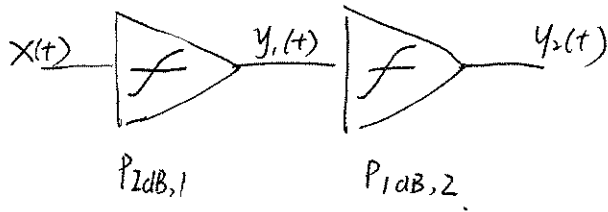


2-1 Solu:



$$y_1(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$$

$$y_2(t) = \beta_1 y_1(t) + \beta_2 y_1^2(t) + \beta_3 y_1^3(t)$$

then.

$$y_2(t) = \beta_1 [\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)] + \beta_2 [\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)]^2 + \beta_3 [\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)]^3$$

Considering only the first - and third - order terms,

$$y_2(t) = \alpha_1 \beta_1 x(t) + (\alpha_3 \beta_1 + 2\alpha_1 \alpha_2 \beta_2 + \alpha_1^3 \beta_3) x^3(t) + \dots$$

$$= [\alpha_1 \beta_1 + \frac{3}{4} (\alpha_3 \beta_1 + 2\alpha_1 \alpha_2 \beta_2 + \alpha_1^3 \beta_3) A_{in,1dB}^2] x(t) + \dots$$

$$P_{1dB,1} : A_{in,1dB} = \sqrt{0.145 \left| \frac{\alpha_1}{\alpha_3} \right|} ; P_{1dB,2} : A_{in,2,1dB} = \sqrt{0.145 \left| \frac{\beta_1}{\beta_3} \right|}$$

$$P_{1dB} \Rightarrow 20 \log \left| \alpha_1 \beta_1 + \frac{3}{4} (\alpha_3 \beta_1 + 2\alpha_1 \alpha_2 \beta_2 + \alpha_1^3 \beta_3) A_{in,1dB}^2 \right| = 20 \log |\alpha_1 \beta_1| - 1dB$$

$$A_{in,1dB} = \sqrt{0.145 \left| \frac{\alpha_1 \beta_1}{\alpha_3 \beta_1 + 2\alpha_1 \alpha_2 \beta_2 + \alpha_1^3 \beta_3} \right|}$$

Represented by the  $P_{1dB}$  of first and second stage.

$$\frac{1}{A_{in,1dB}^2} = \frac{1}{0.145} \left| \frac{\alpha_3}{\alpha_1} + \frac{2\alpha_2 \beta_2}{\beta_1} + \frac{\alpha_1^2 \beta_3}{\beta_1} \right|$$

$$= \left| \frac{1}{A_{in,1dB}^2} + \frac{2}{0.145} \frac{\alpha_2 \beta_2}{\beta_1} + \frac{\alpha_1^2}{A_{in,2,1dB}^2} \right|$$

2.2 solve:

assuming  $-3 \text{ dBm}$   $A_1$  at  $2.42 \text{ GHz}$   
 $-35 \text{ dBm}$   $A_2$  at  $2.43 \text{ GHz}$ .

$$\text{IM product} : \frac{3}{4} \delta_3 A_1^2 A_2$$

$$-3 \text{ dBm} \Rightarrow A_1 = \sqrt{2.50 \cdot 10^{-0.3} \times 10^{-3}} = 223.9 \text{ mV}$$

$$-35 \text{ dBm} \Rightarrow A_2 = \sqrt{2.50 \times 10^{-3.5} \times 10^{-3}} = 5.6 \text{ mV}$$

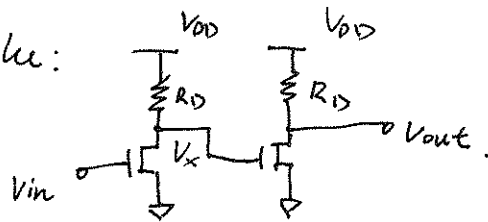
We can write at LNA output:

$$20 \lg |\delta_1 \cdot A_{\text{sig}}| - 20 \text{ dB} = 20 \lg \left| \frac{3}{4} \delta_3 A_1^2 A_2 \right|$$

$$\Rightarrow \lg |\delta_1 \cdot A_{\text{sig}}| = \lg \left| \frac{30}{4} \delta_3 A_1^2 A_2 \right|$$

$$\text{IIP}_3 = \sqrt{\frac{4}{3} \left| \frac{\delta_1}{\delta_3} \right|} = \sqrt{\frac{4}{3} \cdot \frac{30}{4} \cdot \frac{A_1^2 \cdot A_2}{A_{\text{sig}}}} = 9.43 \text{ V}_p$$
$$= 29.5 \text{ dBm}$$

f 2.3 solve:



$$I_D = K \cdot (V_{GS} - V_T)^2$$

$$V_{out} = V_{DD} - K \cdot R_D (V_x - V_T)^2$$

$$V_x = V_{DD} - K \cdot R_D (V_{in} - V_T)^2$$

$$V_{out} = V_{DD} - K \cdot R_D [V_{DD} - K \cdot R_D (V_{in} - V_T)^2 - V_T]^2$$

$$= V_{DD} - K \cdot R_D [(V_{DD} - V_T) - K \cdot R_D (V_{in} - V_T)^2]^2$$

$$= V_{DD} - K \cdot R_D [(V_{DD} - V_T)^2 + K^2 R_D^2 (V_{in} - V_T)^4 - 2 K R_D (V_{DD} - V_T) (V_{in} - V_T)^2]$$

1st order of  $V_{in}$

$$\Rightarrow [4 \cdot (V_{DD} - V_T) \cdot K \cdot R_D V_T - 4 K^2 R_D^2 \cdot V_T^3] \cdot V_{in}$$

3rd order of  $V_{in}$

$$\Rightarrow [-4 K^2 R_D^2 V_T] \cdot V_{in}^3$$

$$A_{IP3} = \sqrt{\frac{4}{3} \cdot \frac{4 \cdot (V_{DD} - V_T) K \cdot R_D V_T - 4 K^2 R_D^2 V_T^3}{4 K^2 R_D^2 V_T}}$$

2.4. Solu.

$$y(t) = \partial_1 X(t) + \partial_2 X^2(t) + \partial_3 X^3(t) + \partial_4 X^4(t) + \partial_5 X^5(t)$$

$$1^\circ \cos^3 \omega t = \frac{3}{4} \cos \omega t + \frac{1}{4} \cos 3\omega t$$

$$2^\circ \cos^2 \omega t = \frac{1 + \cos 2\omega t}{2}$$

$$3^\circ \cos^4 \omega t = \frac{1 + \cos^2 2\omega t + 2\cos 2\omega t}{2^2}$$

$$4^\circ \cos^5 \omega t = \left( \frac{3}{4} \cos \omega t + \frac{1}{4} \cos 3\omega t \right) \cdot \left( \frac{1 + \cos 2\omega t}{2} \right)$$

$$= \left( \frac{3}{8} \cos \omega t + \frac{1}{8} \cos 3\omega t + \frac{3}{8} \cos \omega t \cdot \cos 2\omega t + \frac{1}{8} \cos 3\omega t \cdot \cos 2\omega t \right)$$

$$= \frac{3}{16} [\cos \omega t + \cos 3\omega t] + \frac{1}{16} [\cos \omega t + \cos 5\omega t]$$

1st order  $\Rightarrow \partial_1 A + \frac{3}{4} \partial_3 A^3 + \left( \frac{3}{8} + \frac{3}{16} + \frac{1}{16} \right) \partial_5 A^5$

3rd order  $\Rightarrow \frac{1}{4} \partial_3 A^3 + \left( \frac{1}{8} + \frac{3}{16} \right) \partial_5 A^5$

(1) PIdB  $\Rightarrow 20 \lg |\partial_1 + \frac{3}{4} \partial_3 A^2 + \frac{5}{8} \partial_5 A^4| = 20 \lg |\partial_1| - 1 \text{ dB}$

$$\Rightarrow A_{in, 1 \text{ dB}} = \sqrt{\frac{0.8 \cdot (0.5625 \partial_3^2 - 0.27175 \partial_1 \cdot \partial_5) \pm 0.6 \partial_3}{\partial_5}}$$

(2) IIP3 doesn't change.

$$A_{IIP3} = \sqrt{\frac{4}{3} \left| \frac{\partial_1}{\partial_3} \right|}$$

2.5 (a) Solu:  $A_{sig} = \frac{0.1mV}{\times 0.7943} \Leftarrow -2dB = 10^{-0.1} = 0.7943$

$A_2 = 10mV \times 0.1413 \Leftarrow -17dB = 10^{-0.85} = 0.1413$

$A_3 = 10mV \times 0.0141 \Leftarrow -37dB = 10^{-1.85} = 0.0141$

at the output of amplifier:

$$20 \lg |\sigma_1 \cdot A_{sig}| - 20dB = 20 \lg \left| \frac{3}{4} \sigma_3 \cdot A_2^3 \cdot A_3 \right|$$

$$\Rightarrow |\sigma_1 \cdot 0.07943m| = \left| \frac{3}{4} \sigma_3 \cdot 1.413m^2 \cdot 0.141m \right|$$

$$\therefore A_{IIP3} = \sqrt{\frac{\frac{3}{4} \cdot 1.413m^2 \cdot 0.141}{0.07943} \cdot \frac{4}{3}} = 5.95mV_p = -34.5dBm$$

Neglect the nonlinearity of BPF.

(b).  $y_1(t) = \sigma_1 x(t) + \sigma_3 x^3(t)$

$y_2(t) = \beta_1 y_1(t) + \beta_2 y_1^3(t)$

Only considering the first and third order:

$$y_2(t) = \sigma_1 \beta_1 x(t) + (\sigma_3 \beta_1 + \sigma_1^3 \beta_3) x^3(t) + \dots$$

$$\therefore A_{zP3} = \sqrt{\frac{4}{3} \left| \frac{\sigma_1 \beta_1}{\sigma_3 \beta_1 + \sigma_1^3 \beta_3} \right|}$$

$$\frac{1}{A_{zP3}^2} = \frac{1}{A_{zP3,1}^2} + \frac{\sigma_1^2}{A_{zP3,2}^2}$$

$$\Rightarrow \frac{1}{A_{zP3}^2} = \frac{1}{500m^2} + \frac{10}{5.95m^2}$$

$$\Rightarrow A_{zP3} = 1.875mV_p$$

2.6 Solu:

Let  $x(t)$  be a random signal (wide-sense stationary process)

Auto correlation function:  $R_x(\tau) = E[x(t) \cdot x(t+\tau)]$

Let me prove that:  $S_x(f) = \int_{-\infty}^{\infty} R_x(\tau) e^{-j2\pi f\tau} d\tau$

$$\text{Proof: } X_T(f) \triangleq \int_{-T/2}^{T/2} x(t) e^{-j2\pi ft} dt$$

$$S_T(f) \triangleq E\left[\frac{1}{T} |X_T(f)|^2\right]$$

$$S_x(f) = \lim_{T \rightarrow \infty} S_T(f)$$

$$E[|X_T(f)|^2] = E\left[\int_{-T/2}^{T/2} x(t) e^{-j2\pi ft} dt\right]^2$$

$$= E\left[\int_{-T/2}^{T/2} x(t) e^{-j2\pi ft} dt \cdot \int_{-T/2}^{T/2} x(\tau) e^{-j2\pi f\tau} d\tau\right]$$

$$= E\left[\int_{-T/2}^{T/2} \int_{-T/2}^{T/2} x(t)x(\tau) e^{-j2\pi f(t-\tau)} dt d\tau\right]$$

$$= \int_{-T/2}^{T/2} \int_{-T/2}^{T/2} E[x(t)x(\tau)] e^{-j2\pi f(t-\tau)} dt d\tau$$

$$= \int_{-T/2}^{T/2} \int_{-T/2}^{T/2} R_x(t-\tau) e^{-j2\pi f(t-\tau)} dt d\tau$$

$$= \int_{-T}^T (T-|\tau|) R_x(\tau) e^{-j2\pi f\tau} d\tau$$

$$E\left[\frac{1}{T} |X_T(f)|^2\right] = \int_{-T}^T \left(1 - \frac{|\tau|}{T}\right) R_x(\tau) e^{-j2\pi f\tau} d\tau$$

Therefore:  $S_x(f) = \lim_{T \rightarrow \infty} E\left[\frac{1}{T} |X_T(f)|^2\right] = \int_{-\infty}^{\infty} R_x(\tau) e^{-j2\pi f\tau} d\tau$

2.7. Solu:

$$\text{Assume } y(t) = \delta_1 x(t) + \delta_2 x^2(t) + \delta_3 x^3(t).$$

$$x(t) = V_0 \cos \omega_0 t.$$

$$\text{3rd - harmonic : } \frac{\delta_3 V_0^3}{4} = V_3$$

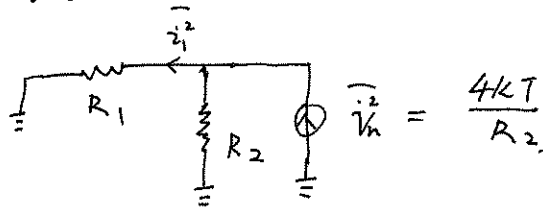
$$\Rightarrow \delta_3 = \frac{4V_3}{V_0^3}$$

$$A_{in, 1dB} = \sqrt{0.145 \left| \frac{\delta_1}{\delta_3} \right|}$$

$$= \sqrt{0.145 \cdot \left| \frac{\delta_1}{4V_3/V_0^3} \right|}$$

$$= \sqrt{\frac{0.145}{4} \left| \frac{\delta_1 V_0^3}{V_3} \right|}$$

12.8 soln:



$$\begin{aligned} P_{R_1} &= \overline{i_1^2} \cdot R_1 \\ &= \left( \sqrt{\frac{4kT}{R_2}} \cdot \frac{R_2}{R_1 + R_2} \right)^2 \cdot R_1 \\ &= \frac{4kT}{(R_1 + R_2)^2} \cdot R_1 R_2 \end{aligned}$$

$$\therefore P_{R_1} = P_{R_2}$$

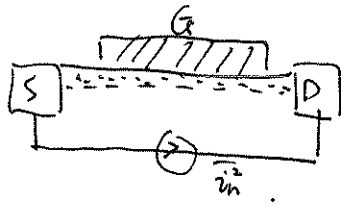
So it proves that the noise power delivered by  $R_1$  to  $R_2$  is equal to that delivered by  $R_2$  to  $R_2$  at the same temperature.

$\therefore$  If it is not the truth, the energy would not be conserved.



2.9 Solu:

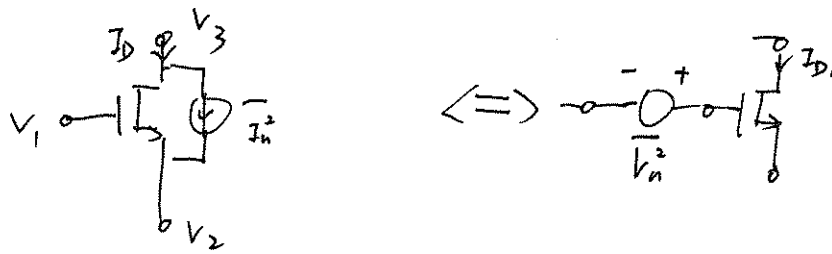
Why the channel thermal noise of a MOSFET is model by a current source bw. S & D, rather than G & D.



Firstly, from the figure we can find the channel resistor is between source & drain. As a result, it is reasonable to model the noise by a current source between source and drain.

Secondly, MOSFET has the function of transconductance. It's easy to transfer the current source from between S & D to the voltage source at the gate.

2.10 Solu:



Proof: transconductance :  $g_m$ .

Assume the transistor is in saturation region.

For small-signal analysis,  $V_1 = 0$

$$I_D = \sqrt{\overline{I_n^2}} = \sqrt{4KT\gamma g_m}$$

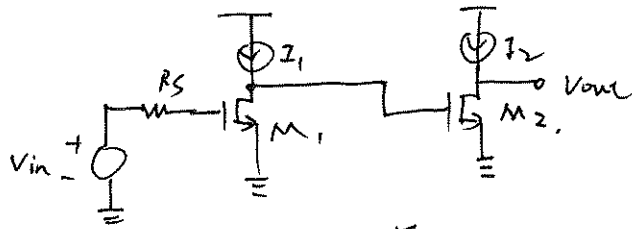
At the same time, for voltage source model.

$$I_D = V_{in} \cdot g_m = \sqrt{4KT\gamma \cdot g_m}$$

$$\Rightarrow V_{in} = \sqrt{\frac{4KT\gamma}{g_m}}$$

$$\Rightarrow \overline{V_n^2} = \frac{4KT\gamma}{g_m}$$

2.11 Solu:



$$NF_1 = 1 + \frac{r}{g_{m1}R_S} \quad (2.122)$$

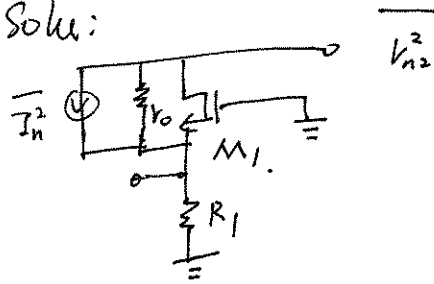
$$NF_2 = 1 + \frac{r}{g_{m2}r_{o1}} ; \quad A_{p1} = \frac{P_{out,av,1}}{P_{in,av,1}} = \frac{V_{in}^2 \cdot A_{v1}^2 \cdot \frac{1}{4r_{o1}}}{V_{in}^2 \cdot \frac{1}{4R_S}}$$

$$\therefore NF_{tot} = 1 + (NF_1 - 1) + \frac{NF_2 - 1}{A_{p1}} = g_{m1}^2 \cdot r_{o1} \cdot R_S$$

$$= 1 + \frac{r}{g_{m1}R_S} + \frac{r}{g_{m2} \cdot r_{o1}} \Big/ g_{m1}^2 r_{o1} R_S$$

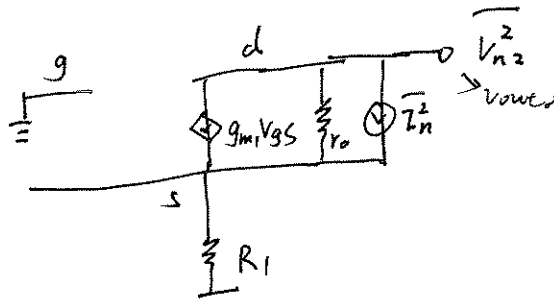
$$= 1 + \frac{r}{g_{m1}R_S} + \frac{r}{g_{m1}^2 r_{o1}^2 g_{m2} \cdot R_S}$$

2.12. Solu:



assume  $I_n$  is ideal, and neglect the noise of  $R_1$ .

Proof:



For small-signal analysis,

$$g_m(-v_s) + \frac{v_{out} - v_s}{r_o} + I_n = \frac{v_s}{R_1}$$

Because we cannot find any loop for the current through  $R_1$ ,

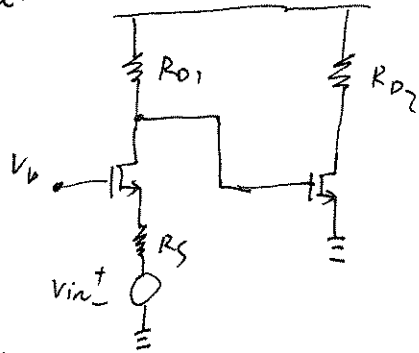
$$v_s = 0$$

$$\Rightarrow \frac{v_{out}}{r_o} = -I_n$$

$$v_{out} = -I_n \cdot r_o$$

$$\therefore \overline{v_{n2}^2} = \overline{I_n^2} \cdot r_o^2$$

2.13. Solu:

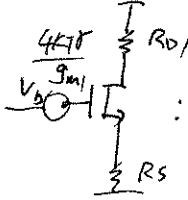


Neglect. transistor cap.  
flicker noise.  
CLM.  
body effect.

For 1st stage:

$$R_{in1} = \frac{1}{g_{m1}}, \quad R_{th2} = \infty,$$

$$\overline{V_{n1}^2} = 4kTR_{D1} + \frac{4kT\gamma}{g_{m1}} \left( \frac{R_{D1}}{\frac{1}{g_{m1}} + R_S} \right)^2 \leftarrow \text{unloaded output noise.}$$



For 2nd stage:

$$\overline{V_{n2}^2} = 4kTR_{D2} + 4kT\gamma \cdot g_{m2} \cdot R_{D2}^2.$$

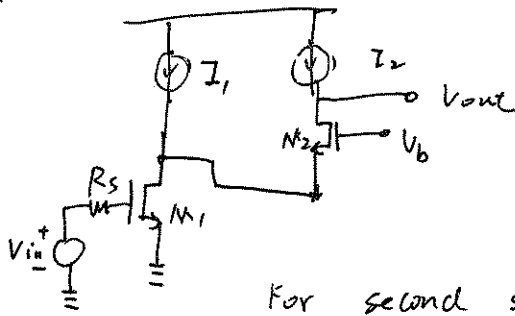
We now substitute these values in Eq. (2.126)

$$NF_{tot} = 1 + \frac{4kTR_{D1} + \frac{4kT\gamma}{g_{m1}} \left( \frac{R_{D1}}{\frac{1}{g_{m1}} + R_S} \right)^2}{\left( \frac{1}{\frac{1}{g_{m1}} + R_S} \right)^2 (g_{m1} \cdot R_{D1})^2} \cdot \frac{1}{4kTR_S} + \frac{4kTR_{D2} + 4kT\gamma \cdot g_{m2} \cdot R_{D2}^2}{\left( \frac{1}{g_{m1} + R_S} \right)^2 (g_{m1} \cdot R_{D1})^2 \cdot (g_{m2} \cdot R_{D2})^2} \cdot \frac{1}{4kTR_S}.$$

This result is different from the CS + CG configuration because, the first stage's NF and input impedance are different, which affect the NF<sub>tot</sub>.

2.14. Solve:

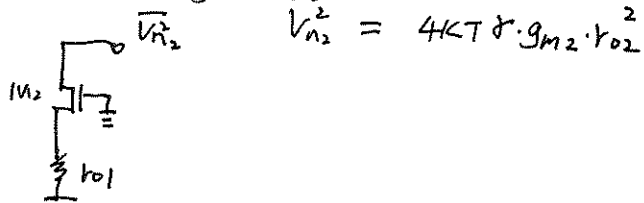
Consider CLM.



$$\overline{V_{n1}^2} = \frac{4kT\gamma}{g_{m1}} \cdot (g_{m1}r_{o1})^2$$

$$= 4kT\gamma \cdot g_{m1}r_{o1}^2$$

For second stage:



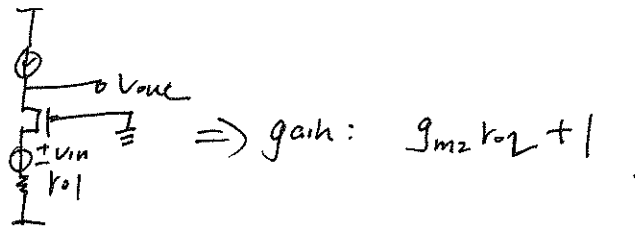
$$\overline{V_{n2}^2} = 4kT\gamma \cdot g_{m2} \cdot r_{o2}^2$$

We now substitute these values in Eq. (126).

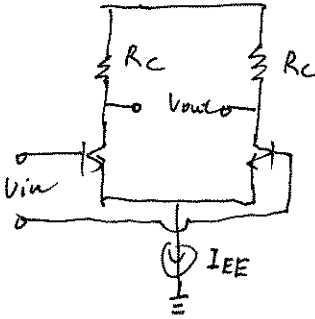
$$NF_{tot} = 1 + \frac{4kT\gamma \cdot g_{m1}r_{o1}^2}{(g_{m1}r_{o1})^2} \cdot \frac{1}{4kTR_S}$$

$$+ \frac{4kT\gamma g_{m2}r_{o2}^2}{(g_{m1}r_{o1})^2 \cdot \left(\frac{1}{g_{m2} + r_{o1}}\right)^2 \cdot (g_{m2}r_{o2} + 1)^2} \cdot \frac{1}{4kTR_S}$$

Note:



2.15 solve:



IP3.

$$\tanh x = x - \frac{x^3}{3} + \frac{2x^5}{15} - \dots$$

$$V_{out} = -2R_c I_{EE} \tanh \left[ \frac{V_{in}}{2V_T} \right]$$

Only consider the first and third order.

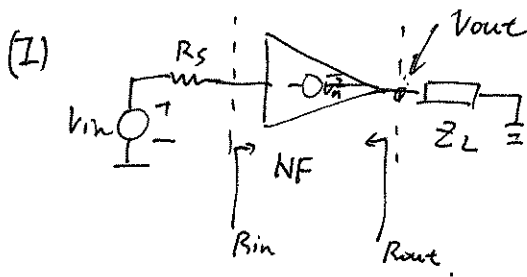
$$V_{out} = -2R_c I_{EE} \left( \frac{V_{in}}{2V_T} - \frac{1}{3} \left( \frac{V_{in}}{2V_T} \right)^3 \right)$$

$$A_{in, IP3} = \sqrt{\frac{4}{3} \left| \frac{\partial^2}{\partial^3} \right|}$$

$$= \sqrt{\frac{4}{3} \cdot \frac{\frac{1}{2V_T}}{\frac{1}{3} \left( \frac{1}{2V_T} \right)^3}}$$

$$= 4V_T = 4 \frac{kT}{q} = 4 \times 26mV = 104mV$$

2.16 Solu:



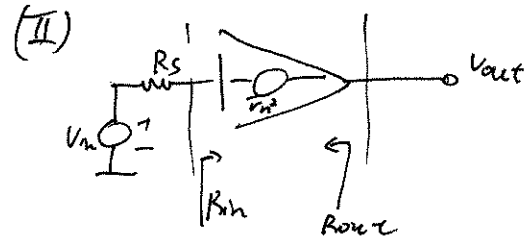
$$\frac{V_{out}}{V_{in}} = A_o \frac{Z_L}{Z_L + R_{out}}$$

$$\overline{V_{n,out}^2} = \overline{V_n^2} \cdot \left( \frac{Z_L}{Z_L + R_{out}} \right)^2$$

$$NF = 1 + \frac{\overline{V_{n,out}^2}}{\frac{V_{out}}{V_{in}}}$$

$$= 1 + \frac{A_o \overline{V_n^2} \left( \frac{Z_L}{Z_L + R_{out}} \right)^2}{A_o^2 \left( \frac{Z_L}{Z_L + R_{out}} \right)^2} \cdot \frac{1}{4kTR_S}$$

$$= 1 + \frac{\overline{V_n^2}}{A_o^2} \cdot \frac{1}{4kTR_S}$$



$$NF = 1 + \frac{\overline{V_n^2}}{A_o^2} \times \frac{1}{4kTR_S} \text{ (unloaded gain)}$$

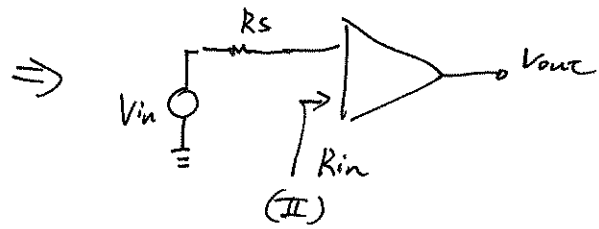
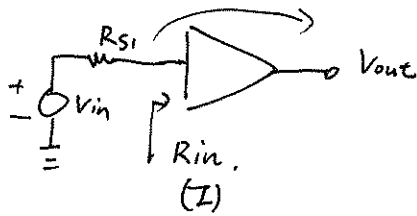
$$A_o = \frac{V_{out}}{V_{in}}$$

unloaded noise at output.

Compared with (I) and (II), we find noise figures of two situations are the same. i.e. output load doesn't affect the circuit's noise figure.



2.17 Solu:  $A_v$



$A_v$  is unloaded voltage gain from input to output of amplifier.

$\therefore$  In Figure (I),

$$NF_1 = 1 + \frac{\overline{V_n^2}}{\left(\frac{R_{in}}{R_{in} + R_{s1}}\right)^2 \cdot A_v^2} \cdot \frac{1}{4kTR_{s1}} \quad (1)$$

In Figure (II)

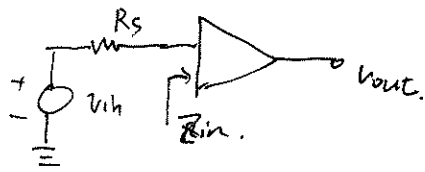
$$NF_2 = 1 + \frac{\overline{V_n^2}}{\left(\frac{R_{in}}{R_{in} + R_{s2}}\right)^2 \cdot A_v^2} \cdot \frac{1}{4kTR_{s2}} \quad (2)$$

$$\frac{(1)}{(2)} \Rightarrow \frac{NF_1 - 1}{NF_2 - 1} = \frac{R_{s2}}{R_{s1}} \cdot \left(\frac{R_{in} + R_{s2}}{R_{in} + R_{s1}}\right)^2$$

So if we know the input impedance and  $R_{s1}$ ,  $R_{s2}$ , it's possible to compute the noise figure for another source impedance  $R_{s2}$ .

2.18 soln:

$$NF = \frac{1}{g_m R_S} + 1 \quad \text{Eq. (2.122)}$$



$$\alpha = \frac{Z_{in}}{Z_{in} + R_S}; \quad \overline{V_{RS}^2} = 4kTR_S$$

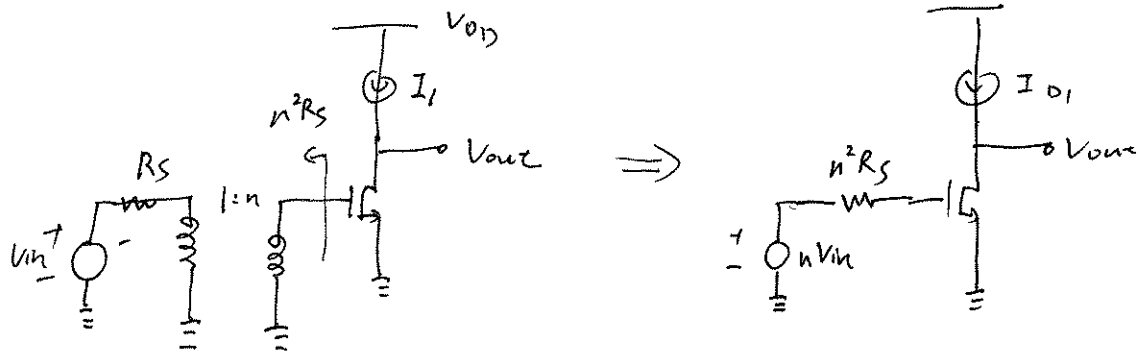
$$SNR_{in} = \frac{V_{in}^2}{\overline{V_{RS}^2}}$$

$$SNR_{out} = \frac{V_{in}^2 |\alpha|^2 A_v^2}{\overline{V_{RS}^2} |\alpha|^2 A_v^2 + \overline{V_n^2}}$$

So, if  $R_S$  increases,  $SNR_{out}$  will fall.

It seems that the result is contradicted with  $NF$ 's falling. Because  $SNR_{in}$  will also fall if  $R_S$  increases, the ratio of  $SNR_{in}$  and  $SNR_{out}$  is reasonable.

2.19 Solu:



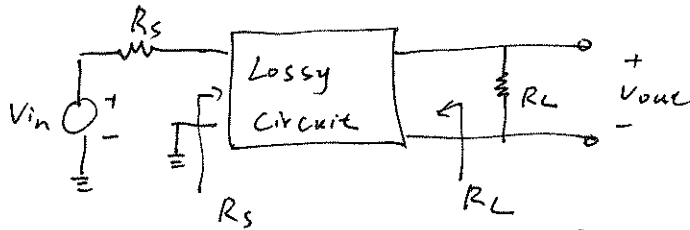
$$\overline{V_n^2} = 4kT \gamma \cdot g_m r_o^2$$

$$A = \frac{V_{out}}{V_{in}} = n \cdot g_m r_o$$

$$\begin{aligned} \therefore NF &= 1 + \frac{4kT \gamma g_m r_o^2}{(n \cdot g_m r_o)^2} \cdot \frac{1}{4kT n^2 R_s} \\ &= 1 + \frac{1}{n^4 g_m R_s} \end{aligned}$$

From the result, we can find transformer improves the noise performance of amplifier greatly.

2.20 Solu:



Proof: 
$$L = \frac{P_{in}}{P_{out}} = \frac{\frac{V_{in}^2}{4R_s}}{\frac{V_{out}^2}{R_L}} = \frac{V_{in}^2}{4R_s} \cdot \frac{R_L}{V_{out}^2} = \frac{R_L}{4R_s} \cdot \frac{V_{in}^2}{V_{out}^2}$$

Theorem: For a passive (reciprocal) network, the PSD of thermal noise is given by  $\overline{V_n^2} = 4kT \operatorname{Re}\{Z_{out}\}$

$$\overline{V_{n,out}^2} = 4kT \cdot R_L \left(\frac{1}{2}\right)^2 \text{ — all the noise} \quad (1)$$

$$A_v = \frac{V_{out}}{V_{in}} \quad (2)$$

$$\begin{aligned} \therefore NF &= \frac{(1)}{(2)^2} = \frac{4kT \cdot R_L \cdot \frac{1}{4}}{\frac{V_{out}^2}{V_{in}^2}} \cdot \frac{1}{4kTR_s} = \frac{R_L}{4R_s} \cdot \frac{V_{in}^2}{V_{out}^2} \\ &= L. \end{aligned}$$

2.21. Solu: Neglect CLM & Body effect.

$$(a) \overline{V_{n,out}^2} = \frac{4kT\gamma}{g_{m1}} \left( \frac{g_{m1}}{g_{m2}} \right)^2 + 4kT\gamma \cdot g_{m2}$$

$$\left| \frac{V_{out}}{V_{in}} \right| = \frac{g_{m1}}{g_{m2}}$$

$$NF = 1 + \frac{\overline{V_{n,out}^2}}{\left( \frac{V_{out}}{V_{in}} \right)^2} \cdot \frac{1}{4kTR_S}$$

$$= 1 + 4kT\gamma \left( \frac{1}{g_{m1}} + \frac{g_{m2}^3}{g_{m1}^2} \right) \cdot \frac{1}{4kTR_S}$$

$$(b) \overline{V_{n,out}^2} = \frac{4kT\gamma}{g_{m1}} \cdot \left( \frac{g_{m1}}{g_{m3}} \right)^2 + 4kT\gamma \cdot g_{m3}$$

$$+ \frac{4kT\gamma}{g_{m2}} \cdot \left( \frac{g_{m2}}{g_{m3}} \right)^2$$

$$\left| \frac{V_{out}}{V_{in}} \right| = \frac{g_{m1}}{g_{m3}}$$

$$NF = 1 + 4kT\gamma \left( \frac{1}{g_{m1}} + \frac{g_{m3} \cdot g_{m2}^2}{g_{m3}^2} + \frac{1}{g_{m2}} \cdot \frac{g_{m2}^2}{g_{m1}^2} \right) \cdot \frac{1}{4kTR_S}$$

$$(c) \overline{V_{n,out}^2} = \frac{4kT\gamma}{g_{m1}} \left( \frac{g_{m1}}{g_{m3}} \right)^2 + \frac{4kT\gamma}{g_{m2}} \left( \frac{g_{m2}}{g_{m3}} \right)^2 + 4kT\gamma \cdot g_{m3}$$

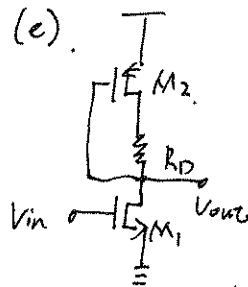
$$\left| \frac{V_{out}}{V_{in}} \right| = \frac{g_{m1}}{g_{m3}}$$

$$NF = 1 + 4kT\gamma \left( \frac{1}{g_{m1}} + \frac{1}{g_{m2}} \frac{g_{m2}^2}{g_{m1}^2} + \frac{g_{m3}^3}{g_{m1}^2} \right) \cdot \frac{1}{4kTR_S}$$

$$(d) \overline{V_{n,out}^2} = \frac{4kT\gamma}{g_{m2}} \left( \frac{g_{m2}}{g_{m3}} \right)^2 + 4kT\gamma \cdot g_{m3} + \frac{4kT\gamma}{g_{m1}} \left( \frac{g_{m1}}{g_{m3}} \right)^2$$

$$\left| \frac{V_{out}}{V_{in}} \right| = \frac{g_{m2}}{g_{m3}}$$

$$NF = 1 + 4kT\gamma \left( \frac{1}{g_{m2}} + \frac{g_{m3}^3}{g_{m2}^2} + \frac{1}{g_{m1}} \frac{g_{m1}^2}{g_{m2}^2} \right) \cdot \frac{1}{4kTR_S}$$



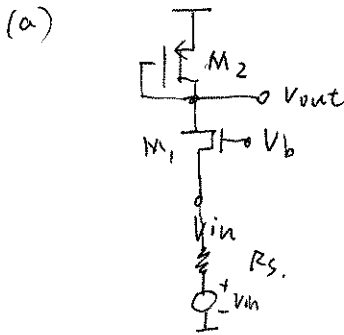
$$\left| \frac{V_{out}}{V_{in}} \right| = \frac{g_{m1}}{g_{m2}}$$

$$\overline{V_{n,out}^2} = \frac{4kT\gamma}{g_{m1}} \left( \frac{g_{m1}}{g_{m2}} \right)^2 + 4kT\gamma R_D$$

$$+ \frac{4kT\gamma}{g_{m2}} \Rightarrow M_2 \text{'s contribution.}$$

$$\therefore NF = 1 + 4kT\gamma \left( \frac{1}{g_{m1}} + \frac{R_D \cdot g_{m2}^2}{g_{m1}^2} + \frac{1}{g_{m2}} \cdot \frac{g_{m2}^2}{g_{m1}^2} \right) \cdot \frac{1}{4kTR_S}$$

2.22 Part Solu:

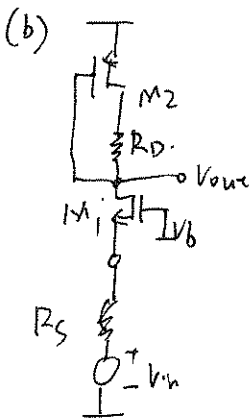


$$\overline{V_{n,out}^2} = 4KT\gamma \cdot g_{m2} + \frac{4KT\gamma}{g_{m1}} \left( \frac{1}{R_s g_{m2}} \right)^2$$

$$\left| \frac{V_{out}}{V_{in}} \right| = \frac{1}{R_s g_{m2}}$$

$$NF = 1 + 4KT\gamma \left( g_{m2} R_s g_{m2} + \frac{1}{g_{m1}} \right) \cdot \frac{1}{4KTR_s}$$

$$= 1 + \frac{\gamma}{R_s} \left( R_s^2 g_{m2}^3 + \frac{1}{g_{m1}} \right)$$



noise by M2:

$$4KT\gamma/g_{m2}$$

noise by RD:

$$4KTR_D$$

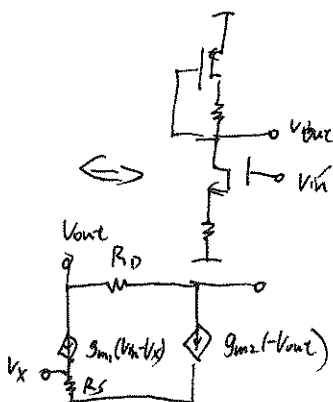
noise by M1:

$$\frac{V_{out}}{V_{in}} = \frac{g_{m1}}{g_{m2} + g_{m1} g_{m2} R_s}$$

$$\frac{4KT\gamma}{g_{m1}} \left( \frac{V_{out}}{V_{in}} \right)^2$$

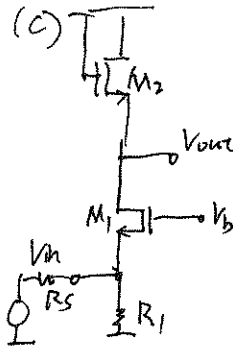
gain of circuit

$$\frac{V_{out}}{V_{in}} = - \frac{g_{m1}}{g_{m2}}$$



$$NF = 1 + 4KT\gamma \left[ \frac{1}{g_{m2}} + R_D + \frac{\gamma \cdot g_{m1}}{(g_{m2} + g_{m1} g_{m2} R_s)^2} \right]$$

$$\times \frac{1}{4KTR_s} \times \frac{g_{m2}^2}{g_{m1}^2}$$



noise of M2:  $\frac{4KT\gamma}{g_{m2}}$  (1)

noise of M1:  $\frac{4KT\gamma}{g_{m1}} \cdot \left( \frac{1}{R_L || R_s g_{m2}} \right)^2$  (2)

noise of R1:

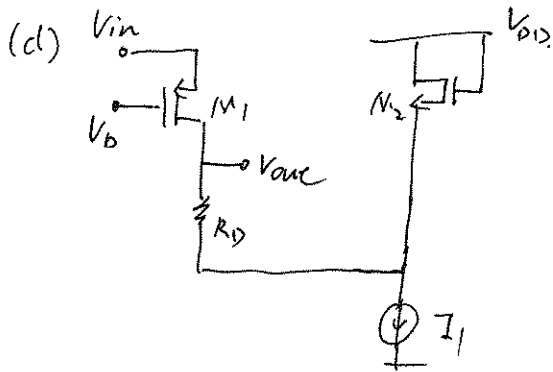
$$4KTR_1 \left( \frac{g_{m1}^{-1}}{R_s + g_{m1}^{-1}} \right)^2 \left( \frac{1}{R_s g_{m2}} \right)^2$$
 (3)

$$NF = 1 + \frac{(1) + (2) + (3)}{A_v} \frac{1}{4KTR_s}$$

Note:

$$A_v = \frac{R_L || \frac{1}{g_{m1}}}{R_s + R_L || \frac{1}{g_{m1}}} \cdot \frac{1}{R_L || R_s \cdot g_{m2}}$$

2.22 Part 2. Solu:

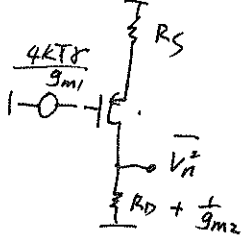


$$\left| \frac{V_{out}}{V_{in}} \right| = \frac{g_{m1}}{R_D + \frac{1}{g_{m2}}}$$

noise of  $R_D$  :  $4KT R_D$  ①

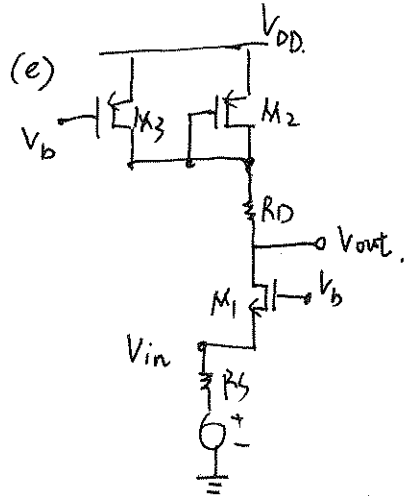
noise of  $M_2$  :  $\frac{4KT\gamma}{g_{m2}}$  ②

noise of  $M_1$  :



$$\frac{4KT\gamma}{g_{m1}} \cdot \frac{(R_D + \frac{1}{g_{m2}})^2}{R_S} \quad \text{③}$$

$$NF = 1 + \frac{\text{①} + \text{②} + \text{③}}{\left| \frac{g_{m1}}{R_D + \frac{1}{g_{m2}}} \right|^2} \times \frac{1}{4KTR_S}$$



$$\left| \frac{V_{out}}{V_{in}} \right| = \frac{R_D + \frac{1}{g_{m2}}}{R_S}$$

noise of  $R_D$  :  $4KT R_D$  ①

noise of  $M_1$  :

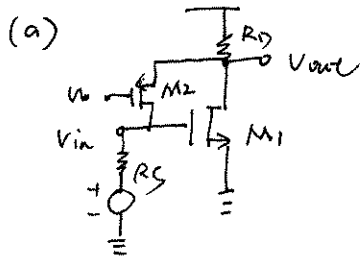
$$\frac{4KT\gamma}{g_{m1}} \cdot \left( \frac{R_D + \frac{1}{g_{m2}}}{R_S} \right)^2 \quad \text{②}$$

noise of  $M_2$  :  $\frac{4KT\gamma}{g_{m2}}$  ③

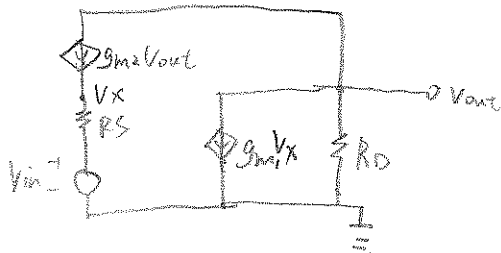
noise of  $M_3$  :  $\frac{4KT\gamma}{g_{m3}} \left( \frac{g_{m3}}{g_{m2}} \right)^2$  ④

$$NF = 1 + \frac{\text{①} + \text{②} + \text{③} + \text{④}}{\left( \frac{R_D + \frac{1}{g_{m2}}}{R_S} \right)^2} \cdot \frac{1}{4KTR_S}$$

part ①  
2.23 solu:



Through small-signal analysis,

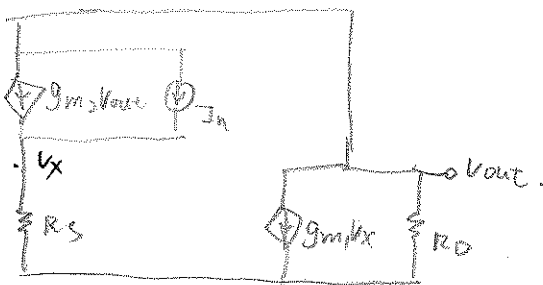


$$\Rightarrow \frac{V_{out}}{V_{in}} = - \frac{g_{m1}}{g_{m2} + \frac{1}{R_D} + g_{m1}g_{m2}R_S}$$

noise from  $R_D = 4kTR_D$  ①

noise from  $M_1$ :  $\frac{4kT \cdot g_{m1}}{g_{m2} + g_{m1}g_{m2}R_S + \frac{1}{R_D}}$  ②

noise from  $M_2$ :

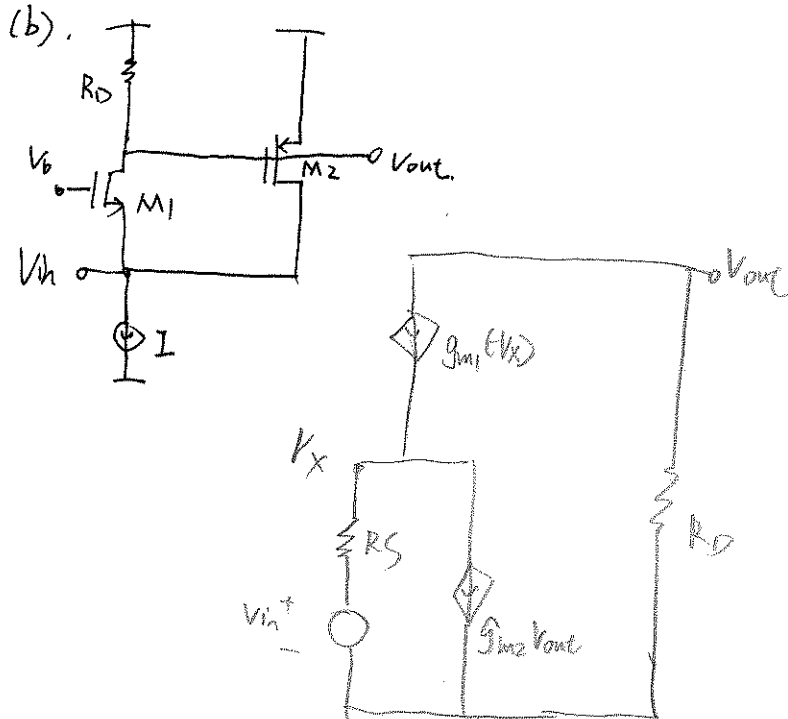


$$g_{m1}V_x + \frac{V_{out}}{R_D} = - \frac{V_x}{R_x} = -(Z_n + g_{m2}V_{out})$$

$$\Rightarrow \left| \frac{V_{out}}{Z_n} \right| = \frac{g_{m1}R_D + 1}{g_{m2} + \frac{1}{R_D} + g_{m1}g_{m2}R_S}$$

$$\Rightarrow \left| \frac{V_{out}}{I_n} \right|^2 \cdot 4kT \cdot g_{m1} \quad ③$$

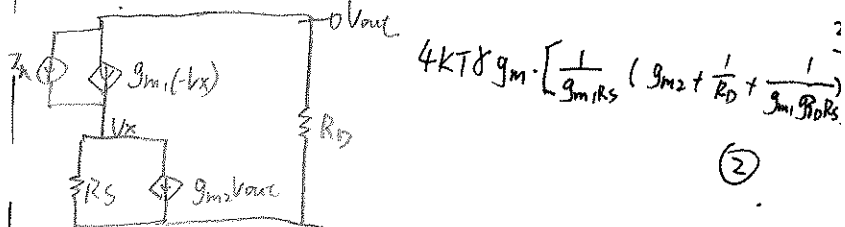
$$NF = 1 + \frac{① + ② + ③}{\left( \frac{g_{m1}}{g_{m2} + \frac{1}{R_D} + g_{m1}g_{m2}R_S} \right)^2} \cdot \frac{1}{4kT \cdot R_S}$$



$$\frac{V_{out}}{V_{in}} = \frac{R_D}{\frac{1}{g_{m1}} + R_S}$$

noise from  $R_D = 4kTR_D$  ①

noise from  $M_1$ :



$$4kT \cdot g_{m1} \cdot \left[ \frac{1}{g_{m1}R_S} \left( g_{m2} + \frac{1}{R_D} + \frac{1}{g_{m1}R_D R_S} \right) \right]^2$$

②

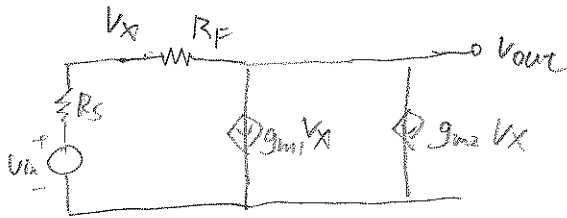
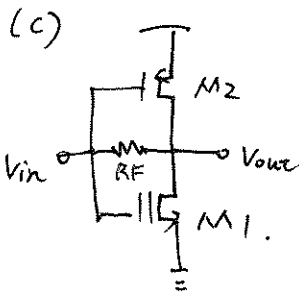
noise from  $M_2$ :

$$4kT \cdot g_{m2} \left[ g_{m2} + \frac{1}{R_D} + \frac{1}{g_{m1}R_D R_S} \right]^2 \quad ③$$

$$NF = 1 + \frac{① + ② + ③}{\left( \frac{R_D}{\frac{1}{g_{m1}} + R_S} \right)^2} \cdot \frac{1}{4kT \cdot R_S}$$

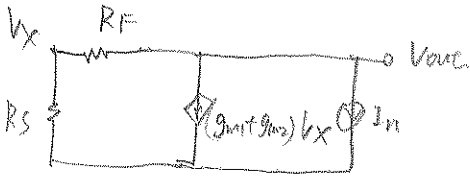


2.23 Part ②  
Solu:



$$\frac{V_{out}}{V_{in}} = \frac{(g_{m1} + g_{m2}) R_F - 1}{(g_{m1} + g_{m2}) R_S + 1}$$

noise from  $M_1 \Rightarrow 4kT \delta \cdot g_{m1} \cdot \left( \frac{R_S + R_S}{(g_{m1} + g_{m2}) R_S + 1} \right)^2$  ①



noise from  $M_2 \Rightarrow 4kT \delta \cdot g_{m2} \cdot \left( \frac{R_S + R_F}{(g_{m1} + g_{m2}) \cdot R_S + 1} \right)^2$  ②

noise from  $R_F \Rightarrow 4kT \delta R_F$  ③

$$\therefore NF = 1 + \frac{① + ② + ③}{\left( \frac{(g_{m1} + g_{m2}) R_F - 1}{(g_{m1} + g_{m2}) R_S + 1} \right)^2} \cdot \frac{1}{4kT R_S}$$