

## An analytical approach for sizing and siting of DGs in balanced radial distribution networks for loss minimization



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### ABSTRACT

This paper presents a novel analytical approach to determine the optimal siting and sizing of distributed generation (DG) units in balanced radial distribution network to minimize the power loss of the system. The proposed analytical expressions are based on a minimizing the loss associated with the active and reactive component of branch currents by placing the DG at various locations. This method first identifies a sequence of nodes where DG units are to be placed. The optimal sizes of DG units at the identified nodes are then evaluated by optimizing the loss saving equations and need only the results of base case load flow. To find out the best location for DG placement, a computational method is also developed. The proposed method has been tested and validated on two IEEE test distribution systems (DSs) consisting of 15 and 33-buses and it has been found that a significant loss saving can be obtained by placing DG units in the system using proposed analytical method.

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### Introduction

R/X ratio in distribution networks is much higher rather than transmission systems, and result of higher power losses and gradually loss of electrical energy along the distribution feeders [1–5]. Consequently, for many utilities all over the world loss minimization is one of the biggest question. Two extensive methods for loss minimization in distribution networks are network reconfiguration and capacitors placement well known and used frequently [2,3–6].

In recent past, DG has attained significant interest and can be defended, aspects such as environmental concerns, the restructuring of electricity market, the development in advance technologies for small-scale power generation, power electronics, and energy storage devices for transient backup into electric power DS [6,7]. However, this inclination has extended considerable opportunities but devised several confrontations in planning and operations of DSs. DGs are defined as electric power generations directly connected to loads or distribution networks; they range from a few kW to a few MWs [7,8]. Today, there are many DG technologies in trend cover conventional (such as micro turbines, combustion turbine, combined cycle, and internal combustion engines) to non-conventional (such as ocean, photovoltaic solar, fuel cell, wind, and geothermal) [1,7,8].

The essential objective of DG units is energy injection; despite, strategically placed and operated DG units can offer several other benefits (i.e. technical and economical) to utilities as well as to customers [9]. Typical cases of such benefits are the application of DG units for loss reduction, voltage and loadability improvement, enhanced system reliability and security, improved power quality, increased overall energy efficiency, and relieved transmission and distribution (T&D) [6–8]. While, economical benefits cover saving world fuel, saving T&D cost and reducing whole sale electricity price. Deferred investments for upgrades of facilities, reduced operational and maintenance (O&M) costs, enhanced productivity, reduced fuel costs due to increased overall efficiency, reduced reserve requirements and the associated costs, lower operating costs due to peak shaving are the additional economical benefits [8,10,11]. As far concern to the electricity market security today's deregulation of power industry, DG units play an important role in ancillary services such as reactive power support, spinning reserve, loss compensation, frequency control and other fast response services [9,12]. Moreover, in achieving the benefits of these ancillary services; DG units have been come into view as an integral part of DS. Although, inadequately operated and poorly planned DG units may also have some adverse effect on the functioning of the DS; based on the size, location, and infiltration level they can lead to reverse power flows, voltage rise, increased fault levels, more power losses, harmonic distortion, stability problems and consecutive feeder overloads [1,9–11,13].

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It is evident that loss reduction is one of the most substantial and beneficial factor to be treated in DG planning and operation apart from factors discussed above. The major challenges in planning of DG for loss minimization are suitable location, proper sizes, and operating strategies. The DG optimal sizing and siting for minimizing losses has drawn increasing attention of the extensive group of researchers in the recent years. There have been diverse techniques/approaches employed to cover the DG siting and sizing problem in DS for power loss minimization considering different type of DG technologies with their relative advantages and disadvantages, in attaining this distinct objective along with their practicability as examined in [1,7,8,10,11]. Therefore, this paper reports the development of some simple analytical expressions for sizing and siting of DG units, which can be easily implemented in a balanced radial DS.

Remaining of the paper is set out as below: Section 'Loss minimization techniques' describe a concise literature review on earlier loss minimization methods/techniques for DG planning. In Section 'Proposed methodology', proposed analytical method for optimal size and siting for single and multiple DG is discussed. The detailed computational procedure is elaborated in Section 'Computational procedure'. Numerical results and simulation of developed analytical method applied in two IEEE test systems, interesting findings along with discussion addressed in Section 'Numerical results'. Section 'Conclusion', summarizes the major contributions and conclusions.

## Loss minimization techniques

It has been realized that most of the existing work on DG siting and sizing in the DS, discussed different issues such as minimization of system power loss [1,6,10,11,13–25], abatement of harmonic pollution [19], enhancement of system voltage profile and stability [12,13,15–18,22–25], investment minimization or profit maximization [26,27], and loading margin [28] have been intended by researchers in their single or multi-objective problem formulations. Different optimization techniques, such as analytical approach [1,9–11,15,16,19,25], mixed integer non-linear programming (MINLP) [12,13,17,18], evolutionary algorithms (EA) technique [9], metaheuristic approaches: meta-heuristic harmony search algorithm (HSA) [6], particle swarm optimization (PSO) [20], heuristic approaches [27]; trade-off method [20], genetic algorithm (GA) technique [26,28], Kalman filter algorithm [23], multi-period AC optimal power flow (OPF) solver tool [20], and multi-objective non-linear programming (NLP) [15] have been used to solve the optimization problems for DG siting and sizing. Except these, there have been many interesting studies on the DGs siting and sizing of DS for loss minimization.

An analytical technique was noticed in [16] to find out the allocation of a single DG in radial as well as mesh networks to minimize the losses, based on unity power factor. However, optimal sizing is not taken into account. A more faster and precise analytical method than the classical methods [6,20,27] based on the equivalent current injection technique and without the use of impedance or Jacobian matrices for optimum size and location of DG in radial systems has been implemented in [19]. Moreover, this method was in near concurrence with the analytical method inscribed in [15]; in which an exact loss formula based analytical approach has been investigated to identify the optimal size and location of single DG in two load flow solution.

Pursuing the aforesaid work, various analytical expressions based on exact loss formula for optimal allocation of DGs were addressed in [1,10,11]. An efficient solution based on improved analytical (IA) expression to locate and size of four type of (renewable and non-renewable) DGs for loss minimization has been

examined in [1]. Although multiple DGs allocation was not considered. Contrary to this, the same authors in [10] applied the same approach for multiple DG unit placement to get an utmost loss minimization in large size primary DSs. Similar kind of work was also noticed in [11] using three analytical expressions to obtain the optimum sizes and locations of renewable DGs for power loss reduction considering the combination of time-varying demand and different DG output curves.

Moreover, in [12,17,18] technique based on probabilistic planning and formulated as MINLP problem have been acquainted to the readers. In [17,18], this technique enforced to identifying the best supply, unify of various classes of non-conventional DGs (i.e. wind, biomass, and solar) to reduce the power losses yearly in DS; although, DGs competent of bringing active power only is taken into account in both the studies. Similarly, in [12] same approach was implemented on renewable DGs for best location and size so as to enhance the voltage stability margin (VSM).

In the line of above, in [23] the optimal size of DGs is determined using the Kalman filter algorithm so that total power losses are minimized. A multi-objective index-based technique to determine optimal size and location of DG units in DS with non-unity power factor considering different load models has been exposed in [24].

A multi-period AC-OPF solver based method is discussed for to determine optimal power of renewable DG sources and there size to minimize the total energy losses during a period in [20]. Authors in [22] considered an iterative DG placement technique to improve the VSM. Though losses and optimal size of DGs not considered and a fixed value is assumed for all DGs. A multi-objective method is examined in [28] for optimal placement of DGs with for loading margin and profit to be maximized considering network constraints. Although, losses and fixed reactive limits for unknown DG sizes are not studied. Recently, a new multi-objective index (IMO) based analytical expressions to accommodate a combination of photovoltaic and battery energy storage DG units for reducing energy loss and enhancing voltage stability suggested in [25] using self-correction algorithm (SCA), while considering the time-varying demand and probabilistic generation.

Most of the studies reported above, DGs considered as only pure active power source. However, it is more beneficial to improve performance of DS, when the DG units supply reactive power. Depending on the type of DG used; they can able to inject or absorbs reactive power within their capability limits [13]. Furthermore, large number of the commonly used analytical techniques for DG siting and sizing are depend on exact loss formula and expect the evaluation of the Jacobian matrix and computationally demanding more time. Therefore, the above said methods are not quite appropriate due to the intricacy, capacity and the distinct property of the DS. Consequently, the optimal allocation of either type of DG using optimal solution methodology draws added consideration.

To overcome the obstacles of earlier studies and motivated by the work of [3–5,29], this paper proposes to apply a novel and simple analytical approach which is based on the DG active and reactive branch currents and the associated loss saving for allocating the DG units for loss reduction in the radial DS. The procedure first determines the location of the DG in a consecutive way. Erstwhile the DG locations are obtained, the optimal DG capacity at each chosen locations are find out by optimizing the loss saving equation.

## Proposed methodology

This segment set forth on a detailed mathematical formulation of the proposed analytical method. To develop the formulation following are the assumptions and constraints used in this paper:

- i. Considered distribution system is a single source, radial, and balanced.
- ii. The lower and upper voltage limits are set at 0.90 and 1.05 pu.
- iii. Load level is constant.
- iv. Maximum DG capacity for different test systems is assumed to be equal to the total load of the system.
- v. Maximum loss saving by the DGs for different test systems should not be less than zero (i.e. maximum loss saving <0)

The power flow solution will be used to check the limit violations of all above assumption and constraints.

Consider a  $N$ -bus radial DS having  $n$  number of branches as shown in Fig. 1. Here,  $I_i$  is the current in branch  $i$  before DG placement.

The total power loss of the system, i.e.  $P_L$  can be given as

$$P_L = \sum_{i=1}^{N-1} I_i^2 \cdot R_i = \sum_{i=1}^{N-1} (I_{ai}^2 + I_{ri}^2) \cdot R_i \quad (1)$$

Here,  $I_{ai}$  and  $I_{ri}$  are the real and imaginary components of  $I_i (= I_{ai} + jI_{ri})$  respectively, the complex current in branch  $i$  and  $R_i$  is the resistance of  $i$ th branch. The  $I_i$  can be obtained from the load flow solution of the given DS [5]. The loss associated with the active and reactive components of branch currents could be expressed as  $P_{La}$  and  $P_{Lr}$  respectively and given as

$$P_{La} = \sum_{i=1}^{N-1} I_{ai}^2 \cdot R_i \quad (2)$$

$$P_{Lr} = \sum_{i=1}^{N-1} I_{ri}^2 \cdot R_i \quad (3)$$

The procedure of single and multiple DG placement for loss minimization is addressed in the following section.

#### Loss minimization by single DG placement

Fig. 2 demonstrates a  $N$ -bus radial distribution network with DG at  $m$ th bus. Here,  $I_i^{new}$  is the current in branch  $i$  after DG placement in the system.

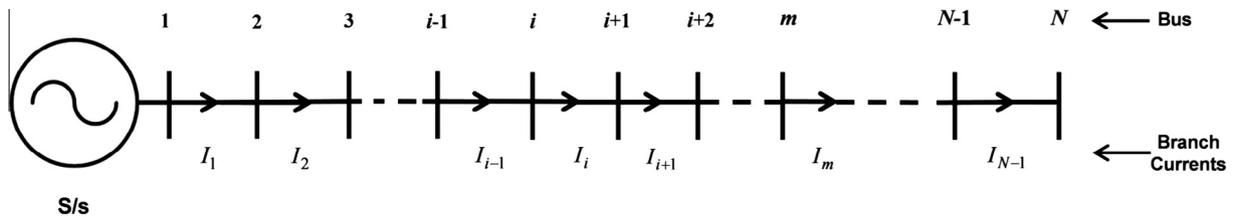


Fig. 1. A  $N$ -bus radial distribution network.

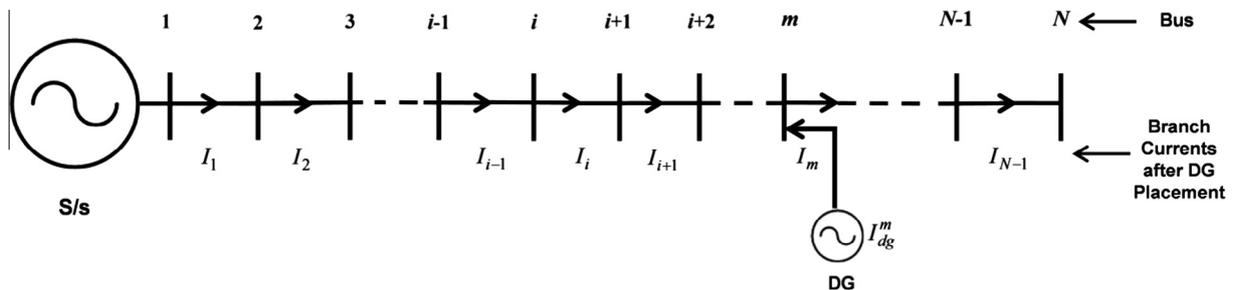


Fig. 2. A  $N$ -bus radial distribution network after DG placement.

When a single DG is placed at bus  $m$ ; injecting a current  $I_{dg}^m$ . This current changes the currents in all the branches connected between substation (bus no. 1) to bus  $m$  and the currents in the remaining branch are unaffected by the DG. Therefore, modified phasor current in  $i$ th branch,  $I_i^{new}$  can be written mathematically as:

$$I_i^{new} = I_i - D_i \cdot I_{dg}^m = (I_{ai} - D_i \cdot I_{dga}^m) + j(I_{ri} - D_i \cdot I_{dgr}^m) \quad (4)$$

where  $I_i = I_{ai} + jI_{ri}$  and  $I_{dg}^m = I_{dga}^m + jI_{dgr}^m$ .

Here,  $I_{dga}^m$  and  $I_{dgr}^m$  are the real and imaginary components of  $I_{dg}^m$  and  $D_i$  can be given as:

$$D_i = \begin{cases} 1; & \text{if branch } i \text{ is between bus 1 and } m \\ 0; & \text{otherwise} \end{cases}$$

Now the total compensated losses  $P_L^{new}$  after DG placement may be expressed as

$$P_L^{new} = \sum_{i=1}^{N-1} (I_i^{new})^2 \cdot R_i = \sum_{i=1}^{N-1} \left[ (I_{ai} - D_i \cdot I_{dga}^m)^2 + (I_{ri} - D_i \cdot I_{dgr}^m)^2 \right] \cdot R_i \quad (5)$$

#### Loss saving in single DG case

The associated loss saving,  $S_L$  may be given using (1) and (5) as:

$$S_L = P_L - P_L^{new} = \sum_{i=1}^{N-1} \left[ (I_{ai}^2 + I_{ri}^2) - \left\{ (I_{ai} - D_i \cdot I_{dga}^m)^2 + (I_{ri} - D_i \cdot I_{dgr}^m)^2 \right\} \right] \cdot R_i \quad (6)$$

Simplifying the above equation, we get

$$S_L = 2 \sum_{i=1}^{N-1} [I_{ai} \cdot I_{dga}^m + I_{ri} \cdot I_{dgr}^m] \cdot D_i \cdot R_i - \sum_{i=1}^{N-1} \left[ (I_{dga}^m)^2 + (I_{dgr}^m)^2 \right] \cdot D_i^2 \cdot R_i \quad (7)$$

The maximum loss saving can be obtained corresponding to the DG currents  $I_{dga}^m$  and  $I_{dgr}^m$  using (7) when the following conditions are to be satisfied

$$\left. \begin{aligned} \frac{\partial S_L}{\partial I_{dga}^m} &= 0 \\ \frac{\partial S_L}{\partial I_{dgr}^m} &= 0 \end{aligned} \right\} \quad (8)$$

Hence from (8) we get

$$\frac{\partial S_L}{\partial I_{dga}^m} = 2 \cdot \sum_{i=1}^{N-1} I_{ai} \cdot D_i \cdot R_i - 2 \sum_{i=1}^{N-1} I_{dga}^m \cdot D_i^2 \cdot R_i = 0 \quad (9)$$

$$\frac{\partial S_L}{\partial I_{dgr}^m} = 2 \cdot \sum_{i=1}^{N-1} I_{ri} \cdot D_i \cdot R_i - 2 \sum_{i=1}^{N-1} I_{dgr}^m \cdot D_i^2 \cdot R_i = 0 \quad (10)$$

Simplifying (9) active DG current  $I_{dga}^m$  corresponding to the maximum loss saving can be given as

$$I_{dga}^m = \frac{\sum_{i=1}^{N-1} D_i \cdot I_{ai} \cdot R_i}{\sum_{i=1}^{N-1} D_i^2 \cdot R_i} \quad (11)$$

Similarly, the reactive current  $I_{dgr}^m$  corresponding the maximum loss saving can be obtained from (10) and given as

$$I_{dgr}^m = \frac{\sum_{i=1}^{N-1} D_i \cdot I_{ri} \cdot R_i}{\sum_{i=1}^{N-1} D_i^2 \cdot R_i} \quad (12)$$

#### Optimal size in single DG case

The corresponding single DG size may be obtained using (10) and (12) as

$$\mathbf{S}_{dg}^m = \mathbf{V}_m \cdot \left( \mathbf{I}_{dg}^m \right)^* = \mathbf{V}_m \cdot \left( I_{dga}^m - I_{dgr}^m \right) \quad (13)$$

In (13)  $\mathbf{S}_{dg}^m$  is the capacity of  $m$ th DG in complex form and  $\mathbf{V}_m$  is the phasor voltage at bus to which  $m$ th DG is connected.

#### Loss minimization by multiple DG placement

This section extends the previously developed approach for placement of multiple DG units simultaneously. Now, suppose in an  $N$ -bus distribution network,  $k$  numbers of DGs are to be placed. Integration of DGs at different buses alters the flow of branch current; therefore the modified current in branch  $i$  can be given mathematically as:

$$\begin{aligned} I_i^{new} &= I_i - \sum_{j=1}^k D_{ij} \cdot I_{dg}^j \\ &= \left( I_{ai} - \sum_{j=1}^k D_{ij} \cdot I_{dga}^j \right) + j \left( I_{ri} - \sum_{j=1}^k D_{ij} \cdot I_{dgr}^j \right) \end{aligned} \quad (14)$$

where  $I_i^{new}$  is the complex current in  $i$ th branch after DGs placement,  $I_{dg}^j$  is the current injected by  $j$ th DG with  $I_{dga}^j$  and  $I_{dgr}^j$  being its real and imaginary parts, respectively.

$$D_{ij} = \begin{cases} 1; & \text{if } i\text{th branch is between S/s bus and bus at} \\ & \text{which } j^{\text{th}} \text{ DG is placed} \\ 0; & \text{otherwise} \end{cases}$$

The detail evaluation of  $D_{ij}$  for multi DG is given in Appendix. When DGs are connected in the DS, the associated compensated losses,  $P_L^{new}$  can be written as

$$\begin{aligned} P_L^{new} &= \sum_{i=1}^{N-1} (I_i^{new})^2 \cdot R_i \\ &= \sum_{i=1}^{N-1} \left( I_{ai} - \sum_{j=1}^k D_{ij} \cdot I_{dga}^j \right)^2 \cdot R_i + \sum_{i=1}^{N-1} \left( I_{ri} - \sum_{j=1}^k D_{ij} \cdot I_{dgr}^j \right)^2 \cdot R_i \end{aligned} \quad (15)$$

#### Loss saving in multi DG case

The associated loss saving in multi DG case can be obtained by subtracting (15) from (1) gives the loss saving  $S_L$  due to integration of the DGs in the DS as:

$$\begin{aligned} S_L &= P_L - P_L^{new} \\ &= \sum_{i=1}^{N-1} \left[ I_{ai}^2 - \left( I_{ai} - \sum_{j=1}^k D_{ij} \cdot I_{dga}^j \right)^2 \right] \cdot R_i \\ &\quad + \sum_{i=1}^{N-1} \left[ I_{ri}^2 - \left( I_{ri} - \sum_{j=1}^k D_{ij} \cdot I_{dgr}^j \right)^2 \right] \cdot R_i \\ &= 2 \cdot \sum_{i=1}^{N-1} \left[ I_{ai} \cdot \sum_{j=1}^k D_{ij} \cdot I_{dga}^j + I_{ri} \cdot \sum_{j=1}^k D_{ij} \cdot I_{dgr}^j \right] \cdot R_i \\ &\quad - \sum_{i=1}^{N-1} \left[ \left( \sum_{j=1}^k D_{ij} \cdot I_{dga}^j \right)^2 + \left( \sum_{j=1}^k D_{ij} \cdot I_{dgr}^j \right)^2 \right] \cdot R_i \end{aligned} \quad (16)$$

The maximum loss saving could be achieved by placing multiple DGs if following conditions are to be satisfied:

$$\left. \begin{aligned} \frac{\partial S_L}{\partial I_{dga}^1} &= 0 \\ \frac{\partial S_L}{\partial I_{dgr}^1} &= 0 \\ &\vdots \\ \frac{\partial S_L}{\partial I_{dga}^k} &= 0 \\ \frac{\partial S_L}{\partial I_{dgr}^k} &= 0 \end{aligned} \right\} \quad (17)$$

The partial derivative of  $S_L$  with respect to  $I_{dga}^m$  and  $I_{dgr}^m$  can be given as:

$$\frac{\partial S_L}{\partial I_{dga}^m} = 2 \cdot \sum_{i=1}^{N-1} D_{im} \cdot \left[ I_{ai} - \sum_{j=1}^k D_{ij} \cdot I_{dga}^j \right] \cdot R_i = 0 \quad (18)$$

$$\frac{\partial S_L}{\partial I_{dgr}^m} = 2 \cdot \sum_{i=1}^{N-1} D_{im} \cdot \left[ I_{ri} - \sum_{j=1}^k D_{ij} \cdot I_{dgr}^j \right] \cdot R_i = 0 \quad (19)$$

Corresponding to (17), there will be total  $2k$  linear algebraic equations,  $k$  out of which are similar to (18) and remaining are similar to (19). These two sets of equations can be represented as:

$$[\mathbf{A}]_{k \times k} \cdot [\mathbf{I}_{dga}]_{k \times 1} = [\mathbf{B}]_{k \times 1} \quad (20)$$

$$[\mathbf{A}]_{k \times k} \cdot [\mathbf{I}_{dgr}]_{k \times 1} = [\mathbf{C}]_{k \times 1} \quad (21)$$

The elements of  $\mathbf{A}$ ,  $\mathbf{B}$ , and  $\mathbf{C}$  can be calculated as:

$$A_{pq} = \sum_{i=1}^{N-1} D_{ip} \cdot D_{iq} \cdot R_i \quad (22)$$

$$A_{pp} = \sum_{i=1}^{N-1} D_{ip}^2 \cdot R_i \quad (23)$$

$$B_p = \sum_{i=1}^{N-1} D_{ip} \cdot I_{ai} \cdot R_i \quad (24)$$

$$C_p = \sum_{i=1}^{N-1} D_{ip} \cdot I_{ri} \cdot R_i \quad (25)$$

where  $A_{pq}$  and  $A_{pp}$  are the off-diagonal and diagonal elements of matrix  $\mathbf{A}$ . Similarly,  $B_p$  and  $C_p$  are the respective elements of matrix  $\mathbf{B}$  and  $\mathbf{C}$ . It is evident from the (22)–(25), that only the branch resistance, active and reactive components of branch current of base case are required to find the elements  $\mathbf{A}$ ,  $\mathbf{B}$  and  $\mathbf{C}$ . From (20) and

(21), the active and reactive components of DG currents for maximum loss saving can be computed as:

$$[I_{dga}]_{k \times 1} = [A]_{k \times k}^{-1} \cdot [B]_{k \times 1} \quad (26)$$

$$[I_{dgr}]_{k \times 1} = [A]_{k \times k}^{-1} \cdot [C]_{k \times 1} \quad (27)$$

**Optimal size in multi DG case**

Once the active and reactive components of DG currents are known from (26) and (27), the optimal size of DGs can be calculated as:

$$S_{dg}^m = V_m \cdot (I_{dg}^m)^* = V_m \cdot (I_{dga}^m - jI_{dgr}^m) \quad (28)$$

**Computational procedure**

To find out the optimal DG size and location for minimization of loss in radial DS, following computational steps are involved.

*Step 1:* Read the system data and run the base load flow program for the original uncompensated system; obtain the branch currents, bus voltages, real power losses other necessary data. The load flow program for the proposed methodology implementation is taken from [5]. The detail description of the base load flow interpreted in flow chart in Fig. 3.

*Step 2:* Assume that every node is candidate node. Calculate the loss saving and consequent DG size using (7) and (13) at each bus except the source bus.

*Step 3:* Select the bus that yields the maximum loss saving and it is corresponding DG size for compensation and is called a sensitive bus/node. This is case of single DG.

*Step 4:* Place the DG obtained from step (3) at the bus which has maximum loss saving and repeat step (1)–(3) again to get the next DG bus. Find out the sequence of nodes which is to be replaced/compensated until no such convincing loss saving achieved or reached to zero value by further DG placement otherwise stop the program.

*Step 5:* As from step (4), the sequence of nodes is known now. Calculate the optimal DG sizes and loss saving using (28) and

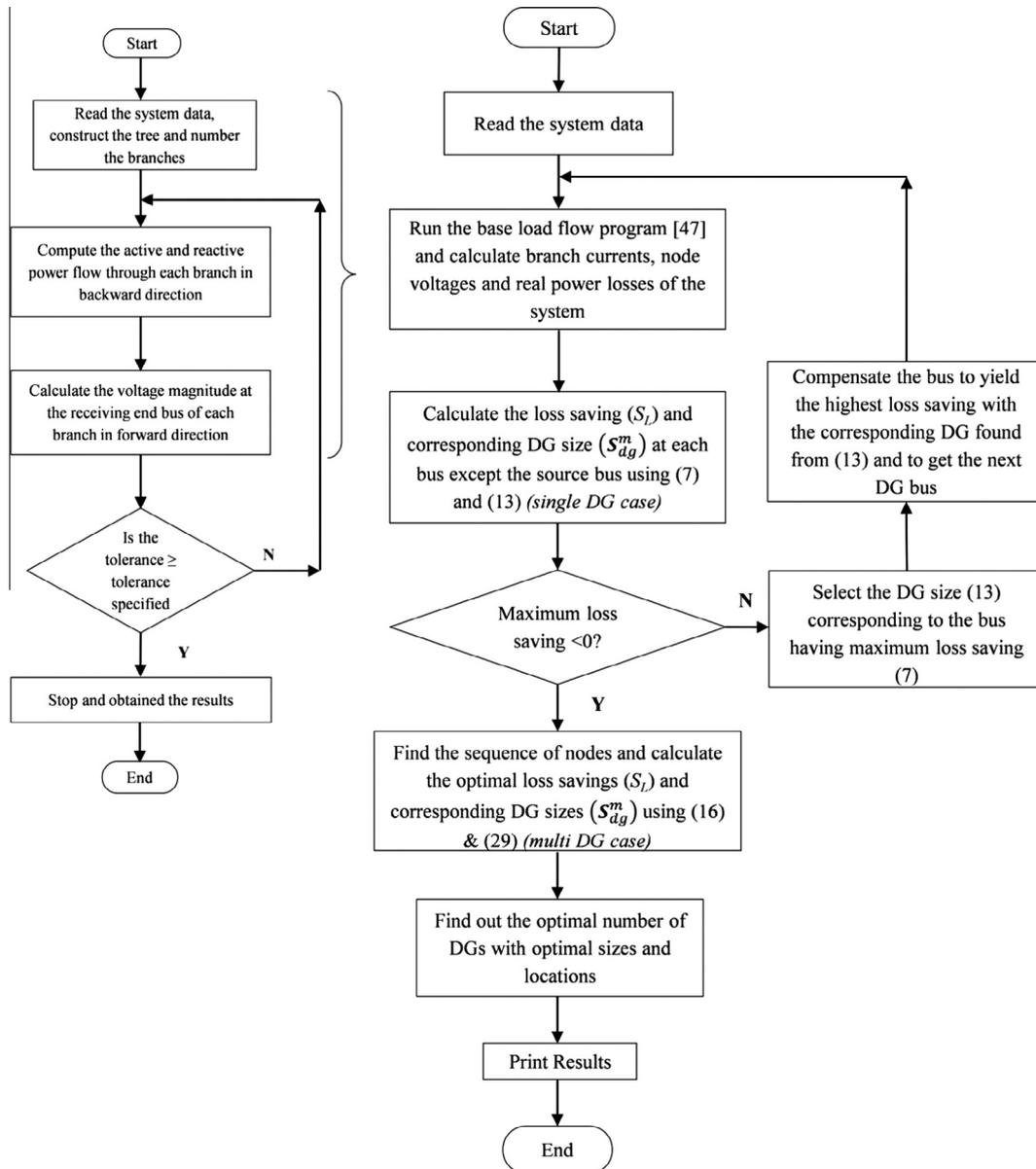


Fig. 3. Flow chart of the proposed analytical method to allocate single and multiple DGs.

(16) respectively. This is the case of multi DG. Finally, obtain the optimal number of DGs to be placed.

This process is repeated iteratively until the total loss saving reach zero value or no further loss saving could be achieved. The method of obtaining the optimal DG sizing and siting is outlined in flowchart in Fig. 3.

It is worth mentioning here that the proposed technique is a SCA (i.e. after setting two or more DGs in the system). The next decision could be to reduce the size of DG that is already set at a certain node to obtain more loss saving. This implies that this bus is overcompensated and we have to reduce the DG size placed at this bus to obtain further reduction in loss. This is an advantage in the proposed technique. Additionally, another advantage of SCA implementation is that it requires less number of iteration to achieve the convergence because the self-correction process is only implemented at the selected buses obtained earlier. Another benefit is that the total number of load flow used normally remains unchanged for larger system such as 69 or 118 bus system [25,29].

## Numerical results

### Test systems

Two test systems have been employed to test the proposed analytical approach for optimal sizing and placement of DGs. The first system is an 11 kV; IEEE 15-bus radial DS is taken into consideration with the total load of  $(1.2264 + j1.2512)$  MV A and the total  $I^2R$  losses are 61.79 kW [30]. The second system is 12.66 kV, IEEE 33-bus test system with total load of  $(3.715 + j2.300)$  MV A radial DS and the total  $I^2R$  losses of the base system are 197.94 kW [2]. To implement the above algorithm an analytical software tool has been developed in MATLAB environment to run load flow, determine losses and optimal sizes of DG.

### Simulation results

#### 15-bus IEEE test system

The single-line diagram of this system is shown in Fig. A.1 in Appendix (considering no DGs). The data of the system are taken

from [30]. The losses associated with the active and reactive elements of branch currents prevailed with the power flow method are 30.42 kW and 31.37 kW respectively.

First, the optimal loss saving and corresponding DG size is determined. Figs. 4 and 5 depicts the DG size and loss saving respectively for all the buses in the system excluding the source bus. It can be observed from Fig. 5 that the highest loss saving of 24.55 kW can be realized by placing a DG of 1421.26 kV A (Fig. 4) at bus 3 in the first iteration. The detailed summary of results obtained by the proposed method is illustrated in Table 1. When the above procedure is applied again, after placing 1421.26 kV A of DG at bus 3, it was evident that a second DG of 728.73 kV A at bus 6, will give a further loss saving of 3.67 kW in the same iteration.

Again, when 728.73 kV A of DG is placed at bus 6, we achieve a loss saving of 1.77 kW with next DG size of 575.38 kV A at bus 7 in the first iteration. Once again, repeating the same process and place 575.38 kV A of DG at bus 7, it was found that no further loss saving could be obtain because the optimization problem was solved to find the size of a singly located DG (as described in technique of Section 'Loss minimization by single DG placement') and loss saving till reaches zero value. Thus the sequence of buses to be redeemed is 3, 6 and 7. The total power losses  $P_L$  retrieved in case of aforementioned single DG placement reduced from 61.79 kW to 36.69 kW with the percentage loss reduction of 40.62%. As said above, the optimization technique is a SCA, therefore important to mention here that in case of single DG, the MATLAB codes are set in such a way that it automatically disconnects the previously connected DG and consider only the next single DG available in the system except the DG size achieve in base case and this operation takes only one iteration as illustrated in Table 1.

However, when multiple DG is to be placed in the system, technique reported in Section 'Loss minimization by multiple DG placement' has to be implemented. The above techniques contribute a total loss saving of 29.94 kW by placing the two DG 1129.69 kV A and 544.64 kV A at buses 3 and 6 respectively in the second iteration. When the original base system is redeemed with above DGs, the load flow result pointed out that the total power losses  $P_L$  reduced by 61.79–36.60 kW with the percentage loss reduction of 40.76%.

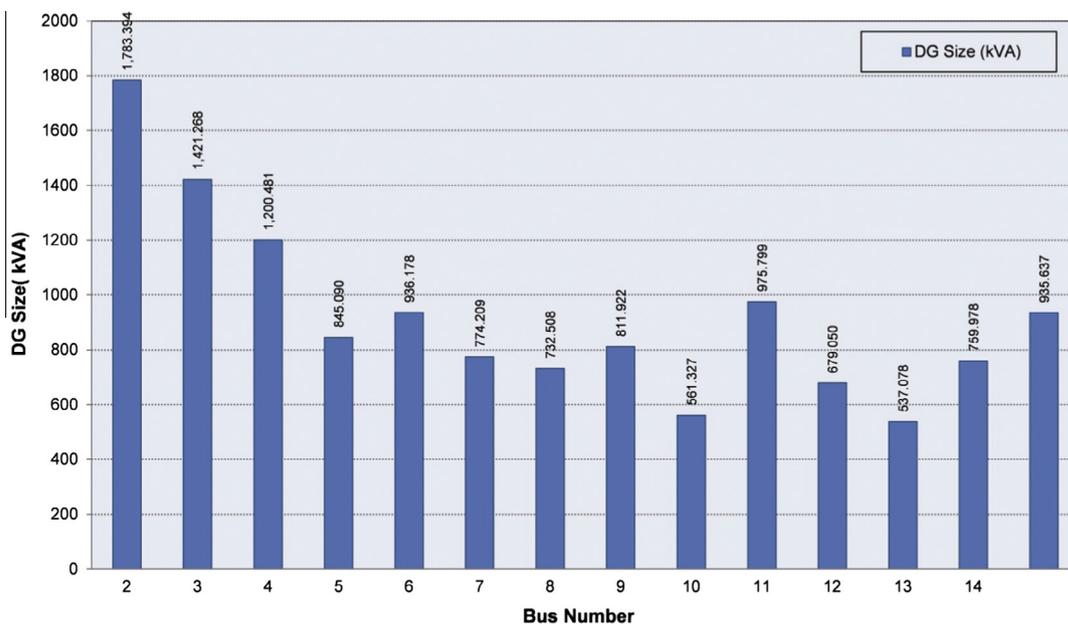


Fig. 4. Optimal size of singly located DG of 15-bus system.

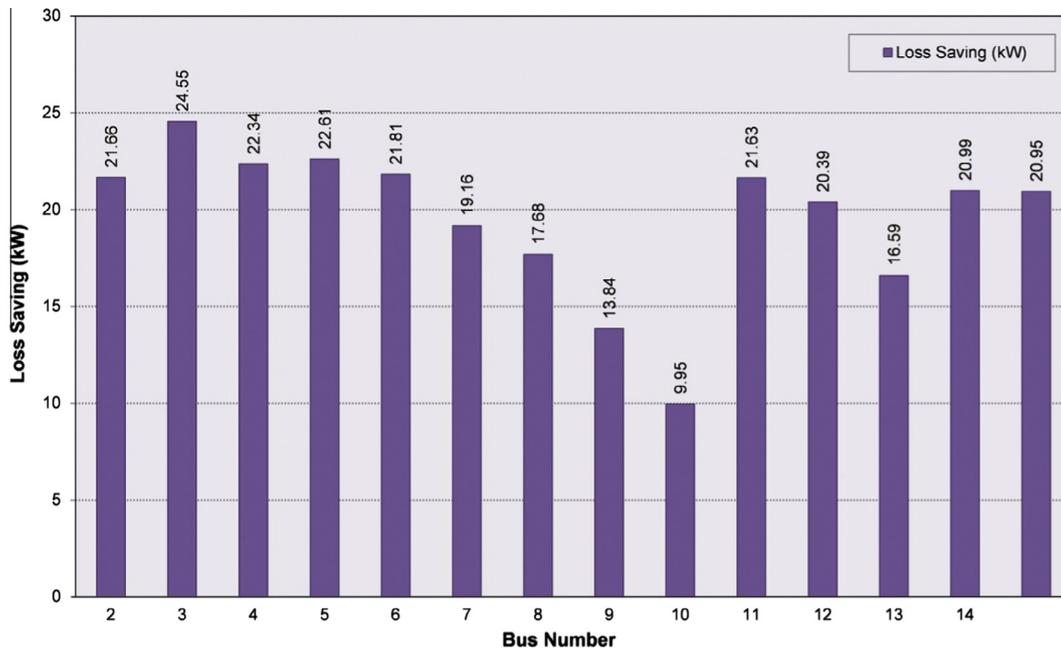


Fig. 5. Estimated loss saving in 15-bus system for a singly located DG.

However, when all three buses (3, 6 and 7) are considered for DG placement it was observed that a total 1673.86 kV A of DGs (1129.69 kV A at bus 3, 344.18 kV A at bus 6, and 199.99 kV A at bus 7) contribute a loss saving of 30.02 kW in third iteration. The associated power losses  $P_L$  by placement of all three DGs are reduced by 61.79–35.63 kW with the percentage loss reduction of 42.33%. Consequently, the respective loss reduction associated with the active and reactive component of branch currents are 30.42–29.00 kW and 31.37–6.63 kW respectively. It is also realized that the proposed technique able to improve the voltage profile of the system under consideration as represented in Fig. 6, with and without DG in single DG case. Similarly, Table 2 highlighted the minimum and maximum voltages deviations before and after DG by proposed method for 15-bus system. It could be examined that the voltage at various buses maintain within the acceptable constraints' limits. In the end, the results obtained by the proposed method are compared based on size, location, percentage loss reduction and computation time, with those methods addressed

in the literature for 15-bus test system and illustrated in Table 3 for validation purpose. The results indicate in this table are for siting and sizing of single DG to minimize the real power loss only. It is evident from this table that better active power loss is possible to reduce by the proposed method as distinguished in the other methods revealed in the literature.

33-bus IEEE test system

Fig. 7 shows the single line diagram of the IEEE 33-bus system. The data of the system acquired from [2]. Whereas the losses linked with the active and reactive components of branch currents about with the power flow in base case are 131.41 kW and 66.52 kW, respectively. Enforced the methodology addressed in Section 'Loss minimization by single DG placement' for single DG placement and find the DG size and loss saving. Similar to the 15-bus system, first the optimal loss saving and corresponding DG size is calculated and delineate in Figs. 9 and 8. It can be recognized from Fig. 9 that the highest loss saving of 53.30 kW can be

Table 1  
Summary of results of 15-bus system.

S. no.	System description	System losses			DG size			DG location (bus no.)	Number of iteration	Total loss saving $S_L$ (kW)	% Loss reduction
		$P_{La}$ (kW)	$P_{Lr}$ (kW)	$P_L$ (kW)	$P_{dga}$ (kW)	$Q_{dga}$ (kVAr)	$S$ (kV A)				
1	Base system	30.42	31.37	61.79	996.99	-1012.92	1421.26	3	1	24.55	-
2	Single DG placement 1421.26 kV A DG is placed at bus 3 728.73 kV A DG is placed at bus 6 575.38 kV A DG is placed at bus 7	29.12	7.98	37.11	651.38	-326.72	728.73	6	1	3.67	39.94
		29.01	7.67	36.69	539.02	-201.31	575.38	7	1	1.77	40.62
		28.89	7.58	36.47	486.23	-187.56	520.93	3	1	-43.23	40.97
3	Base system	30.42	31.37	61.79	792.19	-805.39	1129.69	3	2	-	-
4	Multi DG placement Placed 1129.69 kV A DG at bus 3 and 544.64 kV A DG at bus 6	29.04	7.56	36.60	1174.74	-1193.04	1674.33	3 and 6	-	29.94	40.76
					382.56	-387.66	544.64	6	-	-	
5	Base system	30.42	31.37	61.79	792.19	-805.39	1129.69	3	3	-	-
6	Multi DG placement Placed 1129.69 kV A DG at bus 3, 344.18 kV A DG at bus 6 and 199.99 kV A DG at bus 7	29.00	6.63	35.63	1174.42	-1192.71	1673.86	3, 6 and 7	-	30.02	42.33
					241.69	-245.04	344.18	6	-	-	
					140.54	-142.29	199.99	7	-	-	

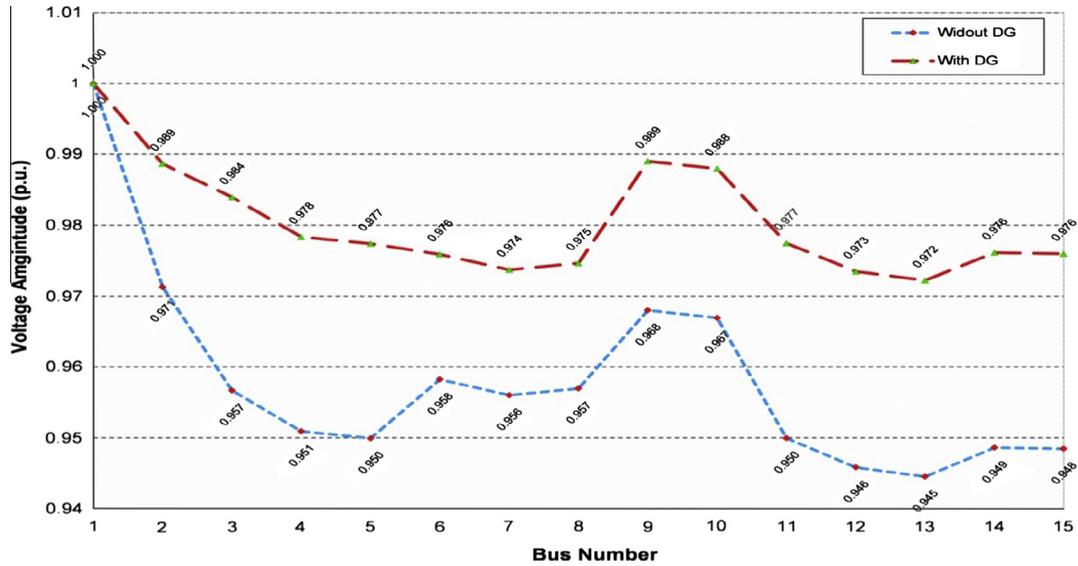


Fig. 6. Voltage profile of 15-bus test system with and without DG.

Table 2  
Voltage before and after DG at 15-bus system.

System	Voltage at bus before DG		Voltage at bus after DG	
	Min	Max	Min	Max
15-bus	0.9452 at 13	1.000 at 1	0.9721 at 13	0.9899 at 9

Table 3  
Comparison of results for 15-bus test system.

Particulars	Gozel and Hocaoglu [19]	Murthy and Kumar [31]	Proposed method
Optimal bus	3	3	3
DG size (MVA)	1.418	1.411	1.421
% Loss reduction	39.041	42.008	42.331
CPU time (s)	0.032	0.041	0.023

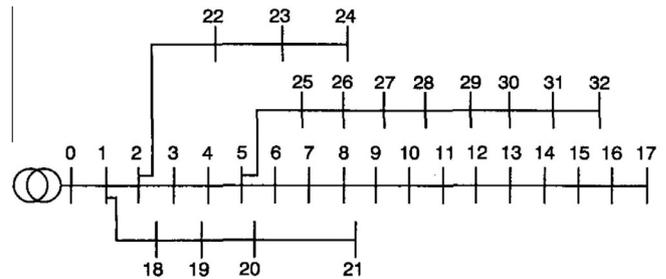


Fig. 7. Single line diagram of the IEEE 33-bus test system.

realized by placing a DG of 2968.53 kVA (Fig. 8) at bus 6. The detailed summary of results accessed by the suggested method is demonstrated in Table 4.

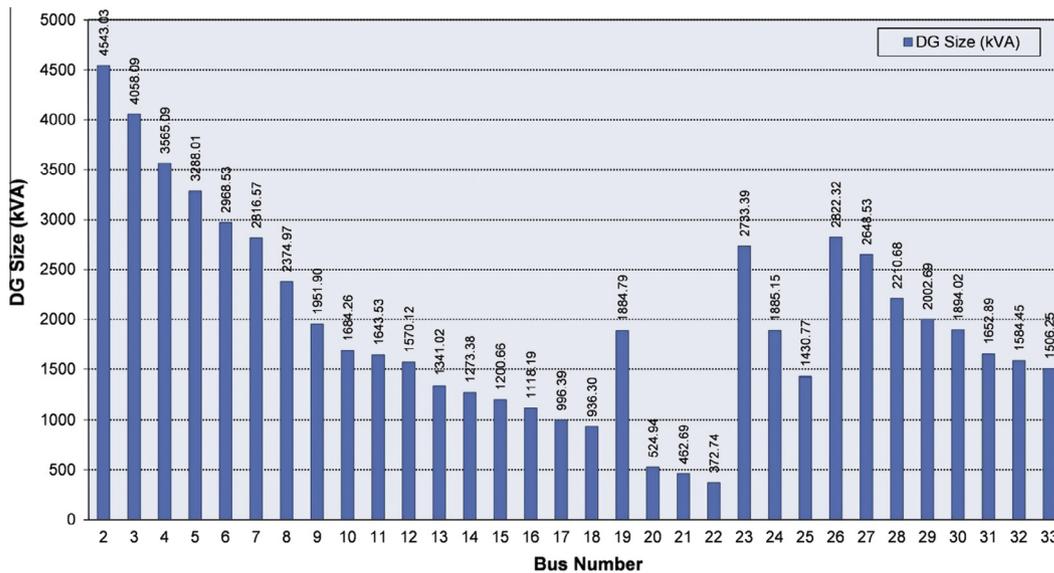


Fig. 8. Optimal size of DG in 33-bus system for a singly located.

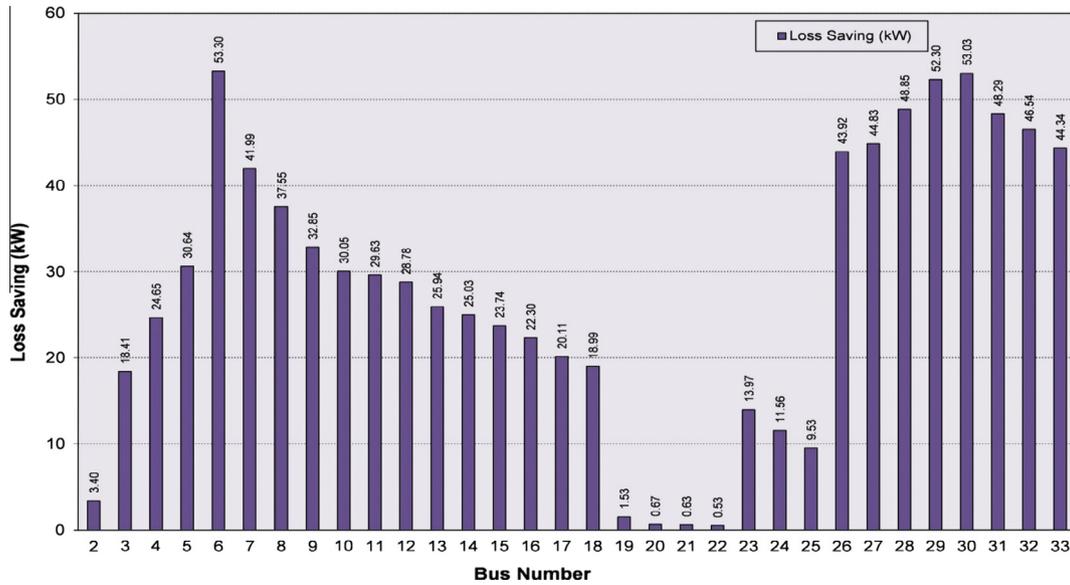


Fig. 9. Estimated loss saving in 33-bus system for a singly located DG.

Table 4  
Summary of results of 33-bus system.

S. no.	System description	System losses			DG size			DG location (bus no.)	Number of iteration	Total loss saving $S_L$ (kW)	% Loss reduction	
		$P_{La}$ (kW)	$P_{Lr}$ (kW)	$P_L$ (kW)	$P_{dga}$ (kW)	$Q_{dga}$ (kVAr)	$S$ (kV A)					
1	Base system	131.41	66.52	197.94	2428.40	-1707.23	2968.53	6	1	53.30	-	
2	Single DG placement	2968.33 kV A DG is placed at bus 6	126.13	13.03	139.16	1077.94	-350.22	1133.41	14	1	6.79	29.69
		1133.41 kV A DG is placed at bus 14	125.67	6.26	131.92	1609.24	-483.33	1680.25	24	1	2.98	33.35
		1680.25 kV A DG is placed at bus 24	124.68	6.18	130.86	1585.16	-479.56	1656.11	-	1	-	33.88
3	Base system	131.41	66.52	197.94	1120.26	-1053.75	1537.97	6	2	-	-	
					774.97	-370.29	858.89	14	-	-	-	
4	Multi DG placement	Placed 1537.97 kV A DG at bus 6 and 858.89 kV A DG at bus 14	125.58	5.95	131.53	1895.23	-1424.04	2396.86	6 and 14	-	58.45	33.55
5	Base system	131.41	66.52	197.94	1023.31	-1006.51	1435.35	6	3	-	-	
					725.22	-346.05	803.55	14	-	-	-	
					1068.39	-520.56	1188.46	24	-	-	-	
6	Multi DG placement	Placed 1435.35 kV A DG at bus 6, 803.55 kV A DG at bus 14 and 1188.46 kV A DG at bus 24	125.28	2.58	127.86	2816.92	-1873.12	3427.36	6, 14 and 24	-	62.94	35.40

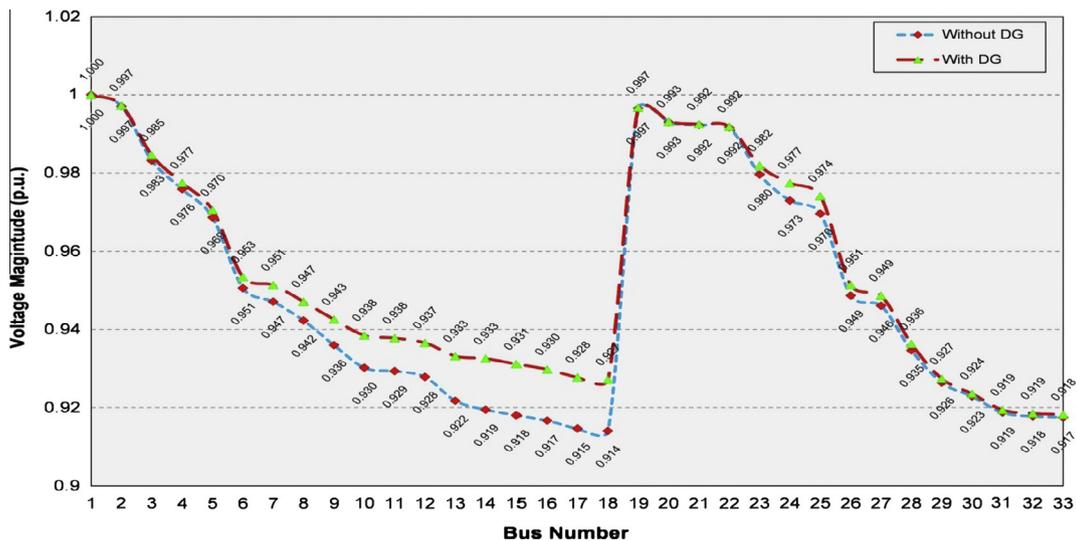


Fig. 10. Voltage profile of 33-bus test system with and without DG.

It is evident from Table 4, that we get two more single DG at buses 14 and 24 in first iteration. The corresponding DG size and loss saving are 1133.41 kV A, 1680.25 kV A and 6.79 kW, 2.98 kW at buses 14 and 24 respectively. It is also seen from Table 2 that after 2.98 kW no further loss saving could be obtain in first iteration. Thus the sequence of buses to be redeemed for this system is 6, 14 and 24. The total power losses  $P_L$  claim in case of aforementioned single DG placement diminished from 197.94 kW to 131.92 kW with the percentage loss reduction of 33.35%. When only first two buses (6 and 14) are to be redeemed in the second iteration using technique reported in Section ‘Loss minimization by multiple DG placement’; it contribute a total loss saving of 58.45 kW. When the original base system is redeemed with above DGs, the load flow result indicated that the total power losses  $P_L$  reduced by 197.94–131.53 kW with the percentage loss reduction of 33.55%.

Further, when all three buses (6, 14 and 24) are treated for DG placement it was detected that a total 3427.36 kV A of DGs (1435.35 kV A at bus 6, 803.55 kV A at bus 14, and 1188.46 kV A at bus 24) add a loss saving of 62.94 kW in third iteration. The associated power losses  $P_L$  by placement of all three DGs are shortened by 197.94–127.86 kW with the percentage loss reduction of 35.40%. The respective loss reduction associated with the active and reactive component of branch currents can also be observed from Table 4. The enhancement of voltage profile can also be

examine as depicted in Fig. 10, with and without DG in single DG case. In the same sequence, the voltage deviation of various buses for this test system can be examine in Table 5. Moreover, it is realized from this table that the voltage at various buses manage within the acceptable constraints limits. Table 6 display the comparison of results achieve by dissimilar methods for allocation of single DG to lessen the real power loss in 33-bus system. It could be realized from this table that extra active power loss saving by the proposed method is in close compromise with the other methods as bring out in the literature.

**Conclusion**

In this paper a simple and novel analytical method for minimizing the loss associated with the active and reactive components of DG branch current is proposed. The proposed method is tested on two IEEE 15 and 33 distribution networks. In both the test system it was demonstrated that on which optimal bus, the optimal number of sizes of DG is to be placed so that the maximum loss saving could achieve. In the 15-bus system it was found that by placing the optimal DGs at buses 3, 6 and 7 the total power losses can be reduced from 61.79 kW to 35.63 kW (whereas,  $P_{La}$  and  $P_{Lr}$  29.00 kW and 6.63 kW respectively due to  $I_{dga}$  and  $I_{dgr}$ ). In the 33-bus system it was found that by placing optimal DGs at buses 6, 14 and 24 the total power losses associated can be reduced from 197.94 kW to 127.86 kW (whereas,  $P_{La}$  and  $P_{Lr}$  125.28 kW and 2.58 kW respectively due to  $I_{dga}$  and  $I_{dgr}$ ). It may also conclude that using proposed method, the voltage profile is also improved. Further, the proposed method compared with those other methods reported in literature to show the credibility. Additionally, since the proposed method uses the SCA, which requires less number of iteration (as only 3 iteration used for each system) to achieve the convergence and hence demands less computation time. Another benefit of the suggested method is that the total number of load flow used normally remains unchanged for larger system such as 69 or 118 bus system. Besides that, the proposed method not applicable for unbalanced and meshed distribution system.

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**Appendix**

Consider a 15-bus radial system as in Fig. A.1 [30], if four DGs (i.e.  $k = 4$ ) are to be placed at buses 7, 10, 12 and 15, then the DG branch set  $\alpha_j$  and the transpose of matrix  $D$  can be written as  $D^T$ . For this set of DG buses the matrix  $D^T$  can be evaluated as given below:

$$k = 4, \text{ number of DGs buses}$$

$$\alpha_{1(\text{at DG bus } 7)} = [1, 7, 8]$$

$$\alpha_{2(\text{at DG bus } 10)} = [1, 5, 6]$$

$$\alpha_{3(\text{at DG bus } 12)} = [1, 2, 10, 11]$$

$$\alpha_{4(\text{at DG bus } 15)} = [1, 2, 3, 14]$$

For these set of branches the formulation of matrix  $D^T$  is as following

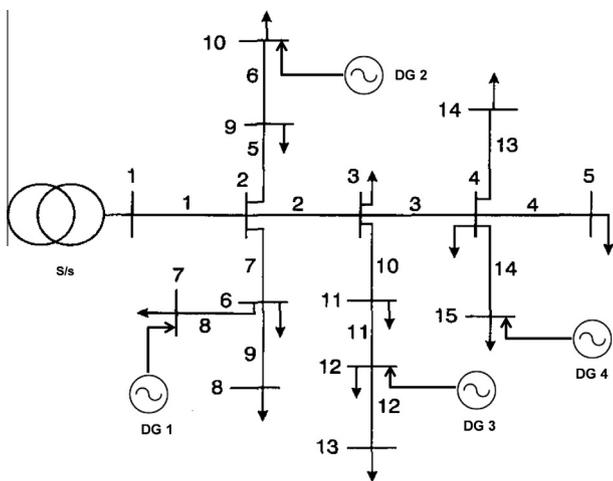


Fig. A.1. Single-line diagram of 15-bus test system with DG.

Table 5  
Voltage before and after DG at 33-bus system.

System	Voltage at bus before DG		Voltage at bus after DG	
	Min	Max	Min	Max
33-bus	0.9141 at 18	1.000 at 1	0.9183 at 33	0.9971 at 19

Table 6  
Comparison of results for 33-bus test system.

Particulars	Gozel and Hocaoglu [19]	Murthy and Kumar [31]	Proposed method
Optimal bus	6	6	6
DG size (MV A)	2.988	2.536	2.968
% Loss reduction	31.599	34.868	35.405
CPU time (s)	0.043	0.053	0.038

$$D^T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}_{k \times N-1}$$

In matrix  $D^T$  rows and columns represent the number of branches and buses of DG respectively. In above case  $D^T$  is the  $(k \times N - 1$  or  $4 \times 14)$  matrix. Similarly, the  $D_{ij}$  can be evaluated for  $n$  number of DG.

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