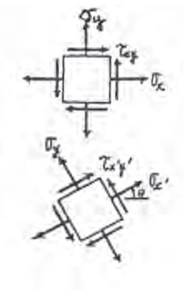
9–1. Prove that the sum of the normal stresses $\sigma_x + \sigma_y = \sigma_{x'} + \sigma_{y'}$ is constant. See Figs. 9–2*a* and 9–2*b*.

Stress Transformation Equations: Applying Eqs. 9-1 and 9-3 of the text.

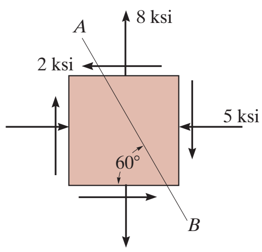
$$\sigma_{x'} + \sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$+ \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\sigma_{x'} + \sigma_{y'} = \sigma_x + \sigma_y \qquad \text{(Q.E.D.)}$$



9–2. The state of stress at a point in a member is shown on the element. Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 9.1.



Referring to Fig *a*, if we assume that the areas of the inclined plane AB is ΔA, then the area of the horizontal and vertical of the triangular element are $\Delta A \cos 60°$ and $\Delta A \sin 60°$ respectively. The forces act acting on these two faces indicated on the FBD of the triangular element, Fig. *b*.

$$+\nearrow \Sigma F_{x'} = 0; \qquad \Delta F_{x'} + 2\Delta A \sin 60° \cos 60° + 5\Delta A \sin 60° \sin 60°$$
$$+ 2\Delta A \cos 60° \sin 60° - 8\Delta A \cos 60° \cos 60° = 0$$
$$\Delta F_{x'} = -3.482\,\Delta A$$

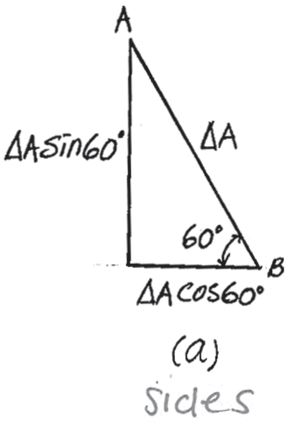
$$+\nwarrow \Sigma F_{y'} = 0; \qquad \Delta F_{y'} + 2\Delta A \sin 60° \sin 60° - 5\Delta A \sin 60° \cos 60°$$
$$- 8\Delta A \cos 60° \sin 60° - 2\Delta A \cos 60° \cos 60° = 0$$
$$\Delta F_{y'} = 4.629\,\Delta A$$

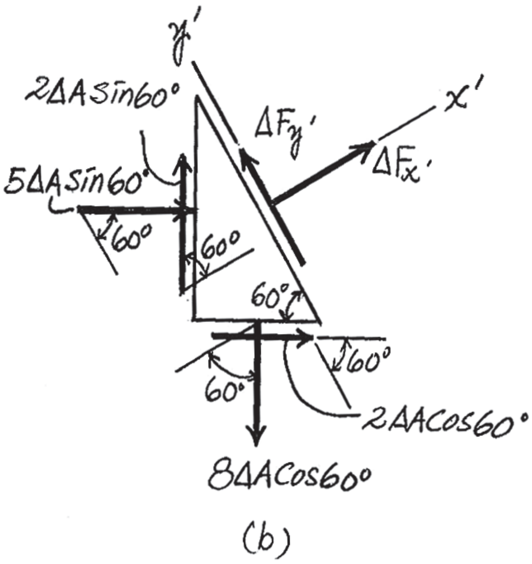
From the definition,

$$\sigma_{x'} = \lim_{\Delta A \to 0} \frac{\Delta F_{x'}}{\Delta A} = -3.48 \text{ ksi} \qquad \textbf{Ans.}$$

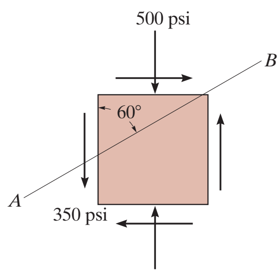
$$\tau_{x'y'} = \lim_{\Delta A \to 0} \frac{\Delta F_{y'}}{\Delta A} = 4.63 \text{ ksi} \qquad \textbf{Ans.}$$

The negative sign indicates that $\sigma_{x'}$, is a compressive stress.





9–3. The state of stress at a point in a member is shown on the element. Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 9.1.



Referring to Fig. a, if we assume that the area of the inclined plane AB is ΔA, then the areas of the horizontal and vertical surfaces of the triangular element are $\Delta A \sin 60°$ and $\Delta A \cos 60°$ respectively. The force acting on these two faces are indicated on the FBD of the triangular element, Fig. b

$$+\searrow \Sigma F_{x'} = 0; \qquad \Delta F_{x'} + 500\,\Delta A \sin 60° \sin 60° + 350 \Delta A \sin 60° \cos 60°$$

$$+350 \Delta A \cos 60° \sin 60° = 0$$

$$\Delta F_{x'} = -678.11\,\Delta A$$

$$+\nearrow \Sigma F_{y'} = 0; \qquad \Delta F_{y'} + 350 \Delta A \sin 60° \sin 60° - 500 \Delta A \sin 60° \cos 60°$$

$$-350 \Delta A \cos 60° \cos 60° = 0$$

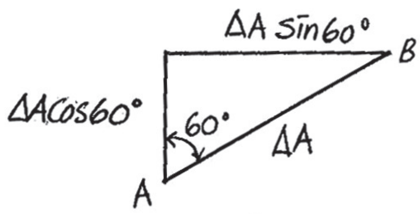
$$\Delta F_{y'} = 41.51\,\Delta A$$

From the definition

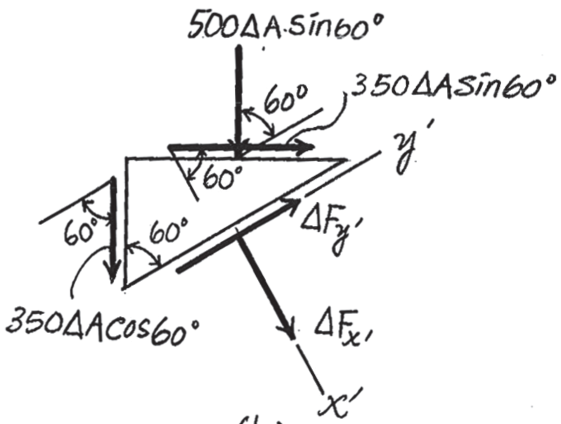
$$\sigma_{x'} = \lim_{\Delta A \to 0} \frac{\Delta F_{x'}}{\Delta A} = -6.78 \text{ psi} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = \lim_{\Delta A \to 0} \frac{\Delta F_{y'}}{\Delta A} = 41.5 \text{ psi} \qquad \textbf{Ans.}$$

The negative sign indicates that $\sigma_{x'}$, is a compressive stress.

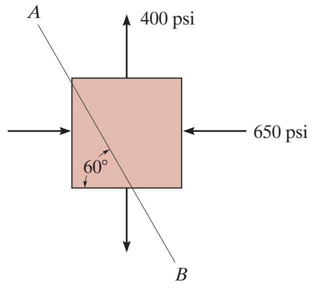


(a)



(b)

***9–4.** The state of stress at a point in a member is shown on the element. Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 9.1.



$\nearrow + \Sigma F_{x'} = 0 \qquad \Delta F_{x'} - 400(\Delta A\cos 60°)\cos 60° + 650(\Delta A \sin 60°)\cos 30° = 0$

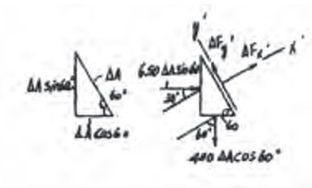
$$\Delta F_{x'} = -387.5\Delta A$$

$\nwarrow + \Sigma F_{y'} = 0 \qquad \Delta F_{y'} - 650(\Delta A\sin 60°)\sin 30° - 400(\Delta A \cos 60°)\sin 60° = 0$

$$\Delta F_{y'} = 455\ \Delta A$$

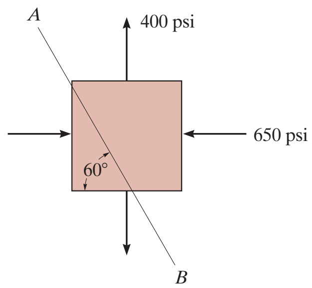
$$\sigma_{x'} = \lim_{\Delta A \to 0} \frac{\Delta F_{x'}}{\Delta A} = -388 \text{ psi} \qquad \textbf{Ans.}$$

$$\sigma_{x'y'} = \lim_{\Delta A \to 0} \frac{\Delta F_{y'}}{\Delta A} = 455 \text{ psi} \qquad \textbf{Ans.}$$



The negative sign indicates that the sense of $\sigma_{x'}$, is opposite to that shown on FBD.

•9–5. Solve Prob. 9–4 using the stress-transformation equations developed in Sec. 9.2.



$\sigma_x = -650 \text{ psi} \qquad \sigma_y = 400 \text{ psi} \qquad \tau_{xy} = 0 \qquad \theta = 30°$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

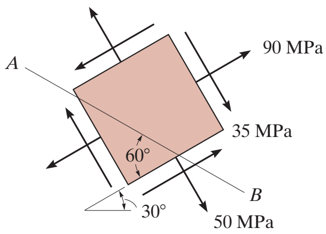
$$= \frac{-650 + 400}{2} + \frac{-650 - 400}{2}\cos 60° + 0 = -388 \text{ psi} \qquad \textbf{Ans.}$$

The negative sign indicates $\sigma_{x'}$, is a compressive stress.

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\left(\frac{-650 - 400}{2}\right)\sin 60° = 455 \text{ psi} \qquad \textbf{Ans.}$$

9–6. The state of stress at a point in a member is shown on the element. Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 9.1.

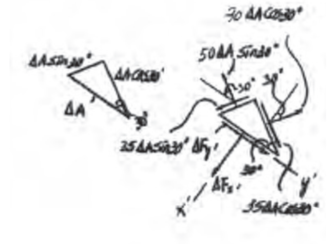


$$\searrow + \Sigma F_{y'} = 0 \qquad \Delta F_{y'} - 50\Delta A \sin 30° \cos 30° - 35\Delta A \sin 30° \cos 60° + 90\Delta A \cos 30° \sin 30° + 35\Delta A \cos 30° \sin 60° = 0$$

$$\Delta F_{y'} = -34.82\Delta A$$

$$\swarrow + \Sigma F_{x'} = 0 \qquad \Delta F_{x'} - 50\Delta A \sin 30° \sin 30° + 35\Delta A \sin 30° \sin 60° - 90\Delta A \cos 30° \cos 30° + 35\Delta A \cos 30° \cos 60° = 0$$

$$\Delta F_{x'} = 49.69\ \Delta A$$

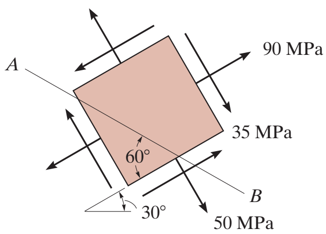


$$\sigma_{x'} = \lim_{\Delta A \to 0} \frac{\Delta F_{x'}}{\Delta A} = 49.7 \text{ MPa}$$ **Ans.**

$$\tau_{x'y'} = \lim_{\Delta A \to 0} \frac{\Delta F_{y'}}{\Delta A} = -34.8 \text{ MPa}$$ **Ans.**

The negative signs indicate that the sense of $\sigma_{x'}$, and $\tau_{x'y'}$ are opposite to the shown on FBD.

9–7. Solve Prob. 9–6 using the stress-transformation equations developed in Sec. 9.2. Show the result on a sketch.



$\sigma_x = 90$ MPa $\qquad \sigma_y = 50$ MPa $\qquad \tau_{xy} = -35$ MPa $\qquad \theta = -150°$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$= \frac{90 + 50}{2} + \frac{90 - 50}{2} \cos(-300°) + (-35) \sin(-300°)$$

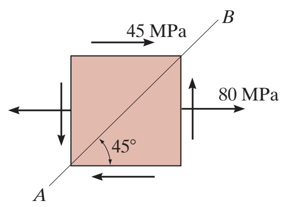
$$= 49.7 \text{ MPa}$$ **Ans.**

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

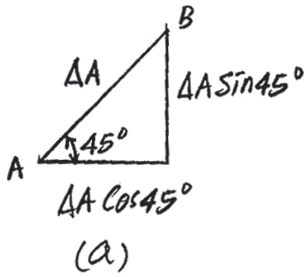
$$= -\left(\frac{90 - 50}{2}\right) \sin(-300°) + (-35) \cos(-300°) = -34.8 \text{ MPa}$$ **Ans.**

The negative sign indicates $\tau_{x'y'}$ acts in $-y'$ direction.

***9–8.** Determine the normal stress and shear stress acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 9.1.



Force Equllibrium: Referring to Fig. *a*, if we assume that the area of the inclined plane AB is ΔA, then the area of the vertical and horizontal faces of the triangular sectioned element are $\Delta A \sin 45°$ and $\Delta A \cos 45°$, respectively. The forces acting on the free-body diagram of the triangular sectioned element, Fig. *b*, are



$$\Sigma F_{x'} = 0; \quad \Delta F_{x'} + \left[45\left(10^6\right)\Delta A \sin 45°\right]\cos 45° + \left[45\left(10^6\right)\Delta A \cos 45°\right]\sin 45° - \left[80\left(10^6\right)\Delta A \sin 45°\right]\cos 45° = 0$$

$$\Delta F_{x'} = -5\left(10^6\right)\Delta A$$

$$\Sigma F_{y'} = 0; \quad \Delta F_{y'} + \left[45\left(10^6\right)\Delta A \cos 45°\right]\cos 45° - \left[45\left(10^6\right)\Delta A \sin 45°\right]\sin 45° - \left[80\left(10^6\right)\Delta A \sin 45°\right]\sin 45° = 0$$

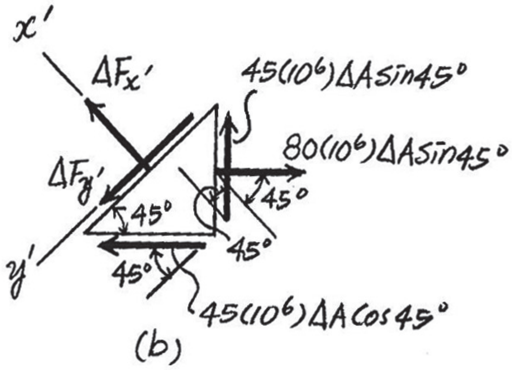
$$\Delta F_{y'} = 40\left(10^6\right)\Delta A$$

Normal and Shear Stress: From the definition of normal and shear stress,

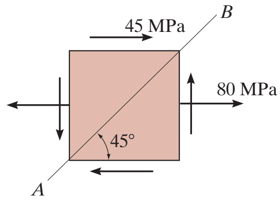
$$\sigma_{x'} = \lim_{\Delta A \to 0} \frac{\Delta F_{x'}}{\Delta A} = -5 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = \lim_{\Delta A \to 0} \frac{\Delta F_{y'}}{\Delta A} = 40 \text{ MPa} \qquad \textbf{Ans.}$$

The negative sign indicates that $\sigma_{x'}$ is a compressive stress.



•9–9. Determine the normal stress and shear stress acting on the inclined plane AB. Solve the problem using the stress transformation equations. Show the result on the sectioned element.



Stress Transformation Equations:

$\theta = +135°$ (Fig. *a*) $\quad \sigma_x = 80$ MPa $\quad \sigma_y = 0 \quad \tau_{xy} = 45$ MPa

we obtain,

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{80 + 0}{2} + \frac{80 - 0}{2}\cos 270 + 45\sin 270°$$

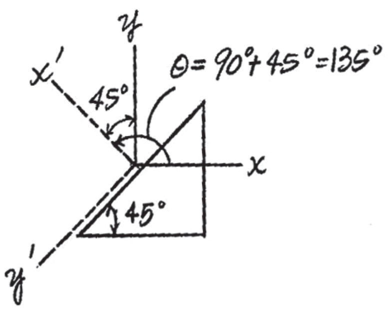
$$= -5 \text{ MPa}$$ **Ans.**

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin\theta + \tau_{xy}\cos 2\theta$$

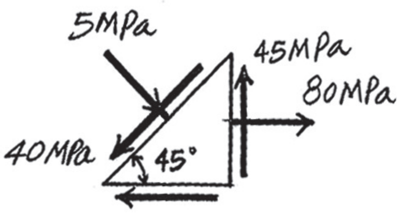
$$= -\frac{80 - 0}{2}\sin 270° + 45\cos 270°$$

$$= 40 \text{ MPa}$$ **Ans.**

The negative sign indicates that $\sigma_{x'}$ is a compressive stress. These results are indicated on the triangular element shown in Fig. *b*.

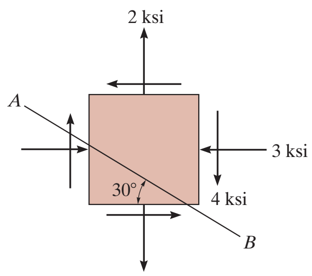


(a)

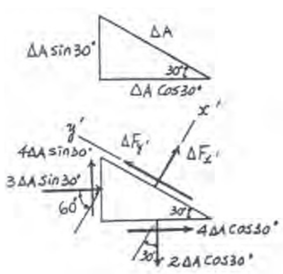


(b)

9–10. The state of stress at a point in a member is shown on the element. Determine the stress components acting on the inclined plane AB. Solve the problem using the method of equilibrium described in Sec. 9.1.



Force Equllibrium: For the sectioned element,



$$\nwarrow + \Sigma F_{y'} = 0; \qquad \Delta F_{y'} - 3(\Delta A \sin 30°)\sin 60° + 4(\Delta\ A \sin 30°)\sin 30°$$

$$-2(\Delta A \cos 30°)\sin 30° - 4(\Delta A \cos 30°)\sin 60° = 0$$

$$\Delta F_{y'} = 4.165\ \Delta A$$

$$\nearrow + \Sigma F_{x'} = 0; \qquad \Delta F_{x'} + 3(\Delta A \sin 30°)\cos 60° + 4(\Delta\ A \sin 30°)\cos 30°$$

$$-2(\Delta A \cos 30°)\cos 30° + 4(\Delta A \cos 30°)\cos 60° = 0$$

$$\Delta F_{x'} = -2.714\ \Delta A$$

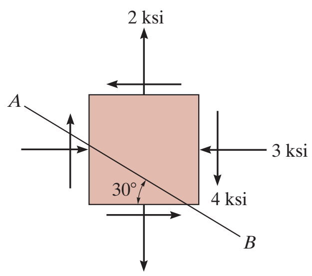
Normal and Shear Stress: For the inclined plane.

$$\sigma_x = \lim_{\Delta A \to 0} \frac{\Delta F_{x'}}{\Delta A} = -2.71 \text{ ksi} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = \lim_{\Delta A \to 0} \frac{\Delta F_{y'}}{\Delta A} = 4.17 \text{ ksi} \qquad \textbf{Ans.}$$

Negative sign indicates that the sense of $\sigma_{x'}$, is opposite to that shown on FBD.

9–11. Solve Prob. 9–10 using the stress-transformation equations developed in Sec. 9.2. Show the result on a sketch.



Normal and Shear Stress: In accordance with the established sign convention,

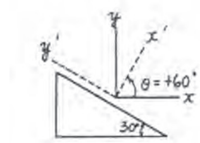
$\theta = +60° \qquad \sigma_x = -3 \text{ ksi} \qquad \sigma_y = 2 \text{ ksi} \qquad \tau_{xy} = -4 \text{ ksi}$

Stress Transformation Equations: Applying Eqs. 9-1 and 9-2.

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{-3+2}{2} + \frac{-3-2}{2}\cos 120° + (-4 \sin 120°)$$

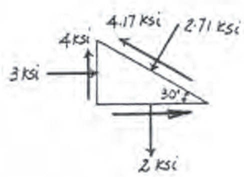
$$= -2.71 \text{ ksi} \qquad \textbf{Ans.}$$



$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

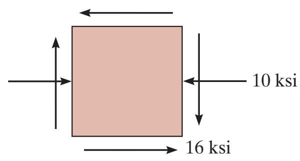
$$= -\frac{-3-2}{2}\sin 120° + (-4 \cos 120°)$$

$$= 4.17 \text{ ksi} \qquad \textbf{Ans.}$$



Negative sign indicates $\sigma_{x'}$, is a *compressive* stress

***9–12.** Determine the equivalent state of stress on an element if it is oriented 50° counterclockwise from the element shown. Use the stress-transformation equations.



$\sigma_x = -10 \text{ ksi} \qquad \sigma_y = 0 \qquad \tau_{xy} = -16 \text{ ksi}$

$\theta = +50°$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

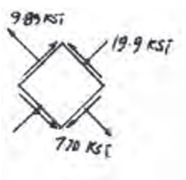
$$= \frac{-10 + 0}{2} + \frac{-10 - 0}{2}\cos 100° + (-16)\sin 100° = -19.9 \text{ ksi} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

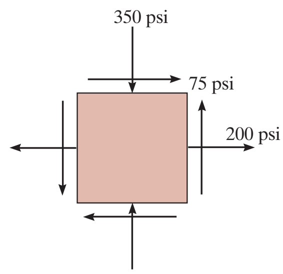
$$= -\left(\frac{-10 - 0}{2}\right)\sin 100° + (-16)\cos 100° = 7.70 \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{-10 + 0}{2} - \left(\frac{-10 - 0}{2}\right)\cos 100° - (-16)\sin 100° = 9.89 \text{ ksi} \qquad \textbf{Ans.}$$



•9–13. Determine the equivalent state of stress on an element if the element is oriented 60° clockwise from the element shown. Show the result on a sketch.



In accordance to the established sign covention,

$\theta = -60°$ (Fig. *a*) $\quad \sigma_x = 200$ psi $\quad \sigma_y = -350$ psi $\quad \tau_{xy} = 75$ psi

Applying Eqs 9-1, 9-2 and 9-3,

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{200 + (-350)}{2} + \frac{200 - (-350)}{2}\cos(-120°) + 75\sin(-120°)$$

$$= -277.45 \text{ psi} = -277 \text{ psi} \qquad \textbf{Ans.}$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{200 + (-350)}{2} - \frac{200 - (-350)}{2}\cos(-120°) - 75\sin(-120°)$$

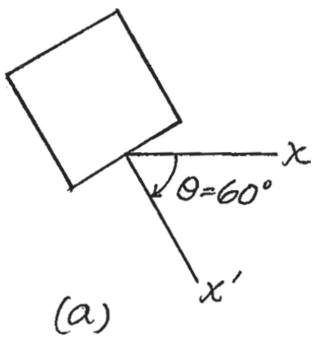
$$= 127.45 \text{ psi} = 127 \text{ psi} \qquad \textbf{Ans.}$$

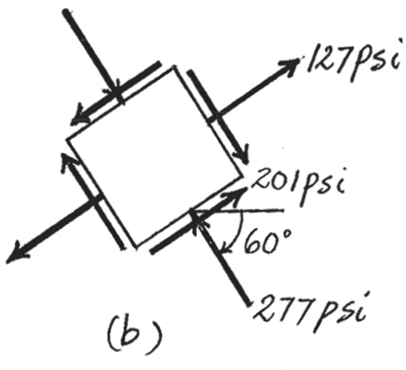
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{200 - (-350)}{2}\sin(-120°) + 75\cos(-120°)$$

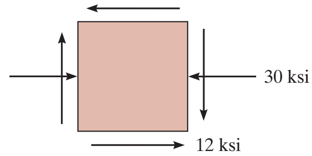
$$= 200.66 \text{ psi} = 201 \text{ psi} \qquad \textbf{Ans.}$$

Negative sign indicates that $\sigma_{x'}$ is a compressive stress. These result, can be represented by the element shown in Fig. *b*.

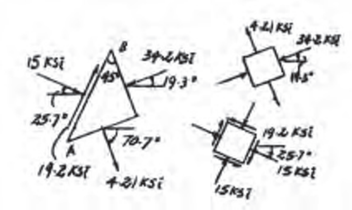




9–14. The state of stress at a point is shown on the element. Determine (a) the principal stress and (b) the maximum in-plane shear stress and average normal stress at the point. Specify the orientation of the element in each case. Show the results on each element.



$\sigma_x = -30$ ksi $\qquad \sigma_y = 0 \qquad \tau_{xy} = -12$ ksi



a)

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2} = \frac{-30 + 0}{2} \pm \sqrt{\left(\frac{-30 - 0}{2}\right)^2 + (-12)^2}$$

$\sigma_1 = 4.21$ ksi **Ans.**

$\sigma_2 = -34.2$ ksi **Ans.**

Orientation of principal stress:

$$\tan 2\theta_P = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-12}{(-30-0)/2} = 0.8$$

$\theta_P = 19.33°$ and $-70.67°$

Use Eq. 9-1 to determine the principal plane of σ_1 and σ_2.

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$\theta = 19.33°$

$$\sigma_{x'} = \frac{-30 + 0}{2} + \frac{-30 - 0}{2}\cos 2(19.33°) + (-12)\sin 2(19.33°) = -34.2 \text{ ksi}$$

Therefore $\theta_{P_2} = 19.3°$ **Ans.**

and $\theta_{P_1} = -70.7°$ **Ans.**

b)

$$\tau_{\max_{\text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2} = \sqrt{\left(\frac{-30 - 0}{2}\right)^2 + (-12)^2} = 19.2 \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{-30 + 0}{2} = -15 \text{ ksi} \qquad \textbf{Ans.}$$

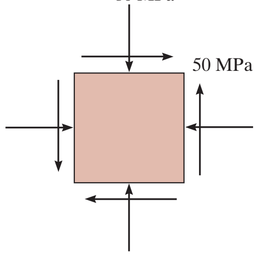
Orientation of max, in - plane shear stress:

$$\tan 2\theta_P = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \frac{-(-30 - 0)/2}{-12} = -1.25$$

$\theta_P = -25.2°$ and $64.3°$ **Ans.**

By observation, in order to preserve equllibrium along AB, $\tau_{\max}$ has to act in the direction shown in the figure.

9–15. The state of stress at a point is shown on the element. Determine (a) the principal stress and (b) the maximum in-plane shear stress and average normal stress at the point. Specify the orientation of the element in each case. Show the results on each element.



In accordance to the established sign convention,

$$\sigma_x = -60 \text{ MPa} \qquad \sigma_y = -80 \text{ MPa} \qquad \tau_{xy} = 50 \text{ MPa}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{-60 + (-80)}{2} \pm \sqrt{\left[\frac{-60 - (-80)}{2}\right]^2 + 50^2}$$

$$= -70 \pm \sqrt{2600}$$

$$\sigma_1 = -19.0 \text{ MPa} \qquad \sigma_2 = -121 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tan 2\theta_P = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{50}{[-60 - (-80)]/2} = 5$$

$$\theta_P = 39.34° \quad \text{and} \quad -50.65°$$

Substitute $\theta = 39.34°$ into Eq. 9-1,

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$= \frac{-60 + (-80)}{2} + \frac{-60 - (-80)}{2} \cos 78.69° + 50 \sin 78.69°$$

$$= -19.0 \text{ MPa} = \sigma_1$$

Thus,

$$(\theta_P)_1 = 39.3° \qquad (\theta_P)_2 = -50.7° \qquad \textbf{Ans.}$$

The element that represents the state of principal stress is shown in Fig. *a*.

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2} = \sqrt{\left[\frac{-60 - (-80)}{2}\right]^2 + 50^2} = 51.0 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tan 2\theta_S = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \frac{-[-60 - (-80)]/2}{50} = -0.2$$

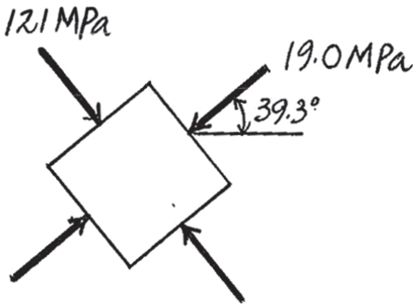
$$\theta_S = -5.65° \quad \text{and} \quad 84.3° \qquad \textbf{Ans.}$$

By Inspection, $\tau_{\substack{\max \\ \text{in-plane}}}$ has to act in the sense shown in Fig. *b* to maintain equilibrium.

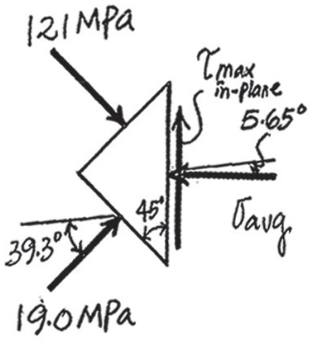
$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-60 + (-80)}{2} = -70 \text{ MPa}$$

The element that represents the state of maximum in - plane shear stress is shown in Fig. *c*.

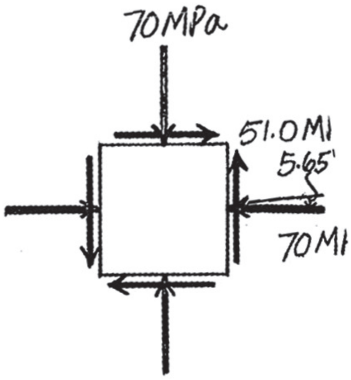
9–15. Continued



(a)

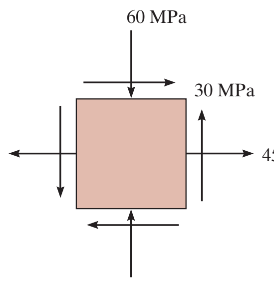


(b)



(c)

***9–16.** The state of stress at a point is shown on the element. Determine (a) the principal stress and (b) the maximum in-plane shear stress and average normal stress at the point. Specify the orientation of the element in each case. Sketch the results on each element.



$\sigma_x = 45 \text{ MPa} \qquad \sigma_y = -60 \text{ MPa} \qquad \tau_{xy} = 30 \text{ MPa}$

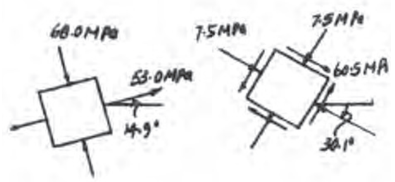
a)

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{45 - 60}{2} \pm \sqrt{\left(\frac{45 - (-60)}{2}\right)^2 + (30)^2}$$

$$\sigma_1 = 53.0 \text{ MPa} \qquad \textbf{Ans.}$$

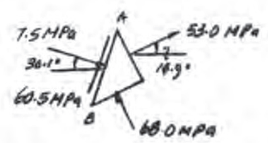
$$\sigma_2 = -68.0 \text{ MPa} \qquad \textbf{Ans.}$$



Orientation of principal stress:

$$\tan 2\theta_P = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{30}{(45 - (-60))/2} = 0.5714$$

$$\theta_P = 14.87, \qquad -75.13$$



Use Eq. 9-1 to determine the principal plane of σ_1 and σ_2:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta, \qquad \text{where } \theta = 14.87°$$

$$= \frac{45 + (-60)}{2} + \frac{45 - (-60)}{2}\cos 29.74° + 30 \sin 29.74° = 53.0 \text{ MPa}$$

Therefore $\theta_{P1} = 14.9°$ **Ans.**

and $\theta_{P2} = -75.1°$ **Ans.**

b)

$$\tau_{\max_{\text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2} = \sqrt{\left(\frac{45 - (-60)}{2}\right)^2 + 30^2} = 60.5 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_{\text{avg}} = \frac{\sigma_x - \sigma_y}{2} = \frac{45 + (-60)}{2} = -7.50 \text{ MPa} \qquad \textbf{Ans.}$$

Orientation of maximum in - plane shear stress:

$$\tan 2\theta_S = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \frac{-(45 - (-60))/2}{30} = -1.75$$

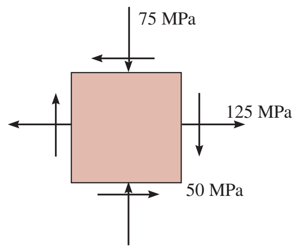
$$\theta_S = -30.1° \qquad \textbf{Ans.}$$

and

$$\theta_S = 59.9° \qquad \textbf{Ans.}$$

By observation, in order to preserve equilibrium along AB, $\tau_{\max}$ has to act in the direction shown.

•9–17. Determine the equivalent state of stress on an element at the same point which represents (a) the principal stress, and (b) the maximum in-plane shear stress and the associated average normal stress. Also, for each case, determine the corresponding orientation of the element with respect to the element shown. Sketch the results on each element.



Normal and Shear Stress:

$\sigma_x = 125$ MPa $\qquad \sigma_y = -75$ MPa $\qquad \tau_{xy} = -50$ MPa

In - Plane Principal Stresses:

$$\sigma_{1,2} = \frac{\sigma_x - \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{125 + (-75)}{2} \pm \sqrt{\left(\frac{125 - (-75)}{2}\right)^2 + (-50)^2}$$

$$= 25 \pm \sqrt{12500}$$

$\sigma_1 = 137$ MPa $\qquad \sigma_2 = -86.8$ MPa **Ans.**

Orientation of Principal Plane:

$$\tan 2\theta_P = \frac{\tau_{xy}}{\left(\sigma_x - \sigma_y\right)/2} = \frac{-50}{\left(125-(-75)\right)/2} = -0.5$$

$$\theta_p = -13.28^\circ \text{ and } 76.72^\circ$$

Substitute $\theta = -13.28^\circ$ into

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{125 + (-75)}{2} + \frac{125 - (-75)}{2}\cos(-26.57^\circ) + (-50)\sin(-26.57^\circ)$$

$$= 137 \text{ MPa} = \sigma_1$$

Thus,

$$\left(\theta_p\right)_1 = -13.3^\circ \text{ and } \left(\theta_p\right)_2 = 76.7^\circ \qquad \textbf{Ans.}$$

$$125 - (-75)/(-50)$$

The element that represents the state of principal stress is shown in Fig. *a*.

Maximum In - Plane Shear Stress:

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2} = \sqrt{\left(\frac{-100 - 0}{2}\right)^2 + 25^2} = 112 \text{ MPa} \qquad \textbf{Ans.}$$

Orientation of the Plane of Maximum In - Plane Shear Stress:

$$\tan 2\theta_s = -\frac{\left(\sigma_x - \sigma_y\right)/2}{\tau_{xy}} = -\frac{\left(125 - (-75)\right)/2}{-50} = 2$$

$$\theta_s = 31.7^\circ \text{ and } 122^\circ$$

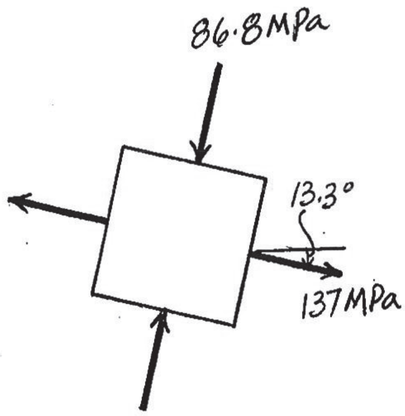
9–17. Continued

By inspection, $\tau_{\substack{\text{max}\\\text{in-plane}}}$ has to act in the same sense shown in Fig. *b* to maintain equilibrium.

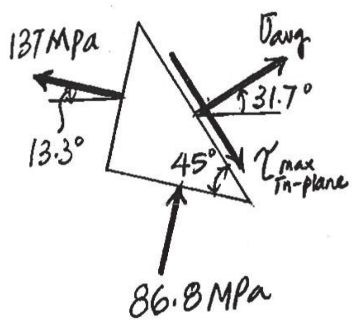
Average Normal Stress:

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{125 + (-75)}{2} = 25 \text{ MPa} \qquad \textbf{Ans.}$$

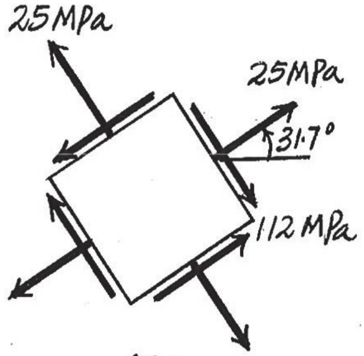
The element that represents the state of maximum in - plane shear stress is shown in Fig. *c*.



(a)

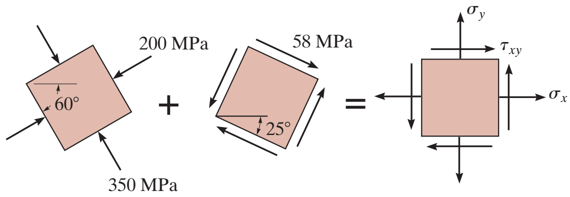


(b)

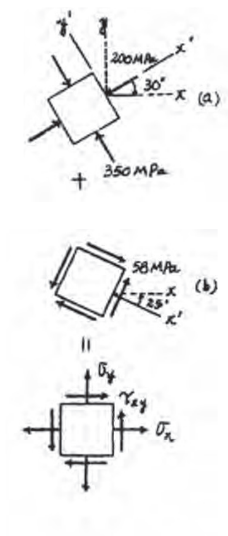


(c)

9–18. A point on a thin plate is subjected to the two successive states of stress shown. Determine the resultant state of stress represented on the element oriented as shown on the right.



Stress Transformation Equations: Applying Eqs. 9-1, 9-2, and 9-3 to element (a) with $\theta = -30°$, $\sigma_{x'} = -200$ MPa, $\sigma_{y'} = -350$ MPa and $\tau_{x'y'} = 0$.



$$(\sigma_x)_a = \frac{\sigma_{x'} + \sigma_{y'}}{2} + \frac{\sigma_{x'} - \sigma_{y'}}{2}\cos 2\theta + \tau_{x'y'}\sin 2\theta$$

$$= \frac{-200 + (-350)}{2} + \frac{-200 - (-350)}{2}\cos(-60°) + 0$$

$$= -237.5 \text{ MPa}$$

$$(\sigma_y)_a = \frac{\sigma_{x'} + \sigma_{y'}}{2} - \frac{\sigma_{x'} - \sigma_{y'}}{2}\cos 2\theta - \tau_{x'y'}\sin 2\theta$$

$$= \frac{-200 + (-350)}{2} - \frac{-200 - (-350)}{2}\cos(-60°) - 0$$

$$= -312.5 \text{ MPa}$$

$$(\tau_{xy})_a = -\frac{\sigma_{x'} - \sigma_{y'}}{2}\sin 2\theta + \tau_{x'y'}\cos 2\theta$$

$$= -\frac{-200 - (-350)}{2}\sin(-60°) + 0$$

$$= 64.95 \text{ MPa}$$

For element (b), $\theta = 25°$, $\sigma_{x'} = \sigma_{y'} = 0$ and $\sigma_{x'y'} = 58$ MPa.

$$(\sigma_x)_b = \frac{\sigma_{x'} + \sigma_{y'}}{2} + \frac{\sigma_{x'} - \sigma_{y'}}{2}\cos 2\theta + \tau_{x'y'}\sin 2\theta$$

$$= 0 + 0 + 58\sin 50°$$

$$= 44.43 \text{ MPa}$$

$$(\sigma_y)_b = \frac{\sigma_{x'} + \sigma_{y'}}{2} - \frac{\sigma_{x'} - \sigma_{y'}}{2}\cos 2\theta - \tau_{x'y'}\sin 2\theta$$

$$= 0 - 0 - 58\sin 50°$$

$$= -44.43 \text{ MPa}$$

$$(\tau_{xy})_b = -\frac{\sigma_{x'} - \sigma_{y'}}{2}\sin 2\theta + \tau_{x'y'}\cos 2\theta$$

$$= -0 + 58\cos 50°$$

$$= 37.28 \text{ MPa}$$

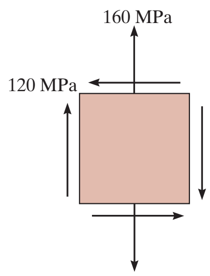
Combining the stress components of two elements yields

$$\sigma_s = (\sigma_x)_a + (\sigma_x)_b = -237.5 + 44.43 = -193 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_y = (\sigma_y)_a + (\sigma_y)_b = -312.5 - 44.43 = -357 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_{xy} = (\tau_{xy})_a + (\tau_{xy})_b = 64.95 + 37.28 = 102 \text{ MPa} \qquad \textbf{Ans.}$$

9–19. The state of stress at a point is shown on the element. Determine (a) the principal stress and (b) the maximum in-plane shear stress and average normal stress at the point. Specify the orientation of the element in each case. Sketch the results on each element.



In accordance to the established sign Convention,

$$\sigma_x = 0 \qquad \sigma_y = 160 \text{ MPa} \qquad \tau_{xy} = -120 \text{ MPa}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{0 + 160}{2} \pm \sqrt{\left(\frac{0 - 160}{2}\right)^2 + (-120)^2}$$

$$= 80 \pm \sqrt{20800}$$

$$\sigma_1 = 224 \text{ MPa} \qquad \sigma_2 = -64.2 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-120}{(0 - 160)/2} = 1.5$$

$$\theta_p = 28.15^\circ \quad \text{and } -61.85^\circ$$

Substitute $\theta = 28.15^\circ$ into Eq. 9-1,

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{0 + 160}{2} + \frac{0 - 160}{2}\cos 56.31^\circ + (-120)\sin 56.31^\circ$$

$$= -64.22 = \sigma_2$$

Thus,

$$(\theta_p)_1 = -61.8^\circ \qquad (\theta_p)_2 = 28.2^\circ \qquad \textbf{Ans.}$$

The element that represents the state of principal stress is shown in Fig. *a*

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{0 - 160}{2}\right)^2 + (-120)^2} = 144 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}} = \frac{-(0 - 160)/2}{-120} = -0.6667$$

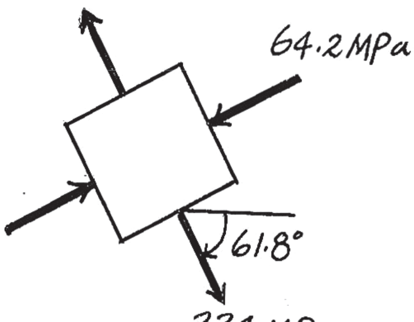
$$\theta_s = -16.8^\circ \quad \text{and} \quad 73.2^\circ \qquad \textbf{Ans.}$$

By inspection, $\tau_{\substack{\max \\ \text{in-plane}}}$ has to act in the sense shown in Fig. *b* to maintain equilibrium.

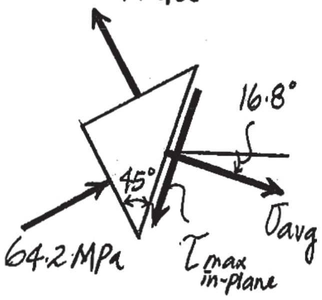
$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + 160}{2} = 80 \text{ MPa} \qquad \textbf{Ans.}$$

The element that represents the state of Maximum in - plane shear stress is shown in Fig. (c)

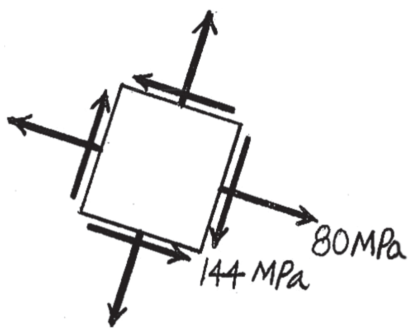
9–19. Continued



(a)

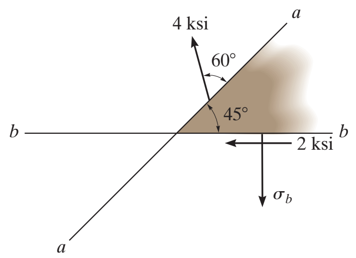


(b)



(c)

***9–20.** The stress acting on two planes at a point is indicated. Determine the normal stress σ_b and the principal stresses at the point.



Stress Transformation Equations: Applying Eqs. 9-2 and 9-1 with $\theta = -135°$, $\sigma_y = 3.464$ ksi, $\tau_{xy} = 2.00$ ksi, $\tau_{x'y'} = -2$ ksi, and $\sigma_{x'} = \sigma_{b'}$.,

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$-2 = -\frac{\sigma_x - 3.464}{2}\sin(-270°) + 2\cos(-270°)$$

$$\sigma_x = 7.464 \text{ ksi}$$

$$\sigma_{x'} = \frac{\sigma_x - \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_y = \frac{7.464 + 3.464}{2} + \frac{7.464 - 3.464}{2}\cos(-270°) + 2\sin(-270°)$$

$$= 7.46 \text{ ksi} \qquad \textbf{Ans.}$$

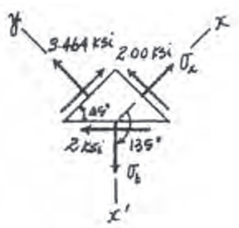
In - Plane Principal Stress: Applying Eq. 9-5.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

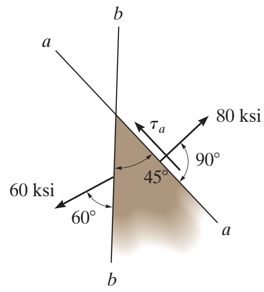
$$= \frac{7.464 + 3.464}{2} \pm \sqrt{\left(\frac{7.464 - 3.464}{2}\right)^2 + 2^2}$$

$$= 5.464 \pm 2.828$$

$$\sigma_1 = 8.29 \text{ ksi} \qquad \sigma_2 = 2.64 \text{ ksi} \qquad \textbf{Ans.}$$



•9–21. The stress acting on two planes at a point is indicated. Determine the shear stress on plane *a–a* and the principal stresses at the point.



$\sigma_x = 60 \sin 60° = 51.962$ ksi

$\tau_{xy} = 60 \cos 60° = 30$ ksi

$$\sigma_a = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$80 = \frac{51.962 + \sigma_y}{2} + \frac{51.962 - \sigma_y}{2}\cos(90°) + 30\sin(90°)$$

$$\sigma_y = 48.038 \text{ ksi}$$

$$\tau_a = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos\theta$$

$$= -\left(\frac{51.962 - 48.038}{2}\right)\sin(90°) + 30\cos(90°)$$

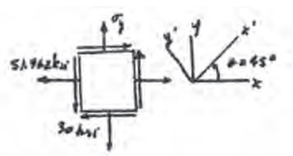
$\tau_a = -1.96$ ksi **Ans.**

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

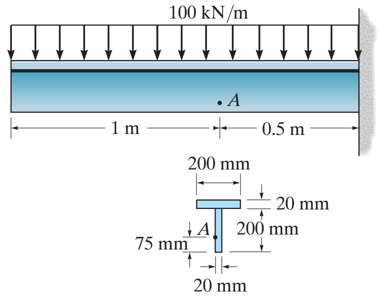
$$= \frac{51.962 + 48.038}{2} \pm \sqrt{\left(\frac{51.962 - 48.038}{2}\right)^2 + (30)^2}$$

$\sigma_1 = 80.1$ ksi **Ans.**

$\sigma_2 = 19.9$ ksi **Ans.**

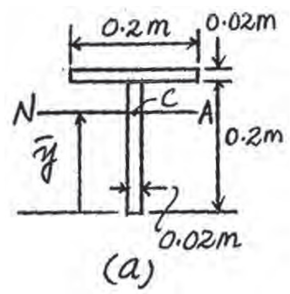


9–22. The T-beam is subjected to the distributed loading that is applied along its centerline. Determine the principal stress at point *A* and show the results on an element located at this point.



The location of the centroid *c* of the T cross-section, Fig. *a*, is

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{0.1(0.2)(0.02) + 0.21(0.02)(0.2)}{0.2(0.02) + 0.02(0.2)} = 0.155 \text{ m}$$



$$I = \frac{1}{12}(0.02)(0.2^3) + 0.02(0.2)(0.155 - 0.1)^2$$

$$+ \frac{1}{12}(0.2)(0.02^3) + 0.2(0.02)(0.21 - 0.155)^2$$

$$= 37.6667(10^{-6}) \text{ m}^4$$

Referring to Fig. *b*,

$$Q_A = \bar{y}'A' = 0.1175(0.075)(0.02) = 0.17625(10^{-3}) \text{ m}^3$$

Using the method of sections and considering the FBD of the left cut segment of the beam, Fig. *c*,

$+\uparrow\Sigma F_y = 0;\qquad V - 100(1) = 0 \qquad V = 100 \text{ kN}$

$\circlearrowleft + \Sigma M_C = 0;\qquad 100(1)(0.5) - M = 0 \qquad M = 50 \text{ kN}\cdot\text{m}$

The normal stress developed is contributed by bending stress only. For point A, $y = 0.155 - 0.075 = 0.08$ m. Thus

$$\sigma = \frac{My}{I} = \frac{50(10^3)(0.08)}{37.6667(10^{-6})} = 106 \text{ MPa}$$

The shear stress is contributed by the transverse shear stress only. Thus,

$$\tau = \frac{VQ_A}{It} = \frac{100(10^3)[0.17625(10^{-3})]}{37.6667(10^{-6})(0.02)} = 23.40(10^6)\text{Pa} = 23.40 \text{ MPa}$$

The state of stress of point *A* can be represented by the element shown in Fig. *c*.

Here, $\sigma_x = -106.19$ MPa, $\sigma_y = 0$ and $\tau_{xy} = 23.40$ MPa.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{-106.19 + 0}{2} \pm \sqrt{\left(\frac{-106.19 - 0}{2}\right)^2 + 23.40^2}$$

$$= -53.10 \pm 58.02$$

$\sigma_1 = 4.93$ MPa $\qquad \sigma_2 = -111$ MPa **Ans.**

9–22. Continued

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{23.40}{(-106.19 - 0)/2} = -0.4406$$

$$\theta_p = -11.89° \qquad \text{ans} \quad 78.11°$$

Substitute $\theta = -11.89°$,

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

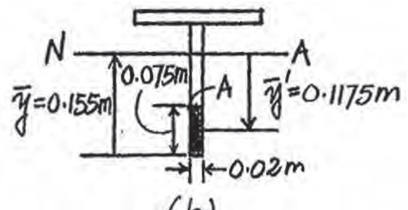
$$= \frac{-106.19 + 0}{2} + \frac{-106.19 - 0}{2}\cos(-23.78°) + 23.40 \;\; 5\text{m}\,(-23.78°)$$

$$= -111.12 \text{ MPa} = \sigma_2$$

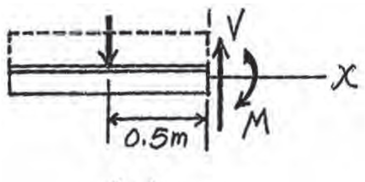
Thus,

$$(\theta_p)_1 = 78.1° \qquad (\theta_p)_2 = -11.9° \qquad \textbf{Ans.}$$

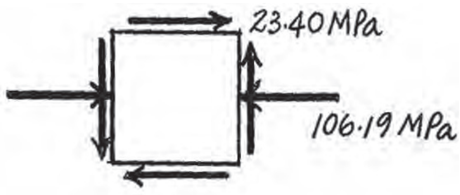
The state of principal stress can be represented by the element shown in Fig. *e*.



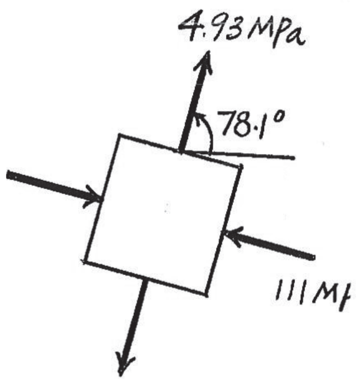
(b)



(c)

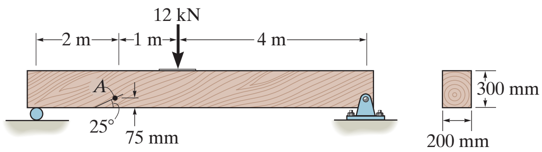


(d)



(e)

•9–23. The wood beam is subjected to a load of 12 kN. If a grain of wood in the beam at point A makes an angle of 25° with the horizontal as shown, determine the normal and shear stress that act perpendicular and parallel to the grain due to the loading.



$$I = \frac{1}{12}(0.2)(0.3)^3 = 0.45(10^{-3})\ \text{m}^4$$

$$Q_A = \bar{y}A' = 0.1125(0.2)(0.075) = 1.6875(10^{-3})\ \text{m}^3$$

$$\sigma_A = \frac{My_A}{I} = \frac{13.714(10^3)(0.075)}{0.45(10^{-3})} = 2.2857\ \text{MPa (T)}$$

$$\tau_A = \frac{VQ_A}{It} = \frac{6.875(10^3)(1.6875)(10^{-3})}{0.45(10^{-3})(0.2)} = 0.1286\ \text{MPa}$$

$$\sigma_x = 2.2857\ \text{MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = -0.1286\ \text{MPa} \qquad \theta = 115°$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

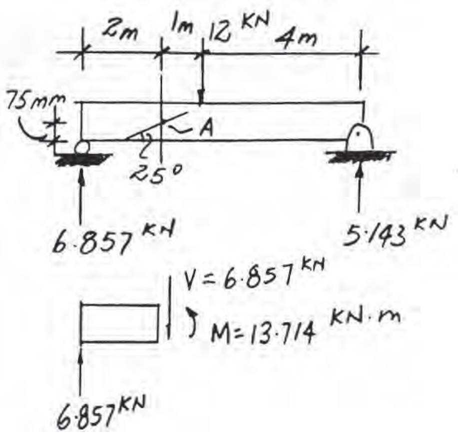
$$\sigma_{x'} = \frac{2.2857 + 0}{2} + \frac{2.2857 - 0}{2}\cos 230° + (-0.1286)\sin 230°$$

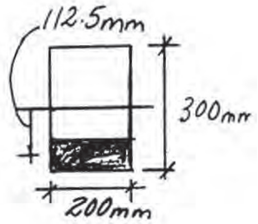
$$= 0.507\ \text{MPa}$$ **Ans.**

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

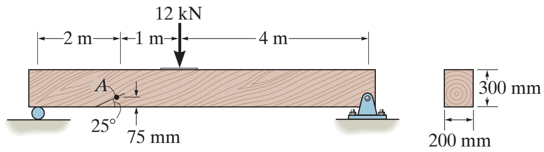
$$= -\left(\frac{2.2857 - 0}{2}\right)\sin 230° + (-0.1286)\cos 230°$$

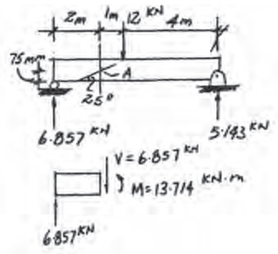
$$= 0.958\ \text{MPa}$$ **Ans.**





***9–24.** The wood beam is subjected to a load of 12 kN. Determine the principal stress at point A and specify the orientation of the element.



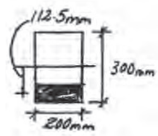


$$I = \frac{1}{12}(0.2)(0.3)^3 = 0.45(10^{-3})\ \text{m}^4$$

$$Q_A = \bar{y}A' = 0.1125(0.2)(0.075) = 1.6875(10^{-3})\ \text{m}^3$$

$$\sigma_A = \frac{My_A}{I} = \frac{13.714(10^3)(0.075)}{0.45(10^{-3})} = 2.2857\ \text{MPa (T)}$$

$$\tau_A = \frac{VQ_A}{It} = \frac{6.875(10^3)(1.6875)(10^{-3})}{0.45(10^{-3})(0.2)} = 0.1286\ \text{MPa}$$



$\sigma_x = 2.2857$ MPa $\quad \sigma_y = 0 \quad \tau_{xy} = -0.1286$ MPa

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{2.2857 + 0}{2} \pm \sqrt{\left(\frac{2.2857 - 0}{2}\right)^2 + (-0.1286)^2}$$

$\sigma_1 = 2.29$ MPa **Ans.**

$\sigma_2 = -7.20$ kPa **Ans.**

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{-0.1286}{(2.2857 - 0)/2}$$

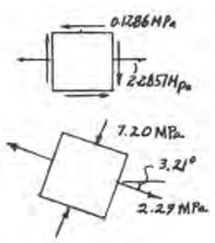
$\theta_p = -3.21°$

Check direction of principal stress:

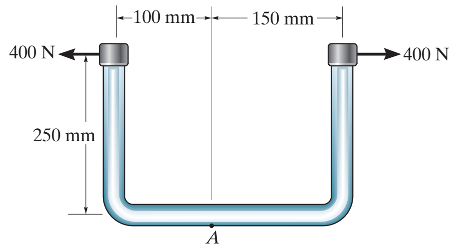
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{2.2857 + 0}{2} + \frac{2.2857 - 0}{2}\cos(-6.42°) - 0.1285\sin(-6.42)$$

$$= 2.29\ \text{MPa}$$



•9–25. The bent rod has a diameter of 20 mm and is subjected to the force of 400 N. Determine the principal stress and the maximum in-plane shear stress that is developed at point *A*. Show the results on a properly oriented element located at this point.



Using the method of sections and consider the FBD of the rod's left cut segment, Fig. *a*.

$\xrightarrow{+} \Sigma F_x = 0; \qquad N - 400 = 0 \quad N = 400 \text{ N}$

$\circlearrowleft + \Sigma M_C = 0; \qquad 400(0.25) - M = 0 \quad M = 100 \text{ N} \cdot \text{m}$

$$A = \pi(0.01^2) = 0.1(10^{-3})\,\pi \text{ m}^2$$

$$I = \frac{\pi}{4}(0.01^4) = 2.5(10^{-9})\pi \text{ m}^4$$

The normal stress developed is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

For point A, $y = C = 0.01$ m.

$$\sigma = \frac{400}{0.1(10^{-3})\pi} - \frac{100(0.01)}{2.5(10^{-9})\pi}$$

$$= -126.05\,(10^6)\text{Pa} = 126.05 \text{ MPa (C)}$$

Since no torque and transverse shear acting on the cross - section,

$$\tau = 0$$

The state of stress at point *A* can be represented by the element shown in Fig. *b*

Here, $\sigma_x = -126.05$ MPa, $\sigma_y = 0$ and $\tau_{xy} = 0$. Since no shear stress acting on the element

$$\sigma_1 = \sigma_y = 0 \qquad \sigma_2 = \sigma_x = -126 \text{ MPa} \qquad \textbf{Ans.}$$

Thus, the state of principal stress can also be represented by the element shown in Fig. *b*.

$$\tau_{\substack{\text{max}\\\text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{-126.05 - 0}{2}\right)^2 + 0^2} = 63.0 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tan 2\theta_s = -\frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}} = -\frac{(-126.05 - 0)/2}{0} = \infty$$

$$\theta_s = 45° \qquad \text{and } -45°$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{-126.05 - 0}{2}\sin 90° + 0\cos 90°$$

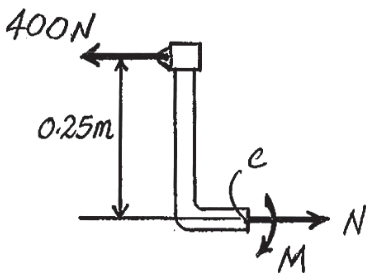
$$= 63.0 = \tau_{\substack{\text{max}\\\text{in-plane}}}$$

9–25. Continued

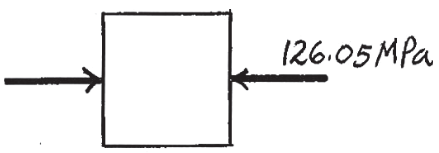
This indicates that $\tau_{\substack{\max \\ \text{in-plane}}}$ acts toward the positive sense of y' axis at the face of element defined by $\theta_s = 45°$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-126.05 + 0}{2} = -63.0 \text{ MPa} \qquad \textbf{Ans.}$$

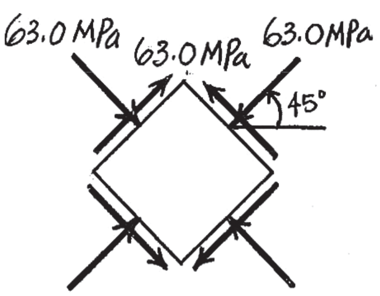
The state of maximum In - plane shear stress can be represented by the element shown in Fig. *c*



(a)

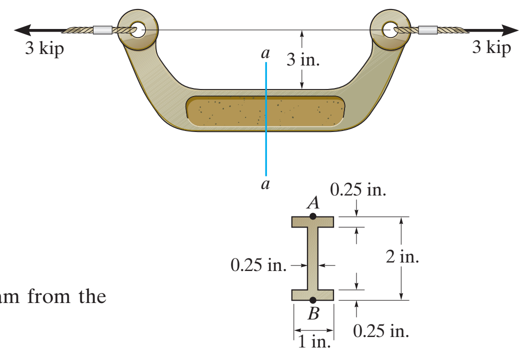


(b)



(c)

9–26. The bracket is subjected to the force of 3 kip. Determine the principal stress and maximum in-plane shear stress at point A on the cross section at section a–a. Specify the orientation of this state of stress and show the results on elements.



Internal Loadings: Consider the equilibrium of the free - body diagram from the bracket's left cut segment, Fig. *a*.

$$\xrightarrow{+} \Sigma F_x = 0; \qquad N - 3 = 0 \qquad N = 3 \text{ kip}$$

$$\Sigma M_O = 0; 3(4) - M = 0 \qquad M = 12 \text{ kip} \cdot \text{in}$$

Normal and Shear Stresses: The normal stress is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} - \frac{My}{I}$$

The cross - sectional area and the moment of inertia about the z axis of the bracket's cross section is

$$A = 1(2) - 0.75(1.5) = 0.875 \text{ in}^2$$

$$I = \frac{1}{12}(1)\left(2^3\right) - \frac{1}{12}(0.75)\left(1.5^3\right) = 0.45573 \text{ in}^4$$

For point A, $y = 1$ in. Then

$$\sigma_A = \frac{3}{0.875} - \frac{(-12)(1)}{0.45573} = 29.76 \text{ ksi}$$

Since no shear force is acting on the section,

$$\tau_A = 0$$

The state of stress at point A can be represented on the element shown in Fig. *b*.

In - Plane Principal Stress: $\sigma_x = 29.76$ ksi, $\sigma_y = 0$, and $\tau_{xy} = 0$. Since no shear stress acts on the element,

$$\sigma_1 = \sigma_x = 29.8 \text{ ksi} \qquad \sigma_2 = \sigma_y = 0 \qquad \textbf{Ans.}$$

The state of principal stresses can also be represented by the elements shown in Fig. *b*

Maximum In - Plane Shear Stress:

$$\tau_{\substack{\text{max} \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2} = \sqrt{\left(\frac{29.76 - 0}{2}\right)^2 + 0^2} = 14.9 \text{ ksi} \qquad \textbf{Ans.}$$

Orientation of the Plane of Maximum In - Plane Shear Stress:

$$\tan 2\theta_s = -\frac{\left(\sigma_x - \sigma_y\right)/2}{\tau_{xy}} = -\frac{(29.76 - 0)/2}{0} = -\infty$$

$$\theta_s = -45° \text{ and } 45° \qquad \textbf{Ans.}$$

9–26. Continued

Substituting $\theta = -45°$ into

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{29.76 - 0}{2}\sin(-90°) + 0$$

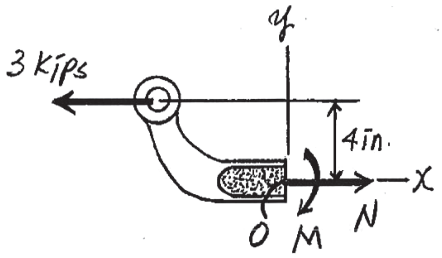
$$= 14.9 \text{ ksi} = \tau_{\substack{\text{max}\\ \text{in-plane}}}$$

This indicates that $\tau_{\substack{\text{max}\\ \text{in-plane}}}$ is directed in the positive sense of the y' axes on the ace of the element defined by $\theta_s = -45°$.

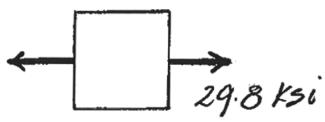
Average Normal Stress:

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{29.76 + 0}{2} = 14.9 \text{ ksi} \qquad \textbf{Ans.}$$

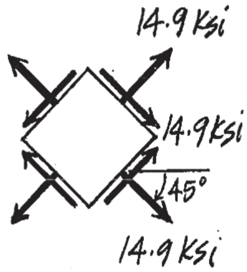
The state of maximum in - plane shear stress is represented by the element shown in Fig. *c*.



(a)

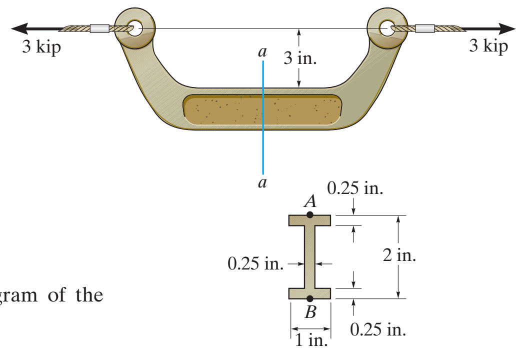


(b)



(c)

9–27. The bracket is subjected to the force of 3 kip. Determine the principal stress and maximum in-plane shear stress at point B on the cross section at section a–a. Specify the orientation of this state of stress and show the results on elements.



Section a – a

Internal Loadings: Consider the equilibrium of the free - body diagram of the bracket's left cut segment, Fig. a.

$$\xrightarrow{+} \Sigma F_x = 0; \qquad N - 3 = 0 \qquad N = 3\,\text{kip}$$

$$\Sigma M_O = 0; \; 3(4) - M = 0 \qquad M = 12\,\text{kip}\cdot\text{in}$$

Normal and Shear Stresses: The normal stress is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} - \frac{My}{I}$$

The cross - sectional area and the moment of inertia about the z axis of the bracket's cross section is

$$A = 1(2) - 0.75(1.5) = 0.875\ \text{in}^2$$

$$I = \frac{1}{12}(1)\left(2^3\right) - \frac{1}{12}(0.75)\left(1.5^3\right) = 0.45573\ \text{in}^4$$

For point B, $y = -1$ in. Then

$$\sigma_B = \frac{3}{0.875} - \frac{(-12)(-1)}{0.45573} = -22.90\ \text{ksi}$$

Since no shear force is acting on the section,

$$\tau_B = 0$$

The state of stress at point A can be represented on the element shown in Fig. b.

In - Plane Principal Stress: $\sigma_x = -22.90$ ksi, $\sigma_y = 0$, and $\tau_{xy} = 0$. Since no shear stress acts on the element,

$$\sigma_1 = \sigma_y = 0 \qquad \sigma_2 = \sigma_x = -22.90\ \text{ksi} \qquad \textbf{Ans.}$$

The state of principal stresses can also be represented by the elements shown in Fig. b.

Maximum In - Plane Shear Stress:

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2} = \sqrt{\left(\frac{-22.90 - 0}{2}\right)^2 + 0^2} = 11.5\ \text{ksi} \qquad \textbf{Ans.}$$

Orientation of the Plane of Maximum In - Plane Shear Stress:

$$\tan 2\theta_s = -\frac{\left(\sigma_x - \sigma_y\right)/2}{\tau_{xy}} = -\frac{(-22.9 - 0)/2}{0} = -\infty$$

$$\theta_s = 45° \text{ and } 135° \qquad \textbf{Ans.}$$

9–27. Continued

Substituting $\theta = 45°$ into

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{-22.9 - 0}{2}\sin 90° + 0$$

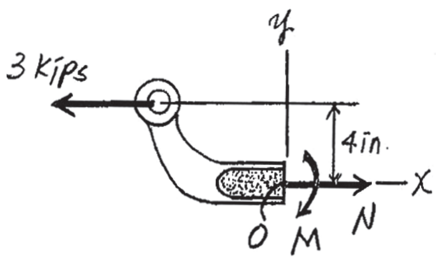
$$= 11.5 \text{ ksi} = \tau_{\substack{\text{max}\\\text{in-plane}}}$$

This indicates that $\tau_{\substack{\text{max}\\\text{in-plane}}}$ is directed in the positive sense of the y' axes on the element defined by $\theta_s = 45°$.

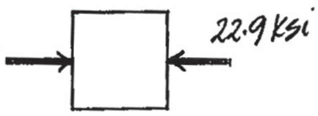
Average Normal Stress:

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{-22.9 + 0}{2} = -11.5 \text{ ksi} \qquad \textbf{Ans.}$$

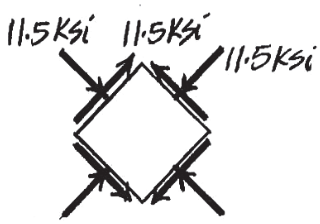
The state of maximum in - plane shear stress is represented by the element shown in Fig. *c*.



(a)

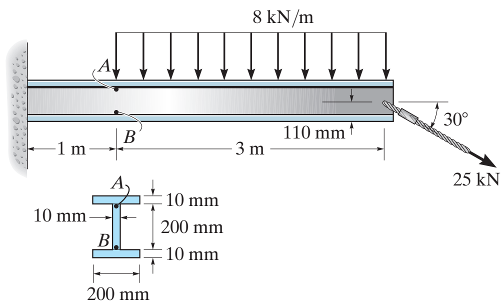


(b)

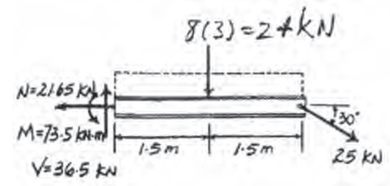


(c)

***9–28.** The wide-flange beam is subjected to the loading shown. Determine the principal stress in the beam at point A and at point B. These points are located at the top and bottom of the web, respectively. Although it is not very accurate, use the shear formula to determine the shear stress.



Internal Forces and Moment: As shown on FBD(a).

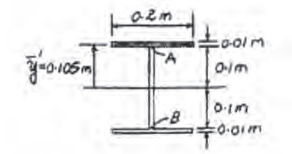


Section Properties:

$$A = 0.2(0.22) - 0.19(0.2) = 6.00\left(10^{-3}\right)\text{ m}^2$$

$$I = \frac{1}{12}(0.2)\left(0.22^3\right) - \frac{1}{12}(0.19)\left(0.2^2\right) = 50.8\left(10^{-6}\right)\text{ m}^4$$

$$Q_A = Q_B = \bar{y}'A' = 0.105(0.01)(0.2) = 0.210\left(10^{-3}\right)\text{ m}^3$$



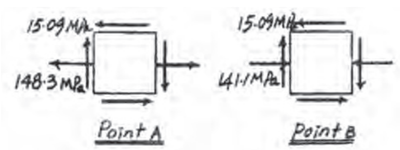
Normal Stress:

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

$$= \frac{21.65(10^3)}{6.00(10^{-3})} \pm \frac{73.5(10^3)(0.1)}{50.8(10^{-6})}$$

$$\sigma_A = 3.608 + 144.685 = 148.3\text{ MPa}$$

$$\sigma_B = 3.608 - 144.685 = -141.1\text{ MPa}$$



Shear Stress: Applying the shear formula $\tau = \dfrac{VQ}{It}$.

$$\tau_A = \tau_B = \frac{36.5(10^3)\left[0.210(10^{-3})\right]}{50.8(10^{-6})(0.01)} = 15.09\text{ MPa}$$

In - Plane Principal Stress: $\sigma_x = 148.3$ MPa, $\sigma_y = 0$, and $\tau_{xy} = -15.09$ MPa for point A. Applying Eq. 9-5.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{148.3 + 0}{2} \pm \sqrt{\left(\frac{148.3 - 0}{2}\right)^2 + (-15.09)^2}$$

$$= 74.147 \pm 75.666$$

$\sigma_1 = 150$ MPa $\qquad \sigma_2 = -1.52$ MPa **Ans.**

$\sigma_x = -141.1$ MPa, $\sigma_y = 0$, and $\tau_{xy} = -15.09$ MPa for point B. Applying Eq. 9-5.

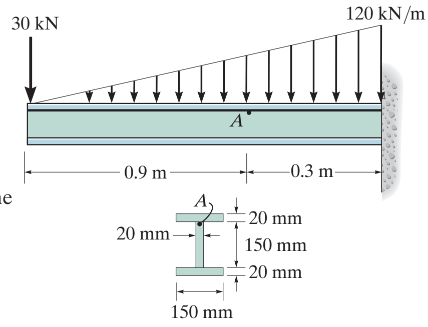
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-141.1 + 0}{2} \pm \sqrt{\left(\frac{(-141.1) - 0}{2}\right)^2 + (-15.09)^2}$$

$$= -70.538 \pm 72.134$$

$\sigma_1 = 1.60$ MPa $\qquad \sigma_2 = -143$ MPa **Ans.**

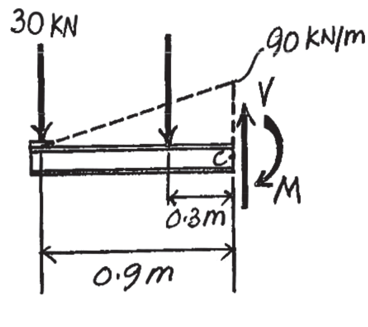
•9–29. The wide-flange beam is subjected to the loading shown. Determine the principal stress in the beam at point A, which is located at the top of the web. Although it is not very accurate, use the shear formula to determine the shear stress. Show the result on an element located at this point.



Using the method of sections and consider the FBD of the left cut segment of the bean, Fig. *a*

$+\uparrow \Sigma F_y = 0; \quad V - \frac{1}{2}(90)(0.9) - 30 = 0 \qquad V = 70.5 \text{ kN}$

$\circlearrowleft + \Sigma M_C = 0; \quad \frac{1}{2}(90)(0.9)(0.3) + 30(0.9) - M = 0 \qquad M = 39.15 \text{ kN} \cdot \text{m}$



(a)

The moment of inertia of the cross - section about the bending axis is

$$I = \frac{1}{12}(0.15)(0.19^3) - \frac{1}{12}(0.13)(0.15^3) = 49.175(10^{-6}) \text{ m}^4$$

Referring to Fig. *b*,

$$Q_A = \bar{y}'A' = 0.085\,(0.02)(0.15) = 0.255\,(10^{-3}) \text{ m}^3$$

The normal stress developed is contributed by bending stress only. For point A, $y = 0.075$ m. Thus,

$$\sigma = \frac{My}{I} = \frac{39.15(10^3)(0.075)}{49.175(10^{-6})} = 59.71(10^6)\text{Pa} = 59.71 \text{ MPa (T)}$$

The shear stress is contributed by the transverse shear stress only. Thus

$$\tau = \frac{VQ_A}{It} = \frac{70.5(10^3)\left[0.255(10^{-3})\right]}{49.175(10^{-6})\,(0.02)} = 18.28(10^6)\text{Pa} = 18.28 \text{ MPa}$$

Here, $\sigma_x = 59.71$ MPa, $\sigma_y = 0$ and $\tau_{xy} = 18.28$ MPa.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}}$$

$$= \frac{59.71 + 0}{2} \pm \sqrt{\left(\frac{59.71 - 0}{2}\right)^2 + 18.28^2}$$

$$= 29.86 \pm 35.01$$

$\sigma_1 = 64.9 \text{ MPa} \qquad \sigma_2 = -5.15 \text{ MPa}$ **Ans.**

$$\tan 2\theta_P = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{18.28}{(59.71 - 0)/2} = 0.6122$$

$$\theta_P = 15.74° \quad \text{and} \quad -74.26°$$

Substitute $\theta = 15.74°$,

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{59.71 + 0}{2} + \frac{59.71 - 0}{2}\cos 31.48° + 18.28 \sin 31.48°$$

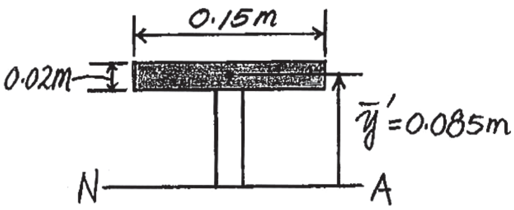
$$= 64.9 \text{ MPa} = \sigma_1$$

9–29. Continued

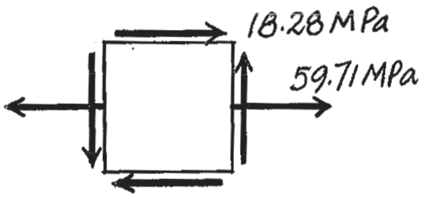
Thus,

$$\left(\theta_P\right)_1 = 15.7^\circ \qquad \left(\theta_P\right)_2 = -74.3^\circ \qquad \textbf{Ans.}$$

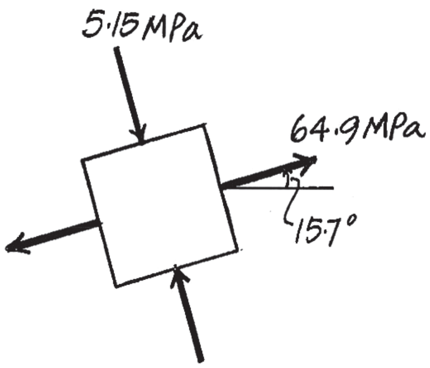
The state of principal stress can be represented by the element shown in Fig. *d*



(b)

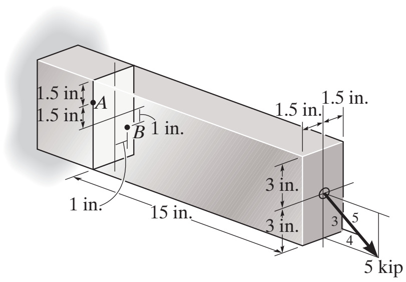


(C)



(d)

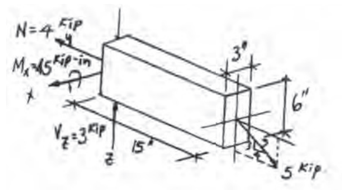
9–30. The cantilevered rectangular bar is subjected to the force of 5 kip. Determine the principal stress at points A and B.



$$I = \frac{1}{12}(3)(6^3) = 54 \text{ in}^4 \qquad A = (6)(3) = 18 \text{ in}^2$$

$$Q_A = 2.25(1.5)(3) = 10.125 \text{ in}^3 \qquad Q_B = 2(2)(3) = 12 \text{ in}^3$$

Point A:



$$\sigma_A = \frac{P}{A} + \frac{M_x z}{I} = \frac{4}{18} + \frac{45(1.5)}{54} = 1.472 \text{ ksi}$$

$$\tau_A = \frac{V_z Q_A}{It} = \frac{3(10.125)}{54(3)} = 0.1875 \text{ ksi}$$

$$\sigma_x = 1.472 \text{ ksi} \qquad \sigma_y = 0 \qquad \tau_{xy} = 0.1875 \text{ ksi}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{1.472 + 0}{2} \pm \sqrt{\left(\frac{1.472 - 0}{2}\right)^2 + 0.1875^2}$$

$\sigma_1 = 1.50$ ksi **Ans.**

$\sigma_2 = -0.0235$ ksi **Ans.**

Point B:

$$\sigma_B = \frac{P}{A} - \frac{M_x z}{I} = \frac{4}{18} - \frac{45(1)}{54} = -0.6111 \text{ ksi}$$

$$\tau_B = \frac{V_z Q_B}{It} = \frac{3(12)}{54(3)} = 0.2222 \text{ ksi}$$

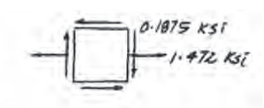
$$\sigma_x = -0.6111 \text{ ksi} \qquad \sigma_y = 0 \qquad \tau_{xy} = 0.2222 \text{ ksi}$$

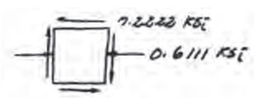
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-0.611 + 0}{2} \pm \sqrt{\left(\frac{-0.6111 - 0}{2}\right)^2 + 0.222^2}$$

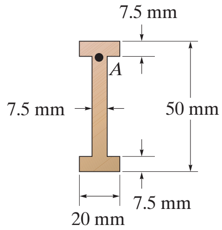
$\sigma_1 = 0.0723$ ksi **Ans.**

$\sigma_2 = -0.683$ ksi **Ans.**

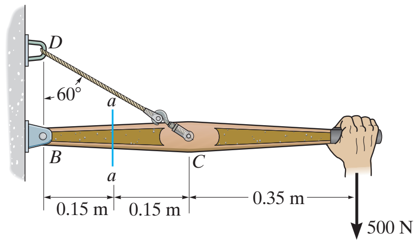




9–31. Determine the principal stress at point A on the cross section of the arm at section a–a. Specify the orientation of this state of stress and indicate the results on an element at the point.



Section $a-a$



Support Reactions: Referring to the free - body diagram of the entire arm shown in Fig. a,

$\Sigma M_B = 0;\ F_{CD}\sin 30°(0.3) - 500(0.65) = 0 \qquad F_{CD} = 2166.67\,\text{N}$

$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad B_x - 2166.67\cos 30° = 0 \qquad B_x = 1876.39\,\text{N}$

$+\uparrow\Sigma F_y = 0; \qquad 2166.67\sin 30° - 500 - B_y = 0 \qquad B_y = 583.33\,\text{N}$

Internal Loadings: Consider the equilibrium of the free - body diagram of the arm's left segment, Fig. b.

$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad 1876.39 - N = 0 \qquad N = 1876.39\,\text{N}$

$+\uparrow\Sigma F_y = 0; \qquad V - 583.33 = 0 \qquad V = 583.33\text{ N}$

$+\Sigma M_O = 0; \qquad 583.33(0.15) - M = 0 \qquad M = 87.5\text{N}\cdot\text{m}$

Section Properties: The cross - sectional area and the moment of inertia about the z axis of the arm's cross section are

$$A = 0.02(0.05) - 0.0125(0.035) = 0.5625\left(10^{-3}\right)\text{m}^2$$

$$I = \frac{1}{12}(0.02)\left(0.05^3\right) - \frac{1}{12}(0.0125)\left(0.035^3\right) = 0.16367\left(10^{-6}\right)\text{m}^4$$

Referring to Fig. b,

$$Q_A = \bar{y}'A' = 0.02125(0.0075)(0.02) = 3.1875\left(10^{-6}\right)\text{ m}^3$$

Normal and Shear Stress: The normal stress is a combination of axial and bending stress. Thus,

$$\sigma_A = \frac{N}{A} + \frac{My_A}{I}$$

$$= \frac{-1876.39}{0.5625\left(10^{-3}\right)} + \frac{87.5(0.0175)}{0.16367\left(10^{-6}\right)} = 6.020\text{ MPa}$$

The shear stress is caused by transverse shear stress.

$$\tau_A = \frac{VQ_A}{It} = \frac{583.33\left[3.1875\left(10^{-6}\right)\right]}{0.16367\left(10^{-6}\right)(0.0075)} = 1.515\text{ MPa}$$

The share of stress at point A can be represented on the element shown in Fig. d.

In - Plane Principal Stress: $\sigma_x = 6.020$ MPa, $\sigma_y = 0$, and $\tau_{xy} = 1.515$ MPa. We have

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{6.020 + 0}{2} \pm \sqrt{\left(\frac{6.020 - 0}{2}\right)^2 + 1.515^2}$$

$\sigma_1 = 6.38\text{ MPa} \qquad \sigma_2 = -0.360\text{ MPa}$ **Ans.**

9–31. Continued

Orientation of the Principal Plane:

$$\tan 2\theta_P = \frac{\tau_{xy}}{\left(\sigma_x - \sigma_y\right)/2} = \frac{1.515}{(6.020 - 0)/2} = 0.5032$$

$$\theta_p = 13.36° \text{ and } 26.71°$$

Substituting $\theta = 13.36°$ into

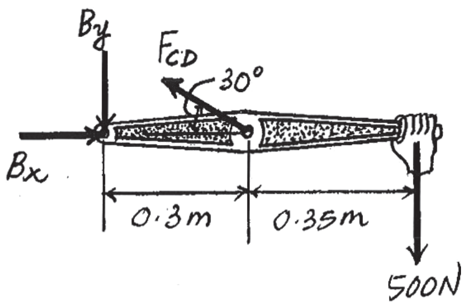
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{6.020 - 0}{2} + \frac{6.020 + 0}{2}\cos 26.71° + 1.515\sin 26.71°$$

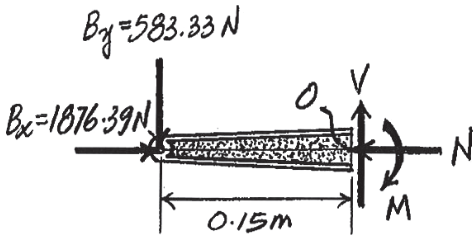
$$= 6.38 \text{ MPa} = \sigma_1$$

Thus, $\left(\theta_P\right)_1 = 13.4$ and $\left(\theta_P\right)_2 = 26.71°$ **Ans.**

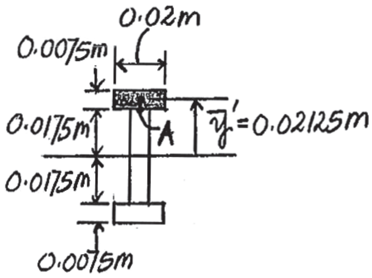
The state of principal stresses is represented by the element shown in Fig. *e*.



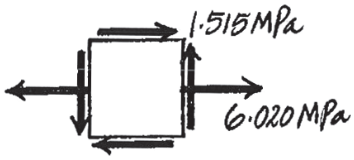
(a)



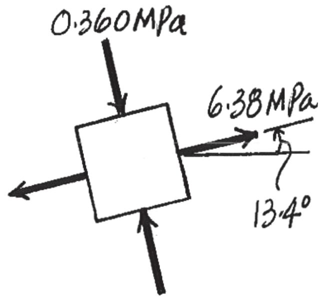
(b)



(c)

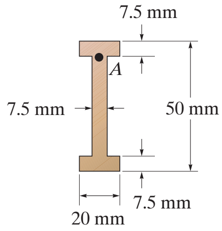


(d)

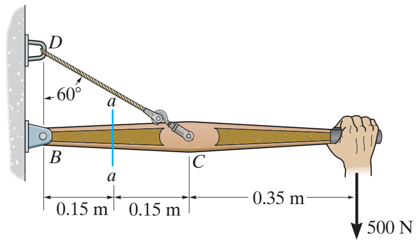


(e)

***9–32.** Determine the maximum in-plane shear stress developed at point A on the cross section of the arm at section a–a. Specify the orientation of this state of stress and indicate the results on an element at the point.



Section $a-a$



Support Reactions: Referring to the free - body diagram of the entire arm shown in Fig. a,

$$\Sigma M_B = 0;\ F_{CD} \sin 30°(0.3) - 500(0.65) = 0 \qquad F_{CD} = 2166.67\text{ N}$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad B_x - 2166.67 \cos 30° = 0 \qquad B_x = 1876.39\text{ N}$$

$$+\uparrow\Sigma F_y = 0; \qquad 2166.67 \sin 30° - 500 - B_y = 0 \qquad B_y = 583.33\text{ N}$$

Internal Loadings: Considering the equilibrium of the free - body diagram of the arm's left cut segment, Fig. b,

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad 1876.39 - N = 0 \qquad N = 1876.39\text{ N}$$

$$+\uparrow\Sigma F_y = 0; \qquad V - 583.33 = 0 \qquad V = 583.33\text{ N}$$

$$+\Sigma M_O = 0; \qquad 583.33(0.15) - M = 0 \qquad M = 87.5\text{ N}\cdot\text{m}$$

Section Properties: The cross - sectional area and the moment of inertia about the z axis of the arm's cross section are

$$A = 0.02(0.05) - 0.0125(0.035) = 0.5625\left(10^{-3}\right)\text{m}^2$$

$$I = \frac{1}{12}(0.02)\left(0.05^3\right) - \frac{1}{12}(0.0125)\left(0.035^3\right) = 0.16367\left(10^{-6}\right)\text{ m}^4$$

Referring to Fig. b,

$$Q_A = \bar{y}'A' = 0.02125(0.0075)(0.02) = 3.1875\left(10^{-6}\right)\text{m}^3$$

Normal and Shear Stress: The normal stress is a combination of axial and bending stress. Thus,

$$\sigma_A = \frac{N}{A} + \frac{My_A}{I}$$

$$= \frac{-1876.39}{0.5625\left(10^{-3}\right)} + \frac{87.5(0.0175)}{0.16367\left(10^{-6}\right)} = 6.020\text{ MPa}$$

The shear stress is contributed only by transverse shear stress.

$$\tau_A = \frac{VQ_A}{It} = \frac{583.33\left[3.1875\left(10^{-6}\right)\right]}{0.16367\left(10^{-6}\right)(0.0075)} = 1.515\text{ MPa}$$

Maximum In - Plane Shear Stress: $\sigma_x = 6.020$ MPa, $\sigma_y = 0$, and $\tau_{xy} = 1.515$ MPa.

$$\tau_{\substack{\text{max}\\\text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\ 2}} = \sqrt{\left(\frac{6.020 - 0}{2}\right)^2 + 1.515^2} = 3.37\text{ MPa} \qquad \textbf{Ans.}$$

9–32. Continued

Orientation of the Plane of Maximum In - Plane Shear Stress:

$$\tan 2\theta_s = -\frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}} = -\frac{(6.020 - 0)/2}{1.515} = -1.9871$$

$$\theta_s = -31.6^\circ \text{ and } 58.4^\circ \qquad \textbf{Ans.}$$

Substituting $\theta = -31.6^\circ$ into

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{6.020 - 0}{2}\sin(-63.29^\circ) + 1.515\cos(-63.29^\circ)$$

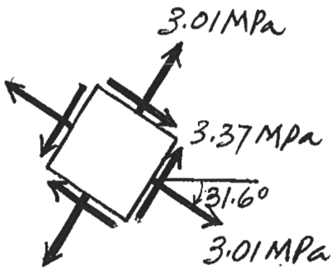
$$= 3.37 \text{ MPa} = \tau_{\substack{\max \\ \text{in-plane}}}$$

This indicates that $\tau_{\substack{\max \\ \text{in-plane}}}$ is directed in the positive sense of the y' axis on the face of the element defined by $\theta_s = -31.6^\circ$.

Average Normal Stress:

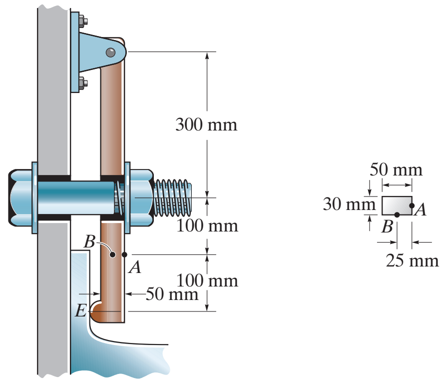
$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{6.020 + 0}{2} = 3.01 \text{ MPa} \qquad \textbf{Ans.}$$

The state of maximum in - plane shear stress is represented on the element shown in Fig. *e*.



(e)

•9–33. The clamp bears down on the smooth surface at E by tightening the bolt. If the tensile force in the bolt is 40 kN, determine the principal stress at points A and B and show the results on elements located at each of these points. The cross-sectional area at A and B is shown in the adjacent figure.



Support Reactions: As shown on FBD(a).

Internal Forces and Moment: As shown on FBD(b).

Section Properties:

$$I = \frac{1}{12}(0.03)\left(0.05^3\right) = 0.3125\left(10^{-6}\right)\text{ m}^4$$

$$Q_A = 0$$

$$Q_B = \bar{y}'A' = 0.0125(0.025)(0.03) = 9.375\left(10^{-6}\right)\text{ m}^3$$

Normal Stress: Applying the flexure formula $\sigma = -\dfrac{My}{I}$.

$$\sigma_A = -\frac{2.40(10^3)(0.025)}{0.3125(10^{-6})} = -192\text{ MPa}$$

$$\sigma_B = -\frac{2.40(10^3)(0)}{0.3125(10^{-6})} = 0$$

Shear Stress: Applying the shear formula $\tau = \dfrac{VQ}{It}$

$$\tau_A = \frac{24.0(10^3)(0)}{0.3125(10^{-6})(0.03)} = 0$$

$$\tau_B = \frac{24.0(10^3)\left[9.375(10^{-6})\right]}{0.3125(10^{-6})(0.03)} = 24.0\text{ MPa}$$

In - Plane Principal Stresses: $\sigma_x = 0$, $\sigma_y = -192$ MPa, and $\tau_{xy} = 0$ for point A. Since no shear stress acts on the element.

$$\sigma_1 = \sigma_x = 0 \qquad \textbf{Ans.}$$

$$\sigma_2 = \sigma_y = -192\text{ MPa} \qquad \textbf{Ans.}$$

$\sigma_x = \sigma_y = 0$ and $\tau_{xy} = -24.0$ MPa for point B. Applying Eq. 9-5

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= 0 \pm \sqrt{0 + (-24.0)^2}$$

$$= 0 \pm 24.0$$

$$\sigma_1 = 24.0 \qquad \sigma_2 = -24.0\text{ MPa} \qquad \textbf{Ans.}$$

9–33. Continued

Orientation of Principal Plane: Applying Eq. 9-4 for point B.

$$\tan 2\theta_p = \frac{\tau_{xy}}{\left(\sigma_x - \sigma_y\right)/2} = \frac{-24.0}{0} = -\infty$$

$$\theta_p = -45.0^\circ \qquad \text{and} \qquad 45.0^\circ$$

Subsututing the results into Eq. 9-1 with $\theta = -45.0^\circ$ yields

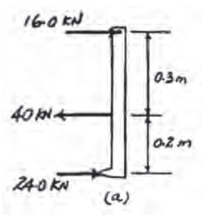
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

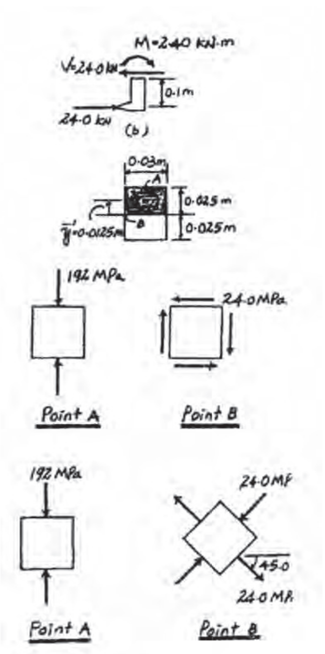
$$= 0 + 0 + [-24.0 \sin(-90.0^\circ)]$$

$$= 24.0 \text{ MPa} = \sigma_1$$

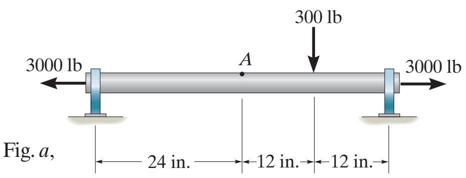
Hence,

$$\theta_{p1} = -45.0^\circ \qquad \theta_{p2} = 45.0^\circ$$ **Ans.**





9–34. Determine the principal stress and the maximum in-plane shear stress that are developed at point A in the 2-in.-diameter shaft. Show the results on an element located at this point. The bearings only support vertical reactions.



Using the method of sections and consider the FBD of shaft's left cut segment, Fig. *a*,

$\xrightarrow{+} \Sigma F_x = 0; \qquad N - 3000 = 0 \qquad N = 3000 \text{ lb}$

$+\uparrow \Sigma F_y = 0; \qquad 75 - V = 0 \qquad V = 75 \text{ lb}$

$\circlearrowleft + \Sigma M_C = 0; \qquad M - 75(24) = 0 \qquad M = 1800 \text{ lb} \cdot \text{in}$

$$A = \pi(1^2) = \pi \text{ in}^2 \qquad I = \frac{\pi}{4}(1^4) = \frac{\pi}{4} \text{ in}^4$$

Also,

$$Q_A = 0$$

The normal stress developed is the combination of axial and bending stress. Thus

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

For point A, $y = C = 1$ in. Then

$$\sigma = \frac{3000}{\pi} - \frac{1800(1)}{\pi/4}$$

$$= -1.337\,(10^3) \text{ psi} = 1.337 \text{ ksi (c)}$$

The shear stress developed is due to transverse shear force. Thus,

$$\tau = \frac{VQ_A}{It} = 0$$

The state of stress at point A, can be represented by the element shown in Fig. *b*.

Here, $\sigma_x = -1.337$ ksi, $\sigma_y = 0$ is $\tau_{xy} = 0$. Since no shear stress acting on the element,

$$\sigma_1 = \sigma_y = 0 \qquad \sigma_2 = \sigma_x = -1.34 \text{ ksi} \qquad \textbf{Ans.}$$

Thus, the state of principal stress can also be represented by the element shown in Fig. *b*.

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{-1.337 - 0}{2}\right)^2 + 0^2} = 0.668 \text{ ksi} - 668 \text{ psi} \quad \textbf{Ans.}$$

$$\tan 2\theta_s = -\frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}} = -\frac{(-1.337 - 0)/2}{0} = \infty$$

$$\theta_s = 45° \quad \text{and} \quad -45° \qquad \textbf{Ans.}$$

Substitute $\theta = 45°$,

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$= -\frac{-1.337 - 0}{2} \sin 90° + 0$$

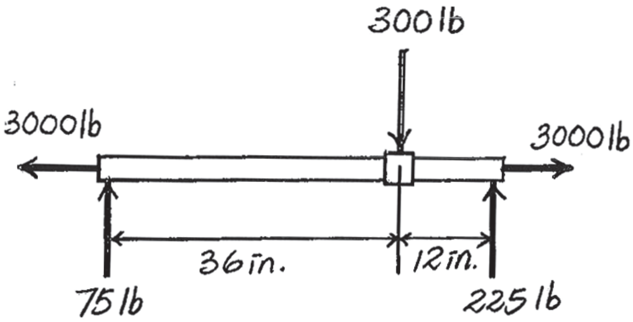
$$= 0.668 \text{ ksi} = 668 \text{ psi} = \tau_{\substack{\max \\ \text{in-plane}}}$$

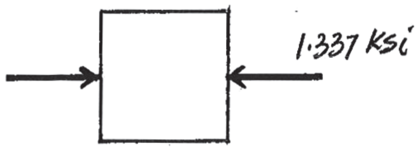
9–34. Continued

This indicates that $\tau_{\substack{\text{max}\\\text{in-plane}}}$ acts toward the positive sense of y' axis at the face of the element defined by $\theta_s = 45°$.

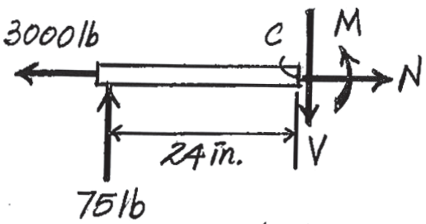
Average Normal Stress.

The state of maximum in - plane shear stress can be represented by the element shown in Fig. *c*.

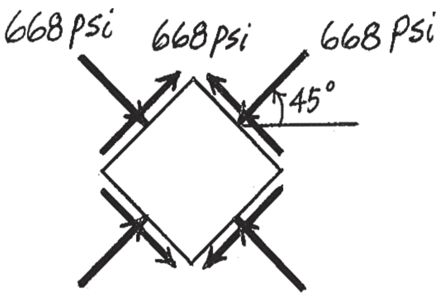




(b)

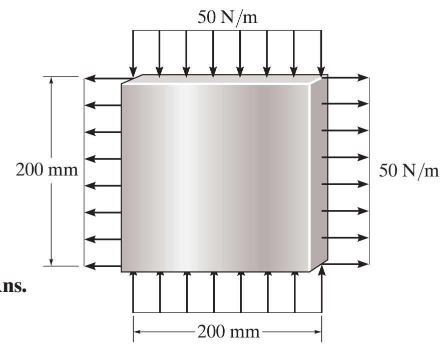


(a)



(c)

9–35. The square steel plate has a thickness of 10 mm and is subjected to the edge loading shown. Determine the maximum in-plane shear stress and the average normal stress developed in the steel.

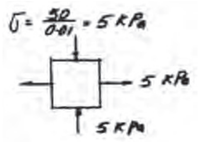


$\sigma_x = 5\text{ kPa} \qquad \sigma_y = -5\text{ kPa} \qquad \tau_{xy} = 0$

$$\tau_{\substack{\text{max}\\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{5+5}{2}\right)^2 + 0} = 5\text{ kPa} \qquad \textbf{Ans.}$$

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{3} = \frac{5-5}{2} = 0 \qquad \textbf{Ans.}$$

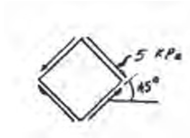


Note:

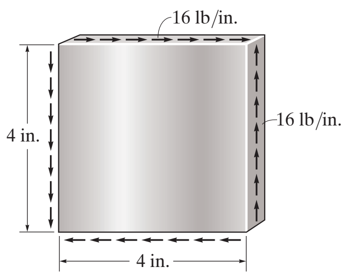
$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)/2}{\tau_{xy}}$$

$$\tan 2\theta_s = \frac{-(5+5)/2}{0} = \infty$$

$$\theta_s = 45^\circ$$



***9–36.** The square steel plate has a thickness of 0.5 in. and is subjected to the edge loading shown. Determine the principal stresses developed in the steel.



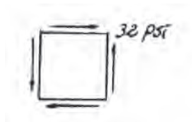
$\sigma_x = 0 \qquad \sigma_y = 0 \qquad \tau_{xy} = 32\text{ psi}$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= 0 \pm \sqrt{0 + 32^2}$$

$$\sigma_1 = 32\text{ psi} \qquad \textbf{Ans.}$$

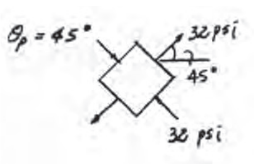
$$\sigma_2 = -32\text{ psi} \qquad \textbf{Ans.}$$



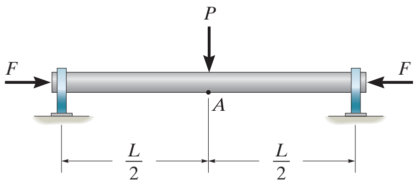
Note:

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{32}{0} = \infty$$

$$\theta_p = 45^\circ$$

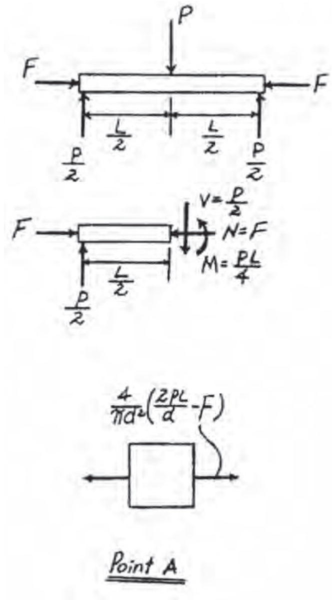


•9–37. The shaft has a diameter d and is subjected to the loadings shown. Determine the principal stress and the maximum in-plane shear stress that is developed at point A. The bearings only support vertical reactions.



Support Reactions: As shown on FBD(a).

Internal Forces and Moment: As shown on FBD(b).



Section Properties:

$$A = \frac{\pi}{4}d^2 \qquad I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{64}d^4 \qquad Q_A = 0$$

Normal Stress:

$$\sigma = \frac{N}{A} \pm \frac{Mc}{I}$$

$$= \frac{-F}{\frac{\pi}{4}d^2} \pm \frac{\frac{pL}{4}\left(\frac{d}{2}\right)}{\frac{\pi}{64}d^4}$$

$$\sigma_A = \frac{4}{\pi d^2}\left(\frac{2PL}{d} - F\right)$$

Shear Stress: Since $Q_A = 0, \tau_A = 0$

In - Plane Principal Stress: $\sigma_x = \dfrac{4}{\pi d^2}\left(\dfrac{2PL}{d} - F\right)$.

$\sigma_y = 0$ and $\tau_{xy} = 0$ for point A. Since no shear stress acts on the element,

$$\sigma_1 = \sigma_x = \frac{4}{\pi d^2}\left(\frac{2PL}{d} - F\right) \qquad \textbf{Ans.}$$

$$\sigma_2 = \sigma_y = 0 \qquad \textbf{Ans.}$$

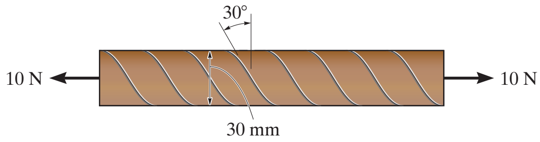
Maximum In - Plane Shear Stress: Applying Eq. 9-7 for point A,

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{\frac{4}{\pi d^2}\left(\frac{2PL}{d} - F\right) - 0}{2}\right)^2 + 0}$$

$$= \frac{2}{\pi d^2}\left(\frac{2PL}{d} - F\right) \qquad \textbf{Ans.}$$

9–38. A paper tube is formed by rolling a paper strip in a spiral and then gluing the edges together as shown. Determine the shear stress acting along the seam, which is at 30° from the vertical, when the tube is subjected to an axial force of 10 N. The paper is 1 mm thick and the tube has an outer diameter of 30 mm.

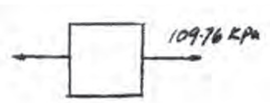


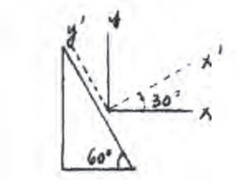
$$\sigma = \frac{P}{A} = \frac{10}{\frac{\pi}{4}(0.03^2 - 0.028^2)} = 109.76 \text{ kPa}$$

$\sigma_x = 109.76 \text{ kPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = 0 \qquad \theta = 30°$

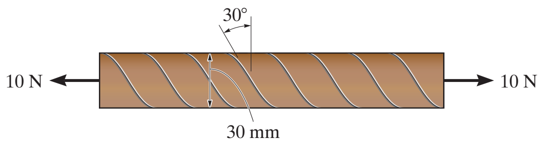
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{106.76 - 0}{2}\sin 60° + 0 = -47.5 \text{ kPa}$$ **Ans.**





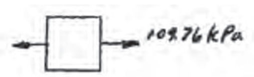
9–39. Solve Prob. 9–38 for the normal stress acting perpendicular to the seam.

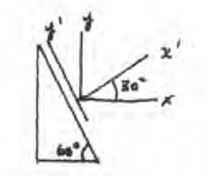


$$\sigma = \frac{P}{A} = \frac{10}{\frac{\pi}{4}(0.03^2 - 0.028^2)} = 109.76 \text{ kPa}$$

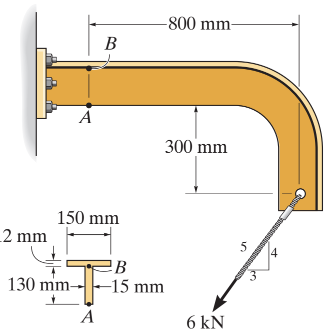
$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

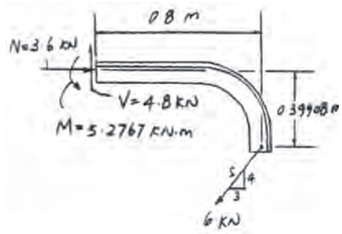
$$= \frac{109.76 + 0}{2} + \frac{109.76 - 0}{2}\cos(60°) + 0 = 82.3 \text{ kPa}$$ **Ans.**





***9–40.** Determine the principal stresses acting at point A of the supporting frame. Show the results on a properly oriented element located at this point.





$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.065(0.13)(0.015) + 0.136(0.15)(0.012)}{0.13(0.015) + 0.15(0.012)} = 0.0991 \text{ m}$$

$$I = \frac{1}{12}(0.015)(0.13^3) + 0.015(0.13)(0.0991 - 0.065)^2$$

$$+ \frac{1}{12}(0.15)(0.012^3) + 0.15(0.012)(0.136 - 0.0991)^2 = 7.4862(10^{-6}) \text{ m}^4$$

$$Q_A = 0$$

$$A = 0.13(0.015) + 0.15(0.012) = 3.75(10^{-3}) \text{ m}^2$$

Normal stress:

$$\sigma = \frac{P}{A} + \frac{M\,c}{I}$$

$$\sigma_A = \frac{-3.6(10^3)}{3.75(10^{-3})} - \frac{5.2767(10^3)(0.0991)}{7.4862(10^{-6})} = -70.80 \text{ MPa}$$

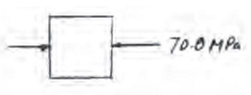
Shear stress:

$$\tau_A = 0$$

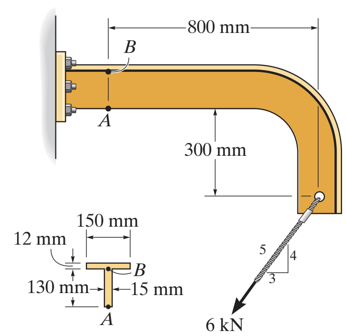
Principal stress:

$\sigma_1 = 0$ **Ans.**

$\sigma_2 = -70.8 \text{ MPa}$ **Ans.**



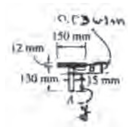
•9–41. Determine the principal stress acting at point B, which is located just on the web, below the horizontal segment on the cross section. Show the results on a properly oriented element located at this point. Although it is not very accurate, use the shear formula to calculate the shear stress.



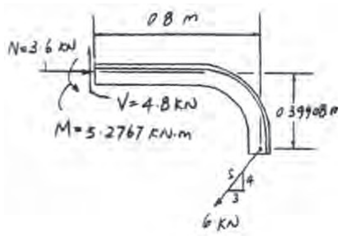
$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.065(0.13)(0.015) + 0.136(0.15)(0.012)}{0.13(0.015) + 0.15(0.012)} = 0.0991 \text{ m}$$

$$I = \frac{1}{12}(0.015)(0.13^3) + 0.015(0.13)(0.0991 - 0.065)^2$$

$$+ \frac{1}{12}(0.15)(0.012^3) + 0.15(0.012)(0.136 - 0.0991)^2 = 7.4862(10^{-6}) \text{ m}^4$$



$$A = 0.13(0.015) + 0.15(0.012) = 3.75(10^{-3}) \text{ m}^2$$



Normal stress:

$$\sigma = \frac{P}{A} + \frac{M\,c}{I}$$

$$\sigma_B = -\frac{3.6(10^3)}{3.75(10^{-3})} + \frac{5.2767(10^3)(0.130 - 0.0991)}{7.4862(10^{-6})} = 20.834 \text{ MPa}$$

Shear stress:

$$\tau_B = \frac{VQ}{I\,t} = \frac{-4.8(10^3)(0.0369)(0.15)(0.012)}{7.4862(10^{-6})(0.015)} = -2.84 \text{ MPa}$$

Principal stress:

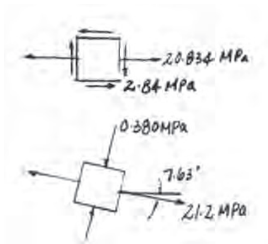
$$\sigma_{1,2} = \left(\frac{20.834 + 0}{2}\right) \pm \sqrt{\left(\frac{20.834 - 0}{2}\right)^2 + (-2.84)^2}$$

$\sigma_1 = 21.2$ MPa **Ans.**

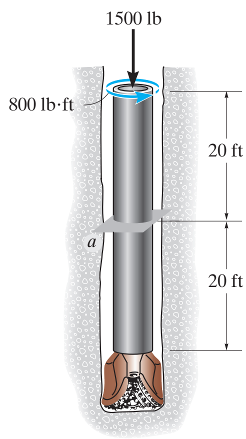
$\sigma_2 = -0.380$ MPa **Ans.**

$$\tan 2\theta_p = \frac{-2.84}{\left(\frac{20.834 - 0}{2}\right)}$$

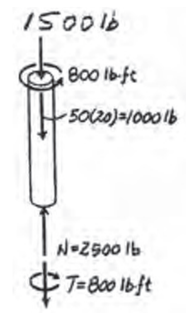
$\theta_p = -7.63°$ **Ans.**



9–42. The drill pipe has an outer diameter of 3 in., a wall thickness of 0.25 in., and a weight of 50 lb/ft. If it is subjected to a torque and axial load as shown, determine (a) the principal stress and (b) the maximum in-plane shear stress at a point on its surface at section *a*.



Internal Forces and Torque: As shown on FBD(a).



Section Properties:

$$A = \frac{\pi}{4}\left(3^2 - 2.5^2\right) = 0.6875\pi \text{ in}^2$$

$$J = \frac{\pi}{2}\left(1.5^4 - 1.25^4\right) = 4.1172 \text{ in}^4$$

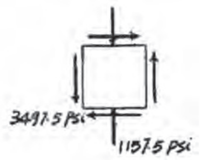
Normal Stress:

$$\sigma = \frac{N}{A} = \frac{-2500}{0.6875\pi} = -1157.5 \text{ psi}$$

Shear Stress: Applying the torsion formula.

$$\tau = \frac{T\,c}{J} = \frac{800(12)(1.5)}{4.1172} = 3497.5 \text{ psi}$$

a) ***In - Plane Principal Stresses:*** $\sigma_x = 0, \sigma_y = -1157.5$ psi and $\tau_{xy} = 3497.5$ psi for any point on the shaft's surface. Applying Eq. 9-5.



$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{0 + (-1157.5)}{2} \pm \sqrt{\left(\frac{0 - (-1157.5)}{2}\right)^2 + (3497.5)^2}$$

$$= -578.75 \pm 3545.08$$

$\sigma_1 = 2966 \text{ psi} = 2.97 \text{ ksi}$ **Ans.**

$\sigma_2 = -4124 \text{ psi} = -4.12 \text{ ksi}$ **Ans.**

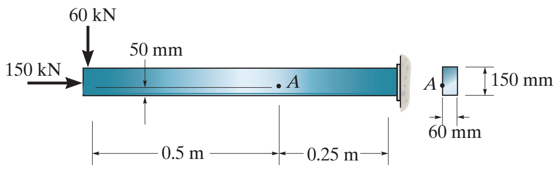
b) ***Maximum In - Plane Shear Stress:*** Applying Eq. 9-7

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{0 - (-1157.5)}{2}\right)^2 + (3497.5)^2}$$

$$= 3545 \text{ psi} = 3.55 \text{ ksi}$$ **Ans.**

9–43. Determine the principal stress in the beam at point A.



Using the method of sections and consider the FBD of the beam's left cut segment, Fig. a,

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 150 - N = 0 \quad N = 150 \text{ kN}$

$+\uparrow \Sigma F_y = 0; \quad V - 60 = 0 \quad V = 60 \text{ kN}$

$\circlearrowleft + \Sigma M_C = 0; \quad 60(0.5) - M = 0 \quad M = 30 \text{ kN} \cdot \text{m}$

$$A = 0.06(0.15) = 0.009 \text{ m}^2$$

$$I = \frac{1}{12}(0.06)(0.15^3) = 16.875(10^{-6}) \text{ m}^4$$

Referring to Fig. b,

$$Q_A = \bar{y}'A' = 0.05\,(0.05)(0.06) = 0.15(10^{-3}) \text{ m}^3$$

The normal stress developed is the combination of axial and bending stress. Thus

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

For point A, $y = 0.075 - 0.05 = 0.025$ m. Then

$$\sigma = \frac{-150(10^3)}{0.009} - \frac{30(10^3)(0.025)}{16.875(10^{-6})}$$

$$= -61.11(10^6) \text{ Pa} = 61.11 \text{ MPa (c)}$$

The shear stress developed is due to the transverse shear, Thus,

$$\tau = \frac{VQ_A}{It} = \frac{60(10^3)\left[0.15(10^{-3})\right]}{16.875(10^{-6})\,(0.06)} = 8.889 \text{ MPa}$$

Here, $\sigma_x = -61.11$ MPa, $\sigma_y = 0$ and $\tau_{xy} = 8.889$ MPa,

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-61.11 + 0}{2} \pm \sqrt{\left(\frac{-61.11 - 0}{2}\right)^2 + 8.889^2}$$

$$= -30.56 \pm 31.82$$

$\sigma_1 = 1.27$ MPa $\quad \sigma_2 = -62.4$ MPa **Ans.**

$$\tan 2\theta_P = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{8.889}{(-61.11 - 0)/2} = -0.2909$$

$$\theta_P = -8.11° \quad \text{and} \quad 81.89°$$

9–43. Continued

Substitute $\theta = -8.11°$,

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

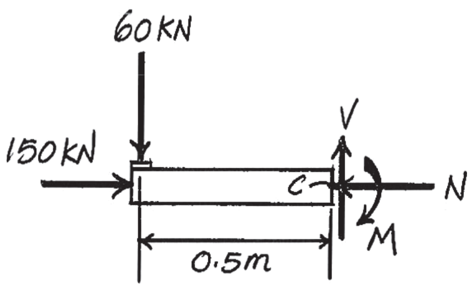
$$= \frac{-61.11 + 0}{2} + \frac{-61.11 - 0}{2}\cos(-16.22°) + 8.889\sin(-16.22°)$$

$$= -62.4 \text{ MPa} = \sigma_2$$

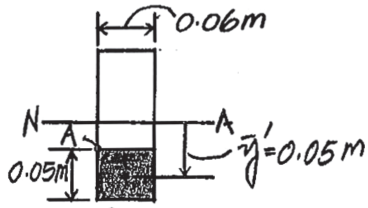
Thus,

$$(\theta_P)_1 = 81.9° \qquad (\theta_P)_2 = -8.11°$$

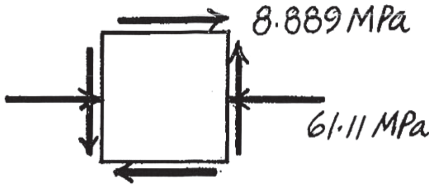
The state of principal stresses can be represented by the elements shown in Fig. (*c*)



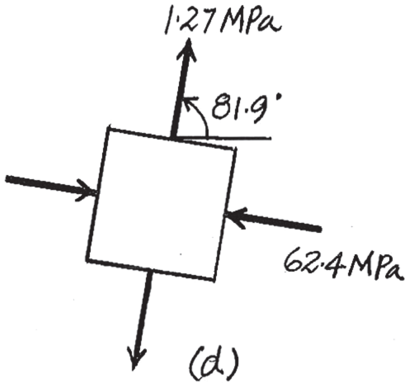
(a)



(b)

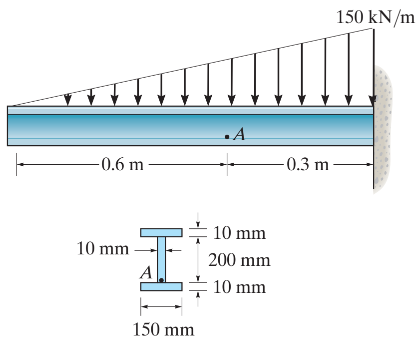


(c)



(d)

***9–44.** Determine the principal stress at point A which is located at the bottom of the web. Show the results on an element located at this point.



Using the method of sections, consider the FBD of the bean's left cut segment, Fig. *a*,

$+\uparrow \Sigma F_y = 0; \qquad V - \frac{1}{2}(100)(0.6) = 0 \qquad V = 30 \text{ kN}$

$\zeta + \Sigma M_C = 0; \qquad \frac{1}{2}(100)(0.6)(0.2) - M = 0 \qquad M = 6 \text{ kN} \cdot \text{m}$

$$I = \frac{1}{12}(0.15)(0.22^3) - \frac{1}{12}(0.14)(0.2^3) = 39.7667(10^{-6}) \text{ m}^4$$

Referring to Fig. *b*

$$Q_A = \bar{y}'A' = 0.105\,(0.01)(0.15) = 0.1575(10^{-3}) \text{ m}^3$$

The normal stress developed is due to bending only. For point A, $y = 0.1$ m. Then

$$\sigma = \frac{M_y}{I} = \frac{6(10^3)(0.1)}{39.7667(10^{-6})} = 15.09(10^6)\text{Pa} = 15.09 \text{ MPa (c)}$$

The shear stress developed is due to the transverse shear. Thus,

$$\tau = \frac{VQ_A}{It} = \frac{30(10^3)\left[0.1575(10^{-3})\right]}{39.7667(10^{-6})(0.01)} = 11.88(10^6)\text{Pa} = 11.88 \text{ MPa}$$

Here, $\sigma_x = -15.09$ MPa, $\sigma_y = 0$ And $\tau_{xy} = 11.88$ MPa.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-15.09 + 0}{2} \pm \sqrt{\left(\frac{-15.09 - 0}{2}\right)^2 + 11.88^2}$$

$$= -7.544 \pm 14.074$$

$\sigma_1 = 6.53$ MPa $\qquad \sigma_2 = -21.6$ MPa **Ans.**

$$\tan 2\theta_P = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{11.88}{(-15.09 - 0)/2} = -1.575$$

$$\theta_P = -28.79° \quad \text{and} \quad 61.21°$$

Substitute $\theta = 61.21°$,

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{-15.09 + 0}{2} + \frac{-15.09 - 0}{2}\cos 122.42° + 11.88 \sin 122.42°$$

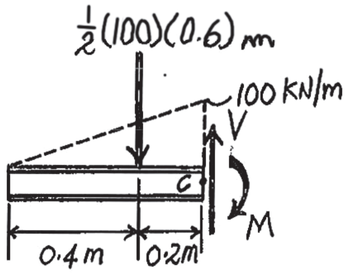
$$= 6.53 \text{ MPa} = \sigma_1$$

Thus,

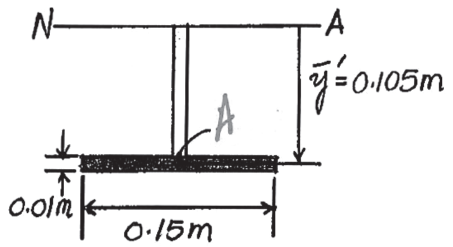
$(\theta_P)_1 = 61.2° \qquad (\theta_P)_2 = -28.8°$ **Ans.**

The state of principal stresses can be represented by the element shown in Fig. *d*.

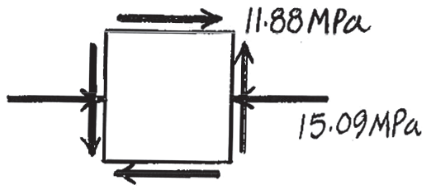
9–44. Continued



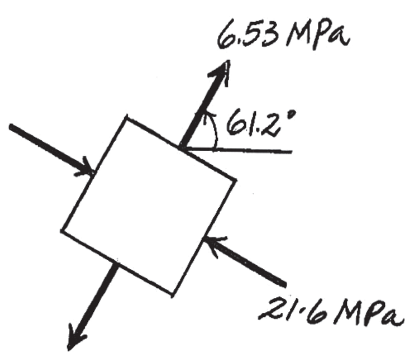
(a)



(b)

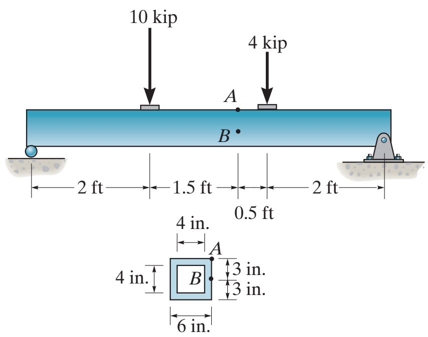


(c)



(d)

•9–45. Determine the maximum in-plane shear stress in the box beam at point A. Show the results on an element located at this point.



Using the method of section, consider the FBD, of bean's left cut segment, Fig. *a*,

$+\uparrow \Sigma F_y = 0; \qquad 8 - 10 + V = 0 \qquad V = 2 \text{ kip}$

$\circlearrowleft + \Sigma M_C = 0; \qquad M + 10(1.5) - 8(3.5) = 0 \qquad M = 13 \text{ kip} \cdot \text{ft}$

The moment of inertia of the cross - section about the neutral axis is

$$I = \frac{1}{12}(6)(6^3) - \frac{1}{12}(4)(4^3) = 86.6667 \text{ in}^4$$

Referring to Fig. *b*,

$$Q_A = 0$$

The normal stress developed is contributed by the bending stress only. For point A, $y = C = 3$ in.

$$\sigma = \frac{M_y}{I} = \frac{13(12)(3)}{86.6667} = 5.40 \text{ ksi (c)}$$

The shear stress is contributed by the transverse shear stress only. Thus

$$\tau = \frac{VQ_A}{It} = 0$$

The state of stress at point A can be represented by the element shown in Fig. *c*

Here, $\sigma_x = -5.40$ ksi, $\sigma_y = 0$ and $\tau_{xy} = 0$.

$$\tau_{\substack{\text{max} \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2} = \sqrt{\left(\frac{-5.40 - 0}{2}\right)^2 + 0^2} = 2.70 \text{ ksi} \qquad \textbf{Ans.}$$

$$\tan 2\theta_s = -\frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}} = -\frac{(-5.40 - 0)/2}{2} = \infty$$

$$\theta_s = 45° \qquad \text{and} \qquad -45°$$

Substitute $\theta = 45°$,

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{-5.40 - 0}{2}\sin 90° + 0$$

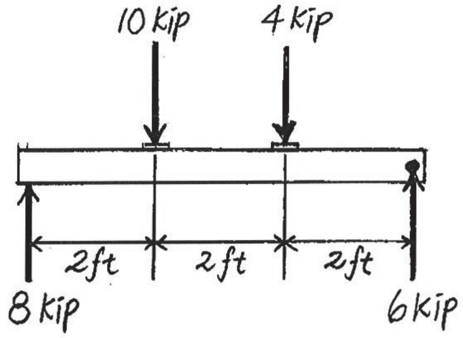
$$= 2.70 \text{ ksi} = \tau_{\substack{\text{max} \\ \text{in-plane}}}$$

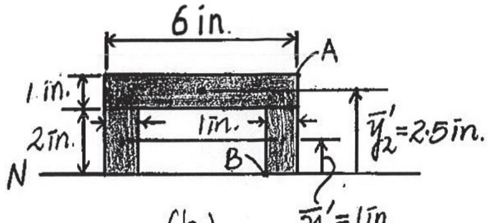
This indicates that $\tau_{\substack{\text{max} \\ \text{in-plane}}}$ acts toward the positive sense of y' axis at the face of element defined by $\theta_s = 45°$

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{-5.40 + 0}{2} = -2.70 \text{ ksi}$$

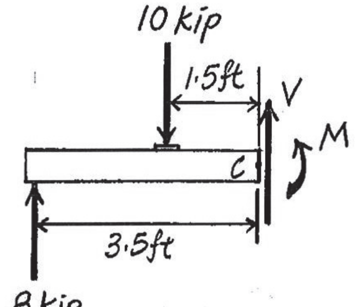
The state of maximum In - plane shear stress can be represented by the element shown in Fig. *d*.

9–45. Continued

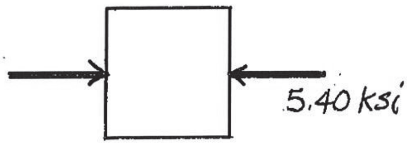




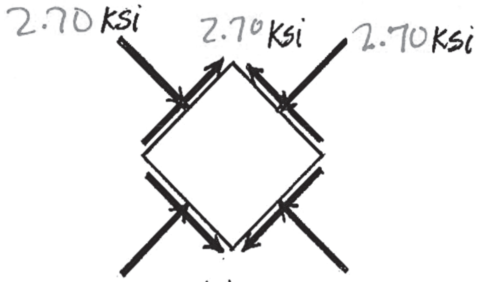
(b)



(a)

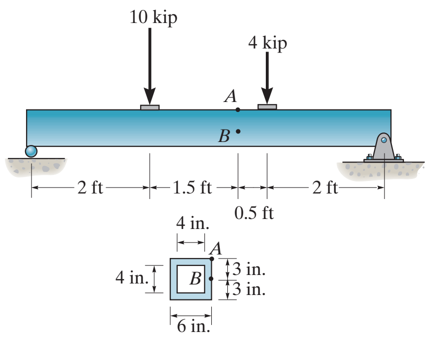


(c)



(d)

9–46. Determine the principal stress in the box beam at point B. Show the results on an element located at this point.



Using the method of sections, consider the FBD of bean's left cut segment, Fig. *a*,

$+\uparrow\Sigma F_y = 0; \qquad 8 - 10 + V = 0 \qquad V = 2\text{ kip}$

$\zeta + \Sigma M_C = 0; \qquad M + 10(1.5) - 8(3.5) = 0 \qquad M = 13\text{ kip}\cdot\text{ft}$

$$I = \frac{1}{12}(6)(6^3) - \frac{1}{12}(4)(4^3) = 86.6667\text{ in}^4$$

Referring to Fig. *b*,

$$Q_B = 2\bar{y}_1'A_1' + \bar{y}_2'A_2' = 2\left[1(2)(1)\right] + 2.5(1)(6) = 19\text{ in}^3$$

The normal stress developed is contributed by the bending stress only. For point B, $y = 0$.

$$\sigma = \frac{M_y}{I} = 0$$

The shear stress is contributed by the transverse shear stress only. Thus

$$\tau = \frac{VQ_B}{It} = \frac{2(10^3)(19)}{86.6667(2)} = 219.23\text{ psi}$$

The state of stress at point B can be represented by the element shown in Fig. *c*

Here, $\sigma_x = \sigma_y = 0$ and $\tau_{xy} = 219.23$ psi.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= 0 \pm \sqrt{0 + 219.23^2}$$

$$\sigma_1 = 219\text{ psi} \qquad \sigma_2 = -219\text{ psi} \qquad \textbf{Ans.}$$

$$\tan 2\theta_P = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2} = \frac{219.23}{0} = \infty$$

$$\theta_P = 45^\circ \qquad \text{and} \qquad -45^\circ$$

Substitute $\theta = 45^\circ$,

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= 0 + 0 + 219.23\sin 90^\circ$$

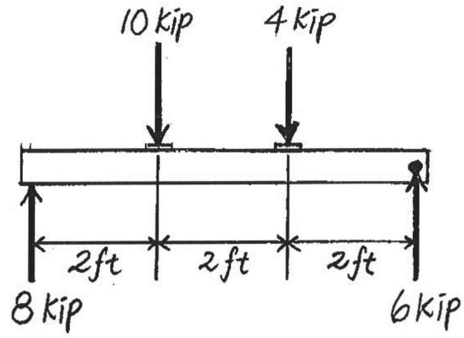
$$= 219\text{ psi} = \sigma_1$$

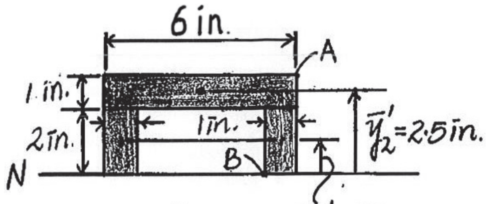
Thus,

$$(\theta_P)_1 = 45^\circ \qquad (\theta_P)_2 = -45^\circ \qquad \textbf{Ans.}$$

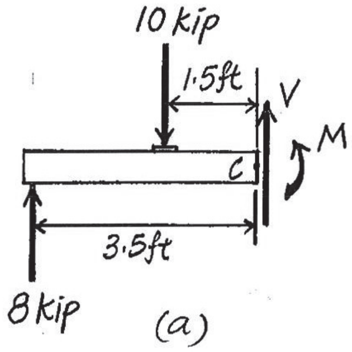
The state of principal stress can be represented by the element shown in Fig. *d*.

9–46. Continued

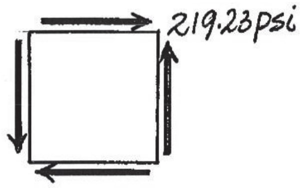




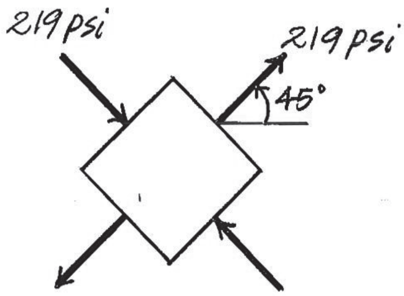
(b)



(a)

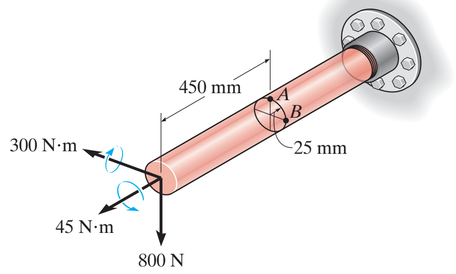


(c)



(d)

9–47. The solid shaft is subjected to a torque, bending moment, and shear force as shown. Determine the principal stresses acting at point A.



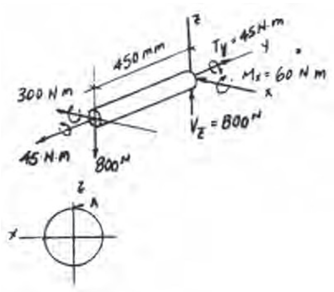
$$I_x = I_y = \frac{\pi}{4}(0.025)^4 = 0.306796(10^{-6})\ \text{m}^4$$

$$J = \frac{\pi}{2}(0.025)^4 = 0.613592(10^{-6})\ \text{m}^4$$

$$Q_A = 0$$

$$\sigma_A = \frac{M_x c}{I} = \frac{60(0.025)}{0.306796(10^{-6})} = 4.889\ \text{MPa}$$

$$\tau_A = \frac{T_y c}{J} = \frac{45(0.025)}{0.613592(10^{-6})} = 1.833\ \text{MPa}$$



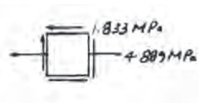
$$\sigma_x = 4.889\ \text{MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = -1.833\ \text{MPa}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

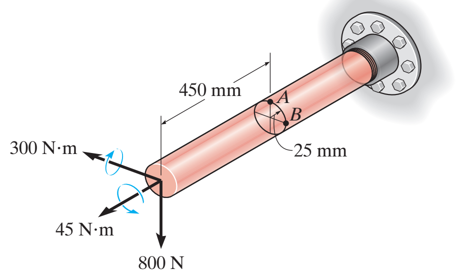
$$= \frac{4.889 + 0}{2} \pm \sqrt{\left(\frac{4.889 - 0}{2}\right)^2 + (-1.833)^2}$$

$$\sigma_1 = 5.50\ \text{MPa} \qquad \textbf{Ans.}$$

$$\sigma_2 = -0.611\ \text{MPa} \qquad \textbf{Ans.}$$



***9–48.** Solve Prob. 9–47 for point B.



$$I_x = I_y = \frac{\pi}{4}(0.025)^4 = 0.306796(10^{-6})\ \text{m}^4$$

$$J = \frac{\pi}{2}(0.025)^4 = 0.613592(10^{-6})\ \text{m}^4$$

$$Q_B = \bar{y}A' = \frac{4(0.025)}{3\pi}\left(\frac{1}{2}\right)\pi\,(0.025^2) = 10.4167(10^{-6})\ \text{m}^3$$

$$\sigma_B = 0$$

$$\tau_B = \frac{V_z Q_B}{It} - \frac{T_y c}{J} = \frac{800(10.4167)(10^{-6})}{0.306796(10^{-6})(0.05)} - \frac{45(0.025)}{0.61359(10^{-6})} = -1.290\ \text{MPa}$$

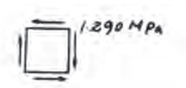
$$\sigma_x = 0 \qquad \sigma_y = 0 \qquad \tau_{xy} = -1.290\ \text{MPa}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

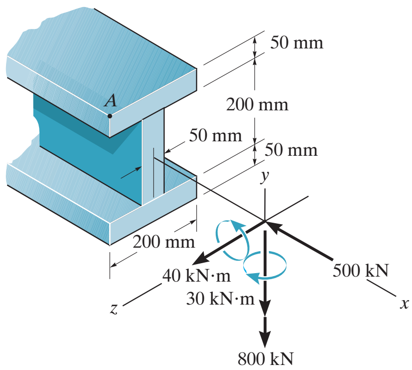
$$= 0 \pm \sqrt{(0)^2 + (-1.290)^2}$$

$$\sigma_1 = 1.29\ \text{MPa} \qquad \textbf{Ans.}$$

$$\sigma_2 = -1.29\ \text{MPa} \qquad \textbf{Ans.}$$



•**9–49.** The internal loadings at a section of the beam are shown. Determine the principal stress at point A. Also compute the maximum in-plane shear stress at this point.



Section Properties:

$$A = 0.2(0.3) - 0.15(0.2) = 0.030 \text{ m}^4$$

$$I_z = \frac{1}{12}(0.2)\left(0.3^3\right) - \frac{1}{12}(0.15)\left(0.2^3\right) = 0.350\left(10^{-3}\right) \text{ m}^4$$

$$I_y = \frac{1}{12}(0.1)\left(0.2^3\right) + \frac{1}{12}(0.2)\left(0.05^3\right) = 68.75\left(10^{-6}\right) \text{ m}^4$$

$$(Q_A)_y = 0$$

Normal Stress:

$$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_A = \frac{-500(10^3)}{0.030} - \frac{40(10^3)(0.15)}{0.350(10^{-3})} + \frac{-30(10^3)(0.1)}{68.75(10^{-6})}$$

$$= -77.45 \text{ MPa}$$

Shear Stress: Since $(Q_A)_y = 0$, $\tau_A = 0$.

In - Plane Principal Stresses: $\sigma_x = -77.45$ MPa. $\sigma_y = 0$. and $\tau_{xy} = 0$ for point A. Since no shear stress acts on the element.

$$\sigma_1 = \sigma_y = 0 \qquad \textbf{Ans.}$$

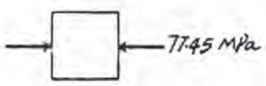
$$\sigma_2 = \sigma_z = -77.4 \text{ MPa} \qquad \textbf{Ans.}$$

Maximum In-Plane Shear Stress: Applying Eq. 9–7.

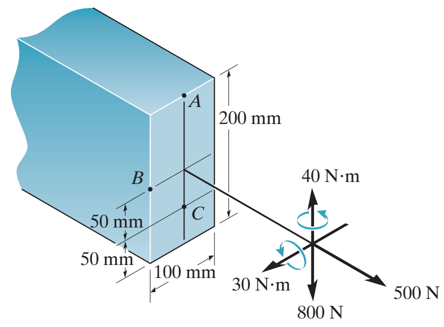
$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{-77.45 - 0}{2}\right)^2 + 0}$$

$$= 38.7 \text{ MPa} \qquad \textbf{Ans.}$$



9–50. The internal loadings at a section of the beam consist of an axial force of 500 N, a shear force of 800 N, and two moment components of 30 N·m and 40 N·m. Determine the principal stress at point A. Also calculate the maximum in-plane shear stress at this point.

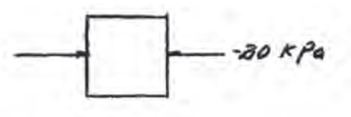


$$I_x = \frac{1}{12}(0.1)(0.2)^3 = 66.67(10^{-6})\ \text{in}^4$$

$$Q_A = 0$$

$$\sigma_A = \frac{P}{A} - \frac{Mz}{I_x} = \frac{500}{(0.1)(0.2)} - \frac{30(0.1)}{66.67(10^{-6})} = -20\ \text{kPa}$$

$$\tau_A = 0$$



Here, the principal stresses are

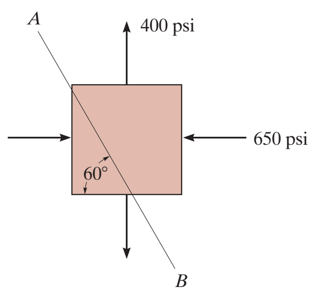
$$\sigma_1 = \sigma_y = 0$$ **Ans.**

$$\sigma_2 = \sigma_x = -20\ \text{kPa}$$ **Ans.**

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \sqrt{\left(\frac{-20 - 0}{2}\right)^2 + 0} = 10\ \text{kPa}$$ **Ans.**

9–51. Solve Prob. 9–4 using Mohr's circle.



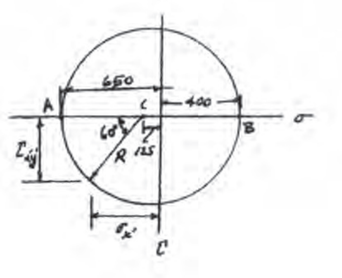
$$\frac{\sigma_x + \sigma_y}{2} = \frac{-650 + 400}{2} = -125$$

$A(-650, 0)$ $\quad$ $B(400, 0)$ $\quad$ $C(-125, 0)$

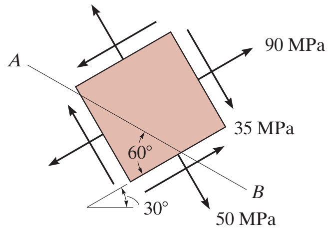
$$R = CA = \ = 650 - 125 = 525$$

$$\sigma_{x'} = -125 - 525 \cos 60° = -388\ \text{psi}$$ **Ans.**

$$\tau_{x'y'} = 525 \sin 60° = 455\ \text{psi}$$ **Ans.**



***9–52.** Solve Prob. 9–6 using Mohr's circle.



$\sigma_x = 90 \text{ MPa} \qquad \sigma_y = 50 \text{ MPa} \qquad \tau_{xy} = -35 \text{ MPa} \qquad A(90, -35)$

$$\frac{\sigma_x + \sigma_y}{2} = \frac{90 + 50}{2} = 70$$

$$R = \sqrt{(90 - 70)^2 + (35)^2} = 40.311$$

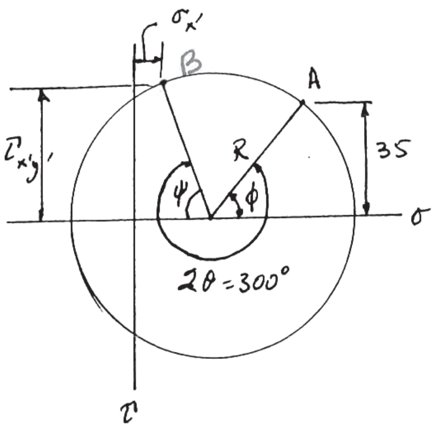
Coordinates of point B:

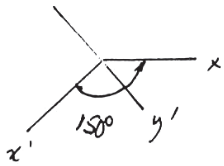
$$\phi = \tan^{-1}\left(\frac{35}{20}\right) = 60.255°$$

$$\psi = 300° - 180° - 60.255° = 59.745°$$

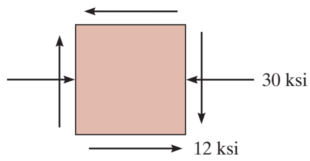
$\sigma_{x'} = 70 - 40.311 \cos 59.745° = 49.7 \text{ MPa}$ **Ans.**

$\tau_{x'} = -40.311 \sin 59.745° = -34.8 \text{ MPa}$ **Ans.**





•9–53. Solve Prob. 9–14 using Mohr's circle.



$$\frac{\sigma_x + \sigma_y}{2} = \frac{-30 + 0}{2} = -15$$

$$R = \sqrt{(30 - 15)^2 + (12)^2} = 19.21 \text{ ksi}$$

$\sigma_1 = 19.21 - 15 = 4.21 \text{ ksi}$ **Ans.**

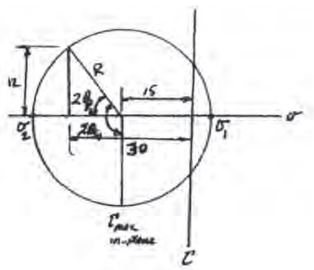
$\sigma_2 = -19.21 - 15 = -34.2 \text{ ksi}$ **Ans.**

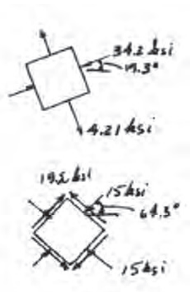
$2\theta_{P2} = \tan^{-1}\dfrac{12}{(30 - 15)}; \qquad \theta_{P2} = 19.3°$ **Ans.**

$\tau_{\substack{\text{max}\\\text{in-plane}}} = R = 19.2 \text{ ksi}$ **Ans.**

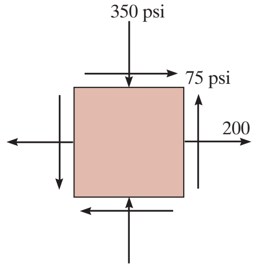
$\sigma_{\text{avg}} = -15 \text{ ksi}$ **Ans.**

$2\theta_{P2} = \tan^{-1}\dfrac{12}{(30 - 15)} + 90°; \qquad \theta_s = 64.3°$ **Ans.**





9–54. Solve Prob. 9–16 using Mohr's circle.



$$\frac{\sigma_x + \sigma_y}{2} = \frac{45 - 60}{2} = -7.5 \text{ MPa}$$

$$R = \sqrt{(45 + 7.5)^2 + (30)^2} = 60.467 \text{ MPa}$$

$\sigma_1 = 60.467 - 7.5 = 53.0 \text{ MPa}$ **Ans.**

$\sigma_2 = -60.467 - 7.5 = -68.0 \text{ MPa}$ **Ans.**

$$2\theta_{P1} = \tan^{-1}\frac{30}{(45 + 7.5)}$$

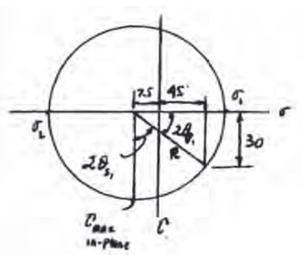
$\theta_{P1} = 14.9°$ counterclockwise **Ans.**

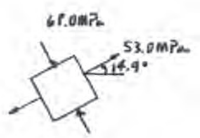
$\tau_{\substack{\max \\ \text{in-plane}}} = 60.5 \text{ MPa}$ **Ans.**

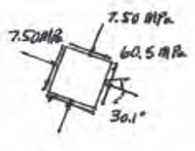
$\sigma_{avg} = -7.50 \text{ MPa}$ **Ans.**

$$2\theta_{P1} = 90° - \tan^{-1}\frac{30}{(45 + 7.5)}$$

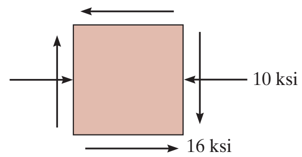
$\theta_{s1} = 30.1°$ clockwise **Ans.**







9–55. Solve Prob. 9–12 using Mohr's circle.



$$\frac{\sigma_x + \sigma_y}{2} = \frac{-10 + 0}{2} = -5 \text{ ksi}$$

$$R = \sqrt{(10 - 5)^2 + (16)^2} = 16.763 \text{ ksi}$$

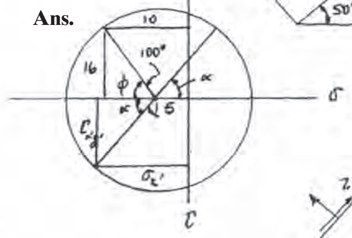
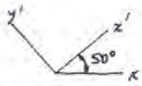
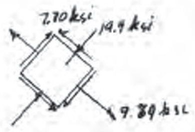
$$\phi = \tan^{-1}\frac{16}{(10 - 5)} = 72.646°$$

$$\alpha = 100 - 72.646 = 27.354°$$

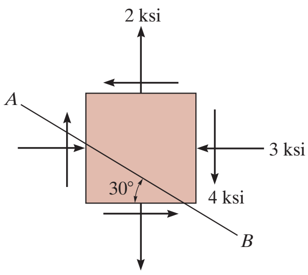
$$\sigma_{x'} = -5 - 16.763 \cos 27.354° = -19.9 \text{ ksi}$$ **Ans.**

$$\tau_{x'y'} = 16.763 \sin 27.354° = 7.70 \text{ ksi}$$ **Ans.**

$$\sigma_{y'} = 16.763 \cos 27.354° - 5 = 9.89 \text{ ksi}$$

***9–56.** Solve Prob. 9–11 using Mohr's circle.



Construction of the Circle: In accordance with the sign convention, $\sigma_x = -3$ ksi, $\sigma_y = 2$ ksi, and $\tau_{xy} = -4$ ksi. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-3 + 2}{2} = -0.500 \text{ ksi}$$

The coordinates for reference point A and C are

$$A(-3, -4) \qquad C(-0.500, 0)$$

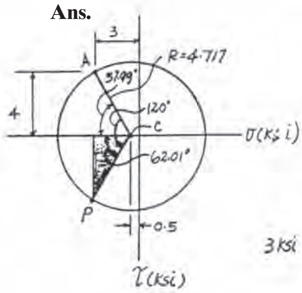
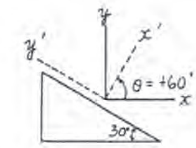
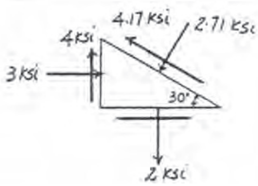
The radius of circle is

$$R = \sqrt{(3 - 0.5)^2 + 4^2} = 4.717 \text{ ksi}$$

Stress on the Inclined Plane: The normal and shear stress components $(\sigma_{x'}$ and $\tau_{x'y'})$ are represented by the coordinates of point P on the circle.

$$\sigma_{x'} = -0.500 - 4.717 \cos 62.01° = -2.71 \text{ ksi}$$ **Ans.**

$$\tau_{x'y'} = 4.717 \sin 62.01° = 4.17 \text{ ksi}$$ **Ans.**

9–57. Mohr's circle for the state of stress in Fig. 9–15*a* is shown in Fig. 9–15*b*. Show that finding the coordinates of point $P(\sigma_{x'}, \tau_{x'y'})$ on the circle gives the same value as the stress-transformation Eqs. 9–1 and 9–2.

$$A(\sigma_x, \tau_{xy}) \qquad B(\sigma_y, -\tau_{xy}) \qquad C\left(\left(\frac{\sigma_x + \sigma_y}{2}\right), 0\right)$$

$$R = \sqrt{\left[\sigma_x - \left(\frac{\sigma_x + \sigma_y}{2}\right)\right]^2 + \tau_{xy}^2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma'_x = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}\cos\theta' \qquad \textbf{(1)}$$

$$\theta' = 2\theta_P - 2\theta$$

$$\cos(2\theta_P - 2\theta) = \cos 2\theta_P \cos 2\theta + \sin 2\theta_p \sin 2\theta \qquad \textbf{(2)}$$

From the circle:

$$\cos 2\theta_P = \frac{\sigma_x - \frac{\sigma_x + \sigma_y}{2}}{\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}} \qquad \textbf{(3)}$$

$$\sin 2\theta_P = \frac{\tau_{xy}}{\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}} \qquad \textbf{(4)}$$

Substitute Eq. (2), (3) and into Eq. (1)

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \qquad \textbf{QED}$$

$$\tau_{x'y'} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}\sin\theta' \qquad \textbf{(5)}$$

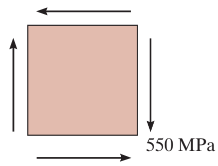
$$\sin\theta' = \sin(2\theta_P - 2\theta)$$

$$= \sin 2\theta_P \cos 2\theta - \sin 2\theta \cos 2\theta_P \qquad \textbf{(6)}$$

Substitute Eq. (3), (4), (6) into Eq. (5),

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \qquad \textbf{QED}$$

9–58. Determine the equivalent state of stress if an element is oriented 25° counterclockwise from the element shown.



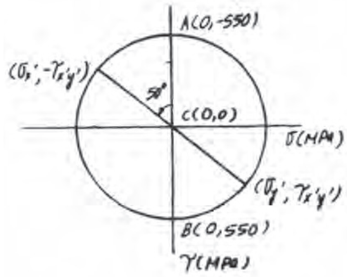
$A(0, -550) \qquad B(0, 550) \qquad C(0, 0)$

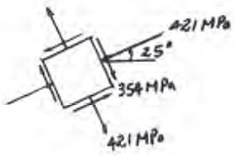
$R = CA = CB = 550$

$\sigma_{x'} = -550 \sin 50° = -421 \text{ MPa}$ **Ans.**

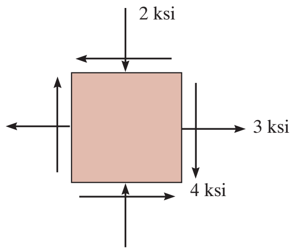
$\tau_{x'y'} = -550 \cos 50° = -354 \text{ MPa}$ **Ans.**

$\sigma_{y'} = 550 \sin 50° = 421 \text{ MPa}$ **Ans.**





9–59. Determine the equivalent state of stress if an element is oriented 20° clockwise from the element shown.



Construction of the Circle: In accordance with the sign convention, $\sigma_x = 3$ ksi, $\sigma_y = -2$ ksi, and $\tau_{x'y'} = -4$ ksi. Hence,

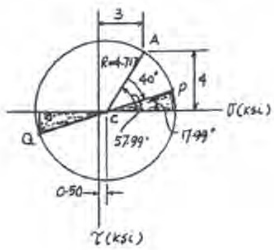
$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{3 + (-2)}{2} = 0.500 \text{ ksi}$$

The coordinates for reference points A and C are

$$A(3, -4) \qquad C(0.500, 0)$$

The radius of the circle is

$$R = \sqrt{(3 - 0.500)^2 + 4^2} = 4.717 \text{ ksi}$$

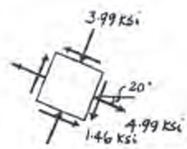


Stress on the Rotated Element: The normal and shear stress components $(\sigma_{x'}$ and $\tau_{x'y'})$ are represented by the coordinate of point P on the circle, $\sigma_{y'}$, can be determined by calculating the coordinates of point Q on the circle.

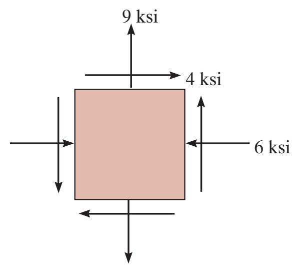
$\sigma_{x'} = 0.500 + 4.717 \cos 17.99° = 4.99 \text{ ksi}$ **Ans.**

$\tau_{x'y'} = -4.717 \sin 17.99° = -1.46 \text{ ksi}$ **Ans.**

$\sigma_{y'} = 0.500 - 4.717 \cos 17.99° = -3.99 \text{ ksi}$ **Ans.**



***9–60.** Determine the equivalent state of stress if an element is oriented 30° clockwise from the element shown. Show the result on the element.



In accordance to the established sign convention, $\sigma_x = -6$ ksi, $\sigma_y = 9$ ksi and $\tau_{xy} = 4$ ksi. Thus,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{-6 + 9}{2} = 1.50 \text{ ksi}$$

Then, the coordinates of reference point A and C are

$$A(-6, 4) \qquad C(1.5, 0)$$

The radius of the circle is

$$R = CA = \sqrt{(-6 - 1.5)^2 + 4^2} = 8.50 \text{ ksi}$$

Using these results, the circle shown in Fig. *a* can be constructed.

Referring to the geometry of the circle, Fig. *a*,

$$\alpha = \tan^{-1}\left(\frac{4}{6 + 1.5}\right) = 28.07^\circ \qquad \beta = 60^\circ - 28.07^\circ = 31.93^\circ$$

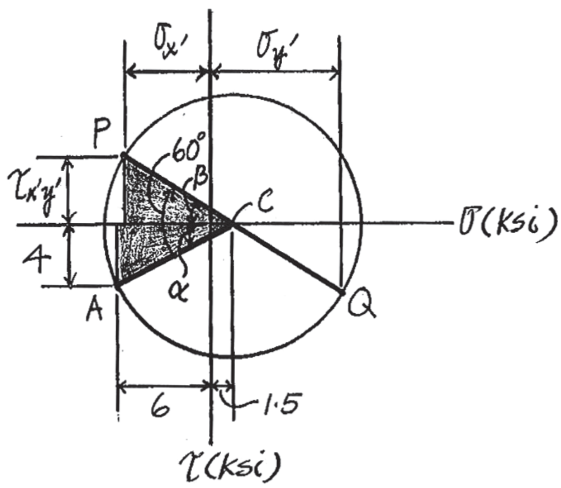
Then,

$$\sigma_{x'} = 1.5 - 8.50 \cos 31.93^\circ = -5.71 \text{ ksi} \qquad \textbf{Ans.}$$

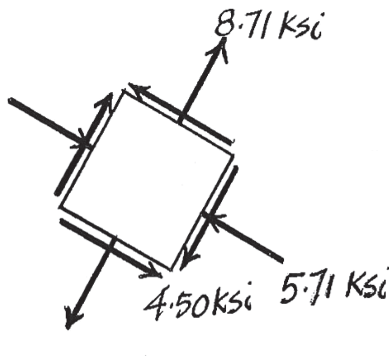
$$\tau_{x'y'} = -8.5 \sin 31.95^\circ = -4.50 \text{ ksi}$$

$$\sigma_{y'} = 8.71 \text{ ksi} \qquad \textbf{Ans.}$$

The results are shown in Fig. *b*.

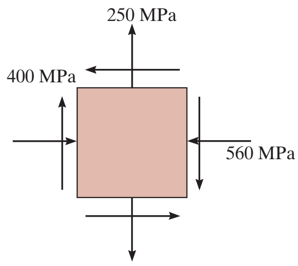


(a)



(b)

•9–61. Determine the equivalent state of stress for an element oriented 60° counterclockwise from the element shown. Show the result on the element.



In accordance to the established sign convention, $\sigma_x = -560$ MPa, $\sigma_y = 250$ MPa and $\tau_{xy} = -400$ MPa. Thus,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-560 + 250}{2} = -155 \text{ MPa}$$

Then, the coordinate of reference points A and C are

$$A(-560, -400) \qquad C(-155, 0)$$

The radius of the circle is

$$R = CA = \sqrt{\left[-560 - (-155)\right]^2 + (-400)^2} = 569.23 \text{ MPa}$$

Using these results, the circle shown in Fig. *a* can be constructed.

Referring to the geometry of the circle, Fig. *a*

$$\alpha = \tan^{-1}\left(\frac{400}{560 - 155}\right) = 44.64° \qquad \beta = 120° - 44.64° = 75.36°$$

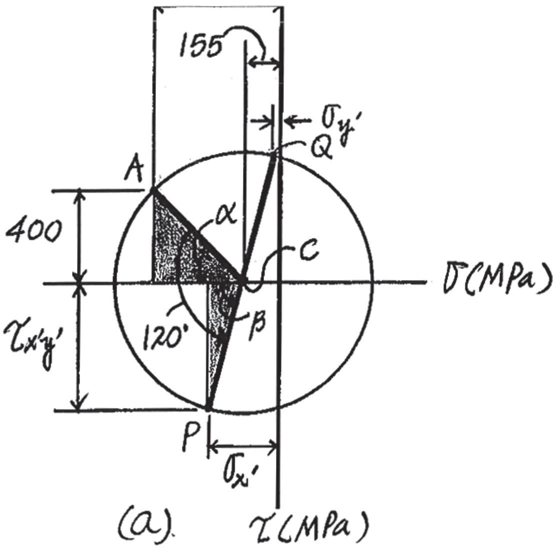
Then,

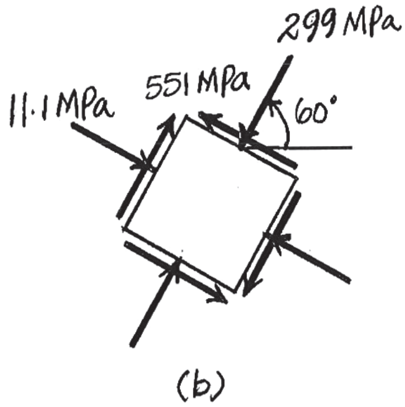
$$\sigma_{x'} = -155 - 569.23 \cos 75.36° = -299 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = 569.23 \sin 75.36° = 551 \text{ MPa} \qquad \textbf{Ans.}$$

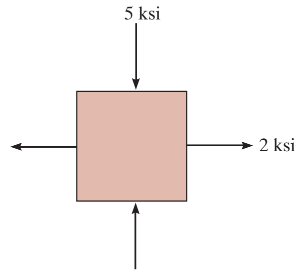
$$\sigma_{y'} = -155 + 569.23 \cos 75.36° = -11.1 \text{ MPa} \qquad \textbf{Ans.}$$

The results are shown in Fig. *b*.





9–62. Determine the equivalent state of stress for an element oriented 30° clockwise from the element shown. Show the result on the element.



In accordance to the established sign convention, $\sigma_x = 2$ ksi, $\sigma_y = -5$ ksi and $\tau_{xy} = 0$. Thus,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{2 + (-5)}{2} = -1.50 \text{ ksi}$$

Then, the coordinate of reference points A and C are

$$A(2, 0) \qquad C(-1.5, 0)$$

The radius of the circle is

$$R = CA = \sqrt{[2 - (-1.5)]^2 + 0^2} = 3.50 \text{ ksi}$$

Using these results, the circle shown in Fig. *a* can be constructed.

Referring to the geometry of the circle, Fig. *a*,

$\beta = 60°$

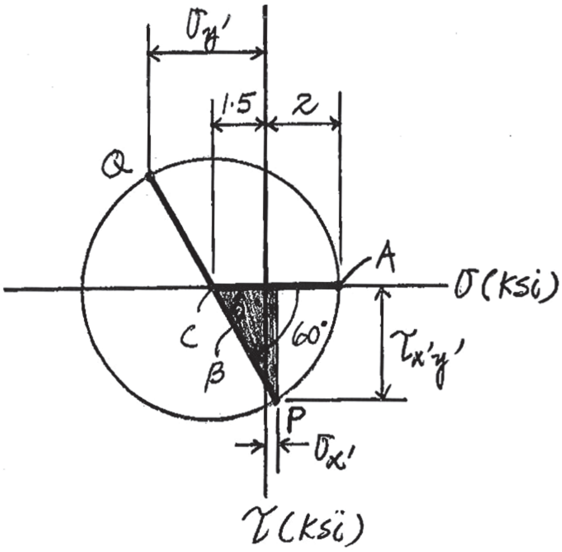
Then,

$$\sigma_{x'} = -1.50 + 3.50 \cos 60° - 0.250 \text{ ksi} \qquad \textbf{Ans.}$$

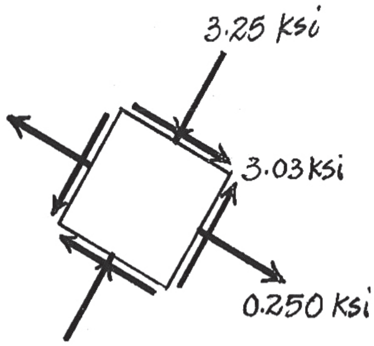
$$\tau_{x'y'} = 3.50 \sin 60° = 3.03 \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_{y'} = -3.25 \text{ ksi} \qquad \textbf{Ans.}$$

The results are shown in Fig *b*.

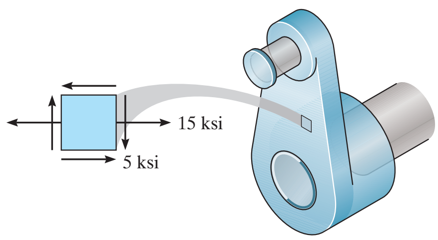


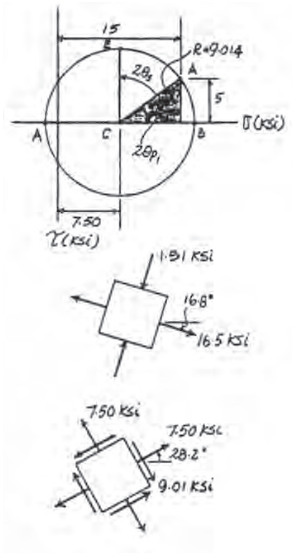
(a)



(b)

9–63. Determine the principal stress, the maximum in-plane shear stress, and average normal stress. Specify the orientation of the element in each case.





Construction of the Circle: In accordance with the sign convention, $\sigma_x = 15$ ksi, $\sigma_y = 0$ and $\tau_{xy} = -5$ ksi. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{15 + 0}{2} = 7.50 \text{ ksi} \qquad \textbf{Ans.}$$

The coordinates for reference point A and C are

$$A(15, -5) \qquad C(7.50, 0)$$

The radius of the circle is

$$R = \sqrt{(15 - 7.50)^2 + 5^2} = 9.014 \text{ ksi}$$

a)

In - Plane Principal Stress: The coordinates of points B and D represent σ_1 and σ_2, respectively.

$$\sigma_1 = 7.50 + 9.014 = 16.5 \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_2 = 7.50 - 9.014 = -1.51 \text{ ksi} \qquad \textbf{Ans.}$$

Orientation of Principal Plane: From the circle

$$\tan 2\theta_{P1} = \frac{5}{15 - 7.50} = 0.6667$$

$$\theta_{P1} = 16.8° \ (\textit{Clockwise}) \qquad \textbf{Ans.}$$

b)

Maximum In - Plane Shear Stress: Represented by the coordinates of point E on the circle.

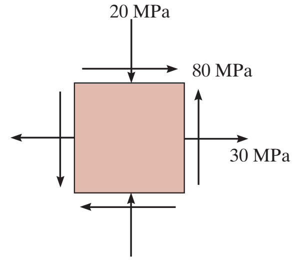
$$\tau_{\substack{max \\ in\text{-}plane}} = -R = -9.01 \text{ ksi} \qquad \textbf{Ans.}$$

Orientation of the Plane for Maximum In - Plane Shear Stress: From the circle

$$\tan 2\theta_s = \frac{15 - 7.50}{5} = 1.500$$

$$\theta_s = 28.2° \quad (\textit{Counterclockwise}) \qquad \textbf{Ans.}$$

***9–64.** Determine the principal stress, the maximum in-plane shear stress, and average normal stress. Specify the orientation of the element in each case.



In accordance to the established sign convention, $\sigma_x = 30$ MPa, $\sigma_y = -20$ MPa and $\tau_{xy} = 80$ MPa. Thus,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{30 + (-20)}{2} = 5 \text{ MPa}$$

Then, the coordinates of reference point A and the center C of the circle is

$$A(30, 80) \qquad C(5, 0)$$

Thus, the radius of circle is given by

$$R = CA = \sqrt{(30 - 5)^2 + (80 - 0)^2} = 83.815 \text{ MPa}$$

Using these results, the circle shown in Fig. a, can be constructed.

The coordinates of points B and D represent σ_1 and σ_2 respectively. Thus

$$\sigma_1 = 5 + 83.815 = 88.8 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_2 = 5 - 83.815 = -78.8 \text{ MPa} \qquad \textbf{Ans.}$$

Referring to the geometry of the circle, Fig. a

$$\tan 2(\theta_P)_1 = \frac{80}{30 - 5} = 3.20$$

$$\theta_P = 36.3° \text{ (}\textit{Counterclockwise}\text{)} \qquad \textbf{Ans.}$$

The state of maximum in - plane shear stress is represented by the coordinate of point E. Thus

$$\tau_{\substack{max \\ in\text{-}plane}} = R = 83.8 \text{ MPa} \qquad \textbf{Ans.}$$

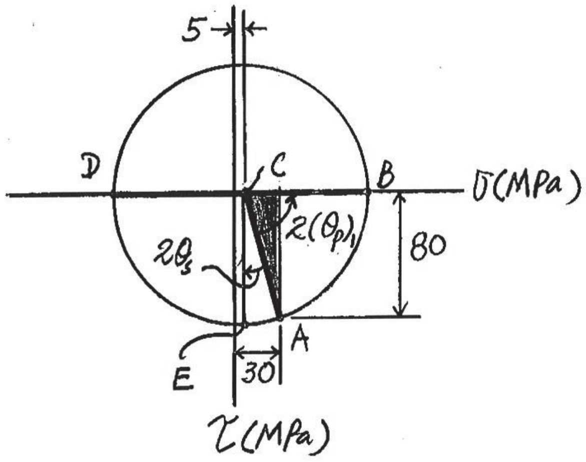
From the geometry of the circle, Fig. a,

$$\tan 2\theta_s = \frac{30 - 5}{80} = 0.3125$$

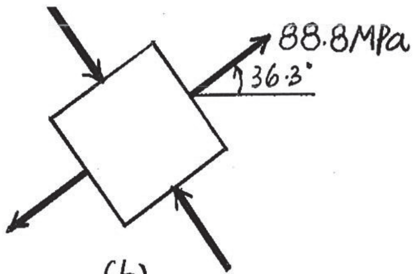
$$\theta_s = 8.68° \quad \text{(}\textit{Clockwise}\text{)} \qquad \textbf{Ans.}$$

The state of maximum in - plane shear stress is represented by the element in Fig. c

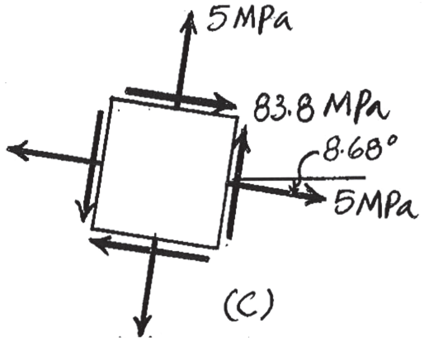
9–64. Continued



(a)

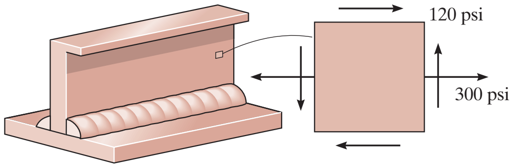


(b)



(c)

•9–65. Determine the principal stress, the maximum in-plane shear stress, and average normal stress. Specify the orientation of the element in each case.

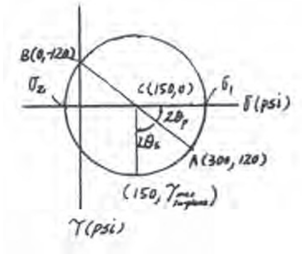


$A(300, 120) \qquad B(0, -120) \qquad C(150, 0)$

$R = \sqrt{(300 - 150)^2 + 120^2} = 192.094$

$\sigma_1 = 150 + 192.094 = 342 \text{ psi}$ **Ans.**

$\sigma_2 = 150 - 192.094 = -42.1 \text{ psi}$ **Ans.**



$$\tan 2\theta_P = \frac{120}{300 - 150} = 0.8$$

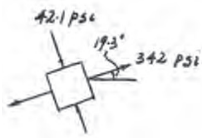
$\theta_{P_1} = 19.3° \quad$ *Counterclockwise* **Ans.**

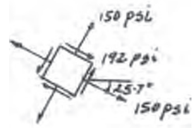
$\sigma_{avg} = 150 \text{ psi}$ **Ans.**

$\tau_{\substack{max \\ in\text{-}plane}} = 192 \text{ psi}$ **Ans.**

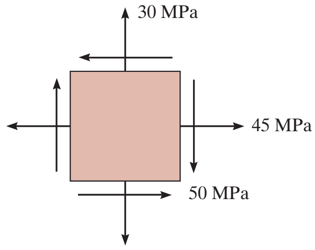
$$\tan 2\theta_s = \frac{300 - 150}{120} = 1.25$$

$\theta_s = -25.7°$ **Ans.**





9–66. Determine the principal stress, the maximum in-plane shear stress, and average normal stress. Specify the orientation of the element in each case.



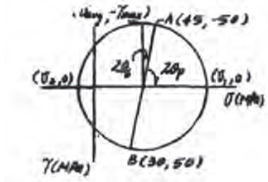
$A(45, -50) \qquad B(30, 50) \qquad C(37.5, 0)$

$R = CA = CB = \sqrt{7.5^2 + 50^2} = 50.56$

a)

$\sigma_1 = 37.5 + 50.56 = 88.1 \text{ MPa}$ **Ans.**

$\sigma_2 = 37.5 - 50.56 = -13.1 \text{ MPa}$ **Ans.**



$$\tan 2\theta_P = \frac{50}{7.5} \qquad 2\theta_P = 81.47° \qquad \theta_P = -40.7°$$

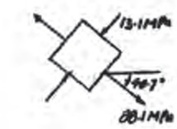
b)

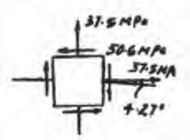
$\tau_{\substack{max \\ in\text{-}plane}} = R = 50.6 \text{ MPa}$ **Ans.**

$\sigma_{avg} = 37.5 \text{ MPa}$ **Ans.**

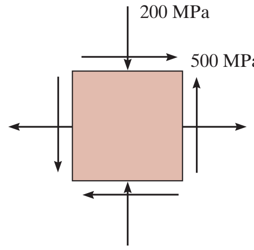
$2\theta_s = 90 - 2\theta_P$

$\theta_s = 4.27°$ **Ans.**



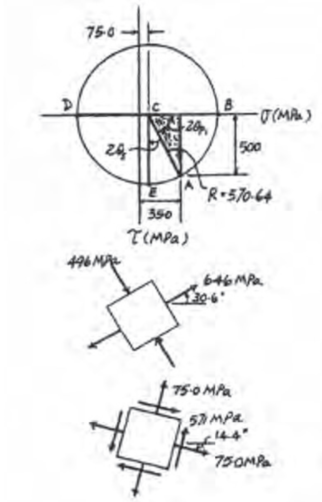


9–67. Determine the principal stress, the maximum in-plane shear stress, and average normal stress. Specify the orientation of the element in each case.



Construction of the Circle: In accordance with the sign convention, $\sigma_x = 350$ MPa, $\sigma_y = -200$ MPa, and $\tau_{xy} = 500$ MPa. Hence,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{350 + (-200)}{2} = 75.0 \text{ MPa} \qquad \textbf{Ans.}$$



The coordinates for reference point A and C are

$$A(350, 500) \qquad C(75.0, 0)$$

The radius of the circle is

$$R = \sqrt{(350 - 75.0)^2 + 500^2} = 570.64 \text{ MPa}$$

a)

In - Plane Principal Stresses: The coordinate of points B and D represent σ_1 and σ_2 respectively.

$$\sigma_1 = 75.0 + 570.64 = 646 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_2 = 75.0 - 570.64 = -496 \text{ MPa} \qquad \textbf{Ans.}$$

Orientaion of Principal Plane: From the circle

$$\tan 2\theta_{P1} = \frac{500}{350 - 75.0} = 1.82$$

$$\theta_{P1} = 30.6° \ (\textit{Counterclockwise}) \qquad \textbf{Ans.}$$

b)

Maximum In - Plane Shear Stress: Represented by the coordinates of point E on the circle.

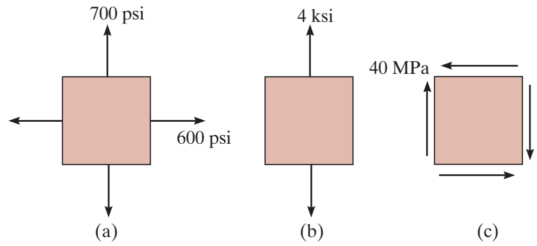
$$\tau_{\substack{\max \\ \text{in-plane}}} = R = 571 \text{ MPa} \qquad \textbf{Ans.}$$

Orientation of the Plane for Maximum In - Plane Shear Stress: From the circle

$$\tan 2\theta_s = \frac{350 - 75.0}{500} = 0.55$$

$$\theta_s = 14.4° \quad (\textit{Clockwise}) \qquad \textbf{Ans.}$$

***9–68.** Draw Mohr's circle that describes each of the following states of stress.



a) Here, $\sigma_x = 600$ psi, $\sigma_y = 700$ psi and $\tau_{xy} = 0$. Thus,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{600 + 700}{2} = 650 \text{ psi}$$

Thus, the coordinate of reference point A and center of circle are

$$A(600, 0) \qquad C(650, 0)$$

Then the radius of the circle is

$$R = CA = 650 - 600 = 50 \text{ psi}$$

The Mohr's circle represents this state of stress is shown in Fig. *a*.

b) Here, $\sigma_x = 0, \sigma_y = 4$ ksi and $\tau_{xy} = 0$. Thus,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + 4}{2} = 2 \text{ ksi}$$

Thus, the coordinate of reference point A and center of circle are

$$A(0, 0) \qquad C(2, 0)$$

Then the radius of the circle is

$$R = CA = 2 - 0 = 2 \text{ psi}$$

c) Here, $\sigma_x = \sigma_y = 0$ and $\tau_{xy} = -40$ MPa. Thus,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = 0$$

Thus, the coordinate of reference point A and the center of circle are

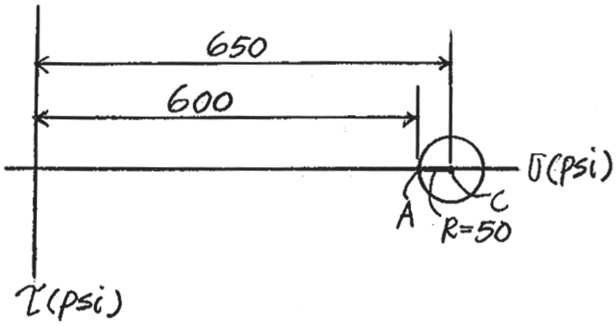
$$A(0, -40) \qquad C(0, 0)$$

Then, the radius of the circle is

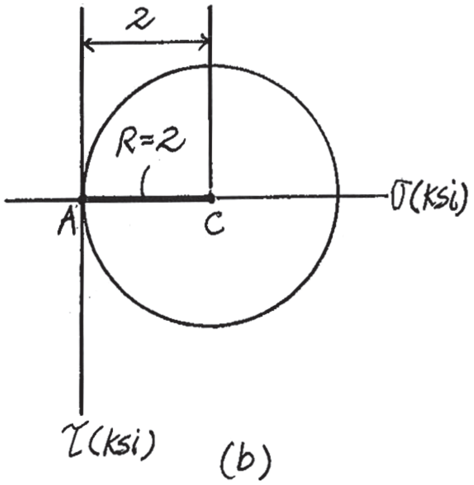
$$R = CA = 40 \text{ MPa}$$

The Mohr's circle represents this state of stress shown in Fig. *c*

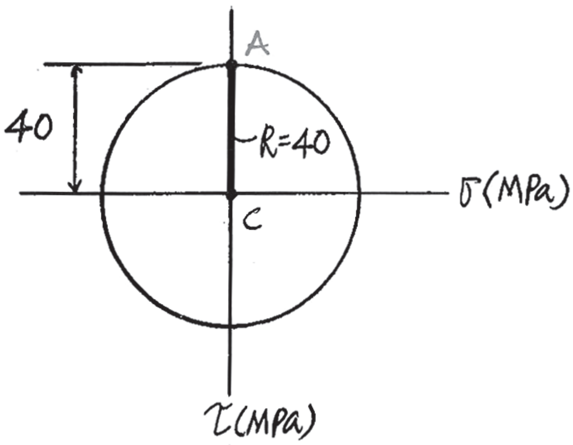
9–68. Continued



(a)

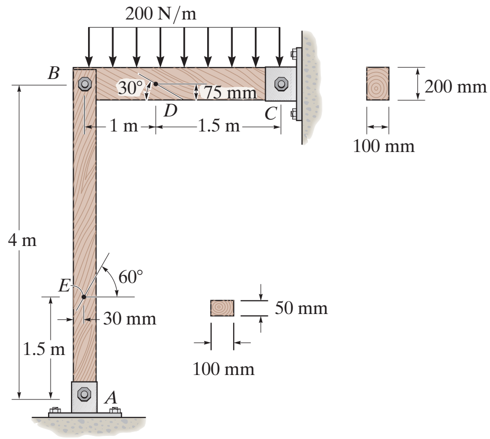


(b)

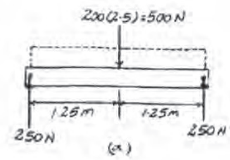


(c)

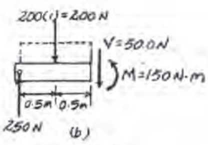
9–69. The frame supports the distributed loading of 200 N/m. Determine the normal and shear stresses at point D that act perpendicular and parallel, respectively, to the grain. The grain at this point makes an angle of 30° with the horizontal as shown.



Support Reactions: As shown on FBD(a).



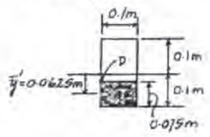
Internal Forces and Moment: As shown on FBD(b).



Section Properties:

$$I = \frac{1}{12}(0.1)\left(0.2^3\right) = 66.667\left(10^{-6}\right)\text{ m}^4$$

$$Q_D = \bar{y}'A' = 0.0625(0.075)(0.1) = 0.46875\left(10^{-3}\right)\text{ m}^3$$

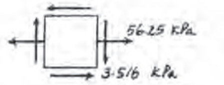


Normal Stress: Applying the flexure formula.

$$\sigma_D = -\frac{My}{I} = -\frac{150(-0.025)}{66.667(10^{-6})} = 56.25\text{ kPa}$$

Shear Stress: Applying the shear formula.

$$\tau_D = \frac{VQ_D}{It} = \frac{50.0\left[0.46875(10^{-3})\right]}{66.667(10^{-6})(0.1)} = 3.516\text{ kPa}$$

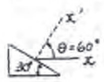


Construction of the Circle: In accordance to the established sign convention, $\sigma_x = 56.25$ kPa, $\sigma_y = 0$ and $\tau_{xy} = -3.516$ kPa. Hence.

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{56.25 + 0}{2} = 28.125\text{ kPa}$$

The coordinates for reference point A and C are

$$A(56.25, -3.516) \qquad C(28.125, 0)$$



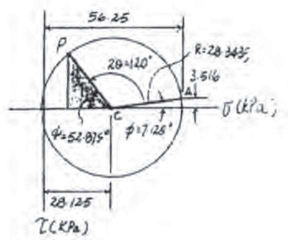
The radius of the circle is

$$R = \sqrt{(56.25 - 28.125)^2 + 3.516^2} = 28.3439\text{ kPa}$$

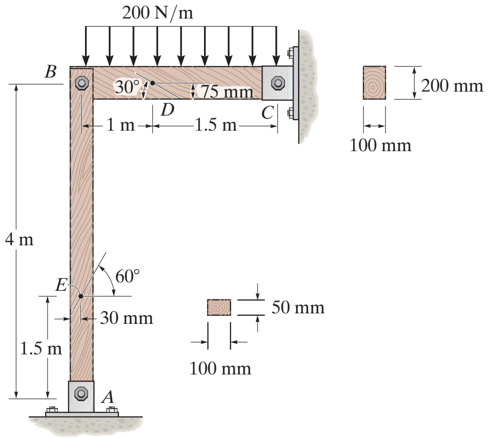
Stresses on The Rotated Element: The normal and shear stress components $\left(\sigma_{x'}\text{ and }\tau_{x'y'}\right)$ are represented by the coordinates of point P on the circle. Here, $\theta = 60°$.

$$\sigma_{x'} = 28.125 - 28.3439\cos 52.875° = 11.0\text{ kPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = -28.3439\sin 52.875° = -22.6\text{ kPa} \qquad \textbf{Ans.}$$



9–70. The frame supports the distributed loading of 200 N/m. Determine the normal and shear stresses at point E that act perpendicular and parallel, respectively, to the grain. The grain at this point makes an angle of 60° with the horizontal as shown.



Support Reactions: As shown on FBD(a).

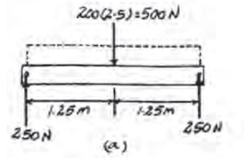
Internal Forces and Moment: As shown on FBD(b).

Section Properties:

$$A = 0.1(0.05) = 5.00\left(10^{-3}\right)\ \text{m}^2$$

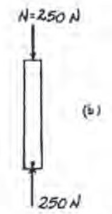
Normal Stress:

$$\sigma_E = \frac{N}{A} = \frac{-250}{5.00(10^{-3})} = -50.0\ \text{kPa}$$



Construction of the Circle: In accordance with the sign convention. $\sigma_x = 0$, $\sigma_y = -50.0$ kPa, and $\tau_{xy} = 0$. Hence.

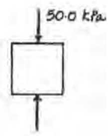
$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + (-50.0)}{2} = -25.0\ \text{kPa}$$



The coordinates for reference points A and C are

$$A(0, 0) \qquad C(-25.0, 0)$$

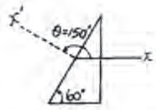
The radius of circle is $R = 25.0 - 0 = 25.0$ kPa

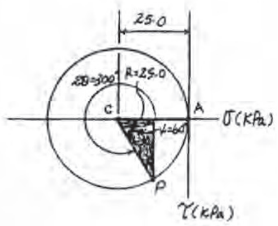


Stress on the Rotated Element: The normal and shear stress components $\left(\sigma_{x'} \text{ and } \tau_{x'y'}\right)$ are represented by coordinates of point P on the circle. Here, $\theta = 150°$.

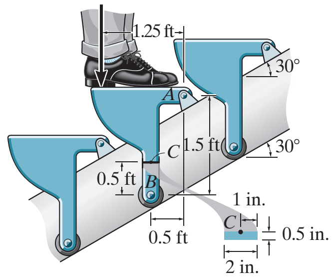
$$\sigma_x = -25.0 + 25.0 \cos 60° = -12.5\ \text{kPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = 25.0 \sin 60° = 21.7\ \text{kPa} \qquad \textbf{Ans.}$$

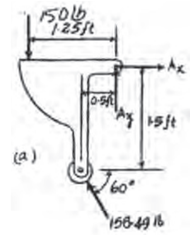




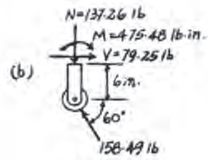
9–71. The stair tread of the escalator is supported on two of its sides by the moving pin at A and the roller at B. If a man having a weight of 300 lb stands in the center of the tread, determine the principal stresses developed in the supporting truck on the cross section at point C. The stairs move at constant velocity.



Support Reactions: As shown on FBD (a).



Internal Forces and Moment: As shown on FBD (b).



Section Properties:

$$A = 2(0.5) = 1.00\ \text{in}^2$$

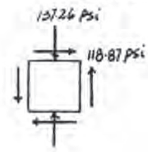
$$I = \frac{1}{12}(0.5)\left(2^3\right) = 0.3333\ \text{in}^4$$

$$Q_B = \bar{y}'A' = 0.5(1)(0.5) = 0.250\ \text{in}^3$$

Normal Stress:

$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

$$\sigma_C = \frac{-137.26}{1.00} + \frac{475.48(0)}{0.3333} = -137.26\ \text{psi}$$

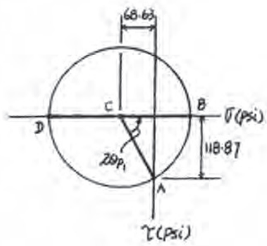


Shear Stress: Applying the shear formula $\tau = \dfrac{VQ}{It}$.

$$\tau_C = \frac{79.25(0.250)}{0.3333(0.5)} = 118.87\ \text{psi}$$

Construction of the Circle: In accordance with the sign convention, $\sigma_x = 0$, $\sigma_y = -137.26$ psi, and $\tau_{xy} = 118.87$ psi. Hence,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + (-137.26)}{2} = -68.63\ \text{psi}$$



The coordinates for reference points A and C are

$$A(0, 118.87) \qquad C(-68.63, 0)$$

The radius of the circle is

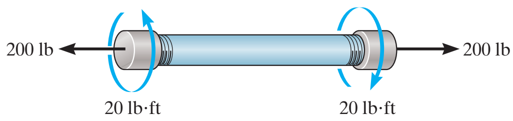
$$R = \sqrt{(68.63 - 0)^2 + 118.87^2} = 137.26\ \text{psi}$$

In - Plane Principal Stress: The coordinates of point B and D represent σ_1 and σ_2, respectively.

$$\sigma_1 = -68.63 + 137.26 = 68.6\ \text{psi} \qquad \textbf{Ans.}$$

$$\sigma_2 = -68.63 - 137.26 = -206\ \text{psi} \qquad \textbf{Ans.}$$

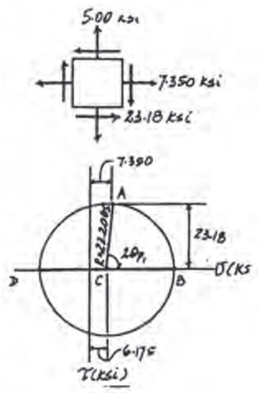
***9–72.** The thin-walled pipe has an inner diameter of 0.5 in. and a thickness of 0.025 in. If it is subjected to an internal pressure of 500 psi and the axial tension and torsional loadings shown, determine the principal stress at a point on the surface of the pipe.



Section Properties:

$$A = \pi\left(0.275^2 - 0.25^2\right) = 0.013125\pi \text{ in}^2$$

$$J = \frac{\pi}{2}\left(0.275^4 - 0.25^4\right) = 2.84768\left(10^{-3}\right) \text{ in}^4$$



Normal Stress: Since $\frac{r}{t} = \frac{0.25}{0.025} = 10$, thin wall analysis is valid.

$$\sigma_{\text{long}} = \frac{N}{A} + \frac{pr}{2t} = \frac{200}{0.013125\pi} + \frac{500(0.25)}{2(0.025)} = 7.350 \text{ ksi}$$

$$\sigma_{\text{hoop}} = \frac{pr}{t} = \frac{500(0.25)}{0.025} = 5.00 \text{ ksi}$$

Shear Stress: Applying the torsion formula,

$$\tau = \frac{Tc}{J} = \frac{20(12)(0.275)}{2.84768(10^{-3})} = 23.18 \text{ ksi}$$

Construction of the Circle: In accordance with the sign convention $\sigma_x = 7.350$ ksi, $\sigma_y = 5.00$ ksi, and $\tau_{xy} = -23.18$ ksi. Hence,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{7.350 + 5.00}{2} = 6.175 \text{ ksi}$$

The coordinates for reference points A and C are

$$A(7.350, -23.18) \qquad C(6.175, 0)$$

The radius of the circle is

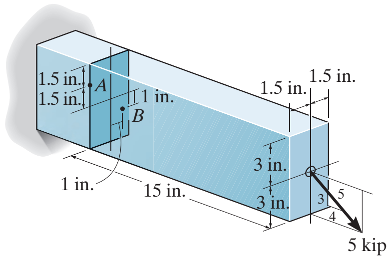
$$R = \sqrt{(7.350 - 6.175)^2 + 23.18^2} = 23.2065 \text{ ksi}$$

In - Plane Principal Stress: The coordinates of point B and D represent σ_1 and σ_2, respectively.

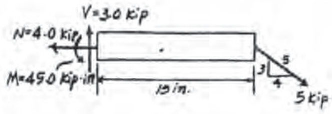
$$\sigma_1 = 6.175 + 23.2065 = 29.4 \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_2 = 6.175 - 23.2065 = -17.0 \text{ ksi} \qquad \textbf{Ans.}$$

•9–73. The cantilevered rectangular bar is subjected to the force of 5 kip. Determine the principal stress at point A.



Internal Forces and Moment: As shown on FBD.

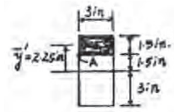


Section Properties:

$$A = 3(6) = 18.0\ \text{in}^2$$

$$I = \frac{1}{12}(3)\left(6^3\right) = 54.0\ \text{in}^4$$

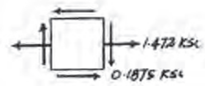
$$Q_A = \bar{y}'A' = 2.25(1.5)(3) = 10.125\ \text{in}^3$$



Normal Stress:

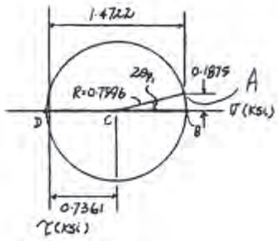
$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

$$\sigma_A = \frac{4.00}{18.0} + \frac{45.0(1.5)}{54.0} = 1.4722\ \text{ksi}$$



Shear Stress: Applying the shear formula $\tau = \dfrac{VQ}{It}$.

$$\tau_A = \frac{3.00(10.125)}{54.0(3)} = 0.1875\ \text{ksi}$$



Construction of the Circle: In accordance with the sign convention, $\sigma_x = 1.4722$ ksi, $\sigma_y = 0$, and $\tau_{xy} = -0.1875$ ksi. Hence,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{1.472 + 0}{2} = 0.7361\ \text{ksi}$$

The coordinates for reference points A and C are

$$A(1.4722, -0.1875) \qquad C(0.7361, 0)$$

The radius of the circle is

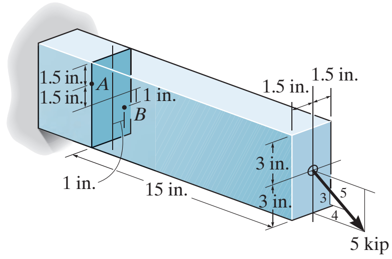
$$R = \sqrt{(1.4722 - 0.7361)^2 + 0.1875^2} = 0.7596\ \text{ksi}$$

In - Plane Principal Stress: The coordinates of point B and D represent σ_1 and σ_2, respectively.

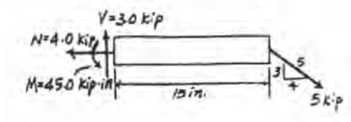
$$\sigma_1 = 0.7361 + 0.7596 = 1.50\ \text{ksi} \qquad \textbf{Ans.}$$

$$\sigma_2 = 0.7361 - 0.7596 = -0.0235\ \text{ksi} \qquad \textbf{Ans.}$$

9–74. Solve Prob. 9–73 for the principal stress at point B.



Internal Forces and Moment: As shown on FBD.

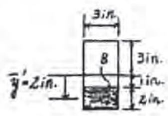


Section Properties:

$$A = 3(6) = 18.0 \text{ in}^2$$

$$I = \frac{1}{12}(3)\left(6^3\right) = 54.0 \text{ in}^4$$

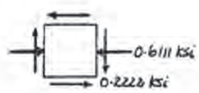
$$Q_B = \bar{y}'A' = 2(2)(3) = 12.0 \text{ in}^3$$



Normal Stress:

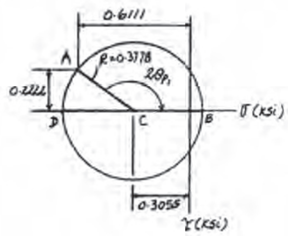
$$\sigma = \frac{N}{A} \pm \frac{My}{I}$$

$$\sigma_B = \frac{4.00}{18.0} - \frac{45.0(1)}{54.0} = -0.6111 \text{ ksi}$$



Shear Stress: Applying the shear formula $\tau = \frac{VQ}{It}$.

$$\tau_B = \frac{3.00(12.0)}{54.0(3)} = 0.2222 \text{ ksi}$$



Construction of the Circle: In accordance with the sign convention, $\sigma_x = -0.6111$ ksi, $\sigma_y = 0$, and $\tau_{xy} = -0.2222$ ksi. Hence.

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{-0.6111 + 0}{2} = -0.3055 \text{ ksi}$$

The coordinates for reference points A and C are

$$A(-0.6111, -0.2222) \qquad C(-0.3055, 0)$$

The radius of the circle is

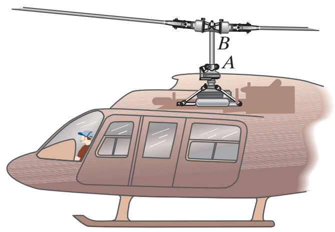
$$R = \sqrt{(0.6111 - 0.3055)^2 + 0.2222^2} = 0.3778 \text{ ksi}$$

In - Plane Principal Stress: The coordinates of point B and D represent σ_1 and σ_2, respectively.

$$\sigma_1 = -0.3055 + 0.3778 = 0.0723 \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_2 = -0.3055 - 0.3778 = -0.683 \text{ ksi} \qquad \textbf{Ans.}$$

9–75. The 2-in.-diameter drive shaft AB on the helicopter is subjected to an axial tension of 10 000 lb and a torque of 300 lb·ft. Determine the principal stress and the maximum in-plane shear stress that act at a point on the surface of the shaft.



$$\sigma = \frac{P}{A} = \frac{10\,000}{\pi(1)^2} = 3.183 \text{ ksi}$$

$$\tau = \frac{Tc}{J} = \frac{300(12)(1)}{\frac{\pi}{2}(1)^4} = 2.292 \text{ ksi}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{3.183 + 0}{2} \pm \sqrt{\left(\frac{3.183 - 0}{2}\right)^2 + (2.292)^2}$$

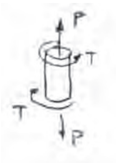
$\sigma_1 = 4.38$ ksi **Ans.**

$\sigma_2 = -1.20$ ksi **Ans.**

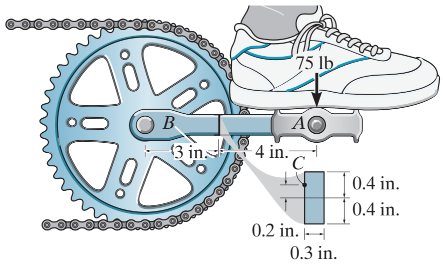
$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{3.183 - 0}{2}\right)^2 + (2.292)^2}$$

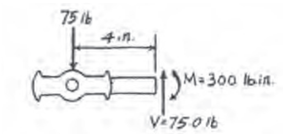
$= 2.79$ ksi **Ans.**



***9–76.** The pedal crank for a bicycle has the cross section shown. If it is fixed to the gear at *B* and does not rotate while subjected to a force of 75 lb, determine the principal stress in the material on the cross section at point *C*.



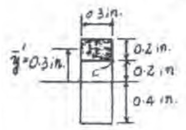
Internal Forces and Moment: As shown on FBD



Section Properties:

$$I = \frac{1}{12}(0.3)\left(0.8^3\right) = 0.0128 \text{ in}^3$$

$$Q_C = \bar{y}'A' = 0.3(0.2)(0.3) = 0.0180 \text{ in}^3$$

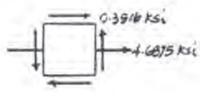


Normal Stress: Applying the flexure formula.

$$\sigma_C = -\frac{My}{I} = -\frac{-300(0.2)}{0.0128} = 4687.5 \text{ psi} = 4.6875 \text{ ksi}$$

Shear Stress: Applying the shear formula.

$$\tau_C = \frac{VQ_C}{It} = \frac{75.0(0.0180)}{0.0128(0.3)} = 351.6 \text{ psi} = 0.3516 \text{ ksi}$$

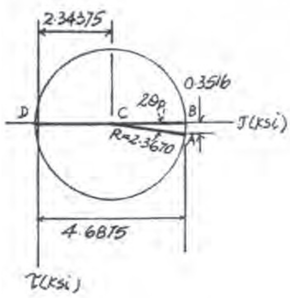


Construction of the Circle: In accordance with the sign convention, $\sigma_x = 4.6875$ ksi, $\sigma_y = 0$, and $\tau_{xy} = 0.3516$ ksi. Hence,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{4.6875 + 0}{2} = 2.34375 \text{ ksi}$$

The coordinates for reference points *A* and *C* are

$$A(4.6875, 0.3516) \qquad C(2.34375, 0)$$



The radius of the circle is

$$R = \sqrt{(4.6875 - 2.34375)^2 + 0.3516^2} = 2.3670 \text{ ksi}$$

In - Plane Principal Stress: The coordinates of point *B* and *D* represent σ_1 and σ_2, respectively.

$$\sigma_1 = 2.34375 + 2.3670 = 4.71 \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_2 = 2.34375 - 2.3670 = -0.0262 \text{ ksi} \qquad \textbf{Ans.}$$

•9–77. A spherical pressure vessel has an inner radius of 5 ft and a wall thickness of 0.5 in. Draw Mohr's circle for the state of stress at a point on the vessel and explain the significance of the result. The vessel is subjected to an internal pressure of 80 psi.

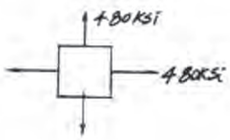
Normal Stress:

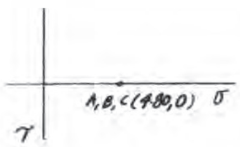
$$\sigma_1 = \sigma_2 = \frac{p\,r}{2\,t} = \frac{80(5)(12)}{2(0.5)} = 4.80 \text{ ksi}$$

Mohr's circle:

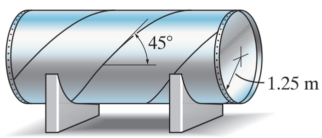
$A(4.80, 0) \qquad B(4.80, 0) \qquad C(4.80, 0)$

Regardless of the orientation of the element, the shear stress is zero and the state of stress is represented by the same two normal stress components.



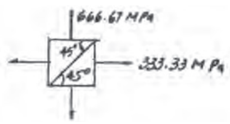


9–78. The cylindrical pressure vessel has an inner radius of 1.25 m and a wall thickness of 15 mm. It is made from steel plates that are welded along the 45° seam. Determine the normal and shear stress components along this seam if the vessel is subjected to an internal pressure of 8 MPa.



$$\sigma_x = \frac{pr}{2t} = \frac{8(1.25)}{2(0.015)} = 333.33 \text{ MPa}$$

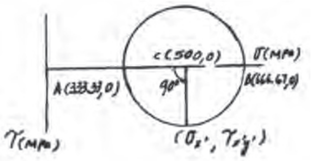
$$\sigma_y = 2\sigma_x = 666.67 \text{ MPa}$$



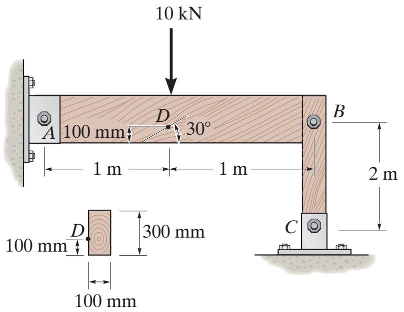
$A(333.33, 0) \qquad B(666.67, 0) \qquad C(500, 0)$

$$\sigma_{x'} = \frac{333.33 + 666.67}{2} = 500 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = R = 666.67 - 500 = 167 \text{ MPa} \qquad \textbf{Ans.}$$



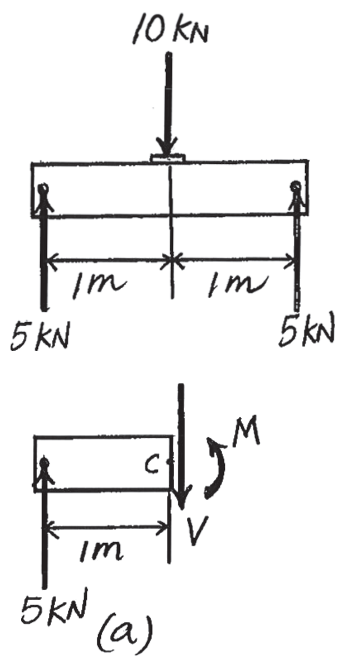
•9–79. Determine the normal and shear stresses at point D that act perpendicular and parallel, respectively, to the grains. The grains at this point make an angle of 30° with the horizontal as shown. Point D is located just to the left of the 10-kN force.



Using the method of section and consider the FBD of the left cut segment, Fig. *a*

$+\uparrow \Sigma F_y = 0; \qquad 5 - V = 0 \qquad V = 5 \text{ kN}$

$\circlearrowleft + \Sigma M_C = 0; \qquad M - 5(1) = 0 \qquad M = 5 \text{ kN} \cdot \text{m}$



The moment of inertia of the rectangular cross - section about the neutral axis is

$$I = \frac{1}{12}(0.1)(0.3^3) = 0.225(10^{-3}) \text{ m}^4$$

Referring to Fig. *b*,

$$Q_D = \bar{y}'A' = 0.1(0.1)(0.1) = 0.001 \text{ m}^3$$

The normal stress developed is contributed by bending stress only. For point D, $y = 0.05$ m. Then

$$\sigma = \frac{My}{I} = \frac{5(10^3)(0.05)}{0.225(10^{-3})} = 1.111 \text{ MPa (T)}$$

The shear stress is contributed by the transverse shear stress only. Thus,

$$\tau = \frac{VQ_D}{It} = \frac{5(10^3)(0.001)}{0.225(10^{-3})(0.1)} = 0.2222 \text{ MPa}$$

The state of stress at point D can be represented by the element shown in Fig. *c*

In accordance to the established sign convention, $\sigma_x = 1.111$ MPa, $\sigma_y = 0$ and $\tau_{xy} = -0.2222$ MPa, Thus.

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{1.111 + 0}{2} = 0.5556 \text{ MPa}$$

Then, the coordinate of reference point A and the center C of the circle are

$$A(1.111, -0.2222) \qquad C(0.5556, 0)$$

Thus, the radius of the circle is given by

$$R = \sqrt{(1.111 - 0.5556)^2 + (-0.2222)^2} = 0.5984 \text{ MPa}$$

Using these results, the circle shown in Fig. *d* can be constructed.

Referring to the geometry of the circle, Fig. *d*,

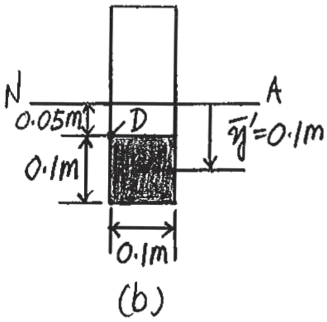
$$\alpha = \tan^{-1}\left(\frac{0.2222}{1.111 - 0.5556}\right) = 21.80° \qquad \beta = 180° - (120° - 21.80°) = 81.80°$$

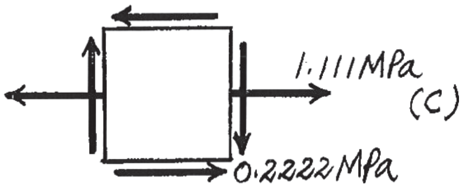
9–79. Continued

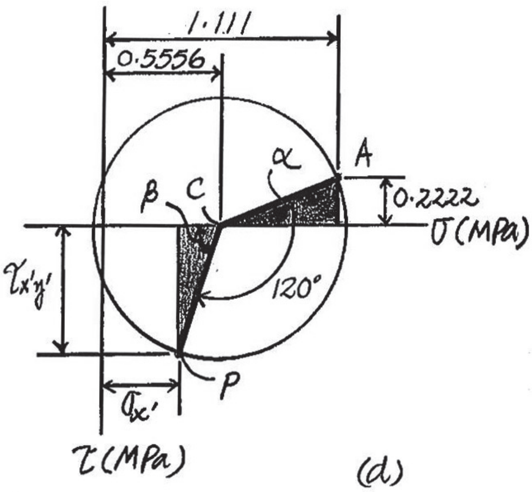
Then

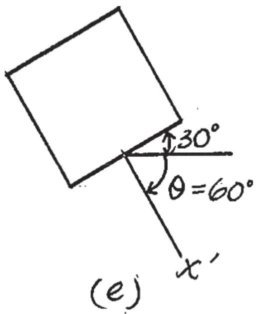
$\sigma_{x'} = 0.5556 - 0.5984 \cos 81.80° = 0.4702 \text{ MPa} = 470 \text{ kPa}$ **Ans.**

$\tau_{x'y'} = 0.5984 \sin 81.80° = 0.5922 \text{ MPa} = 592 \text{ kPa}$ **Ans.**

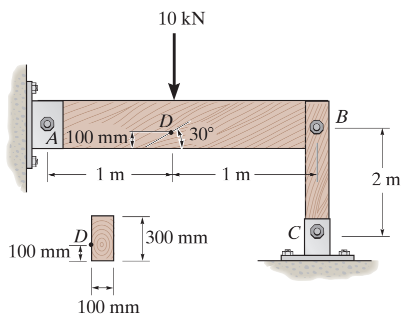








***9–80.** Determine the principal stress at point D, which is located just to the left of the 10-kN force.

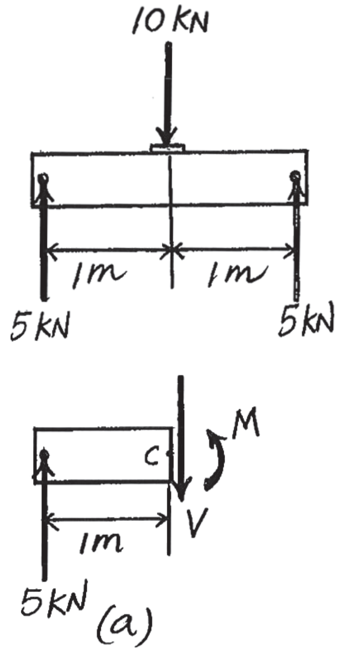


Using the method of section and consider the FBD of the left cut segment, Fig. a,

$+\uparrow\Sigma F_y = 0;\quad 5 - V = 0 \qquad V = 5\text{ kN}$

$\circlearrowleft+\Sigma M_C = 0;\quad M - 5(1) = 0 \qquad M = 5\text{ kN}\cdot\text{m}$

$$I = \frac{1}{12}(0.1)(0.3^3) = 0.225(10^{-3})\text{ m}^4$$



Referring to Fig. b,

$$Q_D = \bar{y}'A' = 0.1(0.1)(0.1) = 0.001\text{ m}^3$$

The normal stress developed is contributed by bending stress only. For point D, $y = 0.05$ m

$$\sigma = \frac{My}{I} = \frac{5(10^3)(0.05)}{0.225(10^{-3})} = 1.111\text{ MPa (T)}$$

The shear stress is contributed by the transverse shear stress only. Thus,

$$\tau = \frac{VQ_D}{It} = \frac{5(10^3)(0.001)}{0.225(10^{-3})(0.1)} = 0.2222\text{ MPa}$$

The state of stress at point D can be represented by the element shown in Fig. c.

In accordance to the established sign convention, $\sigma_x = 1.111$ MPa, $\sigma_y = 0$, and $\tau_{xy} = -0.2222$ MPa. Thus,

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{1.111 + 0}{2} = 0.5556\text{ MPa}$$

Then, the coordinate of reference point A and center C of the circle are

$$A(1.111, -0.2222) \qquad C(0.5556, 0)$$

Thus, the radius of the circle is

$$R = CA = \sqrt{(1.111 - 0.5556)^2 + (-0.2222)^2} = 0.5984\text{ MPa}$$

Using these results, the circle shown in Fig. d.

In-Plane Principal Stresses. The coordinates of points B and D represent σ_1 and σ_2, respectively. Thus,

$\sigma_1 = 0.5556 + 0.5984 = 1.15\text{ MPa}$ **Ans.**

$\sigma_2 = 0.5556 - 0.5984 = -0.0428\text{ MPa}$ **Ans.**

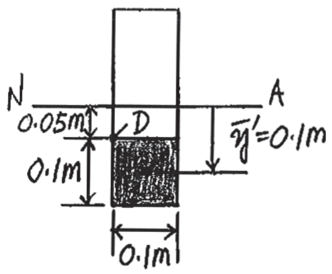
9–80. Continued

Referring to the geometry of the circle, Fig. *d*,

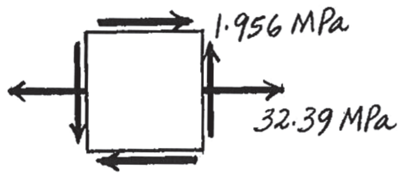
$$\tan(2\theta_P)_1 = \frac{0.2222}{1.111 - 0.5556} = 0.4$$

$$(\theta_P)_1 = 10.9^\circ \ (Clockwise) \qquad \textbf{Ans.}$$

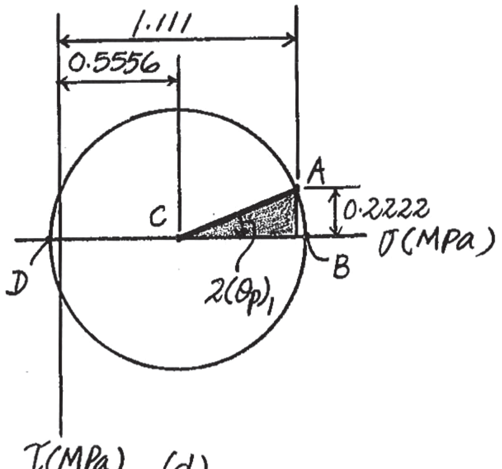
The state of principal stresses is represented by the element show in Fig. *e*.



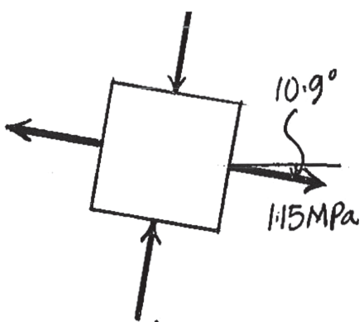
(b)



(C)

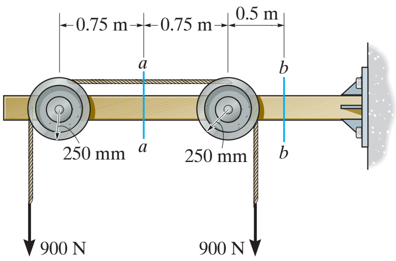


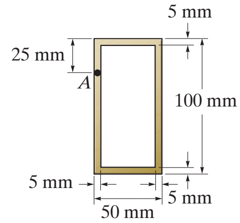
(d)



(e)

•9–81. Determine the principal stress at point A on the cross section of the hanger at section a–a. Specify the orientation of this state of stress and indicate the result on an element at the point.





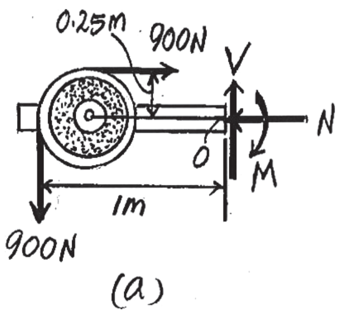
Sections $a-a$ and $b-b$

Internal Loadings: Considering the equilibrium of the free - body diagram of the hanger's left cut segment, Fig. a,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 900 - N = 0 \qquad N = 900\,\text{N}$$

$$+\uparrow \Sigma F_y = 0; \qquad V - 900 = 0 \qquad V = 900\,\text{N}$$

$$\circlearrowleft + \Sigma M_O = 0; \qquad 900(1) - 900(0.25) - M = 0 \qquad M = 675\,\text{N}\cdot\text{m}$$



Section Properties: The cross - sectional area and the moment of inertia about the centroidal axis of the hanger's cross section are

$$A = 0.05(0.1) - 0.04(0.09) = 1.4\left(10^{-3}\right)\text{m}^2$$

$$I = \frac{1}{12}(0.05)\left(0.1^3\right) - \frac{1}{12}(0.04)\left(0.09^3\right) = 1.7367\left(10^{-6}\right)\text{m}^4$$

Referring to Fig. b,

$$Q_A = 2\bar{y}_1'A_1' + \bar{y}_2'A_2' = 2[0.0375(0.025)(0.005)] + 0.0475(0.005)(0.04)$$

$$= 18.875\left(10^{-6}\right)\text{m}^3$$

Normal and Shear Stress: The normal stress is a combination of axial and bending stresses. Thus,

$$\sigma_A = \frac{N}{A} + \frac{My_A}{I} = -\frac{900}{1.4\left(10^{-3}\right)} + \frac{675(0.025)}{1.7367\left(10^{-6}\right)} = 9.074\text{ MPa}$$

The shear stress is caused by the transverse shear stress.

$$\tau_A = \frac{VQ_A}{It} = \frac{900\left[18.875\left(10^{-6}\right)\right]}{1.7367\left(10^{-6}\right)(0.01)} = 0.9782\text{ MPa}$$

The state of stress at point A is represented by the element shown in Fig. c.

Construction of the Circle: $\sigma_x = 9.074$ MPa, $\sigma_y = 0$, and $\tau_{xy} = 0.9782$ MPa. Thus,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{9.074 + 0}{2} = 4.537\text{ MPa}$$

The coordinates of reference points A and the center C of the circle are

$$A(9.074, 0.9782) \qquad C(4.537, 0)$$

Thus, the radius of the circle is

$$R = CA = \sqrt{(9.074 - 4.537)^2 + 0.9782^2} = 4.641\text{ MPa}$$

Using these results, the circle is shown in Fig. d.

9–81. Continued

In - Plane Principal Stress: The coordinates of point *B* and *D* represent σ_1 and σ_2, respectively.

$$\sigma_1 = 4.537 + 4.641 = 9.18 \text{ MPa} \qquad \textbf{Ans.}$$

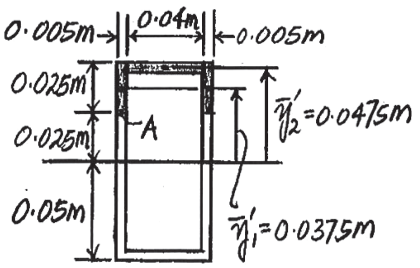
$$\sigma_2 = 4.537 - 4.641 = -0.104 \text{ MPa} \qquad \textbf{Ans.}$$

Orientaion of Principal Plane: Referring to the geometry of the circle, Fig. *d*,

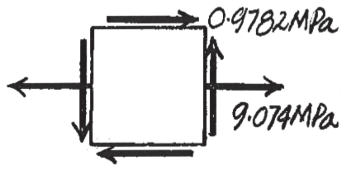
$$\tan 2\left(\theta_P\right)_1 = \frac{0.9782}{9.074 - 4.537} = 0.2156$$

$$\left(\theta_P\right)_1 = 6.08^\circ \text{ (counterclockwise)} \qquad \textbf{Ans.}$$

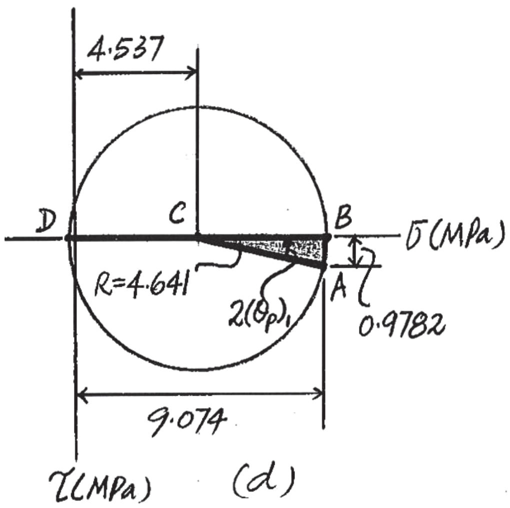
The state of principal stresses is represented on the element shown in Fig. *e*.



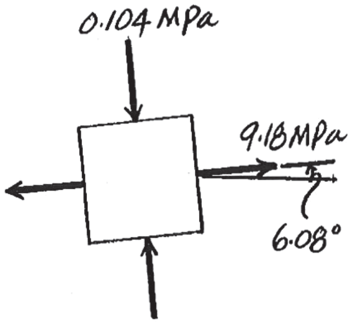
(b)



(c)

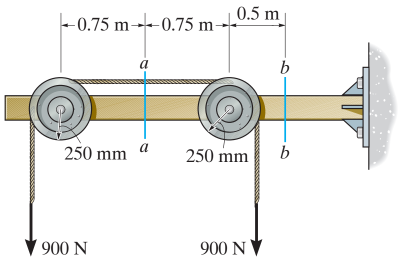


(d)



(e)

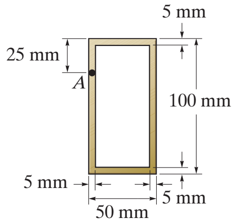
9–82. Determine the principal stress at point A on the cross section of the hanger at section b–b. Specify the orientation of the state of stress and indicate the results on an element at the point.



Internal Loadings: Considering the equilibrium of the free - body diagram of the hanger's left cut segment, Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad V - 900 - 900 = 0 \qquad V = 1800\,\text{N}$$

$$\circlearrowleft + \Sigma M_O = 0; \qquad 900(2.25) + 900(0.25) - M = 0 \qquad M = 2250\,\text{N}\cdot\text{m}$$



Sections $a-a$ and $b-b$

Section Properties: The cross - sectional area and the moment of inertia about the centroidal axis of the hanger's cross section are

$$A = 0.05(0.1) - 0.04(0.09) = 1.4\left(10^{-3}\right)\text{m}^2$$

$$I = \frac{1}{12}(0.05)\left(0.1^3\right) - \frac{1}{12}(0.04)\left(0.09^3\right) = 1.7367\left(10^{-6}\right)\text{m}^4$$

Referring to Fig. *b*.

$$Q_A = 2\bar{y}_1' A_1' + \bar{y}_2' A_2' = 2[0.0375(0.025)(0.005)] + 0.0475(0.005)(0.04)$$

$$= 18.875\left(10^{-6}\right)\text{m}^3$$

Normal and Shear Stress: The normal stress is contributed by the bending stress only.

$$\sigma_A = \frac{My_A}{I} = \frac{2250(0.025)}{1.7367\left(10^{-6}\right)} = 32.39\text{ MPa}$$

The shear stress is contributed by the transverse shear stress only.

$$\tau_A = \frac{VQ_A}{It} = \frac{1800\left[18.875\left(10^{-6}\right)\right]}{1.7367\left(10^{-6}\right)(0.01)} = 1.956\text{ MPa}$$

The state stress at point A is represented by the element shown in Fig. *c*.

Construction of the Circle: $\sigma_x = 32.39$ MPa, $\sigma_y = 0$, and $\tau_{xy} = 1.956$ MPa. Thus,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{32.39 + 0}{2} = 16.19\text{ MPa}$$

The coordinates of reference point A and the center C of the circle are

$A(32.39, 1.956)$ $\qquad$ $C(16.19, 0)$

Thus, the radius of the circle is

$$R = CA = \sqrt{(32.39 - 16.19)^2 + 1.956^2} = 16.313\text{ MPa}$$

Using these results, the cricle is shown in Fig. *d*.

9–82. Continued

In - Plane Principal Stresses: The coordinates of reference point *B* and *D* represent σ_1 and σ_2, respectively.

$$\sigma_1 = 16.19 + 16.313 = 32.5 \text{ MPa} \qquad \textbf{Ans.}$$

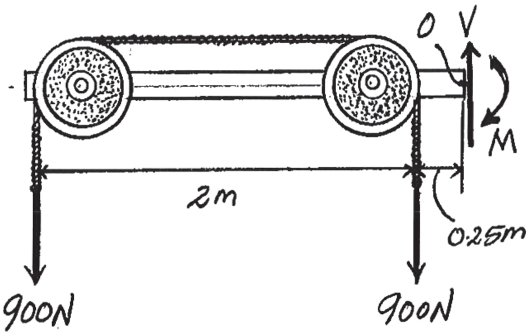
$$\sigma_2 = 16.19 - 16.313 = -0.118 \text{ MPa} \qquad \textbf{Ans.}$$

Orientaion of Principal Plane: Referring to the geometry of the circle, Fig. *d*,

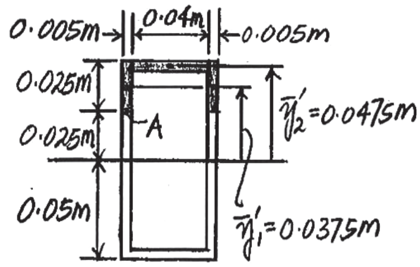
$$\tan 2\left(\theta_P\right)_1 = \frac{1.956}{32.39 - 16.19} = 0.1208$$

$$\left(\theta_P\right)_1 = 3.44° \qquad \text{(counterclockwise)} \qquad \textbf{Ans.}$$

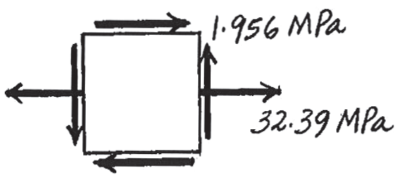
The state of principal stresses is represented on the element shown in Fig. *e*.



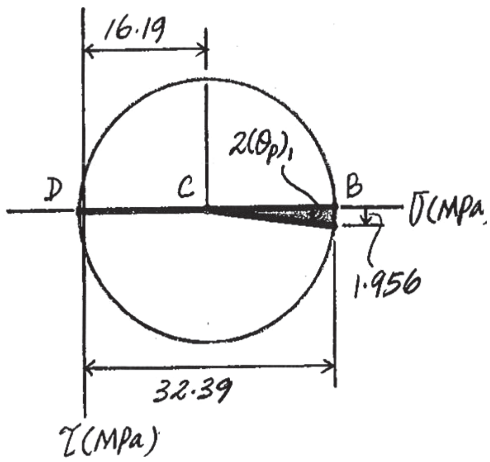
(a)



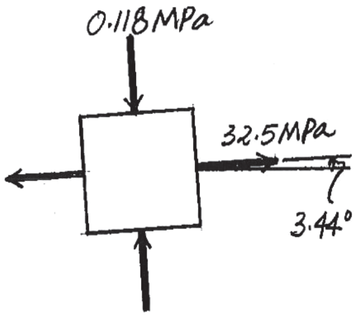
(b)



(C)

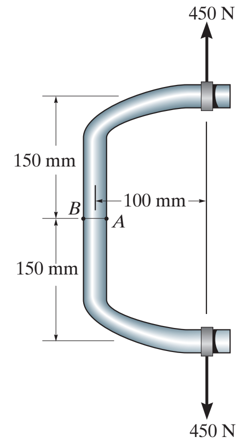


(d)



(e)

9–83. Determine the principal stresses and the maximum in-plane shear stress that are developed at point A. Show the results on an element located at this point. The rod has a diameter of 40 mm.



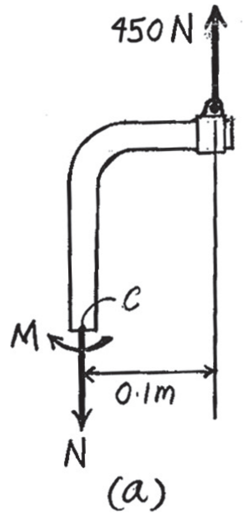
Using the method of sections and consider the FBD of the member's upper cut segment, Fig. a,

$+\uparrow \Sigma F_y = 0; \quad 450 - N = 0 \quad N = 450 \text{ N}$

$\circlearrowleft + \Sigma M_C = 0; \quad 450(0.1) - M = 0 \quad M = 45 \text{ N} \cdot \text{m}$

$$A = \pi(0.02^2) = 0.4(10^{-3})\pi \text{ m}^2$$

$$I = \frac{\pi}{4}(0.02^4) = 40(10^{-9})\pi \text{ m}^4$$



The normal stress is the combination of axial and bending stress. Thus,

$$\sigma = \frac{N}{A} + \frac{My}{I}$$

For point A, $y = C = 0.02$ m.

$$\sigma = \frac{450}{0.4(10^{-3})\pi} + \frac{45\,(0.02)}{40(10^{-9})\pi} = 7.520 \text{ MPa}$$

Since no transverse shear and torque is acting on the cross - section

$$\tau = 0$$

The state of stress at point A can be represented by the element shown in Fig. b.

In accordance to the established sign convention $\sigma_x = 0$, $\sigma_y = 7.520$ MPa and $\tau_{xy} = 0$. Thus

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + 7.520}{2} = 3.760 \text{ MPa}$$

Then, the coordinates of reference point A and the center C of the circle are

$$A(0, 0) \qquad C(3.760, 0)$$

Thus, the radius of the circle is

$$R = CA = 3.760 \text{ MPa}$$

Using this results, the circle shown in Fig. c can be constructed. Since no shear stress acts on the element,

$$\sigma_1 = \sigma_y = 7.52 \text{ MPa} \qquad \sigma_2 = \sigma_x = 0 \qquad \textbf{Ans.}$$

The state of principal stresses can also be represented by the element shown in Fig. b.

The state of maximum in - plane shear stress is represented by point B on the circle, Fig. c. Thus.

$$\tau_{\substack{max \\ in\text{-}plane}} = R = 3.76 \text{ MPa} \qquad \textbf{Ans.}$$

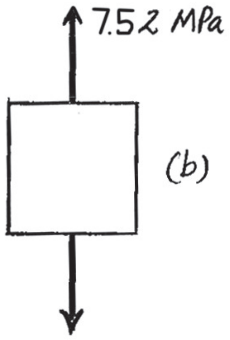
9–83. Continued

From the circle,

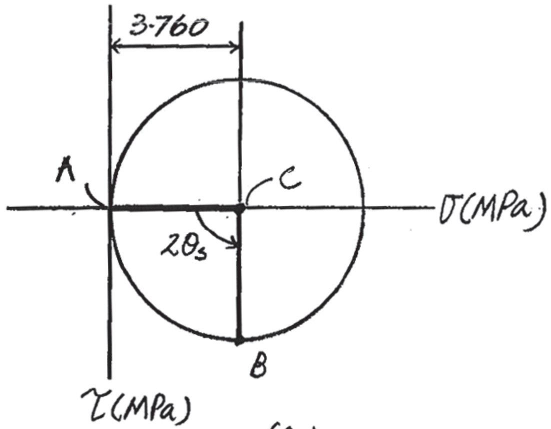
$$2\theta_s = 90°$$

$$\theta_s = 45° \quad (\textit{counter clockwise}) \qquad \textbf{Ans.}$$

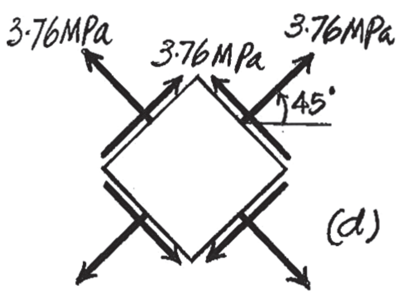
The state of maximum In - Plane shear stress can be represented by the element shown in Fig. *d*.



(b)

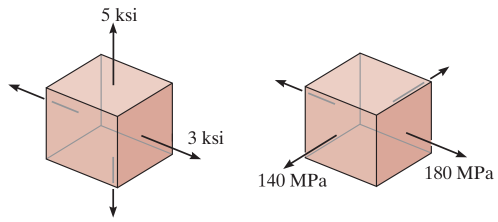


(c)



(d)

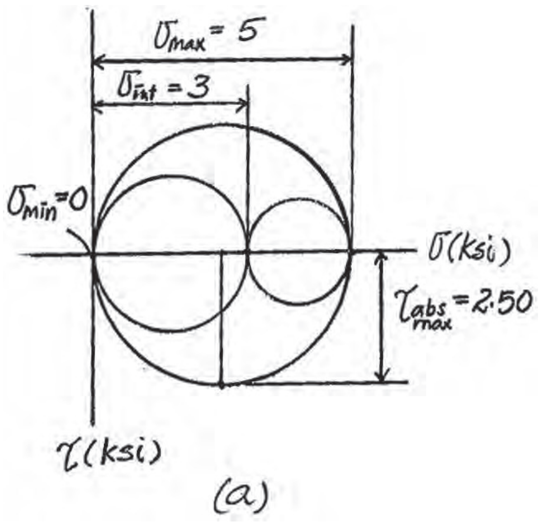
***9–84.** Draw the three Mohr's circles that describe each of the following states of stress.

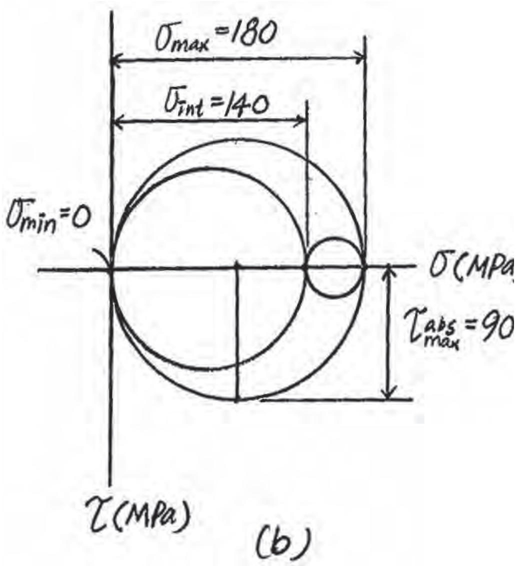


(a) (b)

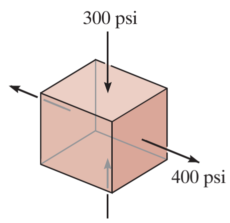
(a) Here, $\sigma_{\min} = 0$, $\sigma_{\text{int}} = 3$ ksi and $\sigma_{\max} = 5$ ksi. The three Mohr's circle of this state of stress are shown in Fig. *a*

(b) Here, $\sigma_{\min} = 0$, $\sigma_{\text{int}} = 140$ MPa and $\sigma_{\max} = 180$ MPa. The three Mohr's circle of this state of stress are shown in Fig. *b*

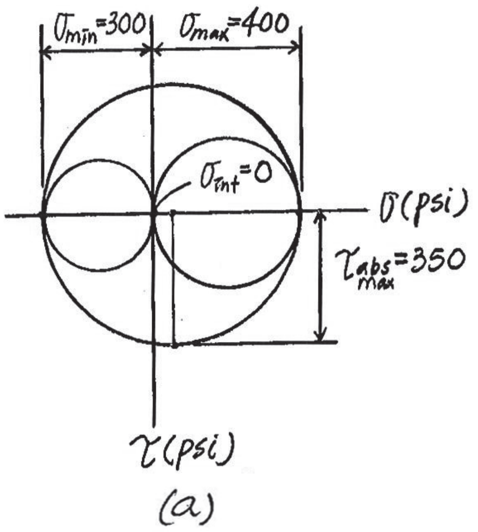




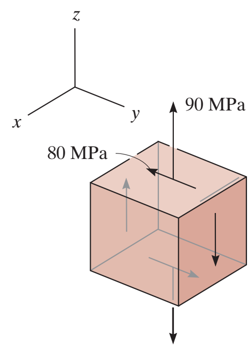
•9–85. Draw the three Mohr's circles that describe the following state of stress.



Here, $\sigma_{\min} = -300$ psi, $\sigma_{\text{int}} = 0$ and $\sigma_{\max} = 400$ psi. The three Mohr's circle for this state of stress is shown in Fig. *a*.



9–86. The stress at a point is shown on the element. Determine the principal stress and the absolute maximum shear stress.



For $y - z$ plane:

$$A(0, -80) \qquad B(90, 80) \qquad C(45, 0)$$

$$R = \sqrt{45^2 + 80^2} = 91.79$$

$$\sigma_1 = 45 + 91.79 = 136.79 \text{ MPa}$$

$$\sigma_2 = 45 - 91.79 = -46.79 \text{ MPa}$$

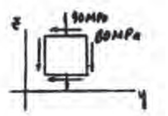
Thus,

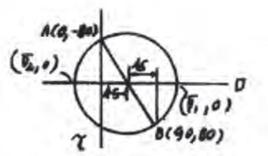
$$\sigma_1 = 0 \qquad \textbf{Ans.}$$

$$\sigma_2 = 137 \text{ MPa} \qquad \textbf{Ans.}$$

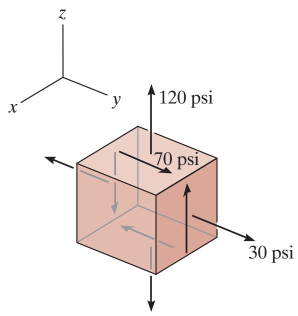
$$\sigma_3 = -46.8 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_{\substack{\text{abs}\\\text{max}}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{136.79 - (-46.79)}{2} = 91.8 \text{ MPa} \qquad \textbf{Ans.}$$





9–87. The stress at a point is shown on the element. Determine the principal stress and the absolute maximum shear stress.



Mohr's circle for the element in *y* - 7 plane, Fig. *a*, will be drawn first. In accordance to the established sign convention, $\sigma_y = 30$ psi, $\sigma_z = 120$ psi and $\tau_{yz} = 70$ psi. Thus

$$\sigma_{avg} = \frac{\sigma_y + \sigma_z}{2} = \frac{30 + 120}{2} = 75 \text{ psi}$$

Thus the coordinates of reference point *A* and the center *C* of the circle are

$$A(30, 70) \qquad C(75, 0)$$

Thus, the radius of the circle is

$$R = CA = \sqrt{(75 - 30)^2 + 70^2} = 83.217 \text{ psi}$$

Using these results, the circle shown in Fig. *b*.

The coordinates of point *B* and *D* represent the principal stresses

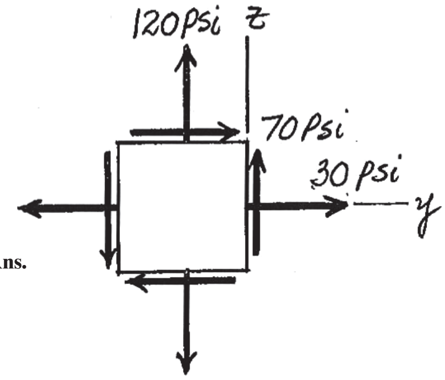
From the results,

$$\sigma_{max} = 158 \text{ psi} \qquad \sigma_{min} = -8.22 \text{ psi} \qquad \sigma_{int} = 0 \text{ psi} \qquad \textbf{Ans.}$$

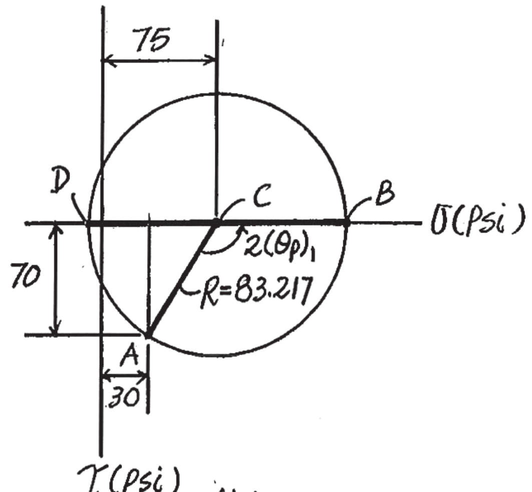
Using these results, the three Mohr's circle are shown in Fig. *c*,

From the geometry of the three circles,

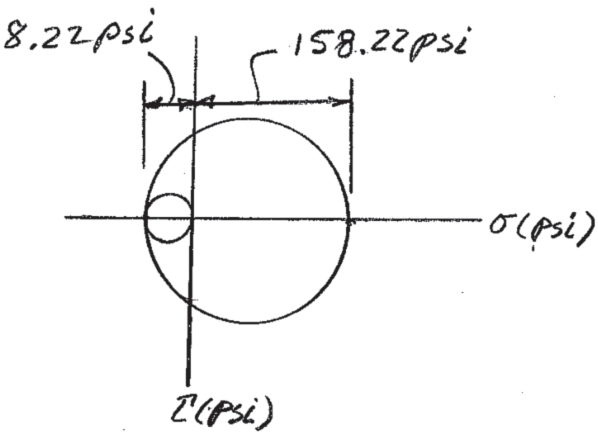
$$\tau_{\substack{abs\\max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{158.22 - (-8.22)}{2} = 83.22 \text{ psi} \qquad \textbf{Ans.}$$



(a)

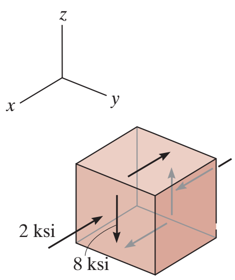


(b)



(c)

***9–88.** The stress at a point is shown on the element. Determine the principal stress and the absolute maximum shear stress.

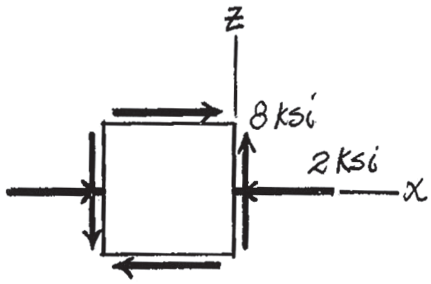


Mohr's circle for the element in x - z plane, Fig. a, will be drawn first. In accordance to the established sign convention, $\sigma_x = -2$ ksi, $\sigma_z = 0$ and $\tau_{xz} = 8$ ksi. Thus

$$\sigma_{avg} = \frac{\sigma_x + \sigma_z}{2} = \frac{-2 + 0}{2} = -1 \text{ ksi}$$

Thus, the coordinates of reference point A and the center C of the circle are

$$A(-2, 8) \qquad C(-1, 0)$$



(a)

Thus, the radius of the circle is

$$R = CA = \sqrt{[-2 - (-1)]^2 + 8^2} = \sqrt{65} \text{ ksi}$$

Using these results, the circle in shown in Fig. b,

The coordinates of points B and D represent σ_1 and σ_2, respectively.

$$\sigma = -1 + \sqrt{65} = 7.062 \text{ ksi}$$

$$\sigma_{max} = 7.06 \text{ ksi}$$

$$\sigma_{int} = 0$$

$$\sigma_{min} = -9.06 \text{ ksi}$$

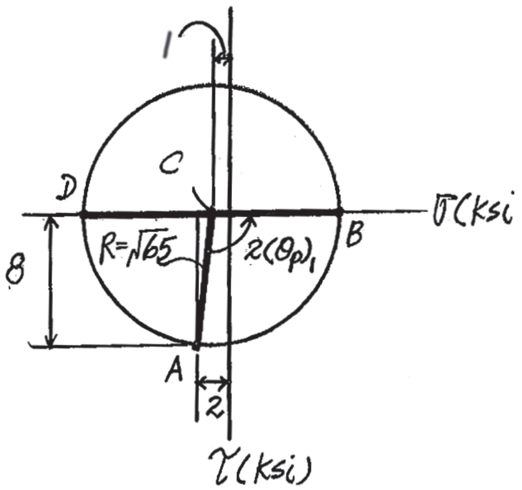
From the results obtained,

$$\sigma_{int} = 0 \text{ ksi} \qquad \sigma_{max} = 7.06 \text{ ksi} \qquad \sigma_{min} = -9.06 \text{ ksi} \qquad \textbf{Ans.}$$

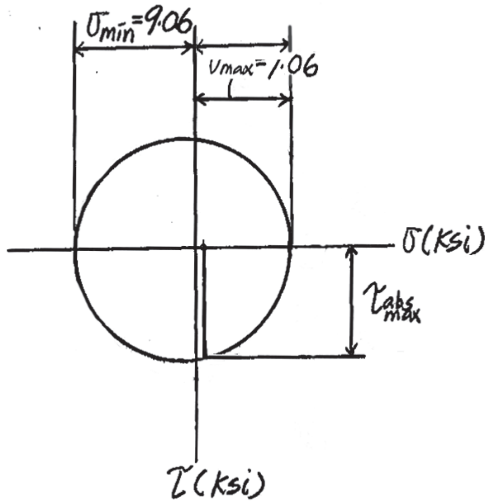
Using these results, the three Mohr's circles are shown in Fig, c.

From the geometry of the cricle,

$$\tau_{\substack{abs \\ max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{7.06 - (-9.06)}{2} = 8.06 \text{ ksi} \qquad \textbf{Ans.}$$

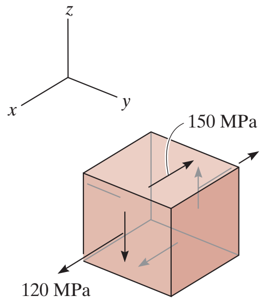


(b)



(c)

•9–89. The stress at a point is shown on the element. Determine the principal stress and the absolute maximum shear stress.



For $x - y$ plane:

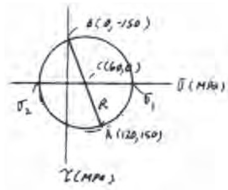
$$R = CA = \sqrt{(120 - 60)^2 + 150^2} = 161.55$$

$$\sigma_1 = 60 + 161.55 = 221.55 \text{ MPa}$$

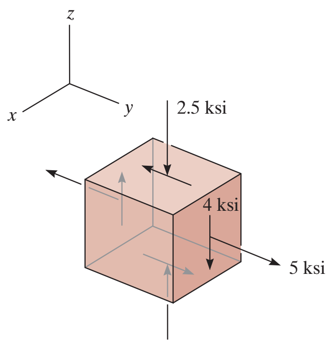
$$\sigma_2 = 60 - 161.55 = -101.55 \text{ MPa}$$

$$\sigma_1 = 222 \text{ MPa} \qquad \sigma_2 = 0 \text{ MPa} \qquad \sigma_3 = -102 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_{\substack{\text{abs}\\\text{max}}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{221.55 - (-101.55)}{2} = 162 \text{ MPa} \qquad \textbf{Ans.}$$



9–90. The state of stress at a point is shown on the element. Determine the principal stress and the absolute maximum shear stress.



For y - z plane:

$$A(5, -4) \qquad B(-2.5, 4) \qquad C(1.25, 0)$$

$$R = \sqrt{3.75^2 + 4^2} = 5.483$$

$$\sigma_1 = 1.25 + 5.483 = 6.733 \text{ ksi}$$

$$\sigma_2 = 1.25 - 5.483 = -4.233 \text{ ksi}$$

Thus,

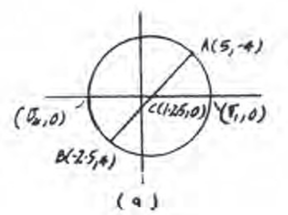
$$\sigma_1 = 6.73 \text{ ksi} \qquad \textbf{Ans.}$$

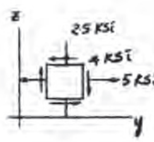
$$\sigma_2 = 0 \qquad \textbf{Ans.}$$

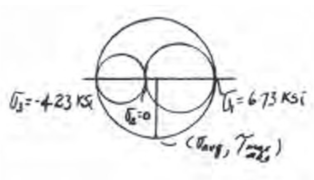
$$\sigma_3 = -4.23 \text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_{avg} = \frac{6.73 + (-4.23)}{2} = 1.25 \text{ ksi}$$

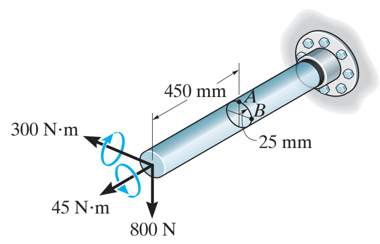
$$\tau_{\substack{\text{abs}\\\text{max}}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{6.73 - (-4.23)}{2} = 5.48 \text{ ksi} \qquad \textbf{Ans.}$$

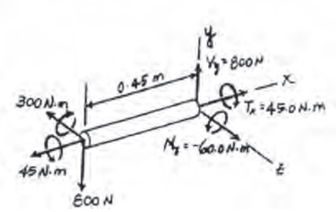






***9–92.** The solid shaft is subjected to a torque, bending moment, and shear force as shown. Determine the principal stress acting at points A and B and the absolute maximum shear stress.





Internal Forces and Moment: As shown on FBD.

Section Properties:

$$I_z = \frac{\pi}{4}\left(0.025^4\right) = 0.306796\left(10^{-6}\right)\text{ m}^4$$

$$J = \frac{\pi}{2}\left(0.025^4\right) = 0.613592\left(10^{-6}\right)\text{ m}^4$$

$$(Q_A)_x = 0$$

$$(Q_B)_y = \bar{y}'A'$$

$$= \frac{4(0.025)}{3\pi}\left[\frac{1}{2}(\pi)\left(0.025^2\right)\right] = 10.417\left(10^{-6}\right)\text{ m}^3$$

Normal stress: Applying the flexure formula.

$$\sigma = -\frac{M_z y}{I_z}$$

$$\sigma_A = -\frac{-60.0(0.025)}{0.306796(10^{-6})} = 4.889\text{ MPa}$$

$$\sigma_B = -\frac{-60.0(0)}{0.306796(10^{-6})} = 0$$

Shear Stress: Applying the torsion formula for point A,

$$\tau_A = \frac{Tc}{J} = \frac{45.0(0.025)}{0.613592(10^{-6})} = 1.833\text{ MPa}$$

The transverse shear stress in the y direction and the torsional shear stress can be obtained using shear formula and torsion formula. $\tau_v = \frac{VQ}{It}$ and $\tau_{\text{twist}} = \frac{T\rho}{J}$, respectively.

$$\tau_B = (\tau_v)_y - \tau_{\text{twist}}$$

$$= \frac{800\left[10.417(10^{-6})\right]}{0.306796(10^{-6})(0.05)} - \frac{45.0(0.025)}{0.613592(10^{-6})} = -1.290\text{ MPa}$$

Construction of the Circle: $\sigma_x = 4.889$ MPa, $\sigma_z = 0$, and $\tau_{xz} = -1.833$ MPa for point A. Hence,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_z}{2} = \frac{4.889 + 0}{2} = 2.445\text{ MPa}$$

The coordinates for reference points A and C are A (4.889, –1.833) and $C(2.445, 0)$.

9–92. Continued

The radius of the circle is

$$R = \sqrt{(4.889 - 2.445)^2 + 1.833^2} = 3.056 \text{ MPa}$$

$\sigma_x = \sigma_y = 0$ and $\tau_{xy} = -1.290$ MPa for point B. Hence,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_z}{2} = 0$$

The coordinates for reference points A and C are $A(0.,–1.290)$ and $C(0,0)$.

The radius of the circle is $R = 1.290$ MPa

In - Plane Principal Stresses: The coordinates of point B and D represent σ_1 and σ_2, respectively. For point A

$$\sigma_1 = 2.445 + 3.056 = 5.50 \text{ MPa}$$

$$\sigma_2 = 2.445 - 3.506 = -0.611 \text{ MPa}$$

For point B

$$\sigma_1 = 0 + 1.290 = 1.29 \text{ MPa}$$

$$\sigma_2 = 0 - 1.290 = -1.290 \text{ MPa}$$

Three Mohr's Circles: From the results obtaired above, the principal stresses for point A are

$$\sigma_{\max} = 5.50 \text{ MPa} \qquad \sigma_{\text{int}} = 0 \qquad \sigma_{\min} = -0.611 \text{ MPa} \qquad \textbf{Ans.}$$

And for point B

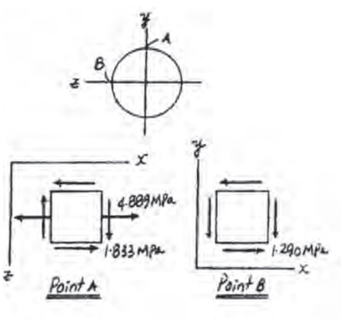
$$\sigma_{\max} = 1.29 \text{ MPa} \qquad \sigma_{\text{int}} = 0 \qquad \sigma_{\min} = -1.29 \text{ MPa} \qquad \textbf{Ans.}$$

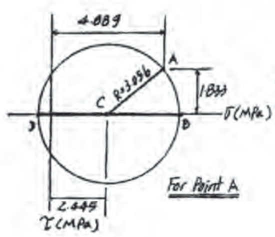
Absolute Maximum Shear Stress: For point A,

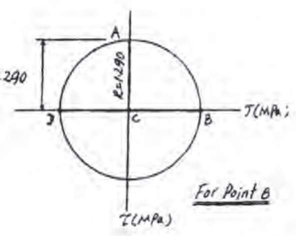
$$\tau_{\substack{\text{abs}\\ \max}} = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{5.50 - (-0.611)}{2} = 3.06 \text{ MPa} \qquad \textbf{Ans.}$$

For point B,

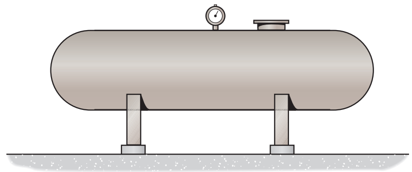
$$\tau_{\substack{\text{abs}\\ \max}} = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{1.29 - (-1.29)}{2} = 1.29 \text{ MPa} \qquad \textbf{Ans.}$$







•9–93. The propane gas tank has an inner diameter of 1500 mm and wall thickness of 15 mm. If the tank is pressurized to 2 MPa, determine the absolute maximum shear stress in the wall of the tank.

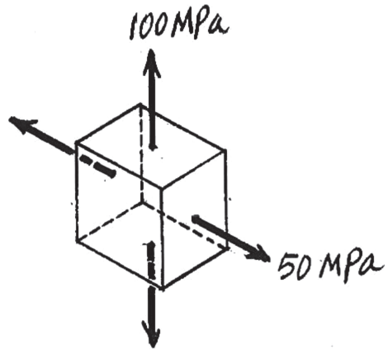


Normal Stress: Since $\frac{r}{t} = \frac{750}{15} = 50 > 10$, thin - wall analysis can be used. We have

$$\sigma_1 = \frac{pr}{t} = \frac{2(750)}{15} = 100 \text{ MPa}$$

$$\sigma_2 = \frac{pr}{2t} = \frac{2(750)}{2(15)} = 50 \text{ MPa}$$

The state of stress of any point on the wall of the tank can be represented on the element shown in Fig. *a*



(a)

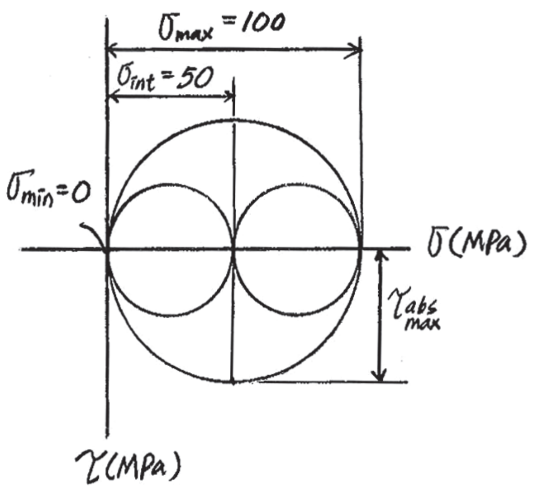
Construction of Three Mohr's Circles: Referring to the element,

$$\sigma_{max} = 100 \text{ MPa} \qquad \sigma_{int} = 50 \text{ MPa} \qquad \sigma_{min} = 0$$

Using these results, the three Mohr's circles are shown in Fig. *b*.

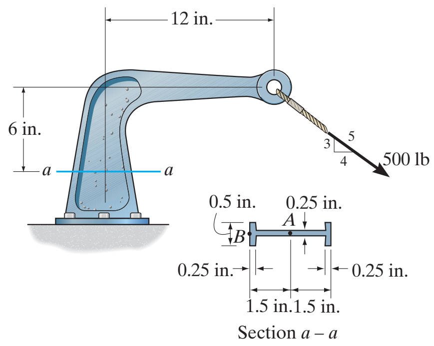
Absolute Maximum Shear Stress: From the geometry of three circles,

$$\tau_{\substack{abs\\max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{100 - 0}{2} = 50 \text{ MPa} \qquad \textbf{Ans.}$$



(b)

9–94. Determine the principal stress and absolute maximum shear stress developed at point A on the cross section of the bracket at section a–a.



Internal Loadings: Considering the equilibrium of the free - body diagram of the bracket's upper cut segment, Fig. a,

$$+\uparrow \Sigma F_y = 0; \qquad N - 500\left(\frac{3}{5}\right) = 0 \qquad N = 300 \text{ lb}$$

$$\overset{+}{\leftarrow} \Sigma F_x = 0; \qquad V - 500\left(\frac{4}{5}\right) = 0 \qquad V = 400 \text{ lb}$$

$$\Sigma M_O = 0; \; M - 500\left(\frac{3}{5}\right)(12) - 500\left(\frac{4}{5}\right)(6) = 0 \qquad M = 6000 \text{ lb}\cdot\text{in}$$

Section Properties: The cross - sectional area and the moment of inertia of the bracket's cross section are

$$A = 0.5(3) - 0.25(2.5) = 0.875 \text{ in}^2$$

$$I = \frac{1}{12}(0.5)\left(3^3\right) - \frac{1}{12}(0.25)\left(2.5^3\right) = 0.79948 \text{ in}^4$$

Referring to Fig. b.

$$Q_A = \bar{x}_1' A_1' + \bar{x}_2' A_2' = 0.625(1.25)(0.25) + 1.375(0.25)(0.5) = 0.3672 \text{ in}^3$$

Normal and Shear Stress: The normal stress is

$$\sigma_A = \frac{N}{A} = -\frac{300}{0.875} = -342.86 \text{ psi}$$

The shear stress is contributed by the transverse shear stress.

$$\tau_A = \frac{VQ_A}{It} = \frac{400(0.3672)}{0.79948(0.25)} = 734.85 \text{ psi}$$

The state of stress at point A is represented by the element shown in Fig. c.

Construction of the Circle: $\sigma_x = 0$, $\sigma_y = -342.86$ psi, and $\tau_{xy} = 734.85$. Thus,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + (-342.86)}{2} = -171.43 \text{ psi}$$

The coordinates of reference point A and the center C of the circle are

$$A(0, 734.85) \qquad C(-171.43, 0)$$

Thus, the radius of the circle is

$$R = CA = \sqrt{[0 - (-171.43)]^2 + 734.85^2} = 754.58 \text{ psi}$$

9–94. Continued

Using these results, the cricle is shown in Fig. *d*.

In - Plane Principal Stresses: The coordinates of reference point *B* and *D* represent σ_1 and σ_2, respectively.

$$\sigma_1 = -171.43 + 754.58 = 583.2 \text{ psi}$$

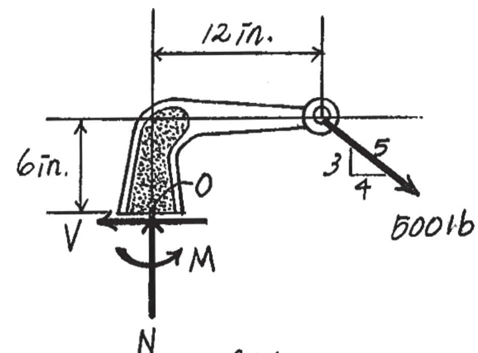
$$\sigma_2 = -171.43 - 754.58 = -926.0 \text{ psi}$$

Three Mohr's Circles: Using these results,

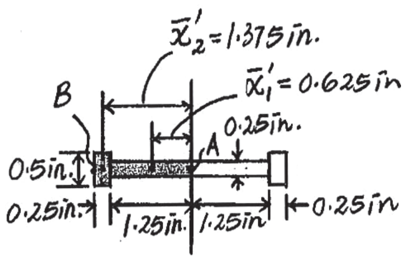
$$\sigma_{max} = 583 \text{ psi} \qquad \sigma_{int} = 0 \quad \sigma_{min} = -926 \text{ psi} \qquad \textbf{Ans.}$$

Absolute Maximum Shear Stress:

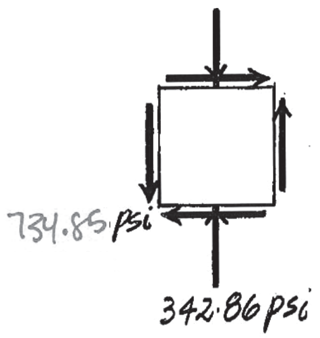
$$\tau_{\substack{abs \\ max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{583.2 - (-926.0)}{2} - 755 \text{ psi} \qquad \textbf{Ans.}$$



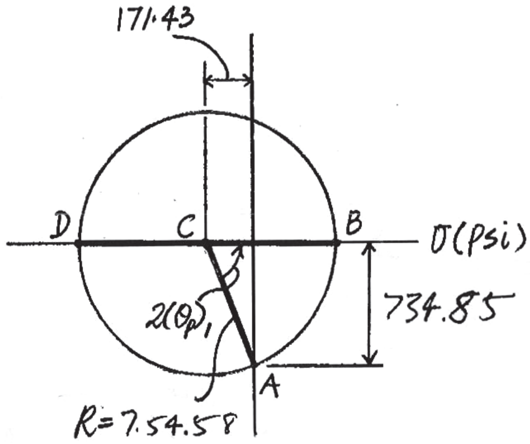
(a)



(b)

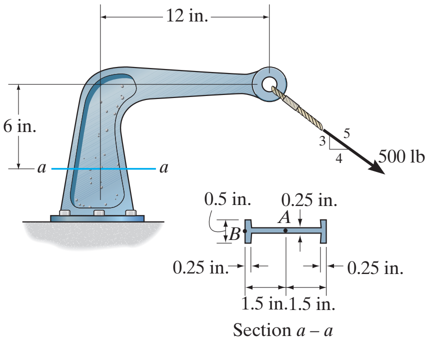


(C)



(d)

9–95. Determine the principal stress and absolute maximum shear stress developed at point B on the cross section of the bracket at section a–a.

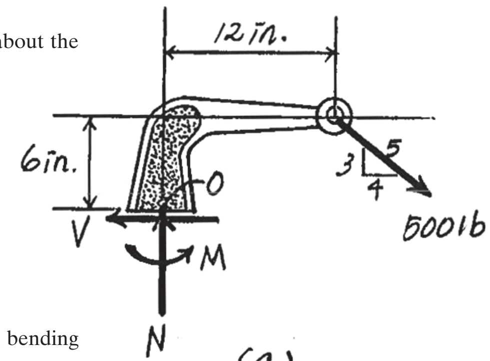


Internal Loadings: Considering the equilibrium of the free - body diagram of the bracket's upper cut segment, Fig. a,

$$+\uparrow \Sigma F_y = 0; \qquad N - 500\left(\frac{3}{5}\right) = 0 \qquad N = 300 \text{ lb}$$

$$\overset{+}{\leftarrow} \Sigma F_x = 0; \qquad V - 500\left(\frac{4}{5}\right) = 0 \qquad V = 400 \text{ lb}$$

$$\Sigma M_O = 0; \; M - 500\left(\frac{3}{5}\right)(12) - 500\left(\frac{4}{5}\right)(6) = 0 \qquad M = 6000 \text{ lb}\cdot\text{in}$$



(a)

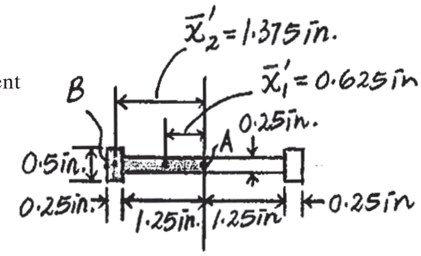
Section Properties: The cross - sectional area and the moment of inertia about the centroidal axis of the bracket's cross section are

$$A = 0.5(3) - 0.25(2.5) = 0.875 \text{ in}^2$$

$$I = \frac{1}{12}(0.5)\left(3^3\right) - \frac{1}{12}(0.25)\left(2.5^3\right) = 0.79948 \text{ in}^4$$

Referring to Fig. b,

$$Q_B = 0$$



(b)

Normal and Shear Stress: The normal stress is a combination of axial and bending stress.

$$\sigma_B = \frac{N}{A} + \frac{Mx_B}{I} = -\frac{300}{0.875} + \frac{6000(1.5)}{0.79948} = 10.9 \text{ ksi}$$

Since $Q_B = 0$, $\tau_B = 0$. The state of stress at point B is represented on the element shown in Fig. c.

In - Plane Principal Stresses: Since no shear stress acts on the element,

$$\sigma_1 = 10.91 \text{ ksi} \qquad \sigma_2 = 0$$

Three Mohr's Circles: Using these results,

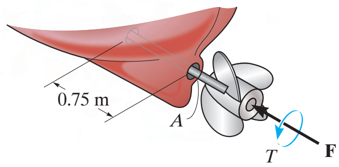
$$\sigma_{max} = 10.91 \text{ ksi} \qquad \sigma_{int} = \sigma_{min} = 0 \qquad \textbf{Ans.}$$

Absolute Maximum Shear Stress:

$$\tau_{\substack{abs \\ max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{10.91 - 0}{2} = 5.46 \text{ ksi} \qquad \textbf{Ans.}$$

(c)

***9–96.** The solid propeller shaft on a ship extends outward from the hull. During operation it turns at $\omega = 15$ rad/s when the engine develops 900 kW of power. This causes a thrust of $F = 1.23$ MN on the shaft. If the shaft has an outer diameter of 250 mm, determine the principal stresses at any point located on the surface of the shaft.



Power Transmission: Using the formula developed in Chapter 5,

$$P = 900 \text{ kW} = 0.900\left(10^6\right) \text{ N} \cdot \text{m/s}$$

$$T_0 = \frac{P}{\omega} = \frac{0.900(10^6)}{15} = 60.0\left(10^3\right) \text{ N} \cdot \text{m}$$

Internal Torque and Force: As shown on FBD.

Section Properties:

$$A = \frac{\pi}{4}\left(0.25^2\right) = 0.015625\pi \text{ m}^2$$

$$J = \frac{\pi}{2}\left(0.125^4\right) = 0.3835\left(10^{-3}\right) \text{ m}^4$$

Normal Stress:

$$\sigma = \frac{N}{A} = \frac{-1.23(10^6)}{0.015625\pi} = -25.06 \text{ MPa}$$

Shear Stress: Applying the torsion formula,

$$\tau = \frac{Tc}{J} = \frac{60.0(10^3)(0.125)}{0.3835(10^{-3})} = 19.56 \text{ MPa}$$

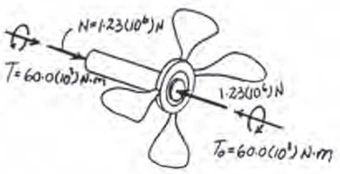
In - Plane Principal Stresses: $\sigma_x = -25.06$ MPa, $\sigma_y = 0$ and $\tau_{xy} = 19.56$ MPa for any point on the shaft's surface. Applying Eq. 9-5,

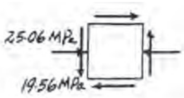
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-25.06 + 0}{2} \pm \sqrt{\left(\frac{-25.06 - 0}{2}\right)^2 + (19.56)^2}$$

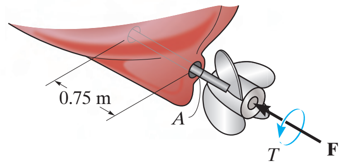
$$= -12.53 \pm 23.23$$

$$\sigma_1 = 10.7 \text{ MPa} \qquad \sigma_2 = -35.8 \text{ MPa}$$ **Ans.**





•9–97. The solid propeller shaft on a ship extends outward from the hull. During operation it turns at $\omega = 15$ rad/s when the engine develops 900 kW of power. This causes a thrust of $F = 1.23$ MN on the shaft. If the shaft has a diameter of 250 mm, determine the maximum in-plane shear stress at any point located on the surface of the shaft.



Power Transmission: Using the formula developed in Chapter 5,

$$P = 900 \text{ kW} = 0.900\left(10^6\right) \text{N} \cdot \text{m/s}$$

$$T_0 = \frac{P}{\omega} = \frac{0.900(10^6)}{15} = 60.0\left(10^3\right) \text{N} \cdot \text{m}$$

Internal Torque and Force: As shown on FBD.

Section Properties:

$$A = \frac{\pi}{4}\left(0.25^2\right) = 0.015625\pi \text{ m}^2$$

$$J = \frac{\pi}{2}\left(0.125^4\right) = 0.3835\left(10^{-3}\right) \text{m}^4$$

Normal Stress:

$$\sigma = \frac{N}{A} = \frac{-1.23(10^6)}{0.015625\pi} = -25.06 \text{ MPa}$$

Shear Stress: Applying the torsion formula.

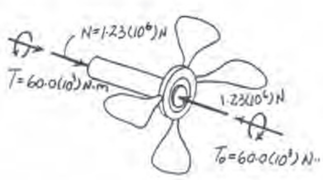
$$\tau = \frac{Tc}{J} = \frac{60.0(10^3)\,(0.125)}{0.3835\,(10^{-3})} = 19.56 \text{ MPa}$$

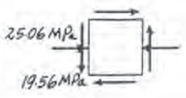
Maximum In - Plane Principal Shear Stress: $\sigma_x = -25.06$ MPa, $\sigma_y = 0$, and $\tau_{xy} = 19.56$ MPa for any point on the shaft's surface. Applying Eq. 9-7,

$$\tau_{\substack{\text{max} \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

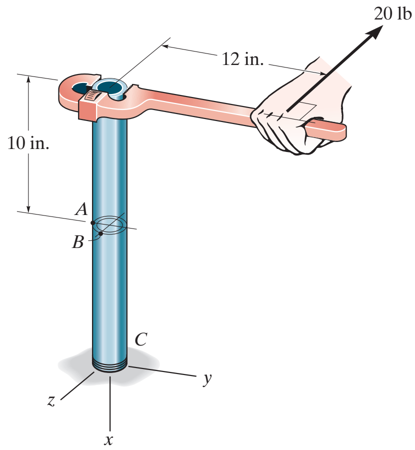
$$= \sqrt{\left(\frac{-25.06 - 0}{2}\right)^2 + (19.56)^2}$$

$$= 23.2 \text{ MPa} \qquad \textbf{Ans.}$$

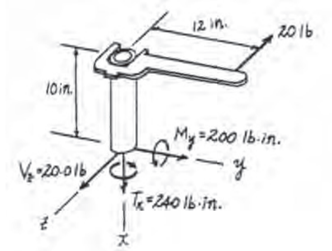




9–98. The steel pipe has an inner diameter of 2.75 in. and an outer diameter of 3 in. If it is fixed at C and subjected to the horizontal 20-lb force acting on the handle of the pipe wrench at its end, determine the principal stresses in the pipe at point A, which is located on the surface of the pipe.



Internal Forces, Torque and Moment: As shown on FBD.



Section Properties:

$$I = \frac{\pi}{4}\left(1.5^4 - 1.375^4\right) = 1.1687 \text{ in}^4$$

$$J = \frac{\pi}{2}\left(1.5^4 - 1.375^4\right) = 2.3374 \text{ in}^4$$

$$(Q_A)_z = \Sigma \bar{y}' A'$$

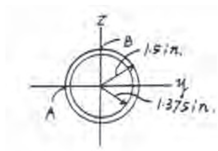
$$= \frac{4(1.5)}{3\pi}\left[\frac{1}{2}\pi\left(1.5^2\right)\right] - \frac{4(1.375)}{3\pi}\left[\frac{1}{2}\pi\left(1.375^2\right)\right]$$

$$= 0.51693 \text{ in}^3$$

Normal Stress: Applying the flexure formula $\sigma = \frac{M_y z}{I_y}$,

$$\sigma_A = \frac{200(0)}{1.1687} = 0$$

Shear Stress: The transverse shear stress in the z direction and the torsional shear stress can be obtained using shear formula and torsion formula, $\tau_v = \frac{VQ}{It}$ and $\tau_{\text{twist}} = \frac{T\rho}{J}$, respectively.



$$\tau_A = (\tau_v)_z - \tau_{\text{twist}}$$

$$= \frac{20.0(0.51693)}{1.1687(2)(0.125)} - \frac{240(1.5)}{2.3374}$$

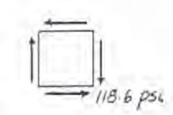
$$= -118.6 \text{ psi}$$

In - Plane Principal Stress: $\sigma_x = 0$, $\sigma_z = 0$ and $\tau_{xz} = -118.6$ psi for point A. Applying Eq. 9-5

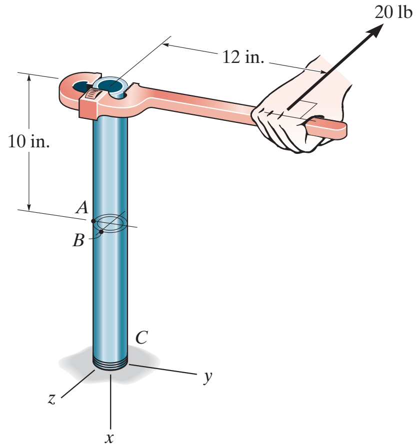
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_z}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2}$$

$$= 0 \pm \sqrt{0 + (-118.6)^2}$$

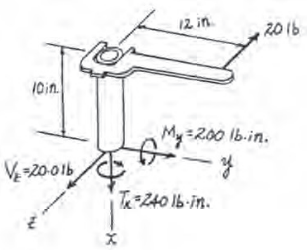
$\sigma_1 = 119$ psi $\qquad \sigma_2 = -119$ psi **Ans.**



9–99. Solve Prob. 9–98 for point B, which is located on the surface of the pipe.



Internal Forces, Torque and Moment: As shown on FBD.



Section Properties:

$$I = \frac{\pi}{4}\left(1.5^4 - 1.375^4\right) = 1.1687 \text{ in}^4$$

$$J = \frac{\pi}{2}\left(1.5^4 - 1.375^4\right) = 2.3374 \text{ in}^4$$

$$(Q_B)_z = 0$$

Normal Stress: Applying the flexure formula $\sigma = \frac{M_y z}{I_y}$,

$$\sigma_B = \frac{200(1.5)}{1.1687} = 256.7 \text{ psi}$$

Shear Stress: Torsional shear stress can be obtained using torsion formula, $\tau_{\text{twist}} = \frac{T\rho}{J}$.

$$\tau_B = \tau_{\text{twist}} = \frac{240(1.5)}{2.3374} = 154.0 \text{ psi}$$

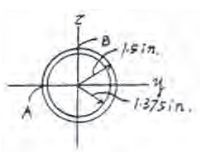
In - Plane Prinicipal Stress: $\sigma_x = 256.7$ psi, $\sigma_y = 0$, and $\tau_{xy} = -154.0$ psi for point B. Applying Eq. 9-5

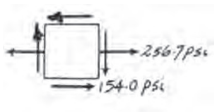
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{256.7 + 0}{2} \pm \sqrt{\left(\frac{256.7 - 0}{2}\right)^2 + (-154.0)^2}$$

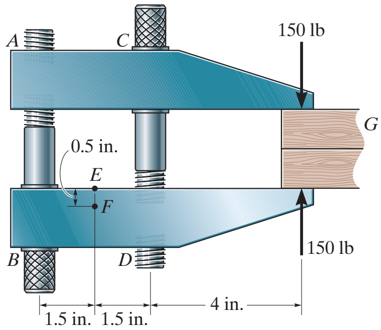
$$= 128.35 \pm 200.49$$

$\sigma_1 = 329$ psi $\qquad \sigma_2 = -72.1$ psi **Ans.**





***9–100.** The clamp exerts a force of 150 lb on the boards at G. Determine the axial force in each screw, AB and CD, and then compute the principal stresses at points E and F. Show the results on properly oriented elements located at these points. The section through EF is rectangular and is 1 in. wide.



Support Reactions: FBD(a).

$\zeta + \Sigma M_B = 0; \quad F_{CD}(3) - 150(7) = 0 \quad F_{CD} = 350 \text{ lb}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad 350 - 150 - F_{AB} = 0 \quad F_{AB} = 200 \text{ lb}$ **Ans.**

Internal Forces and Moment: As shown on FBD(b).

Section Properties:

$$I = \frac{1}{12}(1)\left(1.5^3\right) = 0.28125 \text{ in}^4$$

$$Q_E = 0$$

$$Q_F = \bar{y}'A' = 0.5(0.5)(1) = 0.250 \text{ in}^3$$

Normal Stress: Applying the flexure formula $\sigma = -\dfrac{My}{I}$,

$$\sigma_E = -\frac{-300(0.75)}{0.28125} = 800 \text{ psi}$$

$$\sigma_F = -\frac{-300(0.25)}{0.28125} = 266.67 \text{ psi}$$

Shear Stress: Applying the shear formula $\tau = \dfrac{VQ}{It}$,

$$\tau_E = \frac{200(0)}{0.28125(1)} = 0$$

$$\tau_F = \frac{200(0.250)}{0.28125(1)} = 177.78 \text{ psi}$$

In - Plane Principal Stress: $\sigma_x = 800$ psi, $\sigma_y = 0$ and $\tau_{xy} = 0$ for point E. Since no shear stress acts upon the element.

$$\sigma_1 = \sigma_x = 800 \text{ psi}$$ **Ans.**

$$\sigma_2 = \sigma_y = 0$$ **Ans.**

$\sigma_x = 266.67$ psi, $\sigma_y = 0$, and $\tau_{xy} = 177.78$ psi for point F. Applying Eq. 9-5

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{266.67 + 0}{2} \pm \sqrt{\left(\frac{266.67 - 0}{2}\right)^2 + 177.78^2}$$

$$= 133.33 \pm 222.22$$

$\sigma_1 = 356 \text{ psi} \qquad \sigma_2 = -88.9 \text{ psi}$ **Ans.**

9–100. Continued

Orientation of Principal Plane: Applying Eq. 9-4 for point F,

$$\tan 2\theta_p = \frac{\tau_{xy}}{\left(\sigma_x - \sigma_y\right)/2} = \frac{177.78}{(266.67 - 0)/2} = 1.3333$$

$$\theta_p = 26.57^\circ \qquad \text{and} \qquad -63.43^\circ$$

Substituting the results into Eq. 9-1 with $\theta = 26.57^\circ$ yields

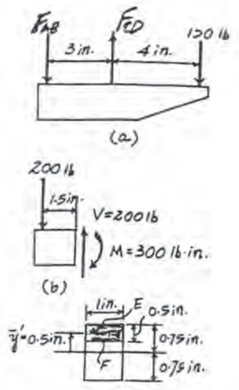
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

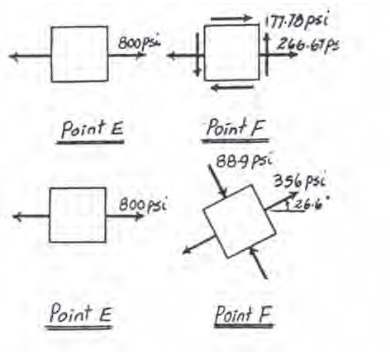
$$= \frac{266.67 + 0}{2} + \frac{266.67 - 0}{2}\cos 53.13^\circ + 177.78 \sin 53.13^\circ$$

$$= 356 \text{ psi} = \sigma_1$$

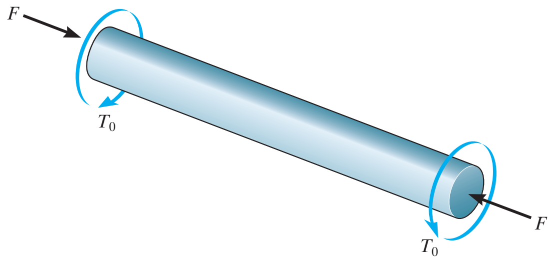
Hence,

$$\theta_{p1} = 26.6^\circ \qquad \theta_{p2} = -63.4^\circ \qquad \textbf{Ans.}$$

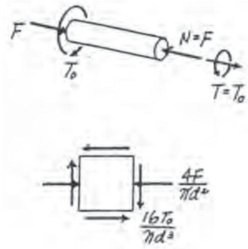




9–101. The shaft has a diameter d and is subjected to the loadings shown. Determine the principal stress and the maximum in-plane shear stress that is developed anywhere on the surface of the shaft.



Internal Forces and Torque: As shown on FBD(b).



Section Properties:

$$A = \frac{\pi}{4} d^2 \qquad J = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi}{32} d^4$$

Normal Stress:

$$\sigma = \frac{N}{A} = \frac{-F}{\frac{\pi}{4} d^2} = -\frac{4F}{\pi d^2}$$

Shear Stress: Applying the shear torsion formula,

$$\tau = \frac{Tc}{J} = \frac{T_0\left(\frac{d}{2}\right)}{\frac{\pi}{32} d^4} = \frac{16T_0}{\pi d^3}$$

In - Plane Principal Stress: $\sigma_x = -\frac{4F}{\pi d^2}$, $\sigma_y = 0$, and $\tau_{xy} = -\frac{16T_0}{\pi d^3}$ for any point on the shaft's surface. Applying Eq. 9-5,

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-\frac{4F}{\pi d^2} + 0}{2} \pm \sqrt{\left(\frac{-\frac{4F}{\pi d^2} - 0}{2}\right)^2 + \left(-\frac{16T_0}{\pi d^3}\right)^2}$$

$$= \frac{2}{\pi d^2}\left(-F \pm \sqrt{F^2 + \frac{64T_0^2}{d^2}}\right)$$

$$\sigma_1 = \frac{2}{\pi d^2}\left(-F + \sqrt{F^2 + \frac{64T_0^2}{d^2}}\right) \qquad \textbf{Ans.}$$

$$\sigma_2 = -\frac{2}{\pi d^2}\left(F + \sqrt{F^2 + \frac{64T_0^2}{d^2}}\right) \qquad \textbf{Ans.}$$

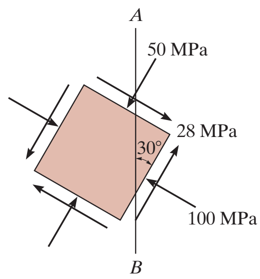
Maximum In - Plane Shear Stress: Applying Eq. 9-7,

$$\tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{-\frac{4F}{\pi d^2} - 0}{2}\right)^2 + \left(-\frac{16T_0}{\pi d^3}\right)^2}$$

$$= \frac{2}{\pi d^2}\sqrt{F^2 + \frac{64T_0^2}{d^2}} \qquad \textbf{Ans.}$$

9–102. The state of stress at a point in a member is shown on the element. Determine the stress components acting on the plane AB.



Construction of the Circle: In accordance with the sign convention, $\sigma_x = -50$ MPa, $\sigma_y = -100$ MPa, and $\tau_{xy} = -28$ MPa. Hence,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{-50 + (-100)}{2} = -75.0 \text{ MPa}$$

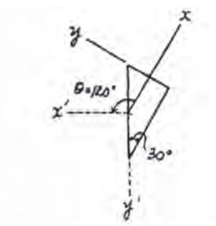
The coordinates for reference points A and C are $A(-50, -28)$ and $C(-75.0, 0)$.

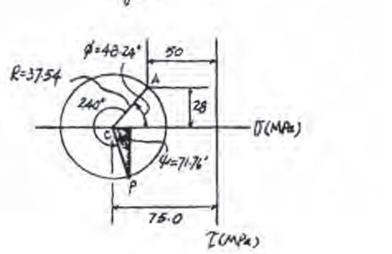
The radius of the circle is $R = \sqrt{(75.0 - 50)^2 + 28^2} = 37.54$ MPa.

Stress on the Rotated Element: The normal and shear stress components $(\sigma_{x'}$ and $\tau_{x'y'})$ are represented by the coordinates of point P on the circle

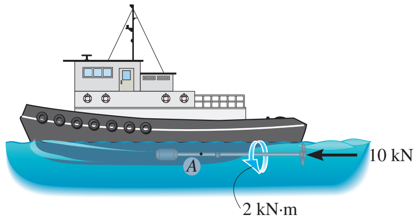
$$\sigma_{x'} = -75.0 + 37.54 \cos 71.76° = -63.3 \text{ MPa} \qquad \textbf{Ans.}$$

$$\tau_{x'y'} = 37.54 \sin 71.76° = 35.7 \text{ MPa} \qquad \textbf{Ans.}$$





9–103. The propeller shaft of the tugboat is subjected to the compressive force and torque shown. If the shaft has an inner diameter of 100 mm and an outer diameter of 150 mm, determine the principal stress at a point A located on the outer surface.



Internal Loadings: Considering the equilibrium of the free - body diagram of the propeller shaft's right segment, Fig. *a*,

$$\Sigma F_x = 0; \quad 10 - N = 0 \qquad\qquad N = 10 \text{ kN}$$

$$\Sigma M_x = 0; \quad T - 2 = 0 \qquad\qquad T = 2 \text{ kN} \cdot \text{m}$$

Section Properties: The cross - sectional area and the polar moment of inertia of the propeller shaft's cross section are

$$A = \pi\left(0.075^2 - 0.05^2\right) = 3.125\pi\left(10^{-3}\right) \text{ m}^2$$

$$J = \frac{\pi}{2}\left(0.075^4 - 0.05^4\right) = 12.6953125\pi\left(10^{-6}\right) \text{ m}^4$$

Normal and Shear Stress: The normal stress is a contributed by axial stress only.

$$\sigma_A = \frac{N}{A} = -\frac{10\left(10^3\right)}{3.125\pi\left(10^{-3}\right)} = -1.019 \text{ MPa}$$

The shear stress is contributed by the torsional shear stress only.

$$\tau_A = \frac{Tc}{J} = \frac{2\left(10^3\right)(0.075)}{12.6953125\pi\left(10^{-6}\right)} = 3.761 \text{ MPa}$$

The state of stress at point A is represented by the element shown in Fig. *b*.

Construction of the Circle: $\sigma_x = -1.019$ MPa, $\sigma_y = 0$, and $\tau_{xy} = -3.761$ MPa. Thus,

$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{-1.019 + 0}{2} = -0.5093 \text{ MPa}$$

The coordinates of reference point A and the center C of the circle are

$$A(-1.019, -3.761) \qquad\qquad C(-0.5093, 0)$$

Thus, the radius of the circle is

$$R = CA = \sqrt{[-1.019 - (-0.5093)]^2 + (-3.761)^2} = 3.795 \text{ MPa}$$

Using these results, the circle is shown is Fig. *c*.

In - Plane Principal Stress: The coordinates of reference points B and D represent σ_1 and σ_2, respectively.

$$\sigma_1 = -0.5093 + 3.795 = 3.29 \text{ MPa} \qquad \textbf{Ans.}$$

$$\sigma_2 = -0.5093 - 3.795 = -4.30 \text{ MPa} \qquad \textbf{Ans.}$$

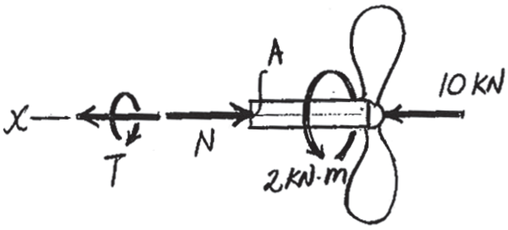
9–103. Continued

Orientation of the Principal Plane: Referring to the geometry of the circle, Fig. *d*,

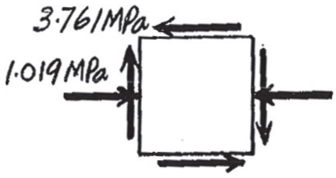
$$\tan 2\left(\theta_p\right)_2 = \frac{3.761}{1.019 - 0.5093} = 7.3846$$

$$\left(\theta_p\right)_2 = 41.1^\circ \text{ (clockwise)} \qquad \textbf{Ans.}$$

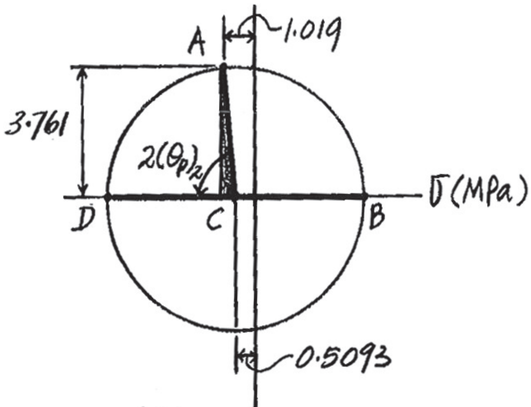
The state of principal stresses is represented on the element shown in Fig. *d*.



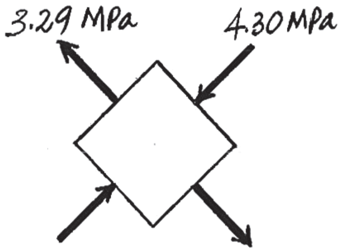
(a)



(b)

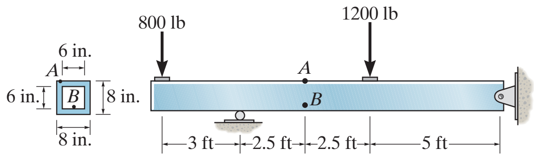


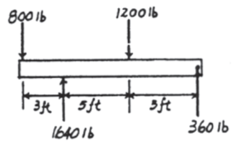
(c)



(d)

***9–104.** The box beam is subjected to the loading shown. Determine the principal stress in the beam at points A and B.





Support Reactions: As shown on FBD(a).

Internal Forces and Moment: As shown on FBD(b).

Section Properties:

$$I = \frac{1}{12}(8)\left(8^3\right) - \frac{1}{12}(6)\left(6^3\right) = 233.33 \text{ in}^4$$

$$Q_A = Q_B = 0$$

Normal Stress: Applying the flexure formula.

$$\sigma = -\frac{M_y}{I}$$

$$\sigma_A = -\frac{-300(12)(4)}{233.33} = 61.71 \text{ psi}$$

$$\sigma_B = -\frac{-300(12)(-3)}{233.33} = -46.29 \text{ psi}$$

Shear Stress: Since $Q_A = Q_B = 0$, then $\tau_A = \tau_B = 0$.

In - Plane Principal Stress: $\sigma_x = 61.71$ psi, $\sigma_y = 0$, and $\tau_{xy} = 0$ for point A. Since no shear stress acts on the element,

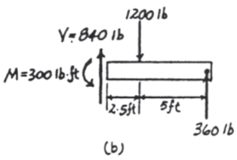
$$\sigma_1 = \sigma_x = 61.7 \text{ psi} \qquad \textbf{Ans.}$$

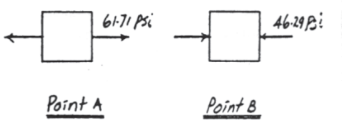
$$\sigma_2 = \sigma_y = 0 \qquad \textbf{Ans.}$$

$\sigma_x = -46.29$ psi, $\sigma_y = 0$, and $\tau_{xy} = 0$ for point B. Since no shear stress acts on the element,

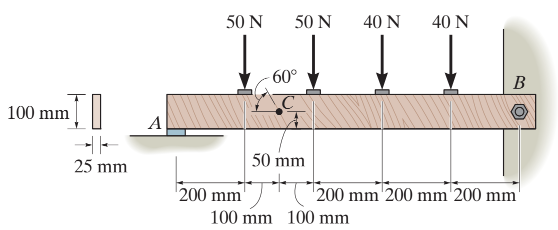
$$\sigma_1 = \sigma_y = 0 \qquad \textbf{Ans.}$$

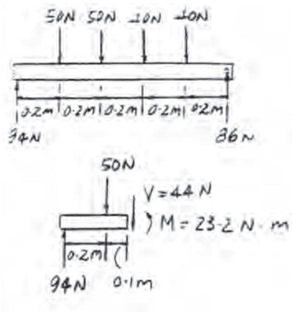
$$\sigma_2 = \sigma_x = -46.3 \text{ psi} \qquad \textbf{Ans.}$$





•9–105. The wooden strut is subjected to the loading shown. Determine the principal stresses that act at point C and specify the orientation of the element at this point. The strut is supported by a bolt (pin) at B and smooth support at A.





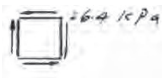
$$Q_C = \bar{y}'A' = 0.025(0.05)(0.025) = 31.25(10^{-6})\ \text{m}^3$$

$$I = \frac{1}{12}(0.025)(0.1^3) = 2.0833(10^{-6})\ \text{m}^4$$

Normal stress: $\sigma_C = 0$

Shear stress:

$$\tau = \frac{VQ_C}{It} = \frac{44(31.25)(10^{-6})}{2.0833(10^{-6})(0.025)} = 26.4\ \text{kPa}$$



Principal stress:

$$\sigma_x = \sigma_y = 0; \qquad \tau_{xy} = -26.4\ \text{kPa}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2{}_{xy}}$$

$$= 0 \pm \sqrt{0 + (26.4)^2}$$

$\sigma_1 = 26.4\ \text{kPa}$; $\sigma_2 = -26.4\ \text{kPa}$ **Ans.**

Orientation of principal stress:

$$\tan 2\theta_p = \frac{\tau_{xy}}{\frac{(\sigma_x - \sigma_y)}{2}} = -\infty$$

$$\theta_p = +45° \text{ and } -45°$$

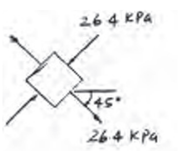
Use Eq. 9-1 to determine the principal plane of σ_1 and σ_2

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

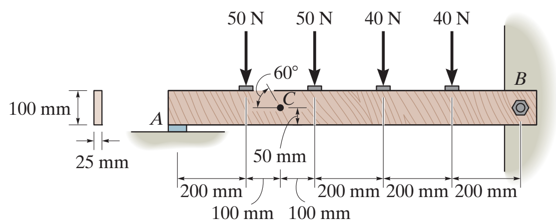
$$\theta = \theta_p = -45°$$

$$\sigma_{x'} = 0 + 0 + (-26.4)\sin(-90°) = 26.4\ \text{kPa}$$

Therefore, $\theta_{p_1} = -45°$; $\theta_{p_2} = 45°$ **Ans.**



9–106. The wooden strut is subjected to the loading shown. If grains of wood in the strut at point C make an angle of 60° with the horizontal as shown, determine the normal and shear stresses that act perpendicular and parallel to the grains, respectively, due to the loading. The strut is supported by a bolt (pin) at B and smooth support at A.



$$Q_C = y'A' = 0.025(0.05)(0.025) = 31.25(10^{-6})\ \text{m}^3$$

$$I = \frac{1}{12}(0.025)(0.1^3) = 2.0833(10^{-6})\ \text{m}^4$$

Normal stress: $\sigma_C = 0$

Shear stress:

$$\tau = \frac{VQ_C}{I\,t} = \frac{44(31.25)(10^{-6})}{2.0833(10^{-6})(0.025)} = 26.4\ \text{kPa}$$

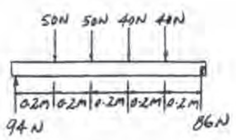
Stress transformation: $\sigma_x = \sigma_y = 0; \qquad \tau_{xy} = -26.4\ \text{kPa}; \qquad \theta = 30°$

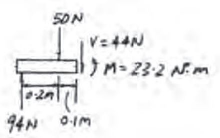
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= 0 + 0 + (-26.4)\sin 60° = -22.9\ \text{kPa}$$ **Ans.**

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -0 + (-26.4)\cos 60° = -13.2\ \text{kPa}$$ **Ans.**





10–1. Prove that the sum of the normal strains in perpendicular directions is constant.

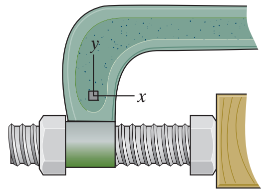
$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta \qquad \textbf{(1)}$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta \qquad \textbf{(2)}$$

Adding Eq. (1) and Eq. (2) yields:

$$\varepsilon_{x'} + \varepsilon_{y'} = \varepsilon_x + \varepsilon_y = \text{constant} \qquad \textbf{QED}$$

10–2. The state of strain at the point has components of $\epsilon_x = 200(10^{-6})$, $\epsilon_y = -300(10^{-6})$, and $\gamma_{xy} = 400(10^{-6})$. Use the strain-transformation equations to determine the equivalent in-plane strains on an element oriented at an angle of 30° counterclockwise from the original position. Sketch the deformed element due to these strains within the x–y plane.



In accordance to the established sign convention,

$$\varepsilon_x = 200(10^{-6}), \qquad \varepsilon_y = -300(10^{-6}) \qquad \gamma_{xy} = 400(10^{-6}) \qquad \theta = 30°$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{200 + (-300)}{2} + \frac{200 - (-300)}{2}\cos 60° + \frac{400}{2}\sin 60°\right](10^{-6})$$

$$= 248\,(10^{-6})$$ **Ans.**

$$\frac{\gamma_{x'y'}}{2} = -\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = \left\{-\left[200 - (-300)\right]\sin 60° + 400\cos 60°\right\}(10^{-6})$$

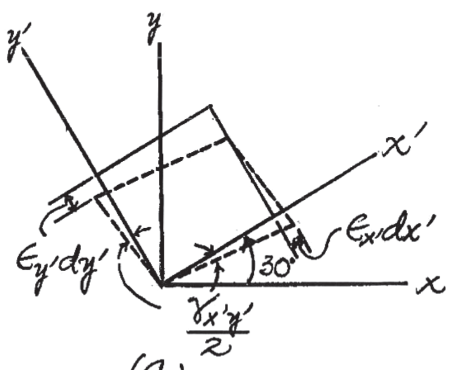
$$= -233(10^{-6})$$ **Ans.**

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{200 + (-300)}{2} - \frac{200 - (-300)}{2}\cos 60° - \frac{400}{2}\sin 60°\right](10^{-6})$$

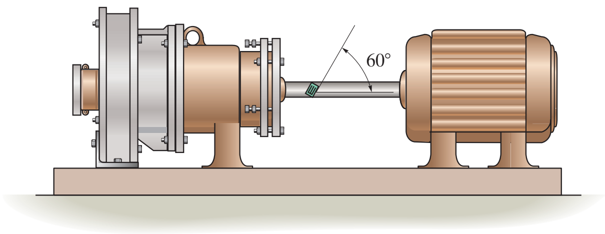
$$= -348(10^{-6})$$ **Ans.**

The deformed element of this equivalent state of strain is shown in Fig. *a*



(a)

10–3. A strain gauge is mounted on the 1-in.-diameter A-36 steel shaft in the manner shown. When the shaft is rotating with an angular velocity of $\omega = 1760$ rev/min, the reading on the strain gauge is $\epsilon = 800(10^{-6})$. Determine the power output of the motor. Assume the shaft is only subjected to a torque.



$$\omega = (1760 \text{ rev/min})\left(\frac{1 \text{ min}}{60 \text{ sec}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 184.307 \text{ rad/s}$$

$$\varepsilon_x = \varepsilon_y = 0$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2} \cos 2\theta + \frac{\gamma_{xy}}{2} \sin 2\theta$$

$$800(10^{-6}) = 0 + 0 + \frac{\gamma_{xy}}{2} \sin 120°$$

$$\gamma_{xy} = 1.848(10^{-3}) \text{ rad}$$

$$\tau = G\,\gamma_{xy} = 11(10^3)(1.848)(10^{-3}) = 20.323 \text{ ksi}$$

$$\tau = \frac{Tc}{J}; \qquad 20.323 = \frac{T(0.5)}{\frac{\pi}{2}(0.5)^4};$$

$$T = 3.99 \text{ kip} \cdot \text{in} = 332.5 \text{ lb} \cdot \text{ft}$$

$$P = T\omega = 0.332.5\,(184.307) = 61.3 \text{ kips} \cdot \text{ft/s} = 111 \text{ hp}$$ **Ans.**

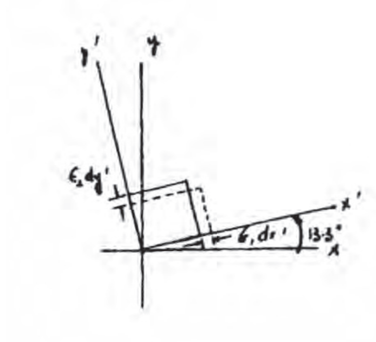
***10–4.** The state of strain at a point on a wrench has components $\epsilon_x = 120(10^{-6})$, $\epsilon_y = -180(10^{-6})$, $\gamma_{xy} = 150(10^{-6})$. Use the strain-transformation equations to determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and average normal strain. In each case specify the orientation of the element and show how the strains deform the element within the *x–y* plane.

$\varepsilon_x = 120(10^{-6}) \qquad \varepsilon_y = -180(10^{-6}) \qquad \gamma_{xy} = 150(10^{-6})$

a) $$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{E_x - E_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left[\frac{120 + (-180)}{2} \pm \sqrt{\left(\frac{120 - (-180)}{2}\right)^2 + \left(\frac{150}{2}\right)^2}\,\right] 10^{-6}$$

$\varepsilon_1 = 138(10^{-6}); \qquad \varepsilon_2 = -198(10^{-6})$ **Ans.**



Orientation of ε_1 and ε_2

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{150}{[120 - (-180)]} = 0.5$$

$\theta_p = 13.28°$ and $-76.72°$

Use Eq. 10.5 to determine the direction of ε_1 and ε_2

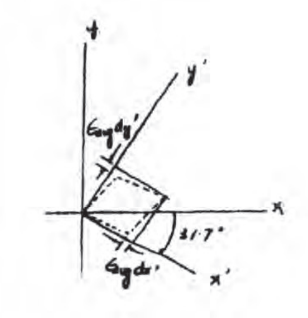
$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2} \cos 2\theta + \frac{\gamma_{xy}}{2} \sin 2\theta$$

$\theta = \theta_p = 13.28°$

$$\varepsilon_{x'} = \left[\frac{120 + (-180)}{2} + \frac{120 - (-180)}{2} \cos(26.56°) + \frac{150}{2} \sin 26.56°\right] 10^{-6}$$

$= 138\,(10^{-6}) = \varepsilon_1$

Therefore $\theta_{p_1} = 13.3°$; $\quad \theta_{p_2} = -76.7°$ **Ans.**



b) $$\frac{\gamma_{\substack{max \\ \text{in-plane}}}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{\substack{max \\ \text{in-plane}}} = 2\left[\sqrt{\left(\frac{120 - (-180)}{2}\right)^2 + \left(\frac{150}{2}\right)^2}\,\right] 10^{-6} = 335\,(10^{-6})$$ **Ans.**

$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left[\frac{120 + (-180)}{2}\right] 10^{-6} = -30.0(10^{-6})$$ **Ans.**

Orientation of γ_{max}

$$\tan 2\theta_s = \frac{-(\varepsilon_x - \varepsilon_y)}{\gamma_{xy}} = \frac{-[120 - (-180)]}{150} = -2.0$$

$\theta_s = -31.7°$ and $58.3°$ **Ans.**

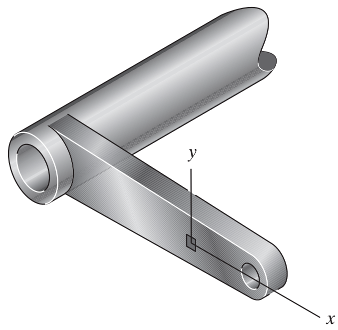
Use Eq. 10–6 to determine the sign of $\gamma_{\substack{max \\ \text{in-plane}}}$

$$\frac{\gamma_{x'y'}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2} \sin 2\theta + \frac{\gamma_{xy}}{2} \cos 2\theta$$

$\theta = \theta_s = -31.7°$

$$\gamma_{x'y'} = 2\left[-\frac{120 - (-180)}{2} \sin(-63.4°) + \frac{150}{2} \cos(-63.4°)\right] 10^{-6} = 335(10^{-6})$$

10–5. The state of strain at the point on the arm has components $\epsilon_x = 250(10^{-6})$, $\epsilon_y = -450(10^{-6})$, $\gamma_{xy} = -825(10^{-6})$. Use the strain-transformation equations to determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and average normal strain. In each case specify the orientation of the element and show how the strains deform the element within the x–y plane.

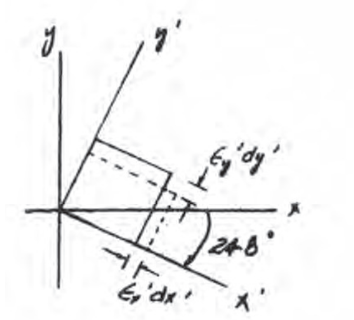


$\varepsilon_x = 250(10^{-6}) \qquad \varepsilon_y = -450(10^{-6}) \qquad \gamma_{xy} = -825(10^{-6})$

a)

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left[\frac{250 - 450}{2} \pm \sqrt{\left(\frac{250 - (-450)}{2}\right)^2 + \left(\frac{-825}{2}\right)^2}\right](10^{-6})$$



$\varepsilon_1 = 441(10^{-6})$ **Ans.**

$\varepsilon_2 = -641(10^{-6})$ **Ans.**

Orientation of ε_1 and ε_2 :

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{-825}{250 - (-450)}$$

$\theta_p = -24.84° \qquad$ and $\qquad \theta_p = 65.16°$

Use Eq. 10–5 to determine the direction of ε_1 and ε_2:

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$\theta = \theta_p = -24.84°$

$$\varepsilon_{x'} = \left[\frac{250 - 450}{2} + \frac{250 - (-450)}{2}\cos(-49.69°) + \frac{-825}{2}\sin(-49.69°)\right](10^{-6}) = 441(10^{-6})$$

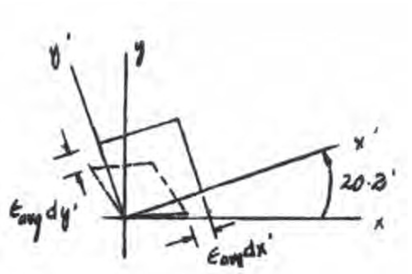
Therefore, $\theta_{p1} = -24.8°$ **Ans.**

$\theta_{p2} = 65.2°$ **Ans.**

b)

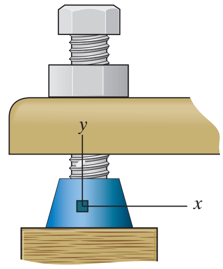
$$\frac{\gamma_{\substack{\max\\ \text{in-plane}}}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{\substack{\max\\ \text{in-plane}}} = 2\left[\sqrt{\left(\frac{250 - (-450)}{2}\right)^2 + \left(\frac{-825}{2}\right)^2}\right](10^{-6}) = 1.08(10^{-3})$$ **Ans.**



$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left(\frac{250 - 450}{2}\right)(10^{-6}) = -100(10^{-6})$$ **Ans.**

10–6. The state of strain at the point has components of $\epsilon_x = -100(10^{-6})$, $\epsilon_y = 400(10^{-6})$, and $\gamma_{xy} = -300(10^{-6})$. Use the strain-transformation equations to determine the equivalent in-plane strains on an element oriented at an angle of 60° counterclockwise from the original position. Sketch the deformed element due to these strains within the x–y plane.



In accordance to the established sign convention,

$$\varepsilon_x = -100(10^{-6}) \qquad \varepsilon_y = 400(10^{-6}) \qquad \gamma_{xy} = -300(10^{-6}) \qquad \theta = 60°$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{-100 + 400}{2} + \frac{-100 - 400}{2}\cos 120° + \frac{-300}{2}\sin 120°\right](10^{-6})$$

$$= 145(10^{-6})$$ **Ans.**

$$\frac{\gamma_{x'y'}}{2} = -\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = \left[-(-100 - 400)\sin 120° + (-300)\cos 120°\right](10^{-6})$$

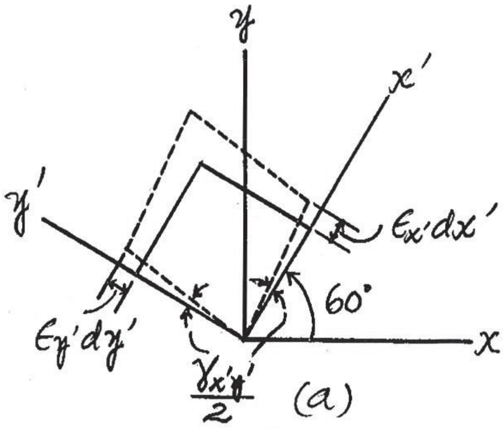
$$= 583(10^{-6})$$ **Ans.**

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

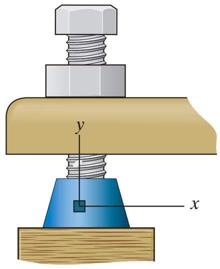
$$= \left[\frac{-100 + 400}{2} - \frac{-100 - 400}{2}\cos 120° - \frac{-300}{2}\sin 120°\right](10^{-6})$$

$$= 155\,(10^{-6})$$ **Ans.**

The deformed element of this equivalent state of strain is shown in Fig. *a*



10–7. The state of strain at the point has components of $\epsilon_x = 100(10^{-6})$, $\epsilon_y = 300(10^{-6})$, and $\gamma_{xy} = -150(10^{-6})$. Use the strain-transformation equations to determine the equivalent in-plane strains on an element oriented $\theta = 30°$ clockwise. Sketch the deformed element due to these strains within the x–y plane.



In accordance to the established sign convention,

$$\varepsilon_x = 100(10^{-6}) \qquad \varepsilon_y = 300(10^{-6}) \qquad \gamma_{xy} = -150(10^{-6}) \qquad \theta = -30°$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{100 + 300}{2} + \frac{100 - 300}{2}\cos(-60°) + \frac{-150}{2}\sin(-60°)\right](10^{-6})$$

$$= 215(10^{-6})$$ **Ans.**

$$\frac{\gamma_{x'y'}}{2} = -\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = \left[-(100 - 300)\sin(-60°) + (-150)\cos(-60°)\right](10^{-6})$$

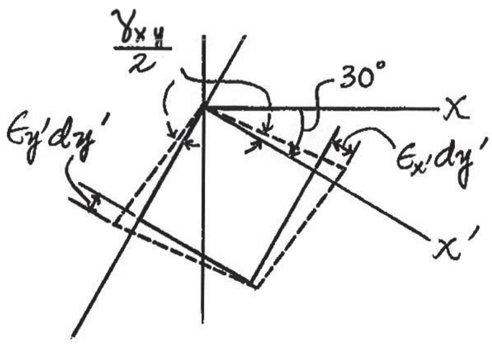
$$= -248\,(10^{-6})$$ **Ans.**

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{100 + 300}{2} - \frac{100 - 300}{2}\cos(-60°) - \frac{-150}{2}\sin(-60°)\right](10^{-6})$$

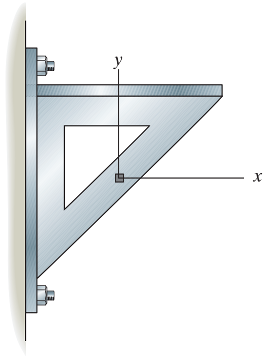
$$= 185(10^{-6})$$ **Ans.**

The deformed element of this equivalent state of strain is shown in Fig. *a*



(a)

***10–8.** The state of strain at the point on the bracket has components $\epsilon_x = -200(10^{-6})$, $\epsilon_y = -650(10^{-6})$, $\gamma_{xy} = -175(10^{-6})$. Use the strain-transformation equations to determine the equivalent in-plane strains on an element oriented at an angle of $\theta = 20°$ counterclockwise from the original position. Sketch the deformed element due to these strains within the x–y plane.



$$\varepsilon_x = -200(10^{-6}) \qquad \varepsilon_y = -650(10^{-6}) \qquad \gamma_{xy} = -175(10^{-6}) \qquad \theta = 20°$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{-200 + (-650)}{2} + \frac{(-200) - (-650)}{2}\cos(40°) + \frac{(-175)}{2}\sin(40°)\right](10^{-6})$$

$$= -309(10^{-6}) \qquad \textbf{Ans.}$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

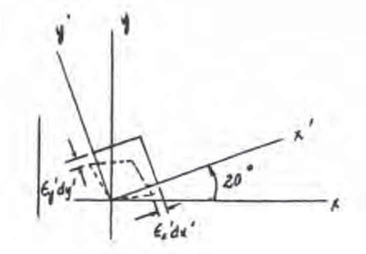
$$= \left[\frac{-200 + (-650)}{2} - \frac{-200 - (-650)}{2}\cos(40°) - \frac{(-175)}{2}\sin(40°)\right](10^{-6})$$

$$= -541(10^{-6}) \qquad \textbf{Ans.}$$

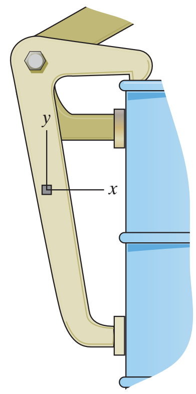
$$\frac{\gamma_{x'y'}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = [-(-200 - (-650))\sin(40°) + (-175)\cos(40°)](10^{-6})$$

$$= -423(10^{-6}) \qquad \textbf{Ans.}$$



10–9. The state of strain at the point has components of $\epsilon_x = 180(10^{-6})$, $\epsilon_y = -120(10^{-6})$, and $\gamma_{xy} = -100(10^{-6})$. Use the strain-transformation equations to determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and average normal strain. In each case specify the orientation of the element and show how the strains deform the element within the x–y plane.



a) In accordance to the established sign convention, $\varepsilon_x = 180(10^{-6})$, $\varepsilon_y = -120(10^{-6})$ and $\gamma_{xy} = -100(10^{-6})$.

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left\{\frac{180 + (-120)}{2} \pm \sqrt{\left[\frac{180 - (-120)}{2}\right]^2 + \left(\frac{-100}{2}\right)^2}\right\}(10^{-6})$$

$$= \left(30 \pm 158.11\right)(10^{-6})$$

$\varepsilon_1 = 188(10^{-6})$ $\quad\varepsilon_2 = -128(10^{-6})$ **Ans.**

$$\tan 2\theta_P = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{-100(10^{-6})}{\left[180 - (-120)\right](10^{-6})} = -0.3333$$

$$\theta_P = -9.217° \quad \text{and} \quad 80.78°$$

Substitute $\theta = -9.217°$,

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{180 + (-120)}{2} + \frac{180 - (-120)}{2}\cos(-18.43°) + \frac{-100}{2}\sin(-18.43)\right](10^{-6})$$

$$= 188(10^{-6}) = \varepsilon_1$$

Thus,

$(\theta_P)_1 = -9.22°$ $\quad(\theta_P)_2 = 80.8°$ **Ans.**

The deformed element is shown in Fig (a).

b) $$\frac{\gamma_{\substack{\max \\ \text{in-plane}}}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{\substack{\max \\ \text{in-plane}}} = \left\{2\sqrt{\left[\frac{180 - (-120)}{2}\right]^2 + \left(\frac{-100}{2}\right)^2}\right\}(10^{-6}) = 316\left(10^{-6}\right)$$ **Ans.**

$$\tan 2\theta_s = -\left(\frac{\varepsilon_x - \varepsilon_y}{\gamma_{xy}}\right) = -\left\{\frac{\left[180 - (-120)\right](10^{-6})}{-100(10^{-6})}\right\} = 3$$

$$\theta_s = 35.78° = 35.8° \quad \text{and} \quad -54.22° = -54.2°$$ **Ans.**

10–9. Continued

The algebraic sign for $\gamma_{\substack{max \\ in\text{-}plane}}$ when $\theta = 35.78°$.

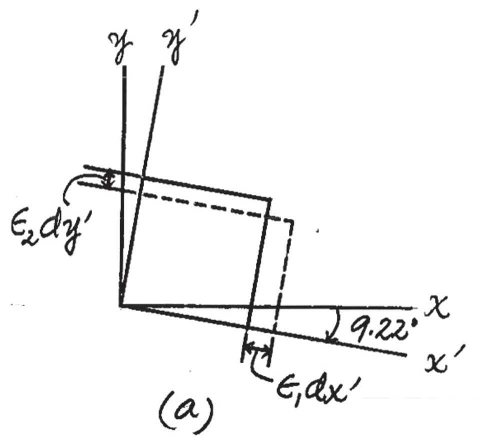
$$\frac{\gamma_{x'y'}}{2} = -\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

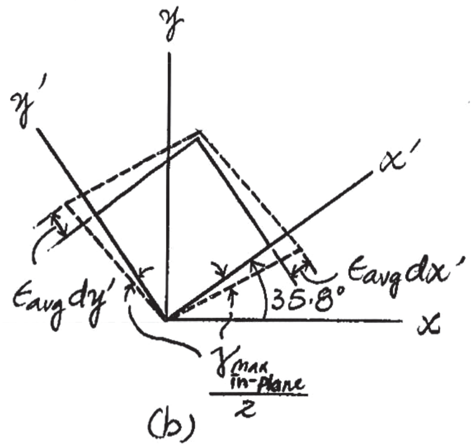
$$\gamma_{x'y'} = \left\{-\left[180 - (-120)\right]\sin 71.56° + (-100)\cos 71.56°\right\}(10^{-6})$$

$$= -316(10^{-6})$$

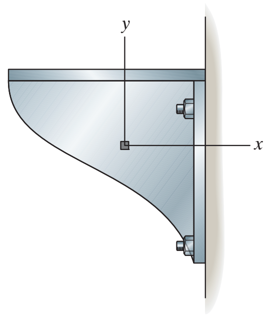
$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left[\frac{180 + (-120)}{2}\right](10^{-6}) = 30(10^{-6}) \qquad \textbf{Ans.}$$

The deformed element for the state of maximum In-plane shear strain is shown is shown in Fig. *b*





10–10. The state of strain at the point on the bracket has components $\epsilon_x = 400(10^{-6})$, $\epsilon_y = -250(10^{-6})$, $\gamma_{xy} = 310(10^{-6})$. Use the strain-transformation equations to determine the equivalent in-plane strains on an element oriented at an angle of $\theta = 30°$ clockwise from the original position. Sketch the deformed element due to these strains within the x–y plane.



$\varepsilon_x = 400(10^{-6}) \qquad \varepsilon_y = -250(10^{-6}) \qquad \gamma_{xy} = 310(10^{-6}) \qquad \theta = -30°$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{400 + (-250)}{2} + \frac{400 - (-250)}{2}\cos(-60°) + \left(\frac{310}{2}\right)\sin(-60°)\right](10^{-6})$$

$$= 103(10^{-6}) \qquad \textbf{Ans.}$$

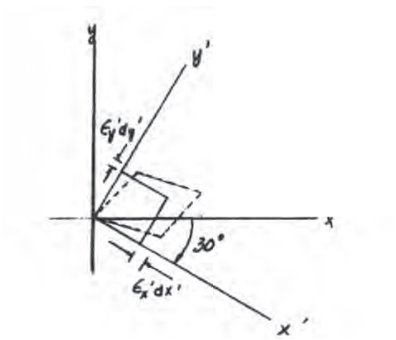
$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{400 + (-250)}{2} - \frac{400 - (-250)}{2}\cos(60°) - \frac{310}{2}\sin(-60°)\right](10^{-6})$$

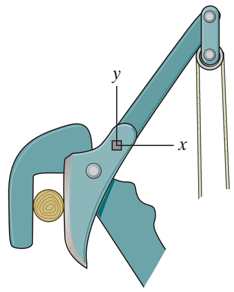
$$= 46.7(10^{-6}) \qquad \textbf{Ans.}$$

$$\frac{\gamma_{x'y'}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = [-(400 - (-250))\sin(-60°) + 310\cos(-60°)](10^{-6}) = 718(10^{-6}) \qquad \textbf{Ans.}$$



10–11. The state of strain at the point has components of $\epsilon_x = -100(10^{-6})$, $\epsilon_y = -200(10^{-6})$, and $\gamma_{xy} = 100(10^{-6})$. Use the strain-transformation equations to determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and average normal strain. In each case specify the orientation of the element and show how the strains deform the element within the x–y plane.



In accordance to the established sign convention, $\varepsilon_x = -100(10^{-6})$, $\varepsilon_y = -200(10^{-6})$ and $\gamma_{xy} = 100(10^{-6})$.

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left\{\frac{-100 + (-200)}{2} \pm \sqrt{\left[\frac{-100 - (-200)}{2}\right]^2 + \left(\frac{100}{2}\right)^2}\right\}(10^{-6})$$

$$= \left(-150 \pm 70.71\right)(10^{-6})$$

$$\varepsilon_1 = -79.3(10^{-6}) \qquad \varepsilon_2 = -221(10^{-6})$$ **Ans.**

$$\tan 2\theta_P = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{100(10^{-6})}{\left[-100 - (-200)\right](10^{-6})} = 1$$

$$\theta_P = 22.5^\circ \quad \text{and} \quad -67.5^\circ$$

Substitute $\theta = 22.5$,

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{-100 + (-200)}{2} + \frac{-100 - (-200)}{2}\cos 45^\circ + \frac{100}{2}\sin 45^\circ\right](10^{-6})$$

$$= -79.3(10^{-6}) = \varepsilon_1$$

Thus,

$$(\theta_P)_1 = 22.5^\circ \qquad (\theta_P)_2 = -67.5^\circ$$ **Ans.**

The deformed element of the state of principal strain is shown in Fig. *a*

$$\frac{\gamma_{\substack{\max \\ \text{in-plane}}}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{\substack{\max \\ \text{in-plane}}} = \left\{2\sqrt{\left[\frac{-100 - (-200)}{2}\right]^2 + \left(\frac{100}{2}\right)^2}\right\}(10^{-6}) = 141(10^{-6})$$ **Ans.**

$$\tan 2\theta_s = -\left(\frac{\varepsilon_x - \varepsilon_y}{\gamma_{xy}}\right) = -\left\{\frac{\left[-100 - (-200)\right](10^{-6})}{100(10^{-6})}\right\} = -1$$

$$\theta_s = -22.5^\circ \quad \text{and} \quad 67.5^\circ$$ **Ans.**

The algebraic sign for $\gamma_{\substack{\max \\ \text{in-plane}}}$ when $\theta = -22.5^\circ$.

$$\frac{\gamma_{x'y'}}{2} = -\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

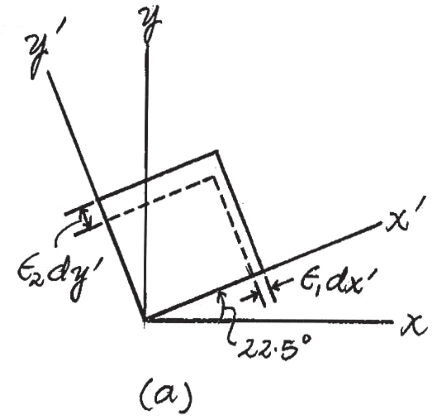
$$\gamma_{x'y'} = -\left[-100 - (-200)\right]\sin(-45^\circ) + 100\cos(-45^\circ)$$

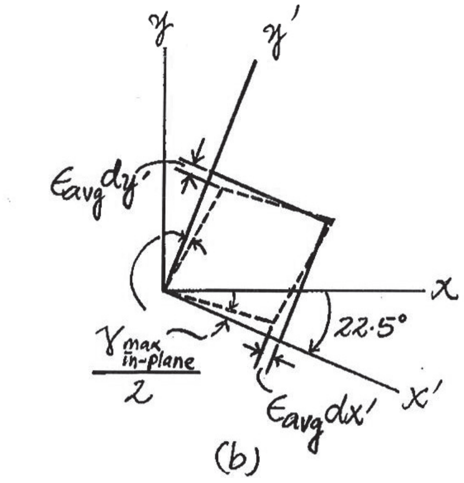
$$= 141(10^{-6})$$

$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left[\frac{-100 + (-200)}{2}\right](10^{-6}) = -150(10^{-6})$$ **Ans.**

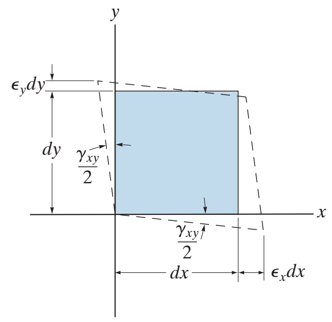
The deformed element for the state of maximum In-plane shear strain is shown in Fig. *b*.

10–11. Continued





***10–12.** The state of plane strain on an element is given by $\epsilon_x = 500(10^{-6})$, $\epsilon_y = 300(10^{-6})$, and $\gamma_{xy} = -200(10^{-6})$. Determine the equivalent state of strain on an element at the same point oriented 45° clockwise with respect to the original element.



Strain Transformation Equations:

$\varepsilon_x = 500\left(10^{-6}\right) \qquad \varepsilon_y = 300\left(10^{-6}\right) \qquad \gamma_{xy} = -200\left(10^{-6}\right) \qquad \theta = -45°$

We obtain

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{500 + 300}{2} + \frac{500 - 300}{2}\cos(-90°) + \left(\frac{-200}{2}\right)\sin(-90°)\right]\left(10^{-6}\right)$$

$$= 500\left(10^{-6}\right) \qquad \textbf{Ans.}$$

$$\frac{\gamma_{x'y'}}{2} = -\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = [-(500 - 300)\sin(-90°) + (-200)\cos(-90°)]\left(10^{-6}\right)$$

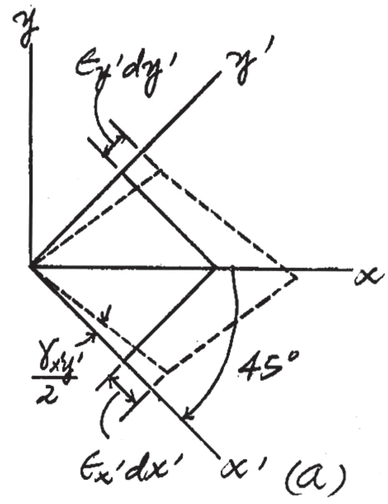
$$= 200\left(10^{-6}\right) \qquad \textbf{Ans.}$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

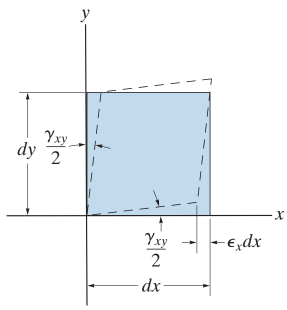
$$= \left[\frac{500 + 300}{2} - \frac{500 - 300}{2}\cos(-90°) - \left(\frac{-200}{2}\right)\sin(-90°)\right]\left(10^{-6}\right)$$

$$= 300\left(10^{-6}\right) \qquad \textbf{Ans.}$$

The deformed element for this state of strain is shown in Fig. *a*.



10–13. The state of plane strain on an element is $\epsilon_x = -300(10^{-6})$, $\epsilon_y = 0$, and $\gamma_{xy} = 150(10^{-6})$. Determine the equivalent state of strain which represents (a) the principal strains, and (b) the maximum in-plane shear strain and the associated average normal strain. Specify the orientation of the corresponding elements for these states of strain with respect to the original element.



In-Plane Principal Strains: $\varepsilon_x = -300\left(10^{-6}\right)$, $\varepsilon_y = 0$, and $\gamma_{xy} = 150\left(10^{-6}\right)$. We obtain

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left[\frac{-300 + 0}{2} \pm \sqrt{\left(\frac{-300 - 0}{2}\right)^2 + \left(\frac{150}{2}\right)^2}\right]\left(10^{-6}\right)$$

$$= (-150 \pm 167.71)\left(10^{-6}\right)$$

$\varepsilon_1 = 17.7\left(10^{-6}\right)$ $\qquad\qquad$ $\varepsilon_2 = -318\left(10^{-6}\right)$ **Ans.**

Orientation of Principal Strain:

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{150\left(10^{-6}\right)}{(-300 - 0)\left(10^{-6}\right)} = -0.5$$

$\theta_P = -13.28°$ and $76.72°$

Substituting $\theta = -13.28°$ into Eq. 9-1,

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{-300 + 0}{2} + \frac{-300 - 0}{2}\cos(-26.57°) + \frac{150}{2}\sin(-26.57°)\right]\left(10^{-6}\right)$$

$$= -318\left(10^{-6}\right) = \varepsilon_2$$

Thus,

$\left(\theta_P\right)_1 = 76.7°$ and $\left(\theta_P\right)_2 = -13.3°$ **Ans.**

The deformed element of this state of strain is shown in Fig. *a*.

Maximum In-Plane Shear Strain:

$$\frac{\gamma_{\substack{\max\\ \text{in-plane}}}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{\substack{\max\\ \text{in-plane}}} = \left[2\sqrt{\left(\frac{-300 - 0}{2}\right)^2 + \left(\frac{150}{2}\right)^2}\right]\left(10^{-6}\right) = 335\left(10^{-6}\right)$$ **Ans.**

Orientation of the Maximum In-Plane Shear Strain:

$$\tan 2\theta_s = -\left(\frac{\varepsilon_x - \varepsilon_y}{\gamma_{xy}}\right) = -\left[\frac{(-300 - 0)\left(10^{-6}\right)}{150\left(10^{-6}\right)}\right] = 2$$

$\theta_s = 31.7°$ and $122°$ **Ans.**

10–13. Continued

The algebraic sign for $\gamma_{\substack{\text{max}\\\text{in-plane}}}$ when $\theta = \theta_s = 31.7°$ can be obtained using

$$\frac{\gamma_{x'y'}}{2} = -\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

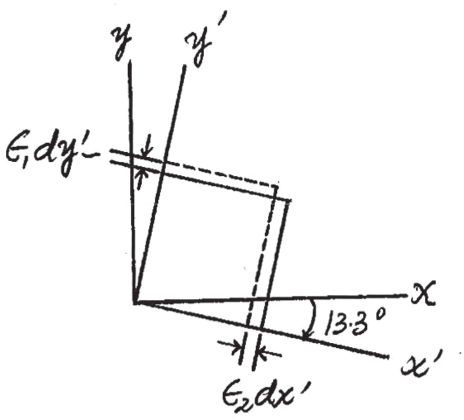
$$\gamma_{x'y'} = [-(-300 - 0)\sin 63.43° + 150\cos 63.43°]\left(10^{-6}\right)$$

$$= 335\left(10^{-6}\right)$$

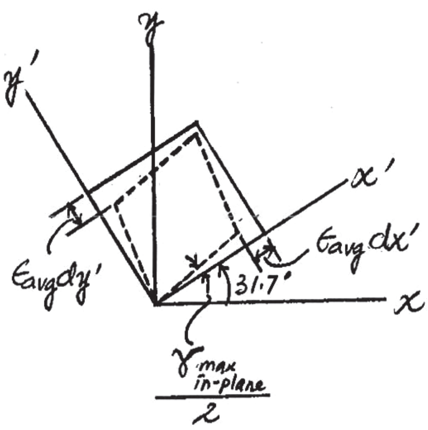
Average Normal Strain:

$$\varepsilon_{\text{avg}} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left(\frac{-300 + 0}{2}\right)\left(10^{-6}\right) = -150\left(10^{-6}\right) \qquad \textbf{Ans.}$$

The deformed element for this state of strain is shown in Fig. *b*.

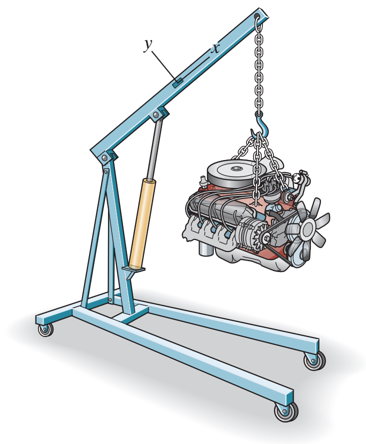


(a)



(b)

10–14. The state of strain at the point on a boom of an hydraulic engine crane has components of $\epsilon_x = 250(10^{-6})$, $\epsilon_y = 300(10^{-6})$, and $\gamma_{xy} = -180(10^{-6})$. Use the strain-transformation equations to determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and average normal strain. In each case, specify the orientation of the element and show how the strains deform the element within the x–y plane.



a)

In-Plane Principal Strain: Applying Eq. 10–9,

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left[\frac{250 + 300}{2} \pm \sqrt{\left(\frac{250 - 300}{2}\right)^2 + \left(\frac{-180}{2}\right)^2}\right](10^{-6})$$

$$= 275 \pm 93.41$$

$\varepsilon_1 = 368(10^{-6}) \qquad \varepsilon_2 = 182(10^{-6})$ **Ans.**

Orientation of Principal Strain: Applying Eq. 10–8,

$$\tan 2\theta_P = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{-180(10^{-6})}{(250 - 300)(10^{-6})} = 3.600$$

$$\theta_P = 37.24° \quad \text{and} \quad -52.76°$$

Use Eq. 10–5 to determine which principal strain deforms the element in the x' direction with $\theta = 37.24°$.

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{250 + 300}{2} + \frac{250 - 300}{2}\cos 74.48° + \frac{-180}{2}\sin 74.48°\right](10^{-6})$$

$$= 182(10^{-6}) = \varepsilon_2$$

Hence,

$\theta_{P1} = -52.8° \quad \text{and} \quad \theta_{P2} = 37.2°$ **Ans.**

b)

Maximum In-Plane Shear Strain: Applying Eq. 10–11,

$$\frac{\gamma_{\substack{\max \\ \text{in-plane}}}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{\substack{\max \\ \text{in-plane}}} = 2\left[\sqrt{\left(\frac{250 - 300}{2}\right)^2 + \left(\frac{-180}{2}\right)^2}\right](10^{-6})$$

$= 187(10^{-6})$ **Ans.**

10–14. Continued

Orientation of the Maximum In-Plane Shear Strain: Applying Eq. 10–10,

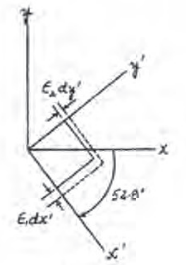
$$\tan 2\theta_s = -\frac{\varepsilon_x - \varepsilon_y}{\gamma_{xy}} = -\frac{250 - 300}{-180} = -0.2778$$

$\theta_s = -7.76°$ and $82.2°$ **Ans.**

The proper sign of $\gamma_{\substack{\text{max}\\ \text{in-plane}}}$ can be determined by substituting $\theta = -7.76°$ into Eq. 10–6.

$$\frac{\gamma_{x'y'}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = \{-[250 - 300]\sin(-15.52°) + (-180)\cos(-15.52°)\}\left(10^{-6}\right)$$
$$= -187\left(10^{-6}\right)$$

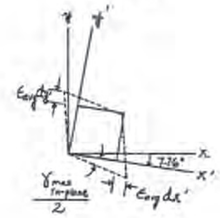


Normal Strain and Shear strain: In accordance with the sign convention,

$\varepsilon_x = 250\left(10^{-6}\right)$ $\quad \varepsilon_y = 300\left(10^{-6}\right)$ $\quad \gamma_{xy} = -180\left(10^{-6}\right)$

Average Normal Strain: Applying Eq. 10–12,

$$\varepsilon_{\text{avg}} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left[\frac{250 + 300}{2}\right]\left(10^{-6}\right) = 275\left(10^{-6}\right)$$ **Ans.**



***10–16.** The state of strain at a point on a support has components of $\epsilon_x = 350(10^{-6})$, $\epsilon_y = 400(10^{-6})$, $\gamma_{xy} = -675(10^{-6})$. Use the strain-transformation equations to determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and average normal strain. In each case specify the orientation of the element and show how the strains deform the element within the x–y plane.

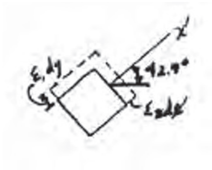
a)

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \frac{350 + 400}{2} \pm \sqrt{\left(\frac{350 - 400}{2}\right)^2 + \left(\frac{-675}{2}\right)^2}$$

$\varepsilon_1 = 713(10^{-6})$ **Ans.**

$\varepsilon_2 = 36.6(10^{-6})$ **Ans.**



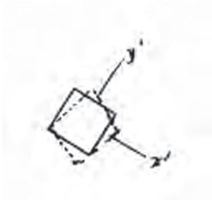
$$\tan 2\theta_P = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{-675}{(350 - 400)}$$

$\theta_P = 42.9°$ **Ans.**

b)

$$\frac{(\gamma_{x'y'})_{\max}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\frac{(\gamma_{x'y'})_{\max}}{2} = \sqrt{\left(\frac{350 - 400}{2}\right)^2 + \left(\frac{-675}{2}\right)^2}$$



$(\gamma_{x'y'})_{\max} = 677(10^{-6})$ **Ans.**

$\varepsilon_{avg} = \dfrac{\varepsilon_x + \varepsilon_y}{2} = \dfrac{350 + 400}{2} = 375(10^{-6})$ **Ans.**

$$\tan 2\theta_s = \frac{-(\varepsilon_x - \varepsilon_y)}{\gamma_{xy}} = \frac{350 - 400}{675}$$

$\theta_s = -2.12°$ **Ans.**

•10–17. Solve part (a) of Prob. 10–4 using Mohr's circle.

$\varepsilon_x = 120(10^{-6}) \qquad \varepsilon_y = -180(10^{-6}) \qquad \gamma_{xy} = 150(10^{-6})$

$A\,(120, 75)(10^{-6}) \quad C\,(-30, 0)(10^{-6})$

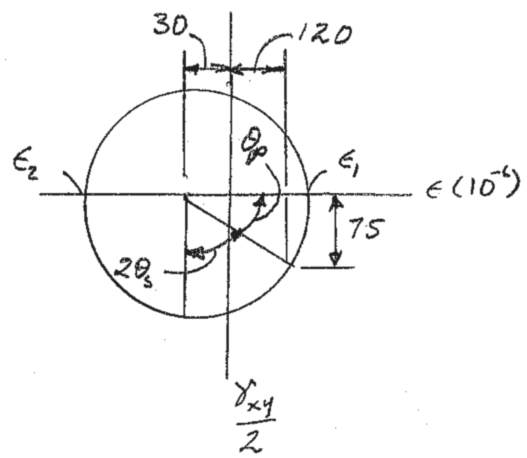
$R = \left[\sqrt{[120 - (-30)]^2 + (75)^2}\right](10^{-6})$

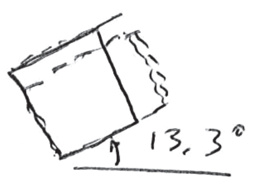
$= 167.71\,(10^{-6})$

$\varepsilon_1 = (-30 + 167.71)(10^{-6}) = 138(10^{-6})$ **Ans.**

$\varepsilon_2 = (-30 - 167.71)(10^{-6}) = -198(10^{-6})$ **Ans.**

$\tan 2\theta_P = \left(\dfrac{75}{30 + 120}\right), \quad \theta_P = 13.3°$ **Ans.**





10–18. Solve part (b) of Prob. 10–4 using Mohr's circle.

$\varepsilon_x = 120(10^{-6}) \qquad \varepsilon_y = -180(10^{-6}) \qquad \gamma_{xy} = 150(10^{-6})$

$A\ (120, 75)(10^{-6}) \quad C\ (-30, 0)(10^{-6})$

$R = \left[\sqrt{[120 - (-30)]^2 + (75)^2}\right](10^{-6})$

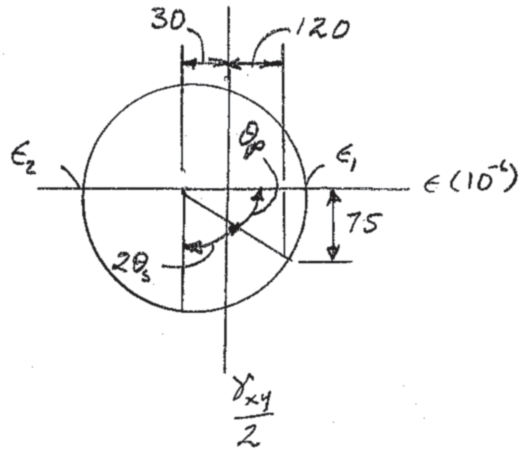
$= 167.71\ (10^{-6})$

$\dfrac{\gamma_{xy}}{2}_{\substack{\text{max}\\ \text{in-plane}}} = R = 167.7(10^{-6})$

$\gamma_{xy}{}_{\substack{\text{max}\\ \text{in-plane}}} = 335(10^{-6})$ **Ans.**

$\varepsilon_{avg} = -30\ (10^{-6})$ **Ans.**

$\tan 2\theta_s = \dfrac{120 + 30}{75} \qquad \theta_s = -31.7°$ **Ans.**



10–19. Solve Prob. 10–8 using Mohr's circle.

$\varepsilon_x = -200(10^{-6}) \qquad \varepsilon_y = -650(10^{-6}) \qquad \gamma_{xy} = -175(10^{-6}) \qquad \dfrac{\gamma_{xy}}{2} = -87.5(10^{-6})$

$\theta = 20^\circ, \quad 2\theta = 40^\circ$

$A(-200, -87.5)(10^{-6}) \qquad C(-425, 0)(10^{-6})$

$R = [\sqrt{(-200 - (-425))^2 + 87.5^2}\,](10^{-6}) = 241.41(10^{-6})$

$\tan\alpha = \dfrac{87.5}{-200 - (-425)}; \qquad \alpha = 21.25^\circ$

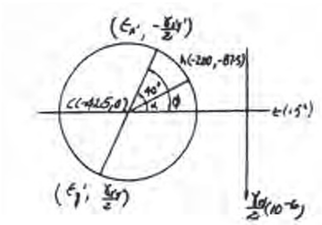
$\phi = 40 + 21.25 = 61.25^\circ$

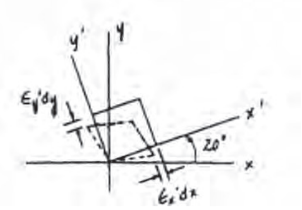
$\varepsilon_{x'} = (-425 + 241.41\ \cos\ 61.25^\circ)(10^{-6}) = -309(10^{-6})$ **Ans.**

$\varepsilon_{y'} = (-425 - 241.41\ \cos\ 61.25^\circ)(10^{-6}) = -541(10^{-6})$ **Ans.**

$\dfrac{-\gamma_{x'y'}}{2} = 241.41(10^{-6})\sin 61.25^\circ$

$\gamma_{x'y'} = -423(10^{-6})$ **Ans.**





***10–20.** Solve Prob. 10–10 using Mohr's circle.

$\varepsilon_x = 400(10^{-6}) \qquad \varepsilon_y = -250(10^{-6}) \qquad \gamma_{xy} = 310(10^{-6}) \qquad \frac{\gamma_{xy}}{2} = 155(10^{-6}) \qquad \theta = 30°$

$A(400, 155)(10^{-6}) \qquad C(75, 0)(10^{-6})$

$R = [\sqrt{(400 - 75)^2 + 155^2}](10^{-6}) = 360.1(10^{-6})$

$\tan \alpha = \frac{155}{400 - 75}; \qquad \alpha = 25.50°$

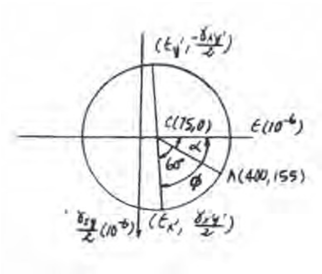
$\phi = 60 + 25.50 = 85.5°$

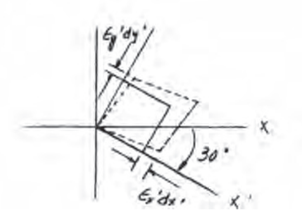
$\varepsilon_{x'} = (75 + 360.1 \cos 85.5°)(10^{-6}) = 103(10^{-6})$ **Ans.**

$\varepsilon_{y'} = (75 - 360.1 \cos 85.5°)(10^{-6}) = 46.7(10^{-6})$ **Ans.**

$\frac{\gamma_{x'y'}}{2} = (360.1 \sin 85.5°)(10^{-6})$

$\gamma_{x'y'} = 718(10^{-6})$ **Ans.**

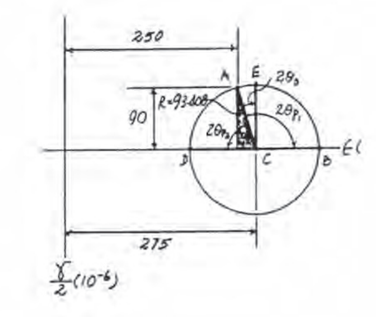




•10–21. Solve Prob. 10–14 using Mohr's circle.

Construction of the Circle: In accordance with the sign convention, $\varepsilon_x = 250(10^{-6})$, $\varepsilon_y = 300(10^{-6})$, and $\dfrac{\gamma_{xy}}{2} = -90(10^{-6})$. Hence,

$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left(\frac{250 + 300}{2}\right)(10^{-6}) = 275(10^{-6}) \qquad \textbf{Ans.}$$



The coordinates for reference points A and C are

$A(250, -90)(10^{-6}) \qquad C(275, 0)(10^{-6})$

The radius of the circle is

$$R = \left(\sqrt{(275 - 250)^2 + 90^2}\right)(10^{-6}) = 93.408$$

In-Plane Principal Strain: The coordinates of points B and D represent ε_1 and ε_2, respectively.

$$\varepsilon_1 = (275 + 93.408)(10^{-6}) = 368(10^{-6}) \qquad \textbf{Ans.}$$

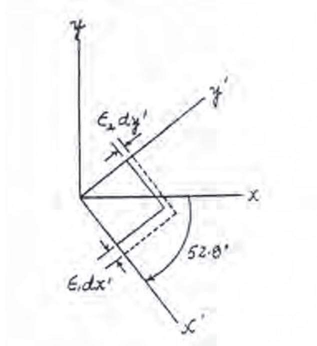
$$\varepsilon_2 = (275 - 93.408)(10^{-6}) = 182(10^{-6}) \qquad \textbf{Ans.}$$

Orientation of Principal Strain: From the circle,

$$\tan 2\theta_{P_2} = \frac{90}{275 - 250} = 3.600 \qquad 2\theta_{P_2} = 74.48^\circ$$

$$2\theta_{P_1} = 180^\circ - 2\theta_{P_2}$$

$$\theta_{P_1} = \frac{180^\circ - 74.78^\circ}{2} = 52.8^\circ \quad (\textbf{\textit{Clockwise}}) \qquad \textbf{Ans.}$$



Maximum In-Plane Shear Strain: Represented by the coordinates of point E on the circle.

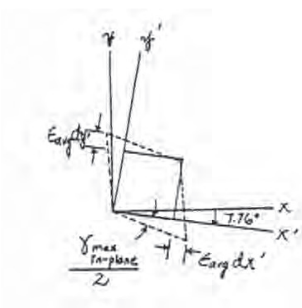
$$\frac{\gamma_{\substack{max \\ in\text{-}plane}}}{2} = -R = -93.408(10^{-6})$$

$$\gamma_{\substack{max \\ in\text{-}plane}} = -187(10^{-6}) \qquad \textbf{Ans.}$$

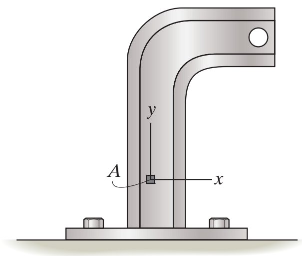
Orientation of the Maximum In-Plane Shear Strain: From the circle,

$$\tan 2\theta_s = \frac{275 - 250}{90} = 0.2778$$

$$\theta_s = 7.76^\circ \quad (\textbf{\textit{Clockwise}}) \qquad \textbf{Ans.}$$



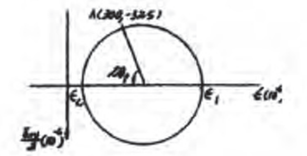
10–22. The strain at point A on the bracket has components $\epsilon_x = 300(10^{-6})$, $\epsilon_y = 550(10^{-6})$, $\gamma_{xy} = -650(10^{-6})$. Determine (a) the principal strains at A in the x–y plane, (b) the maximum shear strain in the x–y plane, and (c) the absolute maximum shear strain.



$\varepsilon_x = 300(10^{-6}) \qquad \varepsilon_y = 550(10^{-6}) \qquad \gamma_{xy} = -650(10^{-6}) \qquad \dfrac{\gamma_{xy}}{2} = -325(10^{-6})$

$A(300, -325)10^{-6} \qquad C(425, 0)10^{-6}$

$R = \left[\sqrt{(425 - 300)^2 + (-325)^2}\right]10^{-6} = 348.2(10^{-6})$



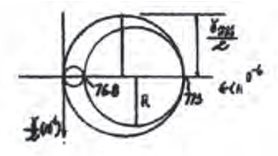
a)

$\varepsilon_1 = (425 + 348.2)(10^{-6}) = 773(10^{-6})$ **Ans.**

$\varepsilon_2 = (425 - 348.2)(10^{-6}) = 76.8(10^{-6})$ **Ans.**

b)

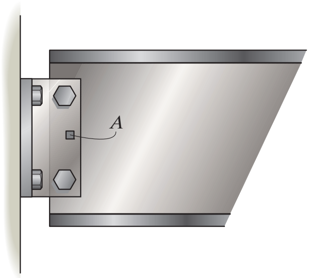
$\gamma_{\substack{\text{max}\\\text{in-plane}}} = 2R = 2(348.2)(10^{-6}) = 696(10^{-6})$ **Ans.**



c)

$\dfrac{\gamma_{\substack{\text{abs}\\\text{max}}}}{2} = \dfrac{773(10^{-6})}{2}; \qquad \gamma_{\substack{\text{abs}\\\text{max}}} = 773(10^{-6})$ **Ans.**

10–23. The strain at point A on the leg of the angle has components $\epsilon_x = -140(10^{-6})$, $\epsilon_y = 180(10^{-6})$, $\gamma_{xy} = -125(10^{-6})$. Determine (a) the principal strains at A in the x–y plane, (b) the maximum shear strain in the x–y plane, and (c) the absolute maximum shear strain.



$\varepsilon_x = -140(10^{-6}) \qquad \varepsilon_y = 180(10^{-6}) \qquad \gamma_{xy} = -125(10^{-6}) \qquad \dfrac{\gamma_{xy}}{2} = -62.5(10^{-6})$

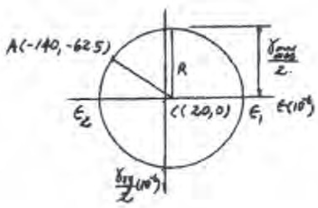
$A(-140, -62.5)10^{-6} \qquad C(20, 0)10^{-6}$

$R = \left(\sqrt{(20 - (-140))^2 + (-62.5)^2}\right)10^{-6} = 171.77(10^{-6})$

a)

$\varepsilon_1 = (20 + 171.77)(10^{-6}) = 192(10^{-6})$ **Ans.**

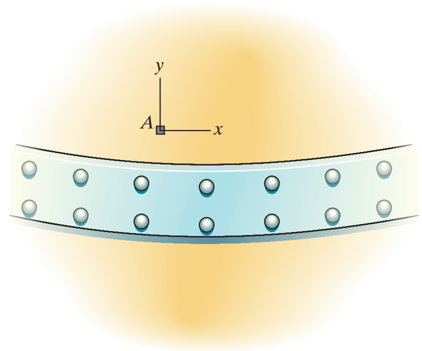
$\varepsilon_2 = (20 - 171.77)(10^{-6}) = -152(10^{-6})$ **Ans.**



(b, c)

$\gamma_{\substack{\text{abs}\\\text{max}}} = \gamma_{\substack{\text{max}\\\text{in-plane}}} = 2R = 2(171.77)(10^{-6}) = 344(10^{-6})$ **Ans.**

***10–24.** The strain at point A on the pressure-vessel wall has components $\epsilon_x = 480(10^{-6})$, $\epsilon_y = 720(10^{-6})$, $\gamma_{xy} = 650(10^{-6})$. Determine (a) the principal strains at A, in the x–y plane, (b) the maximum shear strain in the x–y plane, and (c) the absolute maximum shear strain.



$\varepsilon_x = 480(10^{-6}) \qquad \varepsilon_y = 720(10^{-6}) \qquad \gamma_{xy} = 650(10^{-6}) \qquad \dfrac{\gamma_{xy}}{2} = 325(10^{-6})$

$A(480, 325)10^{-6} \qquad C(600, 0)10^{-6}$

$R = \left(\sqrt{(600 - 480)^2 + 325^2}\right)10^{-6} = 346.44(10^{-6})$

a)

$\varepsilon_1 = (600 + 346.44)10^{-6} = 946(10^{-6})$ **Ans.**

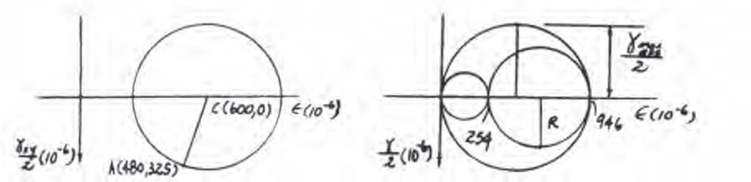
$\varepsilon_2 = (600 - 346.44)10^{-6} = 254(10^{-6})$ **Ans.**

b)

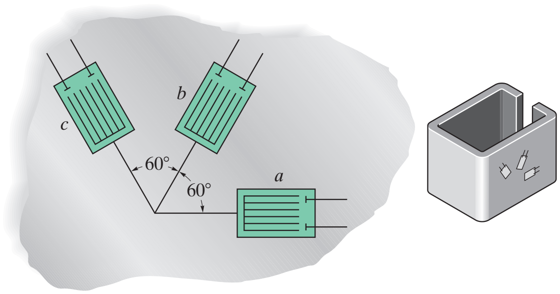
$\gamma_{\substack{\max \\ \text{in-plane}}} = 2R = 2(346.44)10^{-6} = 693(10^{-6})$ **Ans.**

c)

$\dfrac{\gamma_{\substack{\text{abs} \\ \max}}}{2} = \dfrac{946(10^{-6})}{2}; \qquad \gamma_{\substack{\text{abs} \\ \max}} = 946(10^{-6})$ **Ans.**



•**10–25.** The 60° strain rosette is mounted on the bracket. The following readings are obtained for each gauge: $\epsilon_a = -100(10^{-6})$, $\epsilon_b = 250(10^{-6})$, and $\epsilon_c = 150(10^{-6})$. Determine (a) the principal strains and (b) the maximum in-plane shear strain and associated average normal strain. In each case show the deformed element due to these strains.



This is a 60° strain rosette Thus,

$$\varepsilon_x = \varepsilon_a = -100(10^{-6})$$

$$\varepsilon_y = \frac{1}{3}\left(2\varepsilon_b + 2\varepsilon_c - \varepsilon_a\right)$$

$$= \frac{1}{3}\left[2(250) + 2(150) - (-100)\right](10^{-6})$$

$$= 300(10^{-6})$$

$$\gamma_{xy} = \frac{2}{\sqrt{3}}(\varepsilon_b - \varepsilon_c) = \frac{2}{\sqrt{3}}(250 - 150)(10^{-6}) = 115.47(10^{-6})$$

In accordance to the established sign convention, $\varepsilon_x = -100(10^{-6})$, $\varepsilon_y = 300(10^{-6})$ and $\frac{\gamma_{xy}}{2} = 57.74(10^{-6})$.

Thus,

$$\varepsilon_{\text{avg}} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left(\frac{-100 + 300}{2}\right)(10^{-6}) = 100(10^{-6}) \qquad \textbf{Ans.}$$

Then, the coordinates of reference point A and Center C of the circle are

$A(-100, 57.74)(10^{-6}) \qquad C(100, 0)(10^{-6})$

Thus, the radius of the circle is

$$R = CA = \left(\sqrt{(-100 - 100)^2 + 208.16}\right)(10^{-6}) = 208.17(10^{-6})$$

Using these result, the circle is shown in Fig. *a*.

The coordinates of points B and D represent ε_1 and ε_2 respectively.

$$\varepsilon_1 = (100 + 208.17)(10^{-6}) = 308(10^{-6}) \qquad \textbf{Ans.}$$

$$\varepsilon_2 = (100 - 208.17)(10^{-6}) = -108(10^{-6}) \qquad \textbf{Ans.}$$

Referring to the geometry of the circle,

$$\tan 2(\theta_P)_2 = \frac{57.74(10^{-6})}{(100 + 100)(10^{-6})} = 0.2887$$

$$\left(\theta_P\right)_2 = 8.05° \quad (Clockwise) \qquad \textbf{Ans.}$$

The deformed element for the state of principal strain is shown in Fig. *b*.

10–25. Continued

The coordinates for point E represent ε_{avg} and $\dfrac{\gamma_{\substack{max \\ in\text{-}plane}}}{2}$. Thus,

$$\frac{\gamma_{\substack{max \\ in\text{-}plane}}}{2} = R = 208.17(10^{-6})$$

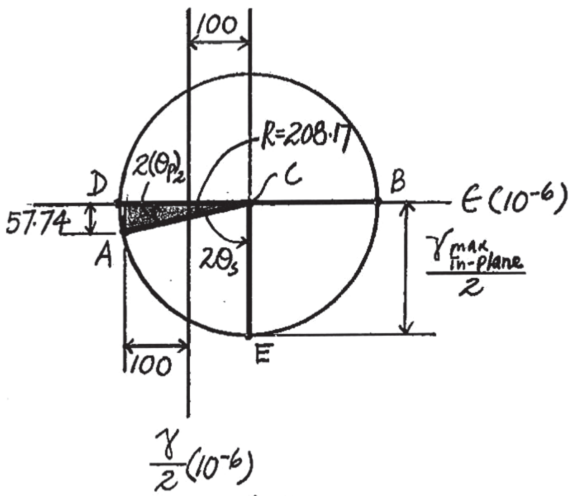
$$\gamma_{\substack{max \\ in\text{-}plane}} = 416(10^{-6}) \qquad \textbf{Ans.}$$

Referring to the geometry of the circle,

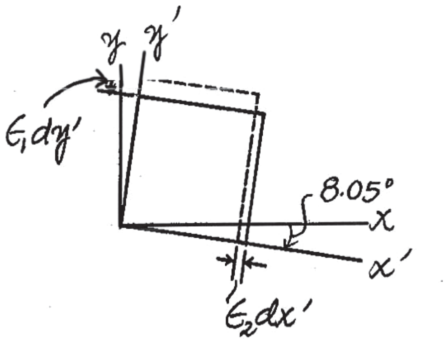
$$\tan 2\theta_s = \frac{100 + 100}{57.74}$$

$$\theta_s = 36.9^\circ \quad (\textit{Counter Clockwise}) \qquad \textbf{Ans.}$$

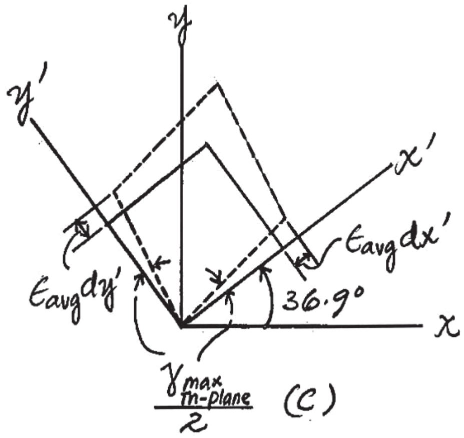
The deformed element for the state of maximum In-plane shear strain is shown in Fig. c.



(a)

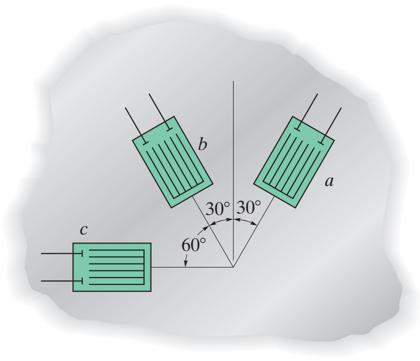


(b)



(c)

10–26. The 60° strain rosette is mounted on a beam. The following readings are obtained for each gauge: $\epsilon_a = 200(10^{-6})$, $\epsilon_b = -450(10^{-6})$, and $\epsilon_c = 250(10^{-6})$. Determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and average normal strain. In each case show the deformed element due to these strains.



With $\theta_a = 60°, \theta_b = 120°$ and $\theta_c = 180°$,

$$\varepsilon_a = \varepsilon_x \cos^2\theta_a + \varepsilon_y \sin^2\theta_a + \gamma_{xy}\sin\theta_a\cos\theta_a$$

$$200(10^{-6}) = \varepsilon_x \cos^2 60° + \varepsilon_y \sin^2 60° + \gamma_{xy}\sin 60°\cos 60°$$

$$0.25\varepsilon_x + 0.75\varepsilon_y + 0.4330 \quad \gamma_{xy} = 200(10^{-6}) \tag{1}$$

$$\varepsilon_b = \varepsilon_x \cos^2\theta_b + \varepsilon_y \sin^2\theta_b + \gamma_{xy}\sin\theta_b\cos\theta_b$$

$$-450(10^{-6}) = \varepsilon_x \cos^2 120° + \varepsilon_y \sin^2 120° + \gamma_{xy}\sin 120°\cos 120°$$

$$0.25\varepsilon_x + 0.75\varepsilon_y - 0.4330 \quad \gamma_{xy} = -450(10^{-6}) \tag{2}$$

$$\varepsilon_c = \varepsilon_x \cos^2\theta_c + \varepsilon_y \sin^2\theta_c + \gamma_{xy}\sin\theta_c\cos\theta_c$$

$$250(10^{-6}) = \varepsilon_x \cos^2 180° + \varepsilon_y \sin^2 180° + \gamma_{xy}\sin 180°\cos 180°$$

$$\varepsilon_x = 250(10^{-6})$$

Substitute this result into Eqs. (1) and (2) and solve them,

$$\varepsilon_y = -250\,(10^{-6}) \qquad \gamma_{xy} = 750.56\,(10^{-6})$$

In accordance to the established sign convention, $\varepsilon_x = 250(10^{-6})$, $\varepsilon_y = -250(10^{-6})$, and $\dfrac{\gamma_{xy}}{2} = 375.28(10^{-6})$, Thus,

$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left[\frac{250 + (-250)}{2}\right](10^{-6}) = 0 \qquad \textbf{Ans.}$$

Then, the coordinates of the reference point A and center C of the circle are

$$A(250, 375.28)(10^{-6}) \qquad C(0, 0)$$

Thus, the radius of the circle is

$$R = CA = \left(\sqrt{(250 - 0)^2 + 375.28^2}\right)(10^{-6}) = 450.92(10^{-6})$$

Using these results, the circle is shown in Fig. *a*.

The coordinates for points B and D represent ε_1 and ε_2, respectively. Thus,

$$\varepsilon_1 = 451(10^{-6}) \qquad \varepsilon_2 = -451(10^{-6}) \qquad \textbf{Ans.}$$

Referring to the geometry of the circle,

$$\tan 2(\theta_P)_1 = \frac{375.28}{250} = 1.5011$$

$$(\theta_P)_1 = 28.2° \quad (\textit{Counter Clockwise}) \qquad \textbf{Ans.}$$

The deformed element for the state of principal strains is shown in Fig. *b*.

10–26. Continued

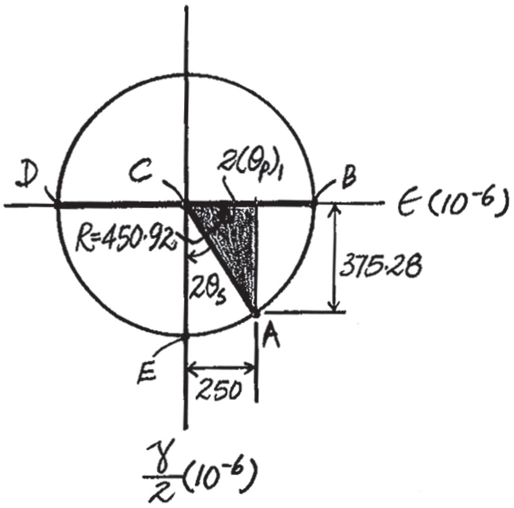
The coordinates of point E represent ε_{avg} and $\dfrac{\gamma_{\substack{max \\ in\text{-}plane}}}{2}$. Thus,

$$\frac{\gamma_{\substack{max \\ in\text{-}plane}}}{2} = R = 450.92(10^{-6}) \qquad \gamma_{\substack{max \\ in\text{-}plane}} = 902(10^{-6})$$ **Ans.**

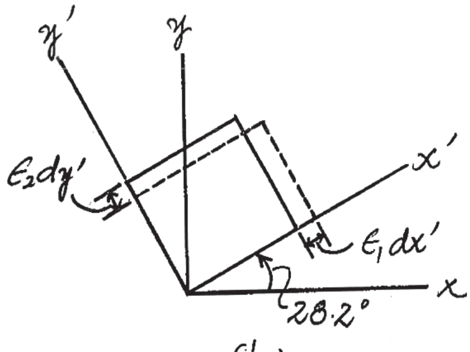
Referring to the geometry of the circle,

$$\tan 2\theta_s = \frac{250}{375.28} = 0.6662$$

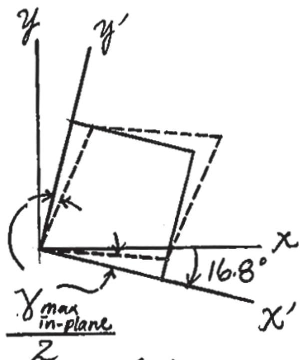
$$\theta_s = 16.8^\circ \quad (Clockwise)$$ **Ans.**



(a)

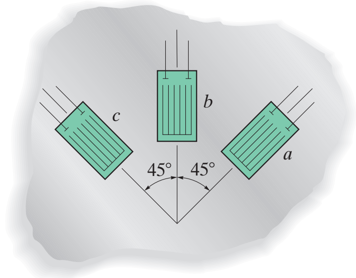


(b)



(c)

10–27. The 45° strain rosette is mounted on a steel shaft. The following readings are obtained from each gauge: $\epsilon_a = 300(10^{-6})$, $\epsilon_b = -250(10^{-6})$, and $\epsilon_c = -450(10^{-6})$. Determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and average normal strain. In each case show the deformed element due to these strains.



With $\theta_a = 45°, \theta_b = 90°$ and $\theta_c = 135°$,

$$\varepsilon_a = \varepsilon_x \cos^2\theta_a + \varepsilon_y \sin^2\theta_a + \gamma_{xy}\sin\theta_a\cos\theta_a$$

$$300(10^{-6}) = \varepsilon_x \cos^2 45° + \varepsilon_y \sin^2 45° + \gamma_{xy}\sin 45°\cos 45°$$

$$\varepsilon_x + \varepsilon_y + \gamma_{xy} = 600(10^{-6}) \qquad \textbf{(1)}$$

$$\varepsilon_b = \varepsilon_x \cos^2\theta_b + \varepsilon_y \sin^2\theta_b + \gamma_{xy}\sin\theta_b\cos\theta_b$$

$$-250(10^{-6}) = \varepsilon_x \cos^2 90° + \varepsilon_y \sin^2 90° + \gamma_{xy}\sin 90°\cos 90°$$

$$\varepsilon_y = -250(10^{-6})$$

$$\varepsilon_c = \varepsilon_x \cos^2\theta_c + \varepsilon_y \sin^2\theta_c + \gamma_{xy}\sin\theta_c\cos\theta_c$$

$$-450(10^{-6}) = \varepsilon_x \cos^2 135° + \varepsilon_y \sin^2 135° + \gamma_{xy}\sin 135°\cos 135°$$

$$\varepsilon_x + \varepsilon_y - \gamma_{xy} = -900(10^{-6}) \qquad \textbf{(2)}$$

Substitute the result of ε_y into Eq. (1) and (2) and solve them

$$\varepsilon_x = 100(10^{-6}) \qquad \gamma_{xy} = 750(10^{-6})$$

In accordance to the established sign convention, $\varepsilon_x = 100(10^{-6})$, $\varepsilon_y = -250(10^{-6})$ and $\frac{\gamma_{xy}}{2} = 375(10^{-6})$. Thus,

$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left[\frac{100 + (-250)}{2}\right](10^{-6}) = -75(10^{-6}) \qquad \textbf{Ans.}$$

Then, the coordinates of the reference point A and the center C of the circle are

$A(100, 375)(10^{-6}) \qquad C(-75, 0)(10^{-6})$

Thus, the radius of the circle is

$$R = CA = \left(\sqrt{[100 - (-75)]^2 + 375^2}\right)(10^{-6}) = 413.82(10^{-6})$$

Using these results, the circle is shown in Fig. *a*.

The Coordinates of points B and D represent ε_1 and ε_2, respectively. Thus,

$$\varepsilon_1 = \left(-75 + 413.82\right)(10^{-6}) = 339(10^{-6}) \qquad \textbf{Ans.}$$

$$\varepsilon_2 = \left(-75 - 413.82\right)(10^{-6}) = -489(10^{-6}) \qquad \textbf{Ans.}$$

Referring to the geometry of the circle

$$\tan 2(\theta_P)_1 = \frac{375}{100 + 75} = 2.1429$$

$$(\theta_P)_1 = 32.5° \quad (\textit{Counter Clockwise}) \qquad \textbf{Ans.}$$

10–27. Continued

The deformed element for the state of principal strains is shown in Fig. *b*.

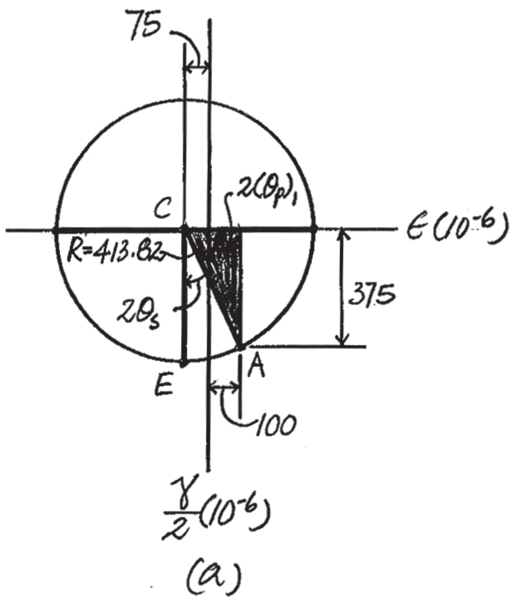
The coordinates of point *E* represent ε_{avg} and $\dfrac{\gamma_{\substack{max \\ in\text{-}plane}}}{2}$. Thus

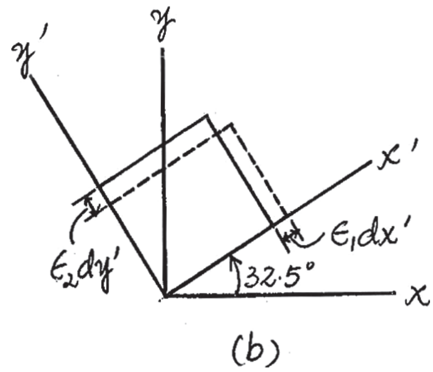
$$\frac{\gamma_{\substack{max \\ in\text{-}plane}}}{2} = R = 413.82(10^6) \qquad \gamma_{\substack{max \\ in\text{-}plane}} = 828(10^{-6}) \qquad \textbf{Ans.}$$

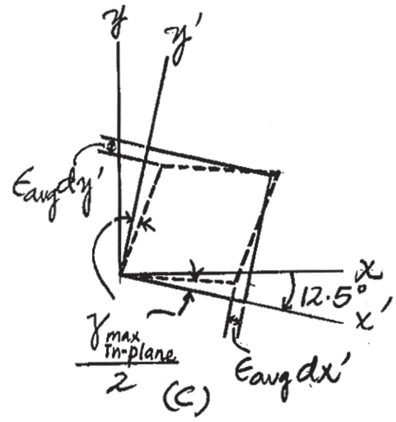
Referring to the geometry of the circle

$$\tan 2\theta_s = \frac{-100 + 75}{375} = 0.4667$$

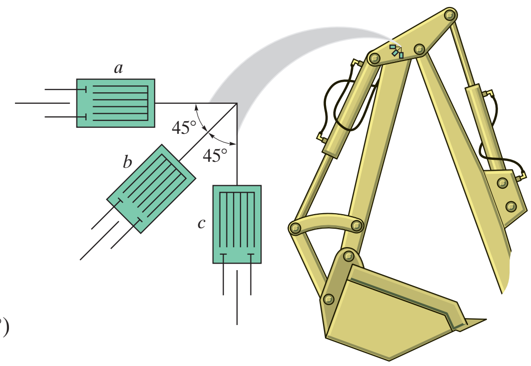
$$\theta_s = 12.5° \quad (Clockwise) \qquad \textbf{Ans.}$$







***10–28.** The 45° strain rosette is mounted on the link of the backhoe. The following readings are obtained from each gauge: $\epsilon_a = 650(10^{-6})$, $\epsilon_b = -300(10^{-6})$, $\epsilon_c = 480(10^{-6})$. Determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and associated average normal strain.



$\varepsilon_a = 650(10^{-6})$; $\quad \varepsilon_b = -300(10^{-6})$; $\quad \varepsilon_c = 480(10^{-6})$

$\theta_a = 180°$; $\quad \theta_b = 225°$ $\quad \theta_c = 270°$

Applying Eq. 10–16, $\varepsilon = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy} \sin\theta\cos\theta$

$$650(10^{-6}) = \varepsilon_x \cos^2(180°) + \varepsilon_y \sin^2(180°) + \gamma_{xy} \sin(180°)\cos(180°)$$

$$\varepsilon_x = 650\,(10^{-6})$$

$$480\,(10^{-6}) = \varepsilon_x \cos^2(270°) + \varepsilon_y \sin^2(270°) + \gamma_{xy} \sin(270°)\cos(270°)$$

$$\varepsilon_y = 480\,(10^{-6})$$

$$-300\,(10^{-6}) = 650\,(10^{-6})\cos^2(225°) + 480\,(10^{-6})\sin^2(225°) + \gamma_{xy}\sin(225°)\cos(225°)$$

$$\gamma_{xy} = -1730\,(10^{-6})$$

Therefore, $\varepsilon_x = 650\,(10^{-6})$ $\quad \varepsilon_y = 480\,(10^{-6})$ $\quad \gamma_{xy} = -1730\,(10^{-6})$

$$\frac{\gamma_{xy}}{2} = -865\,(10^{-6})$$

Mohr's circle:

$A(650, -865)\,10^{-6}$ $\quad C(565, 0)\,10^{-6}$

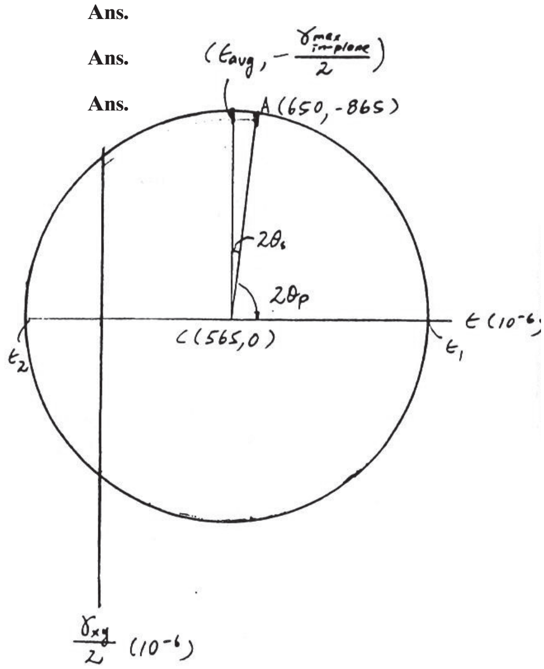
$$R = CA = \left[\sqrt{(650 - 565)^2 + 865^2}\right]10^{-6} = 869.17\,(10^{-6})$$

(a) $\varepsilon_1 = [565 + 869.17]10^{-6} = 1434\,(10^{-6})$ **Ans.**

$\varepsilon_2 = [565 - 869.17]10^{-6} = -304\,(10^{-6})$ **Ans.**

(b) $\gamma_{\substack{\max \\ \text{in-plane}}} = 2R = 2(869.17)\,(10^{-6}) = 1738\,(10^{-6})$ **Ans.**

$\varepsilon_{avg} = 565(10^{-6})$ **Ans.**



10–30. For the case of plane stress, show that Hooke's law can be written as

$$\sigma_x = \frac{E}{(1-\nu^2)}(\epsilon_x + \nu\epsilon_y), \quad \sigma_y = \frac{E}{(1-\nu^2)}(\epsilon_y + \nu\epsilon_x)$$

Generalized Hooke's Law: For plane stress, $\sigma_z = 0$. Applying Eq. 10–18,

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - v\,\sigma_y\right)$$

$$vE\varepsilon_x = \left(\sigma_x - v\,\sigma_y\right)v$$

$$vE\varepsilon_x = v\,\sigma_x - v^2\,\sigma_y \qquad [1]$$

$$\varepsilon_y = \frac{1}{E}\left(\sigma_y - v\,\sigma_x\right)$$

$$E\,\varepsilon_y = -v\,\sigma_x + \sigma_y \qquad [2]$$

Adding Eq [1] and Eq.[2] yields.

$$vE\,\varepsilon_x - E\,\varepsilon_y = \sigma_y - v^2\,\sigma_y$$

$$\sigma_y = \frac{E}{1-v^2}\left(v\varepsilon_x + \varepsilon_y\right) \qquad \textbf{\textit{(Q.E.D.)}}$$

Substituting σ_y into Eq. [2]

$$E\,\varepsilon_y = -v\sigma_x + \frac{E}{1-v^2}\left(v\,\varepsilon_x + \varepsilon_y\right)$$

$$\sigma_x = \frac{E\left(v\,\varepsilon_x + \varepsilon_y\right)}{v\,(1-v^2)} - \frac{E\varepsilon_y}{v}$$

$$= \frac{E\,v\,\varepsilon_x + E\,\varepsilon_y - E\,\varepsilon_y + E\varepsilon_y\,v^2}{v(1-v^2)}$$

$$= \frac{E}{1-v^2}\left(\varepsilon_x + v\,\varepsilon_y\right) \qquad \textbf{\textit{(Q.E.D.)}}$$

10–31. Use Hooke's law, Eq. 10–18, to develop the strain-transformation equations, Eqs. 10–5 and 10–6, from the stress-transformation equations, Eqs. 9–1 and 9–2.

Stress transformation equations:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \qquad \textbf{(1)}$$

$$\tau_{x'y'} = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \qquad \textbf{(2)}$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta \qquad \textbf{(3)}$$

Hooke's Law:

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu\,\sigma_y}{E} \qquad \textbf{(4)}$$

$$\varepsilon_y = \frac{-\nu\,\sigma_x}{E} + \frac{\sigma_y}{E} \qquad \textbf{(5)}$$

$$\tau_{xy} = G\,\gamma_{xy} \qquad \textbf{(6)}$$

$$G = \frac{E}{2\,(1+\nu)} \qquad \textbf{(7)}$$

From Eqs. (4) and (5)

$$\varepsilon_x + \varepsilon_y = \frac{(1-\nu)(\sigma_x + \sigma_y)}{E} \qquad \textbf{(8)}$$

$$\varepsilon_x - \varepsilon_y = \frac{(1+\nu)(\sigma_x - \sigma_y)}{E} \qquad \textbf{(9)}$$

From Eqs. (6) and (7)

$$\tau_{xy} = \frac{E}{2\,(1+\nu)}\gamma_{xy} \qquad \textbf{(10)}$$

From Eq. (4)

$$\varepsilon_{x'} = \frac{\sigma_{x'}}{E} - \frac{\nu\,\sigma_{y'}}{E} \qquad \textbf{(11)}$$

Substitute Eqs. (1) and (3) into Eq. (11)

$$\varepsilon_{x'} = \frac{(1-\nu)(\sigma_x - \sigma_y)}{2\,E} + \frac{(1+\nu)(\sigma_x - \sigma_y)}{2\,E}\cos 2\theta + \frac{(1+\nu)\tau_{xy}\sin 2\theta}{E} \qquad \textbf{(12)}$$

By using Eqs. (8), (9) and (10) and substitute into Eq. (12),

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta \qquad \textbf{QED}$$

10–31. Continued

From Eq. (6).

$$\gamma_{x'y'} = G\,\gamma_{x'y'} = \frac{E}{2(1+\nu)}\gamma_{x'y'} \qquad \textbf{(13)}$$

Substitute Eqs. (13), (6) and (9) into Eq. (2),

$$\frac{E}{2(1+\nu)}\gamma_{x'y'} = -\frac{E(\varepsilon_x - \varepsilon_y)}{2(1+\nu)}\sin 2\theta + \frac{E}{2(1+\nu)}\gamma_{xy}\cos 2\theta$$

$$\frac{\gamma_{x'y'}}{2} = -\frac{(\varepsilon_x - \varepsilon_y)}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta \qquad \textbf{QED}$$

***10–32.** A bar of copper alloy is loaded in a tension machine and it is determined that $\epsilon_x = 940(10^{-6})$ and $\sigma_x = 14$ ksi, $\sigma_y = 0$, $\sigma_z = 0$. Determine the modulus of elasticity, E_{cu}, and the dilatation, e_{cu}, of the copper. $\nu_{\text{cu}} = 0.35$.

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$

$$940(10^{-6}) = \frac{1}{E_{\text{cu}}}[14(10^3) - 0.35(0+0)]$$

$$E_{\text{cu}} = 14.9(10^3)\text{ ksi} \qquad \textbf{Ans.}$$

$$\varepsilon_{\text{cu}} = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z) = \frac{1-2(0.35)}{14.9(10^3)}(14+0+0) = 0.282(10^{-3}) \qquad \textbf{Ans.}$$

•10–33. The principal strains at a point on the aluminum fuselage of a jet aircraft are $\epsilon_1 = 780(10^{-6})$ and $\epsilon_2 = 400(10^{-6})$. Determine the associated principal stresses at the point in the same plane. $E_{\text{al}} = 10(10^3)$ ksi, $\nu_{\text{al}} = 0.33$. *Hint:* See Prob. 10–30.

Plane stress, $\sigma_3 = 0$

See Prob 10-30,

$$\sigma_1 = \frac{E}{1-\nu^2}(\varepsilon_1 + \nu\varepsilon_2)$$

$$= \frac{10(10^3)}{1-0.33^2}(780(10^{-6}) + 0.33(400)(10^{-6})) = 10.2\text{ ksi} \qquad \textbf{Ans.}$$

$$\sigma_2 = \frac{E}{1-\nu^2}(\varepsilon_2 + \nu\varepsilon_1)$$

$$= \frac{10(10^3)}{1-0.33^2}(400(10^{-6}) + 0.33(780)(10^{-6})) = 7.38\text{ ksi} \qquad \textbf{Ans.}$$

10–34. The rod is made of aluminum 2014-T6. If it is subjected to the tensile load of 700 N and has a diameter of 20 mm, determine the absolute maximum shear strain in the rod at a point on its surface.

Normal Stress: For uniaxial loading, $\sigma_y = \sigma_z = 0$.

$$\sigma_x = \frac{P}{A} = \frac{700}{\frac{\pi}{4}(0.02^2)} = 2.228 \text{ MPa}$$

Normal Strain: Applying the generalized Hooke's Law.

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right]$$

$$= \frac{1}{73.1(10^9)}\left[2.228\left(10^6\right) - 0\right]$$

$$= 30.48\left(10^{-6}\right)$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$$

$$= \frac{1}{73.1(10^9)}\left[0 - 0.35\left(2.228\left(10^6\right) + 0\right)\right]$$

$$= -10.67\left(10^{-6}\right)$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right]$$

$$= \frac{1}{73.1(10^9)}\left[0 - 0.35\left(2.228\left(10^6\right) + 0\right)\right]$$

$$= -10.67\left(10^{-6}\right)$$

Therefore.

$$\varepsilon_{\max} = 30.48\left(10^{-6}\right) \qquad \varepsilon_{\min} = -10.67\left(10^{-6}\right)$$

Absolute Maximum Shear Strain:

$$\gamma_{\substack{\text{abs} \\ \text{max}}} = \varepsilon_{max} - \varepsilon_{min}$$

$$= [30.48 - (-10.67)]\left(10^{-6}\right) = 41.1\left(10^{-6}\right) \qquad \textbf{Ans.}$$

10–35. The rod is made of aluminum 2014-T6. If it is subjected to the tensile load of 700 N and has a diameter of 20 mm, determine the principal strains at a point on the surface of the rod.

Normal Stress: For uniaxial loading, $\sigma_y = \sigma_z = 0$.

$$\sigma_x = \frac{P}{A} = \frac{700}{\frac{\pi}{4}(0.02^2)} = 2.228 \text{ MPa}$$

Normal Strains: Applying the generalized Hooke's Law.

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right]$$

$$= \frac{1}{73.1(10^9)}\left[2.228\left(10^6\right) - 0\right]$$

$$= 30.48\left(10^{-6}\right)$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$$

$$= \frac{1}{73.1(10^9)}\left[0 - 0.35\left(2.228\left(10^6\right) + 0\right)\right]$$

$$= -10.67\left(10^{-6}\right)$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right]$$

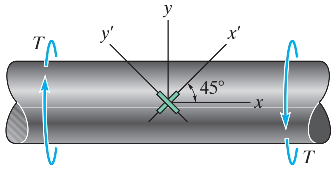
$$= \frac{1}{73.1(10^9)}\left[0 - 0.35\left(2.228\left(10^6\right) + 0\right)\right]$$

$$= -10.67\left(10^{-6}\right)$$

Principal Strains: From the results obtained above,

$$\varepsilon_{max} = 30.5\left(10^{-6}\right) \qquad \varepsilon_{int} = \varepsilon_{min} = -10.7\left(10^{-6}\right)$$ **Ans.**

***10–36.** The steel shaft has a radius of 15 mm. Determine the torque T in the shaft if the two strain gauges, attached to the surface of the shaft, report strains of $\epsilon_{x'} = -80(10^{-6})$ and $\epsilon_{y'} = 80(10^{-6})$. Also, compute the strains acting in the x and y directions. $E_{st} = 200$ GPa, $\nu_{st} = 0.3$.



$\varepsilon_{x'} = -80(10^{-6}) \qquad \varepsilon_{y'} = 80(10^{-6})$

Pure shear

$\varepsilon_x = \varepsilon_y = 0$ **Ans.**

$\varepsilon_{x'} = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy} \sin\theta\cos\theta$

$\theta = 45°$

$-80(10^{-6}) = 0 + 0 + \gamma_{xy} \sin 45° \cos 45°$

$\gamma_{xy} = -160(10^{-6})$ **Ans.**

Also, $\theta = 135°$

$$80(10^{-6}) = 0 + 0 + \gamma \sin 135° \cos 135°$$

$$\gamma_{xy} = -160(10^{-6})$$

$$G = \frac{E}{2(1 + V)} = \frac{200(10^9)}{2(1 + 0.3)} = 76.923(10^9)$$

$$\tau = G\gamma = 76.923(10^9)(160)(10^{-6}) = 12.308(10^6)\text{ Pa}$$

$$T = \frac{\tau J}{c} = \frac{12.308(10^6)\left(\frac{\pi}{2}\right)(0.015)^4}{0.015} = 65.2\text{ N}\cdot\text{m}$$ **Ans.**

10–37. Determine the bulk modulus for each of the following materials: (a) rubber, $E_r = 0.4$ ksi, $\nu_r = 0.48$, and (b) glass, $E_g = 8(10^3)$ ksi, $\nu_g = 0.24$.

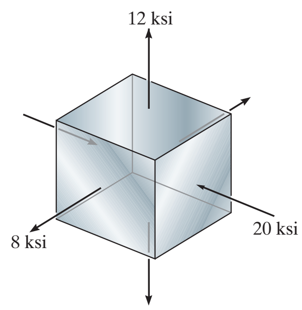
a) For rubber:

$$K_r = \frac{E_r}{3(1 - 2\nu_r)} = \frac{0.4}{3[1 - 2(0.48)]} = 3.33\text{ ksi}$$ **Ans.**

b) For glass:

$$K_g = \frac{E_g}{3(1 - 2\nu_g)} = \frac{8(10^3)}{3[1 - 2(0.24)]} = 5.13\,(10^3)\text{ ksi}$$ **Ans.**

10–38. The principal stresses at a point are shown in the figure. If the material is A-36 steel, determine the principal strains.



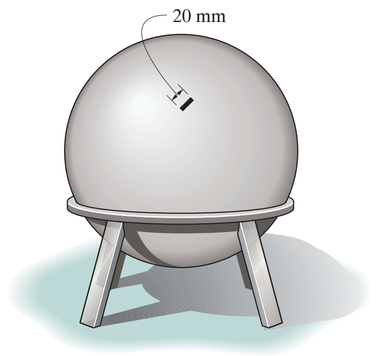
$$\varepsilon_1 = \frac{1}{E}\left[\sigma_1 - \nu(\sigma_2 + \sigma_3)\right] = \frac{1}{29.0(10^3)}\left\{12 - 0.32\left[8 + (-20)\right]\right\} = 546\,(10^{-6})$$

$$\varepsilon_2 = \frac{1}{E}\left[\sigma_2 - \nu(\sigma_1 + \sigma_3)\right] = \frac{1}{29.0(10^3)}\left\{8 - 0.32\left[12 + (-20)\right]\right\} = 364\,(10^{-6})$$

$$\varepsilon_3 = \frac{1}{E}\left[\sigma_3 - \nu(\sigma_1 + \sigma_2)\right] = \frac{1}{29.0(10^3)}\left[-20 - 0.32(12 + 8)\right] = -910\,(10^{-6})$$

$\varepsilon_{max} = 546\,(10^{-6})$ $\quad \varepsilon_{int} = 346\,(10^{-6})$ $\quad \varepsilon_{min} = -910\,(10^{-6})$ **Ans.**

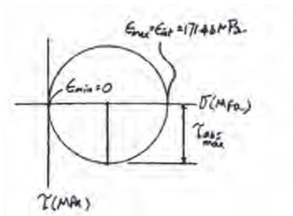
10–39. The spherical pressure vessel has an inner diameter of 2 m and a thickness of 10 mm. A strain gauge having a length of 20 mm is attached to it, and it is observed to increase in length by 0.012 mm when the vessel is pressurized. Determine the pressure causing this deformation, and find the maximum in-plane shear stress, and the absolute maximum shear stress at a point on the outer surface of the vessel. The material is steel, for which $E_{st} = 200$ GPa and $\nu_{st} = 0.3$.



Normal Stresses: Since $\dfrac{r}{t} = \dfrac{1000}{10} = 100 > 10$, the *thin wall* analysis is valid to determine the normal stress in the wall of the spherical vessel. This is a plane stress problem where $\sigma_{min} = 0$ since there is no load acting on the outer surface of the wall.

$$\sigma_{max} = \sigma_{lat} = \frac{pr}{2t} = \frac{p(1000)}{2(10)} = 50.0p \qquad \textbf{[1]}$$

Normal Strains: Applying the generalized Hooke's Law with



$$\varepsilon_{max} = \varepsilon_{lat} = \frac{0.012}{20} = 0.600\left(10^{-3}\right)\text{ mm/mm}$$

$$\varepsilon_{max} = \frac{1}{E}\left[\sigma_{max} - V\,(\sigma_{lat} + \sigma_{min})\right]$$

$$0.600\left(10^{-3}\right) = \frac{1}{200(10^4)}\left[50.0p - 0.3\,(50.0p + 0)\right]$$

$$p = 3.4286\text{ MPa} = 3.43\text{ MPa} \qquad \textbf{Ans.}$$

From Eq.[1] $\sigma_{max} = \sigma_{lat} = 50.0(3.4286) = 171.43$ MPa

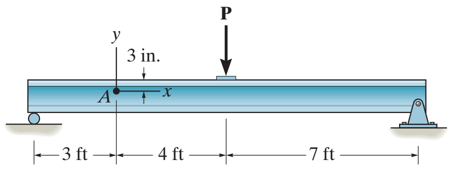
Maximum In-Plane Shear (Sphere's Surface): Mohr's circle is simply a dot. As the result, the state of stress is the same consisting of two normal stresses with zero shear stress regardless of the orientation of the element.

$$\tau_{\substack{max\\ \text{in-plane}}} = 0 \qquad \textbf{Ans.}$$

Absolute Maximum Shear Stress:

$$\tau_{\substack{abs\\ max}} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{171.43 - 0}{2} = 85.7\text{MPa} \qquad \textbf{Ans.}$$

***10–40.** The strain in the x direction at point A on the steel beam is measured and found to be $\epsilon_x = -100(10^{-6})$. Determine the applied load P. What is the shear strain γ_{xy} at point A? $E_{st} = 29(10^3)$ ksi, $\nu_{st} = 0.3$.



$$I_x = \frac{1}{12}(6)(9)^3 - \frac{1}{12}(5.5)(8^3) = 129.833 \text{ in}^4$$

$$Q_A = (4.25)(0.5)(6) + (2.75)(0.5)(2.5) = 16.1875 \text{ in}^3$$

$$\sigma = E\varepsilon_x = 29(10^3)(100)(10^{-6}) = 2.90 \text{ ksi}$$

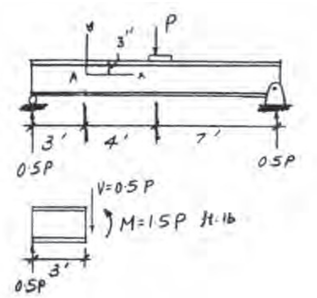
$$\sigma = \frac{My}{I}, \qquad 2.90 = \frac{1.5P(12)(1.5)}{129.833}$$

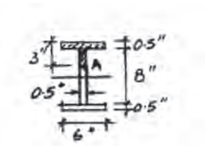
$$P = 13.945 = 13.9 \text{ kip} \qquad \textbf{Ans.}$$

$$\tau_A = \frac{VQ}{It} = \frac{0.5(13.945)(16.1875)}{129.833(0.5)} = 1.739 \text{ ksi}$$

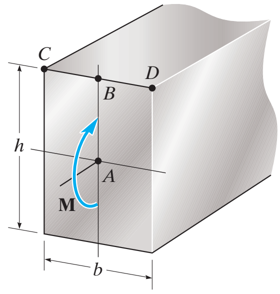
$$G = \frac{E}{2(1+\nu)} = \frac{29(10^3)}{2(1+0.3)} = 11.154(10^3) \text{ ksi}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{1.739}{11.154(10^3)} = 0.156(10^{-3}) \text{ rad} \qquad \textbf{Ans.}$$





•10–41. The cross section of the rectangular beam is subjected to the bending moment **M**. Determine an expression for the increase in length of lines AB and CD. The material has a modulus of elasticity E and Poisson's ratio is ν.



For line AB,

$$\sigma_z = -\frac{My}{I} = \frac{My}{\frac{1}{12}b\,h^3} = -\frac{12My}{b\,h^3}$$

$$\varepsilon_y = -\frac{\nu\,\sigma_z}{E} = \frac{12\,\nu\,My}{E\,b\,h^3}$$

$$\Delta L_{AB} = \int_0^{\frac{h}{2}} \varepsilon_y\,dy = \frac{12\,\nu\,M}{E\,b\,h^3}\int_0^{\frac{h}{2}} y\,dy$$

$$= \frac{3\,\nu\,M}{2\,E\,b\,h} \qquad \textbf{Ans.}$$

For line CD,

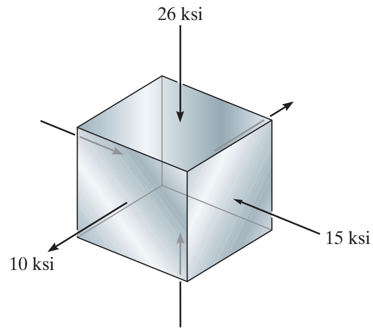
$$\sigma_z = -\frac{Mc}{I} = -\frac{M\frac{h}{2}}{\frac{1}{12}b\,h^3} = -\frac{6M}{bh^2}$$

$$\varepsilon_x = -\frac{\nu\,\sigma_z}{E} = \frac{6\,\nu\,M}{E\,b\,h^2}$$

$$\Delta L_{CD} = \varepsilon_x\,L_{CD} = \frac{6\,\nu\,M}{E\,b\,h^2}(b)$$

$$= \frac{6\,\nu\,M}{E\,h^2} \qquad \textbf{Ans.}$$

10–42. The principal stresses at a point are shown in the figure. If the material is aluminum for which $E_{al} = 10(10^3)$ ksi and $\nu_{al} = 0.33$, determine the principal strains.

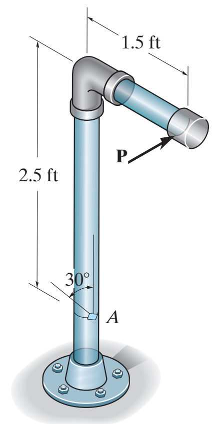


$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu(\sigma_y + \sigma_z)) = \frac{1}{10(10^3)}(10 - 0.33(-15 - 26)) = 2.35(10^{-3}) \qquad \textbf{Ans.}$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu(\sigma_x + \sigma_z)) = \frac{1}{10(10^3)}(-15 - 0.33)(10 - 26)) = -0.972(10^{-3}) \textbf{Ans.}$$

$$\varepsilon_z = \frac{1}{E}(\sigma_z - \nu(\sigma_x + \sigma_y)) = \frac{1}{10(10^3)}(-26 - 0.33(10 - 15)) = -2.44(10^{-3}) \qquad \textbf{Ans.}$$

10–43. A single strain gauge, placed on the outer surface and at an angle of 30° to the axis of the pipe, gives a reading at point A of $\epsilon_a = -200(10^{-6})$. Determine the horizontal force P if the pipe has an outer diameter of 2 in. and an inner diameter of 1 in. The pipe is made of A-36 steel.



Using the method of section and consider the equilibrium of the FBD of the pipe's upper segment, Fig. a,

$$\Sigma F_z = 0; \qquad V_z - p = 0 \qquad V_z = p$$

$$\Sigma M_x = 0; \qquad T_x - p(1.5) = 0 \quad T_x = 1.5p$$

$$\Sigma M_y = 0; \qquad M_y - p(2.5) = 0 \quad M_y = 2.5p$$

The normal strees is due to bending only. For point A, $z = 0$. Thus

$$\sigma_x = \frac{M_y z}{I_y} = 0$$

The shear stress is the combination of torsional shear stress and transverse shear stress. Here, $J = \frac{\pi}{2}(1^4 - 0.5^4) = 0.46875\,\pi \text{ in}^4$. Thus, for point A

$$\tau_t = \frac{T_x c}{J} = \frac{1.5p(12)(1)}{0.46875\pi} = \frac{38.4\,p}{\pi}$$

Referring to Fig. b,

$$(Q_A)_z = \bar{y}_1' A_1' - \bar{y}_2' A_2' = \frac{4\,(1)}{3\pi}\left[\frac{\pi}{2}(1^2)\right] - \frac{4(0.5)}{3\pi}\left[\frac{\pi}{2}(0.5^2)\right]$$

$$= 0.5833 \text{ in}^3$$

$$I_y = \frac{\pi}{4}(1^4 - 0.5^4) = 0.234375\,\pi \text{ in}^4$$

Combine these two shear stress components,

$$\tau = \tau_t + \tau_v = \frac{38.4P}{\pi} + \frac{2.4889P}{\pi} = \frac{40.8889P}{\pi}$$

Since no normal stress acting on point A, it is subjected to pure shear which can be represented by the element shown in Fig. c.

For pure shear, $\varepsilon_x = \varepsilon_z = 0$,

$$\varepsilon_a = \varepsilon_x \cos^3\theta_a + \varepsilon_z \sin^2\theta_a + \gamma_{xz} \sin\theta_a \cos\theta_a$$

$$-200(10^{-6}) = 0 + 0 + \gamma_{xz} \sin 150° \cos 150°$$

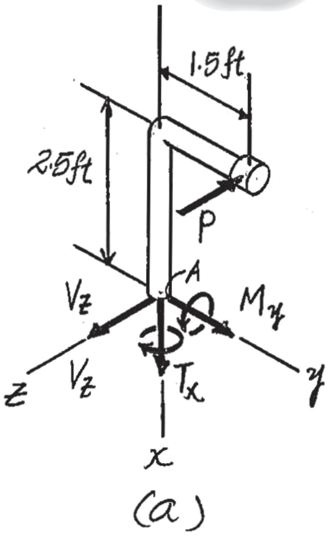
$$\gamma_{xz} = 461.88(10^{-6})$$

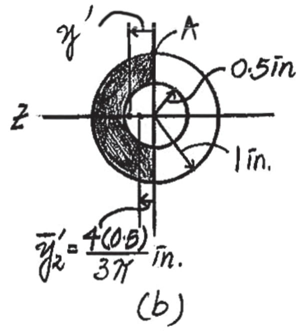
Applying the Hooke's Law for shear,

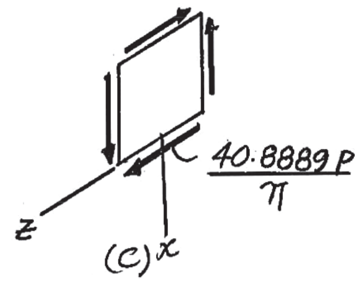
$$\tau_{xz} = G\,\gamma_{xz}$$

$$\frac{40.8889P}{\pi} = 11.0(10^3)\left[461.88(10^{-6})\right]$$

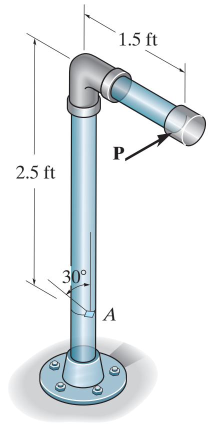
$$P = 0.3904 \text{ kip} = 390 \text{ lb} \qquad \textbf{Ans.}$$







***10–44.** A single strain gauge, placed in the vertical plane on the outer surface and at an angle of 30° to the axis of the pipe, gives a reading at point A of $\epsilon_a = -200(10^{-6})$. Determine the principal strains in the pipe at point A. The pipe has an outer diameter of 2 in. and an inner diameter of 1 in. and is made of A-36 steel.



Using the method of sections and consider the equilibrium of the FBD of the pipe's upper segment, Fig. a,

$$\Sigma F_z = 0; \qquad V_z - P = 0 \qquad V_z = P$$

$$\Sigma M_x = 0; \qquad T_x - P(1.5) = 0 \quad T_x = 1.5P$$

$$\Sigma M_y = 0; \qquad M_y - P(2.5) = 0 \quad M_y = 2.5P$$

By observation, no normal stress acting on point A. Thus, this is a case of pure shear.

For the case of pure shear,

$$\varepsilon_x = \varepsilon_z = \varepsilon_y = 0$$

$$\varepsilon_a = \varepsilon_x \cos^2\theta_a + \varepsilon_z \sin^2\theta_a + \gamma_{xz}\sin\theta_a\cos\theta_a$$

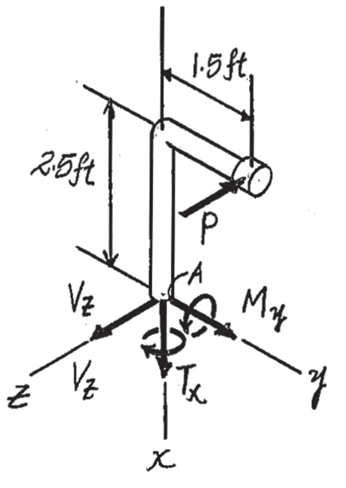
$$-200(10^{-6}) = 0 + 0 + \gamma_{xz}\ \sin 150^\circ \cos 150^\circ$$

$$\gamma_{xz} = 461.88(10^{-6})$$

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_z}{2} + \sqrt{\left(\frac{\varepsilon_x - \varepsilon_z}{2}\right)^2 + \left(\frac{\gamma_{xz}}{2}\right)^2}$$

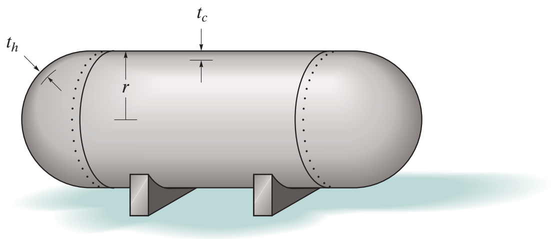
$$= \left[\frac{0+0}{2} \pm \sqrt{\left(\frac{0-0}{2}\right)^2 + \left(\frac{461.88}{2}\right)^2}\right](10^{-6})$$

$$\varepsilon_1 = 231(10^{-6}) \qquad \varepsilon_2 = -231(10^{-6})$$ **Ans.**



(a)

10–45. The cylindrical pressure vessel is fabricated using hemispherical end caps in order to reduce the bending stress that would occur if flat ends were used. The bending stresses at the seam where the caps are attached can be eliminated by proper choice of the thickness t_h and t_c of the caps and cylinder, respectively. This requires the radial expansion to be the same for both the hemispheres and cylinder. Show that this ratio is $t_c/t_h = (2 - \nu)/(1 - \nu)$. Assume that the vessel is made of the same material and both the cylinder and hemispheres have the same inner radius. If the cylinder is to have a thickness of 0.5 in., what is the required thickness of the hemispheres? Take $\nu = 0.3$.



For cylindrical vessel:

$$\sigma_1 = \frac{p\,r}{t_c}; \qquad \sigma_2 = \frac{p\,r}{2\,t_c}$$

$$\varepsilon_1 = \frac{1}{E}[\sigma_1 - v\,(\sigma_2 + \sigma_3)] \qquad \sigma_3 = 0$$

$$= \frac{1}{E}\left(\frac{p\,r}{t_c} - \frac{v\,p\,r}{2\,t_c}\right) = \frac{p\,r}{E\,t_c}\left(1 - \frac{1}{2}v\right)$$

$$d\,r = \varepsilon_1\,r = \frac{p\,r2}{E\,t_c}\left(1 - \frac{1}{2}v\right) \qquad (1)$$

For hemispherical end caps:

$$\sigma_1 = \sigma_2 = \frac{p\,r}{2\,t_h}$$

$$\varepsilon_1 = \frac{1}{E}[\sigma_1 - v\,(\sigma_2 + \sigma_3)]; \qquad \sigma_3 = 0$$

$$= \frac{1}{E}\left(\frac{p\,r}{2\,t_h} - \frac{v\,p\,r}{2\,t_h}\right) = \frac{p\,r}{2\,E\,t_h}(1 - v)$$

$$d\,r = \varepsilon_1\,r = \frac{p\,r^2}{2\,E\,t_h}(1 - v) \qquad (2)$$

Equate Eqs. (1) and (2):

$$\frac{p\,r^2}{E\,t_c}\left(1 - \frac{1}{2}v\right) = \frac{p\,r^2}{2\,E\,t_h}(1 - v)$$

$$\frac{t_c}{t_h} = \frac{2\,(1 - \frac{1}{2}v)}{1 - v} = \frac{2 - v}{1 - v} \qquad \textbf{QED}$$

$$t_h = \frac{(1 - v)\,t_c}{2 - v} = \frac{(1 - 0.3)\,(0.5)}{2 - 0.3} = 0.206 \text{ in.} \qquad \textbf{Ans.}$$

10–46. The principal strains in a plane, measured experimentally at a point on the aluminum fuselage of a jet aircraft, are $\epsilon_1 = 630(10^{-6})$ and $\epsilon_2 = 350(10^{-6})$. If this is a case of plane stress, determine the associated principal stresses at the point in the same plane. $E_{al} = 10(10^3)$ ksi and $\nu_{al} = 0.33$.

Normal Stresses: For plane stress, $\sigma_3 = 0$.

Normal Strains: Applying the generalized Hooke's Law.

$$\varepsilon_1 = \frac{1}{E}\left[\sigma_1 - \nu\,(\sigma_2 + \sigma_3)\right]$$

$$630\left(10^{-6}\right) = \frac{1}{10(10^3)}\left[\sigma_1 - 0.33(\sigma_2 + 0)\right]$$

$$6.30 = \sigma_1 - 0.33\sigma_2 \qquad [1]$$

$$\varepsilon_2 = \frac{1}{E}\left[\sigma_2 - \nu\,(\sigma_1 + \sigma_3)\right]$$

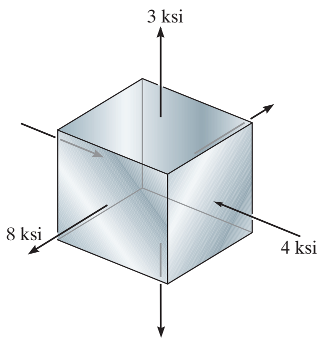
$$350\left(10^{-6}\right) = \frac{1}{10(10^3)}\left[\sigma_2 - 0.33(\sigma_1 + 0)\right]$$

$$3.50 = \sigma_2 - 0.33\sigma_1 \qquad [2]$$

Solving Eqs.[1] and [2] yields:

$$\sigma_1 = 8.37 \text{ ksi} \qquad \sigma_2 = 6.26 \text{ ksi} \qquad \textbf{Ans.}$$

10–47. The principal stresses at a point are shown in the figure. If the material is aluminum for which $E_{al} = 10(10^3)$ ksi and $\nu_{al} = 0.33$, determine the principal strains.



$$\varepsilon_1 = \frac{1}{E}\left[\sigma_1 - \nu(\sigma_2 + \sigma_3)\right] = \frac{1}{10(10^3)}\left\{8 - 0.33\left[3 + (-4)\right]\right\} = 833\,(10^{-6})$$

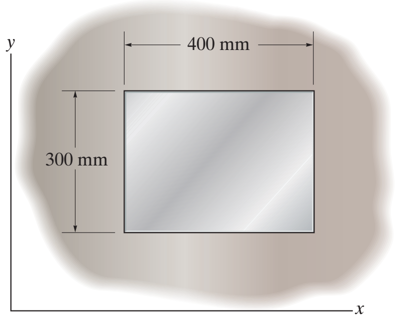
$$\varepsilon_2 = \frac{1}{E}\left[\sigma_2 - \nu(\sigma_1 + \sigma_3)\right] = \frac{1}{10(10^3)}\left\{3 - 0.33\left[8 + (-4)\right]\right\} = 168\,(10^{-6})$$

$$\varepsilon_3 = \frac{1}{E}\left[\sigma_3 - \nu(\sigma_1 + \sigma_2)\right] = \frac{1}{10(10^3)}\left[-4 - 0.33(8 + 3)\right] = -763\,(10^{-6})$$

Using these results,

$$\varepsilon_1 = 833(10^{-6}) \qquad \varepsilon_2 = 168(10^{-6}) \qquad \varepsilon_3 = -763(10^{-6})$$

***10–48.** The 6061-T6 aluminum alloy plate fits snugly into the rigid constraint. Determine the normal stresses σ_x and σ_y developed in the plate if the temperature is increased by $\Delta T = 50°\text{C}$. To solve, add the thermal strain $\alpha \Delta T$ to the equations for Hooke's Law.



Generalized Hooke's Law: Since the sides of the aluminum plate are confined in the rigid constraint along the x and y directions, $\varepsilon_x = \varepsilon_y = 0$. However, the plate is allowed to have free expansion along the z direction. Thus, $\sigma_z = 0$. With the additional thermal strain term, we have

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] + \alpha \Delta T$$

$$0 = \frac{1}{68.9\left(10^9\right)}\left[\sigma_x - 0.35\left(\sigma_y + 0\right)\right] + 24\left(10^{-6}\right)(50)$$

$$\sigma_x - 0.35\sigma_y = -82.68\left(10^6\right) \tag{1}$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right] + \alpha \Delta T$$

$$0 = \frac{1}{68.9\left(10^9\right)}\left[\sigma_y - 0.35(\sigma_x + 0)\right] + 24\left(10^{-6}\right)(50)$$

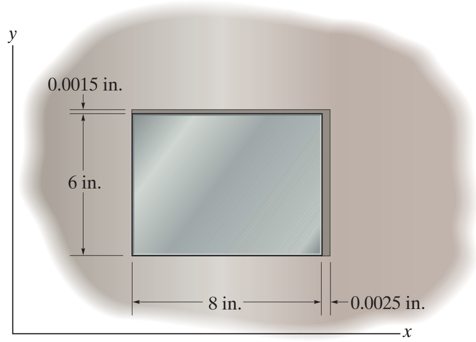
$$\sigma_y - 0.35\sigma_x = -82.68\left(10^6\right) \tag{2}$$

Solving Eqs. (1) and (2),

$$\sigma_x = \sigma_y = -127.2 \text{ MPa} = 127.2 \text{ MPa (C)}$$ **Ans.**

Since $\sigma_x = \sigma_y$ and $\sigma_y < \sigma_Y$, the above results are valid.

•10–49. Initially, gaps between the A-36 steel plate and the rigid constraint are as shown. Determine the normal stresses σ_x and σ_y developed in the plate if the temperature is increased by $\Delta T = 100°\text{F}$. To solve, add the thermal strain $\alpha\Delta T$ to the equations for Hooke's Law.



Generalized Hooke's Law: Since there are gaps between the sides of the plate and the rigid constraint, the plate is allowed to expand before it comes in contact with the constraint. Thus, $\varepsilon_x = \dfrac{\delta_x}{L_x} = \dfrac{0.0025}{8} = 0.3125\left(10^{-3}\right)$ and $\varepsilon_y = \dfrac{\delta_y}{L_y} = \dfrac{0.0015}{6} = 0.25\left(10^{-3}\right)$. However, the plate is allowed to have free expansion along the z direction. Thus, $\sigma_z = 0$. With the additional thermal strain term, we have

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] + \alpha\Delta T$$

$$0.3125\left(10^{-3}\right) = \frac{1}{29.0\left(10^3\right)}\left[\sigma_x - 0.32\left(\sigma_y + 0\right)\right] + 6.60\left(10^{-6}\right)(100)$$

$$\sigma_x - 0.32\sigma_y = -10.0775 \qquad (1)$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right] + \alpha\Delta T$$

$$0.25\left(10^{-3}\right) = \frac{1}{29.0\left(10^3\right)}\left[\sigma_y - 0.32(\sigma_x + 0)\right] + 6.60\left(10^{-6}\right)(100)$$

$$\sigma_y - 0.32\sigma_x = -11.89 \qquad (2)$$

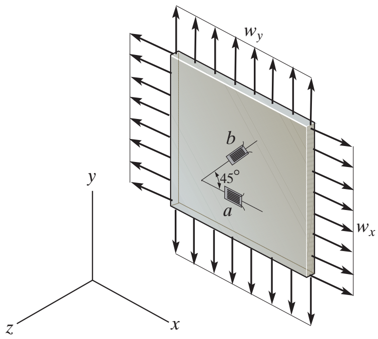
Solving Eqs. (1) and (2),

$$\sigma_x = -15.5 \text{ ksi} = 15.5 \text{ ksi (C)} \qquad \textbf{Ans.}$$

$$\sigma_y = -16.8 \text{ ksi} = 16.8 \text{ ksi (C)} \qquad \textbf{Ans.}$$

Since $\sigma_x < \sigma_Y$ and $\sigma_y < \sigma_Y$, the above results are valid.

10–50. Two strain gauges a and b are attached to a plate made from a material having a modulus of elasticity of $E = 70$ GPa and Poisson's ratio $\nu = 0.35$. If the gauges give a reading of $\epsilon_a = 450(10^{-6})$ and $\epsilon_b = 100(10^{-6})$, determine the intensities of the uniform distributed load w_x and w_y acting on the plate. The thickness of the plate is 25 mm.



Normal Strain: Since no shear force acts on the plane along the x and y axes, $\gamma_{xy} = 0$. With $\theta_a = 0$ and $\theta_b = 45°$, we have

$$\varepsilon_a = \varepsilon_x \cos^2 \theta_a + \varepsilon_y \sin^2 \theta_a + \gamma_{xy} \sin \theta_a \cos \theta_a$$

$$450\left(10^{-6}\right) = \varepsilon_x \cos^2 0° + \varepsilon_y \sin^2 0° + 0$$

$$\varepsilon_x = 450\left(10^{-6}\right)$$

$$\varepsilon_b = \varepsilon_x \cos^2 \theta_b + \varepsilon_y \sin^2 \theta_b + \gamma_{xy} \sin \theta_b \cos \theta_b$$

$$100\left(10^{-6}\right) = 450\left(10^{-6}\right) \cos^2 45° + \varepsilon_y \sin^2 45° + 0$$

$$\varepsilon_y = -250\left(10^{-6}\right)$$

Generalized Hooke's Law: This is a case of plane stress. Thus, $\sigma_z = 0$.

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right]$$

$$450\left(10^{-6}\right) = \frac{1}{70\left(10^9\right)}\left[\sigma_y - 0.35\left(\sigma_y + 0\right)\right]$$

$$\sigma_x - 0.35\sigma_y = 31.5\left(10^6\right) \qquad (1)$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right]$$

$$-250\left(10^{-6}\right) = \frac{1}{70\left(10^9\right)}\left[\sigma_y - 0.35\left(\sigma_y + 0\right)\right]$$

$$\sigma_y - 0.35\sigma_x = -17.5\left(10^6\right) \qquad (2)$$

Solving Eqs. (1) and (2),

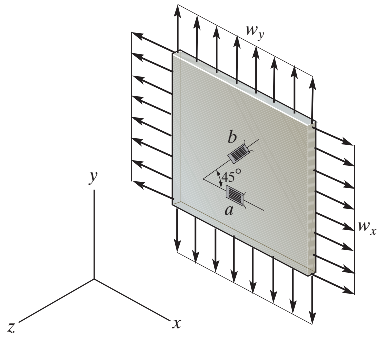
$$\sigma_y = -7.379\left(10^6\right)\text{N/m}^2 \qquad \sigma_x = 28.917\left(10^6\right)\text{N/m}^2$$

Then,

$$w_y = \sigma_y t = -7.379\left(10^6\right)(0.025) = -184 \text{ N/m} \qquad \textbf{Ans.}$$

$$w_x = \sigma_x t = 28.917\left(10^6\right)(0.025) = 723 \text{ N/m} \qquad \textbf{Ans.}$$

10–51. Two strain gauges a and b are attached to the surface of the plate which is subjected to the uniform distributed load $w_x = 700$ kN/m and $w_y = -175$ kN/m. If the gauges give a reading of $\epsilon_a = 450(10^{-6})$ and $\epsilon_b = 100(10^{-6})$, determine the modulus of elasticity E, shear modulus G, and Poisson's ratio ν for the material.



Normal Stress and Strain: The normal stresses along the x, y, and z axes are

$$\sigma_x = \frac{700\left(10^3\right)}{0.025} = 28\left(10^6\right)\text{N/m}^2$$

$$\sigma_y = -\frac{175\left(10^3\right)}{0.025} = -7\left(10^6\right)\text{N/m}^2$$

$$\sigma_z = 0 \text{ (plane stress)}$$

Since no shear force acts on the plane along the x and y axes, $\gamma_{xy} = 0$. With $\theta_a = 0°$ and $\theta_b = 45°$, we have

$$\varepsilon_a = \varepsilon_x \cos^2\theta_a + \varepsilon_y \sin^2\theta_a + \gamma_{xy}\sin\theta_a \cos\theta_a$$

$$450\left(10^{-6}\right) = \varepsilon_x \cos^2 0° + \varepsilon_y \sin^2 0° + 0$$

$$\varepsilon_x = 450\left(10^{-6}\right)$$

$$\varepsilon_b = \varepsilon_x \cos^2\theta_b + \varepsilon_y \sin^2\theta_b + \gamma_{xy}\sin\theta_b \cos\theta_b$$

$$100\left(10^{-6}\right) = 450\left(10^{-6}\right)\cos^2 45° + \varepsilon_y \sin^2 45° + 0$$

$$\varepsilon_y = -250\left(10^{-6}\right)$$

Generalized Hooke's Law:

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right]$$

$$450\left(10^{-6}\right) = \frac{1}{E}\left[28\left(10^6\right) - \nu\left[-7\left(10^6\right) + 0\right]\right]$$

$$450\left(10^{-6}\right)E - 7\left(10^6\right)\nu = 28\left(10^6\right) \qquad (1)$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]$$

$$-250\left(10^{-6}\right) = \frac{1}{E}\left\{-7\left(10^6\right) - \nu\left[28\left(10^6\right) + 0\right]\right\}$$

$$250\left(10^{-6}\right)E - 28\left(10^6\right)\nu = 7\left(10^6\right) \qquad (2)$$

Solving Eqs. (1) and (2),

$$E = 67.74\left(10^9\right)\text{ N/m}^2 = 67.7 \text{ GPa} \qquad \textbf{Ans.}$$

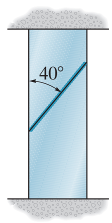
$$\nu = 0.3548 = 0.355 \qquad \textbf{Ans.}$$

Using the above results,

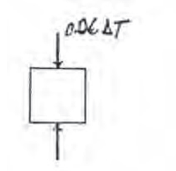
$$G = \frac{E}{2(1+\nu)} = \frac{67.74\left(10^9\right)}{2(1+0.3548)}$$

$$= 25.0\left(10^9\right)\text{N/m}^2 = 25.0 \text{ GPa} \qquad \textbf{Ans.}$$

***10–52.** The block is fitted between the fixed supports. If the glued joint can resist a maximum shear stress of $\tau_{\text{allow}} = 2$ ksi, determine the temperature rise that will cause the joint to fail. Take $E = 10\,(10^3)$ ksi, $\nu = 0.2$, and *Hint:* Use Eq. 10–18 with an additional strain term of $\alpha\Delta T$ (Eq. 4–4).



Normal Strain: Since the aluminum is confined along the *y direction* by the rigid frame, then $\varepsilon_y = 0$ and $\sigma_x = \sigma_z = 0$. Applying the generalized Hooke's Law with the additional thermal strain,

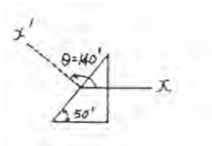


$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right] + \alpha\Delta T$$

$$0 = \frac{1}{10.0(10^3)}\left[\sigma_y - 0.2(0 + 0)\right] + 6.0\left(10^{-6}\right)(\Delta T)$$

$$\sigma_y = -0.06\Delta T$$

Construction of the Circle: In accordance with the sign convention. $\sigma_x = 0$, $\sigma_y = -0.06\Delta T$ and $\tau_{xy} = 0$. Hence.



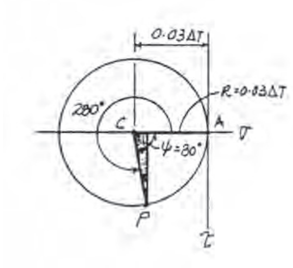
$$\sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} = \frac{0 + (-0.06\Delta T)}{2} = -0.03\Delta T$$

The coordinates for reference points A and C are $A\,(0, 0)$ and $C(-0.03\Delta T, 0)$.

The radius of the circle is $R = \sqrt{(0 - 0.03\Delta T)^2 + 0} = 0.03\Delta T$

Stress on The inclined plane: The shear stress components $\tau_{x'y'}$, are represented by the coordinates of point P on the circle.

$$\tau_{x'y'} = 0.03\Delta T \sin 80^\circ = 0.02954\Delta T$$



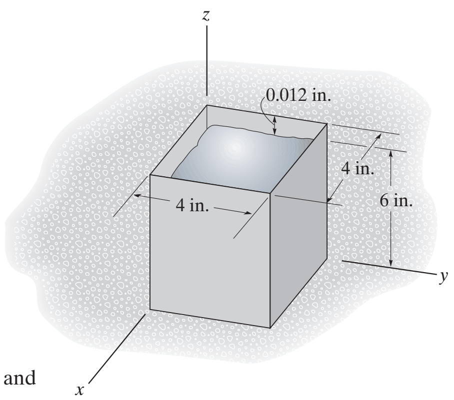
Allowable Shear Stress:

$$\tau_{\text{allow}} = \tau_{x'y'}$$

$$2 = 0.02954\Delta T$$

$$\Delta T = 67.7\ ^\circ\text{F} \qquad \textbf{Ans.}$$

•**10–53.** The smooth rigid-body cavity is filled with liquid 6061-T6 aluminum. When cooled it is 0.012 in. from the top of the cavity. If the top of the cavity is covered and the temperature is increased by 200°F, determine the stress components σ_x, σ_y, and σ_z in the aluminum. *Hint:* Use Eqs. 10–18 with an additional strain term of $\alpha\Delta T$ (Eq. 4–4).



Normal Strains: Since the aluminum is confined at its sides by a rigid container and allowed to expand in the z direction, $\varepsilon_x = \varepsilon_y = 0$; whereas $\varepsilon_z = \dfrac{0.012}{6} = 0.002$. Applying the generalized Hooke's Law with the additional thermal strain,

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right] + \alpha\Delta T$$

$$0 = \frac{1}{10.0(10^3)}\left[\sigma_x - 0.35\left(\sigma_y + \sigma_z\right)\right] + 13.1\left(10^{-6}\right)(200)$$

$$0 = \sigma_x - 0.35\sigma_y - 0.35\sigma_z + 26.2 \qquad [1]$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z) + \alpha\Delta T\right.$$

$$0 = \frac{1}{10.0(10^3)}\left[\sigma_y - 0.35(\sigma_x + \sigma_z)\right] + 13.1\left(10^{-6}\right)(200)$$

$$0 = \sigma_y - 0.35\sigma_x - 0.35\sigma_z + 26.2 \qquad [2]$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right] + \alpha\Delta T$$

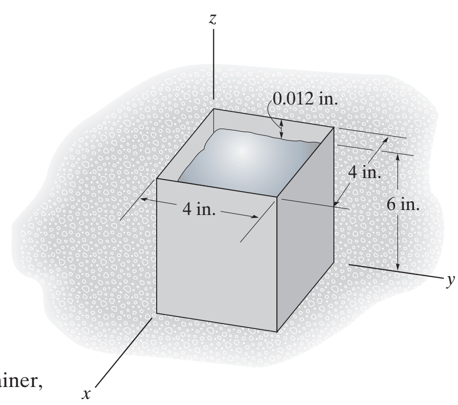
$$0.002 = \frac{1}{10.0(10^3)}\left[\sigma_z - 0.35\left(\sigma_x + \sigma_y\right)\right] + 13.1\left(10^{-6}\right)(200)$$

$$0 = \sigma_z - 0.35\sigma_x - 0.35\sigma_y + 6.20 \qquad [3]$$

Solving Eqs.[1], [2] and [3] yields:

$$\sigma_x = \sigma_y = -70.0 \text{ ksi} \qquad \sigma_z = -55.2 \text{ ksi} \qquad \textbf{Ans.}$$

10–54. The smooth rigid-body cavity is filled with liquid 6061-T6 aluminum. When cooled it is 0.012 in. from the top of the cavity. If the top of the cavity is not covered and the temperature is increased by 200°F, determine the strain components ϵ_x, ϵ_y, and ϵ_z in the aluminum. *Hint:* Use Eqs. 10–18 with an additional strain term of $\alpha \Delta T$ (Eq. 4–4).



Normal Strains: Since the aluminum is confined at its sides by a rigid container, then

$$\varepsilon_x = \varepsilon_y = 0 \qquad \textbf{Ans.}$$

and since it is not restrained in *z direction*, $\sigma_z = 0$. Applying the generalized Hooke's Law with the additional thermal strain,

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] + \alpha \Delta T$$

$$0 = \frac{1}{10.0(10^3)}\left[\sigma_x - 0.35\left(\sigma_y + 0\right)\right] + 13.1\left(10^{-6}\right)(200)$$

$$0 = \sigma_x - 0.35\sigma_y + 26.2 \qquad [1]$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right] + \alpha \Delta T$$

$$0 = \frac{1}{10.0(10^3)}\left[\sigma_y - 0.35(\sigma_x + 0)\right] + 13.1\left(10^{-6}\right)(200)$$

$$0 = \sigma_y - 0.35\sigma_x + 26.2 \qquad [2]$$

Solving Eqs. [1] and [2] yields:

$$\sigma_x = \sigma_y = -40.31 \text{ ksi}$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right] + \alpha \Delta T$$

$$= \frac{1}{10.0(10^3)}\{0 - 0.35[-40.31 + (-40.31)]\} + 13.1\left(10^{-6}\right)(200)$$

$$= 5.44\left(10^{-3}\right) \qquad \textbf{Ans.}$$

10–55. A thin-walled spherical pressure vessel having an inner radius r and thickness t is subjected to an internal pressure p. Show that the increase in the volume within the vessel is $\Delta V = (2p\pi r^4/Et)(1 - \nu)$. Use a small-strain analysis.

$$\sigma_1 = \sigma_2 = \frac{pr}{2t}$$

$$\sigma_3 = 0$$

$$\varepsilon_1 = \varepsilon_2 = \frac{1}{E}(\sigma_1 - v\sigma_2)$$

$$\varepsilon_1 = \varepsilon_2 = \frac{pr}{2t\,E}(1 - v)$$

$$\varepsilon_3 = \frac{1}{E}(-v(\sigma_1 + \sigma_2))$$

$$\varepsilon_3 = -\frac{v\,pr}{t\,E}$$

$$V = \frac{4\pi r^3}{3}$$

$$V + \Delta V = \frac{4\pi}{3}(r + \Delta r)^3 = \frac{4\pi r^3}{3}(1 + \frac{\Delta r}{r})^3$$

where $\Delta V \ll V, \Delta r \ll r$

$$V + \Delta V - \frac{4\pi r^3}{3}\left(1 + 3\frac{\Delta r}{r}\right)$$

$$\varepsilon_{\text{Vol}} = \frac{\Delta V}{V} = 3\left(\frac{\Delta r}{\mathrm{r}}\right)$$

Since $\varepsilon_1 = \varepsilon_2 = \dfrac{2\pi(r + \Delta r) - 2\pi r}{2\pi r} = \dfrac{\Delta r}{r}$

$$\varepsilon_{\text{Vol}} = 3\varepsilon_1 = \frac{3pr}{2t\,E}(1 - v)$$

$$\Delta V = V e_{\text{Vol}} = \frac{2p\pi\, r^4}{E\,t}(1 - v)$$

QED

***10–56.** A thin-walled cylindrical pressure vessel has an inner radius r, thickness t, and length L. If it is subjected to an internal pressure p, show that the increase in its inner radius is $dr = r\epsilon_1 = pr^2(1 - \frac{1}{2}\nu)/Et$ and the increase in its length is $\Delta L = pLr(\frac{1}{2} - \nu)/Et$. Using these results, show that the change in internal volume becomes $dV = \pi r^2(1 + \epsilon_1)^2(1 + \epsilon_2)L - \pi r^2 L$. Since ϵ_1 and ϵ_2 are small quantities, show further that the change in volume per unit volume, called *volumetric strain*, can be written as $dV/V = pr(2.5 - 2\nu)/Et$.

Normal stress:

$$\sigma_1 = \frac{p\,r}{t}; \qquad \sigma_2 = \frac{p\,r}{2\,t}$$

Normal strain: Applying Hooke's law

$$\varepsilon_1 = \frac{1}{E}[\sigma_1 - v(\sigma_2 + \sigma_3)], \qquad \sigma_3 = 0$$

$$= \frac{1}{E}\left(\frac{p\,r}{t} - \frac{v\,p\,r}{2\,t}\right) = \frac{p\,r}{E\,t}\left(1 - \frac{1}{2}\,v\right)$$

$$d\,r = \varepsilon_t\,r = \frac{p\,r^2}{E\,t}\left(1 - \frac{1}{2}\,v\right) \qquad \textbf{QED}$$

$$\varepsilon_2 = \frac{1}{E}[\sigma_2 - v(\sigma_1 + \sigma_3)], \qquad \sigma_3 = 0$$

$$= \frac{1}{E}\left(\frac{p\,r}{2\,t} - \frac{v\,p\,r}{t}\right) = \frac{p\,r}{E\,t}\left(\frac{1}{2} - v\right)$$

$$\Delta L = \varepsilon_2\,L = \frac{p\,L\,r}{E\,t}\left(\frac{1}{2} - v\right) \qquad \textbf{QED}$$

$$V' = \pi(r + \varepsilon_1\,r)^2\,(L + \varepsilon_2 L)\,; \qquad V = \pi\,r^2\,L$$

$$dV = V' - V = \pi r^2\,(1 + \varepsilon_1)^2\,(1 + \varepsilon_2)L - \pi r^2\,L \qquad \textbf{QED}$$

$(1 + \varepsilon_1)^2 = 1 + 2\,\varepsilon_1$ neglect ε_1^2 term

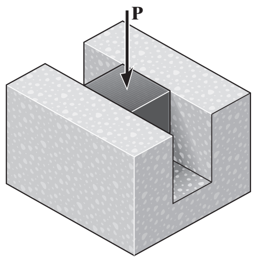
$(1 + \varepsilon_1)^2\,(1 + \varepsilon_2) = (1 + 2\,\varepsilon_1)(1 + \varepsilon_2) = 1 + \varepsilon_2 + 2\,\varepsilon_1$ neglect $\varepsilon_1\varepsilon_2$ term

$$\frac{dV}{V} = 1 + \varepsilon_2 + 2\,\varepsilon_1 - 1 = \varepsilon_2 + 2\,\varepsilon_1$$

$$= \frac{p\,r}{E\,t}\left(\frac{1}{2} - v\right) + \frac{2\,p\,r}{E\,t}\left(1 - \frac{1}{2}\,v\right)$$

$$= \frac{p\,r}{E\,t}\,(2.5 - 2\,v) \qquad \textbf{QED}$$

10–57. The rubber block is confined in the U-shape smooth rigid block. If the rubber has a modulus of elasticity E and Poisson's ratio ν, determine the effective modulus of elasticity of the rubber under the confined condition.



Generalized Hooke's Law: Under this confined condition, $\varepsilon_x = 0$ and $\sigma_y = 0$. We have

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right]$$

$$0 = \frac{1}{E}(\sigma_x - \nu\sigma_z)$$

$$\sigma_x = \nu\sigma_z \tag{1}$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right]$$

$$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + 0)]$$

$$\varepsilon_z = \frac{1}{E}(\sigma_z - \nu\sigma_x) \tag{2}$$

Substituting Eq. (1) into Eq. (2),

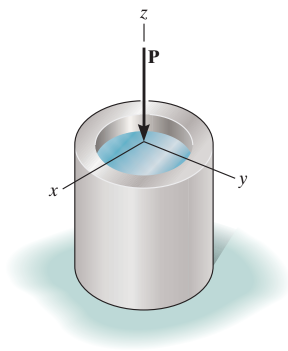
$$\varepsilon_z = \frac{\sigma_z}{E}\left(1 - \nu^2\right)$$

The effective modulus of elasticity of the rubber block under the confined condition can be determined by considering the rubber block as unconfined but rather undergoing the same normal strain of ε_z when it is subjected to the same normal stress σ_z, Thus,

$$\sigma_z = E_{\text{eff}}\,\varepsilon_z$$

$$E_{\text{eff}} = \frac{\sigma_z}{\varepsilon_z} = \frac{\sigma_z}{\dfrac{\sigma_z}{E}\left(1 - \nu^2\right)} = \frac{E}{1 - \nu^2} \qquad \textbf{Ans.}$$

10–58. A soft material is placed within the confines of a rigid cylinder which rests on a rigid support. Assuming that $\epsilon_x = 0$ and $\epsilon_y = 0$, determine the factor by which the modulus of elasticity will be increased when a load is applied if $\nu = 0.3$ for the material.



Normal Strain: Since the material is confined in a rigid cylinder. $\varepsilon_x = \varepsilon_y = 0$. Applying the generalized Hooke's Law,

$$\varepsilon_x = \frac{1}{E}\left[\sigma_z - v(\sigma_y + \sigma_x)\right]$$

$$0 = \sigma_x - v(\sigma_y + \sigma_z) \qquad \textbf{[1]}$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - v(\sigma_x + \sigma_z)\right]$$

$$0 = \sigma_y - v(\sigma_x + \sigma_z) \qquad \textbf{[2]}$$

Solving Eqs.[1] and [2] yields:

$$\sigma_x = \sigma_y = \frac{v}{1 - v}\sigma_z$$

Thus,

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - v(\sigma_x + \sigma_y)\right]$$

$$= \frac{1}{E}\left[\sigma_z - v\left(\frac{v}{1 - v}\sigma_z + \frac{v}{1 - v}\sigma_z\right)\right]$$

$$= \frac{\sigma_z}{E}\left[1 - \frac{2v^2}{1 - v}\right]$$

$$= \frac{\sigma_z}{E}\left[\frac{1 - v - 2v^2}{1 - v}\right]$$

$$= \frac{\sigma_z}{E}\left[\frac{(1 + v)(1 - 2v}{1 - v}\right]$$

Thus, when the material is not being confined and undergoes the same normal strain of ε_z, then the requtred modulus of elasticity is

$$E' = \frac{\sigma_z}{\varepsilon_z} = \frac{1 - v}{(1 - 2v)(1 + v)}E$$

The increased factor is $k = \frac{E'}{E} = \frac{1 - v}{(1 - 2v)(1 + v)}$

$$= \frac{1 - 0.3}{[1 - 2(0.3)](1 + 0.3)}$$

$$= 1.35 \qquad \textbf{Ans.}$$

10–59. A material is subjected to plane stress. Express the distortion-energy theory of failure in terms of σ_x, σ_y, and τ_{xy}.

Maximum distortion energy theory:

$$(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2) = \sigma_Y^2 \qquad (1)$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Let $a = \dfrac{\sigma_x + \sigma_y}{2}$ and $b = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$$\sigma_1 = a + b; \qquad \sigma_2 = a - b$$

$$\sigma_1^2 = a^2 + b^2 + 2\,a\,b; \qquad \sigma_2^2 = a^2 + b^2 - 2\,a\,b$$

$$\sigma_1\sigma_2 = a^2 - b^2$$

From Eq. (1)

$$(a^2 + b^2 + 2\,a\,b - a^2 + b^2 + a^2 + b^2 - 2\,a\,b) = \sigma_y^2$$

$$(a^2 + 3\,b^2) = \sigma_Y^2$$

$$\frac{(\sigma_x + \sigma_y)^2}{4} + 3\,\frac{(\sigma_x - \sigma_y)^2}{4} + 3\,\tau_{xy}^2 = \sigma_Y^2$$

$$\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\,\tau_{xy}^2 = \sigma_Y^2$$ **Ans.**

***10–60.** A material is subjected to plane stress. Express the maximum-shear-stress theory of failure in terms of σ_x, σ_y, and τ_{xy}. Assume that the principal stresses are of different algebraic signs.

Maximum shear stress theory:

$$|\sigma_1 - \sigma_2| = \sigma_Y \qquad (1)$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\left|\sigma_1 - \sigma_2\right| = 2\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

From Eq. (1)

$$4\left[\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right] = \sigma_Y^2$$

$$(\sigma_x - \sigma_y)^2 + 4\,\tau_{xy}^2 = \sigma_Y^2$$ **Ans.**

•10–61. An aluminum alloy 6061-T6 is to be used for a solid drive shaft such that it transmits 40 hp at 2400 rev/min. Using a factor of safety of 2 with respect to yielding, determine the smallest-diameter shaft that can be selected based on the maximum-shear-stress theory.

$$\omega = \left(2400\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)\left(\frac{1\text{ min}}{60\text{s}}\right) = 80\,\pi\text{ rad/s}$$

$$T = \frac{P}{\omega} = \frac{40\,(550)\,(12)}{80\,\pi} = \frac{3300}{\pi}\text{ lb}\cdot\text{in.}$$

Applying $\tau = \dfrac{T\,c}{J}$

$$\tau = \frac{\left(\frac{3300}{\pi}\right)c}{\frac{\pi}{2}c^4} = \frac{6600}{\pi^3\,c^3}$$

The principal stresses:

$$\sigma_1 = \tau = \frac{6600}{\pi^2\,c^3}; \qquad \sigma_2 = -\tau = \frac{6600}{\pi^2\,c^3}$$

Maximum shear stress theory: Both principal stresses have opposite sign, hence,

$$\left|\sigma_1 - \sigma_2\right| = \frac{\sigma_Y}{\text{F.S.}}; \qquad 2\left(\frac{6600}{\pi^2 c^3}\right) = \left|\frac{37\,(10^3)}{2}\right|$$

$c = 0.4166$ in.

$d = 0.833$ in. **Ans.**

10–62. Solve Prob. 10–61 using the maximum-distortion-energy theory.

$$\omega = \left(2400\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\text{p rad}}{\text{rev}}\right)\left(\frac{1\text{ min}}{60\text{s}}\right) = 80\,\pi\text{ rad/s}$$

$$T = \frac{P}{\omega} = \frac{40\,(550)\,(12)}{80\,\pi} = \frac{3300}{\pi}\text{ lb.in.}$$

Applying $\tau = \dfrac{T\,c}{J}$

$$\tau = \frac{\left(\frac{3300}{\pi}\right)c}{\frac{\pi}{2}c^4} = \frac{6600}{\pi^2\,c^3}$$

The principal stresses:

$$\sigma_1 = \tau = \frac{6600}{\pi^2\,c^3}; \qquad \sigma_2 = -\tau = -\frac{6600}{\pi^2\,c^3}$$

The maximum distortion-energy theory:

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \left(\frac{\sigma_Y}{\text{F.S.}}\right)^2$$

$$3\left[\frac{6600}{\pi^2 c^3}\right]^2 = \left(\frac{37(10^3)}{2}\right)^2$$

$c = 0.3971$ in.

$d = 0.794$ in. **Ans.**

10–63. An aluminum alloy is to be used for a drive shaft such that it transmits 25 hp at 1500 rev/min. Using a factor of safety of 2.5 with respect to yielding, determine the smallest-diameter shaft that can be selected based on the maximum-distortion-energy theory. $\sigma_Y = 3.5$ ksi.

$$T = \frac{P}{\omega} \qquad \omega = \frac{1500(2\pi)}{60} = 50\pi$$

$$T = \frac{25(550)(12)}{50\pi} = \frac{3300}{\pi}$$

$$\tau = \frac{Tc}{J}, \qquad J = \frac{\pi}{2}c^4$$

$$\tau = \frac{\frac{3300}{\pi}c}{\frac{\pi}{2}c^4} = \frac{6600}{\pi^2 c^3}$$

$$\sigma_1 = \frac{6600}{\pi^2 c^3} \qquad \sigma_2 = \frac{-6600}{\pi^2 c^3}$$

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \left(\frac{\sigma_Y}{\text{F.S.}}\right)^2$$

$$3\left(\frac{6600}{\pi^2 c^3}\right)^2 = \left(\frac{3.5(10^3)}{2.5}\right)^2$$

$$c = 0.9388 \text{ in.}$$

$$d = 1.88 \text{ in.}$$ **Ans.**

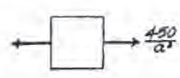
***10–64.** A bar with a square cross-sectional area is made of a material having a yield stress of $\sigma_Y = 120$ ksi. If the bar is subjected to a bending moment of 75 kip · in., determine the required size of the bar according to the maximum-distortion-energy theory. Use a factor of safety of 1.5 with respect to yielding.

Normal and Shear Stress: Applying the flexure formula,

$$\sigma = \frac{Mc}{I} = \frac{75\left(\frac{a}{2}\right)}{\frac{1}{12}a^4} = \frac{450}{a^3}$$

In-Plane Principal Stress: Since no shear stress acts on the element

$$\sigma_1 = \sigma_x = \frac{450}{a^3} \qquad \sigma_2 = \sigma_y = 0$$



Maximum Distortion Energy Theory:

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_{\text{allow}}^2$$

$$\left(\frac{450}{a^3}\right)^2 - 0 + 0 = \left(\frac{120}{1.5}\right)^2$$

$$a = 1.78 \text{ in.}$$ **Ans.**

•10–65. Solve Prob. 10–64 using the maximum-shear-stress theory.

Normal and Shear Stress: Applying the flexure formula,

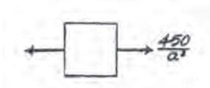
$$\sigma = \frac{Mc}{I} = \frac{75\left(\frac{a}{2}\right)}{\frac{1}{12}a^4} = \frac{450}{a^3}$$

In-Plane Principal Stress: Since no shear stress acts on the element.

$$\sigma_1 = \sigma_x = \frac{450}{a^3} \qquad \sigma_2 = \sigma_x = 0$$

Maximum Shear Stress Theory:

$$|\sigma_2| = 0 < \sigma_{\text{allow}} = \frac{120}{1.5} = 80.0 \text{ ksi} \qquad \textit{(O.K!)}$$



$$|\sigma_1| = \sigma_{\text{allow}}$$

$$\frac{450}{a^3} = \frac{120}{1.5}$$

$$a = 1.78 \text{ in.} \qquad \textbf{Ans.}$$

10–66. Derive an expression for an equivalent torque T_e that, if applied alone to a solid bar with a circular cross section, would cause the same energy of distortion as the combination of an applied bending moment M and torque T.

$$\tau = \frac{T_e\, c}{J}$$

Principal stress:

$$\sigma_1 = \tau_x\,' \quad \sigma_2 = -\tau$$

$$u_d = \frac{1 + v}{3\,E}\left(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2\right)$$

$$(u_d)_1 = \frac{1 + v}{3\,E}\left(3\,\tau^2\right) = \frac{1 + v}{3\,E}\left(\frac{3\,T_x^2\,c^2}{J^2}\right)$$

Bending moment and torsion:

$$\sigma = \frac{M\,c}{I}; \qquad \tau = \frac{T\,c}{J}$$

Principal stress:

$$\sigma_{1,2} = \frac{\sigma + 0}{2} \pm \sqrt{\left(\frac{\sigma - 0}{2}\right)^2 + \tau^2}$$

$$\sigma_1 = \frac{\sigma}{2} + \sqrt{\frac{\sigma^2}{4} + \tau^2}; \qquad \sigma_2 = \frac{\sigma}{2} - \sqrt{\frac{\sigma^2}{4} + \tau^2}$$

10–66. Continued

Let $a = \dfrac{\sigma}{2} \qquad b = \sqrt{\dfrac{\sigma^2}{4} + \tau^2}$

$$\sigma_1^2 = a^2 + b^2 + 2\,a\,b$$

$$\sigma_1\sigma_2 = a^2 - b^2$$

$$\sigma_2^2 = a^2 + b^2 - 2\,a\,b$$

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = 3\,b^2 + a^2$$

$$u_d = \frac{1+v}{3\,E}(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2)$$

$$(u_d)_2 = \frac{1+v}{3\,E}(3\,b^2 + a^2) = \frac{1+v}{3\,E}\left(\frac{3\,\sigma^2}{4} + 3\tau^2 + \frac{\sigma^2}{4}\right)$$

$$= \frac{1+v}{3\,E}(\sigma^2 + 3\,\tau^2) = \frac{c^2(1+v)}{3\,E}\left(\frac{M^2}{I^2} + \frac{3\,T^2}{J^2}\right)$$

$$(u_d)_1 = (u_d)_2$$

$$\frac{c^3(1+v)}{3\,E}\frac{3\,T_x{}^2}{J^2} = \frac{c^2(1+v)}{3\,E}\left(\frac{M^2}{I^2} + \frac{3\,T^2}{J^2}\right)$$

For circular shaft

$$\frac{J}{I} = \frac{\frac{\pi}{3}c^4}{\frac{\pi}{4}c^4} = 2$$

$$T_e = \sqrt{\frac{J^2}{I^2}\frac{M^2}{3} + T^2}$$

$$T_e = \sqrt{\frac{4}{3}M^2 + T^2} \qquad \textbf{Ans.}$$

10–67. Derive an expression for an equivalent bending moment M_e that, if applied alone to a solid bar with a circular cross section, would cause the same energy of distortion as the combination of an applied bending moment M and torque T.

Principal stresses:

$$\sigma_1 = \frac{M_e\,c}{I}; \qquad \sigma_2 = 0$$

$$u_d = \frac{1+v}{3\,E}(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2)$$

$$(u_d)_1 = \frac{1+v}{3\,E}\left(\frac{M_e^2c^2}{I^2}\right) \qquad (1)$$

10–67. Continued

Principal stress:

$$\sigma_{1,2} = \frac{\sigma + 0}{2} \pm \sqrt{\left(\frac{\sigma - 0}{2}\right)^2 + \tau^3}$$

$$\sigma_1 = \frac{\sigma}{2} + \sqrt{\frac{\sigma^2}{4} + \tau^2}; \qquad \sigma_2 = \frac{\sigma}{2} - \sqrt{\frac{\sigma^2}{4} + \tau^2}$$

Distortion Energy:

Let $a = \frac{\sigma}{2}, b = \sqrt{\frac{\sigma^2}{4} + \tau^2}$

$$\sigma_1^2 = a^2 + b^2 + 2\,a\,b$$

$$\sigma_1 \sigma_2 = a^2 - b^2$$

$$\sigma_2^2 = a^2 + b^2 - 2\,a\,b$$

$$\sigma_2^2 - \sigma_1\sigma_2 + \sigma_2^2 = 3\,b^2 + a^2$$

Apply $\sigma = \frac{M\,c}{I}; \qquad \tau = \frac{T\,c}{J}$

$$(u_d)_2 = \frac{1 + v}{3\,E}(3\,b^2 + a^2) = \frac{1 + v}{3\,E}\left(\frac{\sigma^2}{4} + \frac{3\sigma^2}{4} + 3\,\tau^2\right)$$

$$= \frac{1 + v}{3\,E}(\sigma^2 + 3\,\tau^2) = \frac{1 + v}{3\,E}\left(\frac{M^2c^2}{I^2} + \frac{3\,T^2c^2}{J^2}\right) \qquad (2)$$

Equating Eq. (1) and (2) yields:

$$\frac{(1 + v)}{3\,E}\left(\frac{M_e\,c^2}{I^2}\right) = \frac{1 + v}{3\,E}\left(\frac{M^2c^2}{I^2} + \frac{3T^2c^2}{J^2}\right)$$

$$\frac{M_e^2}{I^2} = \frac{M^1}{I^2} + \frac{3\,T^2}{J^2}$$

$$M_e^2 = M^1 + 3\,T^2\left(\frac{I}{J}\right)^2$$

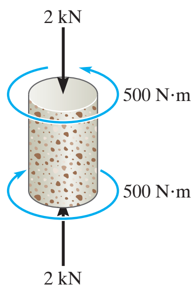
For circular shaft

$$\frac{I}{J} = \frac{\frac{\pi}{4}c^4}{\frac{\pi}{2}c^4} = \frac{1}{2}$$

Hence, $M_e^2 = M^2 + 3\,T^2\left(\frac{1}{2}\right)^2$

$$M_e = \sqrt{M^2 + \frac{3}{4}T^2} \qquad \textbf{Ans.}$$

***10–68.** The short concrete cylinder having a diameter of 50 mm is subjected to a torque of 500 N · m and an axial compressive force of 2 kN. Determine if it fails according to the maximum-normal-stress theory. The ultimate stress of the concrete is $\sigma_{\text{ult}} = 28$ MPa.

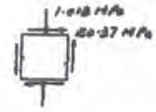


$$A = \frac{\pi}{4}(0.05)^2 = 1.9635(10^{-3})\ \text{m}^2$$

$$J = \frac{\pi}{2}(0.025)^4 = 0.61359(10^{-4})\ \text{m}^4$$

$$\sigma = \frac{P}{A} = \frac{2(10^3)}{1.9635(10^{-3})} = 1.019\ \text{MPa}$$

$$\tau = \frac{Tc}{J} = \frac{500(0.025)}{0.61359(10^{-6})} = 20.372\ \text{MPa}$$



$$\sigma_x = 0 \qquad \sigma_y = -1.019\ \text{MPa} \qquad \tau_{xy} = 20.372\ \text{MPa}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$\sigma_{1,2} = \frac{0 - 1.018}{2} \pm \sqrt{\left(\frac{0 - (-1.019)}{2}\right)^2 + 20.372^2}$$

$$\sigma_1 = 19.87\ \text{MPa} \qquad \sigma_2 = -20.89\ \text{MPa}$$

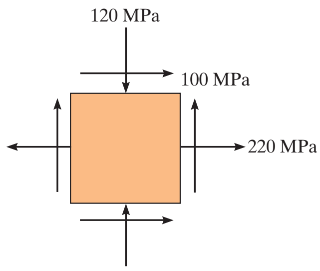
Failure criteria:

$|\sigma_1| < \sigma_{\text{alt}} = 28$ MPa **OK**

$|\sigma_2| < \sigma_{\text{alt}} = 28$ MPa **OK**

No. **Ans.**

•10–69. Cast iron when tested in tension and compression has an ultimate strength of $(\sigma_{\text{ult}})_t = 280$ MPa and $(\sigma_{\text{ult}})_c = 420$ MPa, respectively. Also, when subjected to pure torsion it can sustain an ultimate shear stress of $\tau_{\text{ult}} = 168$ MPa. Plot the Mohr's circles for each case and establish the failure envelope. If a part made of this material is subjected to the state of plane stress shown, determine if it fails according to Mohr's failure criterion.



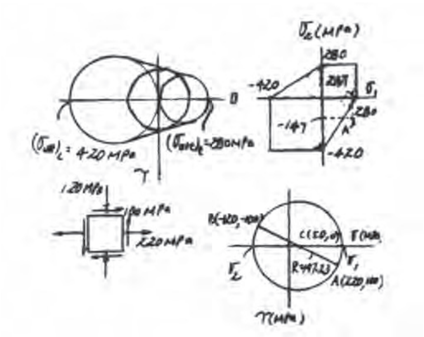
$$\sigma_1 = 50 + 197.23 = 247\ \text{MPa}$$

$$\sigma_2 = 50 - 197.23 = -147\ \text{MPa}$$

The principal stress coordinate is located at point A which is outside the shaded region. Therefore the material fails according to Mohr's failure criterion.

Yes. **Ans.**

10–69. Continued



10–70. Derive an expression for an equivalent bending moment M_e that, if applied alone to a solid bar with a circular cross section, would cause the same maximum shear stress as the combination of an applied moment M and torque T. Assume that the principal stresses are of opposite algebraic signs.

Bending and Torsion:

$$\sigma = \frac{M\,c}{I} = \frac{M\,c}{\frac{\pi}{4}c^4} = \frac{4\,M}{\pi\,c^3}; \qquad \tau = \frac{T\,c}{J} = \frac{T\,c}{\frac{\pi}{2}c^4} = \frac{2\,T}{\pi\,c^3}$$

The principal stresses:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{\frac{4\,M}{\pi c^3} + 0}{2} \pm \sqrt{\left(\frac{\frac{4\,M}{\pi\,c^3} - 0}{2}\right)^2 + \left(\frac{2T}{\pi\,c^3}\right)^2}$$

$$= \frac{2\,M}{\pi\,c^3} \pm \frac{2}{\pi\,c^3}\sqrt{M^2 + T^2}$$

$$\tau_{\substack{\text{abs}\\ \text{max}}} = \sigma_1 - \sigma_2 = 2\left[\frac{2}{\pi\,c^3}\sqrt{M^2 + T^2}\right] \qquad (1)$$

Pure bending:

$$\sigma_1 = \frac{M\,c}{I} = \frac{M_e\,c}{\frac{\pi}{4}c^4} = \frac{4\,M_e}{\pi\,c^3}; \qquad \sigma_2 = 0$$

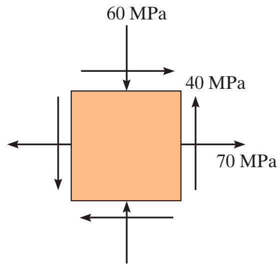
$$\tau_{\substack{\text{abs}\\ \text{max}}} = \sigma_1 - \sigma_2 = \frac{4\,M_e}{\pi\,c^3} \qquad (2)$$

Equating Eq. (1) and (2) yields:

$$\frac{4}{\pi\,c^3}\sqrt{M^2 + T^2} = \frac{4\,M_e}{\pi\,c^3}$$

$$M_e = \sqrt{M^2 + T^2}$$ **Ans.**

10–71. The components of plane stress at a critical point on an A-36 steel shell are shown. Determine if failure (yielding) has occurred on the basis of the maximum-shear-stress theory.



In accordance to the established sign convention, $\sigma_x = 70$ MPa, $\sigma_y = -60$ MPa and $\tau_{xy} = 40$ MPa.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{70 + (-60)}{2} \pm \sqrt{\left[\frac{70 - (-60)}{2}\right]^2 + 40^2}$$

$$= 5 \pm \sqrt{5825}$$

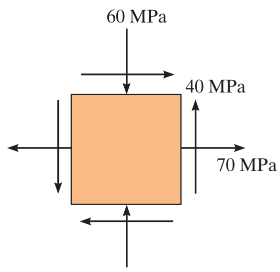
$$\sigma_1 = 81.32 \text{ MPa} \qquad \sigma_2 = -71.32 \text{ MPa}$$

In this case, σ_1 and σ_2 have opposite sign. Thus,

$$|\sigma_1 - \sigma_2| = |81.32 - (-71.32)| = 152.64 \text{ MPa} < \sigma_y = 250 \text{ MPa}$$

Based on this result, **the steel shell does not yield according to the maximum shear stress theory**.

***10–72.** The components of plane stress at a critical point on an A-36 steel shell are shown. Determine if failure (yielding) has occurred on the basis of the maximum-distortion-energy theory.



In accordance to the established sign convention, $\sigma_x = 70$ MPa, $\sigma_y = -60$ MPa and $\tau_{xy} = 40$ MPa.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{70 + (-60)}{2} \pm \sqrt{\left[\frac{70 - (-60)}{2}\right]^2 + 40^2}$$

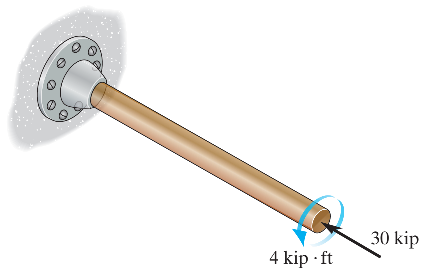
$$= 5 \pm \sqrt{5825}$$

$$\sigma_1 = 81.32 \text{ MPa} \qquad \sigma_2 = -71.32 \text{ MPa}$$

$$\sigma_1{}^2 - \sigma_1\sigma_2 + \sigma_2{}^2 = 81.32^2 - 81.32(-71.32) + (-71.32)^2 = 17{,}500 < \sigma_y{}^2 = 62500$$

Based on this result, **the steel shell does not yield according to the maximum distortion energy theory**.

•10–73. If the 2-in. diameter shaft is made from brittle material having an ultimate strength of $\sigma_{ult} = 50$ ksi for both tension and compression, determine if the shaft fails according to the maximum-normal-stress theory. Use a factor of safety of 1.5 against rupture.

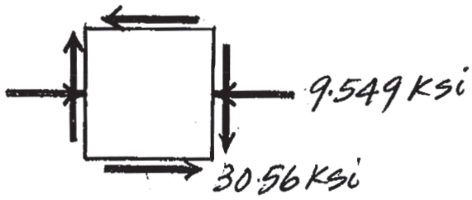


Normal Stress and Shear Stresses. The cross-sectional area and polar moment of inertia of the shaft's cross-section are

$$A = \pi\left(1^2\right) = \pi \text{in}^2 \qquad J = \frac{\pi}{2}\left(1^4\right) = \frac{\pi}{2}\text{ in}^4$$

The normal stress is caused by axial stress.

$$\sigma = \frac{N}{A} = -\frac{30}{\pi} = -9.549 \text{ ksi}$$



(a)

The shear stress is contributed by torsional shear stress.

$$\tau = \frac{Tc}{J} = \frac{4(12)(1)}{\dfrac{\pi}{2}} = 30.56 \text{ ksi}$$

The state of stress at the points on the surface of the shaft is represented on the element shown in Fig. *a*.

In-Plane Principal Stress. $\sigma_x = -9.549$ ksi, $\sigma_y = 0$ and $\tau_{xy} = -30.56$ ksi. We have

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{-9.549 + 0}{2} \pm \sqrt{\left(\frac{-9.549 - 0}{2}\right)^2 + (-30.56)^2}$$

$$= (-4.775 \pm 30.929) \text{ ksi}$$

$$\sigma_1 = 26.15 \text{ ksi} \qquad \sigma_2 = -35.70 \text{ ksi}$$

Maximum Normal-Stress Theory.

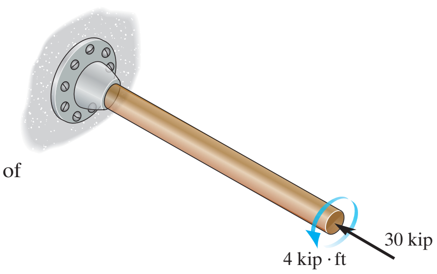
$$\sigma_{\text{allow}} = \frac{\sigma_{\text{ult}}}{F.S.} = \frac{50}{1.5} = 33.33 \text{ ksi}$$

$$|\sigma_1| = 26.15 \text{ ksi} < \sigma_{\text{allow}} = 33.33 \text{ ksi} \qquad \text{(O.K.)}$$

$$|\sigma_2| = 35.70 \text{ ksi} > \sigma_{\text{allow}} = 33.33 \text{ ksi} \qquad \text{(N.G.)}$$

Based on these results, the material *fails* according to the maximum normal-stress theory.

10–74. If the 2-in. diameter shaft is made from cast iron having tensile and compressive ultimate strengths of $(\sigma_{ult})_t = 50$ ksi and $(\sigma_{ult})_c = 75$ ksi, respectively, determine if the shaft fails in accordance with Mohr's failure criterion.



Normal Stress and Shear Stresses. The cross-sectional area and polar moment of inertia of the shaft's cross-section are

$$A = \pi\left(1^2\right) = \pi \text{ in}^2 \qquad J = \frac{\pi}{2}\left(1^4\right) = \frac{\pi}{2} \text{ in}^4$$

The normal stress is contributed by axial stress.

$$\sigma = \frac{N}{A} = -\frac{30}{\pi} = -9.549 \text{ ksi}$$

The shear stress is contributed by torsional shear stress.

$$\tau = \frac{Tc}{J} = \frac{4(12)(1)}{\frac{\pi}{2}} = 30.56 \text{ ksi}$$

The state of stress at the points on the surface of the shaft is represented on the element shown in Fig. *a*.

In-Plane Principal Stress. $\sigma_x = -9.549$ ksi, $\sigma_y = 0$, and $\tau_{xy} = -30.56$ ksi. We have

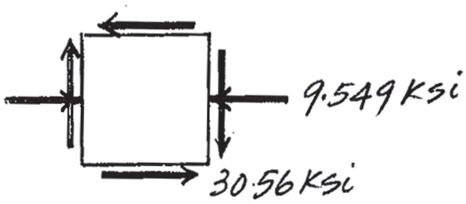
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{-9.549 + 0}{2} \pm \sqrt{\left(\frac{-9.549 - 0}{2}\right)^2 + (-30.56)^2}$$

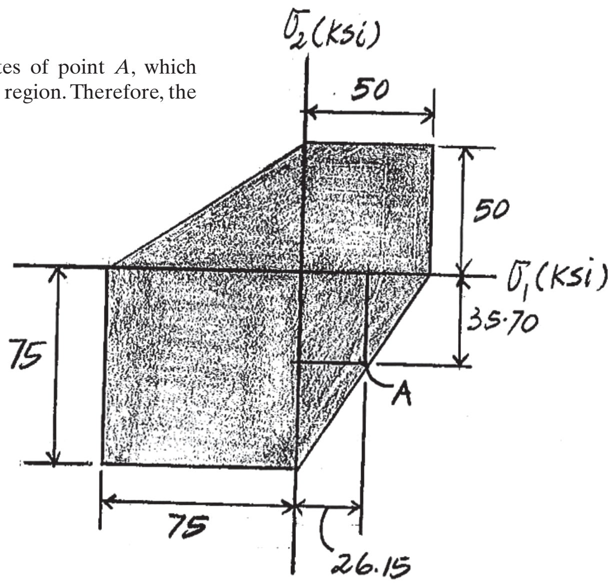
$$= (-4.775 \pm 30.929) \text{ ksi}$$

$$\sigma_1 = 26.15 \text{ ksi} \qquad \sigma_2 = -35.70 \text{ ksi}$$

Mohr's Failure Criteria. As shown in Fig. *b*, the coordinates of point *A*, which represent the principal stresses, are located inside the shaded region. Therefore, the material *does not fail* according to Mohr's failure criteria.

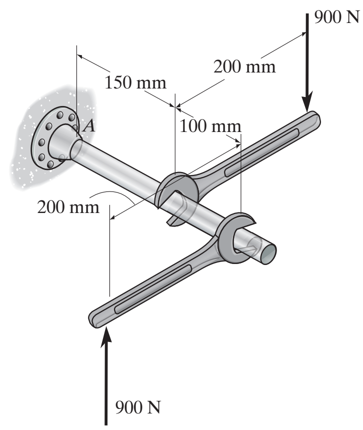


(a)



(b)

10–75. If the A-36 steel pipe has outer and inner diameters of 30 mm and 20 mm, respectively, determine the factor of safety against yielding of the material at point A according to the maximum-shear-stress theory.



Internal Loadings. Considering the equilibrium of the free - body diagram of the post's right cut segment Fig. a,

$\Sigma F_y = 0;\quad V_y + 900 - 900 = 0 \qquad\qquad V_y = 0$

$\Sigma M_x = 0;\ T + 900(0.4) = 0 \qquad\qquad T = -360\,\mathrm{N\cdot m}$

$\Sigma M_z = 0;\ M_z + 900(0.15) - 900(0.25) = 0\ \ M_z = 90\,\mathrm{N\cdot m}$

Section Properties. The moment of inertia about the z axis and the polar moment of inertia of the pipe's cross section are

$$I_z = \frac{\pi}{4}\left(0.015^4 - 0.01^4\right) = 10.15625\pi\left(10^{-9}\right)\,\mathrm{m}^4$$

$$J = \frac{\pi}{2}\left(0.015^4 - 0.01^4\right) = 20.3125\pi\left(10^{-9}\right)\,\mathrm{m}^4$$

Normal Stress and Shear Stress. The normal stress is contributed by bending stress. Thus,

$$\sigma_Y = -\frac{My_A}{I_z} = -\frac{90(0.015)}{10.15625\pi\left(10^{-9}\right)} = -42.31\mathrm{MPa}$$

The shear stress is contributed by torsional shear stress.

$$\tau = \frac{Tc}{J} = \frac{360(0.015)}{20.3125\pi\left(10^{-9}\right)} = 84.62\ \mathrm{MPa}$$

The state of stress at point A is represented by the two - dimensional element shown in Fig. b.

In - Plane Principal Stress. $\sigma_x = -42.31$ MPa, $\sigma_z = 0$ and $\tau_{xz} = 84.62$ MPa. We have

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_z}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}{}^2}$$

$$= \frac{-42.31 + 0}{2} \pm \sqrt{\left(\frac{-42.31 - 0}{2}\right)^2 + 84.62^2}$$

$$= (-21.16 \pm 87.23)\ \mathrm{MPa}$$

$$\sigma_1 = 66.07\ \mathrm{MPa} \qquad\qquad \sigma_2 = -108.38\ \mathrm{MPa}$$

10–75. Continued

Maximum Shear Stress Theory. σ_1 and σ_2 have opposite signs. This requires

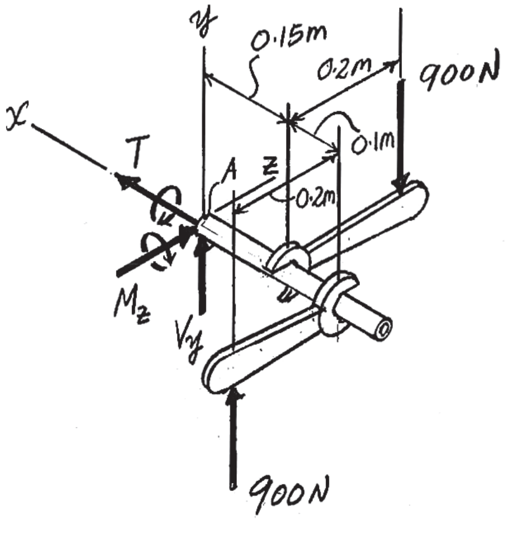
$$|\sigma_1 - \sigma_2| = \sigma_{\text{allow}}$$

$$66.07 - (-108.38) = \sigma_{\text{allow}}$$

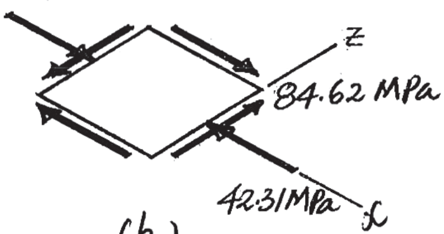
$$\sigma_{\text{allow}} = 174.45 \text{ MPa}$$

The factor of safety is

$$F.S. = \frac{\sigma_Y}{\sigma_{\text{allow}}} = \frac{250}{174.45} = 1.43 \qquad \textbf{Ans.}$$

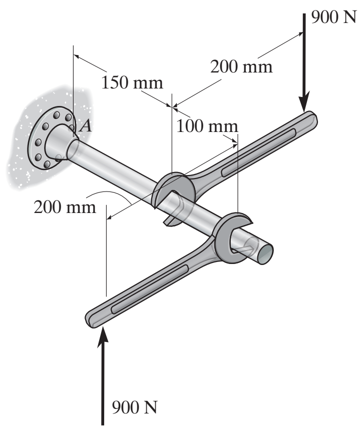


(a)



(b)

***10–76.** If the A-36 steel pipe has an outer and inner diameter of 30 mm and 20 mm, respectively, determine the factor of safety against yielding of the material at point A according to the maximum-distortion-energy theory.



Internal Loadings: Considering the equilibrium of the free - body diagram of the pipe's right cut segment Fig. *a*,

$\Sigma F_y = 0;\quad V_y + 900 - 900 = 0 \qquad V_y = 0$

$\Sigma M_x = 0;\; T + 900(0.4) = 0 \qquad T = -360\,\text{N}\cdot\text{m}$

$\Sigma M_z = 0;\; M_z + 900(0.15) - 900(0.25) = 0 \;\; M_z = 90\,\text{N}\cdot\text{m}$

Section Properties. The moment of inertia about the z axis and the polar moment of inertia of the pipe's cross section are

$$I_z = \frac{\pi}{4}\left(0.015^4 - 0.01^4\right) = 10.15625\pi\left(10^{-9}\right)\,\text{m}^4$$

$$J = \frac{\pi}{2}\left(0.015^4 - 0.01^4\right) = 20.3125\pi\left(10^{-9}\right)\,\text{m}^4$$

Normal Stress and Shear Stress. The normal stress is caused by bending stress. Thus,

$$\sigma_Y = -\frac{My_A}{I_z} = -\frac{90(0.015)}{10.15625\pi\left(10^{-9}\right)} = -42.31\text{MPa}$$

The shear stress is caused by torsional stress.

$$\tau = \frac{Tc}{J} = \frac{360(0.015)}{20.3125\pi\left(10^{-9}\right)} = 84.62\text{ MPa}$$

The state of stress at point A is represented by the two -dimensional element shown in Fig. *b*.

In - Plane Principal Stress. $\sigma_x = -42.31$ MPa, $\sigma_z = 0$ and $\tau_{xz} = 84.62$ MPa. We have

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_z}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}{}^2}$$

$$= \frac{-42.31 + 0}{2} \pm \sqrt{\left(\frac{-42.31 - 0}{2}\right)^2 + 84.62^2}$$

$$= (-21.16 \pm 87.23)\text{ MPa}$$

$\sigma_1 = 66.07$ MPa $\qquad \sigma_2 = -108.38$ MPa

10–76. Continued

Maximum Distortion Energy Theory.

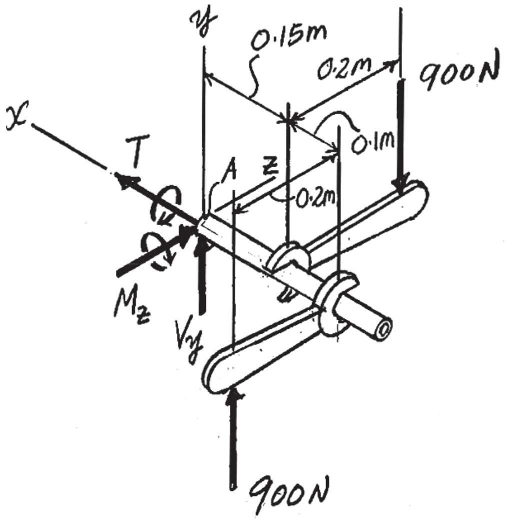
$$\sigma_1{}^2 - \sigma_1\sigma_2 + \sigma_2{}^2 = \sigma_{allow}{}^2$$

$$66.07^2 - 66.07(-108.38) + (-108.38)^2 = \sigma_{allow}{}^2$$

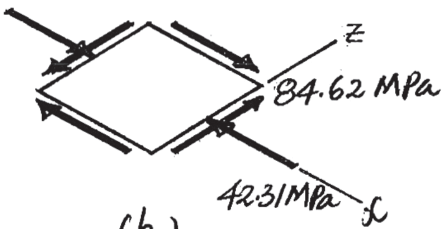
$$\sigma_{allow} = 152.55 \text{ MPa}$$

Thus, the factor of safety is

$$F.S. = \frac{\sigma_Y}{\sigma_{allow}} = \frac{250}{152.55} = 1.64 \qquad \textbf{Ans.}$$

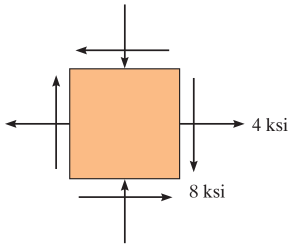


(a)



(b)

•10–77. The element is subjected to the stresses shown. If $\sigma_Y = 36$ ksi, determine the factor of safety for the loading based on the maximum-shear-stress theory.



$\sigma_x = 4$ ksi $\quad \sigma_y = -12$ ksi $\quad \tau_{xy} = -8$ ksi

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{4 - 12}{2} \pm \sqrt{\left(\frac{4 - (-12)}{2}\right)^2 + (-8)^2}$$

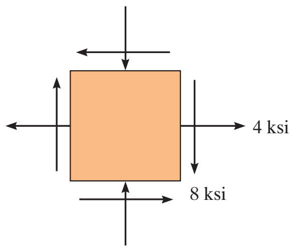
$\sigma_1 = 7.314$ ksi $\quad \sigma_2 = -15.314$ ksi

$$\tau_{\text{abs}_{\text{max}}} = \frac{\sigma_1 - \sigma_2}{2} = \frac{7.314 - (-15.314)}{2} = 11.314 \text{ ksi}$$

$$\tau_{\text{allow}} = \frac{\sigma_Y}{2} = \frac{36}{2} = 18 \text{ ksi}$$

$$F.S. = \frac{\tau_{\text{allow}}}{\tau_{\text{max}}^{\text{abs}}} = \frac{18}{11.314} = 1.59$$ **Ans.**

10–78. Solve Prob. 10–77 using the maximum-distortion-energy theory.



$\sigma_x = 4$ ksi $\quad \sigma_y = -12$ ksi $\quad \tau_{xy} = -8$ ksi

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{4 - 12}{2} \pm \sqrt{\left(\frac{4 - (-12)}{2}\right)^2 + (-8)^2}$$

$\sigma_1 = 7.314$ ksi $\quad \sigma_2 = -15.314$ ksi

$$\sigma_1{}^2 - \sigma_1 \sigma_2 + \sigma_2{}^2 = \left(\frac{\sigma_Y}{\text{F.S.}}\right)^2$$

$$\text{F.S.} = \sqrt{\frac{36^2}{(7.314)^2 - (7.314)(-15.314) + (-15.314)^2}} = 1.80$$ **Ans.**

10–79. The yield stress for heat-treated beryllium copper is $\sigma_Y = 130$ ksi. If this material is subjected to plane stress and elastic failure occurs when one principal stress is 145 ksi, what is the smallest magnitude of the other principal stress? Use the maximum-distortion-energy theory.

Maximum Distortion Energy Theory : With $\sigma_1 = 145$ ksi,

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_Y^2$$

$$145^2 - 145\sigma_2 + \sigma_2^2 = 130^2$$

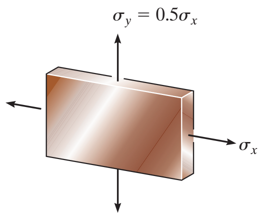
$$\sigma_2^2 - 145\sigma_2 + 4125 = 0$$

$$\sigma_2 = \frac{-(-145) \pm \sqrt{(-145)^2 - 4(1)(4125)}}{2(1)}$$

$$= 72.5 \pm 33.634$$

Choose the smaller root, $\sigma_2 = 38.9$ ksi **Ans.**

***10–80.** The plate is made of hard copper, which yields at $\sigma_Y = 105$ ksi. Using the maximum-shear-stress theory, determine the tensile stress σ_x that can be applied to the plate if a tensile stress $\sigma_y = 0.5\sigma_x$ is also applied.

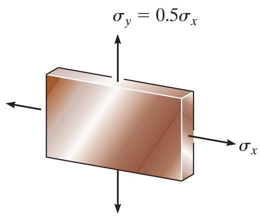


$$\sigma_1 = \sigma_x \qquad \sigma_2 = \frac{1}{2}\sigma_x$$

$$|\sigma_1| = \sigma_Y$$

$\sigma_x = 105$ ksi **Ans.**

•10–81. Solve Prob. 10–80 using the maximum-distortion-energy theory.



$$\sigma_1 = \sigma_x$$

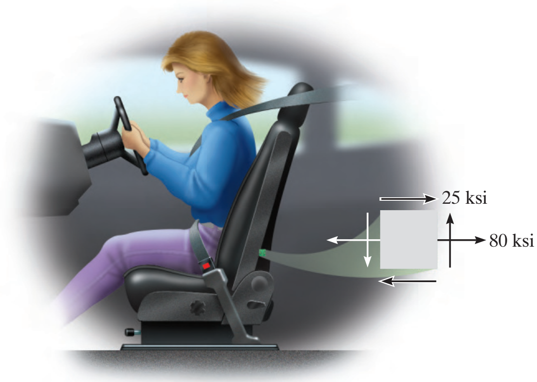
$$\sigma_2 = \frac{\sigma_x}{2}$$

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_Y^2$$

$$\sigma_x^2 - \frac{\sigma_x^2}{2} + \frac{\sigma_x^2}{4} = (105)^2$$

$\sigma_x = 121$ ksi **Ans.**

10–82. The state of stress acting at a critical point on the seat frame of an automobile during a crash is shown in the figure. Determine the smallest yield stress for a steel that can be selected for the member, based on the maximum-shear-stress theory.



Normal and Shear Stress: In accordance with the sign convention.

$$\sigma_x = 80 \text{ ksi} \qquad \sigma_y = 0 \qquad \tau_{xy} = 25 \text{ ksi}$$

In - Plane Principal Stress: Applying Eq. 9-5.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{80 + 0}{2} \pm \sqrt{\left(\frac{80 - 0}{2}\right)^2 + 25^2}$$

$$= 40 \pm 47.170$$

$$\sigma_1 = 87.170 \text{ ksi} \qquad \sigma_2 = -7.170 \text{ ksi}$$

Maximum Shear Stress Theory: σ_1 and σ_2 have opposite signs so

$$|\sigma_1 - \sigma_2| = \sigma_Y$$

$$|87.170 - (-7.170)| = \sigma_Y$$

$$\sigma_Y = 94.3 \text{ ksi} \qquad \textbf{Ans.}$$

10–83. Solve Prob. 10–82 using the maximum-distortion-energy theory.

Normal and Shear Stress: In accordance with the sign convention.

$$\sigma_x = 80 \text{ ksi} \qquad \sigma_y = 0 \qquad \tau_{xy} = 25 \text{ ksi}$$

In - Plane Principal Stress: Applying Eq. 9-5.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{80 + 0}{2} \pm \sqrt{\left(\frac{80 - 0}{2}\right)^2 + 25^2}$$

$$= 40 \pm 47.170$$

$$\sigma_1 = 87.170 \text{ ksi} \qquad \sigma_2 = -7.170 \text{ ksi}$$

Maximum Distortion Energy Theory:

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_Y^2$$

$$87.170^2 - 87.170(-7.170) + (-7.170)^2 = \sigma_Y^2$$

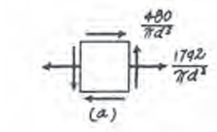
$$\sigma_Y = 91.0 \text{ ksi} \qquad \textbf{Ans.}$$

***10–84.** A bar with a circular cross-sectional area is made of SAE 1045 carbon steel having a yield stress of $\sigma_Y = 150$ ksi. If the bar is subjected to a torque of 30 kip · in. and a bending moment of 56 kip · in., determine the required diameter of the bar according to the maximum-distortion-energy theory. Use a factor of safety of 2 with respect to yielding.

Normal and Shear Stresses: Applying the flexure and torsion formulas.

$$\sigma = \frac{Mc}{I} = \frac{56\left(\frac{d}{2}\right)}{\frac{\pi}{4}\left(\frac{d}{2}\right)^4} = \frac{1792}{\pi d^3}$$

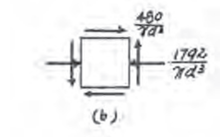
$$\tau = \frac{Tc}{J} = \frac{30\left(\frac{d}{2}\right)}{\frac{\pi}{2}\left(\frac{d}{2}\right)^4} = \frac{480}{\pi d^3}$$



The critical state of stress is shown in Fig. (a) or (b), where

$$\sigma_x = \frac{1792}{\pi d^3} \qquad \sigma_y = 0 \qquad \tau_{xy} = \frac{480}{\pi d^3}$$

In - Plane Principal Stresses : Applying Eq. 9-5,



$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{\frac{1792}{\pi d^3} + 0}{2} \pm \sqrt{\left(\frac{\frac{1792}{\pi d^3} - 0}{2}\right)^2 + \left(\frac{480}{\pi d^3}\right)^2}$$

$$= \frac{896}{\pi d^3} \pm \frac{1016.47}{\pi d^3}$$

$$\sigma_1 = \frac{1912.47}{\pi d^3} \qquad \sigma_2 = -\frac{120.47}{\pi d^3}$$

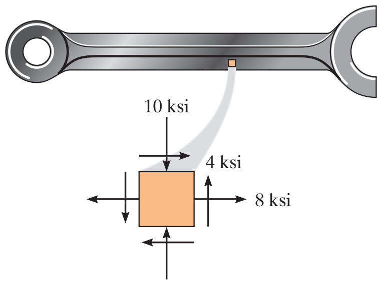
Maximum Distortion Energy Theory :

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_{\text{allow}}^2$$

$$\left(\frac{1912.47}{\pi d^3}\right)^2 - \left(\frac{1912.47}{\pi d^3}\right)\left(-\frac{120.47}{\pi d^3}\right) + \left(-\frac{120.47}{\pi d^3}\right)^2 = \left(\frac{150}{2}\right)^2$$

$$d = 2.30 \text{ in.}$$ **Ans.**

•10–85. The state of stress acting at a critical point on a machine element is shown in the figure. Determine the smallest yield stress for a steel that might be selected for the part, based on the maximum-shear-stress theory.



The Principal stresses:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{8 - 10}{2} \pm \sqrt{\left(\frac{8 - (-10)}{2}\right)^2 + 4^2}$$

$\sigma_1 = 8.8489 \text{ ksi} \qquad \sigma_2 = -10.8489 \text{ ksi}$

Maximum shear stress theory: Both principal stresses have opposite sign. hence,

$|\sigma_1 - \sigma_2| = \sigma_Y \qquad 8.8489 - (-10.8489) = \sigma_Y$

$\sigma_Y = 19.7 \text{ ksi}$ **Ans.**

10–86. The principal stresses acting at a point on a thin-walled cylindrical pressure vessel are $\sigma_1 = pr/t$, $\sigma_2 = pr/2t$, and $\sigma_3 = 0$. If the yield stress is σ_Y, determine the maximum value of p based on (a) the maximum-shear-stress theory and (b) the maximum-distortion-energy theory.

a) Maximum Shear Stress Theory: σ_1 and σ_2 have the same signs, then

$|\sigma_2| = \sigma_\gamma \qquad \left|\frac{pr}{2t}\right| = \sigma_\gamma \qquad p = \frac{2t}{r}\sigma_\gamma$

$|\sigma_1| = \sigma_\gamma \qquad \left|\frac{pr}{t}\right| = \sigma_\gamma \qquad p = \frac{t}{r}\sigma_\gamma\ (Controls!)$ **Ans.**

b) Maximum Distortion Energy Theory :

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_\gamma^2$$

$$\left(\frac{pr}{t}\right)^2 - \left(\frac{pr}{t}\right)\left(\frac{pr}{2t}\right) + \left(\frac{pr}{2t}\right)^2 = \sigma_\gamma^2$$

$$p = \frac{2t}{\sqrt{3}r}\sigma_\gamma$$ **Ans.**

10–87. If a solid shaft having a diameter d is subjected to a torque **T** and moment **M**, show that by the maximum-shear-stress theory the maximum allowable shear stress is $\tau_{allow} = (16/\pi d^3)\sqrt{M^2 + T^2}$. Assume the principal stresses to be of opposite algebraic signs.

Section properties :

$$I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{64}; \qquad J = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{32}$$

Thus,

$$\sigma = \frac{Mc}{I} = \frac{M\left(\frac{d}{2}\right)}{\frac{\pi d^4}{64}} = \frac{32\,M}{\pi d^3}$$

$$\tau = \frac{T\,c}{J} = \frac{T\left(\frac{d}{2}\right)}{\frac{\pi d^4}{32}} = \frac{16\,T}{\pi d^3}$$

The principal stresses :

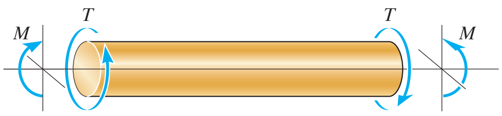
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{16\,M}{\pi\,d^3} \pm \sqrt{\left(\frac{16\,M}{\pi\,d^3}\right)^2 + \left(\frac{16\,T}{\pi\,d^3}\right)^2} = \frac{16\,M}{\pi d^3} \pm \frac{16}{\pi\,d^3}\sqrt{M^2 + T^2}$$

Assume σ_1 and σ_2 have opposite sign, hence,

$$\tau_{allow} = \frac{\sigma_1 - \sigma_2}{2} = \frac{2\left[\frac{16}{\pi d^3}\sqrt{M^2 + T^2}\right]}{2} = \frac{16}{\pi d^3}\sqrt{M^2 + T^2} \qquad \textbf{QED}$$

***10–88.** If a solid shaft having a diameter d is subjected to a torque **T** and moment **M**, show that by the maximum-normal-stress theory the maximum allowable principal stress is $\sigma_{allow} = (16/\pi d^3)(M + \sqrt{M^2 + T^2})$.



Section properties :

$$I = \frac{\pi\,d^4}{64}; \qquad J = \frac{\pi\,d^4}{32}$$

Stress components :

$$\sigma = \frac{M\,c}{I} = \frac{M\left(\frac{d}{2}\right)}{\frac{\pi}{64}\,d^4} = \frac{32\,M}{\pi\,d^3}; \qquad \tau = \frac{T\,c}{J} = \frac{T\left(\frac{d}{2}\right)}{\frac{\pi}{32}\,d^4} = \frac{16\,T}{\pi\,d^3}$$

The principal stresses :

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{\frac{32\,M}{\pi\,d^3} + 0}{2} \pm \sqrt{\left(\frac{\frac{32\,M}{\pi\,d^3} - 0}{2}\right)^2 + \left(\frac{16\,T}{\pi\,d^3}\right)^2}$$

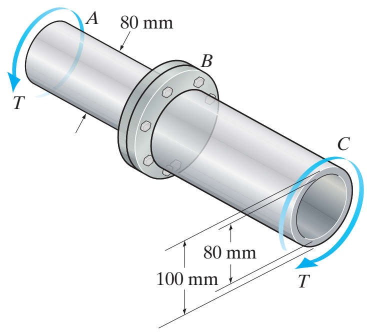
$$= \frac{16\,M}{\pi\,d^3} \pm \frac{16}{\pi\,d^3}\sqrt{M^2 + T^2}$$

Maximum normal stress theory. Assume $\sigma_1 > \sigma_2$

$$\sigma_{allow} = \sigma_1 = \frac{16\,M}{\pi\,d^3} + \frac{16}{\pi\,d^3}\sqrt{M^2 + T^2}$$

$$= \frac{16}{\pi\,d^3}\left[M + \sqrt{M^2 + T^2}\right] \qquad \textbf{QED}$$

•10–89. The shaft consists of a solid segment AB and a hollow segment BC, which are rigidly joined by the coupling at B. If the shaft is made from A-36 steel, determine the maximum torque T that can be applied according to the maximum-shear-stress theory. Use a factor of safety of 1.5 against yielding.



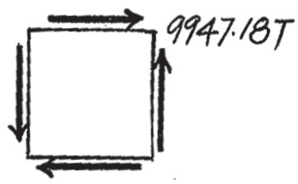
Shear Stress: This is a case of pure shear, and the shear stress is contributed by torsion. For the hollow segment, $J_h = \frac{\pi}{2}\left(0.05^4 - 0.04^4\right) = 1.845\pi\left(10^{-6}\right)\text{ m}^4$. Thus,

$$(\tau_{\max})_h = \frac{Tc_h}{J_h} = \frac{T(0.05)}{1.845\pi\left(10^{-6}\right)} = 8626.28T$$

For the solid segment, $J_s = \frac{\pi}{2}\left(0.04^4\right) = 1.28\pi\left(10^{-6}\right)\text{ m}^4$. Thus,

$$(\tau_{\max})_s = \frac{Tc_s}{J_s} = \frac{T(0.04)}{1.28\pi\left(10^{-6}\right)} = 9947.18T$$

By comparision, the points on the surface of the solid segment are critical and their state of stress is represented on the element shown in Fig. *a*.



(a)

In - Plane Principal Stress. $\sigma_x = \sigma_y = 0$ and $\tau_{xy} = 9947.18T$. We have

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{0+0}{2} \pm \sqrt{\left(\frac{0-0}{2}\right)^2 + (9947.18T)^2}$$

$$\sigma_1 = 9947.18T \qquad \sigma_2 = -9947.18T$$

Maximum Shear Stress Theory.

$$\sigma_{\text{allow}} = \frac{\sigma_Y}{F.S.} = \frac{250}{1.5} = 166.67\text{ MPa}$$

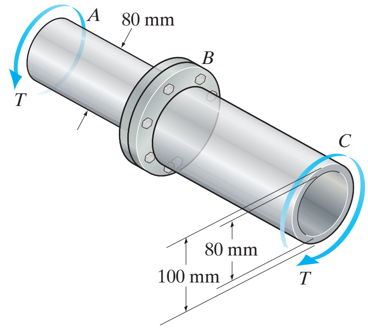
Since σ_1 and σ_2 have opposite sings,

$$|\sigma_1 - \sigma_2| = \sigma_{\text{allow}}$$

$$9947.18T - (-9947.18T) = 166.67\left(10^6\right)$$

$$T = 8377.58\text{ N}\cdot\text{m} = 8.38\text{ kN}\cdot\text{m}$$ **Ans.**

10–90. The shaft consists of a solid segment AB and a hollow segment BC, which are rigidly joined by the coupling at B. If the shaft is made from A-36 steel, determine the maximum torque T that can be applied according to the maximum-distortion-energy theory. Use a factor of safety of 1.5 against yielding.



Shear Stress. This is a case of pure shear, and the shear stress is contributed by torsion. For the hollow segment, $J_h = \frac{\pi}{2}\left(0.05^4 - 0.04^4\right) = 1.845\pi\left(10^{-6}\right)\text{ m}^4$. Thus,

$$(\tau_{\max})_h = \frac{Tc_h}{J_h} = \frac{T(0.05)}{1.845\pi\left(10^{-6}\right)} = 8626.28T$$

For the solid segment, $J_s = \frac{\pi}{2}\left(0.04^4\right) = 1.28\pi\left(10^{-6}\right)\text{ m}^4$. Thus,

$$(\tau_{\max})_s = \frac{Tc_s}{J_s} = \frac{T(0.04)}{1.28\pi\left(10^{-6}\right)} = 9947.18T$$

By comparision, the points on the surface of the solid segment are critical and their state of stress is represented on the element shown in Fig. *a*.

In - Plane Principal Stress. $\sigma_x = \sigma_y = 0$ and $\tau_{xy} = 9947.18T$. We have

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{0+0}{2} \pm \sqrt{\left(\frac{0-0}{2}\right)^2 + (9947.18T)^2}$$

$$\sigma_1 = 9947.18T \qquad \sigma_2 = -9947.18T$$

Maximum Distortion Energy Theory.

$$\sigma_{\text{allow}} = \frac{\sigma_Y}{F.S.} = \frac{250}{1.5} = 166.67\text{ MPa}$$

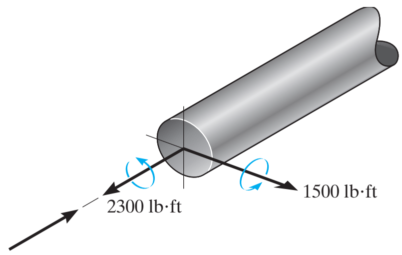
Then,

$$\sigma_1{}^2 - \sigma_1\sigma_2 + \sigma_2{}^2 = \sigma_{\text{allow}}{}^2$$

$$(9947.18T)^2 - (9947.18T)(-9947.18T) + (-9947.18T)^2 = \left[166.67\left(10^6\right)\right]^2$$

$T = 9673.60\text{ N}\cdot\text{m} = 9.67\text{ kN}\cdot\text{m}$ **Ans.**

10–91. The internal loadings at a critical section along the steel drive shaft of a ship are calculated to be a torque of 2300 lb · ft, a bending moment of 1500 lb · ft, and an axial thrust of 2500 lb. If the yield points for tension and shear are $\sigma_Y = 100$ ksi and $\tau_Y = 50$ ksi, respectively, determine the required diameter of the shaft using the maximum-shear-stress theory.



$$A = \pi c^2 \qquad I = \frac{\pi}{4} c^4 \qquad J = \frac{\pi}{2} c^4$$

$$\sigma_A = \frac{P}{A} + \frac{Mc}{I} = -\left(\frac{2500}{\pi c^2} + \frac{1500(12)(c)}{\frac{\pi c^4}{4}}\right) = -\left(\frac{2500}{\pi c^2} + \frac{72\,000}{\pi c^3}\right)$$

$$\tau_A = \frac{Tc}{J} = \frac{2300(12)(c)}{\frac{\pi c^4}{2}} = \frac{55\,200}{\pi c^3}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= -\left(\frac{2500\,c + 72\,000}{2\pi c^3}\right) \pm \sqrt{\left(\frac{2500c + 72\,000}{2\pi c^3}\right)^2 + \left(\frac{55200}{\pi c^3}\right)^2} \qquad (1)$$

Assume σ_1 and σ_2 have opposite signs:

$$|\sigma_1 - \sigma_2| = \sigma_Y$$

$$2\sqrt{\left(\frac{2500c + 72\,000}{2\pi c^3}\right)^2 + \left(\frac{55\,200}{\pi c^3}\right)^2} = 100(10^3)$$

$$(2500c + 72000)^2 + 110400^2 = 10\,000(10^6)\pi^2 c^6$$

$$6.25c^2 + 360c + 17372.16 - 10\,000\pi^2 c^6 = 0$$

By trial and error:

$c = 0.750\,57$ in.

Substitute c into Eq. (1):

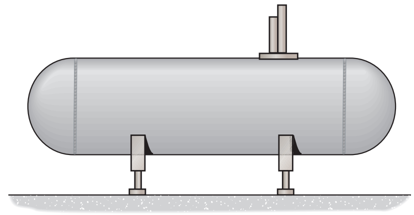
$\sigma_1 = 22\,193$ psi $\qquad \sigma_2 = -77\,807$ psi

σ_1 and σ_2 are of opposite signs **OK**

Therefore,

$d = 1.50$ in. **Ans.**

***10–92.** The gas tank has an inner diameter of 1.50 m and a wall thickness of 25 mm. If it is made from A-36 steel and the tank is pressured to 5 MPa, determine the factor of safety against yielding using (a) the maximum-shear-stress theory, and (b) the maximum-distortion-energy theory.



(a) **Normal Stress.** Since $\frac{r}{t} = \frac{0.75}{0.025} = 30 > 10$, thin - wall analysis can be used. We have

$$\sigma_1 = \sigma_h = \frac{pr}{t} = \frac{5(0.75)}{0.025} = 150 \text{ MPa}$$

$$\sigma_2 = \sigma_{\text{long}} = \frac{pr}{2t} = \frac{5(0.75)}{2(0.025)} = 75 \text{ MPa}$$

Maximum Shear Stress Theory. σ_1 and σ_2 have the sign. Thus,

$$|\sigma_1| = \sigma_{\text{allow}}$$

$$\sigma_{\text{allow}} = 150 \text{ MPa}$$

The factor of safety is

$$F.S. = \frac{\sigma_Y}{\sigma_{\text{allow}}} = \frac{250}{150} = 1.67$$ **Ans.**

(b) **Maximum Distortion Energy Theory.**

$$\sigma_1{}^2 - \sigma_1\sigma_2 + \sigma_2{}^2 = \sigma_{\text{allow}}{}^2$$

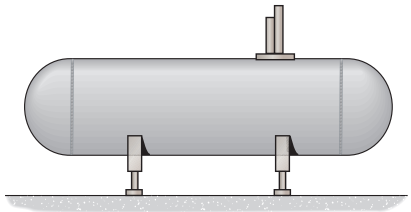
$$150^2 - 150(75) + 75^2 = \sigma_{\text{allow}}{}^2$$

$$\sigma_{\text{allow}} = 129.90 \text{ MPa}$$

The factor of safety is

$$F.S. = \frac{\sigma_Y}{\sigma_{\text{allow}}} = \frac{250}{129.90} = 1.92$$ **Ans.**

•**10–93.** The gas tank is made from A-36 steel and has an inner diameter of 1.50 m. If the tank is designed to withstand a pressure of 5 MPa, determine the required minimum wall thickness to the nearest millimeter using (a) the maximum-shear-stress theory, and (b) maximum-distortion-energy theory. Apply a factor of safety of 1.5 against yielding.



(a) **Normal Stress.** Assuming that thin - wall analysis is valid, we have

$$\sigma_1 = \sigma_h = \frac{pr}{t} = \frac{5(10^6)(0.75)}{t} = \frac{3.75(10^6)}{t}$$

$$\sigma_2 = \sigma_{\text{long}} = \frac{pr}{2t} = \frac{5(10^6)(0.75)}{2t} = \frac{1.875(10^6)}{t}$$

Maximum Shear Stress Theory.

$$\sigma_{\text{allow}} = \frac{\sigma_Y}{F.S.} = \frac{250(10^6)}{1.5} = 166.67(10^6)\text{Pa}$$

σ_1 and σ_2 have the same sign. Thus,

$$|\sigma_1| = \sigma_{\text{allow}}$$

$$\frac{3.75(10^6)}{t} = 166.67(10^6)$$

$$t = 0.0225 \text{ m} = 22.5 \text{ mm}$$ **Ans.**

Since $\frac{r}{t} = \frac{0.75}{0.0225} = 33.3 > 10$, thin - wall analysis is valid.

(b) **Maximum Distortion Energy Theory.**

$$\sigma_{\text{allow}} = \frac{\sigma_Y}{F.S.} = \frac{250(10^6)}{1.5} = 166.67(10^6)\text{Pa}$$

Thus,

$$\sigma_1{}^2 - \sigma_1\sigma_2 + \sigma_2{}^2 = \sigma_{\text{allow}}{}^2$$

$$\left[\frac{3.75(10^6)}{t}\right]^2 - \left[\frac{3.75(10^6)}{t}\right]\left[\frac{1.875(10^6)}{t}\right] + \left[\frac{1.875(10^6)}{t}\right]^2 = \left[166.67(10^6)\right]^2$$

$$\frac{3.2476(10^6)}{t} = 166.67(10^6)$$

$$t = 0.01949 \text{ m} = 19.5 \text{ mm}$$ **Ans.**

Since $\frac{r}{t} = \frac{0.75}{0.01949} = 38.5 > 10$, thin - wall analysis is valid.

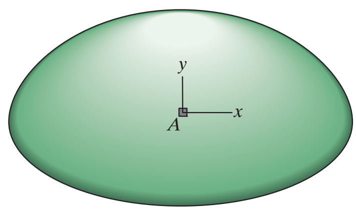
10–94. A thin-walled spherical pressure vessel has an inner radius r, thickness t, and is subjected to an internal pressure p. If the material constants are E and ν, determine the strain in the circumferential direction in terms of the stated parameters.

$$\sigma_1 = \sigma_2 = \frac{pr}{2t}$$

$$\varepsilon_1 = \varepsilon_2 = \varepsilon = \frac{1}{E}(\sigma - \nu\sigma)$$

$$\varepsilon = \frac{1-\nu}{E}\sigma = \frac{1-\nu}{E}\left(\frac{pr}{2t}\right) = \frac{pr}{2Et}(1-\nu)$$ **Ans.**

10–95. The strain at point A on the shell has components $\epsilon_x = 250(10^{-6})$, $\epsilon_y = 400(10^{-6})$, $\gamma_{xy} = 275(10^{-6})$, $\epsilon_z = 0$. Determine (a) the principal strains at A, (b) the maximum shear strain in the x–y plane, and (c) the absolute maximum shear strain.



$$\varepsilon_x = 250(10^{-6}) \qquad \varepsilon_y = 400(10^{-6}) \qquad \gamma_{xy} = 275(10^{-6}) \qquad \frac{\gamma_{xy}}{2} = 137.5(10^{-6})$$

$A(250, 137.5)10^{-6} \qquad C(325, 0)10^{-6}$

$$R = \left(\sqrt{(325-250)^2 + (137.5)^2}\right)10^{-6} = 156.62(10^{-6})$$

a)

$$\varepsilon_1 = (325 + 156.62)10^{-6} = 482(10^{-6})$$ **Ans.**

$$\varepsilon_2 = (325 - 156.62)10^{-6} = 168(10^{-6})$$ **Ans.**

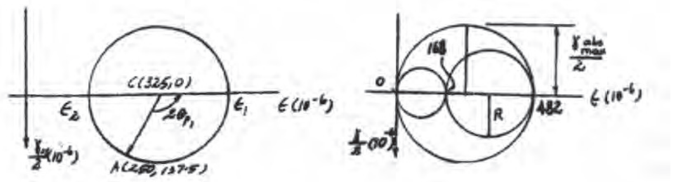
b)

$$\gamma_{\substack{\max \\ \text{in-plane}}} = 2R = 2(156.62)(10^{-6}) = 313(10^{-6})$$ **Ans.**

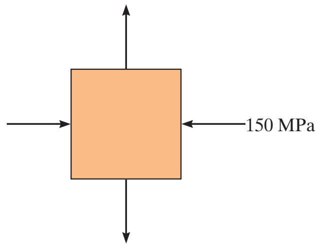
c)

$$\frac{\gamma_{\substack{\text{abs} \\ \max}}}{2} = \frac{482(10^{-6})}{2}$$

$$\gamma_{\substack{\text{abs} \\ \max}} = 482(10^{-6})$$ **Ans.**



***10–96.** The principal plane stresses acting at a point are shown in the figure. If the material is machine steel having a yield stress of $\sigma_Y = 500$ MPa, determine the factor of safety with respect to yielding if the maximum-shear-stress theory is considered.



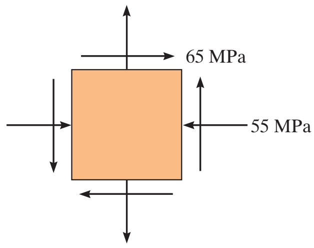
Have, the in plane principal stresses are

$$\sigma_1 = \sigma_y = 100 \text{ MPa} \qquad \sigma_2 = \sigma_x = -150 \text{ MPa}$$

Since σ_1 and σ_2 have same sign,

$$F.S = \frac{\sigma_y}{|\sigma_1 - \sigma_2|} = \frac{500}{|100 - (-150)|} = 2$$ **Ans.**

•10–97. The components of plane stress at a critical point on a thin steel shell are shown. Determine if failure (yielding) has occurred on the basis of the maximum-distortion-energy theory. The yield stress for the steel is $\sigma_Y = 650$ MPa.



$\sigma_x = -55$ MPa $\qquad \sigma_y = 340$ MPa $\qquad \tau_{xy} = 65$ MPa

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{-55 + 340}{2} \pm \sqrt{\left(\frac{-55 - 340}{2}\right)^2 + 65^2}$$

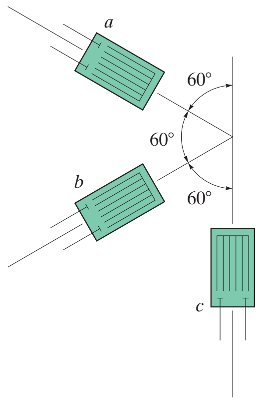
$\sigma_1 = 350.42$ MPa $\qquad \sigma_2 = -65.42$ MPa

$$(\sigma_1{}^2 - \sigma_1\sigma_2 + \sigma_2) = [350.42^2 - 350.42(-65.42) + (-65.42)^2]$$

$$= 150\,000 < \sigma_Y^2 = 422\,500$$ **OK**

No. **Ans.**

10–98. The 60° strain rosette is mounted on a beam. The following readings are obtained for each gauge: $\epsilon_a = 600(10^{-6})$, $\epsilon_b = -700(10^{-6})$, and $\epsilon_c = 350(10^{-6})$. Determine (a) the in-plane principal strains and (b) the maximum in-plane shear strain and average normal strain. In each case show the deformed element due to these strains.



Strain Rosettes (60º): Applying Eq. 10-15 with $\varepsilon_x = 600\left(10^{-6}\right)$,

$\varepsilon_b = -700\left(10^{-6}\right), \varepsilon_c = 350\left(10^{-6}\right), \theta_a = 150°, \theta_b = -150°$ and $\theta_c = -90°$,

$$350\left(10^{-6}\right) = \varepsilon_x \cos^2(-90°) + \varepsilon_y \sin^2(-90°) + \gamma_{xy} \sin(-90°) \cos(-90°)$$

$$\varepsilon_y = 350\left(10^{-6}\right)$$

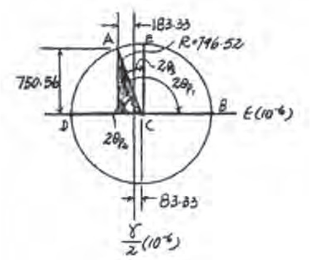
$$600\left(10^{-6}\right) = \varepsilon_x \cos^2 150° + 350\left(10^{-6}\right) \sin^2 150° + \gamma_{xy} \sin 150° \cos 150°$$

$$512.5\left(10^{-6}\right) = 0.75\, \varepsilon_x - 0.4330\, \gamma_{xy} \qquad \textbf{[1]}$$

$$-700\left(10^{-6}\right) = \varepsilon_x \cos^2(-150°) + 350\left(10^{-6}\right) \sin^2(-150°) + \gamma_{xy} \sin(-150°) \cos(-150°)$$

$$-787.5\left(10^{-6}\right) = 0.75\varepsilon_x + 0.4330\, \gamma_{xy} \qquad \textbf{[2]}$$

Solving Eq. [1] and [2] yields $\varepsilon_x = -183.33\left(10^{-6}\right)$ $\quad \gamma_{xy} = -1501.11\left(10^{-6}\right)$



Construction of she Circle: With $\varepsilon_x = -183.33\left(10^{-6}\right)$, $\varepsilon_y = 350\left(10^{-6}\right)$, and $\dfrac{\gamma_{xy}}{2} = -750.56\left(10^{-6}\right)$.

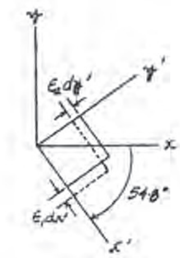
$$\varepsilon_{\text{avg}} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left(\frac{-183.33 + 350}{2}\right)\left(10^{-6}\right) = 83.3\left(10^{-6}\right) \qquad \textbf{Ans.}$$

The coordinates for reference points A and C are

$$A(-183.33, -750.56)\left(10^{-6}\right) \qquad C(83.33, 0)\left(10^{-6}\right)$$

The radius of the circle is

$$R = \left(\sqrt{(183.33 + 83.33)^2 + 750.56^2}\right)\left(10^{-6}\right) = 796.52\left(10^{-6}\right)$$



a)

In-plane Principal Strain: The coordinates of points B and D represent ε_1 and ε_2, respectively.

$$\varepsilon_1 = (83.33 + 796.52)\left(10^{-6}\right) = 880\left(10^{-6}\right) \qquad \textbf{Ans.}$$

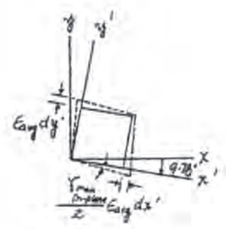
$$\varepsilon_2 = (83.33 - 796.52)\left(10^{-6}\right) = -713\left(10^{-6}\right) \qquad \textbf{Ans.}$$

Orientation of Principal Strain: From the circle,

$$\tan 2\theta_{P1} = \frac{750.56}{183.33 + 83.33} = 2.8145 \qquad 2\theta_{P2} = 70.44°$$

$$2\theta_{P1} = 180° - 2\theta_{P2}$$

$$\theta_P = \frac{180° - 70.44°}{2} = 54.8° \quad (\textit{Clockwise}) \qquad \textbf{Ans.}$$



10–98. Continued

b)

Maximum In - Plane Shear Strain: Represented by the coordinates of point E on the circle.

$$\frac{\gamma_{\substack{\max \\ \text{in-plane}}}}{2} = -R = -796.52\left(10^{-6}\right)$$

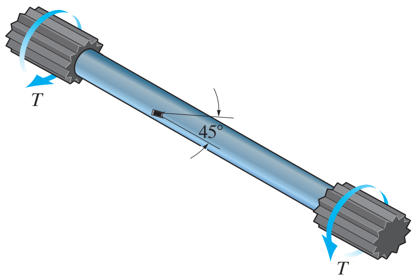
$$\gamma_{\substack{\max \\ \text{in-plane}}} = -1593\left(10^{-6}\right) \qquad \textbf{Ans.}$$

Orientation of Maximum In-Plane Shear Strain: From the circle.

$$\tan 2\theta_P = \frac{183.33 + 83.33}{750.56} = 0.3553$$

$$\theta_P = 9.78^\circ \ (Clockwise) \qquad \textbf{Ans.}$$

10–99. A strain gauge forms an angle of 45° with the axis of the 50-mm diameter shaft. If it gives a reading of $\epsilon = -200(10^{-6})$ when the torque **T** is applied to the shaft, determine the magnitude of **T**. The shaft is made from A-36 steel.



Shear Stress. This is a case of pure shear, and the shear stress developed is contributed by torsional shear stress. Here, $J = \frac{\pi}{2}\left(0.025^4\right) = 0.1953125\pi\left(10^{-6}\right)\text{m}^4$. Then

$$\tau = \frac{Tc}{J} = \frac{T(0.025)}{0.1953125\pi\left(10^{-6}\right)} = \frac{0.128\left(10^6\right)T}{\pi}$$

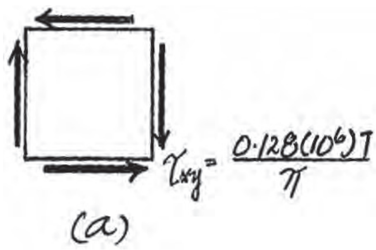
The state of stress at points on the surface of the shaft can be represented by the element shown in Fig. *a*.

Shear Strain: For pure shear $\varepsilon_x = \varepsilon_y = 0$. We obtain,

$$\varepsilon_a = \varepsilon_x \cos^2\theta_a + \varepsilon_y \sin^2\theta_a + \gamma_{xy}\sin\theta_a\cos\theta_a$$

$$-200\left(10^{-6}\right) = 0 + 0 + \gamma_{xy}\sin 45^\circ \cos 45^\circ$$

$$\gamma_{xy} = -400\left(10^{-6}\right)$$



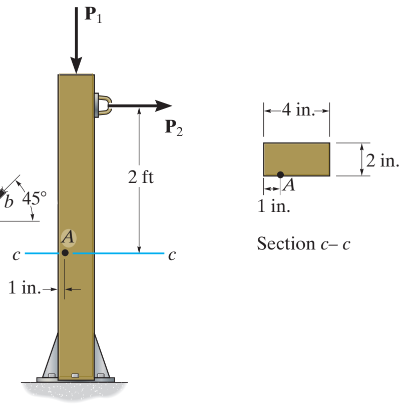
Shear Stress and Strain Relation: Applying Hooke's Law for shear,

$$\tau_{xy} = G\gamma_{xy}$$

$$-\frac{0.128\left(10^6\right)T}{\pi} = 75\left(10^9\right)\left[-400\left(10^{-6}\right)\right]$$

$$T = 736 \text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

***10–100.** The A-36 steel post is subjected to the forces shown. If the strain gauges a and b at point A give readings of $\epsilon_a = 300(10^{-6})$ and $\epsilon_b = 175(10^{-6})$, determine the magnitudes of $\mathbf{P}_1$ and $\mathbf{P}_2$.



Internal Loadings: Considering the equilibrium of the free - body diagram of the post's segment, Fig. a,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad P_2 - V = 0 \qquad V = P_2$$

$$+\uparrow \Sigma F_y = 0; \qquad N - P_1 = 0 \qquad N = P_1$$

$$\circlearrowleft + \Sigma M_O = 0; \qquad M + P_2(2) = 0 \qquad M = 2P_2$$

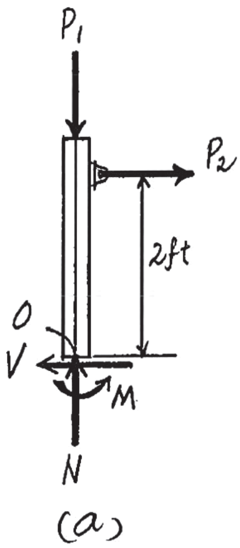
Section Properties: The cross - sectional area and the moment of inertia about the bending axis of the post's cross - section are

$$A = 4(2) = 8 \text{ in}^2$$

$$I = \frac{1}{12}(2)\left(4^3\right) = 10.667 \text{ in}^4$$

Referring to Fig. b,

$$\left(Q_y\right)_A = \bar{x}'A' = 1.5(1)(2) = 3 \text{ in}^3$$



Normal and Shear Stress: The normal stress is a combination of axial and bending stress.

$$\sigma_A = \frac{N}{A} + \frac{Mx_A}{I} = -\frac{P_1}{8} + \frac{2P_2(12)(1)}{10.667} = 2.25P_2 - 0.125P_1$$

The shear stress is caused by transverse shear stress.

$$\tau_A = \frac{VQ_A}{It} = \frac{P_2(3)}{10.667(2)} = 0.140625P_2$$

Thus, the state of stress at point A is represented on the element shown in Fig. c.

Normal and Shear Strain: With $\theta_a = 90°$ and $\theta_b = 45°$, we have

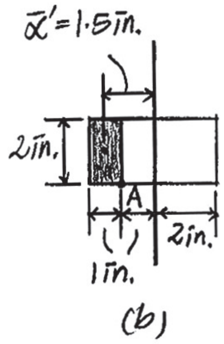
$$\varepsilon_a = \varepsilon_x \cos^2\theta_a + \varepsilon_y \sin^2\theta_a + \gamma_{xy}\sin\theta_a \cos\theta_a$$

$$300\left(10^{-6}\right) = \varepsilon_x \cos^2 90° + \varepsilon_y \sin^2 90° + \gamma_{xy}\sin 90° \cos 90°$$

$$\varepsilon_y = 300\left(10^{-6}\right)\varepsilon_b = \varepsilon_x \cos^2\theta_b + \varepsilon_y \sin^2\theta_b + \gamma_{xy}\sin\theta_b \cos\theta_b$$

$$175\left(10^{-6}\right) = \varepsilon_x \cos^2 45° + 300\left(10^{-6}\right)\sin^2 45° + \gamma_{xy} \sin 45° \cos 45°$$

$$\varepsilon_x + \gamma_{xy} = 50\left(10^{-6}\right) \qquad (1)$$



10–100. Continued

Since $\sigma_y = \sigma_z = 0, \varepsilon_x = -\nu\varepsilon_y = -0.32(300)\left(10^{-6}\right) = -96\left(10^{-6}\right)$

Then Eq. (1) gives

$$\gamma_{xy} = 146\left(10^{-6}\right)$$

Stress and Strain Relation: Hooke's Law for shear gives

$$\tau_x = G\gamma_{xy}$$

$$0.140625P_2 = 11.0\left(10^3\right)\left[146\left(10^{-6}\right)\right]$$

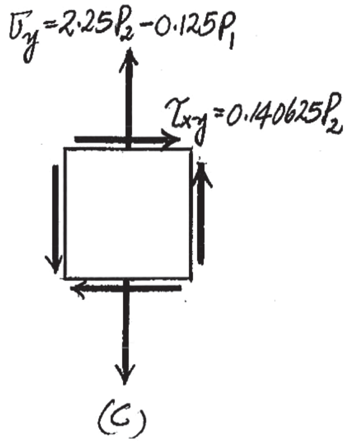
$$P_2 = 11.42\text{ kip} = 11.4\text{ kip} \qquad \textbf{Ans.}$$

Since $\sigma_y = \sigma_z = 0$, Hooke's Law gives

$$\sigma_y = E\varepsilon_y$$

$$2.25(11.42) - 0.125P_1 = 29.0\left(10^3\right)\left[300\left(10^{-6}\right)\right]$$

$$P_1 = 136\text{ kip} \qquad \textbf{Ans.}$$



10–101. A differential element is subjected to plane strain that has the following components: $\epsilon_x = 950(10^{-6})$, $\epsilon_y = 420(10^{-6})$, $\gamma_{xy} = -325(10^{-6})$. Use the strain-transformation equations and determine (a) the principal strains and (b) the maximum in-plane shear strain and the associated average strain. In each case specify the orientation of the element and show how the strains deform the element.

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \gamma_{xy}{}^2}$$

$$= \left[\frac{950 + 420}{2} \pm \sqrt{\left(\frac{950 - 420}{2}\right)^2 + \left(\frac{-325}{2}\right)^2}\right](10^{-6})$$

$$\varepsilon_1 = 996(10^{-6}) \qquad \textbf{Ans.}$$

$$\varepsilon_2 = 374(10^{-6}) \qquad \textbf{Ans.}$$

Orientation of ε_1 and ε_2:

$$\tan 2\theta_P = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{-325}{950 - 420}$$

$$\theta_P = -15.76°, 74.24°$$

Use Eq. 10.5 to determine the direction of ε_1 and ε_2.

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\theta = \theta_P = -15.76°$$

$$\varepsilon_{x'} = \left\{\frac{950 + 420}{2} + \frac{950 - 420}{2}\cos(-31.52°) + \frac{(-325)}{2}\sin(-31.52°)\right\}(10^{-6}) = 996(10^{-6})$$

$$\theta_{P1} = -15.8° \qquad \textbf{Ans.}$$

$$\theta_{P2} = 74.2° \qquad \textbf{Ans.}$$

b)

$$\frac{\gamma_{\substack{\max\\\text{in-plane}}}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{\substack{\max\\\text{in-plane}}} = 2\left[\sqrt{\left(\frac{950 - 420}{2}\right)^2 + \left(\frac{-325}{2}\right)^2}\right](10^{-6}) = 622(10^{-6}) \qquad \textbf{Ans.}$$

$$\varepsilon_{avg} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left(\frac{950 + 420}{2}\right)(10^{-6}) = 685(10^{-6}) \qquad \textbf{Ans.}$$

10–101. Continued

Orientation of $\gamma_{\max}$:

$$\tan 2\theta_P = \frac{-(\varepsilon_x - \varepsilon_y)}{\gamma_{xy}} = \frac{-(950 - 420)}{-325}$$

$\theta_P = 29.2°$ and $\theta_P = 119°$ **Ans.**

Use Eq. 10.6 to determine the sign of $\gamma_{\substack{\max \\ \text{in-plane}}}$:

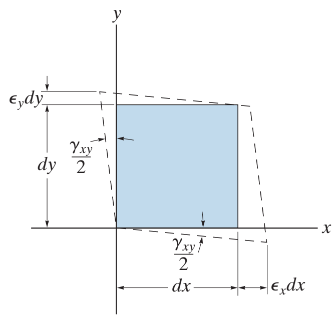
$$\frac{\gamma_{x'y'}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\theta = \theta_P = 29.2°$$

$$\gamma_{x'y'} = 2\left[\frac{-(950 - 420)}{2}\sin(58.4°) + \frac{-325}{2}\cos(58.4°)\right](10^{-6})$$

$$\gamma_{xy} = -622(10^{-6})$$

10–102. The state of plane strain on an element is $\epsilon_x = 400(10^{-6})$, $\epsilon_y = 200(10^{-6})$, and $\gamma_{xy} = -300(10^{-6})$. Determine the equivalent state of strain on an element at the same point oriented 30° clockwise with respect to the original element. Sketch the results on the element.



Stress Transformation Equations:

$\varepsilon_x = 400(10^{-6})$ $\varepsilon_y = 200(10^{-6})$ $\gamma_{xy} = -300(10^{-6})$ $\theta = -30°$

We obtain,

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{400 + 200}{2} + \frac{400 - 200}{2}\cos(-60°) + \left(\frac{-300}{2}\right)\sin(-60°)\right](10^{-6})$$

$= 480(10^{-6})$ **Ans.**

$$\frac{\gamma_{x'y'}}{2} = -\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\gamma_{x'y'} = [-(400 - 200)\sin(-60°) + (-300)\cos(-60°)](10^{-6})$$

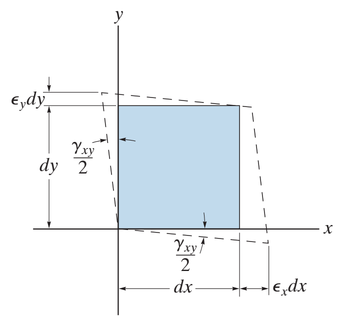
$= 23.2(10^{-6})$ **Ans.**

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{400 + 200}{2} - \frac{400 - 200}{2}\cos(-60°) - \left(\frac{-300}{2}\right)\sin(-60°)\right](10^{-6})$$

$= 120(10^{-6})$ **Ans.**

10–103. The state of plane strain on an element is $\epsilon_x = 400(10^{-6})$, $\epsilon_y = 200(10^{-6})$, and $\gamma_{xy} = -300(10^{-6})$. Determine the equivalent state of strain, which represents (a) the principal strains, and (b) the maximum in-plane shear strain and the associated average normal strain. Specify the orientation of the corresponding element at the point with respect to the original element. Sketch the results on the element.



Construction of the Circle: $\varepsilon_x = 400\left(10^{-6}\right)$, $\varepsilon_y = 200\left(10^{-6}\right)$, and $\dfrac{\gamma_{xy}}{2} = -150\left(10^{-6}\right)$. Thus,

$$\varepsilon_{\text{avg}} = \frac{\varepsilon_x + \varepsilon_y}{2} = \left(\frac{400 + 200}{2}\right)\left(10^{-6}\right) = 300\left(10^{-6}\right) \qquad \textbf{Ans.}$$

The coordinates for reference points A and the center C of the circle are

$$A(400, -150)\left(10^{-6}\right) \qquad C(300, 0)\left(10^{-6}\right)$$

The radius of the circle is

$$R = CA = \sqrt{(400 - 300)^2 + (-150)^2} = 180.28\left(10^{-6}\right)$$

Using these results, the circle is shown in Fig. *a*.

In - Plane Principal Stresses: The coordinates of points B and D represent ε_1 and ε_2, respectively. Thus,

$$\varepsilon_1 = (300 + 180.28)\left(10^{-6}\right) = 480\left(10^{-6}\right) \qquad \textbf{Ans.}$$

$$\varepsilon_2 = (300 - 180.28)\left(10^{-6}\right) = 120\left(10^{-6}\right) \qquad \textbf{Ans.}$$

Orientation of Principal Plane: Referring to the geometry of the circle,

$$\tan 2\left(\theta_p\right)_1 = \frac{150}{400 - 300} = 1.5$$

$$\left(\theta_p\right)_1 = 28.2^\circ \text{ (clockwise)} \qquad \textbf{Ans.}$$

The deformed element for the state of principal strains is shown in Fig. *b*.

Maximum In - Plane Shear Stress: The coordinates of point E represent ε_{avg} and $\gamma_{\substack{\max \\ \text{in-plane}}}$. Thus

$$\frac{\gamma_{\substack{\max \\ \text{in-plane}}}}{2} = -R = -180.28\left(10^{-6}\right)$$

$$\gamma_{\substack{\max \\ \text{in-plane}}} = -361\left(10^{-6}\right) \qquad \textbf{Ans.}$$

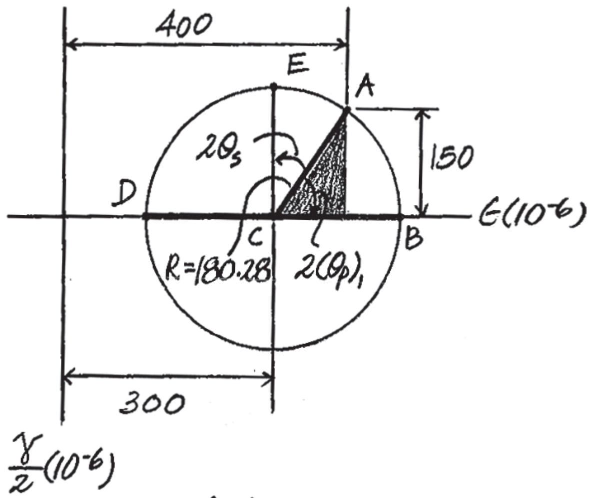
Orientation of the Plane of Maximum In - Plane Shear Strain: Referring to the geometry of the circle,

$$\tan 2\theta_s = \frac{400 - 300}{150} = 0.6667$$

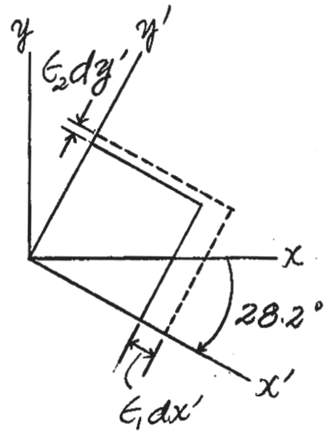
$$\theta_S = 16.8^\circ \text{ (counterclockwise)} \qquad \textbf{Ans.}$$

The deformed element for the state of maximum in - plane shear strain is shown in Fig. *c*.

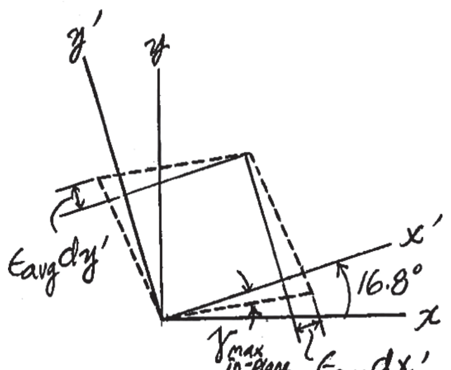
10–103. Continued



(a)



(b)



(c)