Reports on
"Making Sense of Quantum Mechanics"
Chapter 2

The First Mystery: *In the beginning was* 'measurement' ?!

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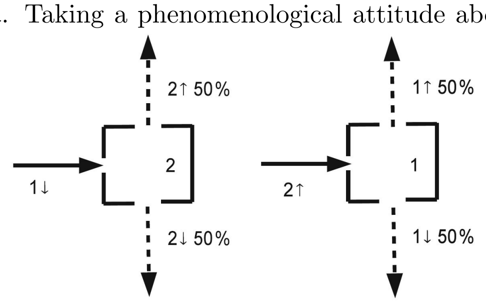
1 The Spin

> *The notion that there is an intrinsic property of a particle corresponding to its spin in a given direction and that is being measured when one "measures its spin" is untenable.*
>
> - Jean Bricmont

Quantum Mechanics is meant to be an overarching theory of reality; its necessity comes from the deficiencies of classical theories of physics. we have a debatable experimental situation and our acclaimed theory should explain it meticulously; some particles possess a property called "spin" which can be measured in different directions and takes, in each direction, only two values, denoted $\uparrow$ and $\downarrow$. Taking a phenomenological attitude about spin, the situation is as follows:



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In the above figure, we send particles we know are 1↓ through box 2 which measures the spin in direction 2; we select the particles having spin 2↑ and send them through another box (labeled 1) which measures the spin in direction 1. we expect that 100% of particles be 1↓ but what we observe is that 50% are 1↓ (and 50% 1↑, consequently)! This is an example of "complementarity": measuring spin in one direction is complementary to another thus destroys the result of another. Complementarity in Quantum Mechanics was first used by Niels Bohr but not in the common meaning; here, according to Bell it means contradictariness.

2 The Mach-Zehnder Interferometer

> *Whether one path is open or not seems to influence the behavior of the particles following the other path. This is the essence of the first quantum mystery.*
>
> \- Jean Bricmont

The example in the last section was an example of situation where particles "forget" what their spin was; we might wonder is there any situation where they "remember" their spin? Yes:

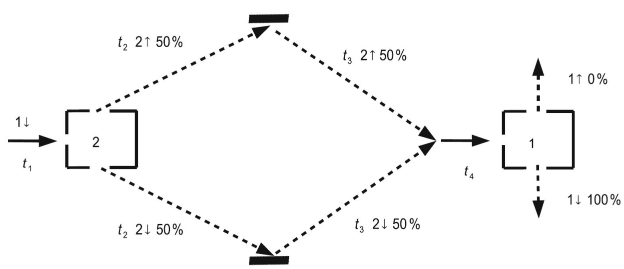


Fig. 2.3 Interference

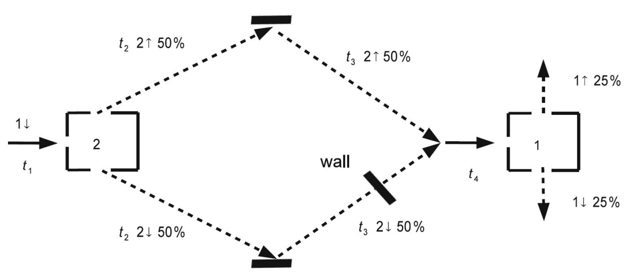


Fig. 2.4 Interference with a wall

We send particles that are 1↓ through box 2, it reflects from the mirror and at t_4 again its spin in direction 1 is measured; surprisingly, here the particle "remembers" its spin and we will get 100% of the particles in the $1\downarrow$ state. Another mystery is unleashed if we put a wall in $2\downarrow$ path; what do we expect? 50% of particles will collide with the wall and not reach the detector. of remaining 50% we expect all of them to be $1\downarrow$ but shockingly we observe that 25% (half of the remaining 50%) are $1\downarrow$! we have put a wall in path $2\downarrow$ and this action has affected the particles that has not taken this path! this is interference and our first mystery!

3 The Quantum Formalism

We associate with each particle a "state", which is a vector in $\mathbb{C}^2$ –its basis is of length 2–.

$$|1\uparrow\rangle = \begin{bmatrix}1\\0\end{bmatrix}$$

$$|1\downarrow\rangle = \begin{bmatrix}0\\1\end{bmatrix}$$

$$|2\uparrow\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$$

$$|2\downarrow\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$$

Quantum Algorithm –as Bricmont calls conventional quantum mechanics– is governed by two disjoint sets of rules:

1. When no measurements are made the state of a particle which is a linear combination of the basis

$$|state(t)\rangle = c_1(t)|1\uparrow\rangle + c_2(t)|1\downarrow\rangle = d_1(t)|2\uparrow\rangle + d_2(t)|2\downarrow\rangle \tag{1}$$

   has coefficients that evolve *continuously* such that $|c_1(t)|^2 + |c_2(t)|^2 = 1$ and $|d_1(t)|^2 + |d_2(t)|^2 = 1$ for all t.
   The evolution is *deterministic* meaning that given any initial state $|state(0)\rangle$ the state is uniquely determined for all t.
   the evolution is linear.

2. But, if a measurement is made, for state (1), for example, if one measures the spin in direction 2 at time t, one finds $1\uparrow$ with probability $|d_1(t)|^2$ and $\downarrow$ with probability $|d_2(t)|^2$, where $|d_1(t)|^2 + |d_2(t)|^2 = 1$.
   After the measurement in direction 2, if one "sees" the result $\uparrow$, the state changes and becomes $|2\uparrow\rangle$, and if one "sees" $\downarrow$, the state changes and becomes $|2\downarrow\rangle$.
   This rule is called the "reduction", or the "collapse" of the state, which is discontinuous in time, non-deterministic and non-linear.

Spin is a part of our picture to discribe a particle, another part is the *wave function* of the particle which is the analouge of position, 'the position part': $\Psi(\vec{r},t)$. A product of this two parts, e.g., $\Psi(\vec{r},t)|2\downarrow\rangle$ is called the *quantum state* and whether it gives a complete discription of the reality of our particle is an issue to be discussed in section 5.

4 How Does it Work?

In this part we see how quantum formalism accounts for what we observe in figure 2.3:
At time t_1 we have

$$|1\downarrow\rangle = \frac{1}{\sqrt{2}}(|2\uparrow\rangle - |2\downarrow\rangle)$$

and at t_2 and t_3

$$\frac{1}{\sqrt{2}}(|2\uparrow\rangle|path\ 2\uparrow\rangle - |2\downarrow\rangle|path\ 2\downarrow\rangle)$$

Let us assume that, at time t_4, the black arrow is a device that is able to recombine the paths of two particles and send them in direction $\rightarrow$; thus at t_4 we have

$$\frac{1}{\sqrt{2}}\bigg(|2\uparrow\rangle|path\ 2\uparrow\rangle - |2\downarrow\rangle|path\ 2\downarrow\rangle\bigg)|path\rightarrow\rangle = |1\downarrow\rangle|path\rightarrow\rangle$$

which is 100% in direction 1$\downarrow$.

5 The Meaning of the Quantum State?

In the last section we saw that if we accept the dualistic laws of quantum mechanics explained in section 3, everything else follows clearly, But the acceptance is not an obvious one! Considering the Mach-Zehnder interferometer experiment, after developing an algorithm allowing us to predict the results of experiments, we need to find out the meaning of such quantum states.

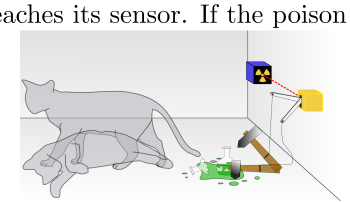
There are four reactions considering the old problem of measurement and the meaning of quantum states:
- Copenhagenean answer
- Urge for a complete theory (e.g. the de Broglie-Bohm theory)
- Naive statistical Interpretation, and
- Measurement is a part of Q.M. Formalism

The first two reactions need some further discussion, both technical and philosophical and are pondered in next chapters, so Bricmont turns to refute the last two:

5.1 The Measurement Process Within the Quantum Formalism

> *The assumption that we can identify a macroscopic quantum state with a physical situation in the three-dimensional world, such as a pointer pointing up is hard to justify...*
>
> \- Jean Bricmont

Following the second reaction we analyze the famous Schrödinger's Cat thought experiment; suppose a cat is in a sealed box and there is a purely classical mechanism linking a pointer to a hammer that will break a bottle containing some deadly poison if the pointer is up, but not if it is down; the pointer, in turn, triggers down if a radioactive particle from the source disintegrates and reaches its sensor. If the poison is released, it kills the cat.



The initial wave function of the system has three parts, one corresponding to the cat, one to a radioactive particle which may disintegrate or not at the chance of 50% uniformly in the interval that the box is closed, and one to a pointer which is connected to the hammer:

$$\Psi_0 = |cat\rangle \left(c_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right) \varphi_0(z)$$

After the measurement (opening the box) we have

$$c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} \varphi^{\uparrow}(z)|\text{cat alive}\rangle + c_2 \begin{bmatrix} 1 \\ 0 \end{bmatrix} \varphi^{\downarrow}(z)|\text{cat dead}\rangle$$

=

(particle not disintegrate)(pointer up)(cat alive)+(particle disintegrate)(pointer down)(cat dead)

We see that ordinary quantum mechanics predicts unambiguously that the cat is alive *and* dead, while we always see it either alive *or* dead. It is only by switching from *and* to *or* that one can "eliminate" that problem.
Therefore, clearly, this reaction is untenable. Either we should accept incompleteness of ordinary quantum mechanics or we should consider the Everett's idea of many-worlds which is not considered in this chapter.

5.2 The "Naive" Statistical Interpretation

In this interpretation measurement reveals something pre-existing, the probabilities reflect our ignorance and so does the quantum state; the collapse of the

quantum state would be similar to the adjustment (adding to our information) of probabilities.
Often here the term "Hidden Variables" is used since this view assumes that quantum mechanics is incomplete: each individual system is characterized by variables other than the quantum state, variables called "Hidden" and whose statistical distribution would be determined by the quantum state. As an example, if we prepare an ensemble of particles in state (1) a fraction $|c_1|^2$ of particles would be in state $|1 \uparrow\rangle$ and a fraction $|c_2|^2$ of them in $|1 \downarrow\rangle$, but this interpretation faces a serious problem of inconsistency for we have a theorem which says the mere assumption of the existence of hidden variables is impossible:

Theorem 1. *No Hidden Variables*

1. *There does not exist a function $v : \mathcal{O} \to \mathbb{R}$ where $\mathcal{O}$ is a collection of quantitites related to "spin", such that $\forall A \in \mathcal{O}, v(A)$ agrees with the predictions of quantum mechanics.*

2. *There does not exist a function $v : \mathcal{O} \to \mathbb{R}$ where $\mathcal{O}$ is the set of functions of the four quantities representing the positions and the momenta of two particles moving on a line, such that $\forall A \in \mathcal{O}, v(A)$ agrees with the predictions of quantum mechanics.*

6 Conclusions

Bricmont emphasizes that the quantum algorithm is an unambiguous method for accurately predicting results of measurements, and nothing else.In particular,it should not be associated with any mental picture of what is "really going on. The main issue of course is whether one should consider this algorithm as satisfactory or as being, in some sense, the "end of physics, or whether one should try to go beyond it.
As with its relation with reality (interpretation) it is probable that the statistical interpretation lies in the back of the mind of many physicists. As with the second and third reaction, either the quantum state represents an ensemble of systems or we should treat measurement process as a part of our theory but neither of these positions are defensible, either because the linearity of Schrödinger equation leads to macroscopic superpositions or because of the no hidden variables theorem. So it remains to see whether the Copenhagenean answer is acceptable or not.

7 Appendices (Mathematical Expository)

Personal Note: Following parts are examples of the elegance of mathematical physics; covering a conventional course on quantum mechanics in 10 pages yet in more detail and a unified framework!

7.1 The Schrödinger Equation

$$i\hbar\frac{\partial}{\partial t}\Psi(\vec{r},t) = \hat{H}\Psi(\vec{r},t)$$
$$\Psi : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{C}$$

in which $\hbar$ is a physical constant called Planck Constant and $\hat{H}$ is a linear and usually self-adjoint operator called Hamiltonian.
Schrödinger Equation is at the basis of non-relativistic quantum mechanics and determines the evolution of the quantum state of a system.
Given an initial value $\Psi(\vec{x},0)$ we can attempt to solve it:

$$\Psi(x,t) = e^{-i\hat{H}t}\Psi(x,0)$$

we are talking of the exponential map of an operator which is meaningful since we are working in a Banach space.Ψ would be easily at hand if $\hat{H}$ is self-adjoint, for we can use the spectral theorem and find an orthonormal basis of eigenvectors in which the exponential could be easily computed.
Bricmont, here, beautifully introduces the Fourier Series as an example of such basis for periodic functions. Another beauty is the introduction of fourier transform for the sake of self-adjointness...

7.2 "Uncertainty" Relations and "Complementarity"

If we let

$$\text{Var(x)} = \int_{\mathbb{R}} x\Psi(x)dx$$

and

$$\text{Var(p)} = \int_{\mathbb{R}} p\hat{\Psi}(p)dp$$

where $\hat{\Psi}$ is the Fourier transform of Ψ, we will have the following purely mathematical relation:

$$\text{Var(x)Var(p)} \geq \frac{1}{4} \tag{2}$$

(2) is usually called Heisenberg's Uncertainty Principle and its physical interpretation has always been the subject of many debates...
Since (2) is a lower bound on variances of results of measurement, it implies nothing about the intrinsic properties of quantum particles. one could think in accordance with the statistical interpretation, that each individual particle has a well-defined position and momentum but when we prepare an ensemble of particles they have certain statistical distributions whose variances satisfy (2). But, as we saw in 5.2, due to the No Hidden Variables theorem, this view is untenable.
As to the Bohr's disciples, we are talking of complementarity here: we have two pictures of a quantum system, one described by x and one by p; the two pictures are incompatible for we cannot know both simultaneously. Anyway, Bohr and his followers, despite what they always claim are way too vague about such words to be taken seriously.