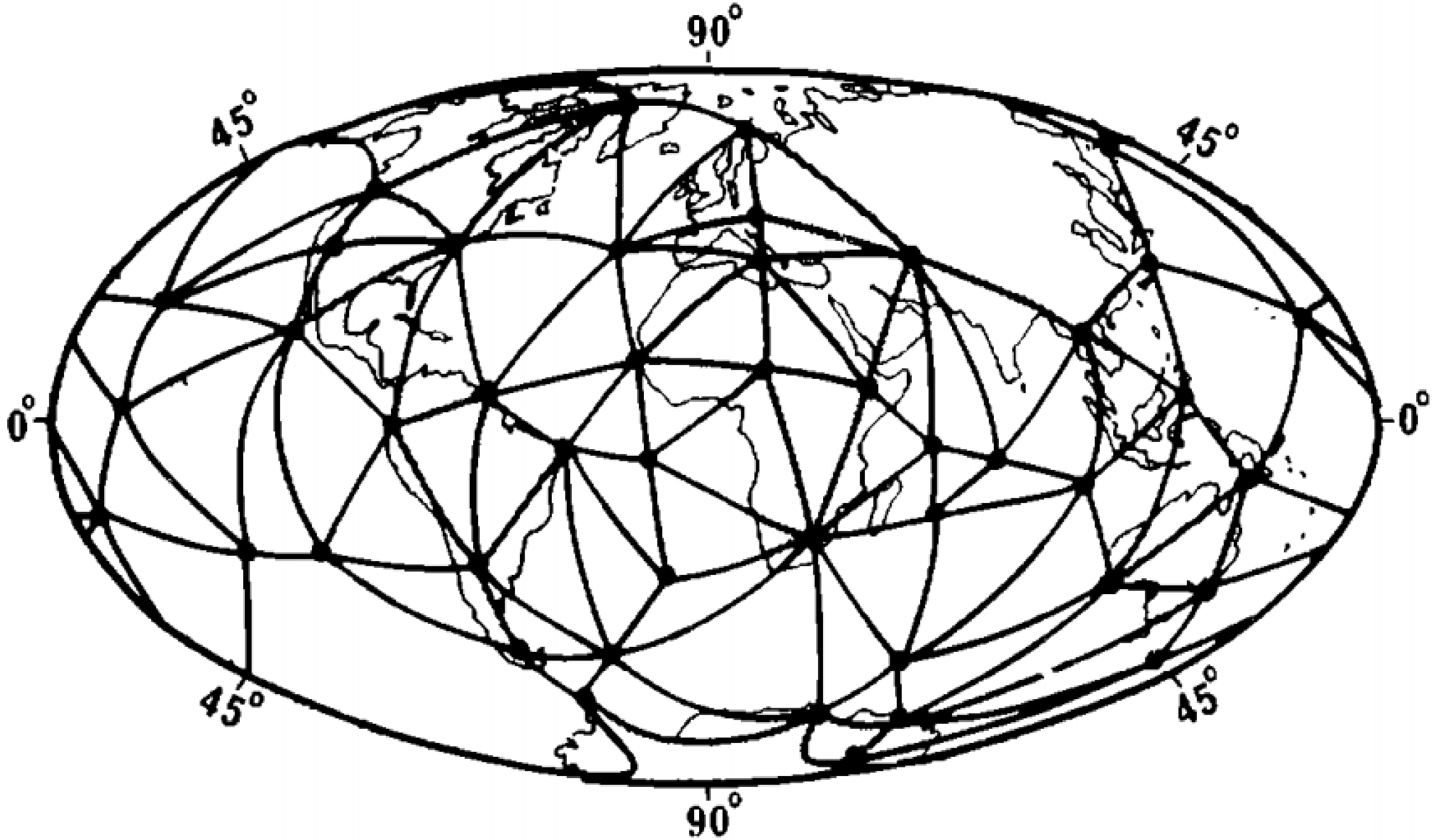
$$e = \frac{\sqrt{a^2 - b^2}}{a} \;;$$

$$e' = \frac{\sqrt{a^2 - b^2}}{b} \;.$$



z^G
H
h
N
TERRAIN
GEOID
REF. ELLIPSOID
E
ϕ
λ
x^G
y^G

three-dimensional network



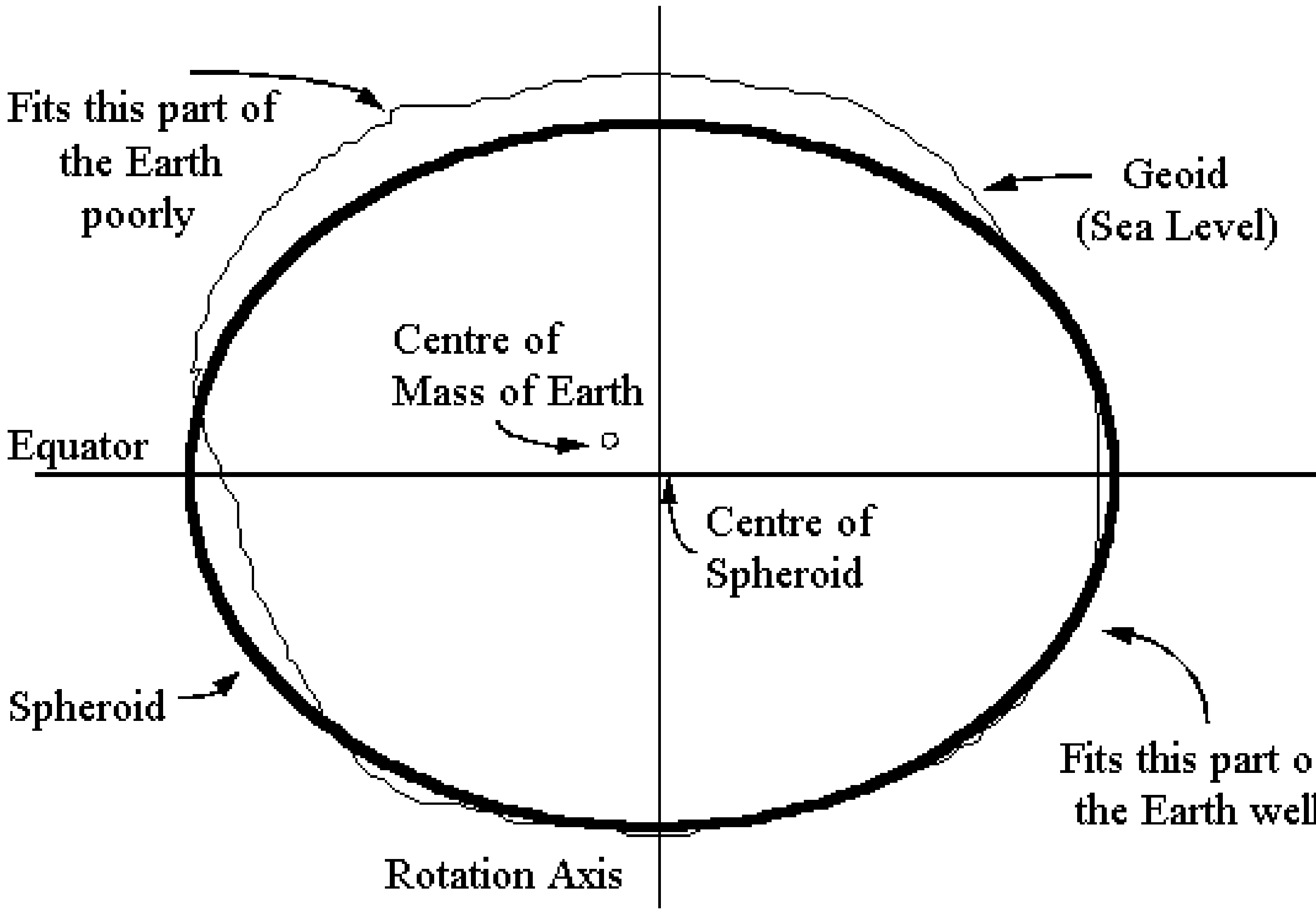
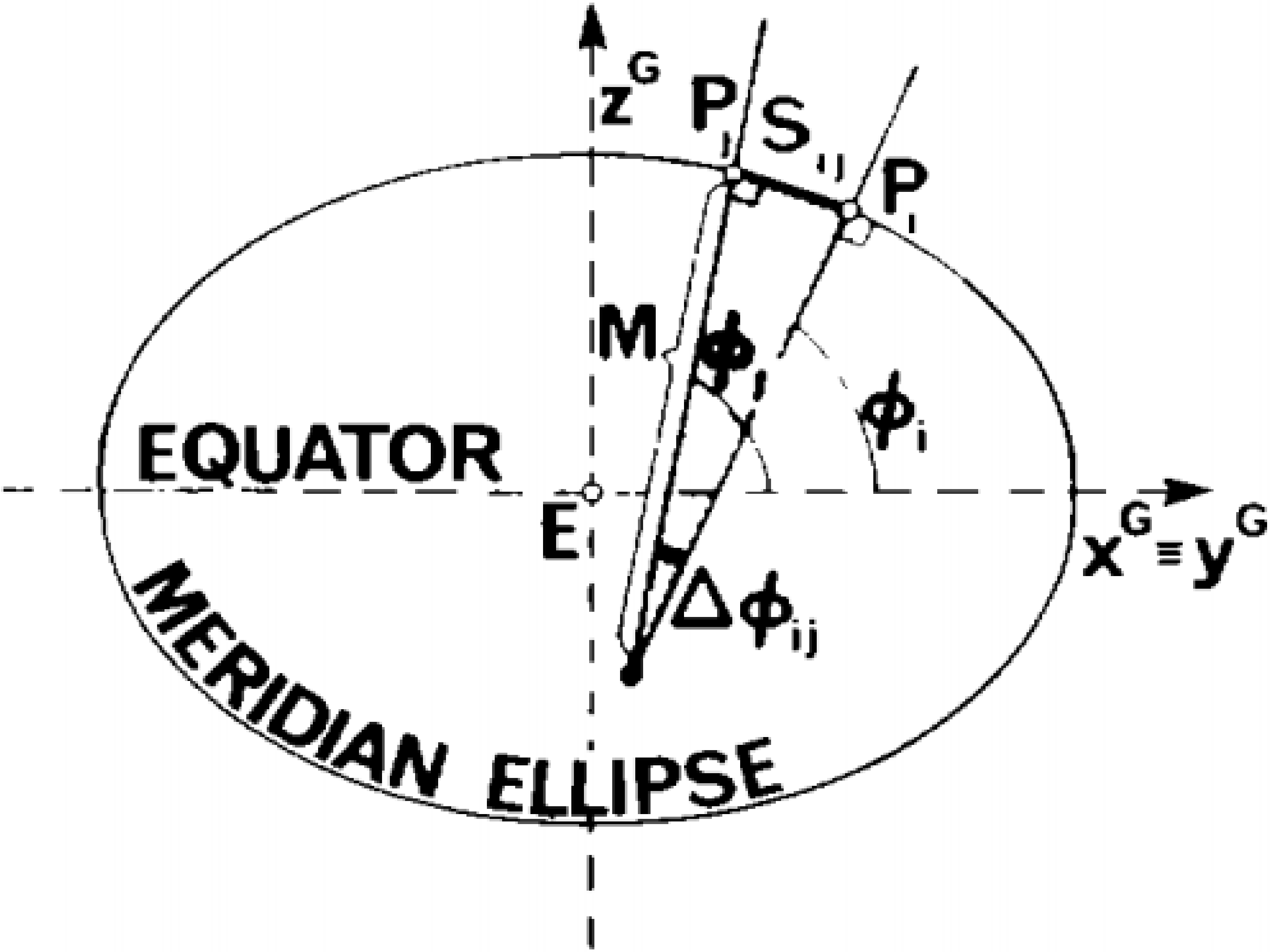

Fits this part of
the Earth
poorly
Geoid
(Sea Level)
Centre of
Mass of Earth
Equator
Centre of
Spheroid
Spheroid
Fits this part of
the Earth well
Rotation Axis

Table 2.1: Terrestrial Ellipsoids.

Ellipsoid Name (year computed)	Semi-Major Axis, a, [m]	Inverse Flattening, $1/f$
Airy (1830)	6377563.396	299.324964
Everest (1830)	6377276.345	300.8017
Bessel (1841)	6377397.155	299.152813
Clarke (1866)	6378206.4	294.978698
Clarke (1880)	6378249.145	293.465
Modified Clarke (1880)	6378249.145	293.4663
International (1924)	6378388.	297.
Krassovski (1940)	6378245.	298.3
Mercury (1960)	6378166.	298.3
Geodetic Reference System (1967), GRS67	6378160.	298.2471674273
Modified Mercury (1968)	6378150.	298.3
Australian National	6378160.	298.25
South American (1969)	6378160.	298.25
World Geodetic System (1966), WGS66	6378145.	298.25
World Geodetic System (1972), WGS72	6378135.	298.26
Geodetic Reference System (1980), GRS80	6378137.	298.257222101
World Geodetic System (1984), WGS84	6378137.	298.257223563
TOPEX/Poseidon (1992) (IERS recom.)[2]	6378136.3	298.257

Radius in the Meridian

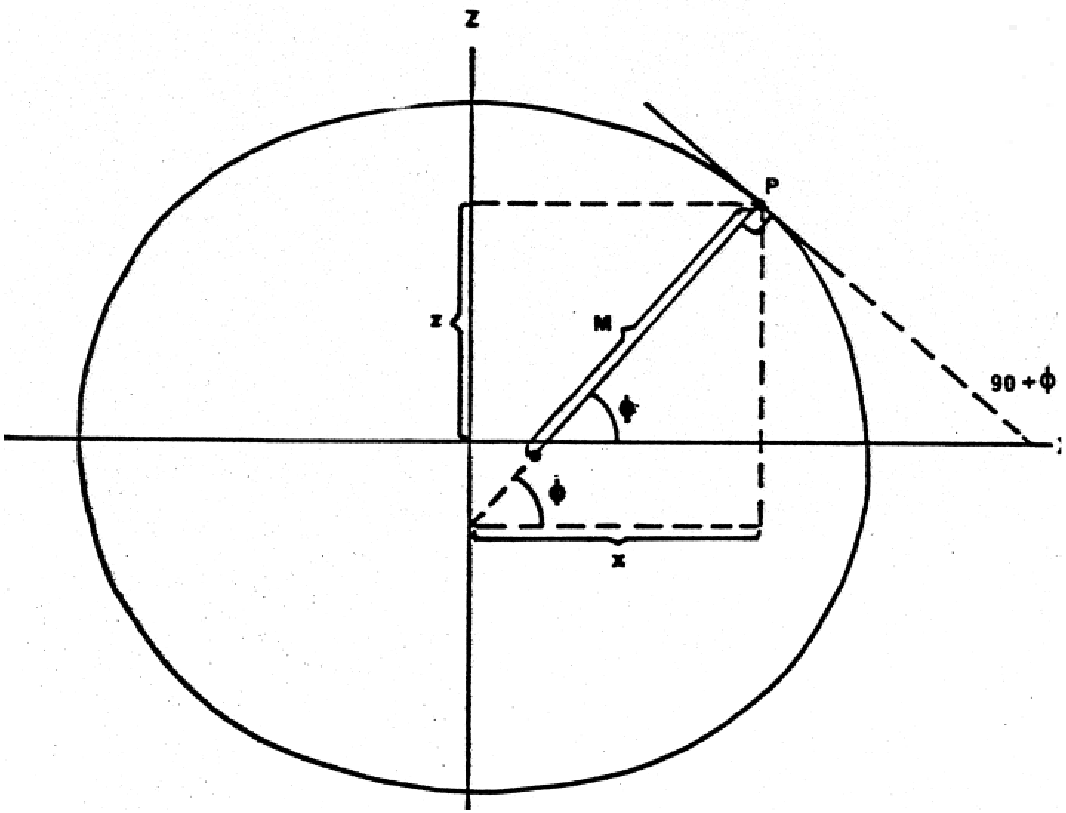


Radius in the Meridian is the radius of the great circle that passes through the observer's station and the Earth's Polar axes.

$$M = \frac{\left(1 + \left(\frac{dz}{dx}\right)^2\right)^{3/2}}{\frac{d^2z}{dx^2}}$$

$$\frac{dz}{dx} = -\frac{x}{z}\frac{b^2}{a^2}$$

$$\frac{d^2z}{dx^2} = -\frac{b^2}{a^2}\left(\frac{z - x\frac{dz}{dx}}{z^2}\right)$$



$$\tan(90+\phi) = \frac{dz}{dx}$$

$$b = a(1-e^2)^{1/2}$$

$$z = x(1-e^2)\tan\phi$$

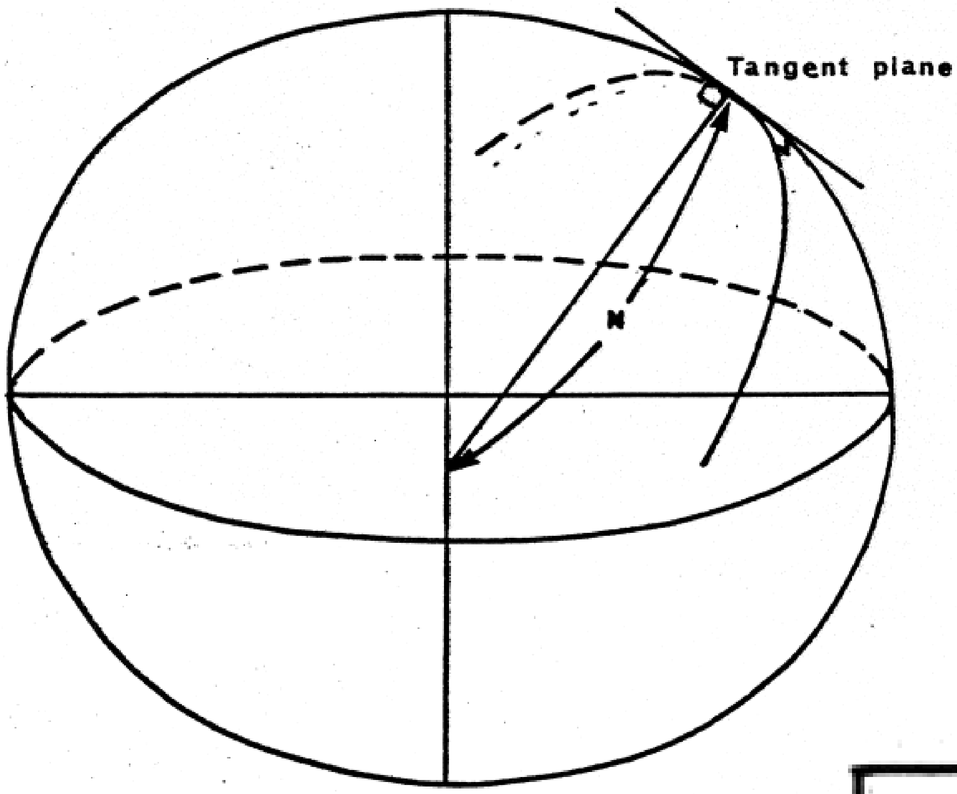
$$\frac{x^2}{a^2} + \frac{z^2}{b^2} = 1.$$

$$z = x(1-e^2) \tan \phi$$

$$x = \frac{a \cos \phi}{(1-e^2\sin^2\phi)^{1/2}} \qquad z = \frac{a(1-e^2)\sin \phi}{(1-e^2\sin^2\phi)^{1/2}}$$

$$\boxed{M = \frac{a(1-e^2)}{(1-e^2\sin^2\phi)^{3/2}}} .$$

Prime Vertical Radius



$$\cos \phi = \frac{X}{N}$$

$$N = \frac{X}{\cos \phi}$$

$$N = \frac{a}{(1-e^2 \sin^2 \phi)^{1/2}} .$$

Also, at any point M and N are, respectively, the minimum and maximum radii of curvature for all normal sections through that point. M and N are known as the *principal radii of curvature* at a point of the ellipsoid.

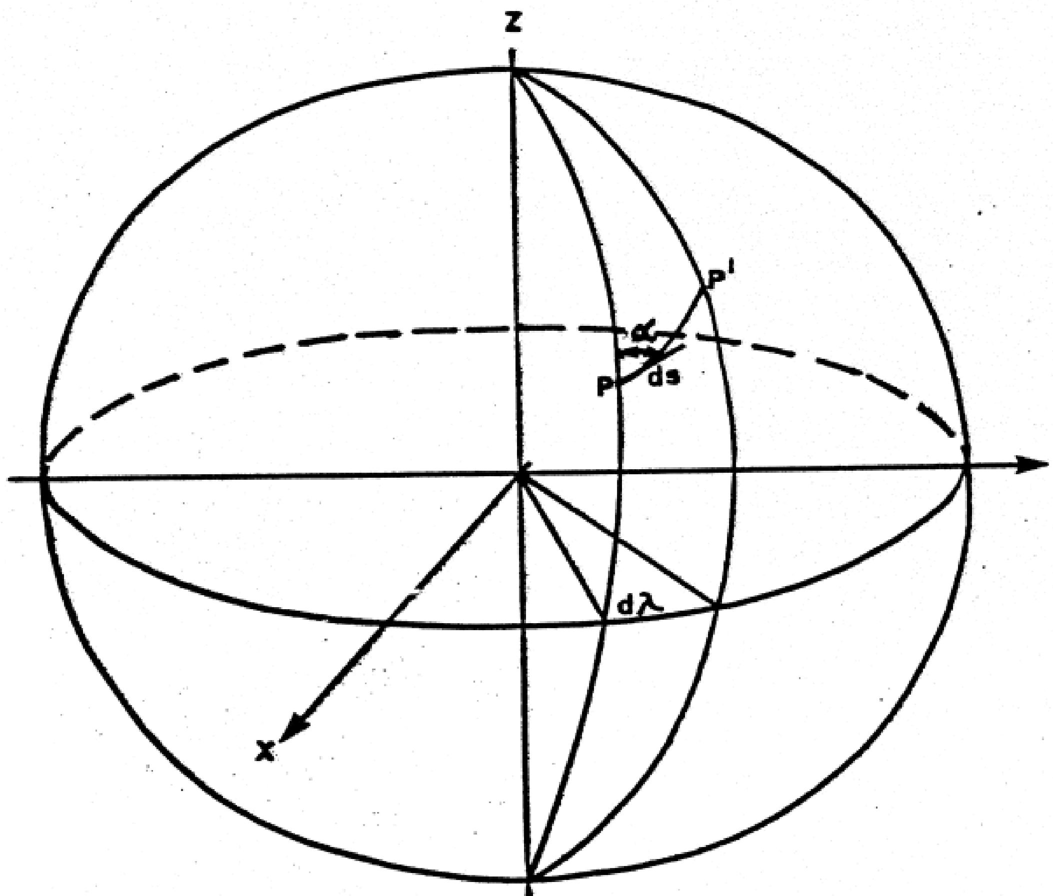
Gaussian mean radius

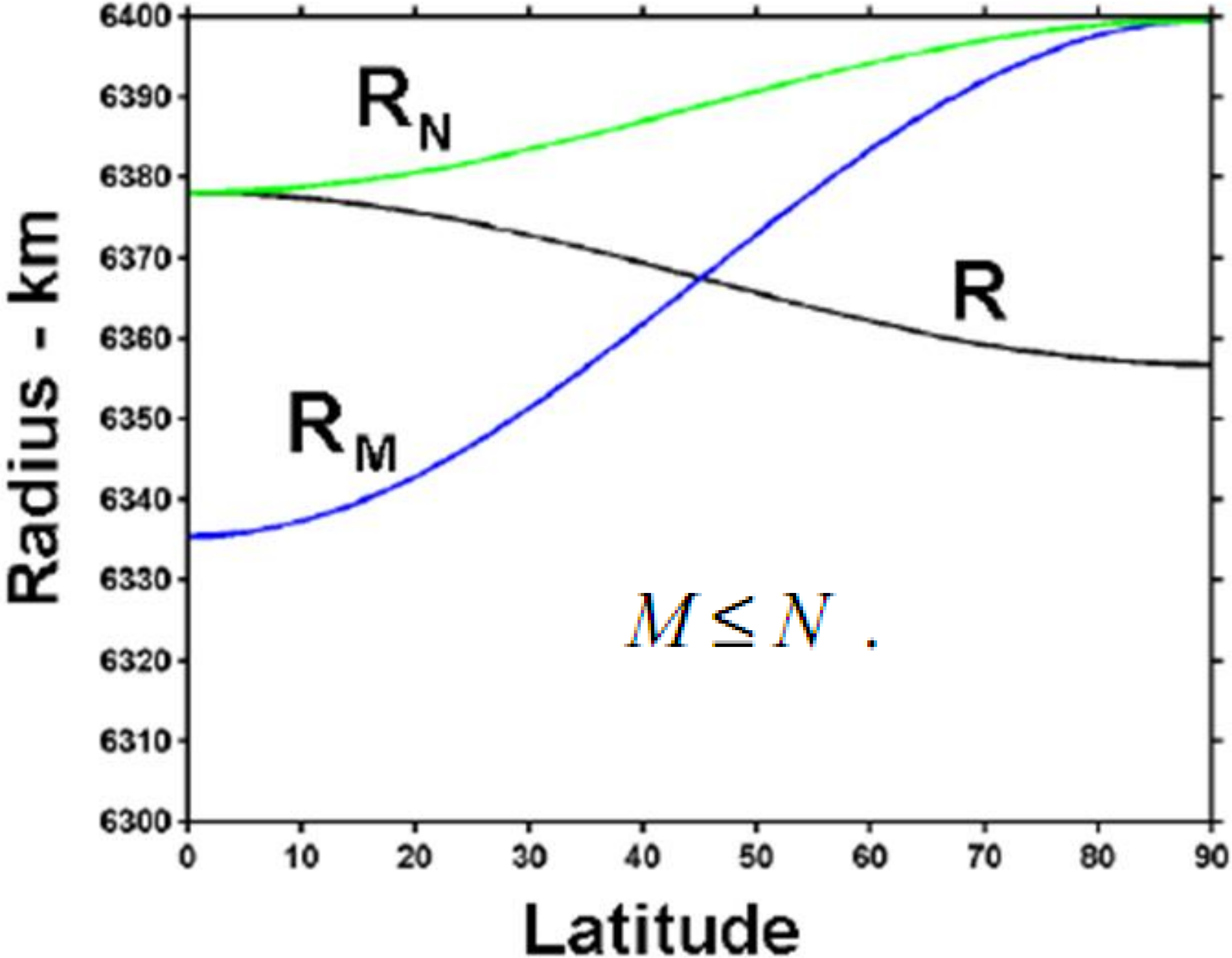
$$R_G = \frac{1}{2\pi}\int_0^{2\pi} R_\alpha \, d\alpha = \int_0^{2\pi} \frac{d\alpha}{\frac{\sin^2\alpha}{N} + \frac{\cos^2\alpha}{M}}$$

$$= \sqrt{MN} = \frac{a\,(1-f)}{1-e^2\sin^2\phi},$$

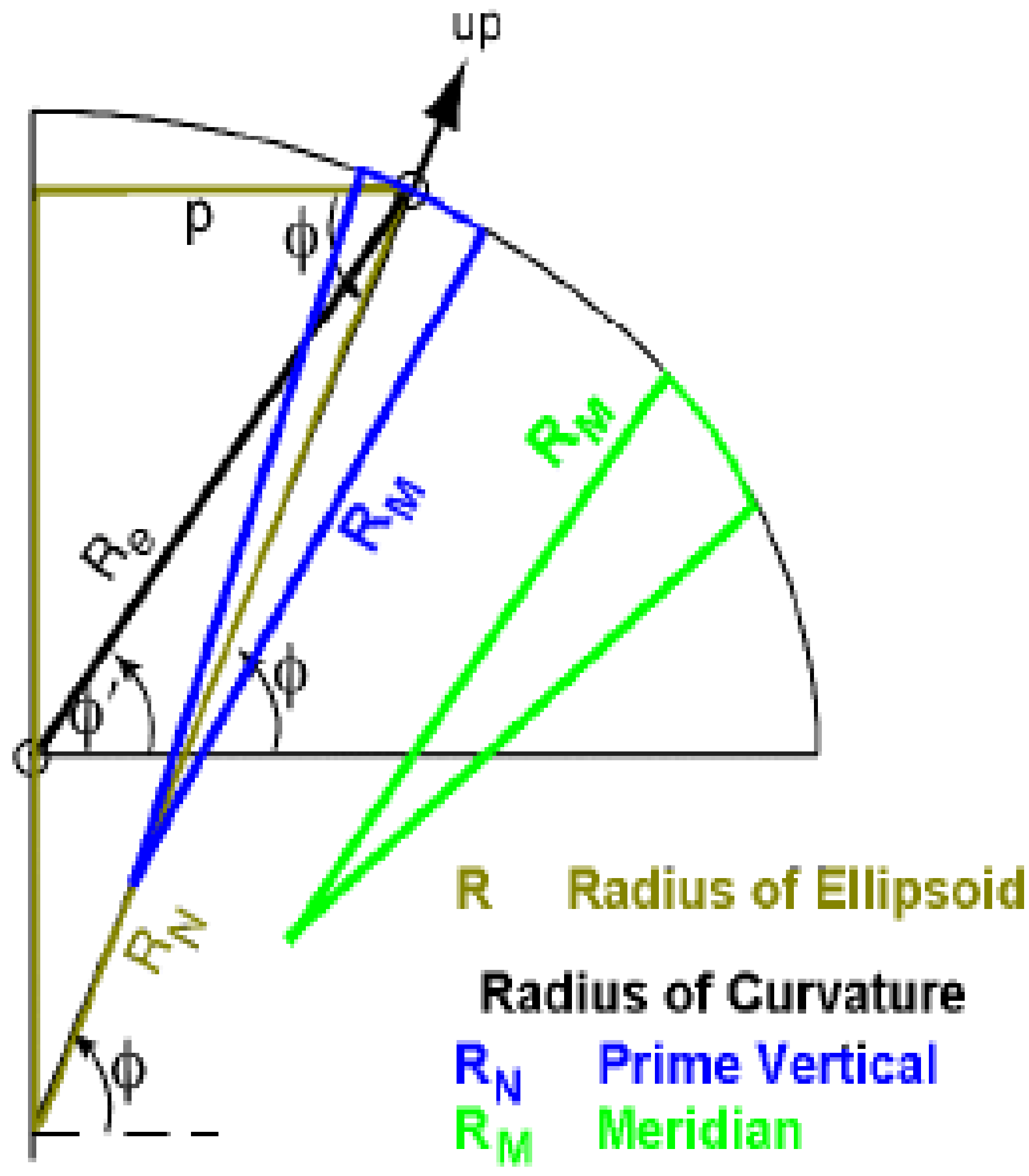
Euler's formula

$$\frac{1}{R_\alpha} = \frac{\sin^2\alpha}{N} + \frac{\cos^2\alpha}{M}$$

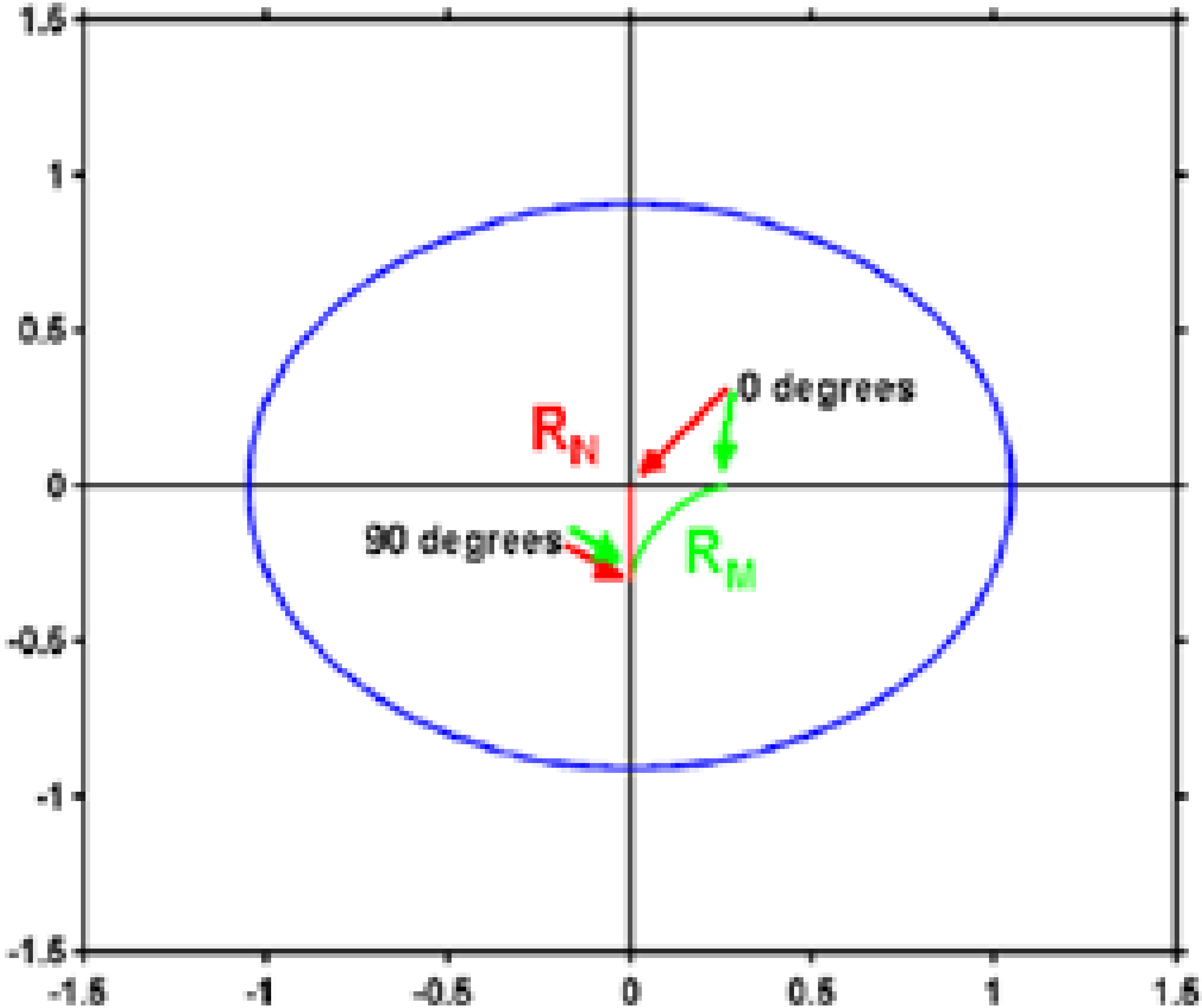




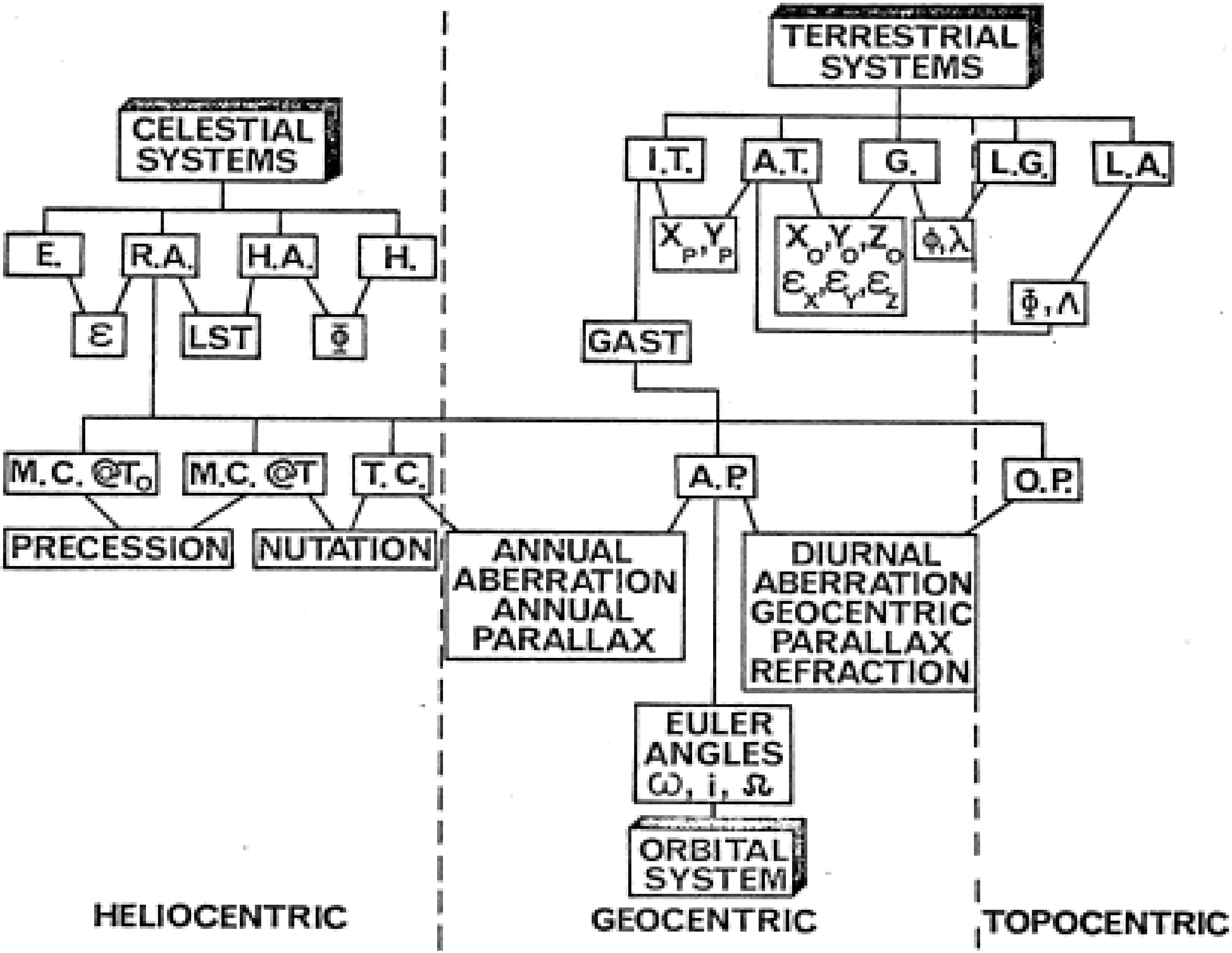
Radius - km
6400
6390
6380
6370
6360
6350
6340
6330
6320
6310
6300
R_N
R
R_M
$M \leq N$.
0
10
20
30
40
50
60
70
80
90
Latitude



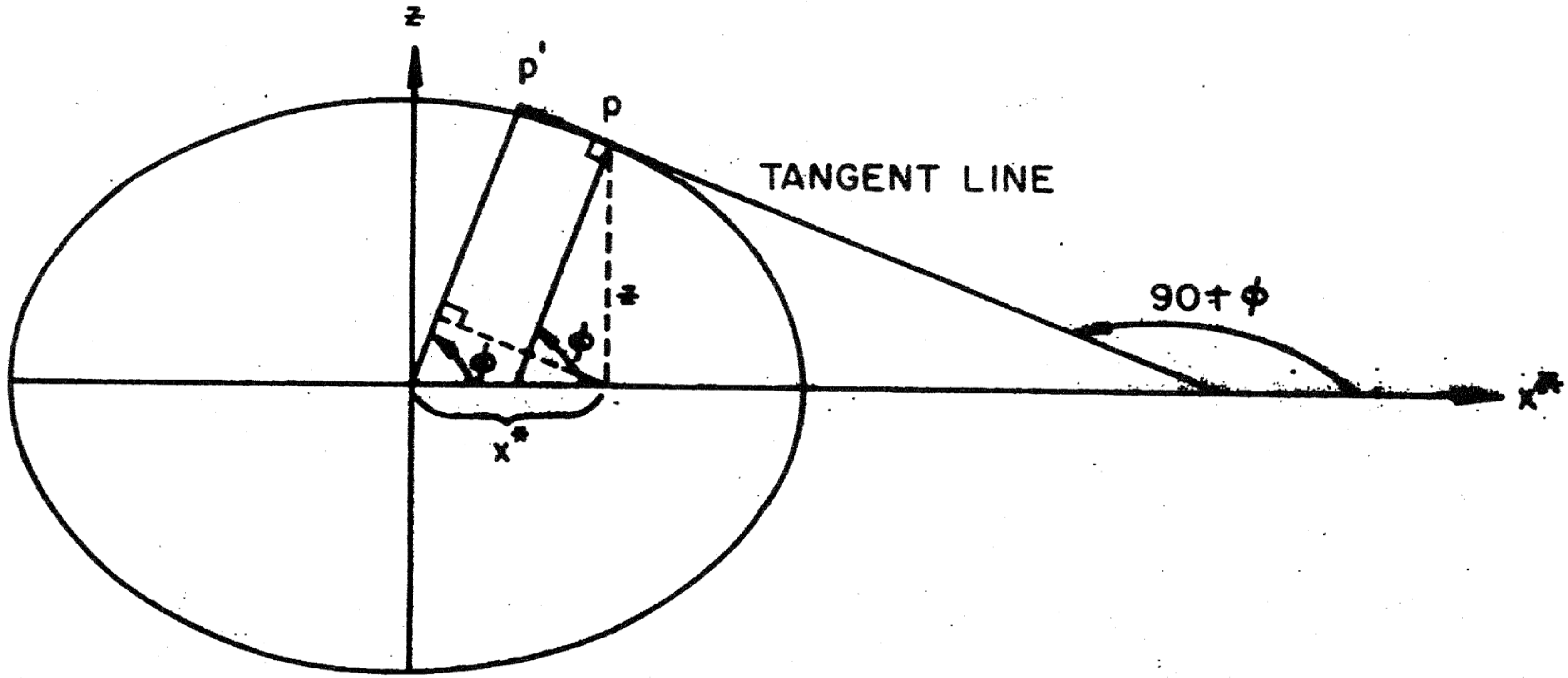
up
p
φ'
Re
φ'
φ
RM
RM
RN
φ
R Radius of Ellipsoid
Radius of Curvature
RN Prime Vertical
RM Meridian



Center of Radii of Curvature



COORDINATE SYSTEMS USED IN GEODESY


z
p'
p
TANGENT LINE
z
φ
φ
90+φ
x*
x*

Consider a point P on the surface of the ellipsoid. The coordinates of P referred to a system with the primary axis (denoted x*) in the meridian plane of P are

$$\bar{r} = \begin{bmatrix} x^* \\ 0 \\ z \end{bmatrix} . \qquad 2\text{-}11$$

The plane perpendicular to the ellipsoid normal at P, and passing through P is called the <u>tangent plane</u> at P. From Figure 2-7 the slope of the tangent plane is

$$\frac{dz}{dx^*} = \tan\,(90^\circ+\phi) = -\,\frac{\cos\phi}{\sin\phi} . \qquad 2\text{-}12$$

The slope can also be computed from the equation of the meridian ellipse as follows:

$$\frac{(x^*)^2}{a^2} + \frac{z^2}{b^2} = 1 \qquad 2\text{-}13$$

or

$$b^2 (x^*)^2 + a^2 z^2 = a^2 b^2 \qquad 2\text{-}14$$

$$2b^2 x^* \, dx^* + 2a^2 z dz = 0 \qquad 2\text{-}15$$

$$\frac{dz}{dx^*} = - \frac{b^2 x^*}{a^2 z} \ . \qquad 2\text{-}16$$

It follows from the above two equations for the slope, that

$$\frac{b^2 x^*}{a^2 z} = \frac{\cos\phi}{\sin\phi} \qquad 2\text{-}17$$

or

or

$$b^2 (x^*) \sin\phi = a^2 z \cos\phi \qquad 2\text{-}18$$

and after squaring the above

$$b^4 (x^*)^2 \sin^2\phi = a^4 z^2 \cos^2\phi \ . \qquad 2\text{-}19$$

Expressing equations 2-14 and 2-19 in matrix form

$$\begin{bmatrix} b^4 \sin^2\phi & -a^4 \cos^2\phi \\ b^2 & a^2 \end{bmatrix} \begin{bmatrix} (x^*)^2 \\ z^2 \end{bmatrix} = \begin{bmatrix} 0 \\ a^2 b^2 \end{bmatrix} . \qquad 2\text{-}20$$

The inverse of the coefficient matrix is

$$\frac{1}{a^2 b^2 (a^2 \cos^2\phi + b^2 \sin^2\phi)} \begin{bmatrix} a^2 & a^4 \cos^2\phi \\ -b^2 & b^4 \sin^2\phi \end{bmatrix} ,$$

therefore

$$\begin{bmatrix}(x^*)^2 \\ z^2\end{bmatrix} = \frac{1}{a^2 \cos^2\phi + b^2 \sin^2\phi} \begin{bmatrix}a^4 \cos^2\phi \\ b^4 \sin^2\phi\end{bmatrix}$$

and finding the square root

$$\begin{bmatrix}x^* \\ z\end{bmatrix} = \frac{1}{(a^2 \cos^2\phi + b^2 \sin^2\phi)^{1/2}} \begin{bmatrix}a^2 \cos\phi \\ b^2 \sin\phi\end{bmatrix} . \qquad 2\text{-}21$$

From Figure 2-6

$$\cos\phi = \frac{x^*}{N} ,$$

but from equation 2-21

$$x^* = \frac{a^2 \cos\phi}{(a^2 \cos^2\phi + b^2 \sin^2\phi)^{1/2}} ,$$

therefore

$$\boxed{N = \frac{a^2}{(a^2\cos^2\phi + b^2\sin^2\phi)^{1/2}}} \qquad 2\text{-}22$$

therefore

$$N = \frac{a^2}{(a^2\cos^2\phi + b^2\sin^2\phi)^{1/2}} \qquad 2\text{-}22$$

$$\bar{r} = \begin{bmatrix} x^* \\ 0 \\ z \end{bmatrix} = \begin{bmatrix} N\cos\phi \\ 0 \\ N\, b^2/a^2 \sin\phi \end{bmatrix} . \qquad 2\text{-}23$$

$$\bar{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = R_3(-\lambda) \begin{bmatrix} x^* \\ 0 \\ z \end{bmatrix}$$

$$= \begin{bmatrix} \cos(-\lambda) & \sin(-\lambda) & 0 \\ -\sin(-\lambda) & \cos(-\lambda) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} N\cos\phi \\ 0 \\ N\, b^2/a^2 \sin\phi \end{bmatrix}$$

$$\bar{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = N \begin{bmatrix} \cos\phi \ \cos\lambda \\ \cos\phi \ \sin\lambda \\ b^2/a^2 \ \sin\phi \end{bmatrix}.$$

Coordinate Conversion
Geodetic Latitude, Longitude, and Height to ECEF, X, Y, Z

$$X = (N + h)\cos\phi\cos\lambda$$

$$Y = (N + h)\cos\phi\sin\lambda$$

$$Z = [N(1 - e^2) + h]\sin\phi$$

where:

ϕ, λ, h = geodetic latitude, longitude, and height above ellipsoid

X, Y, Z = Earth Centered Earth Fixed Cartesian Coordinates

and:

$N(\phi) = a / \sqrt{1 - e^2 \sin^2\phi}$ = radius of curvature in prime vertical

a = semi-major earth axis (ellipsoid equatorial radius)

b = semi-minor earth axis (ellipsoid polar radius)

$f = \dfrac{a - b}{a}$ = flattening

$e^2 = 2f - f^2$ = eccentricity squared

Peter H. Dana 8/3/96

$$e^2 = 1 - b^2/a^2$$

$$p = (x^2 + y^2)^{1/2}$$

$$N_o = a$$

$$h_o = (x^2+y^2+z^2)^{1/2} - (ab)^{1/2}$$

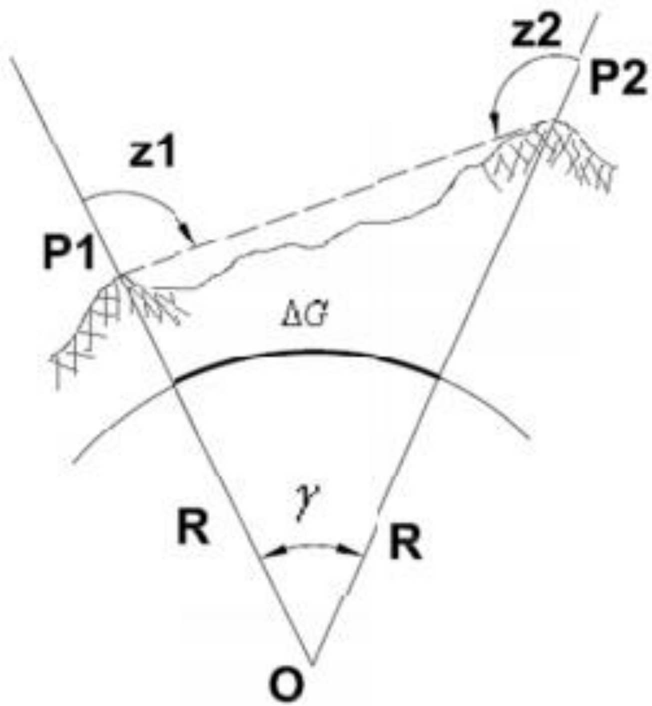
$$\phi_o = \tan^{-1} \left[\left(\frac{z}{p}\right) \left(1 - \frac{e^2 N_o}{N_o + h_o}\right)^{-1} \right] .$$

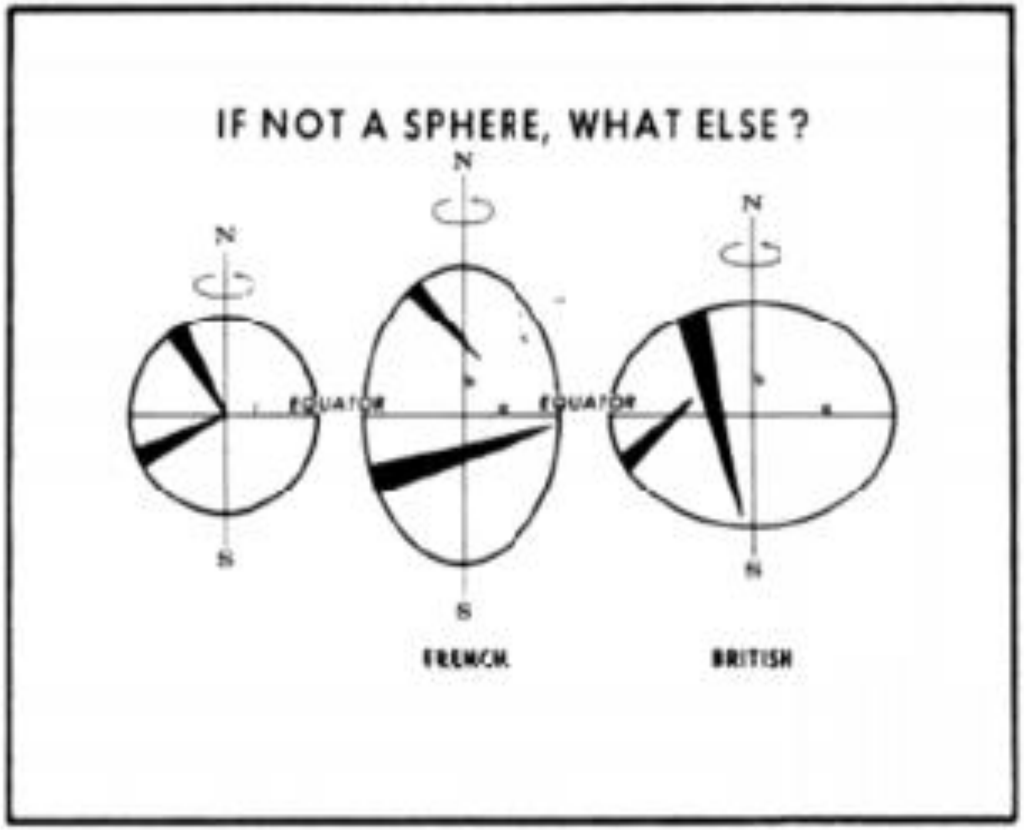
Each iteration then consists of evaluating in order

$$N_i = \frac{a}{\cos^2\phi_{i-1} + b^2/a^2 \sin^2\phi_{i-1})^{1/2}}$$

$$h_i = \frac{p}{\cos\phi_{i-1}} - N_i$$

$$\phi_i = \tan^{-1} \left[\left(\frac{z}{p}\right) \left(1 - \frac{e^2 N_i}{N_i + h_i}\right)^{-1} \right] .$$





Clairaut's theorem:

$$f = \frac{5}{2}\frac{\omega^2 a}{\gamma_E} - \frac{\gamma_P - \gamma_E}{\gamma_E}$$

بهترین بیضوی

۱- سرعت دوران مساوی سرعت دوران زمین

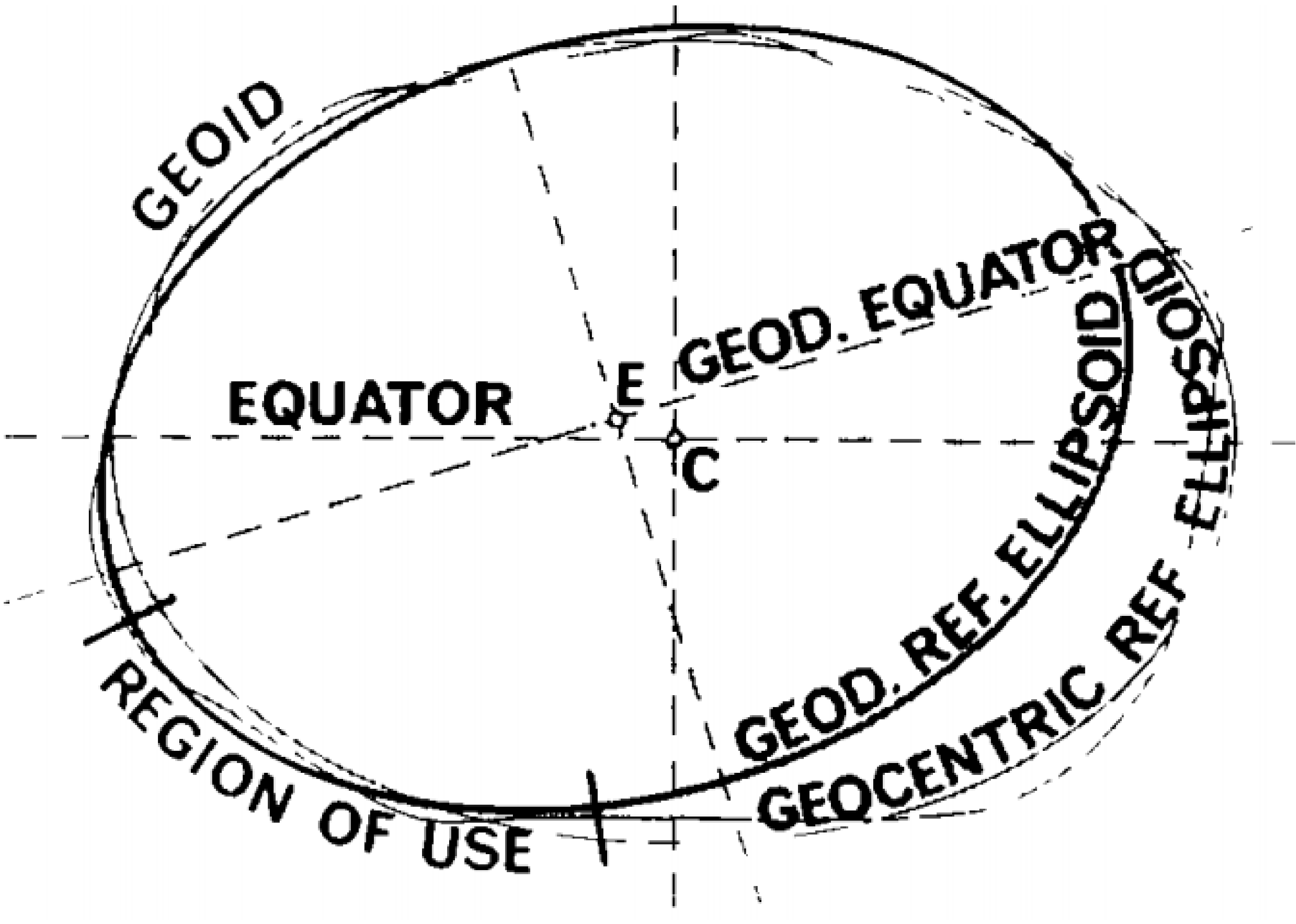
۲- شکل و اندازهٔ نزدیک به زمین

۳- پتانسیل روی سطح مساوی با پتانسیل ژئوئید

۴- جرم برابر با جرم زمین

$$\min_{f} \oiint_{S} (\xi^2 + \eta^2)\, dS ,$$

$$\boxed{\min_{a,f} \oiint_{S} N^2\, dS ,}$$


GEOID
EQUATOR
E
C
GEOD. EQUATOR
GEOD. REF. ELLIPSOID
GEOCENTRIC REF ELLIPSOID
REGION OF USE

Telluroid

The surface designed to approximate the physical surface of the earth is the *telluroid*. The telluroid is defined as the surface whose height above a geocentric reference ellipsoid is the same as the height of the terrain above the geoid [HIRVONEN, 1960]. FIG. 18 shows the relationship between the telluroid, terrain, geoid, and geocentric ellipsoid.

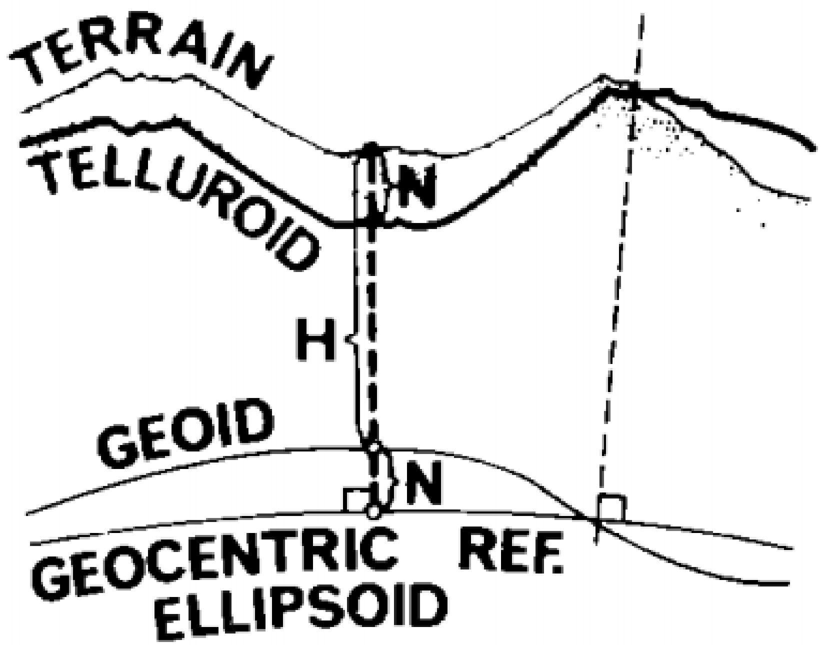
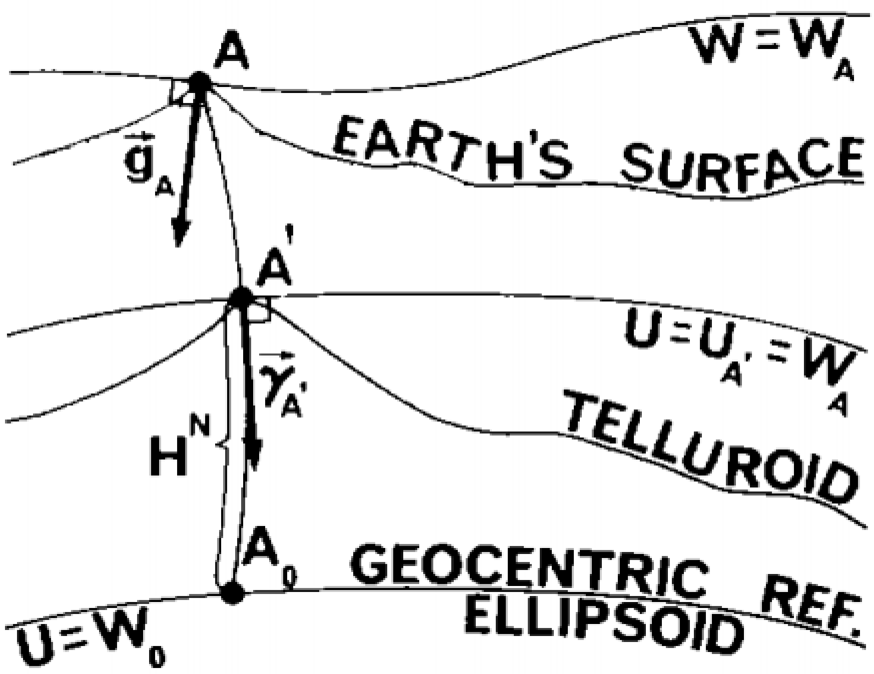
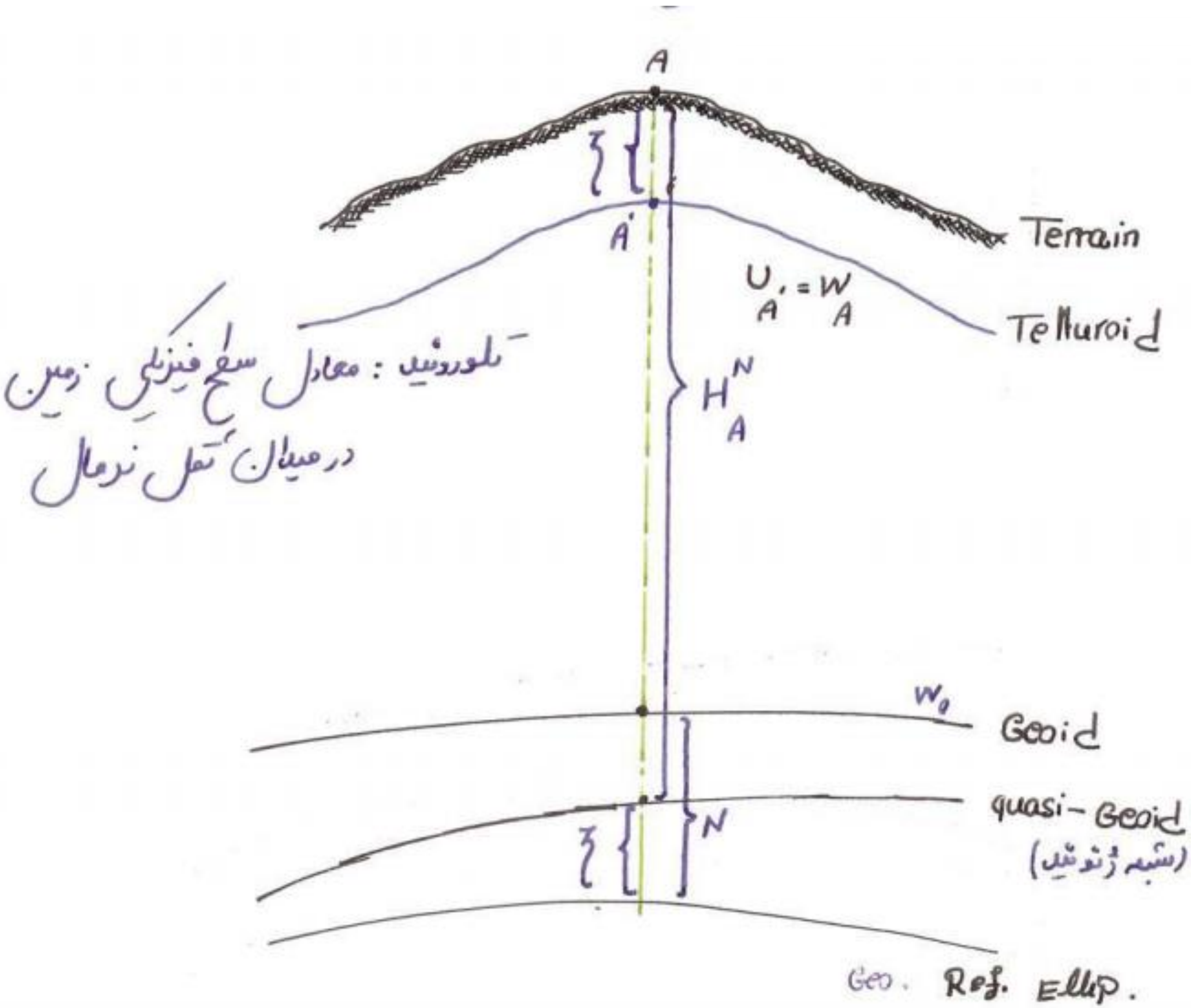


FIG. 7.18. Telluroid.

- **Telluroid: Surface whose normal potential *U* is equal to the actual potential *W* at the Earth's surface along the ellipsoidal normal. The telluroid is not an equipotential surface. The telluroid was proposed by Molodenskii**



شبه ژئوئید یک سطح فرضی فاقد نقش فیزیکی است که مبنای ارتفاعات نرمال میباشد و یک سطح هم پتانسیل نمی باشد.



World Geodetic System

- U.S. military references an ellipsoid known as the GRS80 ellipsoid
- Refined the values slightly in 1984 to make the world geodetic system (WGS84)
- This datum is significant because it is the datum referenced by the GPS system

WGS 84

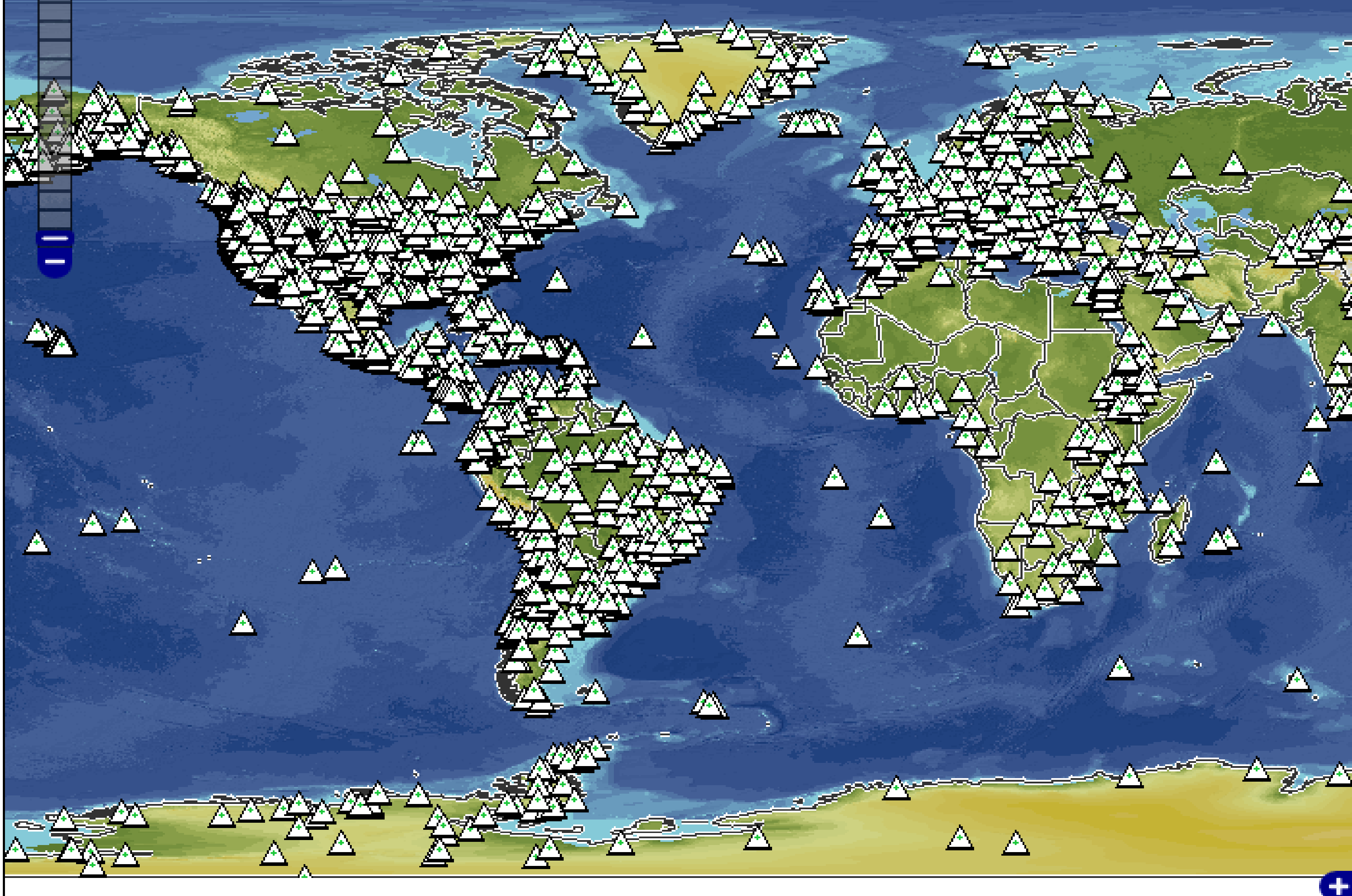
- The WGS 84 defining parameters include
- Semi-Major axis
  - 6378137 meters
- Ellipsoid flattening
  - 1/298.257223563
- Angular velocity of the earth
  - 7292115×10^{-11} radians/second
- Earth's gravitational constant
  - $9.81\ m/s^2$

International Terrestrial Reference System

- In 1991, the International Association of Geodesy established the International GPS Service (IGS) to promote and support activities related to the GPS system
- A network was established that included an international collaboration of 50 core tracking stations located around the world and supplemented by over 200 other stations (some working continuously, others intermittently)
- The IGS provides the precise orbits of the GPS satellites (can be found on the Internet)

International Terrestrial Reference System

- The definition of the reference system in which the coordinates of the tracking stations are expressed is the responsibility of the Earth Rotation Service
- The reference system is know as the International Terrestrial Reference Frame (ITRF)
- Defined by satellite laser ranging, baseline inferometry, and GPS coordinate results
- Each year, a new combination of precise tracking results is performed and the resulting datum is referred to by "ITRFxx" where "xx" is the current year
- One distinguishing characteristic of the ITRF is the definition of not only the tracking stations but also their velocities due to regional tectonic motion



Scale = 1 : 111M

sopac.ucsd.edu/map.shtml