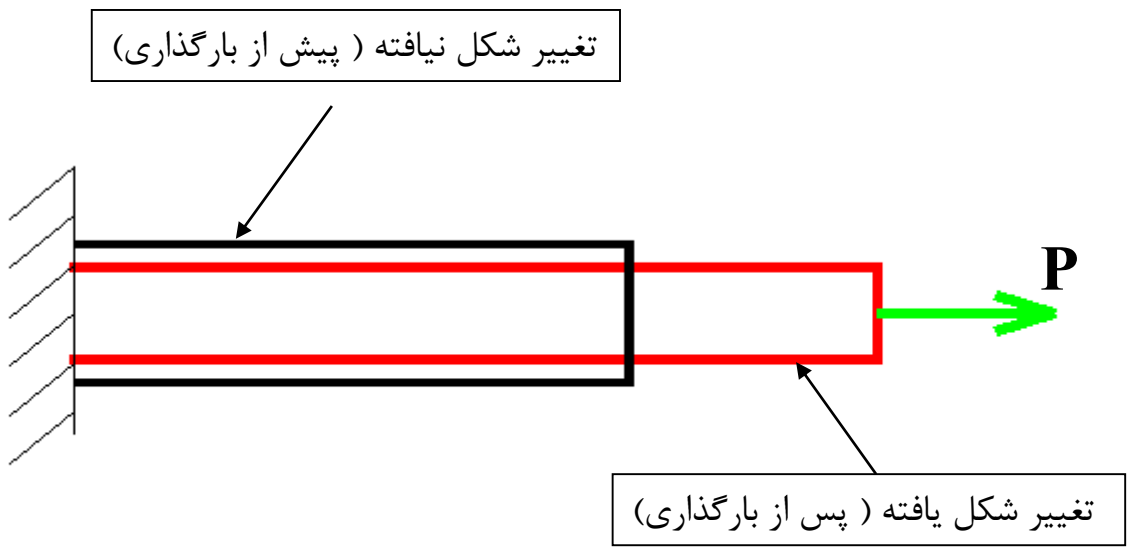
فصل دوم – کرنش

کرنش نرمال – افزایش و یا کاهش طول یک عضو خطی در واحد طول

یک سازه در اثر بارگذاری، تنش و کرنش را تجربه میکند.

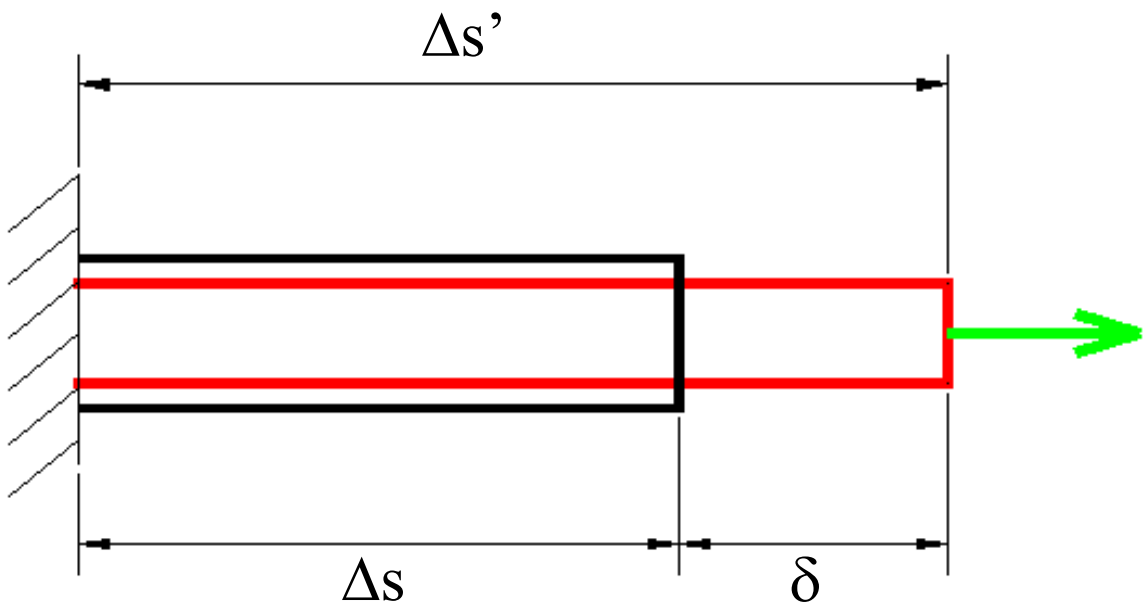
مثال:



پیش از بارگذاری

پس از بارگذاری

تعریف کرنش نرمال:



Δs = طول اولیه

Δs’ = طول نهایی

δ = تغییر شکل = تغییر طول

$\delta = \Delta s' - \Delta s$

$\text{کرنش نرمال} = \varepsilon = \delta/\Delta s = (\Delta s' - \Delta s)/\Delta s$

واحدها = in/in or mm/mm or M/M, etc….

تعریف کرنش نرمال:

کرنش نرمال= $\varepsilon = \delta/\Delta s = (\Delta s' - \Delta s)/\Delta s$

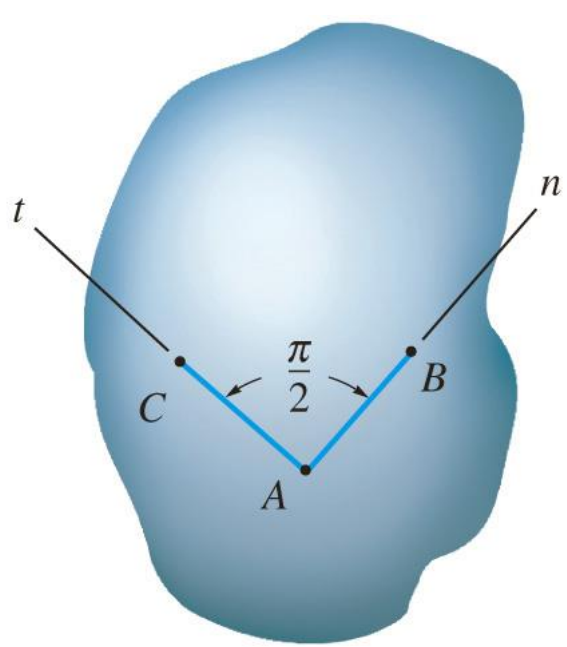
مثال: طول یک شافت با طول اولیه 60.000” پس از بارگذاری به 60.005” می رسد. میزان تغییر شکل و کرنش را تعیین کنید.

جواب:

$\delta = 60.005” - 60.000” = 0.005”$

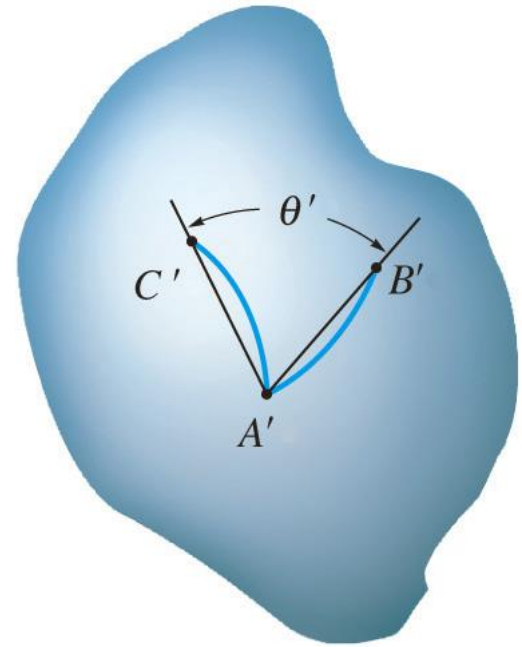
$\varepsilon = 0.005”/60.000 = 83.33 \text{ E-6 in/in} = 83.33 \ \mu\text{in/in}$

تعریف کرنش برشی:



Undeformed body

(a)



Deformed body

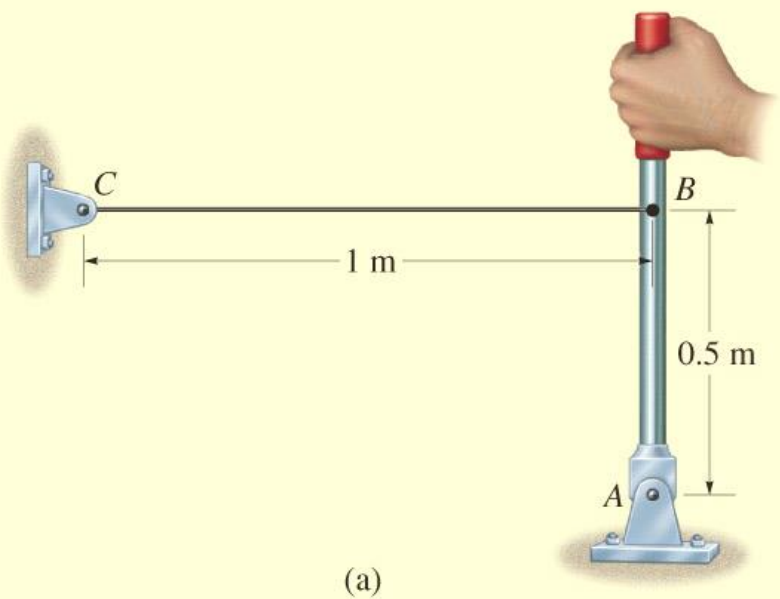
(b)

کرنش برشی= $\gamma = \pi/2 - \theta'$

واحد = rad

EXAMPLE 2.2

A force acting on the grip of the lever arm shown in Fig. 2–5*a* causes the arm to rotate clockwise through an angle of $\theta = 0.002$ rad. Determine the average normal strain developed in the wire BC.



(a)

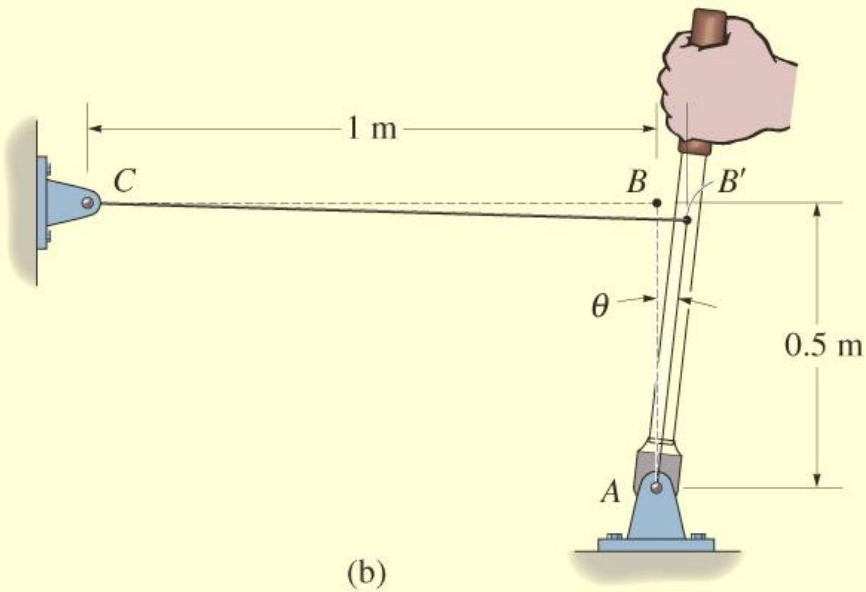


Fig. 2–5

Solution

Since $\theta = 0.002$ rad is small, the stretch in the wire CB, Fig. 2–5*b*, is $BB' = \theta(0.5\text{ m}) = (0.002\text{ rad})(0.5\text{ m}) = 0.001$ m. The average normal strain in the wire is therefore,

$$\epsilon_{\text{avg}} = \frac{BB'}{CB} = \frac{0.001}{1\text{ m}} = 0.001\text{ m/m} \qquad \textit{Ans.}$$

EXAMPLE 2.3

The plate is deformed into the dashed shape shown in Fig. 2–6*a*. If in this deformed shape horizontal lines on the plate remain horizontal and do not change their length, determine (a) the average normal strain along the side AB, and (b) the average shear strain in the plate relative to the x and y axes.

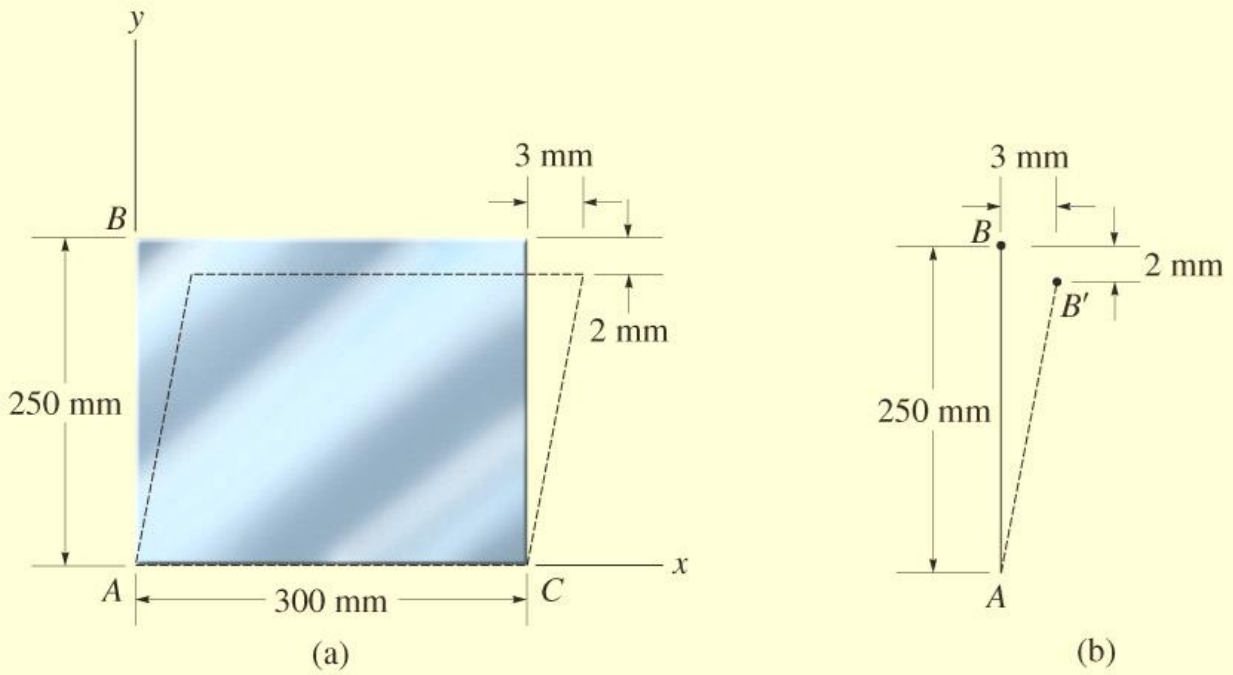
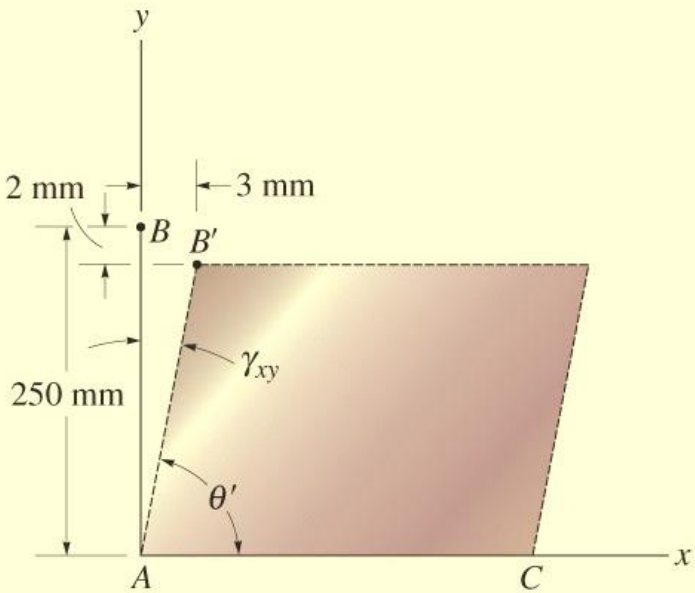


Fig. 2–6



(c)

Solution

Part (a). Line AB, coincident with the y axis, becomes line AB' after deformation, as shown in Fig. 2–6*b*. The length of this line is

$$AB' = \sqrt{(250 - 2)^2 + (3)^2} = 248.018 \text{ mm}$$

The average normal strain for AB is therefore

$$(\epsilon_{AB})_{\text{avg}} = \frac{AB' - AB}{AB} = \frac{248.018 \text{ mm} - 250 \text{ mm}}{250 \text{ mm}}$$

$$= -7.93(10^{-3}) \text{ mm/mm} \qquad \textit{Ans.}$$

The negative sign indicates the strain causes a contraction of AB.

Part (b). As noted in Fig. 2–6*c*, the once 90° angle BAC between the sides of the plate, referenced from the x, y axes, changes to θ' due to the displacement of B to B'. Since $\gamma_{xy} = \pi/2 - \theta'$, then γ_{xy} is the angle shown in the figure. Thus,

$$\gamma_{xy} = \tan^{-1}\left(\frac{3 \text{ mm}}{250 \text{ mm} - 2 \text{ mm}}\right) = 0.0121 \text{ rad} \qquad \textit{Ans.}$$