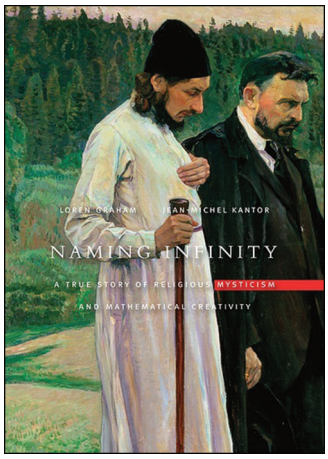

Naming Infinity: A True Story of Religious Mysticism and Mathematical Creativity

Reviewed by Alexey Glutsyuk

Naming Infinity: A True Story of Religious Mysticism and Mathematical Creativity
Loren Graham and Jean-Michel Kantor, Belknap Press of Harvard University Press, 2009. ISBN-13: 978-06740-329-34.

During the nineteenth century, a foundational crisis in mathematics led to signal events of fundamental importance. The first was the creation of set theory by Georg Cantor at the end of the nineteenth century. The second was the creation of the theory of functions and of measure theory and integration theory by the French trio Emile Borel, René Baire, and Henri Lebesgue. Their works relied heavily on Cantor's set theory. A major contribution to the further development of set theory, function theory, and topology was made by Russian mathematicians: Dmitry Egorov, Nikolai Luzin, and their school, the famous Lusitania.

The book of Jean-Michel Kantor and Loren Graham presents the history of this important period of mathematics through vivid portraits that bring to life the personalities of the mathematicians. The main heros of the book are Cantor, the above-mentioned French trio, and a Russian trio consisting of Egorov, Luzin, and their close friend Pavel Florensky, an extremely talented scientist and engineer and a priest of the Russian Orthodox Church. The book intertwines and links their mathematical research with their cultural and religious backgrounds.

The authors describe the continuous development of mathematics from Cantor to the Russians. Cantor, who was the first to compare different kinds of infinities and prove key results about them, was a Protestant Christian believer and a philosopher of "free mathematics". With Cantor's set theory as a basis, the young French mathematicians Borel, Baire, and Lebesgue created modern measure theory and function theory. But when some difficulties and paradoxes were discovered in the foundations of set theory, they retreated from research in the subject. After that, further research was carried out by the Russians Egorov and Luzin, who were Orthodox Christian believers,[1] and by their students. They were men of great spirit and courage, inspired by their Christian faith, and they attacked difficult classical problems directly. The authors of the book claim that the mathematical research of the Russian trio was inspired by Name Worshipping, which was a heretical current in the Russian Orthodox Church at the beginning of the twentieth century.

More than half of the book is devoted to the Russian mathematicians, who worked during a dramatic period of Russian history: the Revolutions of 1905 and 1917, the Civil War, the Bolsheviks' rise to power, Stalin's terror, ... The book shows how, in these very difficult conditions, Egorov and Luzin managed not only to obtain their famous

Alexey Glutsyuk is chargé de recherche in the Centre National de la Recherche Scientifique at the École Normale Supérieure de Lyon. His email address is aglutsyu@ens-lyon.fr.

DOI: http://dx.doi.org/10.1090/noti1071

[1] *As shown by the lives of Cantor and the Russian trio—and by the book under review—a religion does not contradict science; the two can complement each other in a harmonious way. The same point of view is expressed in the book* Science and Religion *(Moscow, Obraz, 2007) by Archbishop Luka (Voyno-Yasenetsky) of Crimea (1877–1961), a famous Russian and Soviet surgeon. He shows (with detailed historical analysis and citations) that a majority of the most famous scientists were believers. (Archbishop Luka was persecuted by the Bolsheviks for his Christian faith, before he was awarded the Stalin Prize for his achievements in surgery. He was recently canonized by the Russian Orthodox Church.)*

results, but also to create the outstanding Moscow mathematical school "Lusitania". This school put Moscow on the mathematical map of the world and made it one of the world centers with a maximal concentration of outstanding mathematicians. The majority of famous Moscow mathematicians are descendants of Lusitania.

The authors describe the dramatic personal fates of the Russian trio and the Lusitania students after the Revolution of 1917. During Stalin's terror, Egorov, Florensky, and Luzin were persecuted for their Christian faith. All three are highly admirable, especially Egorov and Florensky, who showed great personal bravery. When the Bolsheviks cruelly persecuted Christian believers, these two men remained believers and did not change their habitudes at all. In fact, Florensky's courage only increased: he caused sensations by always wearing his priest's robe at scientific and engineering meetings. Egorov and Florensky were arrested, and their lives ended tragically: Egorov died in detention, and Florensky was executed. Luzin, a believer and a professor of the old generation, barely escaped a similar destiny after he was accused, publicly and wrongly, of being a traitor. He was more productive scientifically than Egorov and Florensky, though more unstable and less brave. The book describes in an honest way how some of the famous Lusitania students were contradictory people, with good and bad aspects.

I liked very much the authors' choice of the picture on the book's cover, a reproduction of the painting *Philosophers* by the famous Russian painter Mikhail Nesterov. The painting represents two great Russian philosophers and priests, Pavel Florensky and Sergei Bulgakov, whose fates were completely different. Bulgakov was expelled from Russia by the Bolsheviks on the Philosophers' Ship, along with many other philosophers. After his expulsion, he remained extremely active as a philosopher and theologist and published a tremendous number of works. He was one of the key creators of the famous Saint Serge Orthodox Institute in Paris. Many other scientists and philosophers decided to leave Russia after the Revolution of 1917. Florensky was one of the very few theologists and philosophers who decided to stay.[2] He was quite aware of the new political situation in Russia and of what his fate would be. Nevertheless, he decided to stay and to serve Russia with all his energy.

It is very impressive that the authors, not being of Russian origin, know the history of Russia, its mathematics, and its church so deeply. They have done an unimaginable amount of work, including many trips to and across Russia, many interviews, and extensive reading in many archives. I would like to add that the author Jean-Michel Kantor greatly helped young mathematicians from the Former Soviet Union during a very difficult period in the 1990s. At that time, in order to have something to eat, many mathematicians from the Former Soviet Union chose either to leave the country or to leave mathematics in order to earn money. Jean-Michel miraculously organized financial assistance from the French government, thereby saving many young mathematicians, including myself, by allowing them to do only mathematics while staying in their country. I wish to take this occasion to thank him a lot once more.

Returning to the book, I would like to make a remark related to my own preferences. I would have been glad if the book had put less emphasis on details about personal lives and philosophical reasonings about inspiration, and more emphasis on mathematics (explained in a way understandable by nonmathematicians) and on history. The reasoning behind my preference is that, while one can check whether a scientist was inspired by some other scientific work, one cannot check in a rational way whether the inspiration for a person's scientific creativity came from outside of science.

I will now describe in more detail the content of the book. The first chapter presents the origin and history of Name Worshipping, which, according to the authors, was a source of inspiration for the Russian mathematical trio. The main part of the chapter is devoted to a dramatic event of February 1913: the storming of St. Pantaleimon Monastery at Mount Athos by the army of Russian Tsar Nikolai II and the cruel expulsion of the Name Worshipping monks from the monastery.

The second chapter describes the life and mathematical achievements of Georg Cantor and the reception of his theory by other famous mathematicians of his time, including a detailed history of the development of Cantor's set theory and of his famous continuum hypothesis (CH), with links to his philosophy.

The third chapter is devoted to the reception of Cantor's theory in France and the French trio of Borel, Baire, and Lebesgue. It starts with an important event in the history of mathematics, the International Congress in Paris in 1900. At the congress, Hilbert made clear that Cantor's theory would play a major role in the future development

[2] *While in prison after his second arrest, Florensky had the opportunity to emigrate to the Czech Republic together with his family. He refused. Florensky's grandson and biographer, Igumen (Father Superior) Andronik (Trubachev) says that he does not know of any other case in which a Gulag camp prisoner refused to leave the camp. The biography, published in Moscow in 2007, would be interesting to those who would like to learn more about Pavel Florensky.*

of mathematics and placed the continuum hypothesis at the top of his famous problem list. The book describes some of the French trio's contributions that were heavily based on Cantor's set theory: the Heine–Borel theorem, the basis of the future "Borel measure"; the introduction of Borelian and measurable sets; the introduction by Baire of the notion of semicontinuity and his classification of discontinuous limits of continuous functions; and the construction of the "Lebesgue integral". The authors intertwine the development of mathematics in France with descriptions of the cultural spirit and important historical events in France at the beginning of the twentieth century. One is the tragic Dreyfus Affair, in which leading French mathematicians, including Henri Poincaré, actively defended Dreyfus. The authors also describe the lives of the members of the French trio, such as the extremely rich and intense life of Borel, who, besides being a mathematician, played many other roles: Navy minister, mayor of his home town, and participant in the Résistance.

The rest of the chapter focuses on contradictions and paradoxes that appeared in the foundations of Cantor's set theory at the beginning of the twentieth century, such as the difficulties found by Cantor[3] himself in 1895 and various paradoxes, including that of Russell. There is also a discussion of Zermelo's Axiom of Choice and the famous exchange of five letters about it by Borel, Baire, Lebesgue, and Hadamard. This exchange confirmed the critical state of the foundations of mathematics and raised important problems that were partially solved later, including famous incompleteness results by Gödel and Cohen. Even now, not all is resolved.

Chapter four is devoted to the Russian trio: Dmitry Egorov, Nikolai Luzin, and Pavel Florensky. At the end of the nineteenth and the beginning of the twentieth century, Russian mathematics was closely related to philosophy and religion, and the chapter describes the spirit of this time in a remarkable way. The authors discuss the creation of Markov chains, which appeared as a result of a philosophical debate between P. A. Nekrasov and A. A. Markov. Nekrasov, who was a Christian believer and a supporter of the Tsar's power, drew motivation from philosophy related to the question of free will and was thereby led to make overly strong claims about probabilities. Markov, an atheist and a critic of both Tsarist power and the Russian church, constructed his famous chains as a counterexample to Nekrasov's statement. Nikolai Bugaev, the teacher of the three members of the Russian trio and the president of the Moscow Mathematical Society, defended free will and connected it to mathematics. While many mathematicians were frightened by discontinuous functions and called them "monsters", Bugaev called them beautiful and morally strengthening because they freed the human being from "fatalism". The opinion of his student Florensky was that the nineteenth century was intellectually a disaster and that one of its main origins was the "governing principle of continuity", which "was cementing everything in one gigantic monolith".

The book describes the lives of the Russian trio before the Revolution of 1917, their mathematical works, and their personal qualities. It briefly discusses Egorov's first famous achievement in differential geometry, after which "Egorov surfaces" appeared. Egorov is described as a deep Christian believer whose modesty mixed in a remarkable way with his courage to express his disagreement on matters of principle. For example, he signed a petition protesting the 1903 pogrom against Jews in Kishinev even though he had not been politically active. The authors describe Luzin's mental crisis and depression after he saw bloody events in the Revolution of 1905, and they discuss how correspondence with Florensky helped Luzin to recover, become a Christian believer, and return to mathematics. This shows that, for Luzin and for the whole Russian trio, Christian belief was the Pillar and Ground of the Truth (to use words from the title of Florensky's book). Florensky converted to the Christian orthodox faith at the age of seventeen. Eventually, after successfully graduating from Moscow University, he left mathematics, studied at the Theological Academy at Sergiev Posad, and became a priest. Florensky protested the execution of Peter Schmidt, a revolutionary lieutenant of the Tsar's army. He did not share Schmidt's political opinions; he simply opposed capital punishment. After that, Florensky was arrested and held in jail for a week, where he wrote one of his mathematical works.

Chapter five describes the relations between Russian mathematics and "mysticism". Henri Lebesgue spoke of "naming a set". Luzin emphasized the significance of naming in his mathematical work. As already mentioned above, the authors of the book relate the creativity of the Russian trio to Name Worshipping. This was a heretical current in the Russian church. Its supporters practiced the Jesus Prayer and claimed that, after repeating it correctly many times, a person achieves a unity with God: roughly speaking, the name of God is God himself. The authors explain the influence of Name Worshipping on the mathematical creativity of the Russian trio in set theory, basically, by noting the importance of naming in both of them.

[3] *As is mentioned in the book, Cantor escaped from contradictions by naming the objects "too big to be sets" as "Absolute".*

Remark. This is the point of view of the authors of the book under review. From my own point of view, a claim that Name Worshipping was a major inspiration for the Russian trio would seem a bit too strong.

Chapter six gives an impressive description of the spirit and life of the mathematical school founded by Egorov and Luzin, the famous Lusitania. The professors created an atmosphere of openness and closeness. Sometimes Luzin's classes finished in his apartment, with discussions about mathematics, culture, arts, religion, etc., that would continue into the night. Most of the students were young, having joined the Lusitania when they were around seventeen years old. The book describes two key achievements of Lusitanians, namely, the proof of the continuum hypothesis for Borelian sets by Pavel Alexandrov (1915) and the creation of descriptive set theory (1916) by Mikhail Suslin and Nikolai Luzin, after Suslin found a fundamental mistake in Lebesgue's seminal paper of 1905.

Chapter seven describes the dramatic fates of the members of the Russian trio after the Revolution of 1917. Egorov and Florensky were persecuted for having courageously confessed their Christian belief. The authors describe the attacks against Egorov, his arrest and imprisonment, his hunger strike in detention, his hospitalization, and finally his death. A highly admirable person appearing in the book is Nikolai Chebatorev, a famous mathematician though not a Lusitanian. Chebatorev was an atheist and a former Red Army soldier. He and his wife tried, at huge risk to themselves, to save the believer Egorov. The authors discuss the fate of Florensky, who was first arrested in 1928 and sent into exile. After his second arrest in 1933, he never came back. It is absolutely remarkable that, even while in detention, Florensky remained very active and made many important scientific and engineering achievements. He spent the last period of his life as a prisoner at the infamous Solovetsky Gulag camp, where he created a famous iodine enterprise. Much later, after Stalin's death, it was found that he had been executed in 1937.

Luzin was much more cautious than Egorov and Florensky: he became a "secret believer". At some point, he even stopped going to church and restarted only at the end of the Second World War. However, his caution did not save him, as the authorities knew he was a believer and a professor of the old generation. The authors of the book describe the "Luzin Affair", initiated by a communist mathematician, Ernst Kolman. Tragically, many of Luzin's former students and friends, including some famous mathematicians, were against him in the Luzin Affair and agreed that Luzin was a traitor. Luckily, Luzin was saved from imprisonment and death by a letter of support from the famous physicist Peter Kapitsa to Stalin.

Chapter 8 describes the fates of the best-known members of the Lusitania school. It starts with the impressive genealogical tree of Luzin's school, his students, grandstudents, etc., which includes the most famous Russian mathematicians. Traditionally, in Soviet times, the Moscow mathematical school was called the "Luzin school". The name of Egorov as one of its founding fathers was not mentioned at all, because of his arrest and subsequent death. I wish to thank the authors for mentioning this fact and for noting that, even after

not given the credit he deserved. Moscow mathematicians have an obligation to correct this. The authors also present portraits of some of Luzin's famous former students, with an emphasis on Andrei Kolmogorov, Pavel Alexandrov, and Pavel Urysohn. Descriptions of some of their mathematical works are intertwined with information about their personal lives. The friendship of Alexandrov and Urysohn included a very productive collaboration in topology as well as swimming, trips abroad, etc. Urysohn wrote one of his famous mathematical papers on the beach at Batz-sur-mer, just a few days before he drowned while swimming. Alexandrov and Kolmogorov were also friends and collaborators, and they both were among the accusers of their former teacher Luzin in the Luzin Affair. Both of them were asked by the police to write a condemnation of Alexander Solzhenitsyn, calling him a traitor, and both did so. Shortly before his death, Kolmogorov confessed that he would fear the secret police to his last day.

Chapter 9 presents the authors' conclusions about, in particular, the relationship between scientific creativity and religion. There is also a discussion of the history of the further development of the descriptive set theory that Luzin and Suslin created.

This book under review weaves mathematics, history, religion, philosophy, and human drama in a remarkable story that will appeal to a wide audience. It is accessible to nonmathematicians and is also well structured, so that readers interested in specific topics can read parts of the book independently of the rest. I highly recommend this unusual and compelling book.