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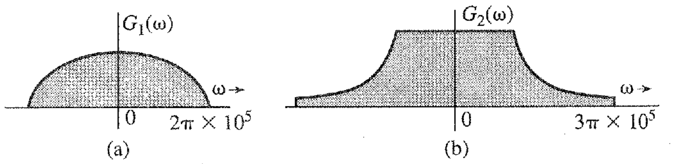
PROBLEMS

6.1-1 Figure P6.1-1 shows Fourier spectra of signals $g_1(t)$ and $g_2(t)$. Determine the Nyquist interval and the sampling rate for signals $g_1(t)$, $g_2(t)$, $g_1^2(t)$, $g_2^m(t)$, and $g_1(t)g_2(t)$.
Hint: Use the frequency convolution and the width property of the convolution.

Figure P.6.1-1



6.1-2 Determine the Nyquist sampling rate and the Nyquist sampling interval for the signals:

(a) sinc $(100\pi t)$
(b) sinc^2 $(100\pi t)$
(c) sinc $(100\pi t)$ + sinc $(50\pi t)$
(d) sinc $(100\pi t)$ + 3 sinc^2 $(60\pi t)$
(e) sinc $(50\pi t)$ sinc $(100\pi t)$

6.1-3 A signal $g(t)$ band-limited to B Hz is sampled by a periodic pulse train $p_{T_s}(t)$ made up of a rectangular pulse of width $1/8B$ second (centered at the origin) repeating at the Nyquist rate ($2B$ pulses per second). Show that the sampled signal $\overline{g}(t)$ is given by

$$\overline{g}(t) = \frac{1}{4}g(t) + \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin\left(\frac{n\pi}{4}\right) g(t) \cos 4n\pi Bt$$

Show that the signal $g(t)$ can be recovered by passing $\bar{g}(t)$ through an ideal low-pass filter of bandwidth B Hz and a gain of 4.

6.1-4 A signal $g(t) = \text{sinc}^2(5\pi t)$ is sampled (using uniformly spaced impulses) at a rate of **(i)** 5 Hz; **(ii)** 10 Hz; **(iii)** 20 Hz. For each of the three cases:

(a) Sketch the sampled signal.

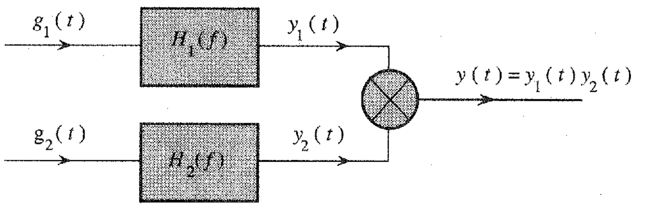
(b) Sketch the spectrum of the sampled signal.

(c) Explain whether you can recover the signal $g(t)$ from the sampled signal.

(d) If the sampled signal is passed through an ideal low-pass filter of bandwidth 5 Hz, sketch the spectrum of the output signal.

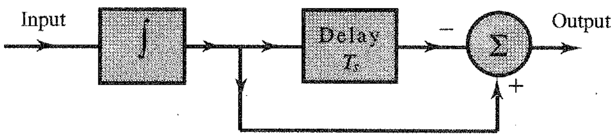
6.1-5 Signals $g_1(t) = 10^4\,\Pi(10^4 t)$ and $g_2(t) = \delta(t)$ are applied at the inputs of ideal low-pass filters $H_1(f) = \Pi(f/20,000)$ and $H_2(f) = \Pi(f/10,000)$ (Fig. P6.1-5). The outputs $y_1(t)$ and $y_2(t)$ of these filters are multiplied to obtain the signal $y(t) = y_1(t)y_2(t)$. Find the Nyquist rate of $y_1(t), y_2(t)$, and $y(t)$. Use the convolution property and the width property of convolution to determine the bandwidth of $y_1(t)y_2(t)$. See also Prob. 6.1-1.

Figure P.6.1-5



6.1-6 A zero-order hold circuit (Fig. P6.1-6) is often used to reconstruct a signal $g(t)$ from its samples.

Figure P.6.1-6



(a) Find the unit impulse response of this circuit.

(b) Find the transfer function $H(f)$ and sketch $|H(f)|$.

(c) Show that when a sampled signal $\bar{g}(t)$ is applied at the input of this circuit, the output is a staircase approximation of $g(t)$. The sampling interval is T_s.

6.1-7 **(a)** A first-order hold circuit can also be used to reconstruct a signal $g(t)$ from its samples. The impulse response of this circuit is

$$h(t) = \Delta\left(\frac{t}{2T_s}\right)$$

where T_s is the sampling interval. Consider a typical sampled signal $\bar{g}(t)$ and show that this circuit performs the linear interpolation. In other words, the filter output consists of sample tops connected by straight-line segments. Follow the procedure discussed in Sec. 6.1.1 (Fig. 6.2b).

(b) Determine the transfer function of this filter and its amplitude response, and compare it with the ideal filter required for signal reconstruction.

(c) This filter, being noncausal, is unrealizable. Suggest a modification that will make this filter realizable. How would such a modification affect the reconstruction of $g(t)$ from its samples? How would it affect the frequency response of the filter?

6.1-8 Prove that a signal cannot be simultaneously time-limited and band-limited.

Hint: Show that the contrary assumption leads to contradiction. Assume a signal simultaneously time-limited and band-limited so that $G(f) = 0$ for $|f| > B$. In this case, $G(f) = G(f)\,\Pi(f/2B')$ for $B' > B$. This means that $g(t)$ is equal to $g(t) * 2B' \operatorname{sinc}(2\pi B' t)$. Show that the latter cannot be time-limited.

6.2-1 The American Standard Code for Information Interchange (ASCII) has 128 characters, which are binary-coded. If a certain computer generates 100,000 characters per second, determine the following:

(a) The number of bits (binary digits) required per character.

(b) The number of bits per second required to transmit the computer output, and the minimum bandwidth required to transmit this signal.

(c) For single error detection capability, an additional bit (parity bit) is added to the code of each character. Modify your answers in parts **(a)** and **(b)** in view of this information.

6.2-2 A compact disc (CD) records audio signals digitally by using PCM. Assume that the audio signal bandwidth equals 15 kHz.

(a) If the Nyquist samples are uniformly quantized into $L = 65,536$ levels and then binary-coded, determine the number of binary digits required to encode a sample.

(b) If the audio signal has average power of 0.1 watt and peak voltage of 1 volt. Find the resulting signal-to-quantization-noise ratio (SQNR) of the uniform quantizer output in part **(a)**.

(c) Determine the number of binary digits per second (bit/s) required to encode the audio signal.

(d) For practical reasons discussed in the text, signals are sampled at a rate well above the Nyquist rate. Practical CDs use 44,100 samples per second. If $L = 65,536$, determine the number of bits per second required to encode the signal, and the minimum bandwidth required to transmit the encoded signal.

6.2-3 A television signal (video and audio) has a bandwidth of 4.5 MHz. This signal is sampled, quantized, and binary coded to obtain a PCM signal.

(a) Determine the sampling rate if the signal is to be sampled at a rate 20% above the Nyquist rate.

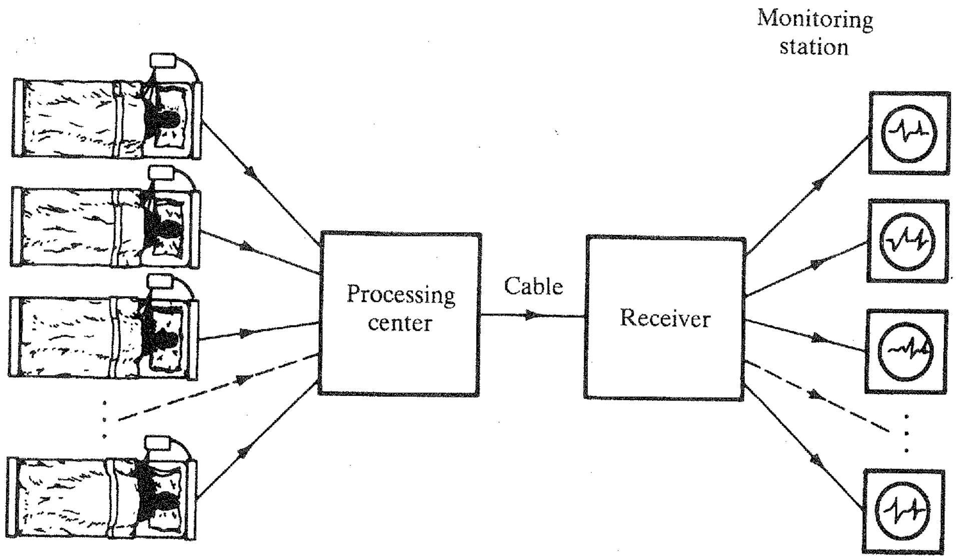
(b) If the samples are quantized into 1024 levels, determine the number of binary pulses required to encode each sample.

(c) Determine the binary pulse rate (bits per second) of the binary-coded signal, and the minimum bandwidth required to transmit this signal.

6.2-4 Five telemetry signals, each of bandwidth 240 Hz, are to be transmitted simultaneously by binary PCM. The signals must be sampled at least 20% above the Nyquist rate. Framing and synchronizing requires an additional 0.5% extra bits. A PCM encoder is used to convert these signals before they are time-multiplexed into a single data stream. Determine the minimum possible data rate (bits per second) that must be transmitted, and the minimum bandwidth required to transmit the multiplex signal.

6.2-5 It is desired to set up a central station for simultaneous monitoring of the electrocardiograms (ECGs) of 10 hospital patients. The data from the 10 patients are brought to a processing center over wires and are sampled, quantized, binary-coded, and time-division-multiplexed. The multiplexed data are now transmitted to the monitoring station (Fig. P6.2-5). The ECG signal bandwidth is 100 Hz. The maximum acceptable error in sample amplitudes is 0.25% of the peak signal amplitude. The sampling rate must be at least twice the Nyquist rate. Determine the minimum cable bandwidth needed to transmit these data.

Figure P.6.2-5

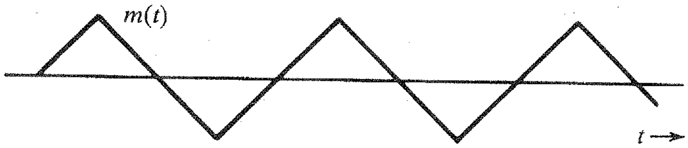


6.2-6 A message signal $m(t)$ is transmitted by binary PCM without compression. If the SQNR is required to be at least 47 dB, determine the minimum value of $L = 2^n$ required, assuming that $m(t)$ is sinusoidal. Determine the actual SQNR obtained with this minimum L.

6.2-7 Repeat Prob. 6.2-6 for $m(t)$ shown in Fig. P6.2-7.

Hint: The power of a periodic signal is its energy averaged over one cycle. In this case, however, because the signal amplitude takes on the same values every quarter cycle, the power can also be found by averaging the signal energy over a quarter cycle.

Figure P.6.2-7



6.2-8 For a PCM signal, determine L if the compression parameter $\mu = 100$ and the minimum SNR required is 45 dB. Determine the output SQNR with this value of L. Remember that L must be a power of 2, that is, $L = 2^n$ for a binary PCM.

6.2-9 A signal band-limited to 1 MHz is sampled at a rate 50% higher than the Nyquist rate and quantized into 256 levels by using a μ-law quantizer with $\mu = 255$.

(a) Determine the signal-to-quantization-noise ratio.

(b) The SQNR (the received signal quality) found in part (a) was unsatisfactory. It must be increased at least by 10 dB. Would you be able to obtain the desired SQNR without increasing the transmission bandwidth if it was found that a sampling rate 20% above the Nyquist rate is adequate? If so, explain how. What is the maximum SQNR that can be realized in this way?

6.2-10 The output SQNR of a 10-bit PCM was found to be insufficient at 30 dB. To achieve the desired SNR of 42 dB, it was decided to increase the number of quantization levels L. Find the fractional increase in the transmission bandwidth required for this increase in L.

6.4-1 In a certain telemetry system, there are four analog signals $m_1(t)$, $m_2(t)$, $m_3(t)$, and $m_4(t)$. The bandwidth of $m_1(t)$ is 3.6 kHz, but for each of the remaining signals it is 1.4 kHz. These signals are to be sampled at rates no less than their respective Nyquist rates and are to be word-by-word multiplexed. This can be achieved by multiplexing the PAM samples of the four signals and then binary coding the multiplexed samples (as in the case of the PCM T1 carrier in Fig. 6.20a). Suggest a suitable multiplexing scheme for this purpose. What is the commutator frequency (in rotations per second)? *Note:* In this case you may have to sample some signal(s) at rates higher than their Nyquist rate(s).

6.4-2 Repeat Prob. 6.4-1 if there are four signals $m_1(t)$, $m_2(t)$, $m_3(t)$, and $m_4(t)$ with bandwidths 1200, 700, 300, and 200 Hz, respectively.

Hint: First multiplex m_2, m_3, and m_4 and then multiplex this composite signal with $m_1(t)$.

6.4-3 A signal $m_1(t)$ is band-limited to 3.6 kHz, and the three other signals $m_2(t)$, $m_3(t)$, and $m_4(t)$ are band-limited to 1.2 kHz each. These signals are sampled at the Nyquist rate and binary coded using 512 levels ($L = 512$). Suggest a suitable bit-by-bit multiplexing arrangement (as in Fig. 6.12). What is the commutator frequency (in rotations per second), and what is the output bit rate?

6.7-1 In a single-integration DM system, the voice signal is sampled at a rate of 64 kHz, similar to PCM. The maximum signal amplitude is normalized as $A_{max} = 1$.

(a) Determine the minimum value of the step size σ to avoid slope overload.

(b) Determine the granular noise power N_o if the voice signal bandwidth is 3.4 kHz.

(c) Assuming that the voice signal is sinusoidal, determine S_o and the SNR.

(d) Assuming that the voice signal amplitude is uniformly distributed in the range $(-1, 1)$, determine S_o and the SNR.

(e) Determine the minimum transmission bandwidth.

7 PRINCIPLES OF DIGITAL DATA TRANSMISSION

Throughout most of the twentieth century, a significant percentage of communication systems was in analog form. However, by the end of the 1990s, the digital format began to dominate most applications. One does not need to look hard to witness the continuous migration from analog to digital communications: from audiocassette tape to MP3 and CD, from NTSC analog TV to digital HDTV, from traditional telephone to VoIP, and from VHS videotape to DVD. In fact, even the last analog refuge of broadcast radio is facing a strong digital competitor in the form of satellite radio. Given the dominating importance of digital communication systems in our lives today, it is never too early to study the basic principles and various aspects of digital data transmission, as we will do in this chapter.

This chapter deals with the problems of transmitting digital data over a channel. Hence, the starting messages are assumed to be digital. We shall begin by considering the binary case, where the data consist of only two symbols: **1** and **0**. We assign a distinct waveform (pulse) to each of these two symbols. The resulting sequence of these pulses is transmitted over a channel. At the receiver, these pulses are detected and are converted back to binary data (**1**s and **0**s).

7.1 DIGITAL COMMUNICATION SYSTEMS

A digital communication system consists of several components, as shown in Fig. 7.1. In this section, we conceptually outline their functionalities in the communication systems. The details of their analysis and design will be given in dedicated sections later in this chapter.

7.1.1 Source

The input to a digital system takes the form of a sequence of digits. The input could be the output from a data set, a computer, or a digitized audio signal (PCM, DM, or LPC), digital facsimile or HDTV, or telemetry data, and so on. Although most of the discussion in this chapter is confined to the binary case (communication schemes using only two symbols), the more general case of M-ary communication, which uses M symbols, will also be discussed in Secs. 7.7 and 7.9.

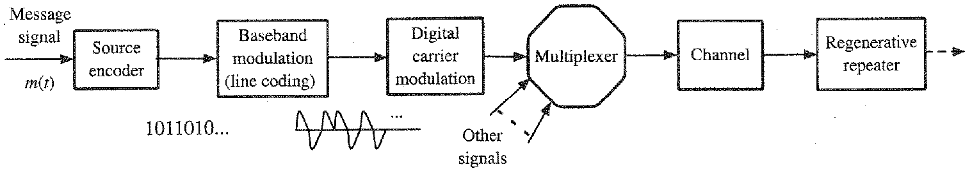


Figure 7.1 Fundamental building blocks of digital communication systems.

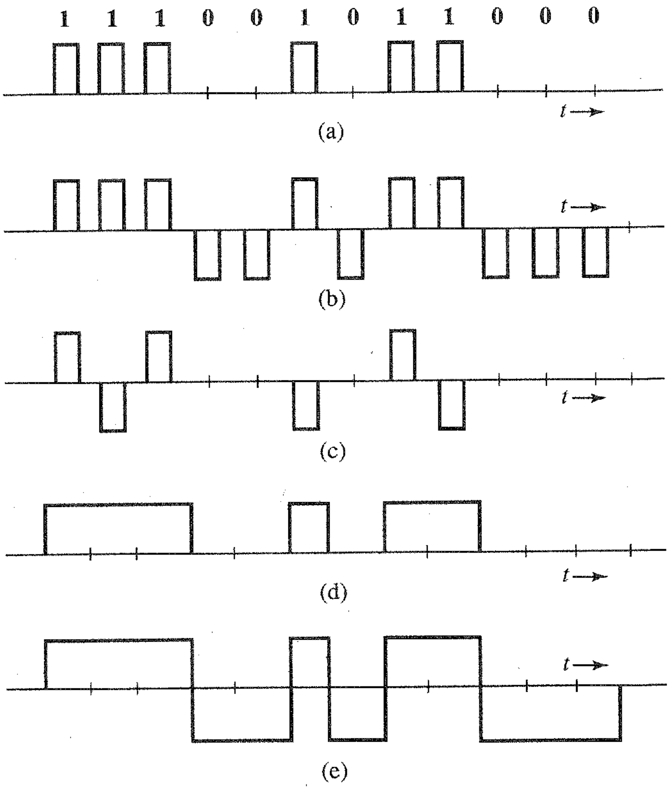


Figure 7.2 Line code examples: (a) on-off (RZ); (b) polar (RZ); (c) bipolar (RZ); (d) on-off (NRZ); (e) polar (NRZ).

7.1.2 Line Coder

The digital output of a source encoder is converted (or coded) into electrical pulses (waveforms) for the purpose of transmission over the channel. This process is called **line coding** or **transmission coding**. There are many possible ways of assigning waveforms (pulses) to the digital data. In the binary case (2 symbols), for example, conceptually the simplest line code is **on-off**, where a **1** is transmitted by a pulse $p(t)$ and a **0** is transmitted by no pulse (zero signal) as shown in Fig. 7.2a. Another commonly used code is **polar**, where **1** is transmitted by a pulse $p(t)$ and **0** is transmitted by a pulse $-p(t)$ (Fig. 7.2b). The polar scheme is the most power-efficient code because it requires the least power for a given noise immunity (error probability). Another popular code in PCM is **bipolar**, also known as **pseudoternary** or **alternate mark inversion (AMI)**, where **0** is encoded by no pulse and **1** is encoded by a pulse $p(t)$ or $-p(t)$ depending on whether the previous **1** is encoded by $-p(t)$ or $p(t)$. In short, pulses representing consecutive **1**s alternate in sign, as shown in Fig. 7.2c. This code has the advantage that if *one single* error is made in the detecting of pulses, the received pulse sequence will violate the bipolar rule and the error can be detected (although not corrected) immediately.*

Another line code that appeared promising earlier is the duobinary (and modified duobinary) proposed by Lender.[1, 2] This code is better than the bipolar in terms of bandwidth efficiency. Its more prominent variant, the *modified duobinary* line code, has seen applications in hard disk drive read channels, in optical 10 Gbit/s transmission for metronetworks, and in the first-generation modems for integrated services digital networks (ISDN). Details of duobinary line codes will be discussed later in this chapter.

In our discussion so far, we have used half-width pulses just for the sake of illustration. We can select other widths also. Full-width pulses are often used in some applications. Whenever full-width pulses are used, the pulse amplitude is held to a constant value throughout the pulse interval (i.e., it does not have a chance to go to zero before the next pulse begins). For this reason, these schemes are called **non-return-to-zero** or **NRZ** schemes, in contrast to **return-to-zero** or **RZ** schemes (Fig. 7.2a–c). Figure 7.2d shows an on-off NRZ signal, whereas Fig. 7.2e shows a polar NRZ signal.

7.1.3 Multiplexer

Generally speaking, the capacity of a physical channel (e.g., coaxial cable, optic fiber) for transmitting data is much larger than the data rate of individual sources. To utilize this capacity effectively, we combine several sources by means of a digital multiplexer. The digital multiplexing can be achieved through frequency division or time division, as we have already discussed. Alternatively, code division is also a practical and effective approach (to be discussed in Chapter 11). Thus a physical channel is normally shared by several messages simultaneously.

7.1.4 Regenerative Repeater

Regenerative repeaters are used at regularly spaced intervals along a digital transmission line to detect the incoming digital signal and regenerate new "clean" pulses for further transmission along the line. This process periodically eliminates, and thereby combats, accumulation of noise and signal distortion along the transmission path. The ability of such regenerative repeaters to effectively eliminate noise and signal distortion effects is one of the biggest advantages of digital communication systems over their analog counterparts.

If the pulses are transmitted at a rate of R_b pulses per second, we require the periodic timing information—the clock signal at R_b Hz—to sample the incoming pulses at a repeater. This timing information can be extracted from the received signal itself if the line code is chosen properly. When the *RZ* polar signal in Fig. 7.2b is rectified, for example, it results in a periodic signal of clock frequency R_b Hz, which contains the desired periodic timing signal of frequency R_b Hz. When this signal is applied to a resonant circuit tuned to frequency R_b, the output, which is a sinusoid of frequency R_b Hz, can be used for timing. The on-off signal can be expressed as a sum of a periodic signal (of clock frequency) and a polar, or random, signal as shown in Fig. 7.3. Because of the presence of the periodic component, we can extract the timing information from this signal by using a resonant circuit tuned to the clock frequency. A bipolar signal, when rectified, becomes an on-off signal. Hence, its timing information can be extracted using the same way as that for an on-off signal.

* This assumes no more than one error in sequence. Multiple errors in sequence could cancel their respective effects and remain undetected. However, the probability of multiple errors is much smaller than that of single errors. Even

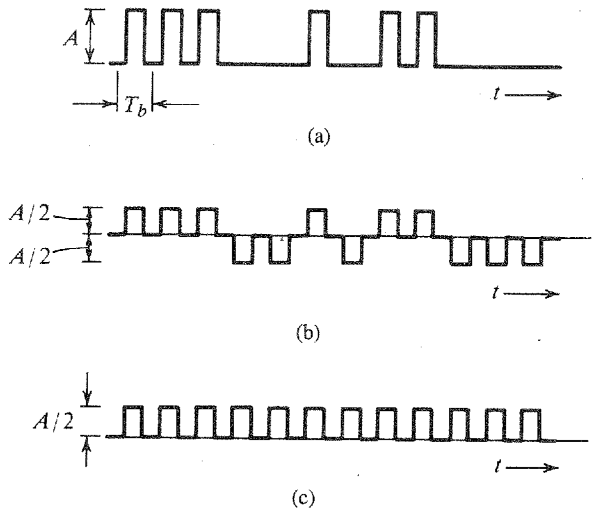


Figure 7.3 An on-off signal (a) is a sum of a random polar signal (b) and a clock frequency periodic signal (c).

The timing signal (the resonant circuit output) is sensitive to the incoming bit pattern. In the on-off or bipolar case, a **0** is transmitted by 'no pulse.' Hence, if there are too many **0**s in a sequence (no pulses), there is no signal at the input of the resonant circuit and the sinusoidal output of the resonant circuit starts decaying, thus causing error in the timing information. We shall discuss later ways of overcoming this problem. A line code in which the bit pattern does not affect the accuracy of the timing information is said to be a **transparent** line code. The *RZ* polar scheme (where each bit is transmitted by some pulse) is transparent, whereas the on-off and bipolar are nontransparent.

7.2 LINE CODING

Digital data can be transmitted by various **transmission** or **line codes**. We have given examples of on-off, polar, and bipolar. Each line code has its advantages and disadvantages. Among other desirable properties, a line code should have the following properties.

- *Transmission bandwidth* should be as small as possible.
- *Power efficiency.* For a given bandwidth and a specified detection error rate, the transmitted power should be as low as possible.
- *Error detection and correction capability.* It is desirable to detect, and preferably correct, detection errors. In a bipolar case, for example, a single error will cause bipolar violation and can easily be detected. Error correcting codes will be discussed in depth in Chapter 14.
- *Favorable power spectral density.* It is desirable to have zero power spectral density (PSD) at $f = 0$ (dc) because ac coupling and transformers are often used at the repeaters.* Significant power in low-frequency components should also be avoided because it causes dc wander in the pulse stream when ac coupling is used.

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for single errors, we cannot tell exactly where the error is located. Therefore, this code can detect the presence of single errors, but it cannot correct them.

* The ac coupling is required because the dc paths provided by the cable pairs between the repeater sites are used to transmit the power needed to operate the repeaters.

- *Adequate timing content.* It should be possible to extract timing or clock information from the signal.
- *Transparency.* It should be possible to correctly transmit a digital signal regardless of the pattern of **1**s and **0**s. We saw earlier that a long string of **0**s could cause problems in timing extraction for the on-off and bipolar cases. A code is transparent if the data are so coded that for every possible sequence of data, the coded signal is received faithfully.

7.2.1 PSD of Various Line Codes

In Example 3.19 we discussed a procedure for finding the PSD of a polar pulse train. We shall use a similar procedure to find a general expression for PSD of the baseband modulation (line coding) output signals as shown in Fig. 7.1. In particular, we directly apply the relationship between the PSD and the autocorrelation function of the baseband modulation signal given in Section 3.8 [Eq. (3.85)].

In the following discussion, we consider a generic pulse $p(t)$ whose corresponding Fourier transform is $P(f)$. We can denote the line code symbol at time k as a_k. When the transmission rate is $R_b = 1/T_b$ pulses per second, the line code generates a pulse train constructed from the basic pulse $p(t)$ with amplitude a_k starting at time $t = kT_b$; in other words, the kth symbol is transmitted as $a_k p(t - kT_b)$. Figure 7.4a provides an illustration of a special pulse $p(t)$, whereas Fig. 7.4b shows the corresponding pulse train generated by the line coder at baseband. As shown

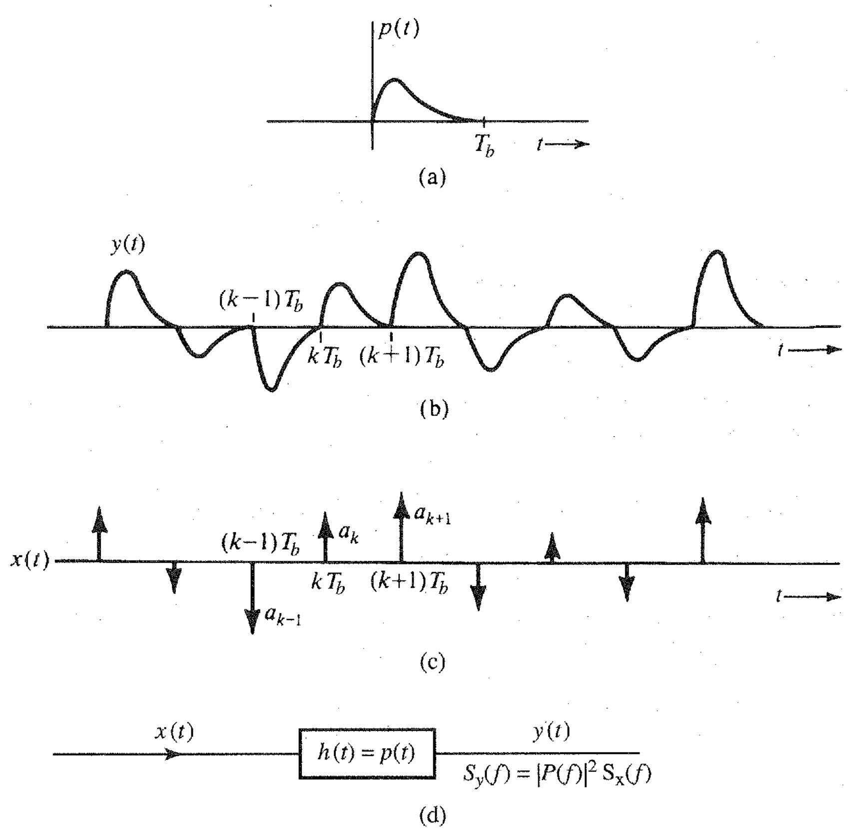


Figure 7.4 Random pulse-amplitude-modulated signal and its generation from a PAM impulse.

in Fig. 7.4b, counting a succession of symbol transmissions T_b second apart, the baseband signal is a pulse train of the form

$$y(t) = \sum a_k p(t - kT_b) \tag{7.1}$$

Note that the line coder determines the symbol $\{a_k\}$ as the amplitude of the pulse $p(t - kT_b)$.

The values a_k are random and depend on the line coder input and the line code itself; $y(t)$ is a pulse-amplitude-modulated (PAM) signal. The on-off, polar, and bipolar line codes are all special cases of this pulse train $y(t)$, where a_k takes on values 0, 1, or -1 randomly, subject to some constraints. We can, therefore, analyze many line codes according to the PSD of $y(t)$. Unfortunately, the PSD of $y(t)$ depends on both a_k and $p(t)$. If the pulse shape $p(t)$ changes, we may have to derive the PSD all over again. This difficulty can be overcome by the simple artifice of selecting a PAM signal $x(t)$ that uses a unit impulse for the basic pulse $p(t)$ (Fig. 7.4c). The impulses are at the intervals of T_b and the strength (area) of the kth impulse is a_k. If $x(t)$ is applied to the input of a filter that has a unit impulse response $h(t) = p(t)$ (Fig. 7.4d), the output will be the pulse train $y(t)$ in Fig. 7.4b. Also, applying Eq. (3.92), the PSD of $y(t)$ is

$$S_y(f) = |P(f)|^2 S_x(f)$$

This relationship allows us to determine $S_y(f)$, the PSD of a line code corresponding to any pulse shape $p(t)$, once we know $S_x(f)$. This approach is attractive because of its generality.

We now need to derive $\mathcal{R}_x(\tau)$, the time autocorrelation function of the impulse train $x(t)$. This can be conveniently done by considering the impulses as a limiting form of the rectangular pulses, as shown in Fig. 7.5a. Each pulse has a width $\epsilon \to 0$, and the kth pulse height

$$h_k = \frac{a_k}{\epsilon} \to \infty$$

This way, we guarantee that the strength of the kth impulse is a_k, or

$$\epsilon h_k = a_k$$

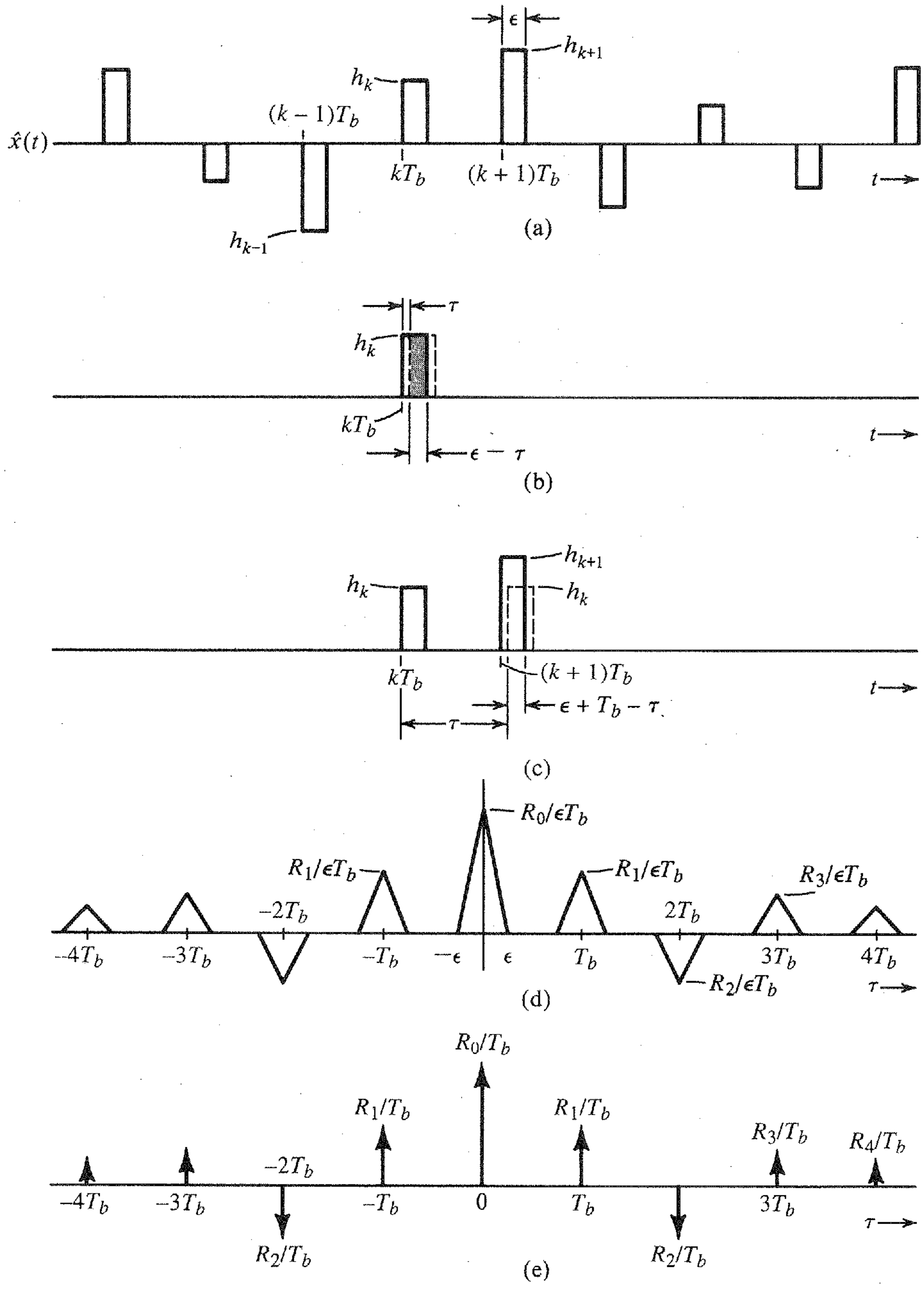
If we designate the corresponding rectangular pulse train by $\hat{x}(t)$, then by definition [Eq. (3.82) in Sec. 3.8]

$$\mathcal{R}_{\hat{x}}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \hat{x}(t)\hat{x}(t-\tau)\, dt \tag{7.2}$$

Because $\mathcal{R}_{\hat{x}}(\tau)$ is an even function of τ [Eq. (3.83)], we need to consider only positive τ. To begin with, consider the case of $\tau < \epsilon$. In this case the integral in Eq. (7.2) is the area under the signal $\hat{x}(t)$ multiplied by $\hat{x}(t)$ delayed by $\tau\,(\tau < \epsilon)$. As seen from Fig. 7.5b, the area associated with the kth pulse is $h_k^2(\epsilon - \tau)$, and

$$\begin{aligned}
\mathcal{R}_{\hat{x}} &= \lim_{T\to\infty} \frac{1}{T} \sum_k h_k^2(\epsilon - \tau) \\
&= \lim_{T\to\infty} \frac{1}{T} \sum_k a_k^2 \left(\frac{\epsilon - \tau}{\epsilon^2}\right) \\
&= \frac{R_0}{\epsilon T_b}\left(1 - \frac{\tau}{\epsilon}\right)
\end{aligned} \tag{7.3a}$$

Figure 7.5 Derivation of PSD of a random PAM signal with a very narrow pulse of width ϵ and height $h_k = a_k/\epsilon$.



where

$$R_0 = \lim_{T\to\infty} \frac{T_b}{T} \sum_k a_k^2 \tag{7.3b}$$

During the averaging interval T ($T \to \infty$), there are N pulses ($N \to \infty$), where

$$N = \frac{T}{T_b} \tag{7.4}$$

and from Eq. (7.3b)

$$R_0 = \lim_{N\to\infty} \frac{1}{N} \sum_k a_k^2 \tag{7.5}$$

Observe that the summation is over N pulses. Hence, R_0 is the time average of the square of the pulse amplitudes a_k. Using our time average notation, we can express R_0 as

$$R_0 = \lim_{N\to\infty} \frac{1}{N} \sum_k a_k^2 = \widetilde{a_k^2} \tag{7.6}$$

We also know that $\mathcal{R}_{\hat{x}}(\tau)$ is an even function of τ [see Eq. (3.83)]. Hence, Eq. (7.3) can be expressed as

$$\mathcal{R}_{\hat{x}}(\tau) = \frac{R_0}{\epsilon T_b}\left(1 - \frac{|\tau|}{\epsilon}\right) \qquad |\tau| < \epsilon \tag{7.7}$$

This is a triangular pulse of height $R_0/\epsilon T_b$ and width 2ϵ centered at $\tau = 0$ (Fig. 7.5d). This is expected because as τ increases beyond ϵ, there is no overlap between the delayed signal $\hat{x}(t-\tau)$ and $\hat{x}(t)$; hence, $\mathcal{R}_{\hat{x}}(\tau) = 0$, as seen from Fig. 7.5d. But as we increase τ further, we find that the kth pulse of $\hat{x}(t-\tau)$ will start overlapping the $(k+1)$th pulse of $\hat{x}(t)$ as τ approaches T_b (Fig. 7.5c). Repeating the earlier argument, we see that $\mathcal{R}_{\hat{x}}(\tau)$ will have another triangular pulse of width 2ϵ centered at $\tau = T_b$ and of height $R_1/\epsilon T_b$ where

$$\begin{aligned} R_1 &= \lim_{T\to\infty} \frac{T_b}{T} \sum_k a_k a_{k+1} \\ &= \lim_{N\to\infty} \frac{1}{N} \sum_k a_k a_{k+1} \\ &= \widetilde{a_k a_{k+1}} \end{aligned}$$

Observe that R_1 is obtained by multiplying every pulse strength (a_k) by the strength of its immediate neighbor (a_{k+1}), adding all these products, and then dividing by the total number of pulses. This is clearly the time average (mean) of the product $a_k a_{k+1}$ and is, in our notation, $\widetilde{a_k a_{k+1}}$. A similar thing happens around $\tau = 2T_b, 3T_b, \ldots$. Hence, $\mathcal{R}_{\hat{x}}(\tau)$ consists of a sequence of triangular pulses of width 2ϵ centered at $\tau = 0, \pm T_b, \pm 2T_b, \ldots$. The height of the pulses centered at $\pm nT_b$ is $R_n/\epsilon T_b$, where

$$\begin{aligned} R_n &= \lim_{T\to\infty} \frac{T_b}{T} \sum_k a_k a_{k+n} \\ &= \lim_{N\to\infty} \frac{1}{N} \sum_k a_k a_{k+n} \\ &= \widetilde{a_k a_{k+n}} \end{aligned}$$

R_n is essentially the discrete autocorrelation function of the line code symbols $\{a_k\}$.

To find $\mathcal{R}_x(\tau)$, we let $\epsilon \to 0$ in $\mathcal{R}_{\hat{x}}(\tau)$. As $\epsilon \to 0$, the width of each triangular pulse $\to 0$ and the height $\to \infty$ in such a way that the area is still finite. Thus, in the limit as $\epsilon \to 0$, the triangular pulses become impulses. For the nth pulse centered at nT_b, the height is $R_n/\epsilon T_b$ and the area is R_n/T_b. Hence, (Fig. 7.5e)

$$\mathcal{R}_x(\tau) = \frac{1}{T_b} \sum_{n=-\infty}^{\infty} R_n \, \delta(\tau - nT_b) \tag{7.8}$$

The PSD $S_x(f)$ is the Fourier transform of $\mathcal{R}_x(\tau)$. Therefore,

$$S_x(f) = \frac{1}{T_b} \sum_{n=-\infty}^{\infty} R_n e^{-jn2\pi f T_b} \tag{7.9}$$

Recognizing that $R_{-n} = R_n$ [because $\mathcal{R}(\tau)$ is an even function of τ], we have

$$S_x(f) = \frac{1}{T_b} \left[R_o + 2 \sum_{n=1}^{\infty} R_n \cos n2\pi f T_b \right] \tag{7.10}$$

The input $x(t)$ to the filter with impulse response $h(t) = p(t)$ results in the output $y(t)$, as shown in Fig. 7.4d. If $p(t) \Longleftrightarrow P(f)$, the transfer function of the filter is $H(f) = P(f)$, and according to Eq. (3.91),

$$S_y(f) = |P(f)|^2 S_x(f) \tag{7.11a}$$

$$= \frac{|P(f)|^2}{T_b} \left[\sum_{n=-\infty}^{\infty} R_n e^{-jn2\pi f T_b} \right] \tag{7.11b}$$

$$= \frac{|P(f)|^2}{T_b} \left[R_o + 2 \sum_{n=1}^{\infty} R_n \cos n2\pi f T_b \right] \tag{7.11c}$$

Thus, the PSD of a line code is fully characterized by its R_n and the pulse-shaping selection $P(f)$. We shall now use this general result to find the PSDs of various specific line codes by first determining the symbol autocorrelation R_n.

7.2.2 Polar Signaling

In polar signaling, **1** is transmitted by a pulse $p(t)$ and **0** is represented by $-p(t)$. In this case, a_k is equally likely to be 1 or -1, and a_k^2 is always 1. Hence,

$$R_0 = \lim_{N \to \infty} \frac{1}{N} \sum_k a_k^2$$

There are N pulses and $a_k^2 = 1$ for each one, and the summation on the right-hand side above is N. Hence,

$$R_0 = \lim_{N \to \infty} \frac{1}{N} (N) = 1 \tag{7.12a}$$

Moreover, both a_k and a_{k+1} are either 1 or -1. Hence, $a_k a_{k+1}$ is either 1 or -1. Because the pulse amplitude a_k is equally likely to be 1 and -1 on the average, out of N terms the product $a_k a_{k+1}$ is equal to 1 for $N/2$ terms and is equal to -1 for the remaining $N/2$ terms. Therefore,

Possible Values of $a_k a_{k+1}$

a_{k+1} \ a_k	-1	$+1$
-1	1	-1
$+1$	-1	1

$$R_1 = \lim_{N\to\infty} \frac{1}{N}\left[\frac{N}{2}(1) + \frac{N}{2}(-1)\right] = 0 \tag{7.12b}$$

Arguing this way, we see that the product $a_k a_{k+n}$ is also equally likely to be 1 or -1. Hence,

$$R_n = 0 \qquad n \geq 1 \tag{7.12c}$$

Therefore from Eq. (7.11c)

$$\begin{aligned} S_y(f) &= \frac{|P(f)|^2}{T_b} R_0 \\ &= \frac{|P(f)|^2}{T_b} \end{aligned} \tag{7.13}$$

For the sake of comparison of various schemes, we shall consider a specific pulse shape. Let $p(t)$ be a rectangular pulse of width $T_b/2$ (half-width rectangular pulse), that is,

$$p(t) = \Pi\left(\frac{t}{T_b/2}\right) = \Pi\left(\frac{2t}{T_b}\right)$$

and

$$P(f) = \frac{T_b}{2}\,\text{sinc}\left(\frac{\pi f T_b}{2}\right) \tag{7.14}$$

Therefore

$$S_y(f) = \frac{T_b}{4}\,\text{sinc}^2\left(\frac{\pi f T_b}{2}\right) \tag{7.15}$$

Figure 7.6 shows the spectrum $S_y(f)$. It is clear that the polar signal has most of its power concentrated in lower frequencies. Theoretically, the spectrum becomes very small as frequency increases but never becomes totally zero above a certain frequency. To define a meaningful measure of bandwidth, we consider its *first non-dc null frequency* to be its **essential bandwidth.***

From polar signal spectrum, the essential bandwidth of the signal is seen to be $2R_b$ Hz (where R_b is the clock frequency). This is 4 times the theoretical bandwidth (Nyquist bandwidth) required to transmit R_b pulses per second. Increasing the pulse width reduces the bandwidth (expansion in the time domain results in compression in the frequency domain).

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* Strictly speaking, the location of the first null frequency above dc is not always a good measure of signal bandwidth. Whether the first non-dc null is a meaningful bandwidth depends on the amount of signal power contained in the main (or first) lobe of the PSD, as we will see later in the PSD comparison of several line codes (Fig. 7.9). In most practical cases, this approximation is acceptable for commonly used line codes and pulse shapes.

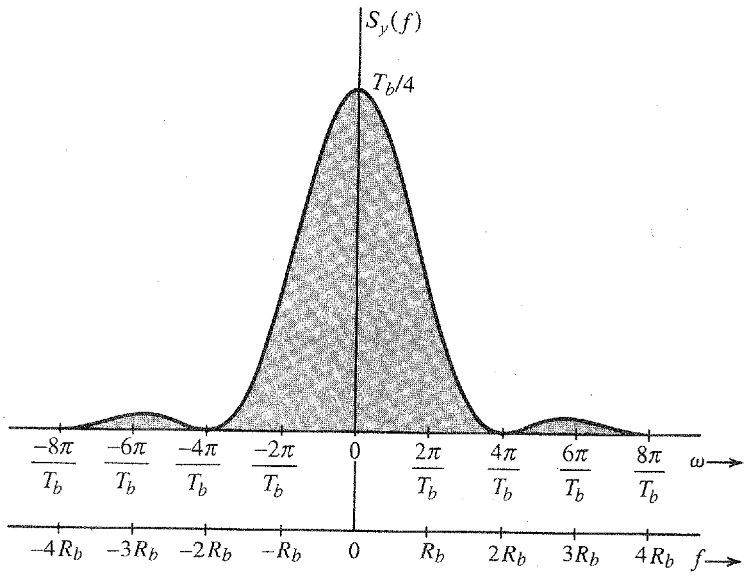

Figure 7.6 Power spectral density of a polar signal.

For a full-width pulse* (maximum possible pulse width), the essential bandwidth is half, that is R_b Hz. This is still twice the theoretical bandwidth. Thus, polar signaling is not the most bandwidth efficient.

Second, polar signaling has no capability for error detection or error correction. A third disadvantage of polar signaling is that it has nonzero PSD at dc ($f = 0$). This will rule out the use of ac coupling during transmission. The ac mode of coupling, which permits transformers and blocking capacitors to aid in impedance matching and bias removal, and allows dc powering of the line repeaters over the cable pairs, is very important in practice. Later, we shall show how a PSD of a line code may be forced to zero at dc by properly shaping $p(t)$.

On the positive side, polar signaling is the most efficient scheme from the power requirement viewpoint. For a given power, it can be shown that the error detection probability for a polar scheme is the lowest among all signaling techniques (see Chapter 10). Polar signaling is also transparent because there is always some pulse (positive or negative) regardless of the bit sequence. There is no discrete clock frequency component in the spectrum of the polar signal. Rectification of the *RZ* polar signal, however, yields a periodic signal of clock frequency and can readily be used to extract timing.

7.2.3 Constructing a DC Null in PSD by Pulse Shaping

Because $S_y(f)$, the PSD of a line code contains a factor $|P(f)|^2$, we can force the PSD to have a dc null by selecting a pulse $p(t)$ such that $P(f)$ is zero at dc ($f = 0$). Because

$$P(f) = \int_{-\infty}^{\infty} p(t)e^{-j2\pi ft}\, dt$$

---

* Scheme using the full-width pulse $p(t) = \Pi(t/T_b)$ is an example of a non-return-to-zero (NRZ) scheme. The half-width pulse scheme, on the other hand, is an example of a return-to-zero (RZ) scheme.

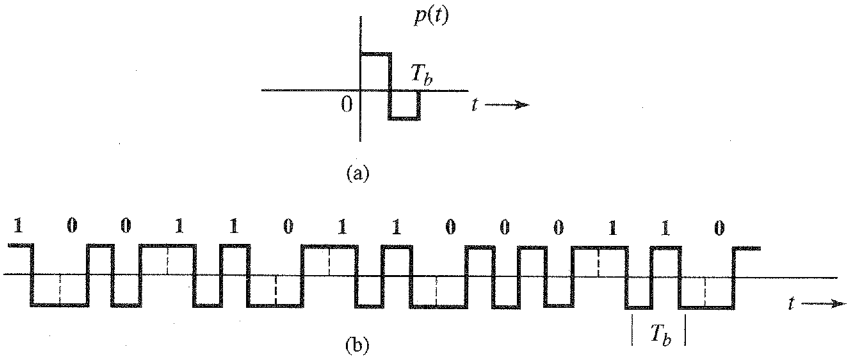

Figure 7.7 Split-phase (Manchester or twinned-binary) signal. (a) Basic pulse $p(t)$ for Manchester signaling. (b) Transmitted waveform for binary data sequence using Manchester signaling.

we have

$$P(0) = \int_{-\infty}^{\infty} p(t)\, dt$$

Hence, if the area under $p(t)$ is made zero, $P(0)$ is zero, and we have a dc null in the PSD. For a rectangular pulse, one possible shape of $p(t)$ to accomplish this is shown in Fig. 7.7a. When we use this pulse with polar line coding, the resulting signal is known as **Manchester code**, or **split-phase** (also called **twinned-binary**), signal. The reader can use Eq. (7.13), to show that for this pulse, the PSD of the Manchester line code has a dc null (see Prob. 7.2-2).

7.2.4 On-Off Signaling

In on-off signaling, a **1** is transmitted by a pulse $p(t)$ and a **0** is transmitted by no pulse. Hence, a pulse strength a_k is equally likely to be 1 or 0. Out of N pulses in the interval of T seconds, a_k is 1 for $N/2$ pulses and is 0 for the remaining $N/2$ pulses on the average. Hence,

$$R_0 = \lim_{N\to\infty} \frac{1}{N}\left[\frac{N}{2}(1)^2 + \frac{N}{2}(0)^2\right] = \frac{1}{2} \tag{7.16}$$

To compute R_n we need to consider the product $a_k a_{k+n}$. Since a_k and a_{k+n} are equally likely to be 1 or 0, the product $a_k a_{k+n}$ is equally likely to be 1×1, 1×0, 0×1 or 0×0, that is, 1, 0, 0, 0. Therefore on the average, the product $a_k a_{k+n}$ is equal to 1 for $N/4$ terms and 0 for $3N/4$ terms and

$$R_n = \lim_{N\to\infty} \frac{1}{N}\left[\frac{N}{4}(1) + \frac{3N}{4}(0)\right] = \frac{1}{4} \qquad n \geq 1 \tag{7.17}$$

Therefore, [Eq. (7.9)]

$$S_x(f) = \frac{1}{2T_b} + \frac{1}{4T_b}\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} e^{-jn2\pi f T_b} \tag{7.18a}$$

$$= \frac{1}{4T_b} + \frac{1}{4T_b}\sum_{n=-\infty}^{\infty} e^{-jn2\pi f T_b} \tag{7.18b}$$

Equation (7.18b) is obtained from Eq. (7.18a) by splitting the term $1/2T_b$ corresponding to R_0 into two: $1/4T_b$ outside the summation and $1/4T_b$ inside the summation (corresponding to $n = 0$). We now use the formula (see the footnote for a proof*)

$$\sum_{n=-\infty}^{\infty} e^{-jn2\pi f T_b} = \frac{1}{T_b} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_b}\right)$$

Substitution of this result in Eq. (7.18b) yields

$$S_x(f) = \frac{1}{4T_b} + \frac{1}{4T_b^2} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_b}\right) \tag{7.19a}$$

and the desired PSD of the on-off waveform $y(t)$ is [from Eq. (7.11a)]

$$S_y(f) = \frac{|P(f)|^2}{4T_b}\left[1 + \frac{1}{T_b} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_b}\right)\right] \tag{7.19b}$$

Note that unlike the continuous PSD spectrum of polar signaling, the on-off PSD of Eq. (7.19b) also has an additional discrete part. This discrete part may be nullified if the pulse shape is chosen such that

$$P\left(\frac{n}{T_b}\right) = 0 \qquad n = 0, \pm 1, \ldots$$

For the example case of a half-width rectangular pulse [see Eq. (7.14)],

$$S_y(f) = \frac{T_b}{16} \operatorname{sinc}^2\left(\frac{\pi f T_b}{2}\right)\left[1 + \frac{1}{T_b} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_b}\right)\right] \tag{7.20}$$

The resulting PSD is shown in Fig. 7.8. The continuous component of the spectrum is $(T_b/16)\operatorname{sinc}^2(\pi f T_b/2)$. This is identical (except for a scaling factor) to the spectrum of the polar signal [Eq. (7.15)]. The discrete component is represented by the product of an impulse train with the continuous component $(T_b/16)\operatorname{sinc}^2(\pi f T_b/2)$. Hence this component appears as periodic impulses with the continuous component as the envelope. Moreover, the impulses repeat at the clock frequency $R_b = 1/T_b$ because its fundamental frequency is $2\pi/T_b$ rad/s, or $1/T_b$ Hz. This is a logical result because as Fig. 7.3 shows, an on-off signal can be expressed as a sum of a polar and a periodic component. The polar component $y_1(t)$ is exactly half

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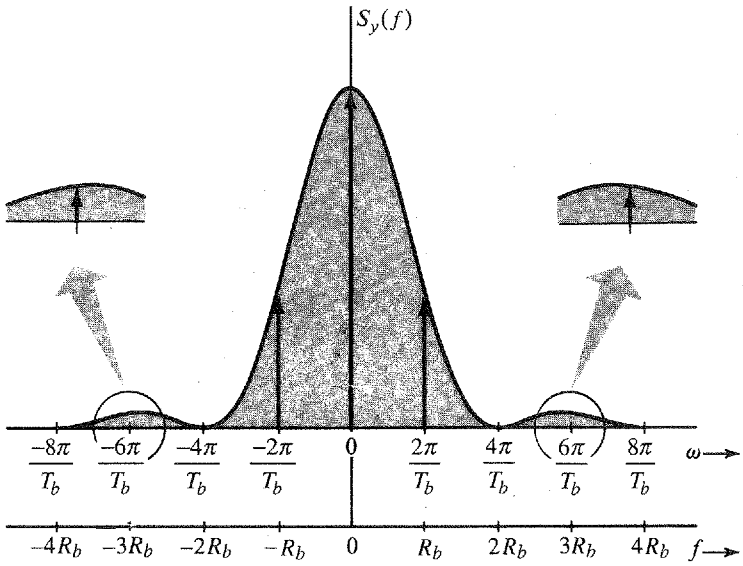
* The impulse train in Fig. 3.23a of Example 3.11 is $\delta_{T_b}(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_b)$. Moreover, the Fourier series for this impulse train as found in Eq. (2.67) is

$$\sum_{n=-\infty}^{\infty} \delta(t - nT_b) = \frac{1}{T_b} \sum_{n=-\infty}^{\infty} e^{jn2\pi R_b t} \qquad R_b = \frac{1}{T_b}$$

We take the Fourier transform of both sides of this equation, and use the fact that $\delta(t - nT_b) \Longleftrightarrow e^{-jn2\pi f T_b}$ and $e^{jn2\pi R_b t} \Longleftrightarrow \delta(f - nR_b)$. This yields

$$\sum_{n=-\infty}^{\infty} e^{-jn2\pi f T_b} = \frac{1}{T_b} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_b}\right)$$

Figure 7.8 Power spectral density (PSD) of an on-off signal.



the polar signal discussed earlier. Hence, the PSD of this component is one-fourth the PSD in Eq. (7.15). The periodic component is of clock frequency R_b; it consists of discrete components of frequency R_b and its harmonics.

On-off signaling has very little to brag about. For a given transmitted power, it is less immune to noise interference than the polar scheme, which uses a positive pulse for **1** and a negative pulse for **0**. This is because the noise immunity depends on the difference of amplitudes representing **1** and **0**. Hence, for the same immunity, if on-off signaling uses pulses of amplitudes 2 and 0, polar signaling need use only pulses of amplitudes 1 and -1. It is simple to show that on-off signaling requires twice as much power as polar signaling. If a pulse of amplitude 1 or -1 has energy E, then the pulse of amplitude 2 has energy $(2)^2E = 4E$. Because $1/T_b$ digits are transmitted per second, polar signal power is $(E)(1/T_b) = E/T_b$. For the on-off case, on the other hand, each pulse energy is $4E$, though on average such a pulse is transmitted over half of the time while nothing is transmitted over the other half. Hence, the average signal power of on-off is

$$\frac{1}{T_b}\left(4E\frac{1}{2} + 0 \cdot \frac{1}{2}\right) = \frac{2E}{T_b}$$

which is twice that required for the polar signal. Moreover, unlike the polar case, on-off signaling is not transparent. A long string of **0**s (or offs) causes the absence of a signal and can lead to errors in timing extraction. In addition, all the disadvantages of polar signaling, (e.g., excessive transmission bandwidth, nonzero power spectrum at dc, no error detection (or correction) capability are also present in on-off signaling.

7.2.5 Bipolar Signaling

The signaling scheme used in PCM for telephone networks is called bipolar (pseudoternary or alternate mark inverted). A **0** is transmitted by no pulse, and a **1** is transmitted by a pulse

$p(t)$ or $-p(t)$, depending on whether the previous **1** was transmitted by $-p(t)$ or $p(t)$. With consecutive pulses alternating, we can avoid dc wander and thus cause a dc null in the PSD. Bipolar signaling actually uses three symbols [$p(t)$, 0, and $-p(t)$], and, hence, it is in reality ternary rather than binary signaling.

To calculate the PSD, we have

$$R_o = \lim_{N\to\infty} \frac{1}{N} \sum_k a_k^2$$

On the average, half of the a_ks are 0, and the remaining half are either 1 or −1, with $a_k^2 = 1$. Therefore,

$$R_o = \lim_{N\to\infty} \frac{1}{N} \left[\frac{N}{2}(\pm 1)^2 + \frac{N}{2}(0)^2\right] = \frac{1}{2}$$

To compute R_1, we consider the pulse strength product $a_k a_{k+1}$. There are four equally likely sequences of two bits: **11, 10, 01, 00**. Since bit **0** is encoded by no pulse ($a_k = 0$), the product $a_k a_{k+1}$ is zero for the last three of these sequences. This means, on the average, that $3N/4$ combinations have $a_k a_{k+1} = 0$ and only $N/4$ combinations have nonzero $a_k a_{k+1}$. Because of the bipolar rule, the bit sequence **11** can be encoded only by two consecutive pulses of opposite polarities. This means the product $a_k a_{k+1} = -1$ for the $N/4$ combinations. Therefore

$$R_1 = \lim_{N\to\infty} \frac{1}{N} \left[\frac{N}{4}(-1) + \frac{3N}{4}(0)\right] = -\frac{1}{4}$$

To compute R_2 in a similar way, we need to observe the product $a_k a_{k+2}$. For this, we need to consider all possible combinations of three bits in sequence. There are eight equally likely combinations: **111, 101, 110, 100, 011, 010, 001, 000**. The last six combinations have either the first and/or the last bit **0**. Hence $a_k a_{k+2} = 0$ for all these six combinations. The first two combinations are the only ones that yield nonzero $a_k a_{k+2}$. From the bipolar rule, the first and the third pulses in the combination **111** are of the same polarity, yielding $a_k a_{k+2} = 1$. But for **101**, the first and the third pulse are of opposite polarity, yielding $a_k a_{k+2} = -1$. Thus, on the average, $a_k a_{k+2} = 1$ for $N/8$ terms, −1 for $N/8$ terms and 0 for $3N/4$ terms. Hence,

$$R_2 = \lim_{N\to\infty} \frac{1}{N} \left[\frac{N}{8}(1) + \frac{N}{8}(-1) + \frac{3N}{8}(0)\right] = 0$$

In general

$$R_n = \lim_{N\to\infty} \frac{1}{N} \sum_k a_k a_{k+n}$$

For $n > 2$, the product $a_k a_{k+n}$ can be 1, −1, or 0. Moreover, an equal number of combinations have values 1 and −1. This causes $R_n = 0$. Thus

$$R_n = 0 \qquad n > 1$$

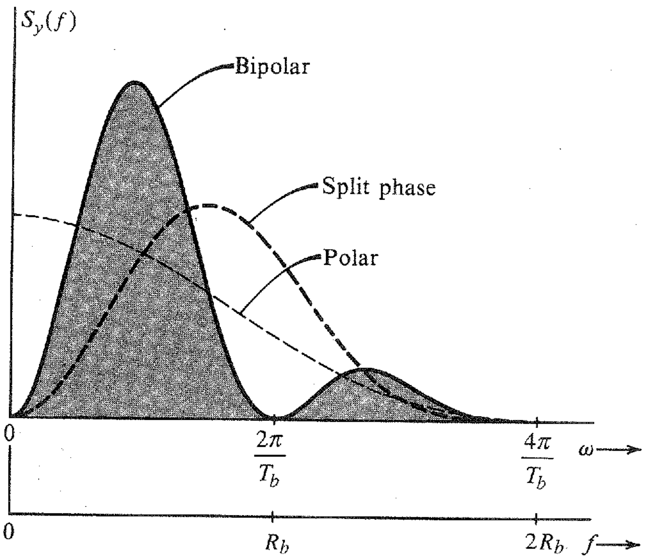


Figure 7.9 PSD of bipolar, polar, and split-phase signals normalized for equal powers. Half-width rectangular pulses are used.

and [see Eq. (7.11c)]

$$S_y(f) = \frac{|P(f)|^2}{2T_b}[1 - \cos 2\pi f T_b] \tag{7.21a}$$

$$= \frac{|P(f)|^2}{T_b} \sin^2 (\pi f T_b) \tag{7.21b}$$

Note that $S_y(f) = 0$ for $f = 0$ (dc), regardless of $P(f)$. Hence, the PSD has a dc null, which is desirable for ac coupling. Moreover, $\sin^2(\pi f T_b) = 0$ at $f = 1/T_b$, that is, at $f = 1/T_b = R_b$ Hz. Thus, regardless of $P(f)$, we are assured of the first non-dc null bandwidth R_b Hz. For the half-width pulse

$$S_y(f) = \frac{T_b}{4} \operatorname{sinc}^2\left(\frac{\pi f T_b}{2}\right) \sin^2(\pi f T_b) \tag{7.22}$$

This is shown in Fig. 7.9. The essential bandwidth of the signal is R_b ($R_b = 1/T_b$), which is half that of polar using the same half-width pulse or on-off signaling and twice the theoretical minimum bandwidth. Observe that we were able to obtain the bandwidth R_b for polar (or on-off) case for full-width pulse. For the bipolar case, the bandwidth is R_b Hz whether the pulse is half-width or full-width.

Bipolar signaling has several advantages: (1) its spectrum has a dc null; (2) its bandwidth is not excessive; (3) it has single-error-detection capability. This is because even single detection error will cause a violation of the alternating pulse rule, and this will be immediately detected. If a bipolar signal is rectified, we get an on-off signal that has a discrete component at the clock frequency. Among the disadvantages of a bipolar signal is the requirement for twice as much power (3 dB) as a polar signal needs. This is because bipolar detection is essentially equivalent to on-off signaling from the detection point of view. One distinguishes between $+p(t)$ or $-p(t)$ from 0 rather than between $\pm p(t)$.

Another disadvantage of bipolar signaling is that it is not transparent. In practice, various substitution schemes are used to prevent long strings of logic zeros from allowing the extracted clock signals to drift away. We shall now discuss two such schemes.

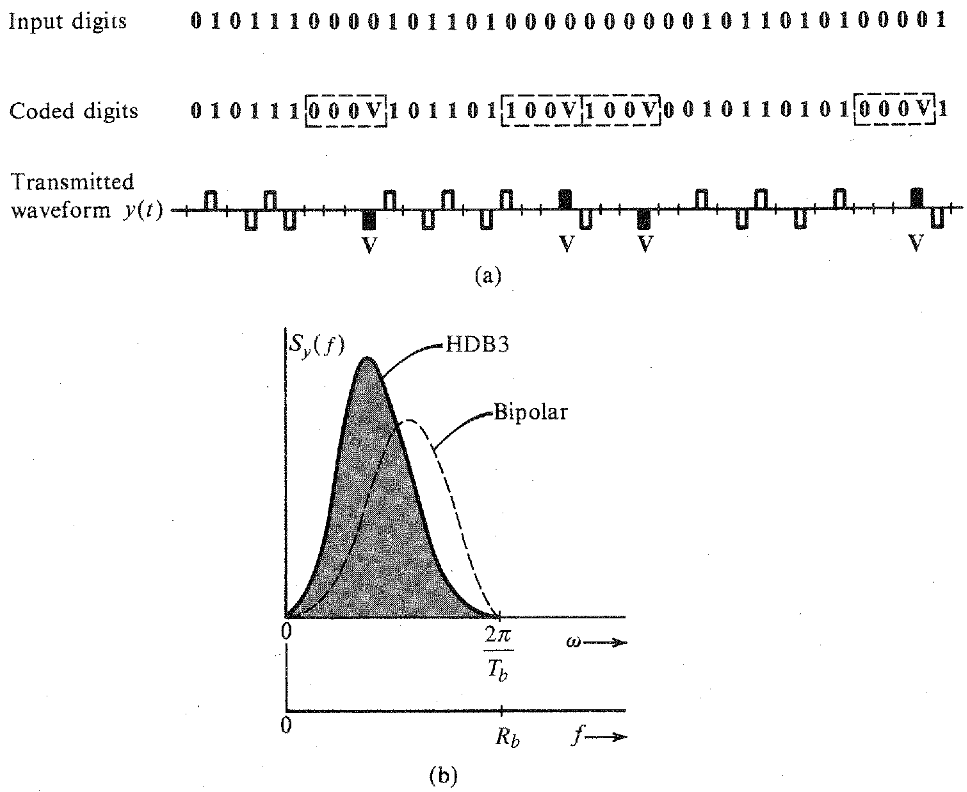


Figure 7.10 (a) HDB3 signal and (b) its PSD.

High-Density Bipolar (HDB) Signaling

The HDB scheme is an ITU (formerly CCITT) standard. In this scheme the problem of nontransparency in bipolar signaling is eliminated by adding pulses when the number of consecutive **0**s exceeds N. Such a modified coding is designated as **high-density bipolar coding** (HDBN), where N can take on any value $1, 2, 3, \ldots$. The most important of the HDB codes is HDB3 format, which has been adopted as an international standard.

The basic idea of the HDBN code is that when a run of $N + 1$ zeros occurs, this group of zeros is replaced by one of the special $N + 1$ binary digit sequences. To increase the timing content of the signal, the sequences are chosen to include some binary **1**s. The **1**s included deliberately violate the bipolar rule for easy identification of the substituted sequence. In HDB3 coding, for example, the special sequences used are **000V** and **B00V** where **B=1** that conforms to the bipolar rule and **V=1** that violates the bipolar rule. The choice of sequence **000V** or **B00V** is made in such a way that consecutive **V** pulses alternate signs to avoid dc wander and to maintain the dc null in the PSD. This requires that the sequence **B00V** be used when there are an even number of **1**s following the last special sequence and the sequence **000V** be used when there are an odd number of **1**s following the last sequence. Figure 7.10a shows an example of this coding. Note that in the sequence **B00V**, both **B** and **V** are encoded by the same pulse. The decoder has to check two things—the bipolar violations and the number of **0**s preceding each violation to determine if the previous **1** is also a substitution.

Despite deliberate bipolar violations, HDB signaling retains error detecting capability. Any single error will insert a spurious bipolar violation (or will delete one of the deliberate violations). This will become apparent when, at the next violation, the alternation of violations does not appear. This also shows that deliberate violations can be detected despite single errors. Figure 7.10b shows the PSD of HDB3 as well as that of a bipolar signal to facilitate comparison.[3]