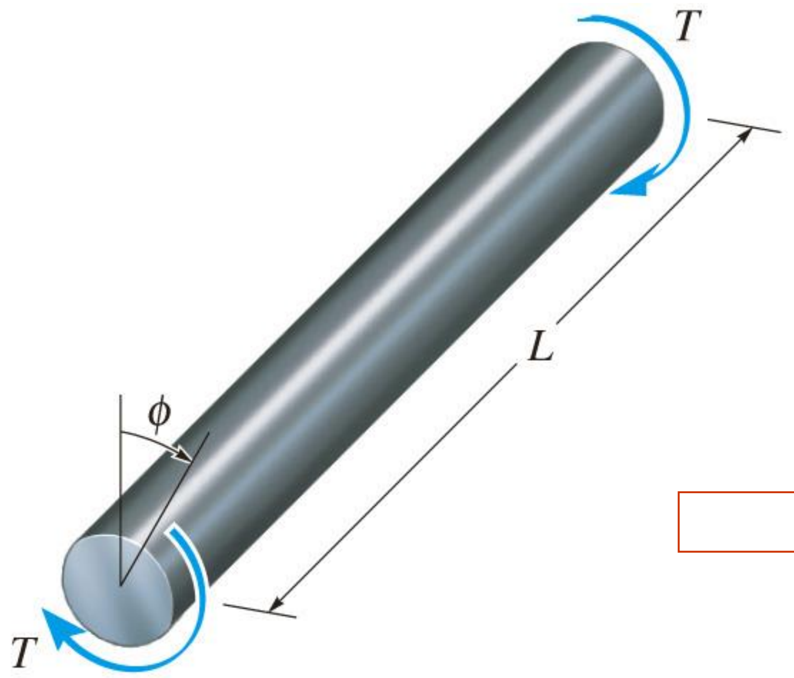
۵-۴: زاویه پیچش

$\phi(x) =$ زاویه پیچش

این زاویه به صورت خطی در امتداد طول محور تغییر می کند

0 در $x = 0$

max در $x = L$



گشتاور

طول

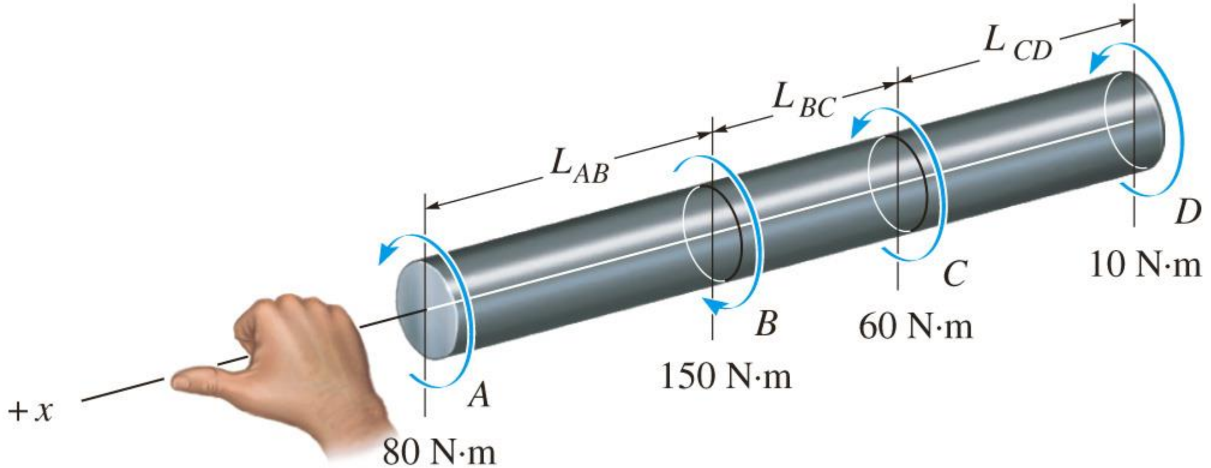
$$\phi = \frac{TL}{JG}$$

مدول صلبیت (مدول برشی)

ممان قطبی اینرسی

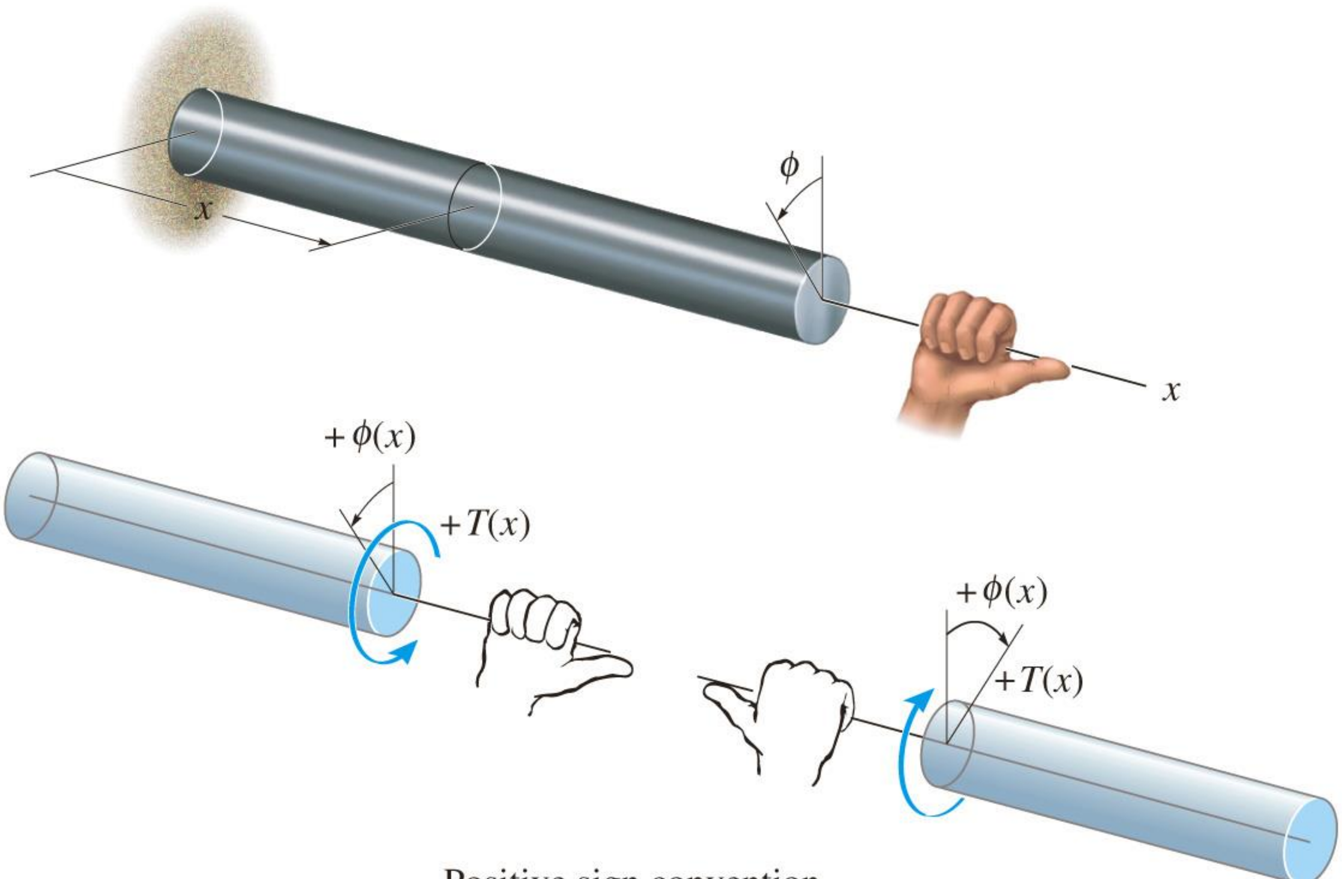
برای مقاطع متعدد

$$\phi = \sum \frac{TL}{JG}$$

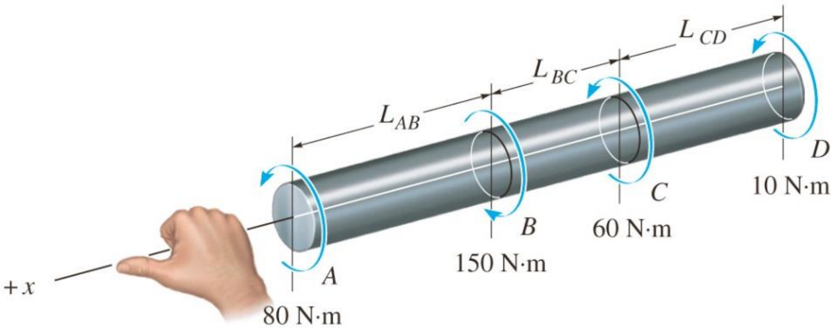


(a)

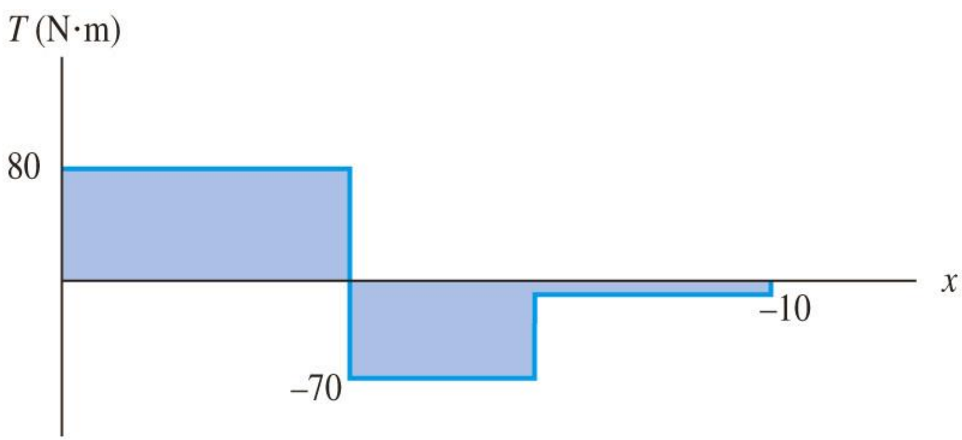
قرارداد علامت ها



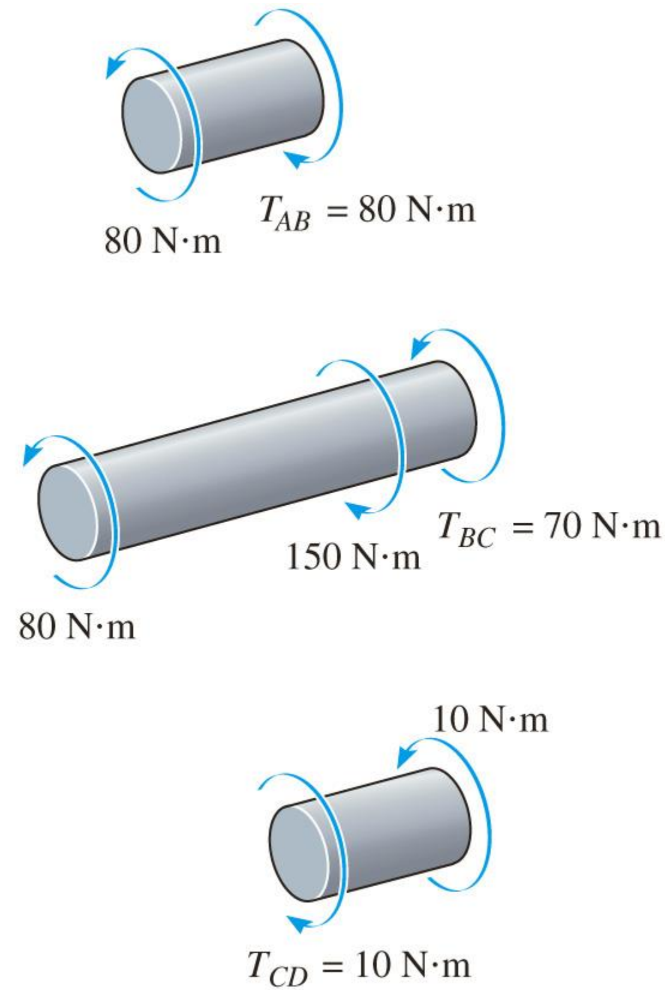
Positive sign convention for T and ϕ.


L_CD
L_BC
L_AB
D
10 N·m
C
60 N·m
B
150 N·m
A
80 N·m
+x

(a)


T (N·m)
80
x
–10
–70

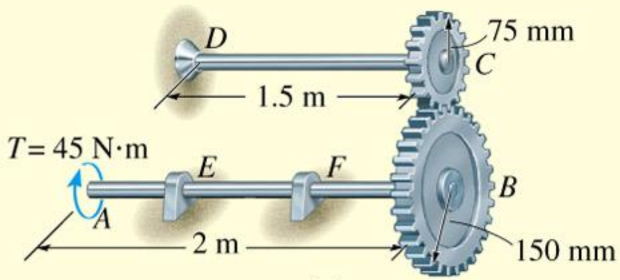
(c)


80 N·m
T_AB = 80 N·m
150 N·m
T_BC = 70 N·m
80 N·m
10 N·m
T_CD = 10 N·m

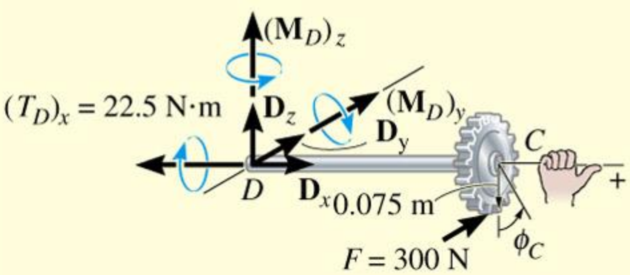
(b)

EXAMPLE 5.8

The two solid steel shafts shown in Fig. 5–21*a* are coupled together using the meshed gears. Determine the angle of twist of end A of shaft AB when the torque $T = 45 \text{ N} \cdot \text{m}$ is applied. Take $G = 80$ GPa. Shaft AB is free to rotate within bearings E and F, whereas shaft DC is fixed at D. Each shaft has a diameter of 20 mm.



(a)



(b)

Fig. 5–21

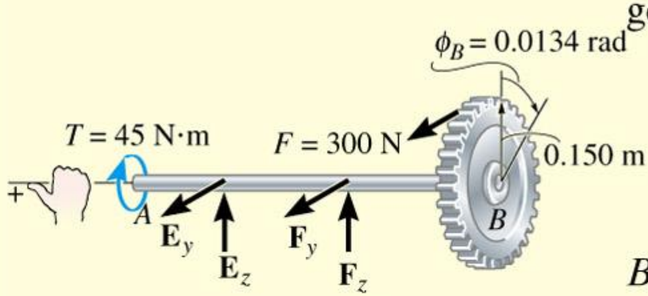
Solution

Internal Torque. Free-body diagrams for each shaft are shown in Fig. 5–21*b* and 5–21*c*. Summing moments along the x axis of shaft AB yields the tangential reaction between the gears of $F = 45 \text{ N} \cdot \text{m}/0.15 \text{ m} = 300 \text{ N}$. Summing moments about the x axis of shaft DC, this force then creates a torque of $(T_D)_x = 300 \text{ N}(0.075 \text{ m}) = 22.5 \text{ N} \cdot \text{m}$ on shaft DC.

Angle of Twist. To solve the problem, we will first calculate the rotation of gear C due to the torque of 22.5 N·m in shaft DC, Fig. 5–21*b*. This angle of twist is

$$\phi_C = \frac{TL_{DC}}{JG} = \frac{(+22.5\ \text{N}\cdot\text{m})(1.5\ \text{m})}{(\pi/2)(0.010\ \text{m})^4[80(10^9)\ \text{N/m}^2]} = +0.0269\ \text{rad}$$

Since the gears at the end of the shaft are in mesh, the rotation ϕ_C of gear C causes gear B to rotate ϕ_B, Fig. 5–21*c*, where



(c)

Fig. 5–21

$$\phi_B(0.15\ \text{m}) = (0.0269\ \text{rad})(0.075\ \text{m})$$
$$\phi_B = 0.0134\ \text{rad}$$

We will now determine the angle of twist of end A with respect to end B of shaft AB caused by the 45 N·m torque, Fig. 5–21*c*. We have

$$\phi_{A/B} = \frac{T_{AB}L_{AB}}{JG} = \frac{(+45\ \text{N}\cdot\text{m})(2\ \text{m})}{(\pi/2)(0.010\ \text{m})^4[80(10^9)\ \text{N/m}^2]} = +0.0716\ \text{rad}$$

The rotation of end A is therefore determined by adding ϕ_B and $\phi_{A/B}$, since both angles are in the *same direction*, Fig. 5–21*c*. We have

$$\phi_A = \phi_B + \phi_{A/B} = 0.0134\ \text{rad} + 0.0716\ \text{rad} = +0.0850\ \text{rad} \quad \textit{Ans.}$$

EXAMPLE 5.9

The 2-in.-diameter solid cast-iron post shown in Fig. 5–22*a* is buried 24 in. in soil. If a torque is applied to its top using a rigid wrench, determine the maximum shear stress in the post and the angle of twist at its top. Assume that the torque is about to turn the post, and the soil exerts a uniform torsional resistance of t lb · in./in. along its 24-in. buried length. $G = 5.5(10^3)$ ksi.

Solution

Internal Torque. The internal torque in segment AB of the post is constant. From the free-body diagram, Fig. 5–22*b*, we have

$$\Sigma M_z = 0; \qquad T_{AB} = 25\text{ lb}(12\text{ in.}) = 300\text{ lb}\cdot\text{in.}$$

The magnitude of the uniform distribution of torque along the buried segment BC can be determined from equilibrium of the entire post, Fig. 5–22*c*. Here

$$\Sigma M_z = 0 \qquad 25\text{ lb}(12\text{ in.}) - t(24\text{ in.}) = 0$$

$$t = 12.5\text{ lb}\cdot\text{in./in.}$$

Hence, from a free-body diagram of a section of the post located at the position x within region BC, Fig. 5–22*d*, we have

$$\Sigma M_z = 0; \qquad T_{BC} - 12.5x = 0$$

$$T_{BC} = 12.5x$$

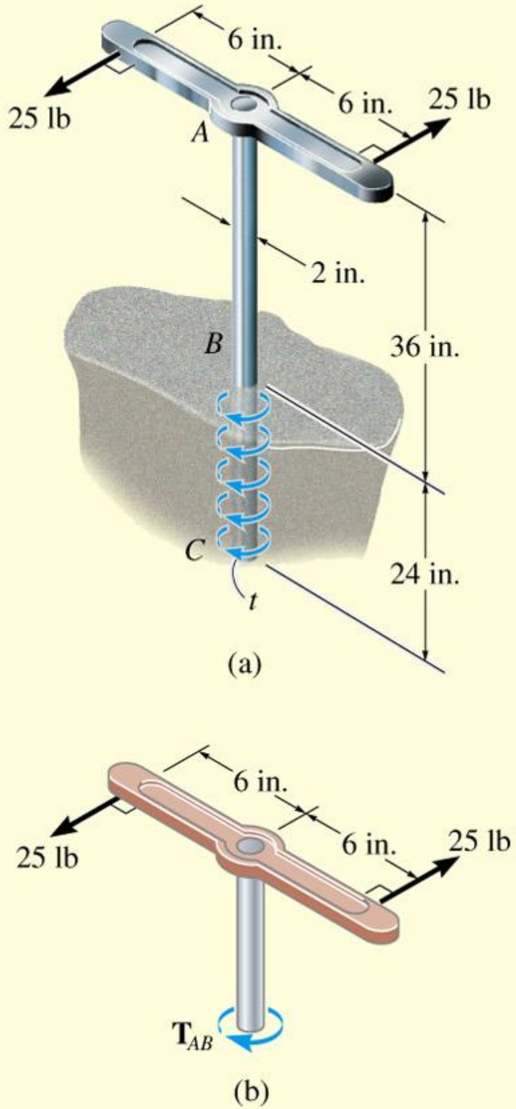


Fig. 5–22

Maximum Shear Stress. The largest shear stress occurs in region AB, since the torque is largest there and J is constant for the post. Applying the torsion formula, we have

$$\tau_{max} = \frac{T_{AB}c}{J} = \frac{(300\ \text{lb}\cdot\text{in.})(1\ \text{in.})}{(\pi/2)(1\ \text{in.})^4} = 191\ \text{psi} \qquad \textit{Ans.}$$

Angle of Twist. The angle of twist at the top can be determined relative to the bottom of the post, since it is fixed and yet is about to turn. Both segments AB and BC twist, and so in this case we have

$$\phi_A = \frac{T_{AB}L_{AB}}{JG} + \int_0^{L_{BC}} \frac{T_{BC}\,dx}{JG}$$

$$= \frac{(300\ \text{lb}\cdot\text{in.})}{JG} + \int_0^{24\text{in.}} \frac{12.5x\,dx}{JG}$$

$$= \frac{10\,800\ \text{lb}\cdot\text{in}^2}{JG} + \frac{12.5[(24)^2/2]\ \text{lb}\cdot\text{in}^2}{JG}$$

$$= \frac{14\,400\ \text{lb}\cdot\text{in}^2}{(\pi/2)(1\ \text{in.})^4 5500(10^3)\ \text{lb/in}^2} = 0.00167\ \text{rad} \qquad \textit{Ans.}$$

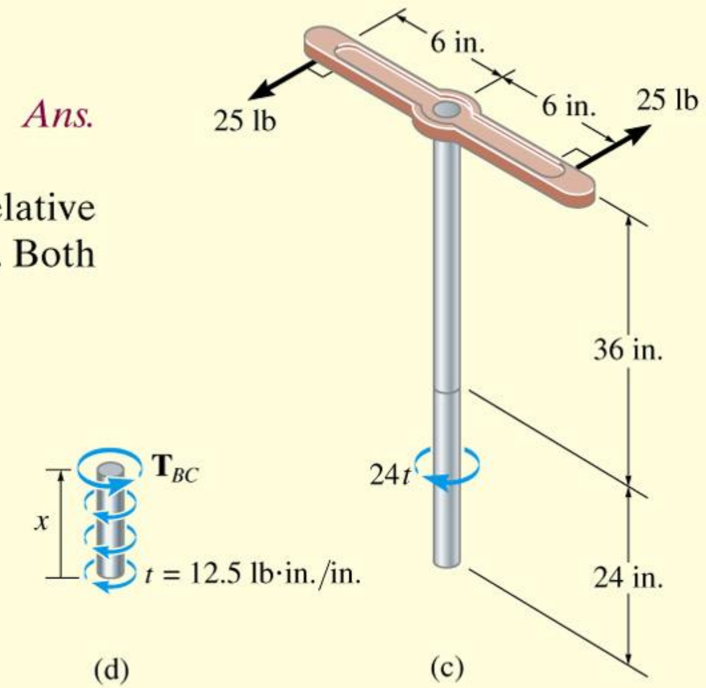


Fig. 5–22