MATH3305 — Problem Sheet 1

Problems 1 and 2 to be handed in **at the lecture** on Friday, 13 October 2017

1. You are given Euclidean 3-space with standard Cartesian coordinates $X^i = \{x, y, z\}$. Now introduce spherical polar coordinates $Y^i = \{r, \theta, \phi\}$ satisfying

$$
\begin{aligned}
x &= r \sin\theta \cos\phi \\
y &= r \sin\theta \sin\phi \\
z &= r \cos\theta.
\end{aligned}
$$

   (i) Find the 3×3 matrix of first derivatives $J^i{}_j = \partial X^i / \partial Y^j$.

   (ii) Show that this matrix is invertible by computing the determinant.

   (iii) For which angle θ is $J^i{}_j$ not invertible?

   (iv) You are given the vector $V^i_{(Y)} = (1, 0, 0)$ in spherical polar coordinates. Find the components of $V^i_{(X)}$ in Cartesian coordinates.

2. You are given Euclidean 3-space $X^i = \{x, y, z\}$ with cylindrical coordinates $Y^i = \{\rho, \varphi, z\}$ such that

$$
\begin{aligned}
x &= \rho \cos\varphi \\
y &= \rho \sin\varphi \\
z &= z.
\end{aligned}
$$

   (i) Find dx, dy, dz in terms of $d\rho, d\varphi, dz$.

   (ii) Show that for a smooth function f we can write

$$
\begin{pmatrix} \frac{\partial f}{\partial r} \\ \frac{\partial f}{\partial \varphi} \\ \frac{\partial f}{\partial z} \end{pmatrix} = \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -r\sin\varphi & r\cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{pmatrix}
$$

   (iii) Express the basis vectors $\boldsymbol{e}_{(Y)i}$ in terms of the basis vectors $\boldsymbol{e}_{(X)i}$