Power System Analysis

Power Flow

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Introduction

A power flow study (**load-flow study**) is a steady-state analysis whose target is to determine the voltages, currents, and real and reactive power flows in a system under a given load conditions.

The purpose of power flow studies is to plan ahead and account for various hypothetical situations. For example, if a transmission line is be taken off line for maintenance, can the remaining lines in the system handle the required loads without exceeding their rated values.

Power-flow study formulation

Once voltages (magnitude and angle) are known for all buses, line flows and losses can be calculated. The starting point of solving power flow problems is to identify the known and unknown variables in the system.

There are 4 variables that are associated with each bus:

- ❑ P
- ❑ Q
- ❑ V
- ❑ δ

Meanwhile, there are two power flow equations associated with each bus.

In a power flow study, two of the four variables are defined an the other two are unknown. That way, we have the same number of equations as the number of unknown.

The known and unknown variables depend on the type of bus.

Power-flow study formulation

Each bus in a power system can be classified as one of three types:

1. Load bus (P-Q bus): a bus at which the real and reactive power are specified, and for which the bus voltage will be calculated. All busses having no generators are load buses. In here, **V and δ are unknown.**

2. Generator bus (P-V bus): a bus at which the magnitude of the voltage is defined and is kept constant by adjusting the field current of a synchronous generator. We also assign real power generation for each generator according to the economic dispatch. In here, **Q and δ are unknown.**

3. Generator bus (P-V bus): a special generator bus serving as the reference bus. Its voltage is assumed to be fixed in both magnitude and phase (for instance, $1\angle 0^{\circ}$ pu). In here, **P and Q are unknown.**

Power-flow study formulation
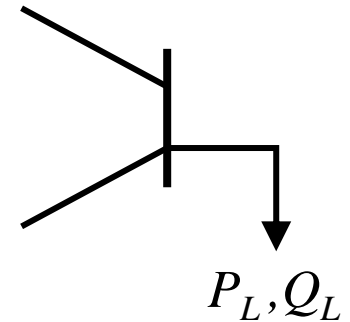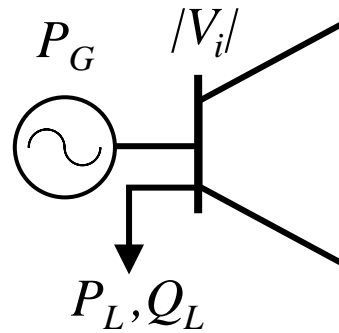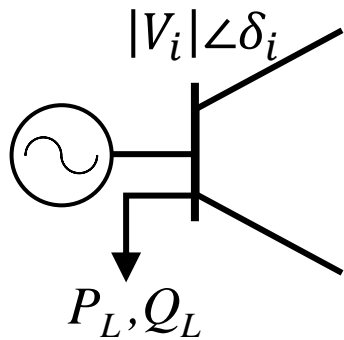
Bus type	Voltage($|V_i|\angle\delta_i$)		Real power			Reactive power		
	Magnitude	Angle	Generation	Load	Net(P_i)	Generation	Load	Net(Q_i)
Slack/ Swing	Specified	Specified	Unknown	Specified	Unknown	Unknown	Specified	Unknown
Generator/ Regulated/ PV	Specified	Unknown	Specified	Specified	Specified	Unknown	Specified	Unknown
Load/PQ	Unknown	Unknown	Specified	Specified	Specified	Specified	Specified	Specified

Type of buses in the power flow study

Power-flow analysis equations
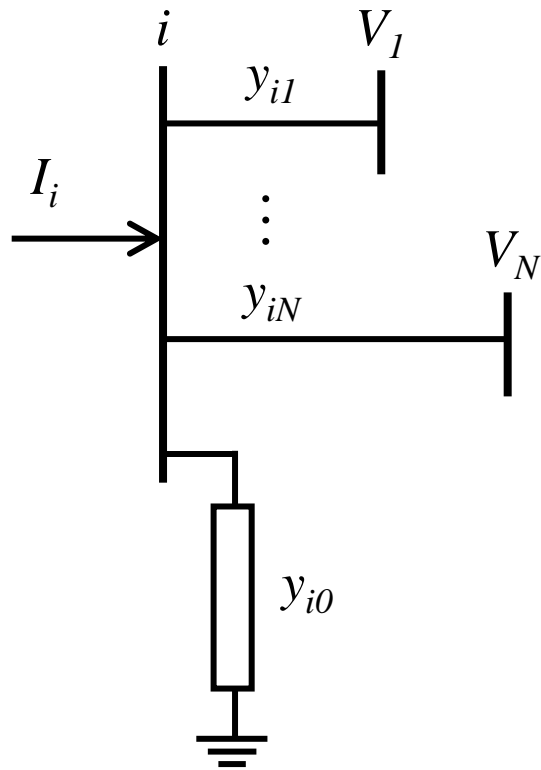
Nodal equations in an N-bus system:

$$[I] = [Y_{bus}][V]$$

$$\begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} Y_{11} & \cdots & Y_{1N} \\ \vdots & \ddots & \vdots \\ Y_{N1} & \cdots & Y_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ \vdots \\ V_N \end{bmatrix} \qquad \textbf{(1)}$$

where

$$Y_{ii} = \sum_{\substack{j=0 \\ j \neq i}}^{N} y_{ij} \qquad \text{and} \qquad Y_{ij} = Y_{ji} = -y_{ij}$$

So the node equation at bus i can be written as

$$I_i = \sum_{n=1}^{N} Y_{in} V_n \qquad \textbf{(2)}$$

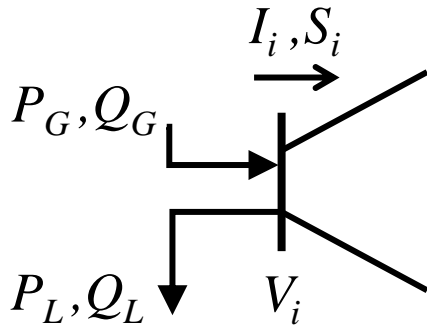# Power-flow analysis equations

Relationship between real and reactive power supplied to the system at bus i and the current injected into the system at that bus:

$$S_i = V_i I_i^* = P_i + jQ_i$$

$$P_i - jQ_i = V_i^* I_i$$

$$\overset{(2)}{\Longrightarrow}$$

$$P_i - jQ_i = V_i^* \sum_{n=1}^{N} Y_{in} V_n \qquad (3)$$

I_i, S_i

P_G, Q_G

P_L, Q_L V_i

Power-flow solution

Note that the power flow equations are non−linear, thus cannot be solved analytically. A numerical iterative algorithm is required to solve such equations. A standard procedure follows:

1. Create a bus admittance matrix Y_{bus} for the power system;

2. Make an initial estimate for the voltages (both magnitude and phase angle) at each bus in the system;

3. Substitute in the power flow equations and determine the deviations from the solution.

4. Update the estimated voltages based on some commonly known numerical algorithms (e.g., Newton-Raphson or Gauss-Seidel).

5. Repeat the above process until the deviations from the solution are minimal.

Gauss-Seidel technique

The Gauss-Seidel (GS) method is the simplest iterative technique used to solve power flow problems. In general, GS method follows the following iterative steps to reach the solution for the function $f(x) = 0$:

➢ Rearrange the function into the form $x = g(x)$ to calculate the unknown variable.

➢ Calculate the value $g(x^{(0)})$ based on initial estimates $x^{(0)}$.

➢ Calculate the improved value $x^{(1)} = g(x^{(0)})$.

➢ Continue solving for improved values until the solution is within acceptable limits $\left|x^{(k+1)} - x^{(k)}\right| \leq \epsilon$

In the context of a power flow problem, the unknown variables are voltages at all buses, but the slack. Both voltage magnitudes and angles are unknown for load buses, whereas voltage angles are unknown for generation buses.

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Gauss-Seidel technique

We need to put the equations in the correct form:

We know:

$$P_i - jQ_i = V_i^* \sum_{n=1}^{N} Y_{in} V_n \qquad \textbf{(3)}$$

$$\Rightarrow \begin{cases} Q_i = -\text{Im}\left\{ V_i^* \sum_{n=1}^{N} Y_{in} V_n \right\} & \textbf{(4)} \\\\ \dfrac{P_i - jQ_i}{V_i^*} = Y_{ii} V_i + \sum_{\substack{n=1 \\ n \neq i}}^{N} Y_{in} V_n & \textbf{(5)} \end{cases}$$

Gauss-Seidel technique

$$\frac{P_i - jQ_i}{V_i^*} = Y_{ii}V_i + \sum_{\substack{n=1 \\ n \neq i}}^{N} Y_{in}V_n \qquad (5)$$

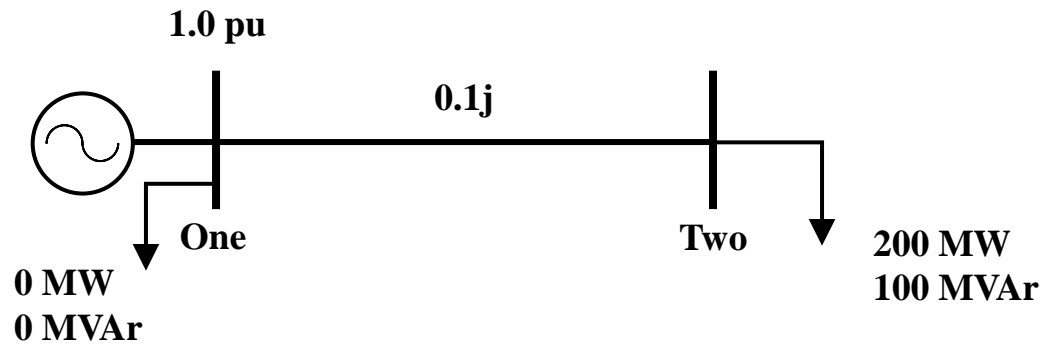The voltage V_i at bus i can be calculated using:

$$V_i = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{n=1 \\ n \neq i}}^{N} Y_{in}V_n\right] \qquad (6)$$

The iterative voltage equation is as follows:

$$V_i^{(k+1)} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{V_i^{*(k)}} - \sum_{\substack{n=1 \\ n \neq i}}^{N} Y_{in}V_n^{(k+1 \ or \ k)}\right] \qquad (7)$$

Two Bus Gauss-Seidel Example

For the two bus power system shown below, use the Gauss-Seidel power flow to determine the voltage magnitude and angle at bus two (5 iterations). Assume that bus one is the slack and $S_{base} = 100$ MVA.

1.0 pu

0.1j

One

0 MW
0 MVAr

Two

200 MW
100 MVAr

Two Bus Gauss-Seidel Example

1.0 pu

$S_b = 100$ MVA

0.1j

One

0 MW
0 MVAr

Two

200 MW
100 MVAr

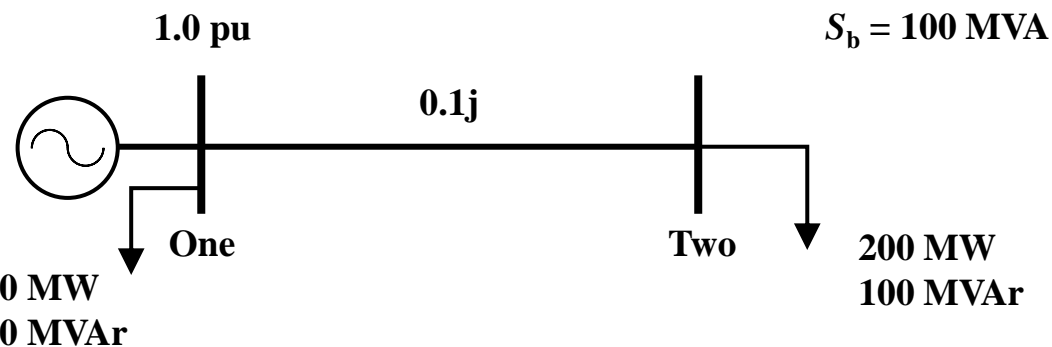| i		Known	Unknown
Slack	1	$V_1 = 1 < 0$	P_1 , Q_2
P-Q	2	$P_2 = -2 , Q_2 = -1$	$V_2 = ? < ?$

$$V_i^{(k+1)} = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{V_i^{*(k)}} - \sum_{\substack{n=1 \\ n \neq i}}^{N} Y_{in} V_n^{(k+1 \; or \; k)} \right] \quad (7)$$

$$Y_{bus} = \begin{bmatrix} -10j & 10j \\ 10j & -10j \end{bmatrix}$$

$k = 0:$ $V_2^{(0)} = 1.0 \angle 0$ *(flat start)*

$$V_2^{(1)} = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{V_i^{*(0)}} - \sum_{\substack{n=1 \\ n \neq i}}^{2} Y_{in} V_n^{(0)} \right] = \frac{1}{-10j} \left[\frac{-2 - j(-1)}{V_2^{*(0)}} - (10j \times V_2^{(0)}) \right] = 0.922 \angle -12.5288°$$

$V_2^{(2)} = 0.8677 \angle -12.5288°$

$V_2^{(3)} = 0.8610 \angle -13.43°$

$V_2^{(4)} = 0.8565 \angle -13.43°$

$V_2^{(5)} = 0.8559 \angle -13.51°$

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Two Bus Gauss-Seidel Example

Once the voltage angle and magnitude at bus 2 are known we can calculate all the other system values, such as the line flows and the generator real and reactive power output.

200 MW
168.1 MVAr
→

-200 MW
-100 MVAr
←

$1\angle 0°$

$0.8559 \angle -13.51°$

0.1j

One

Two

0 MW
0 MVAr

200 MW
100 MVAr

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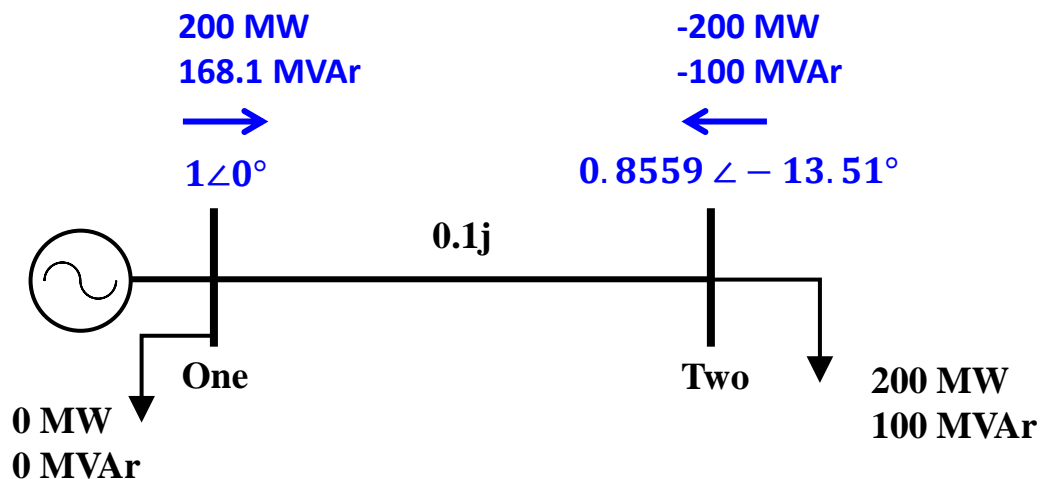
Newton-Raphson technique

The Newton-Raphson (N-R) method is based on Taylor's expansion approximation.

➤ Consider the function: $f(x) = c$, where x is unknown

➤ Let $x^{(0)}$ be an initial estimate, then $\Delta x^{(0)}$ is a small deviation from the correct solution

$$f\left(x^{(0)} + \Delta x^{(0)}\right) = c$$

➤ Expand the left-hand side into a Taylor's series about $x^{(0)}$ yeilds

$$f\left(x^{(0)}\right) + \left(\frac{df(x^{(0)})}{dx}\right)\left(\Delta x^{(0)}\right) + \frac{1}{2}\left(\frac{d^2 f(x^{(0)})}{dx^2}\right)(\Delta x^{(0)})^2 + \cdots = c$$

Newton-Raphson technique

➤ Assuming the error, $\Delta x^{(0)}$, is small, the higher-order terms are neglected, resulting in

$$f\left(x^{(0)}\right) + \left(\frac{df\left(x^{(0)}\right)}{dx}\right)\left(\Delta x^{(0)}\right) \approx c \Rightarrow \Delta c^{(0)} \approx \left(\frac{df\left(x^{(0)}\right)}{dx}\right)\left(\Delta x^{(0)}\right)$$

➤ Where

$$\Delta c^{(0)} = c - f\left(x^{(0)}\right)$$

➤ Rearranging the equations

$$\Delta x^{(0)} = \frac{\Delta c^{(0)}}{\dfrac{df\left(x^{(0)}\right)}{dx}}$$

$$x^{(1)} = x^{(0)} + \Delta x^{(0)}$$

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Newton-Raphson technique

We need to put the equations in the correct form:

We know:
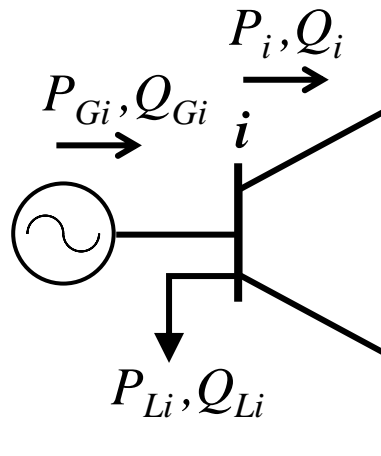
$$P_i - jQ_i = V_i^* \sum_{n=1}^{N} Y_{in} V_n \qquad \textbf{(3)}$$

Let $\qquad Y_{in} = |Y_{in}| \angle \theta_{in} \,, V_i^* = |V_i| \angle - \delta_i \,, V_n = |V_n| \angle \delta_n$

$$\Rightarrow P_i - jQ_i = \sum_{n=1}^{N} |V_i||V_n||Y_{in}| \angle - \delta_i + \delta_n + \theta_{in}$$

$$\Rightarrow \begin{cases} P_i = \sum_{n=1}^{N} |V_i||V_n||Y_{in}| \cos(\delta_i - \delta_n - \theta_{in}) \qquad \textbf{(8)} \\[2em] Q_i = \sum_{n=1}^{N} |V_i||V_n||Y_{in}| \sin(\delta_i - \delta_n - \theta_{in}) \qquad \textbf{(9)} \end{cases}$$

Newton-Raphson technique

Power Balance Equations

$$\Rightarrow \begin{cases} \underbrace{P_i - P_{Gi} + P_{Li}}_{\Delta P_i} = 0 \\ \underbrace{Q_i - Q_{Gi} + Q_{Li}}_{\Delta Q_i} = 0 \end{cases} \qquad \textbf{(10)}$$

Note that P_i and Q_i mean the functions that expresses flow from bus I into the system in terms of voltage magnitudes and angles, while P_{Gi}, P_{Li}, Q_{Gi}, Q_{Li} mean the generations and demand at the bus.

For a system with a slack bus and the rest PQ buses, power flow problem is to use the power balance equations to solve for the unknown voltage magnitudes and angles in terms of the given bus generation and demands, and then use solution to calculate the real and reactive injection at the slack bus.

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Newton-Raphson technique

Power Flow Variables and Solution

Assume the slack bus is the first bus (with a fixed voltage angle/magnitude). We then need to determine the voltage angle/magnitude at the other buses.

We must solve $f(\mathbf{x}) = 0$, where:

$$\mathbf{x} = \begin{bmatrix} [\delta] \\ [|V|] \end{bmatrix} = \begin{bmatrix} \delta_2 \\ \delta_3 \\ \vdots \\ \delta_N \\ |V_2| \\ |V_3| \\ \vdots \\ |V_N| \end{bmatrix} \qquad f(\mathbf{x}) = \begin{bmatrix} [\Delta P] \\ [\Delta Q] \end{bmatrix} = \begin{bmatrix} P_2 - P_{G2} + P_{L2} \\ \vdots \\ P_N - P_{GN} + P_{LN} \\ Q_2 - Q_{G2} + Q_{L2} \\ \vdots \\ Q_N - Q_{GN} + Q_{LN} \end{bmatrix}$$

And the iterative equation is as follows:

$$\begin{bmatrix} [\delta]^{(k+1)} \\ [|V|]^{(k+1)} \end{bmatrix} = \begin{bmatrix} [\delta]^{(k)} \\ [|V|]^{(k)} \end{bmatrix} - \left[J^{(k)} \right]^{-1} \begin{bmatrix} [\Delta P]^{(k)} \\ [\Delta Q]^{(k)} \end{bmatrix} \qquad \textbf{(11)}$$

$[J]$: Jacobian matrix

Newton-Raphson technique

Jacobian matrix

$$[J] = \begin{bmatrix} \left[\dfrac{\partial P}{\partial \delta}\right] & \left[\dfrac{\partial P}{\partial |V|}\right] \\[2em] \left[\dfrac{\partial Q}{\partial \delta}\right] & \left[\dfrac{\partial Q}{\partial |V|}\right] \end{bmatrix} = \begin{bmatrix} \dfrac{\partial P_2}{\partial \delta_2} & \cdots & \dfrac{\partial P_2}{\partial \delta_N} & \dfrac{\partial P_2}{\partial |V_2|} & \cdots & \dfrac{\partial P_2}{\partial |V_N|} \\[1em] \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\[1em] \dfrac{\partial P_N}{\partial \delta_2} & \cdots & \dfrac{\partial P_N}{\partial \delta_N} & \dfrac{\partial P_N}{\partial |V_2|} & \cdots & \dfrac{\partial P_N}{\partial |V_N|} \\[1em] \dfrac{\partial Q_2}{\partial \delta_2} & \cdots & \dfrac{\partial Q_2}{\partial \delta_N} & \dfrac{\partial Q_2}{\partial |V_2|} & \cdots & \dfrac{\partial Q_2}{\partial |V_N|} \\[1em] \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\[1em] \dfrac{\partial Q_N}{\partial \delta_2} & \cdots & \dfrac{\partial Q_N}{\partial \delta_N} & \dfrac{\partial Q_N}{\partial |V_2|} & \cdots & \dfrac{\partial Q_N}{\partial |V_N|} \end{bmatrix}$$

Newton-Raphson technique

Jacobian matrix

$$\frac{\partial P_i}{\partial \delta_n} = |V_i||V_n||Y_{in}| \sin(\delta_i - \delta_n - \theta_{in}) \ \ n \neq i$$

$$\frac{\partial Q_i}{\partial \delta_n} = -|V_i||V_n||Y_{in}| \cos(\delta_i - \delta_n - \theta_{in}) \ \ n \neq i$$

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{\substack{n=1 \\ n \neq i}}^{N} |V_i||V_n||Y_{in}| \sin(-\delta_i + \delta_n + \theta_{in})$$

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{\substack{n=1 \\ n \neq i}}^{N} |V_i||V_n||Y_{in}| \cos(\delta_i - \delta_n - \theta_{in})$$

$$\frac{\partial P_i}{\partial |V_n|} = |V_i||Y_{in}| \cos(\delta_i - \delta_n - \theta_{in}) \ \ n \neq i$$

$$\frac{\partial P_i}{\partial |V_i|} = 2|V_i||Y_{ii}| \cos(\theta_{ii}) + \sum_{\substack{n=1 \\ n \neq i}}^{N} |V_n||Y_{in}| \cos(\delta_i - \delta_n - \theta_{in})$$

$$\frac{\partial Q_i}{\partial |V_n|} = |V_i||Y_{in}| \sin(\delta_i - \delta_n - \theta_{in}) \ \ n \neq i$$
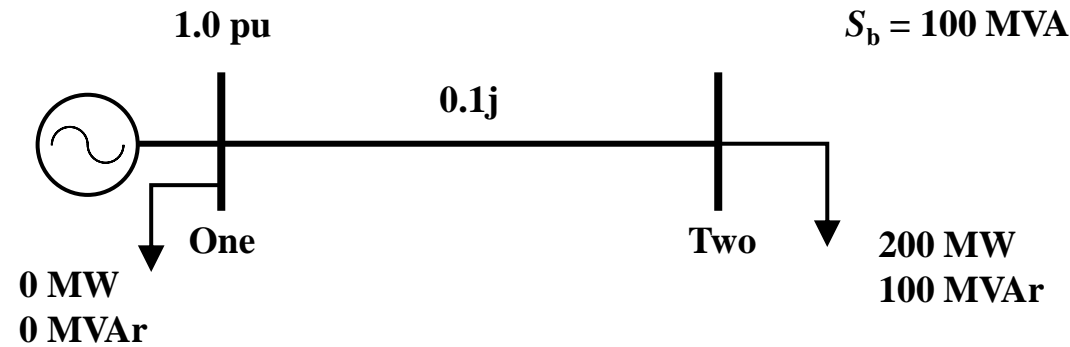
$$\frac{\partial Q_i}{\partial |V_i|} = 2|V_i||Y_{ii}| \sin(-\theta_{ii}) + \sum_{\substack{n=1 \\ n \neq i}}^{N} |V_n||Y_{in}| \sin(\delta_i - \delta_n - \theta_{in})$$

Two Bus Newton-Raphson Example

Now in previous example, use the Newton-Raphson power flow to determine the voltage magnitude and angle at bus two (3 iterations). Assume that bus one is the slack and $S_{\text{base}} = 100$ MVA.

1.0 pu

0.1j

One

Two

0 MW
0 MVAr

200 MW
100 MVAr

Two Bus Newton-Raphson Example

1.0 pu

0.1j

One

0 MW
0 MVAr

Two

200 MW
100 MVAr

$S_b = 100$ MVA

i		Known	Unknown
Slack	1	$V_1 = 1 < 0$	P_1 , Q_2
P-Q	2	$P_2 = -2 , Q_2 = -1$	$V_2 = ? < ?$

$$\mathbf{x} = \begin{bmatrix} \delta_2 \\ |V_2| \end{bmatrix}$$

$$Y_{bus} = \begin{bmatrix} -10j & 10j \\ 10j & -10j \end{bmatrix}$$

(8), (9), (10)
Power balance equations

$$\begin{cases} \sum_{n=1}^{N} |V_i||V_n||Y_{in}| \cos(\delta_i - \delta_n - \theta_{in}) - P_{Gi} + P_{Li} = 0 \\ \\ \sum_{n=1}^{N} |V_i||V_n||Y_{in}| \sin(\delta_i - \delta_n - \theta_{in}) - Q_{Gi} + Q_{Li} = 0 \end{cases}$$

$$\rightarrow \begin{cases} \underbrace{|V_2||V_1|(10 \sin(\delta_2))}_{P_2} + 2 = 0 \\ \\ \underbrace{|V_2||V_1|(-10 \cos(\delta_2)) + |V_2|^2(10)}_{Q_2} + 1 = 0 \end{cases}$$

$$\rightarrow \quad f(\mathbf{x}) = \begin{bmatrix} |V_2|(10 \sin(\delta_2)) + 2 \\ \\ |V_2|(-10 \cos(\delta_2)) + |V_2|^2(10) + 1 \end{bmatrix}$$

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Two Bus Newton-Raphson Example

$$[J] = \begin{bmatrix} \dfrac{\partial P_2}{\partial \delta_2} & \dfrac{\partial P_2}{\partial |V_2|} \\ \dfrac{\partial Q_2}{\partial \delta_2} & \dfrac{\partial Q_2}{\partial |V_2|} \end{bmatrix} = \begin{bmatrix} 10|V_2|\cos\delta_2 & 10\sin\delta_2 \\ 10|V_2|\sin\delta_2 & -10\cos\delta_2 + 20|V_2| \end{bmatrix}$$

$$k=0: \quad \mathbf{x}^{(0)} = \begin{bmatrix} \delta_2^{(0)} \\ \left|V_2^{(0)}\right| \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \overset{\text{(flat start)}}{\longrightarrow} \quad f\left(\mathbf{x}^{(0)}\right) = \begin{bmatrix} \left|V_2^{(0)}\right|\left(10\sin\delta_2^{(0)}\right)+2 \\ \left|V_2^{(0)}\right|\left(-10\cos\delta_2^{(0)}\right)+\left|V_2^{(0)}\right|^2(10)+1 \end{bmatrix} = \begin{bmatrix} 2.0 \\ 1.0 \end{bmatrix}$$

$$\left[J\left(\mathbf{x}^{(0)}\right)\right] = \begin{bmatrix} 10\left|V_2^{(0)}\right|\cos\delta_2^{(0)} & 10\sin\delta_2^{(0)} \\ 10\left|V_2^{(0)}\right|\sin\delta_2^{(0)} & -10\cos\delta_2^{(0)} + 20\left|V_2^{(0)}\right| \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$

$$\mathbf{x}^{(1)} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}^{-1} \begin{bmatrix} 2.0 \\ 1.0 \end{bmatrix} \Rightarrow \begin{bmatrix} \delta_2^{(1)} \\ \left|V_2^{(1)}\right| \end{bmatrix} = \begin{bmatrix} -0.2 \\ 0.9 \end{bmatrix}$$

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Two Bus Newton-Raphson Example

$$\begin{bmatrix} \delta_2^{(1)} \\ \left|V_2^{(1)}\right| \end{bmatrix} = \begin{bmatrix} -0.2 \\ 0.9 \end{bmatrix} \longrightarrow f(\mathbf{x}^{(1)}) = \begin{bmatrix} 0.9(10\sin(-0.2)) + 2 \\ 0.9(-10\cos(-0.2)) + 0.9^2(10) + 1 \end{bmatrix} = \begin{bmatrix} 0.212 \\ 0.279 \end{bmatrix}$$

$$[J(\mathbf{x}^{(1)})] = \begin{bmatrix} 8.82 & -1.986 \\ -1.788 & 8.199 \end{bmatrix}$$

$$\mathbf{x}^{(2)} = \begin{bmatrix} -0.2 \\ 0.9 \end{bmatrix} - \begin{bmatrix} 8.82 & -1.986 \\ -1.788 & 8.199 \end{bmatrix}^{-1} \begin{bmatrix} 0.212 \\ 0.279 \end{bmatrix} = \begin{bmatrix} -0.233 \\ 0.8586 \end{bmatrix}$$

Next iteration

$$f(\mathbf{x}^{(2)}) = \begin{bmatrix} 0.0145 \\ 0.0190 \end{bmatrix} \qquad \mathbf{x}^{(3)} = \begin{bmatrix} -0.236 \\ 0.8554 \end{bmatrix} \qquad f(\mathbf{x}^{(3)}) = \begin{bmatrix} 0.0000906 \\ 0.0001175 \end{bmatrix} \approx \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\longrightarrow V_2 = 0.8554 \angle -13.52°$$

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Two Bus Newton-Raphson Example