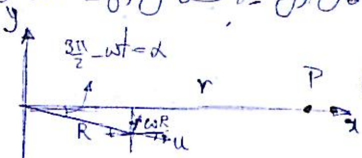
بسمه تعالی

امتحان اول - تابستان 85 - دوره‌ی 19

1)



$$\begin{cases} y = -R\sin\alpha + (\omega R\cos\alpha - u\sin\alpha)(T-t) \ , & \sin\alpha = -\cos(\omega t) \\ x = R\cos\alpha + (\omega R\sin\alpha + u\cos\alpha)(T-t) \ , & \cos\alpha = -\sin(\omega t) \end{cases}$$

$$T - t = t' \to T = t' + t$$

$$x^2 + y^2 = R^2 + (\omega^2R^2+u^2)t'^2 - 2\omega R^2 t'\sin\alpha\cos\alpha + 2Rut'\sin^2\alpha + 2\omega R^2 t'\sin\alpha\cos\alpha + 2Rut'\cos^2\alpha \to$$

$$(\omega^2R^2+u^2)t'^2 + 2Rut' - (r^2-R^2) = 0 \to t' = \frac{-Ru + \sqrt{R^2u^2 + (r^2-R^2)(\omega^2R^2+u^2)}}{\omega^2R^2+u^2} \ ,$$

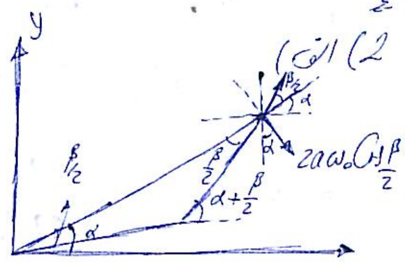
$$\frac{y+R\sin\alpha}{x-R\cos\alpha} = \frac{\omega R\cos\alpha - u\sin\alpha}{\omega R\sin\alpha + u\cos\alpha} \to \omega R^2\sin^2\alpha + Ru\sin\alpha\cos\alpha = r(\omega R\cos\alpha - u\sin\alpha) - \omega R^2\cos^2\alpha + Ru\sin\alpha\cos\alpha$$

$$\to (\omega R\cos\alpha - u\sin\alpha) = \frac{\omega R^2}{r} \quad (1) \to -R\cos(\omega t) = \frac{\omega R^2}{r}t' \to t = \frac{1}{\omega}\cos^{-1}\left(\frac{\omega R\left(Ru - \sqrt{R^2u^2+(r^2-R^2)(\omega^2R^2+u^2)}\right)}{r(\omega^2R^2+u^2)}\right)$$

$$\to \boxed{T = \frac{\sqrt{R^2u^2+(r^2-R^2)(\omega^2R^2+u^2)} - Ru}{\omega^2R^2+u^2} + \frac{1}{\omega}\cos^{-1}\left(\frac{\omega R\left(Ru - \sqrt{R^2u^2+(r^2-R^2)(u^2+\omega^2R^2)}\right)}{r(u^2+\omega^2R^2)}\right)}$$

$u \ll \omega R$ تقریب $\to u = 0 \to T = \frac{\omega R\sqrt{r^2-R^2}}{\omega^2R^2} + \frac{1}{\omega}\cos^{-1}\left(\frac{-\omega R\cdot\omega R\sqrt{r^2-R^2}}{r\cdot\omega^2R^2}\right) \to \boxed{T = \frac{\sqrt{r^2-R^2}}{\omega R} + \frac{1}{\omega}\cos^{-1}\left(-\frac{\sqrt{r^2-R^2}}{r}\right)}$

2) الف)



$$\begin{cases} y = 2a\cos(\frac{\beta}{2})\sin\alpha + \left(v_0\sin(\alpha+\frac{\beta}{2}) - 2a\omega_0\cos(\frac{\beta}{2})\cos\alpha\right)(t-T) \to \\ x = 2a\cos(\frac{\beta}{2})\cos\alpha + \left(v_0\cos(\alpha+\frac{\beta}{2}) + 2a\omega_0\cos(\frac{\beta}{2})\sin\alpha\right)(t-T) \end{cases}$$

$$\begin{cases} y = -2a\cos(\frac{\beta}{2})\sin(\omega T) + \left(\sin(\frac{\beta}{2}-\omega T) - \sin\frac{\beta}{2}\cos(\omega T)\right)v_0(t-T) \to \\ x = 2a\cos\frac{\beta}{2}\cos(\omega T) + \left(\cos(\frac{\beta}{2}-\omega T) + \sin\frac{\beta}{2}\sin(\omega T)\right)v_0(t-T) \end{cases}$$

$$\begin{cases} \alpha + \omega T = 2\pi \to \\ \sin\alpha = -\sin(\omega T) \\ \cos\alpha = \cos(\omega T) \\ \sin(\alpha+\frac{\beta}{2}) = \sin(\frac{\beta}{2}-\omega T) \\ \cos(\alpha+\frac{\beta}{2}) = \cos(\frac{\beta}{2}-\omega T) \end{cases}$$

$$\begin{cases} y = -2a\cos(\frac{\beta}{2})\sin(\omega T) - \cos\frac{\beta}{2}\sin(\omega T)\,v_0(t-T) \\ x = 2a\cos(\frac{\beta}{2})\cos(\omega T) + \cos\frac{\beta}{2}\cos(\omega T)\,v_0(t-T) \end{cases} \to \frac{y + 2a\cos(\frac{\beta}{2})\sin(\omega T)}{x - 2a\cos(\frac{\beta}{2})\cos\omega T} = \frac{-\sin(\omega T)}{\cos(\omega T)}$$

$$\to y\cos\omega T + 2a\cos\frac{\beta}{2}\sin(\omega T)\cos(\omega T) = -x\sin(\omega T) + 2a\cos\frac{\beta}{2}\sin(\omega T)\cos(\omega T)$$

$$\to \boxed{y = -x\tan(\omega T)}$$

ب)

$$\begin{cases} y = -2a\cos(\frac{\beta}{2})\sin(\omega t) - \cos\frac{\beta}{2}\sin(\omega t)\,v_0(T-t) \\ x = 2a\cos(\frac{\beta}{2})\cos(\omega t) + \cos\frac{\beta}{2}\cos(\omega t)\,v_0(T-t) \end{cases} \to \begin{cases} y = -\left(2a + v_0(T-t)\right)\cos\frac{\beta}{2}\sin(\omega t) \\ x = \left(2a + v_0(T-t)\right)\cos\frac{\beta}{2}\cos(\omega t) \end{cases}$$

$$y = -x\tan(\omega t) \to \tan\theta = -\tan(\omega t) \to \theta + \omega t = 2\pi \to t = \frac{2\pi-\theta}{\omega} \to \boxed{\theta = \alpha} \to$$

$$r\cos\theta = \left(2a + v_0\left(T + \frac{\theta-2\pi}{\omega}\right)\right)\cos\frac{\beta}{2}\cos\theta \to \boxed{r = \left(2a + v_0\left(T - \frac{2\pi-\theta}{\omega}\right)\right)\cos\frac{\beta}{2}}$$

4)

$$\begin{cases} \vec{AB} = (15, -4, -7) \ ; & P_1 = (-16+15t,\ 6-4t,\ 4-7t) \\ \vec{CD} = (3, 10, 4) \ ; & P_2 = (1+3u,\ 10u-1,\ 3+4u) \end{cases} \to \begin{cases} \vec{P_1P_2}\cdot\vec{AB} = 0 \\ \vec{P_1P_2}\cdot\vec{CD} = 0 \end{cases}$$

بسمه تعالی

$$\overrightarrow{P_1P_2}=(-17+15t-3u\,,\,7-4t-10u\,,\,1-7t-4u)\longrightarrow$$

$$\begin{cases}15(-17+15t-3u)-4(7-4t-10u)-7(1-7t-4u)=0\\ 3(-17+15t-3u)+10(7-4t-10u)+4(1-7t-4u)=0\end{cases}\longrightarrow$$

$$\begin{cases}-255+225t-45u-28+16t+40u-7+49t+28u=0\\ -51+45t-9u+70-40t-100u+4-28t-16u=0\end{cases}\longrightarrow\begin{cases}290t+23u-290=0\\ -23t-125u-63=0\end{cases}\longrightarrow\ldots$$

روش دوم :

$$\begin{cases}P_1=(-1+15t\,,\,2-4t\,,\,-3-7t)\\ P_2=(1+3u\,,\,-1+10u\,,\,3+4u)\end{cases}\longrightarrow \overrightarrow{P_1P_2}=(15t-3u-2\,,\,3-4t-10u\,,\,-6-7t-4u)\longrightarrow$$

$$\begin{cases}15(15t-3u-2)-4(3-4t-10u)+7(6+7t+4u)=0\\ 3(15t-3u-2)+10(3-4t-10u)-4(6+7t+4u)=0\end{cases}\longrightarrow$$

(5

$$\begin{cases}\dfrac{2v_0^2\sin\theta\cos\theta}{g}\geq R\\ R'\leq R+d\end{cases}\quad -h=\frac{-gR'^2}{2v_0^2\cos^2\theta}+R'\tan\theta\longrightarrow (g)R'^2-(2v_0^2\sin\theta\cos\theta)R'-2hv_0^2\cos^2\theta=0\longrightarrow$$

$$R'=\frac{2v_0^2\sin\theta\cos\theta+\sqrt{4v_0^4\sin^2\theta\cos^2\theta+4g\times 2hv_0^2\cos^2\theta}}{2g}\longrightarrow$$

$$R'=\frac{v_0^2\sin\theta\cos\theta+v_0\cos\theta\sqrt{v_0^2\sin^2\theta+2gh}}{g}\longrightarrow R'=\frac{v_0\cos\theta}{g}\left(v_0\sin\theta+\sqrt{v_0^2\sin^2\theta+2gh}\right)\longrightarrow$$

$$\frac{v_0\cos\theta}{g}\left(v_0\sin\theta+\sqrt{v_0^2\sin^2\theta+2gh}\right)\leq R+d\longrightarrow\sqrt{v_0^2\sin^2\theta+2gh}\leq\frac{g(R+d)}{v_0\cos\theta}-v_0\sin\theta\longrightarrow$$

$$\cancel{v_0^2\sin^2\theta}+2gh\leq\cancel{v_0^2\sin^2\theta}+\frac{g^2(R+d)^2}{v_0^2\cos^2\theta}-\frac{2g(R+d)\sin\theta}{\cos\theta}\longrightarrow 2ghv_0^2\cos^2\theta\leq g^2(R+d)^2-2v_0^2g(R+d)\sin\theta\cos\theta$$

$$\longrightarrow v_0^2\leq\frac{g^2(R+d)^2}{2g\cos\theta\left(h\cos\theta+(R+d)\sin\theta\right)}\longrightarrow v_0\leq(R+d)\sqrt{\frac{g}{2\cos\theta\left(h\cos\theta+(R+d)\sin\theta\right)}}$$

$$v_0\geq\sqrt{\frac{Rg}{2\sin\theta\cos\theta}}\longrightarrow\boxed{\sqrt{\frac{Rg}{2\sin\theta\cos\theta}}\leq v_0\leq(R+d)\sqrt{\frac{g}{2\cos\theta\left(h\cos\theta+(R+d)\sin\theta\right)}}}$$

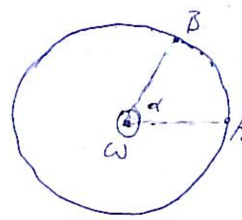
$$\longrightarrow\frac{R\cancel{g}}{2\sin\theta\cos\theta}\leq\frac{(R+d)^2\cancel{g}}{2\cos\theta\left(h\cos\theta+(R+d)\sin\theta\right)}\longrightarrow Rh\cos\theta+R^2\sin\theta+Rd\sin\theta\leq R^2\sin\theta+d^2\sin\theta+2Rd\sin\theta$$

$$\longrightarrow Rh\cos\theta\leq d^2\sin\theta+Rd\sin\theta\longrightarrow\boxed{\tan\theta\geq\frac{Rh}{(R+d)d}}$$

ادامه 4 :

$$\begin{cases}225t-45u-30-12+16t+40u+42+49t+28u=0\\ 45t-9u-6+30-40t-100u-24-28t-16u=0\end{cases}\longrightarrow\begin{cases}290t=-23u\\ -23t=125u\end{cases}\longrightarrow\boxed{u=t=0}\longrightarrow$$

$$\overrightarrow{P_1P_2}=(-2\,,\,3\,,\,-6)\longrightarrow r=\sqrt{4+9+36}\longrightarrow\boxed{r=7}$$



(a (b ≠
ib

$$\begin{cases}\tau=T+\dfrac{\alpha}{2\pi}d\longrightarrow\dfrac{\tau}{d}=\dfrac{T}{d}+\dfrac{\alpha}{2\pi}\\ T=\dfrac{S\alpha R\cos\lambda}{v}\longrightarrow\alpha=\dfrac{SvT}{R\cos\lambda}\end{cases}\longrightarrow\boxed{\frac{\tau}{d}=\frac{T}{d}\left(1+\frac{vSd}{2\pi R\cos\lambda}\right)}\ (a$$

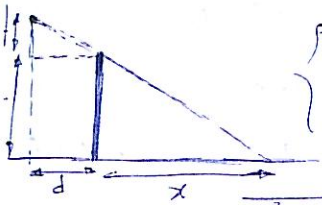
$$\boxed{\frac{\tau}{d}=\frac{\cancel{T}}{\cancel{d}}\frac{\alpha}{2\pi}\left(1+\frac{2\pi RS\cos\lambda}{vd}\right)}\ (b$$

$$\tau=0\longrightarrow\frac{T}{d}+\frac{\alpha}{2\pi}=0\longrightarrow\boxed{T=-\left(\frac{\alpha}{2\pi}\right)d}\,,\,S<0\ \Big/\ \frac{\tau}{d}=1\longrightarrow\boxed{T=\left(1-\frac{\alpha}{2\pi}\right)d}\,,\,S>0\quad (c$$

بسمه تعالی

آزمون دوم - تابستان 89 - دوره ی 19

1) الف)



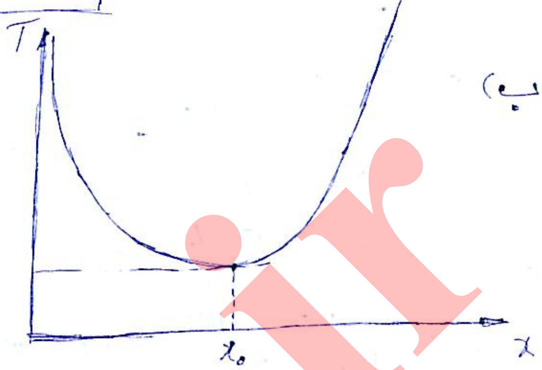
$$\begin{cases} T = t + \dfrac{\sqrt{(d+x)^2+(L+v_0 t)^2}}{c} \\ \dfrac{v_0 t}{L} = \dfrac{d}{x} \to t = \dfrac{Ld}{v_0 x} \end{cases} \to T = \frac{Ld}{v_0 x} + \frac{\sqrt{(d+x)^2+L^2\left(1+\frac{d}{x}\right)^2}}{c} \to$$

$$\boxed{T = \frac{Ld}{v_0 x} + \frac{(x+d)\sqrt{x^2+L^2}}{cx}}$$

ج)

$$1 = \frac{-Ld}{v_0 x^2}\dot{x} + \frac{x\sqrt{x^2+L^2} + \frac{x^2(x+d)}{\sqrt{x^2+L^2}} - (x+d)\sqrt{x^2+L^2}}{cx^2}\dot{x}$$

ب)



$$\frac{1}{\dot{x}} = -\frac{Ld}{v_0 x^2} + \frac{x^3+x^2d-x^2d-L^2d}{cx^2\sqrt{x^2+L^2}} \to v_{(x)} = -\frac{Ld}{v_0 x^2} + \frac{x^3-L^2d}{cx^2\sqrt{x^2+L^2}}$$

د)

$$v_{(x)} = 0 \to \frac{Ld}{v_0} = \frac{x_0^3-L^2d}{c\sqrt{x_0^2+L^2}} \to \boxed{Ld\sqrt{x_0^2+L^2} = \frac{v_0}{c}x_0^3 - \frac{v_0}{c}L^2d} \to$$

ه)

$$\frac{Ld}{x_0^2}\sqrt{1+\frac{L^2}{x_0^2}} = \frac{v_0}{c} - \frac{v_0}{c}\frac{L^2d}{x_0^3} \xrightarrow{x_0 \gg L,\, d} \frac{Ld}{x_0^2} = \frac{v_0}{c} \to \boxed{x_0 = \sqrt{\frac{LCd}{v_0}}}$$

2) الف)

$$\begin{cases} a^2+c^2-2ac\cos\alpha = b^2 \\ b^2+c^2-2bc\cos\beta = a^2 \end{cases} \to \begin{cases} \boxed{\cos\alpha = \dfrac{-b^2+a^2+c^2}{2ac}} \to \boxed{\alpha = \cos^{-1}\left(\dfrac{a^2+c^2-b^2}{2ac}\right)} \\ \boxed{\cos\beta = \dfrac{b^2+c^2-a^2}{2bc}} \to \boxed{\beta = \cos^{-1}\left(\dfrac{b^2+c^2-a^2}{2bc}\right)} \end{cases}$$

ب)

$$\begin{cases} F\sin\gamma = F_1\cos\alpha - F_2\cos\beta \\ F\cos\gamma = F_1\sin\alpha + F_2\sin\beta \end{cases} \to F(\sin\gamma\sin\beta + \cos\gamma\cos\beta) = F_1(\cos\alpha\sin\beta + \sin\alpha\cos\beta) \to \boxed{F_1 = \frac{\cos(\gamma-\beta)}{\sin(\alpha+\beta)}F}$$

$$\to F_2 = \frac{\cos\gamma\cos\alpha - \sin\alpha\sin\alpha}{\sin\beta\cos\alpha + \cos\beta\sin\alpha}F \to \boxed{F_2 = \frac{\cos(\gamma+\alpha)}{\sin(\alpha+\beta)}F}$$

3) الف)

$$[R] = m, \quad [v] = \frac{\frac{kg}{ms}}{\frac{kg}{m^3}} \to [v] = \frac{m^2}{s}, \quad [\rho] = \frac{kg}{m^3}, \quad [\omega] = \frac{1}{s}, \quad [c] = \frac{m}{s}, \quad [u] = \frac{m}{s}, \quad [P] = \frac{kg\cdot\frac{m^2}{s^2}}{s} \to [P] = \frac{kg\,m^2}{s^3}$$

ب)

$$P = u^\alpha \times c^\gamma \times v^\theta \times \omega^z \times R^\beta \times \rho^\delta \to \frac{m^\alpha}{s^\alpha}\cdot\frac{m^\gamma}{s^\gamma}\cdot\frac{m^{2\theta}}{s^\theta}\cdot\frac{1}{s^z}\cdot m^\beta \times \frac{kg^\delta}{m^{3\delta}} = \frac{kg\,m^2}{s^3} \to \boxed{P = \rho R^5\omega^2 f\left(\frac{v}{\omega R^2}, \frac{c}{\omega R}, \frac{u}{\omega R}\right)}$$

الف

$$(\alpha+\gamma)+2\theta+\beta-3\delta = 2 \to \theta = 1 + \frac{3\delta-\beta-(\alpha+\gamma)}{2}$$

$$(\alpha+\gamma)+\theta+z = 3 \to z = 3-\theta-(\alpha+\gamma) \to 3-(\alpha+\gamma) = 1+z+\frac{3}{2}-\frac{\beta+(\alpha+\gamma)}{2} \to \boxed{z = \frac{1-(\alpha+\gamma)+\beta}{2}}$$

$$\boxed{\delta = 1} \to P = u^\alpha c^\gamma \times v^{\frac{5-\beta-(\alpha+\gamma)}{2}} \times \omega^{\frac{1-(\alpha+\gamma)+\beta}{2}} \times R^\beta \times \rho \to P = \frac{u^2}{c^3}\cdot v^{\frac{5-\beta+1}{2}} \times \omega^{\frac{1+1+\beta}{2}} \times R^\beta \times \rho$$

ج)

$$\beta > 0 \to \frac{\beta}{2} > 0 \to 1+\frac{\beta}{2} > 1 \to \boxed{1 < z < 4}$$

$$\frac{5-\beta+1}{2} > 0 \to \frac{\beta}{2} < 3 \to 1+\frac{\beta}{2} < 4$$

4) الف)

$$v_0 = 8\times\frac{kq}{\frac{\sqrt{3}}{2}a} \to \boxed{v_0 = \frac{16kq}{\sqrt{3}a}}$$

ب)

$$V = v_0 + a_1x + a_2y + a_3z + a_4x^2 + a_5y^2 + a_6z^2 + a_7xy + a_8yz + a_9zx$$

$$V_{(x)} = V_{(-x)}, \quad V_{(y)} = V_{(-y)} \to V = v_0 + a_4x^2 + a_5y^2 + a_6z^2, \quad V_z = V_{(-z)}$$

$$\vec{E}_{(0,0,0)} = 0 \to -\vec{E} = (2a_4x)\hat{i} + (2a_5y)\hat{j} + (2a_6z)\hat{k} = 0$$

$$\nabla^2 V = 0 \to 2a_4 + 2a_5 + 2a_6 = 0 \to a_4 + a_5 + a_6 = 0 \to a_4 = a_5 = a_6 = 0$$

موفق باشید

باسمه تعالی

ج) $\boxed{\vec{E}=0}$

$\longrightarrow \boxed{V_{(2)}=V_0}$ جواب (ب)

5) الف) $v=\rho^{\alpha}\times b^{\gamma}\times A^{\beta}$, $[A]=m^2$, $[b]=\frac{kg}{s^2}$, $[v]=\frac{1}{s}$, $[\rho]=\frac{kg}{m^3}$ $\rightarrow \frac{1}{s}=\frac{kg^{\alpha}}{m^{3\alpha}}\cdot\frac{kg^{\gamma}}{s^{2\gamma}}\cdot m^{2\beta} \rightarrow \begin{cases}2\beta=3\alpha\rightarrow\beta=-\frac{3}{4}\\ \alpha=-\gamma\rightarrow\alpha=-\frac{1}{2}\\ 2\gamma=1\rightarrow\gamma=\frac{1}{2}\end{cases}$

$\longrightarrow \boxed{v=\sqrt{\frac{b}{\rho}}\cdot\frac{1}{\sqrt[4]{A^3}}}$

ب) با یک آزمایش آب توری را داخل یک لیوان عادی می‌ریزیم و زمان را اندازه می‌گیریم.

$\frac{dV}{dt}=\frac{dV}{dt}\Rightarrow AV=\frac{\Delta V}{\Delta t}\rightarrow \pi(0.5)^2\cdot V=\frac{\pi\cdot(6.5)^2\times 8}{4\times4.5}\rightarrow V=\frac{8\times(3.25)^2}{4.5\times(0.5)^2}\rightarrow V=\frac{8\times325}{450\times25}\simeq0.7\,\frac{m}{s}$

$v=\sqrt{\frac{0.7}{10^3}}\cdot\frac{1000}{\sqrt[4]{(\pi\cdot(0.5)^2)^3}}\rightarrow v\simeq\frac{\sqrt{7}}{100}\times1190\rightarrow v\simeq30$

$\rightarrow\lambda=\frac{V}{v}\rightarrow\boxed{\lambda\simeq2.3}\,cm\rightarrow\boxed{\lambda=0.023}\,m$

سوال 6 امتحان اول: d) $\boxed{S=-1}\rightarrow\omega=\frac{2\pi}{d}=\frac{v}{R\cos\lambda}\rightarrow\boxed{\cos\lambda=\frac{vd}{2\pi R}}\rightarrow\cos\alpha=0.36\rightarrow\boxed{\alpha\simeq69^\circ}$

e) هواپیما باید 2π نسبت ناظر ساکن بچرخد $\rightarrow\boxed{T=\frac{d}{2}=12h}$ ، λ طوری است که سرعت هواپیما با سرعت دوران زمین برابر است.

6) b) $n=\frac{\omega_p^2 m}{\left(\frac{q^2}{\varepsilon_0}\right)}\rightarrow\frac{db}{dt}=nqV_{(t)}\rightarrow\boxed{\frac{db}{dt}=\frac{m\omega_p^2\varepsilon_0}{q}V}$ a) $E=E_0-\frac{b_{(t)}}{\varepsilon_0}$

c, d) $m\dot{v}=q\left(E_0-\frac{b}{\varepsilon_0}\right)-\frac{2m}{\tau}v\rightarrow\ddot{v}=-\frac{q\dot{b}}{m\varepsilon_0}-\frac{2}{\tau}\dot{v}=-\omega_p^2v-\frac{2}{\tau}\dot{v}\rightarrow\ddot{v}+\left(\frac{2}{\tau}\right)\dot{v}+\omega_p^2v=0\rightarrow$

$v=Ae^{st}\rightarrow s^2+\frac{2}{\tau}s+\omega_p^2=0\rightarrow s=-\frac{1}{\tau}\pm\sqrt{\frac{1}{\tau^2}-\omega_p^2}=\frac{1}{\tau}\left(-1\pm\sqrt{1-\omega_p^2\tau^2}\right)\rightarrow V=A_1e^{s_1t}+A_2e^{s_2t}\rightarrow$

$\begin{cases}\dot{V}_{(0)}=\frac{qE_0}{m}\rightarrow A_1s_1+A_2s_2=0\\ V_{(0)}=0\rightarrow A_1+A_2=0\rightarrow A_2=-A_1\end{cases}\rightarrow A_1(s_1-s_2)=\frac{qE_0}{m}\rightarrow A_1=\frac{2qE_0\tau}{m\sqrt{1-\omega^2\tau^2}}\rightarrow A_2=-\frac{2qE_0\tau}{m\sqrt{1-\omega^2\tau^2}}\rightarrow$

$\boxed{V=\frac{2qE_0\tau}{m\sqrt{1-\omega^2\tau^2}}e^{-\frac{t}{\tau}}\sinh\left(\frac{t}{\tau}\sqrt{1-\omega^2\tau^2}\right)}$

2) ج) $\boxed{\delta a=\frac{F\cos(\beta-\gamma)}{k\sin(\alpha+\gamma)}}$ $\boxed{\Delta b=\frac{F\cos(\alpha+\gamma)}{k\sin(\alpha+\beta)}}$

سوال 3 امتحان اول: 3) الف) $h=\frac{c}{r}\rightarrow h_0+\alpha x^2=\frac{c}{r_{(x)}}\rightarrow\boxed{r_{(x)}=\frac{c}{h_0+\alpha x^2}}$

ب) $\phi=\frac{2\pi x\,dx\cdot f_{(x)}\cdot\pi r_{(x)}^2\cdot h_{(x)}}{2\pi x\,dx\cdot h_{(x)}}\rightarrow f_{(x)}=\frac{\phi}{\pi r_{(x)}^2}\rightarrow\boxed{f_{(x)}=\frac{\phi}{\pi c^2}(h_0+\alpha x^2)^2}$

ج) $n_{(x+dx)}-n_{(x)}=2\pi x f_{(x)}dx\rightarrow n_{(x)}=2\pi\int_0^x f_{(x)}x\,dx\rightarrow n_{(x)}=\frac{2\pi}{2\alpha}\cdot\frac{\phi}{\pi c^2}\int_0^x(h_0+\alpha x^2)^2d(\alpha x^2+h)\rightarrow$

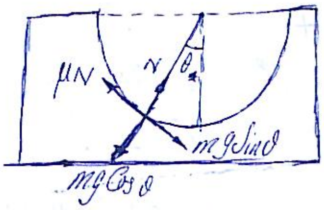
$\boxed{n_{(x)}=\frac{\phi}{3\alpha c^2}\left((h_0+\alpha x^2)^3-h_0^3\right)}$

بسمه تعالی

آزمون سوم - تابستان 85

(a (1

$$\begin{cases} mg\cos\theta - N = mR\dot\theta^2 \\ \mu N - mg\sin\theta = mR\ddot\theta \end{cases} \longrightarrow \mu g\cos\theta + \mu R\dot\theta^2 - g\sin\theta = R\ddot\theta \longrightarrow \boxed{-R\ddot\theta + g(\mu\cos\theta - \sin\theta) + \mu R\dot\theta^2 = 0}$$



(b

$$-\frac{R}{2}\cdot\frac{1}{R^2}\frac{dv^2}{d\theta} + g(\mu\cos\theta - \sin\theta) + \mu\frac{v^2}{R} = 0 \longrightarrow \boxed{\frac{dv^2}{d\theta} - 2\mu v^2 = 2gR(\mu\cos\theta - \sin\theta)}$$

$$\longrightarrow \boxed{C_0 = -2\mu}\ ,\ \boxed{C_1 = 2\mu Rg}\ ,\ \boxed{C_2 = -2gR}$$

(c

$$v^2 = A_0 e^{-C_0\theta} + A_1\cos\theta + A_2\sin\theta \longrightarrow -A_0C_0e^{-C_0\theta} - A_1\sin\theta + A_2\cos\theta - 2\mu A_0 e^{-C_0\theta} - 2\mu A_1\cos\theta - 2\mu A_2\sin\theta = 2\mu Rg\cos\theta - 2gR\sin\theta$$

$$\longrightarrow \begin{cases} A_1 + 2\mu A_2 = +2gR \\ A_2 - 2\mu A_1 = 2\mu gR \end{cases} \longrightarrow A_2(1+4\mu^2) = 6\mu gR \longrightarrow \boxed{A_2 = \frac{6\mu gR}{1+4\mu^2}} \longrightarrow A_1 = 2\left(\frac{gR(1+4\mu^2)}{1+4\mu^2} - \frac{6\mu^2 gR}{1+4\mu^2}\right) \longrightarrow$$

$$\boxed{A_1 = \frac{2gR(1-2\mu^2)}{1+4\mu^2}}$$

$$\theta = \theta_0 : v = 0 \longrightarrow A_0 e^{2\mu\theta_0} + A_1\cos\theta_0 + A_2\sin\theta_0 = 0 \longrightarrow A_0 e^{2\mu\theta_0} + \frac{2gR}{1+4\mu^2}\left((1-2\mu^2)\cos\theta_0 + 3\mu\sin\theta_0\right) = 0 \longrightarrow$$

$$\boxed{A_0 = -\frac{2gR\left((1-2\mu^2)\cos\theta_0 + 3\mu\sin\theta_0\right)}{e^{2\mu\theta_0}(1+4\mu^2)}}$$

(d

$$v^2 = e^{-C_0\theta}\left(A_0 + \frac{A_1\cos\theta + A_2\sin\theta}{e^{-C_0\theta}}\right) \longrightarrow v^2 = e^{-C_0\theta}\left(A_0 + \frac{A_1\cos\theta + A_2\sin\theta}{e^{-C_0\theta}} - A_0 - \frac{A_1\cos\theta_0 + A_2\sin\theta_0}{e^{-C_0\theta_0}}\right) \longrightarrow$$

$$v^2 = e^{-C_0\theta_0}\left(\frac{A_1\cos\theta + A_2\sin\theta}{e^{-C_0\theta}} - \frac{A_1\cos\theta_0 + A_2\sin\theta_0}{e^{-C_0\theta_0}}\right) \longrightarrow \boxed{f_{(\theta)} = \frac{A_1\cos\theta + A_2\sin\theta}{e^{-C_0\theta}}}$$

(e

$$v^2_{(0)} \geq 0 \longrightarrow f_{(0)} \geq f_{(\frac{\pi}{2})} \longrightarrow A_1 \geq \frac{A_2}{e^{\mu\frac{\pi}{2}}} \longrightarrow A_1 e^{\mu\pi} \geq A_2 \longrightarrow 2gR(1-2\mu^2) \geq \frac{6\mu gR}{e^{\mu\pi}} \longrightarrow \boxed{(1-2\mu^2)\cdot e^{\mu\pi} \geq 3\mu}$$

(f : حالت تعادل

$$\mu mg\cos\theta_0 = mg\sin\theta_0 \longrightarrow \mu = \tan\theta_0 \longrightarrow \tan\theta_0 \leq \mu \longrightarrow \boxed{\theta_0 < \tan^{-1}(\mu)}$$

(g

$$f_{(0)} \geq f_{(\theta_0)} \longrightarrow A_1 e^{2\mu\theta_0} \geq A_1\cos\theta_0 + A_2\sin\theta_0 \longrightarrow A_1 \geq -A_0 \longrightarrow \frac{2gR(1-2\mu^2)}{1+4\mu^2} \geq \frac{2gR\left[3\mu\sin\theta_0 + (1-2\mu^2)\cos\theta_0\right]}{e^{2\mu\theta_0}(1+4\mu^2)} \longrightarrow$$

$$(1-2\mu^2)e^{2\mu\theta_0} \geq 3\mu\sin\theta_0 + (1-2\mu^2)\cos\theta_0 \longrightarrow \boxed{\frac{(1-2\mu^2)}{3\mu} \geq \frac{\sin\theta_0}{e^{2\mu\theta_0} - \cos\theta_0}}$$

(h

$$\begin{cases} \theta_0 > \tan^{-1}(\mu) \\ f_{(0)} < f_{(\theta_0)} \end{cases} \longrightarrow \boxed{\begin{cases} \theta_0 > \tan^{-1}(\mu) \\ \dfrac{1-2\mu^2}{3\mu} < \dfrac{\sin\theta_0}{e^{2\mu\theta_0} - \cos\theta_0} \end{cases}}$$

(2

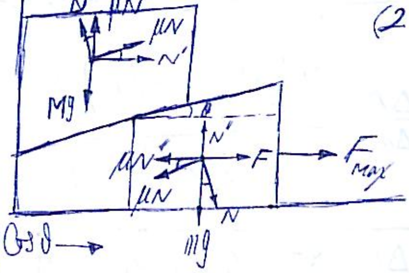
$$\begin{cases} F + N\sin\theta = \mu N\cos\theta + \mu N'' \\ N'' = mg + N\cos\theta + \mu N\sin\theta \end{cases} \longrightarrow F + N\sin\theta - \mu N\cos\theta = \mu mg + \mu N\cos\theta + \mu^2 N\sin\theta \longrightarrow$$

$$F - \mu mg = N\left((\mu^2-1)\sin\theta + 2\mu\cos\theta\right) \quad \text{I}$$

$$Mg = N\left(2\mu\sin\theta + (1-\mu^2)\cos\theta\right) \quad \text{II}$$

$$N' + \mu N\cos\theta = N\sin\theta$$

$$N\cos\theta + \mu N' + \mu N\sin\theta = Mg \longrightarrow \mu N\sin\theta - Mg = \mu^2 N\cos\theta - N\cos\theta - \mu N\sin\theta \longrightarrow$$



$$\text{I},\text{II} \longrightarrow Mg(\mu^2-1)\sin\theta + 2\mu Mg\cos\theta = F\left(2\mu\sin\theta + (1-\mu^2)\cos\theta\right) - 2\mu^2 mg\sin\theta - \mu mg\cos\theta + \mu^3 mg\cos\theta \longrightarrow$$

$$F\left(2\mu\sin\theta + (1-\mu^2)\cos\theta\right) = \mu^2 g(M+2m)\sin\theta + \mu g(2M+m)\cos\theta + \mu^3 mg\cos\theta - Mg\sin\theta \longrightarrow$$

$$\boxed{F_{Max} = \frac{\mu^2(M+2m)\sin\theta + \mu(2M+m)\cos\theta + \mu^3 m\cos\theta - M\sin\theta}{2\mu\sin\theta + (1-\mu^2)\cos\theta}\, g}$$

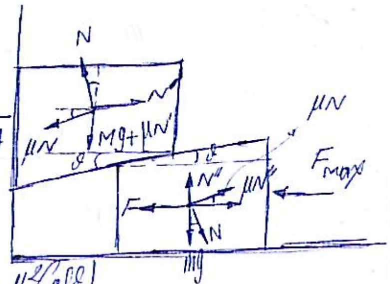
بسمه تعالی

$$\begin{cases} N\cos\theta = \mu N\sin\theta + Mg + \mu N' \\ N\sin\theta + \mu N\cos\theta = N' \\ N'' + \mu N\sin\theta = mg + N\cos\theta \\ F = \mu N'' + \mu N\cos\theta + N\sin\theta \end{cases}$$

$$\rightarrow \mu N\sin\theta + \mu^2 N\cos\theta = N\cos\theta - \mu N\sin\theta - Mg \rightarrow$$

$$\rightarrow \frac{Mg}{\cos\theta - 2\mu\sin\theta - \mu^2\cos\theta} = \frac{F - \mu mg}{2\mu\cos\theta + \sin\theta - \mu^2\sin\theta}$$

$$\rightarrow \mu mg + \mu N\cos\theta - \mu^2 N\sin\theta = F - \mu N\cos\theta - N\sin\theta$$



$$2\mu Mg\cos\theta + Mg\sin\theta - \mu^2 Mg\sin\theta + \mu mg\cos\theta - 2\mu^2 mg\sin\theta - \mu^3 mg\cos\theta = F(\cos\theta - 2\mu\sin\theta - \mu^2\cos\theta)$$

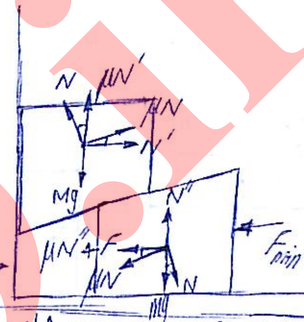
$$\rightarrow \boxed{F_{max} = \frac{\mu(2M+m)\cos\theta + \mu^2\left(\frac{M}{\mu^2} - 2m\right)\sin\theta - \mu^2(M\sin\theta + m\mu\cos\theta)}{\cos\theta - 2\mu\sin\theta - \mu^2\cos\theta}\, g}$$

$$\begin{cases} N\cos\theta + \mu N' + \mu N\sin\theta = Mg \\ N\sin\theta = N' + \mu N\cos\theta \\ N'' = mg + N\cos\theta + \mu N\sin\theta \\ F + \mu N'' + \mu N\cos\theta = N\sin\theta \end{cases}$$

$$\rightarrow Mg - \mu^2 N\sin\theta - N\cos\theta = \mu N\sin\theta - \mu^2 N\cos\theta \rightarrow$$

$$\frac{Mg}{2\mu\sin\theta + \cos\theta - \mu^2\cos\theta} = \frac{F + \mu mg}{\sin\theta - \mu^2\sin\theta - 2\mu\cos\theta} \rightarrow$$

$$\rightarrow \mu mg + \mu N\cos\theta + \mu^2 N\sin\theta = N\sin\theta - \mu N\cos\theta - F$$



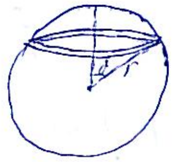
$$Mg\sin\theta - \mu^2 Mg\sin\theta - 2\mu Mg\cos\theta - 2\mu^2 mg\sin\theta - \mu mg\cos\theta + \mu^3 mg\cos\theta = F(2\mu\sin\theta + (1-\mu^2)\cos\theta) \rightarrow$$

$$\boxed{F_{min} = \frac{(M\sin\theta + \mu^3 m\cos\theta) - \mu^2(M+2m)\sin\theta - \mu(2M+m)\cos\theta}{2\mu\sin\theta + (1-\mu^2)\cos\theta}\, g} \quad \boxed{F_{max} = \frac{\mu(2M+m)\cos\theta - \mu^2(2m+M)\sin\theta + M\sin\theta - \mu^3 m\cos\theta}{\cos\theta - 2\mu\sin\theta - \mu^2\cos\theta}\, g}$$

3) الف) [illegible] (د)

$$F_\tau = \tau \times 2\pi r\sin\theta \times \sin\theta = 2\pi\tau r\sin^2\theta \rightarrow \boxed{F_\tau = 2\pi\tau r\theta^2}$$

ب)



$$dF_e = \frac{\sigma^2}{2\varepsilon_0} \times 2\pi r^2\sin\theta\, d\theta \times \cos\theta \rightarrow F_e = \frac{Q^2 \times \pi r^2}{16\pi^2 r^4 \times \varepsilon_0 \times 2} \times \frac{1}{2}\int_0^\theta d\sin^2\theta = \frac{Q^2}{32\pi\varepsilon_0 r^2}\sin^2\theta \rightarrow \boxed{F_e = \frac{Q^2}{32\pi\varepsilon_0 r^2}\theta^2}$$

ج)

$$2\pi\tau r_0 = \frac{Q^2}{32\pi\varepsilon_0 r_0^2} \rightarrow \boxed{r_0 = \sqrt[3]{\frac{Q^2}{64\pi^2\varepsilon_0\tau}}}$$

د)

$$\frac{\Delta F}{\Delta m} = \frac{\left(-2\pi\tau r + \frac{Q^2}{32\pi\varepsilon_0 r^2}\right)\left(\theta - \frac{\theta^3}{3!}\right)^2}{\frac{M \times 4\pi r^2\left(\frac{\theta^2}{2} - \frac{\theta^4}{4!}\right)}{4\pi r^2}} \rightarrow \frac{\Delta F}{\Delta m} = 4\left(\frac{r_0^3}{r^2} - r\right)\frac{\times 2\pi\tau}{M}\left(1 - \frac{\theta^2}{3}\right)\left(1 + \frac{\theta^2}{12}\right) = \frac{8\pi\tau}{M}\left(r_0\left(1 - \frac{\Delta r}{r_0}\right) - r_0 - \Delta r\right)\left(1 - \frac{\theta^2}{3} + \frac{\theta^2}{12}\right) \rightarrow$$

$$\boxed{\frac{\Delta F}{\Delta m} = -\frac{16\pi\tau}{M}(r - r_0)\left(1 - \frac{\theta^2}{4}\right)}$$ مرتبه دوم $$\rightarrow \boxed{\frac{\Delta F}{\Delta m} = -\frac{6\pi\tau}{M}(r - r_0)}$$ مرتبه 0

ه)

$$F'_e = \frac{\sigma'^2}{2\varepsilon_0} \times 2\pi r'^2\sin\theta\cos\theta\, d\theta = \frac{Q^2}{(4\pi r'^2)^2 \times \varepsilon_0}\pi r'^2\sin\theta\cos\theta\, d\theta = V^2\pi\varepsilon_0 r'^2\sin\theta\, d(\sin\theta) \rightarrow F'_e = \frac{V^2\pi\varepsilon_0 r'^2}{2}\sin^2\theta \rightarrow \boxed{F'_e = \frac{V^2\pi\varepsilon_0 r'^2}{2}\theta^2}$$

و)

$$\frac{V^2\pi\varepsilon_0 r_0'^2}{2} = 2\pi r_0'\tau \rightarrow \cancel{r_0' = 4\pi\tau} \quad \boxed{r_0' = \frac{4\tau}{\varepsilon_0 V^2}}$$

ز)

$$\frac{\Delta F}{\Delta m} = \frac{\left(\frac{V^2\pi\varepsilon_0 r'^2}{2} - 2\pi\tau r'\right)\left(\theta - \frac{\theta^3}{3!}\right)^2}{\frac{M}{2}\left(\frac{\theta^2}{2} - \frac{\theta^4}{4!}\right)} \rightarrow \frac{\Delta F}{\Delta m} = \frac{\left(2\pi\tau\frac{r'^2}{r_0'} - 2\pi\tau r'\right)}{M} \times 4\left(1 - \frac{\theta^2}{3}\right)\left(1 + \frac{\theta^2}{12}\right) \rightarrow \frac{\Delta F}{\Delta m} = \frac{8\pi\tau}{M}(r_0' + r')\left(1 - \frac{\theta^2}{3} + \frac{\theta^2}{12}\right) \rightarrow$$

$$\boxed{\frac{\Delta F}{\Delta m} = \frac{8\pi\tau}{M}(r_0' + r')\left(1 - \frac{\theta^2}{4}\right)}$$ مرتبه 2 $$\rightarrow \boxed{\frac{\Delta F}{\Delta m} = \frac{8\pi\tau}{M}(r_0' + r')}$$ مرتبه صفر

1) الف)

$$f(\theta) \geq f\left(\frac{\pi}{2}\right) \rightarrow e^{-2\mu\theta}(A_1\cos\theta + A_2\sin\theta) \geq e^{-2\mu\frac{\pi}{2}}A_2 \rightarrow \sin\left(\theta + \tan^{-1}\left(\frac{A_1}{A_2}\right)\right) > 0$$ نزولی

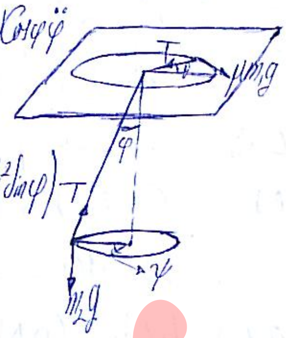
بسمه تعالی

امتحان سوم - تابستان 85

(5

$$\left|z=(l_0-r)\cos\varphi\right.,\quad \left|\dot z=-\dot r\cos\varphi-(l_0-r)\sin\varphi\dot\varphi\right.,\quad \left|\ddot z=-\ddot r\cos\varphi+\dot r\sin\varphi\dot\varphi+\dot r\sin\varphi\dot\varphi-(l_0-r)\cos\varphi\dot\varphi^2-(l_0-r)\sin\varphi\ddot\varphi\right.$$

$$\left|r'=(l_0-r)\sin\varphi\right. \to \left|\dot r'=-\dot r\sin\varphi+(l_0-r)\cos\varphi\dot\varphi\right. \to \left|\ddot r'=-\ddot r\sin\varphi-\dot r\cos\varphi\dot\varphi-\dot r\cos\varphi\dot\varphi-(l_0-r)\sin\varphi\dot\varphi^2+(l_0-r)\cos\varphi\ddot\varphi\right.$$



$$\begin{cases}-T\cos\varphi+m_2g=m_2\ddot z\\ -T\sin\varphi=m_2(\ddot r'-r'\dot\psi^2)\\ r'^2\dot\psi=(l_0-r_0)^2\sin^2\varphi_0\Omega_0\end{cases}\to\begin{cases}-T\cos\varphi+m_2g=m_2\left(-\ddot r\cos\varphi+2\dot r\sin\varphi\dot\varphi-(l_0-r)\cos\varphi\dot\varphi^2-(l_0-r)\sin\varphi\ddot\varphi\right)\\ -T\sin\varphi=m_2\left(-\ddot r\sin\varphi-2\dot r\cos\varphi\dot\varphi-(l_0-r)\sin\varphi\dot\varphi^2+(l_0-r)\cos\varphi\ddot\varphi-(l_0-r)\dot\psi^2\sin\varphi\right)\\ (l_0-r)^2\sin^2\varphi\dot\psi=(l_0-r_0)^2\sin^2\varphi_0\Omega_0\end{cases}$$

$$\dot\psi=\Omega_0+\mu\dot\psi_{(1)},\quad \dot\theta=\omega_0+\mu\dot\theta_{(1)},\quad r=r_0+\mu r_{(1)},\quad \varphi=\varphi_0+\mu\varphi_{(1)}$$

$$\begin{cases}-T=m_1(\ddot r-r\dot\theta^2)\\ -\mu g=2\dot r\dot\theta+r\ddot\theta\end{cases},\quad\begin{cases}\cos\varphi=\cos\varphi_0-\mu\varphi_{(1)}\sin\varphi_0\\ \sin\varphi=\sin\varphi_0+\mu\varphi_{(1)}\cos\varphi_0\end{cases}\to\begin{cases}-T_0-\mu T_{(1)}=m_1\mu\ddot r_{(1)}-m_1(r_0+\mu r_{(1)})(\omega_0^2+2\mu\omega_0\dot\theta_{(1)})\\ -\mu g=2\mu\dot r_{(1)}\omega_0+\mu r_0\ddot\theta_{(1)}\end{cases}\to$$

$$\begin{cases}T_0=m_1\omega_0^2r_0,\quad -T_{(1)}=m_1\ddot r_{(1)}-m_1\omega_0^2r_{(1)}-2m_1r_0\omega_0\dot\theta_{(1)}\\ -g=2\omega_0\dot r_{(1)}+r_0\ddot\theta_{(1)}\to\boxed{r_0\dot\theta_{(1)}=-gt-2\omega_0r_{(1)}}\end{cases}\to\boxed{T_{(1)}=-m_1\ddot r_{(1)}-3m_1\omega_0^2r_{(1)}-2m_1\omega_0gt}$$

$$\begin{cases}-(T_0+\mu T_{(1)})(\cos\varphi_0-\mu\varphi_{(1)}\sin\varphi_0)+m_2g=-\mu m_2\ddot r_{(1)}\cos\varphi_0-\mu(l_0-r_0)\sin\varphi_0\ddot\varphi_{(1)}m_2\\ -(T_0+\mu T_{(1)})(\sin\varphi_0+\mu\varphi_{(1)}\cos\varphi_0)=-\mu m_2\ddot r_{(1)}\sin\varphi_0+\mu m_2(l_0-r_0)\cos\varphi_0\ddot\varphi_{(1)}-(l_0-r_0-\mu r_{(1)})(\Omega_0^2+2\mu\Omega_0\dot\psi_{(1)})(\sin\varphi_0+\mu\varphi_{(1)}\cos\varphi_0)\\ (l_0-r_0-\mu r_{(1)})^2(\sin^2\varphi_0+2\mu\varphi_{(1)}\sin\varphi_0\cos\varphi_0)(\Omega_0+\mu\dot\psi_{(1)})=(l_0-r_0)^2\sin^2\varphi_0\Omega_0\end{cases}$$

$$\begin{cases}-T_0\cos\varphi_0+m_2g=0,\quad -\mu T_{(1)}\cos\varphi_0+\mu T_0\varphi_{(1)}\sin\varphi_0=-\mu m_2\ddot r_{(1)}\cos\varphi_0-\mu(l_0-r_0)m_2\ddot\varphi_{(1)}\sin\varphi_0\\ -T_0\sin\varphi_0=-(l_0-r_0)\Omega_0^2\sin\varphi_0,\quad -T_{(1)}\sin\varphi_0-T_0\varphi_{(1)}\cos\varphi_0=-m_2\ddot r_{(1)}\sin\varphi_0+m_2(l_0-r_0)\cos\varphi_0\ddot\varphi_{(1)}+\Omega_0^2r_{(1)}\sin\varphi_0-2(l_0-r_0)\Omega_0\sin\varphi_0\dot\psi_{(1)}-(l_0-r_0)\Omega_0^2\varphi_{(1)}\cos\varphi_0\\ \left((l_0-r_0)^2-2\mu(l_0-r_0)r_{(1)}\right)\left(\Omega_0\sin^2\varphi_0+\mu\dot\psi_{(1)}\sin^2\varphi_0+2\mu\Omega_0\varphi_{(1)}\sin\varphi_0\cos\varphi_0\right)=(l_0-r_0)^2\sin^2\varphi_0\Omega_0\end{cases}$$

$$\boxed{T_0=\frac{m_2g}{\cos\varphi_0}},\quad \boxed{\Omega_0=\sqrt{\frac{m_2g}{(l_0-r_0)\cos\varphi_0}}},\quad \boxed{\omega_0=\sqrt{\frac{m_2g}{m_1r_0\cos\varphi_0}}}$$

$$\left|m_1\ddot r_{(1)}\cos\varphi_0+3m_1\omega_0^2r_{(1)}\cos\varphi_0+2m_1\omega_0gt\cos\varphi_0+T_0\varphi_{(1)}\sin\varphi_0+m_2\ddot r_{(1)}\cos\varphi_0+m_2(l_0-r_0)\ddot\varphi_{(1)}\sin\varphi_0=0\right.$$

$$m_1\ddot r_{(1)}\sin\varphi_0+3m_1\omega_0^2r_{(1)}\sin\varphi_0+2m_1\omega_0gt\sin\varphi_0-T_0\varphi_{(1)}\cos\varphi_0+m_2\ddot r_{(1)}\sin\varphi_0-m_2(l_0-r_0)\cos\varphi_0\ddot\varphi_{(1)}-\Omega_0^2r_{(1)}\sin\varphi_0+2(l_0-r_0)\Omega_0\sin\varphi_0\dot\psi_{(1)}$$
$$+(l_0-r_0)\Omega_0^2\varphi_{(1)}\cos\varphi_0=0$$

$$(l_0-r_0)^2\dot\psi_{(1)}\sin^2\varphi_0+2(l_0-r_0)^2\Omega_0\varphi_{(1)}\sin\varphi_0\cos\varphi_0-2(l_0-r_0)\Omega_0r_{(1)}\sin^2\varphi_0=0\to\boxed{\dot\psi_{(1)}=\frac{2\Omega_0\left(r_{(1)}\sin\varphi_0-(l_0-r_0)\varphi_{(1)}\cos\varphi_0\right)}{(l_0-r_0)\sin\varphi_0}}$$

$$\to(m_1+m_2)\ddot r_{(1)}\sin\varphi_0+3m_1\omega_0^2r_{(1)}\sin\varphi_0+2m_1\omega_0gt\sin\varphi_0-T_0\varphi_{(1)}\cos\varphi_0-m_2(l_0-r_0)\cos\varphi_0\ddot\varphi_{(1)}-\Omega_0^2r_{(1)}\sin\varphi_0+4\Omega_0^2r_{(1)}\sin\varphi_0-4\Omega_0^2(l_0-r_0)\varphi_{(1)}\cos\varphi_0$$
$$-(l_0-r_0)\Omega_0^2\varphi_{(1)}\cos\varphi_0=0\to$$

$$\left|(m_1+m_2)\ddot r_{(1)}\sin\varphi_0+\frac{3m_2g\,r_{(1)}\sin\varphi_0}{r_0\cos\varphi_0}+2gt\sqrt{\frac{m_1m_2g}{r_0\cos\varphi_0}}\sin\varphi_0-m_2g\cos\varphi_0-m_2(l_0-r_0)\cos\varphi_0\ddot\varphi_{(1)}+\frac{3m_2g\,r_{(1)}\sin\varphi_0}{(l_0-r_0)\cos\varphi_0}-3m_2g\varphi_{(1)}=0\right.$$

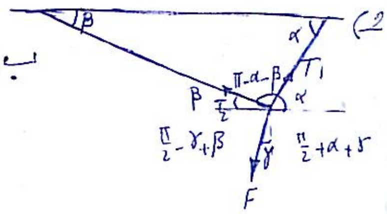
$$(m_1+m_2)\ddot r_{(1)}\cos\varphi_0+\frac{3m_2g\,r_{(1)}}{r_0}+2gt\sqrt{\frac{m_1m_2g\cos\varphi_0}{r_0}}+m_2g\varphi_{(1)}\tan\varphi_0+m_2(l_0-r_0)\ddot\varphi_{(1)}\sin\varphi_0=0$$

باسمه تعالی

امتحان دوم 85 -

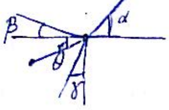
2) الف)

$$\frac{F}{\sin(\alpha+\beta)}=\frac{T_2}{\cos(\alpha+\gamma)}=\frac{T_1}{\cos(\beta-\gamma)}\longrightarrow \boxed{T_1=\frac{\cos(\alpha+\gamma)}{\sin(\alpha+\beta)}F}\quad \boxed{T_2=\frac{\cos(\beta-\gamma)}{\sin(\alpha+\beta)}F}$$



ب)

$$\boxed{\Delta a=\frac{F}{k}\cdot\frac{\cos(\beta-\gamma)}{\sin(\alpha+\beta)}}\ ,\ \boxed{\Delta b=\frac{F}{k}\cdot\frac{\cos(\alpha+\gamma)}{\sin(\alpha+\beta)}}$$



$$\begin{cases}\Delta a=\delta\cos(\alpha-\theta)\\ \Delta b=-\delta\cos(\beta+\theta)\end{cases}\quad \begin{cases}\Delta\beta=\frac{\delta}{b}\sin(\beta+\theta)\\ \Delta\alpha=-\frac{\delta}{a}\sin(\alpha-\theta)\end{cases}\longrightarrow\begin{cases}\Delta\alpha=-\frac{1}{a}\sqrt{\delta^2-\Delta a^2}\\ \Delta\beta=\frac{1}{b}\sqrt{\delta^2-\Delta b^2}\end{cases}$$

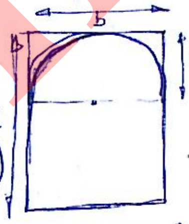
$$a^2+b^2+2ab\cos(\alpha+\beta)=(a+\Delta a)^2+(b+\Delta b)^2+2(a+\Delta a)(b+\Delta b)\cos(\alpha+\beta+\Delta\alpha+\Delta\beta)\longrightarrow$$

$$a\Delta a+b\Delta b+\left(a\Delta b\cos(\alpha+\beta)+b\Delta a\cos(\alpha+\beta)-ab\sin(\alpha+\beta)(\Delta\alpha+\Delta\beta)\right)=0\longrightarrow$$

$$a\Delta a+b\Delta b+(a\Delta b+b\Delta a)\cos(\alpha+\beta)=\sin(\alpha+\beta)\left(a\sqrt{\delta^2-\Delta b^2}-b\sqrt{\delta^2-\Delta a^2}\right)$$

امتحان چهارم 85

1)



$$\frac{1}{2}kl^2+mg(l_0-l)\sin\alpha=mg\left((a-b)\sin\alpha+\frac{b\pi}{2}\sin\alpha\right)\longrightarrow kl^2-2mgl\sin\alpha+mg\left(2(l_0-a+b)-b\pi\right)\sin\alpha=0\longrightarrow$$

$$l=\frac{mg\sin\alpha\pm\sqrt{m^2g^2\sin^2\alpha-kmg\left(2(l_0-a+b)-b\pi\right)\sin\alpha}}{k}\ \xrightarrow{\text{غلط}\ \times}\ \boxed{l=\frac{\sqrt{mg\sin\alpha}}{k}\left(\sqrt{mg\sin\alpha}+\sqrt{mg\sin\alpha+k(b\pi+2(a-b-l_0))}\right)}$$

2) الف)

$$\vec{F}=k\left(3\frac{(P_1P_2r\cos\theta+P_1P_2r\cos\theta)\hat{z}+3P_1P_2r\hat{r}}{r^5}-15\frac{P_1P_2\cos^2\theta\, r^3\hat{r}}{r^7}\right)=\frac{3kP_1P_2}{r^4}\left(2\cos\theta(\hat{r}\cos\theta-\hat{\theta}\sin\theta)-5\cos^2\theta\,\hat{r}+\hat{r}\right)$$

$$\longrightarrow\boxed{\vec{F}=\frac{3kP_1P_2}{r^4}\left((1-3\cos^2\theta)\hat{r}-2\sin\theta\cos\theta\,\hat{\theta}\right)}$$

ب)

$$\sum\vec{F}=0\longrightarrow\begin{cases}\frac{3kP_1P_2}{r^4}(1-3\cos^2\theta)-mg\cos\theta=0\\ -\frac{6kP_1P_2}{r^4}\sin\theta\cos\theta+mg\sin\theta=0\end{cases}\longrightarrow\frac{1}{\cos\theta}=\frac{2\cos\theta}{1-3\cos^2\theta}\longrightarrow 5\cos^2\theta=1\longrightarrow\cos\theta_0=\pm\frac{1}{\sqrt5}\longrightarrow\boxed{\cos\theta_0=\frac{1}{\sqrt5}}$$

$$\longrightarrow\sin\theta_0'=0\longrightarrow\boxed{\theta_0'=\pi}\longrightarrow\boxed{r_0'=\left(\frac{6kP_1P_2}{mg}\right)^{1/4}}\qquad\boxed{r_0=\left(\frac{6kP_1P_2}{\sqrt5\, mg}\right)^{1/4}}$$

ج)

$$\sum F_y=F_r\sin\theta+F_\theta\cos\theta=\frac{3kP_1P_2}{r^4}\left(1-3\cos^2\theta-2\cos^2\theta\right)\sin\theta=\frac{3kP_1P_2(1-5\cos^2\theta)\sin\theta}{r^4}=m\ddot{\delta}$$

$$\begin{cases}r\sin\theta=y\longrightarrow r=\sqrt{h^2+y^2}=\sqrt{h^2+y_0^2}\left(1+\frac{2\delta y_0}{h^2+y_0^2}\right)^{1/2}=\sqrt{h^2+y_0^2}+\frac{\delta y_0}{\sqrt{h^2+y_0^2}}\longrightarrow\Delta r=\frac{\delta y_0}{\sqrt{h^2+y_0^2}}=\delta\sin\theta_0\\ r\cos\theta=h\end{cases}$$

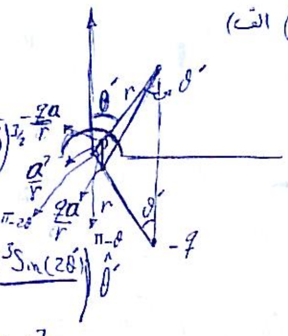
$$\longrightarrow(r_0+\Delta r)(\cos\theta_0-\Delta\theta\sin\theta_0)=h\longrightarrow\Delta r\cos\theta_0=r_0\Delta\theta\sin\theta_0\longrightarrow\Delta\theta=\frac{\cot\theta_0}{r_0}\times\frac{y_0\delta}{\sqrt{h^2+y_0^2}}\longrightarrow\Delta\theta=\frac{\delta\cos\theta_0}{\sqrt{h^2+y_0^2}}=\frac{\delta\cos\theta_0}{r_0}$$

$$\longrightarrow\frac{3kP_1P_2}{r_0^4}\left(1-5\cos^2\theta_0+10\sin\theta_0\cos\theta_0\Delta\theta\right)\left(1-\frac{\Delta r}{r_0}\right)\left(\sin\theta_0+\Delta\theta\cos\theta_0\right)=m\ddot{\delta}\longrightarrow$$

$$m\ddot{\delta}=\frac{3kP_1P_2}{r_0^4}\times10\sin^2\theta_0\cos\theta_0\Delta\theta\longrightarrow\boxed{\text{تعادل ناپایدار}}\ ,\ \cos\theta_0=\frac{1}{\sqrt5}$$

$$m\ddot{\delta}=\frac{3kP_1P_2}{r_0^4}(1-5)\Delta\theta\cos\theta_0\ \ \cancel{\text{تعادل پایدار}\ ,\ \theta_0=\pi}\longrightarrow\omega^2=\frac{3kP_1P_2}{ml^4}\times\frac{4}{l}\longrightarrow\boxed{\omega=\sqrt{\frac{2g}{l}}}$$

3) الف)



$$U=\frac{1}{2}\sum q_iV_i\longrightarrow U=\frac{1}{2}qV\longrightarrow U=\frac{q}{2}\left(-\frac{kq}{2r\cos\theta'}-\frac{kqa}{r^2(r-\frac{a^2}{r})}+\frac{kqa}{r\left(r^2+\frac{a^4}{r^2}+2a^2\cos(2\theta')\right)^{1/2}}\right)$$

$$\vec{F}=-\vec{\nabla}U=-\frac{kq^2}{2}\left(\left(\frac{1}{2r^2\cos\theta'}-a\left(-\frac{1}{r^3(1-\frac{a^2}{r^2})}-\frac{1+\frac{a^2}{r^2}}{r^3(1-\frac{a^2}{r^2})^2}\right)+a\left(-\frac{1}{r^3\left(1+\frac{a^4}{r^4}+\frac{2a^2}{r^2}\cos(2\theta')\right)^{1/2}}-\frac{2r-\frac{2a^4}{r^3}}{2r^4\left(1+\frac{a^4}{r^4}+\frac{2a^2}{r^2}\cos(2\theta')\right)^{3/2}}\right)\right)\hat{r}+\frac{1}{r}\left(-\frac{\sin\theta'}{2r\cos^2\theta'}+\frac{a}{r^2}\times-\frac{-4a^2\sin(2\theta')}{2\left(r^2+\frac{a^4}{r^2}+2a^2\cos(2\theta')\right)^{3/2}}\right)\hat{\theta}\right)\longrightarrow$$

$$\vec{F}=-\frac{kq^2}{2}\left[\left(\frac{1}{2r^2\cos\theta'}+\frac{2a}{r^3}\left(1+\frac{2a^2}{r^2}\right)-\frac{a\left(1+\cancel{\frac{a^4}{r^4}}+\frac{2a^2}{r^2}\cos(2\theta')+1-\cancel{\frac{a^4}{r^4}}\right)}{r^3\left(1+\frac{a^4}{r^4}+\frac{2a^2}{r^2}\cos(2\theta')\right)^{3/2}}\right)\hat{r}+\frac{1}{r}\left(-\frac{\sin\theta'}{2r\cos^2\theta'}+\frac{2a^3\sin(2\theta')}{r^4}\right)\hat{\theta}'\right]$$

$$\longrightarrow\vec{F}=-\frac{kq^2}{2r^2}\left[\left(\frac{1}{2\cos\theta'}+\frac{2a}{r}+\frac{4a^3}{r^3}-\frac{2a}{r}\left(1+\frac{a^2}{r^2}\cos(2\theta')\right)\left(1-\frac{3a^2}{r^2}\cos(2\theta')\right)\right)\hat{r}+\left(-\frac{\sin\theta'}{2\cos^2\theta'}+\frac{2a^3}{r^3}\sin(2\theta')\right)\hat{\theta}'\right]\longrightarrow$$

$$\vec{F}=-\frac{kq^2}{2r^2}\left[\left(\frac{1}{2\cos\theta'}+\cancel{\frac{2a}{r}}+\frac{4a^3}{r^3}-\cancel{\frac{2a}{r}}-\frac{2a^3}{r^3}\cos(2\theta')+\frac{6a^3}{r^3}\cos(2\theta')\right)\hat{r}+\left(-\frac{\sin\theta'}{2\cos^2\theta'}+\frac{2a^3}{r^3}\sin(2\theta')\right)\hat{\theta}'\right]\longrightarrow$$

$$\boxed{\vec{F}=-\frac{kq^2}{2r^2}\left[\left(\frac{1}{2\cos\theta'}+\frac{8a^3\cos^2\theta'}{r^3}\right)\hat{r}+\left(-\frac{\sin\theta'}{2\cos^2\theta'}+\frac{2a^3}{r^3}\sin(2\theta')\right)\hat{\theta}'\right]}$$

باسمه تعالی

امتحان چهارم 85

3) ب) ج)

$$\boxed{\vec F = -\frac{kq^2}{2r^2}\left[\left(\frac{1}{2\sin\theta}+\frac{8a^3\sin^2\theta}{r^3}\right)\hat r + \left(\frac{\cos\theta}{2\sin^2\theta}-\frac{2a^3}{r^3}\sin(2\theta)\right)\hat\theta\right]}$$

$$\begin{cases} N - mg\sin\theta - \frac{kq^2}{2r^2}\left(\frac{1}{2\sin\theta}+\frac{8a^3\sin^2\theta}{r^3}\right) = -mr\dot\theta^2 \\ -mg\cos\theta - \frac{kq^2}{2r^2}\left(\frac{\cos\theta}{2\sin^2\theta}-\frac{2a^3}{r^3}\sin(2\theta)\right) = mr\ddot\theta \end{cases} \longrightarrow +mg(1-\sin\theta) - \frac{kq^2}{2r^2}\left(-\frac{1}{2}\left(\frac{1}{\sin\theta}-1\right)-\frac{8a^3}{r^3}(\sin^2\theta-1)\right) = mr\frac{\dot\theta^2}{2} \longrightarrow$$

$$N = mg\sin\theta + \frac{kq^2}{2r^2}\left(\frac{1}{2\sin\theta}+\frac{8a^3\sin^2\theta}{r^3}-\frac{1}{\sin\theta}+1+\frac{2a^3}{r^3}-\frac{2a^3\sin^2\theta}{r^3}\right) - 2mg + 2mg\sin\theta \longrightarrow$$

$$\boxed{N = mg(-2+3\sin\theta) + \frac{kq^2}{2r^2}\left(\left(\frac{-1}{2\sin\theta}+1\right) + \frac{8a^3\sin^2\theta}{r^3}+\frac{2a^3\cos^2\theta}{r^3}\right)}$$

د) $N=0 \rightarrow mg(-2+3\sin\theta_0+3\theta_{(1)}\cos\theta_0) + \frac{kq^2}{2r^2}\left(\frac{4a^3}{r^3}\frac{(\sin^3\theta_0+3\theta_{(1)}\cos\theta_0)}{\sin^2\theta_0} + \left(1+\frac{-1}{2\sin\theta_0}+\frac{\theta_{(1)}\cos\theta_0}{2\sin^2\theta_0}\right) + \frac{4a^3}{r^3}\right) = 0 \longrightarrow \boxed{\theta_{(1)}=0}$

$, \; mg(3\sin\theta_0-2) + \frac{kq^2}{2r^2}\left(1-\frac{1}{2\sin\theta_0}\right) = 0 \rightarrow 3\sin\theta_0 - 2 + \frac{\eta}{2} - \frac{\eta}{4\sin\theta_0} = 0 \rightarrow 6\sin^2\theta_0 + (\eta-4)\sin\theta_0 - \frac{\eta}{2} = 0 \rightarrow$

$\sin\theta_0 = \frac{\eta-4\pm\sqrt{(\eta-4)^2+12\eta}}{12} \longrightarrow \boxed{\sin\theta_0 = \frac{\eta-4}{12}\left(1\pm\sqrt{1+\frac{12\eta}{\eta-4}}\right)}$ $\quad \boxed{A = \frac{2\eta(1-\sin^2\theta_0)}{3+\frac{\eta}{2\sin\theta_0}}}$ ✓

$3mg\theta_{(2)}\cos\theta_0 + \frac{kq^2}{2r^2}\times\frac{\theta_2\cos\theta_0}{2\sin\theta_0} = 0 \rightarrow \boxed{\theta_{(2)}=0}$ $, \; 3mg\theta_{(3)}\cos\theta_0 + \frac{kq^2}{2r^2}\left(\frac{4a^3}{r^3}(\sin^3\theta_0+1) + \frac{\theta_{(3)}\cos\theta_0}{2\sin^2\theta_0}\right) = 0 \rightarrow$

$\left(3\cos\theta_0 + \frac{\eta\cos\theta_0}{4\sin^2\theta_0}\right)\theta_{(3)} = -\frac{2\eta a^3}{r^3}(1+\sin^3\theta_0) \longrightarrow \boxed{\theta_{(3)} = -\frac{a^3}{r^3}\left(\frac{2\eta(1+\sin^3\theta_0)}{\left(3+\frac{\eta}{4\sin^2\theta_0}\right)\cos\theta_0}\right)} \longrightarrow \boxed{A = -\frac{8\eta\sin^2\theta_0(1+\sin^3\theta_0)}{12\sin^2\theta_0+\eta}}$ ✗

5) ~~آ) +~~

~~$-\mu_s g = -\omega^2 r + 2\omega v$~~ $\rightarrow \boxed{v = \frac{-\mu_s g + \omega^2 r}{2\omega}}$ ~~ب)~~ ~~$-\mu_s g = -\omega^2 r - 2\omega v$~~ $\rightarrow \boxed{v = \frac{\mu_s g - \omega^2 r}{2\omega}}$

$\frac{(v+\omega r)^2}{r} = \mu_s g \longrightarrow \boxed{v = \sqrt{\mu_s r g} - \omega r}$ آ) $\quad \frac{(v-\omega r)^2}{r} = \mu_s g \longrightarrow \boxed{v = \omega r + \sqrt{\mu_s r g}}$ ب) $\quad$ ب)

$\boxed{T = \frac{\sqrt{\mu_s r g}}{\omega\alpha}}$ ✗ $\quad$ پ) $\quad \boxed{\omega < \sqrt{\frac{\mu_s g}{r}}}$ برای $\quad$ ت)

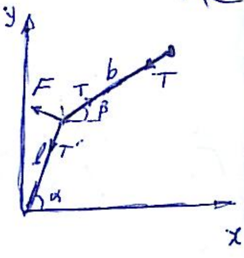
6) الف) و ب)

$$\begin{cases} x = l\cos\alpha + b\cos\beta \\ y = l\sin\alpha + b\sin\beta \end{cases} \longrightarrow \begin{cases} \dot x = -l\sin\alpha\,\dot\alpha - b\sin\beta\,\dot\beta \\ \dot y = l\cos\alpha\,\dot\alpha + b\cos\beta\,\dot\beta \end{cases} \longrightarrow \begin{cases} \ddot x = -l(\cos\alpha\,\dot\alpha^2+\sin\alpha\,\ddot\alpha) - b(\cos\beta\,\dot\beta^2+\sin\beta\,\ddot\beta) \\ \ddot y = l(\cos\alpha\,\ddot\alpha - \sin\alpha\,\dot\alpha^2) + b(\cos\beta\,\ddot\beta - \sin\beta\,\dot\beta^2) \end{cases}$$

ج) $\begin{cases} -T\cos\beta = m\ddot x \\ -T\sin\beta = m\ddot y \end{cases} \longrightarrow \frac{\ddot y}{\ddot x} = \tan\beta \longrightarrow l(\cos\alpha\cos\beta\,\ddot\alpha - \sin\alpha\cos\beta\,\dot\alpha^2) + b(\cos^2\beta\,\ddot\beta - \sin\beta\cos\beta\,\dot\beta^2) = l(-\sin\beta\cos\alpha\,\dot\alpha^2 - \sin\alpha\sin\beta\,\ddot\alpha)$

$+ b(-\sin\beta\cos\beta\,\dot\beta^2 - \sin^2\beta\,\ddot\beta) \longrightarrow \boxed{l(\cos(\beta-\alpha)\ddot\alpha + \sin(\beta-\alpha)\dot\alpha^2) = b(-\ddot\beta)}$

د) $\frac{F}{\sin(\pi-(\alpha-\beta))} = \frac{T}{\sin(\frac{\pi}{2})} = \frac{T'}{\sin(\frac{\pi}{2}-(\beta-\alpha))} \longrightarrow T = \frac{F}{\sin(\alpha-\beta)} \longrightarrow$



~~$\frac{F\cos\beta}{m\sin(\beta-\alpha)}$~~ $= \boxed{\frac{F\cos\beta}{m\sin(\alpha-\beta)} = l(\cos\alpha\,\dot\alpha^2+\sin\alpha\,\ddot\alpha) + b(\cos\beta\,\dot\beta^2+\sin\beta\,\ddot\beta)}$

ز) معادله تیر : $l\sin\gamma\,\omega^2 = -b\ddot\gamma \longrightarrow \boxed{\ddot\gamma + \left(\frac{l\omega^2}{b}\right)\sin\gamma = 0}$ ه) $\ddot\gamma + \left(\frac{l\omega^2}{b}\right)\gamma = 0 \longrightarrow \boxed{\Omega = \omega\sqrt{\frac{l}{b}}}$

امتحان پنجم 85

2) آ)

$\begin{cases} E = \frac{1}{2}m(\dot r^2 + r^2\dot\theta^2) - mgr\cos\theta + \frac{1}{2}k(r-a)^2 \\ \dot r = -a\sin\theta\,\dot\theta \longrightarrow \dot\theta = -\frac{\dot r}{a\sin\theta} \end{cases} \longrightarrow E = \frac{m\dot r^2}{2}\left(1+\left(\frac{r}{a\sin\theta}\right)^2\right) - mgr\cos\theta + \frac{2mg}{a}(r-a)^2 \rightarrow$

$U_{eff} = \frac{1}{2}k(r-a)^2 - mgr\cos\theta$

$\vec F = -\vec\nabla U_{eff} \rightarrow \vec F = -\left(\frac{\partial U}{\partial r}\hat r + \frac{1}{r}\frac{\partial U}{\partial\theta}\hat\theta\right) \quad , \; U = \frac{1}{2}ka^2\cos^2\theta - mga\cos\theta(1+\cos\theta) \Rightarrow U = mga\cos\theta(2\cos\theta-1-\cos\theta)$

$\longrightarrow U = mga\cos\theta(\cos\theta-1) \longrightarrow \frac{\partial U}{\partial\theta} = mga(\sin\theta - 2\cos\theta\sin\theta) \longrightarrow \boxed{\theta_0=0}$ یا $\boxed{\theta_0=\frac{\pi}{3}}$

$\frac{\partial^2 U}{\partial\theta^2} = mga(\cos\theta - 2\cos(2\theta)) \longrightarrow \boxed{\theta_0=0 \rightarrow \text{تعادل ناپایدار}}$, $\boxed{\theta_0=\frac{\pi}{3} \; \text{تعادل پایدار}}$

~~$U = mg(r-a)(\frac{r}{a}-2) \rightarrow \frac{\partial U}{\partial r} = mg(\frac{r}{a}-1+\frac{r}{a}-2) \rightarrow \frac{\partial U}{\partial r} = mg(\frac{2r}{a}-3) \rightarrow \frac{\partial^2 U}{\partial r^2} =$~~ $\quad$ ب)

باسمه تعالی

ب)

$E=\frac12 m(\dot r^2+r^2\dot\theta^2)+\frac12 k(r-a)^2-mgr\cos\theta \longrightarrow E=\frac12 ma^2\dot\theta^2(\sin^2\theta+(1+\cos\theta)^2)+\frac12 k(a^2\cos^2\theta)-mga\cos\theta(1+\cos\theta)\longrightarrow$

$\dot r=-a\sin\theta\,\dot\theta$ $\quad E=ma^2(1+\cos\theta)\dot\theta^2+\frac12 a\cos^2\theta\times\frac{4mg}{a}-mag\cos^2\theta-mag\cos\theta\longrightarrow$

$E=ma^2(1+\cos\theta)\dot\theta^2+mag\cos\theta(\cos\theta-1)\longrightarrow \dot E=2ma^2[(1+\cos\theta)\ddot\theta-\sin\theta\,\dot\theta^2]\dot\theta-2mag\cos\theta\sin\theta\,\dot\theta+mga\sin\theta\,\dot\theta=0\rightarrow$

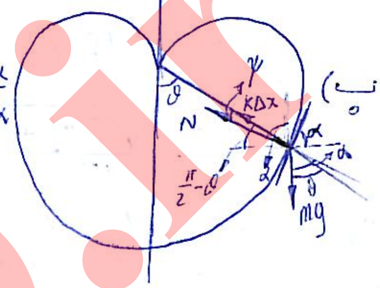
→ تعادل: $\sin\theta_0=0\rightarrow\boxed{\theta_0=0}$ یا $\cos\theta_0=\frac12\rightarrow\boxed{\theta_0=\frac{\pi}{3}}$

$2(1+\cos\theta_0)\ddot\theta_{(1)}+\frac{g}{a}(\sin\theta_0+\theta_{(1)}\cos\theta_0)(1-2\cos\theta_0+2\theta_{(1)}\sin\theta_0)=0\rightarrow$

$4\ddot\theta_{(1)}+\frac{g}{a}(-1)\theta_{(1)}=0\longrightarrow$ هارمونیک نیست

$3\ddot\theta_{(1)}+\frac{g}{a}(2\sin^2\theta_0)\theta_{(1)}=0\rightarrow \ddot\theta_{(1)}+\frac{g}{2a}\theta_{(1)}=0\rightarrow\boxed{\omega=\sqrt{\frac{g}{2a}}}$

$y=-r\cos\theta$, $x=r\sin\theta$ → ~~$dy=r\sin\theta\, d$~~ → $y=-a(1+\cos\theta)\cos\theta$, $x=a(1+\cos\theta)\sin\theta$ → $\frac{dy}{dx}=\frac{\sin\theta+2\cos\theta\sin\theta}{\cos\theta+\cos^2\theta-\sin^2\theta}=\tan\alpha=\frac{\sin\alpha}{\cos\alpha}$



پ)

$\frac{\sin\alpha}{\cos\alpha}=\frac{\sin\theta+\sin(2\theta)}{\cos\theta+\cos(2\theta)}\longrightarrow\sin\alpha\cos\theta-\cos\alpha\sin\theta=\cos\alpha\sin(2\theta)-\sin\alpha\cos(2\theta)\rightarrow$

$\sin(\alpha-\theta)=\sin(2\theta-\alpha)\rightarrow 3\theta=2\alpha\rightarrow\alpha=\frac32\theta,\ \psi=\alpha-\theta\rightarrow\boxed{\psi=\frac{\theta}{2}}$

$N+k(r-a)\cos\psi-mg\cos\alpha=m\frac{v^2}{R}\rightarrow N=mg\cos(\tfrac32\theta)+\frac{3mg\cos\theta\cos(\frac32\theta)(1-\cos\theta)}{\cos\theta+\cos(2\theta)}-4mg\cos\theta\cos\frac{\theta}{2}$ ت)

$R=\frac{(1+f'^2)^{3/2}}{f''}$

$f''=\frac{df'}{dx}=\frac{df'}{d\theta}\times\frac{d\theta}{dx}=\frac{df'/d\theta}{dx/d\theta}=\frac{\frac{1}{\cos^2(\frac32\theta)}\times\frac32}{a(\cos\theta+\cos(2\theta))}\rightarrow f''=\frac{3}{2a\cos^2(\frac32\theta)(\cos\theta+\cos(2\theta))}\rightarrow R=\frac{2a(\cos\theta+\cos(2\theta))}{3\cos(\frac32\theta)}$

$+mgr\cos\theta=\frac12 mv^2+\frac12\times\frac{4mg}{a}\times a^2\cos^2\theta=amg\cos\theta+mag\cos^2\theta\rightarrow v^2=2ag\cos\theta(1-\cos\theta)$

$$\boxed{N=mg\left[\cos(\tfrac32\theta)+\frac{3\cos\theta\cos(\frac32\theta)(1-\cos\theta)}{\cos\theta+\cos(2\theta)}-4\cos\theta\cos\frac{\theta}{2}\right]}$$

1)

~~$\vec F_2=\frac12\oint E_2^2\,ds$~~ ب) ~~$\vec F_1=\frac12\oint E_1^2\,ds$~~ الف)

$\vec F_2=\frac12\oint E_2^2\hat E_2\,ds$ ب) $\quad\vec F_1=\frac12\oint E_1^2\hat E_1\,ds$ الف)

1		2
q_1	$\rightarrow F_1 \leftarrow$	0
0	$\rightarrow F_2 \leftarrow$	q_2
Q_2	$\leftarrow F_3 \rightarrow$	Q_1
Q_2'	$\leftarrow F \rightarrow$	Q_1'

$\vec F_3=\frac12\left(\oint\frac{E_2^2}{q_2^2}Q_2^2\hat E_2\,ds+\oint\frac{E_1^2}{q_1^2}Q_1^2\hat E_1\,ds\right)\rightarrow\boxed{\vec F_3=\frac12\left(\frac{Q_2^2}{q_2^2}\oint E_2^2\hat E_2\,ds+\frac{Q_1^2}{q_1^2}\oint E_1^2\hat E_1\,ds\right)}$ ج)

$\vec F=-\vec\nabla U=-\frac12\vec\nabla(\sum q_iV_i)=-\frac12(P'_{11}q_1^2+2P'_{12}q_1q_2+P'_{22}q_2^2)\rightarrow$ د)

$\vec F_1=-\frac{P'_{11}}{2}q_1^2\rightarrow P'_{11}=\frac{-2\vec F_1}{q_1^2}$, $\quad P'_{22}=-\frac{2\vec F_2}{q_2^2}$, $\quad\vec F_3=\frac{\vec F_1}{q_1^2}Q_1^2-P'_{12}Q_1Q_2+\frac{\vec F_2}{q_2^2}Q_2^2\rightarrow P'_{12}=-\frac{\vec F_3}{Q_1Q_2}+\frac{Q_1}{q_1^2Q_2}\vec F_1+\frac{Q_2}{q_2^2Q_1}\vec F_2$

$\vec F=\frac{\vec F_1}{q_1^2}Q_1'^2+\frac{\vec F_3}{Q_1Q_2}Q_1'Q_2'-\frac{Q_1Q_1'Q_2'}{q_1^2Q_2}\vec F_1-\frac{Q_2Q_1'Q_2'}{q_2^2Q_1}\vec F_2+\frac{\vec F_2}{q_2^2}Q_2'^2\rightarrow$

$\vec F=Q_1'\vec F_1\left(\frac{Q_1'}{q_1^2}-\frac{Q_1Q_2'}{q_1^2Q_2}\right)+\frac{Q_1'Q_2'}{Q_1Q_2}\vec F_3+Q_2'\vec F_2\left(\frac{Q_2'}{q_2^2}-\frac{Q_2Q_1'}{q_2^2Q_1}\right)\rightarrow\vec F=\frac{Q_1'}{q_1^2}\left(Q_1'-\frac{Q_1Q_2'}{Q_2}\right)\vec F_1+\frac{Q_1'Q_2'}{Q_1Q_2}\vec F_3+\frac{Q_2'}{q_2^2}\left(Q_2'-\frac{Q_2Q_1'}{Q_1}\right)\vec F_2$

$\boxed{\vec F=\frac{Q_1'(Q_2Q_1'-Q_1Q_2')}{Q_2q_1^2}\vec F_1+\frac{Q_1'Q_2'}{Q_1Q_2}\vec F_3+\frac{Q_2'(Q_1Q_2'-Q_2Q_1')}{Q_1q_2^2}\vec F_2}$ یا $\boxed{\vec F=(Q_2Q_1'-Q_1Q_2')\left(\frac{Q_1'\vec F_1}{Q_2q_1^2}-\frac{Q_2'\vec F_2}{Q_1q_2^2}\right)+\frac{Q_1'Q_2'}{Q_1Q_2}\vec F_3}$

پاسخ سوالات

امتحان پنجم 85

4) a)

$$\lambda g x + \lambda g (L-x)\sin\theta = \lambda g \mu \cos\theta (L-x) \rightarrow \mu_0 = \frac{x}{L-x} \rightarrow \boxed{\mu_0 = \alpha}$$

b)

$$\alpha + \sin\theta = \mu\cos\theta \rightarrow \sin(\theta - \tan^{-1}(\mu)) = \frac{\alpha}{\sqrt{1+\mu^2}} \rightarrow \boxed{\theta_1 = \sin^{-1}\left(\frac{\alpha}{\sqrt{1+\mu^2}}\right) + \tan^{-1}(\mu)} \quad \boxed{\sin\theta_1 = \frac{\alpha+\mu}{\sqrt{1+\mu^2}}}$$

c)

$$gx + g(L-x)\sin\theta - \mu g (L-x)\cos\theta = L\ddot{x} \rightarrow \ddot x = \frac{g}{L}x(1-\sin\theta+\mu\cos\theta) + g(\sin\theta - \mu\cos\theta) \rightarrow \boxed{A = \frac{g}{L}(1-\sin\theta+\mu\cos\theta)}, \quad \boxed{B = g(\sin\theta-\mu\cos\theta)}$$

$$\cancel{C_0 = B \rightarrow C_0 = g(\sin\theta - \mu} \quad AC_0 + B = 0 \rightarrow C_0 = -\frac{B}{A} \rightarrow \boxed{C_0 = \frac{\sin\theta-\mu\cos\theta}{1-\sin\theta+\mu\cos\theta}L}$$

$$x|_{t=0} = x_0 \rightarrow x_0 = C_0 + C_1 + C_2 \rightarrow C_1 = C_2 = \frac{x_0 - C_0}{2} \qquad \dot x|_{t=0} = 0 \rightarrow C_1 = C_2$$

$$\rightarrow C_1 = C_2 = \left(x_0 + \frac{-\sin\theta+\mu\cos\theta}{1-\sin\theta+\mu\cos\theta}L\right)\frac12 \rightarrow \boxed{C_1=C_2=\frac12\left(x_0+\frac{\mu\cos\theta-\sin\theta}{1+\mu\cos\theta-\sin\theta}\right)}$$

e)

$$\frac12 \frac{d\dot x^2}{dx} = Ax + B \rightarrow \dot x^2 = Ax^2 + 2Bx \rightarrow \dot x|_{x=L} = \sqrt{gL(1-\sin\theta+\mu\cos\theta+2\sin\theta-2\mu\cos\theta)} \rightarrow$$

$$\boxed{\dot x|_{x=L} = \sqrt{gL(1+\sin\theta-\mu\cos\theta)}}$$

5) a)

$$\mu\cos\theta > \sin\theta \rightarrow \boxed{\theta_0 < \tan^{-1}(\mu)}$$

$$\cancel{= \mu g\cos\theta + g\sin\theta = +R\dot\theta^2 \rightarrow \frac{R}{2g}\int_0^{\dot\theta^2} d\dot\theta^2 = -\int_{\theta_0}^{\theta}\cos\theta - \mu\int_{\theta_0}^{\theta}\sin\theta}$$

b)

$$\begin{cases} N - mg\cos\theta = mR\dot\theta^2 \\ -\frac{\mu N}{m} + g\sin\theta = R\ddot\theta \end{cases} \rightarrow g\sin\theta - \mu g\cos\theta + \mu R\dot\theta^2 = R\ddot\theta = \frac R2 \frac{d\dot\theta^2}{d\theta} \rightarrow g\sin\theta - \mu g\cos\theta + \frac{\mu}{R}v^2 = \frac{1}{2R}\frac{dv^2}{d\theta} \rightarrow$$

$$\frac{dv^2}{d\theta} = 2\mu v^2 + 2gR(\sin\theta - \mu\cos\theta)$$

$$2\mu A_0 e^{2\mu\theta} - A_1\sin\theta + A_2\cos\theta = 2\mu A_0 e^{2\mu\theta} + 2\mu A_1\cos\theta + 2\mu A_2\sin\theta + 2gR\sin\theta - 2g\mu R\cos\theta \rightarrow$$

$$-A_1 = 2\mu A_2 + 2gR \rightarrow -A_1 = 4\mu^2 A_1 - 4\mu^2 gR + 2gR \rightarrow \boxed{A_1 = 2gR\frac{2\mu^2-1}{4\mu^2+1}}$$

$$A_2 = 2\mu A_1 - 2\mu g R \rightarrow A_2 = 4\mu^2 A_2 - 2\mu gR - 2\mu gR \rightarrow \boxed{A_2 = -\frac{4\mu gR}{1+4\mu^2}} \times$$

$$v^2|_{\theta=\theta_0} = 0 \rightarrow A_0 e^{2\mu\theta_0} = -2gR\frac{2\mu^2-1}{4\mu^2+1}\times\frac{1}{\sqrt{1+\mu^2}} + \frac{4\mu gR}{1+4\mu^2}\times\frac{\mu}{\sqrt{1+\mu^2}} \rightarrow \boxed{A_0 = \frac{2gR\, e^{-2\mu\tan^{-1}(\mu)}}{(4\mu^2+1)\sqrt{1+\mu^2}}}$$

c)

$$N = mg\cos\theta - m\frac{v^2}{R} = mg\left(-\frac{A_0}{gR}e^{2\mu\theta} - \frac{A_2}{gR}\sin\theta + \left(1-\frac{A_1}{gR}\right)\cos\theta\right)$$

$$1 - \frac{A_1}{gR} = 1 - \frac{4\mu^2-2}{4\mu^2+1} = \frac{3}{4\mu^2+1} \rightarrow N = \frac{mg}{1+4\mu^2}\left(4\mu\sin\theta + 3\cos\theta - \frac{2e^{-2\mu\tan^{-1}(\mu)}}{\sqrt{1+\mu^2}}\right) \rightarrow$$

$$\boxed{N = \frac{mg}{1+4\mu^2}\left(4\mu\sin\theta + 3\cos\theta - \frac{2e^{-2\mu\tan^{-1}(\mu)}}{\sqrt{1+\mu^2}}\right)}$$

d) تقریبی

$$N=0 \rightarrow 4\mu\sin\theta_{(0)} + 3\cos\theta_{(0)} - 3\theta_{(0)}\sin\theta_{(0)} - 2 = 0 \rightarrow \boxed{\theta_{\text{جدا}} = \cos^{-1}\left(\frac23\right) + \frac43\mu} \times$$

امتحان 85 (1 الف)

$$Fx = \frac{\mu\lambda g}{2}(x^2 + (L-x)^2) \Rightarrow F = \frac{\mu\lambda g}{2x}(2x^2 + L^2 - 2xL)$$

$$W = F\cdot a \rightarrow W = \frac{\mu\lambda a g}{2}\left(2x + \frac{L^2}{x} - 2L\right) \rightarrow \frac{dW}{dx}\Big|_{x=x_0} = 0 \rightarrow 2 = \frac{L^2}{x_0^2} \rightarrow \boxed{x_0 = \frac{L}{\sqrt2}}$$

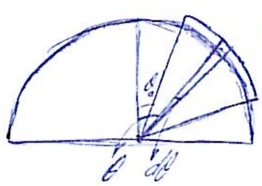
ب)

$$W = F x\theta \rightarrow W = \frac{\mu\lambda g}{2}\theta(2x^2+L^2-2xL) \rightarrow \boxed{x_0 = \frac L2}$$

2)

$$T_{(\theta+d\theta)} + dm\,g\sin\theta = \mu\, dN + T_{(\theta)} \rightarrow dT + \lambda R\, d\theta\, g\sin\theta = \mu\lambda R\, d\theta\, g\cos\theta + \mu T\, d\theta \rightarrow$$

$$dN = (T_{\theta+d\theta} + T_\theta)\frac{d\theta}{2} + dm\,g\cos\theta$$



$$\frac{dT}{d\theta} = \mu T + \lambda Rg(-\sin\theta + \mu\cos\theta) \rightarrow -A_0C_0 e^{-C_0\theta} - A_1\sin\theta + A_2\cos\theta = \mu A_0 e^{-C_0\theta} + \mu A_1\cos\theta +$$

$$\mu A_2\sin\theta - \lambda Rg\sin\theta + \mu\lambda Rg\cos\theta \rightarrow \boxed{C_0 = -\mu}, \quad \boxed{C_1 = \mu\lambda Rg}, \quad \boxed{C_2 = \lambda Rg}$$

باسمه تعالی

$$\begin{cases} -A_1=-\lambda Rg+\mu A_2 \\ A_2=\mu A_1+\mu\lambda Rg \end{cases} \longrightarrow \boxed{A_2=\frac{2\mu\lambda Rg}{1+\mu^2}} \rightarrow A_1=\lambda Rg\left(\frac{1+\mu^2-2\mu^2}{1+\mu^2}\right)\longrightarrow \boxed{A_1=\frac{1-\mu^2}{1+\mu^2}\lambda Rg}$$

درست است. یعنی: $T_{(\theta_0+\frac{m}{\lambda R})}=0$

$$\cancel{T_{(\theta_0+\frac{m}{\lambda R})}=0 \longrightarrow A_0 e^{\mu(\theta_0+\frac{m}{\lambda R})}+A_1\cos(\theta_0+\frac{m}{\lambda R})+A_2\sin(\theta_0+\frac{m}{\lambda R})=0 \longrightarrow} \boxed{A_0=\frac{\lambda Rg}{1+\mu^2}\left((1-\mu^2)\cos(\theta_0+\frac{m}{\lambda R})+2\mu\sin(\theta_0+\frac{m}{\lambda R})\right)e^{-\mu(\theta_0+\frac{m}{\lambda R})}}$$

$$T_{(\theta_0)}=0 \longrightarrow A_0 e^{\mu\theta_0}+A_1\cos\theta_0+A_2\sin\theta_0=0 \longrightarrow \boxed{A_0=-\frac{\lambda Rg e^{-\mu\theta_0}}{1+\mu^2}\left((1-\mu^2)\cos\theta_0+2\mu\sin\theta_0\right)}$$ (c

$$T=A_0e^{\mu\theta}+A_1\cos\theta+A_2\sin\theta-\left(A_0e^{\mu\theta_0}+A_1\cos\theta_0+A_2\sin\theta_0\right)=A_0e^{\mu\theta}+A_1\cos\theta+A_2\sin\theta-\left(A_0+\frac{A_1\cos\theta_0}{e^{\mu\theta_0}}+\frac{A_2\sin\theta_0}{e^{\mu\theta_0}}\right)e^{\mu\theta}=$$ (d

$$e^{\mu\theta}\left(\left(\frac{A_1\cos\theta+A_2\sin\theta}{e^{\mu\theta}}\right)-\left(\frac{A_1\cos\theta_0+A_2\sin\theta_0}{e^{\mu\theta_0}}\right)\right)\longrightarrow \boxed{f_{(\theta)}=\frac{A_1\cos\theta+A_2\sin\theta}{e^{\mu\theta}}}\longrightarrow \boxed{f_{(\theta)}=\frac{(1-\mu^2)\cos\theta+2\mu\sin\theta}{(1+\mu^2)e^{\mu\theta}}\lambda Rg}$$

$$\frac{df_{(\theta)}}{d\theta}=0\longrightarrow(-A_1\sin\theta+A_2\cos\theta)-\mu(A_1\cos\theta+A_2\sin\theta)=0\rightarrow \tan\theta=\frac{A_2-\mu A_1}{A_1+\mu A_2}=\frac{2\mu-\mu+\mu^3}{1-\mu^2+2\mu^2}\rightarrow\boxed{\tan\alpha=\mu}$$

$$T<0\rightarrow f_{(\theta)}<f_{(\theta_0)}\xrightarrow{\theta>\theta_0}\frac{df_{(\theta)}}{d\theta}<0\longrightarrow\boxed{\theta>\alpha}$$ (e

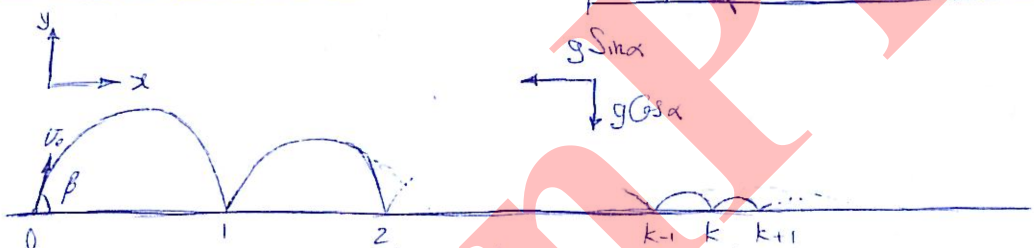
$$\left.\frac{df_{(\theta)}}{d\theta}\right|_{\theta=0}>0\longrightarrow \text{شرط}: f_{(\frac{\pi}{2})}>f_{(0)}\longrightarrow\frac{A_2}{e^{\frac{\mu\pi}{2}}}>A_1\rightarrow\boxed{2\mu>e^{\frac{\mu\pi}{2}}(1-\mu^2)}$$ (f

$$T_{(\frac{\pi}{2})}=\lambda gh\longrightarrow\boxed{A_0e^{\frac{\mu\pi}{2}}+A_2=\lambda gh}\xrightarrow{\theta_0=0}\left(-\frac{\lambda gR}{1+\mu^2}(1-\mu^2)+\frac{2\mu\lambda gR}{1+\mu^2}\right)e^{\frac{\mu\pi}{2}}+\frac{2\mu\lambda gR}{1+\mu^2}=\lambda gh\longrightarrow$$ (g

$$\boxed{h=\frac{(2\mu-1+\mu^2)e^{\frac{\mu\pi}{2}}+2\mu}{1+\mu^2}R}\ ,\ \theta_0=0\ ,\ \boxed{h=\frac{R}{1+\mu^2}\left(2\mu-\left((1-\mu^2)\cos\theta_0+2\mu\sin\theta_0\right)e^{\mu(\frac{\pi}{2}-\theta_0)}\right)}$$

کلی θ_0 با شرط ثابت

$$\boxed{t_{k+1}=\frac{2(v_k^f)_y}{g\cos\alpha}}$$ (3 (آ



$$\begin{cases} \vec{v}_k^{\,i}=\left(v_0\cos\beta-g\sin\alpha\sum_{k=1}^{k}t_k\right)\hat{i}-\left(v_0\sin\beta\, e^{k-1}\right)\hat{j} \\ t_k=\frac{2v_0\sin\beta}{g\cos\alpha}e^{k-1}\longrightarrow\sum_{k=1}^{k}t_k=\frac{2v_0\sin\beta}{g\cos\alpha}\left(1+e+e^2+\dots+e^{k-1}\right)=\boxed{\frac{2v_0\sin\beta}{g\cos\alpha}\frac{1-e^k}{1-e}=T_k} \end{cases}$$ (ب) ، (پ)

$$\boxed{\vec{v}_k^{\,i}=v_0\left(\cos\beta-\frac{2\sin\beta\sin\alpha}{\cos\alpha}\frac{1-e^k}{1-e}\right)\hat{i}-\left(v_0\sin\beta\, e^{k-1}\right)\hat{j}}\ ,\ \boxed{\vec{v}_k^{\,f}=v_0\left(\cos\beta-\frac{2\sin\alpha\sin\beta}{\cos\alpha}\frac{1-e^k}{1-e}\right)\hat{i}+\left(v_0\sin\beta\, e^k\right)\hat{j}}$$

$$R_n=-\frac{g\sin\alpha T_n^2}{2}+v_0\cos\beta T_n\longrightarrow R_n=v_0T_n\left(\cos\beta-\frac{\sin\alpha}{2}\times\frac{2\sin\beta}{\cos\alpha}\frac{1-e^n}{1-e}\right)\longrightarrow$$ (ت

$$\boxed{R_n=\frac{2v_0^2\sin\beta}{g\cos\alpha}\cdot\frac{1-e^n}{1-e}\left(\cos\beta-\frac{\sin\alpha\sin\beta}{\cos\alpha}\frac{1-e^n}{1-e}\right)}$$

$$R_{(2)}=0\longrightarrow\cos\alpha\cos\beta(1-e)=\sin\alpha\sin\beta(1-e^2)\longrightarrow\boxed{\tan\alpha\tan\beta=\frac{1-e}{1-e^2}}$$ (ث

$$(v_2)_x=0\longrightarrow\boxed{2\tan\alpha\tan\beta=\frac{1-e}{1-e^2}}$$ (ج

$$V=kq\times\frac{a\lambda d\theta}{\left(z^2+(\rho-a\cos\theta)^2+a^2\sin^2\theta\right)^{1/2}}\longrightarrow\boxed{V=\frac{kqQ}{2\pi}\int_0^{2\pi}\frac{d\theta}{\sqrt{z^2+\rho^2+a^2-2\rho a\cos\theta}}}$$ (6 (a

فاینال 85

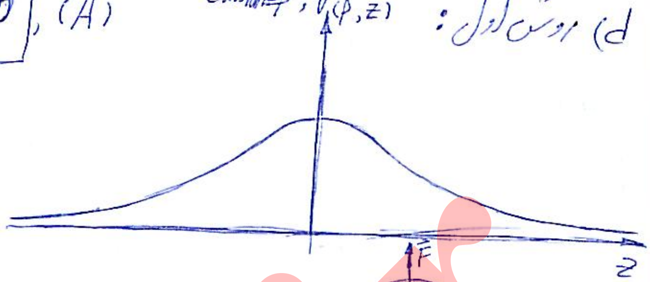
باسمه تعالی

b) پتانسیل تک تک اجزاء ها با میل کردن z به صفر افزایش می یابد ← $\boxed{z_0=0}$

c) با افزایش ρ پتانسیل تک تک اجزاء ها در محل بار رو به صفر میل می کند ← $V_{(\rho,0)}$ نسبت به ρ نزولی (B) است.

d) روش اول:

$$\nabla^2 V=0 \to \frac{\partial^2 V}{\partial\rho^2}+\frac{\partial^2 V}{\partial z^2}=0 \to \frac{\partial^2 V}{\partial\rho^2}=-\frac{\partial^2 V}{\partial z^2} \xrightarrow{\text{طبق نمودار}} \boxed{\frac{\partial^2 V}{\partial\rho^2}>0},\ (A)$$



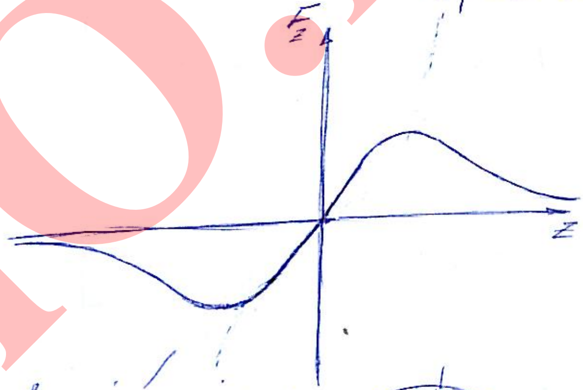
روش دوم:

$$\vec F=-\vec\nabla V=-\frac{\partial V}{\partial\rho}\hat\rho \to \begin{cases}\frac{\partial^2 V}{\partial\rho^2}=-\frac{\partial F}{\partial\rho}\\ \frac{\partial F}{\partial\rho}\Big|_{\rho=\rho_0}<0\end{cases} \to \boxed{\frac{\partial^2 V}{\partial\rho^2}>0}$$

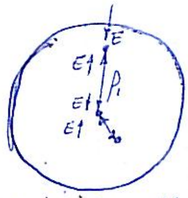
روش سوم:

$$\vec F=-\vec\nabla V=-\frac{\partial V}{\partial\rho}\hat\rho\ ,\quad \vec\nabla\cdot\vec E=0\to\vec\nabla\cdot\vec F=0 \to \cancel{\frac{\partial}{\partial\rho}(\rho F_\rho)=0\to F_\rho+\frac{\partial F}{\partial\rho}\rho=0}$$

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho F_\rho)+\frac{\partial F_z}{\partial z}=0\to\frac{\partial F_\rho}{\partial\rho}=-\frac{\partial F_z}{\partial z}\to\frac{\partial F_\rho}{\partial\rho}<0\to\boxed{\frac{\partial^2 V}{\partial\rho^2}>0}$$



e) ابتدا فرض می کنیم در بیش از یک نقطه $\frac{\partial V}{\partial\rho}=0$ می شود. در نتیجه یکی از نقاط ρ=0 و دیگری ρ_1 در نزدیکی های حلقه است. میدان در نزدیکی محور حلقه در راستای $-\hat\rho$ است. (طبق قانون گاوس و شار). میدان در بالای ρ_1 نیز در راستای $-\hat\rho$ است و در پایین آن در راستای $+\hat\rho$ پس بین ρ_1 و ρ=0 حتما نقطه ی دیگری باید وجود داشته باشد که در آن $\vec E=0$ است. در نتیجه وقتی $\frac{\partial V}{\partial\rho}=0$ است $\frac{\partial^2 V}{\partial\rho^2}$ هم ممکن است مثبت و هم ممکن است منفی شود که در قسمت قبل چنین گزینه ای نداریم. پس $\boxed{\rho_1=0}$ است ← $\vec E_\rho=-|\vec E_\rho|\hat\rho$ ← $-\frac{\partial V}{\partial\rho}\le 0$ ← $\boxed{\frac{\partial V}{\partial\rho}\ge 0}$ (A)



f)

$$\begin{cases}\frac12 mv_0^2=\frac12(M+m)v^2+V_{(\rho,z_0)}\\ mv_0=(M+m)v\end{cases}\to V_{(\rho,z_0)}=\frac12 mv_0^2\left(1-\frac{m}{M+m}\right)\to v_0=\sqrt{\frac{2(M+m)}{Mm}V_{(\rho,z_0)}}\to\boxed{v_0\le\sqrt{\frac{2(M+m)}{Mm}V_{(\rho,z_0)}}}$$

حالت حدی

h)

$$V_{(\rho,z_0)}=\frac{kQq}{2a\pi}\int_0^{2\pi}\left(1+\frac{a}{\rho}\cos\theta\right)d\theta=\frac{kQq}{2a\pi}\times 2\pi=\frac{kQq}{a}$$

تا مرتبه 1

$$V_{(\rho,z_0)}=\frac{kQq}{2\pi a}\int_0^{2\pi}\left(1+\frac{\rho^2}{a^2}-\frac{2\rho}{a}\cos\theta\right)^{-1/2}d\theta=\frac{kQq}{2a\pi}\int_0^{2\pi}\left(1-\frac{\rho^2}{2a^2}+\frac{\rho}{a}\cos\theta+\frac38\times\frac{4\rho^2}{a^2}\cos^2\theta\right)d\theta=\frac{kQq}{2a\pi}\left(1-\frac{\rho^2}{2a^2}+\frac34\frac{\rho^2}{a^2}\right)2\pi$$

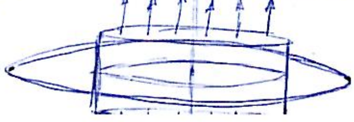
$$\to V_{(\rho,z_0)}=\frac{kQq}{a}\left(1+\frac{\rho^2}{4a^2}\right)\to\boxed{v_0\le\sqrt{\frac{2(M+m)kQq}{aMm}\left(1+\frac{\rho^2}{4a^2}\right)}}$$

g)

$$\boxed{v_0\le\sqrt{\frac{2(M+m)kQq}{\rho Mm}}}$$

d) روش چهارم:

اگر یک سطح گاوسی استوانه ای در نظر بگیریم خطوط میدان مطابق شکل است



$$\Phi=\frac{Q}{\varepsilon}=0\to\Phi_z+\Phi_\rho=0\to\Phi_\rho<0\to E_\rho<0$$

پاسخ سوالات

$\rightarrow \rho_0=0 \Rightarrow E_p<0 \rightarrow \frac{\partial U}{\partial \rho}\Big|_{\rho=0^+}>0 \rightarrow \boxed{\frac{\partial^2 U}{\partial \rho^2}>0}$

4) الف)

$\sigma \times \sigma^{-1}=1 \rightarrow \begin{pmatrix} \alpha & \beta & 0 \\ \beta & \gamma & 0 \\ 0 & 0 & \delta \end{pmatrix}\begin{pmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{pmatrix}=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{cases} \begin{cases} \alpha x_1+\beta y_1=1 \\ \beta x_1+\gamma y_1=0 \end{cases} \\ \begin{cases} \alpha x_2+\beta y_2=0 \\ \beta x_2+\gamma y_2=1 \end{cases} \end{cases}$

$x_1=\frac{\gamma}{\alpha\gamma-\beta^2}$, $y_1=\frac{\beta}{\beta^2-\alpha\gamma}$, $x_2=\frac{\beta}{\beta^2-\alpha\gamma}$, $y_2=\frac{\alpha}{\alpha\gamma-\beta^2}$

$\begin{cases} \alpha x_3+\beta y_3=0 \\ \beta x_3+\gamma y_3=0 \end{cases} \rightarrow x_3=y_3=0 \quad , \quad z_1=z_2=0 \quad , \quad z_3=\frac{1}{\delta}$

$$\rightarrow \boxed{\sigma^{-1}=\begin{pmatrix} \frac{\gamma}{\alpha\gamma-\beta^2} & \frac{\beta}{\beta^2-\alpha\gamma} & 0 \\ \frac{\beta}{\beta^2-\alpha\gamma} & \frac{\alpha}{\alpha\gamma-\beta^2} & 0 \\ 0 & 0 & \frac{1}{\delta} \end{pmatrix}}$$

ب)

$$j=\sigma E \rightarrow E=\sigma^{-1} j \rightarrow \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}=\begin{pmatrix} \frac{\gamma}{\alpha\gamma-\beta^2} & \frac{\beta}{\beta^2-\alpha\gamma} & 0 \\ \frac{\beta}{\beta^2-\alpha\gamma} & \frac{\alpha}{\alpha\gamma-\beta^2} & 0 \\ 0 & 0 & \frac{1}{\delta} \end{pmatrix}\begin{pmatrix} j_x \\ 0 \\ 0 \end{pmatrix} \rightarrow \begin{cases} \boxed{E_x=\frac{\gamma}{\alpha\gamma-\beta^2} j_x} \\ \boxed{E_y=\frac{\beta}{-\alpha\gamma+\beta^2} j_x} \\ \boxed{E_z=0} \end{cases}$$

$U=RI \rightarrow R=\frac{E_x \times a}{j_{(x)} \times bc} \rightarrow \boxed{R=\frac{\gamma}{\gamma\alpha-\beta^2}\frac{a}{bc}}$

ج)

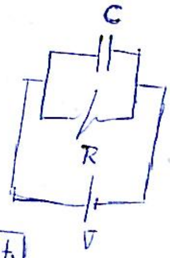
$U'=E_y \times b=\frac{\beta}{\beta^2-\alpha\gamma} b j_x=\frac{\beta}{\beta^2-\alpha\gamma} b \times \frac{\alpha\gamma-\beta^2}{\gamma}\times\frac{-U}{a} \rightarrow \boxed{U'=\frac{\beta b}{\gamma a} U}$

د)

5) a) $\frac{kQ}{a}=U \rightarrow \boxed{Q=(4\pi\varepsilon_0 a)U}$

b) $\boxed{E=\frac{U}{a}}$

c) $P=\frac{U^2}{Z}+U\dot{Q} \rightarrow \boxed{\dot{Q}=\frac{P}{U}-\frac{U}{Z}}$ $\quad\boxed{\dot{U}=\frac{K}{a}\left(\frac{P}{U}-\frac{U}{Z}\right)}$

d) 

$E \leq E_2 \rightarrow U \leq E_2 a \rightarrow U_{max} \leq E_2 a$

e) $\frac{dU}{dt}=\frac{K}{aZU}(PZ-U^2) \rightarrow \int_0^U \frac{U dU}{U^2-PZ}=\int_0^t \frac{K}{aZ} dt \rightarrow \ln\left(\frac{U^2-PZ}{-PZ}\right)=-\frac{2K}{aZ}t \rightarrow \boxed{U^2=PZ\left(1-e^{-\frac{2K}{aZ}t}\right)}$

$\rightarrow \boxed{PZ<E_2^2 a^2} \rightarrow \boxed{\sqrt{PR}<E_2 a}$

$\cancel{P=\frac{U^2}{Z}+U\dot{Q}} \rightarrow \dot{Q}\geq 0 \rightarrow \frac{P}{U}-\frac{U}{Z}\geq 0 \rightarrow U^2\leq PZ \rightarrow \sqrt{PZ}<E_2 a \rightarrow \sqrt{PR}<E_2 a$

روش دوم:

f) $\boxed{U_\infty=\sqrt{PR}}$

g) $\ln\left(\frac{U^2-Pr}{U_0^2-Pr}\right)=-\frac{2K}{ar}\Delta t \rightarrow U^2=Pr+(U_0^2-Pr)e^{-\frac{2K}{ar}\Delta t} \quad , \quad U\geq E_1 a$

و $U_0^2>Pr$; $\boxed{pr>(E_1 a)^2}$ $\quad U_0^2<pr:\ U_0^2>(E_1 a)^2 \rightarrow E_2>E_1 \rightarrow$ برعکس $\rightarrow$

اگر $pr>(E_1a)^2$ در هیچ شرایطی به حالت تخلیه باز نمی‌گردد.

h) $\boxed{U_\infty=\sqrt{pr}}$

i) $\boxed{\cancel{U_1=E_1 a}}$ $\quad\boxed{U_1=E_2 a}$ $\quad\boxed{U_2=\sqrt{PR}}$

7) a) $U=\alpha a^\beta+\frac{kQ^2}{2a} \rightarrow \alpha\beta a_0^{\beta-1}=\frac{kQ^2}{2a_0^2} \rightarrow a_0^{\beta+1}=\frac{kQ^2}{2\alpha\beta} \rightarrow \boxed{a_0=\left(\frac{kQ^2}{2\alpha\beta}\right)^{\frac{1}{\beta+1}}}$, $\boxed{Q=\left(\frac{2\alpha\beta}{k}a_0^{\beta+1}\right)^{\frac{1}{2}}}$

پاسخ سوالات

تابستان 85

7) b ، c)

$$V=\frac{KQ}{a}\longrightarrow \boxed{V=\left(2\alpha\beta K a_0^{\beta-1}\right)^{1/2}}\longrightarrow \boxed{E=\left(2\alpha\beta K a_0^{\beta-3}\right)^{1/2}}$$

d)

$$a_0=\left(\frac{V^2}{2\alpha\beta K}\right)^{\frac{1}{\beta-1}}\longrightarrow E=(2\alpha\beta K)^{\left(1-\frac{\beta-3}{\beta-1}\right)\frac{1}{2}}\times V^{\frac{\beta-3}{\beta-1}}\longrightarrow \boxed{E=(2\alpha\beta K)^{\frac{1}{\beta-1}}\,V^{\frac{\beta-3}{\beta-1}}}$$

$$\frac{\beta-3}{\beta-1}<0\longrightarrow\begin{cases}\beta-3>0\\ \beta-1<0\end{cases}\longrightarrow \text{غ ق ق}\quad \text{یا}\quad \begin{cases}\beta-3<0\\ \beta-1>0\end{cases}\longrightarrow \boxed{1<\beta<3}$$

$$E\Big|_{t=0}=\infty\longrightarrow E>E_1\longrightarrow \frac{4\alpha K}{V}>E_1\longrightarrow V<\frac{4\alpha K}{E_1}\;,\;\boxed{E_1<\frac{4K\alpha}{\sqrt{PR}}}$$ (f

$$\boxed{E=\frac{4\alpha K}{V}}$$ (e

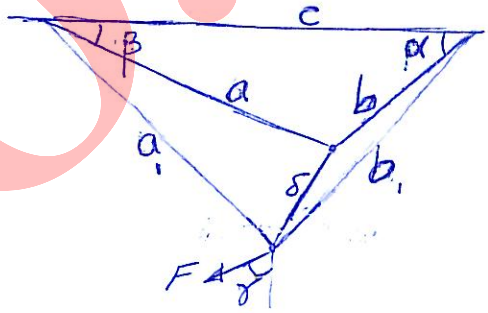
$$\boxed{n=1}$$ (g

$$\begin{cases}\dfrac{F}{K}\dfrac{\cos(\beta-\gamma)}{\sin(\alpha+\beta)}=\cancel{\delta\cos}\;\delta a\\[2ex] \dfrac{F}{K}\dfrac{\cos(\alpha+\gamma)}{\sin(\alpha+\beta)}=\cancel{\delta\cos}\;\delta b\end{cases}$$

امتحان دوم 85

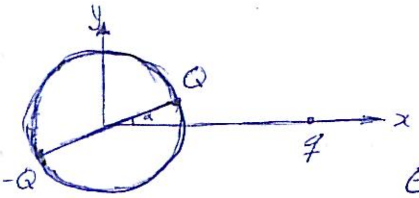
سؤال 2) د

$F\delta$



باسمه تعالی

سال 85



8) الف)

$$\boxed{E_x = \frac{KQ\,(x-a\cos\alpha)}{\left((x-a\cos\alpha)^2+a^2\sin^2\alpha\right)^{3/2}} - \frac{KQ\,(x+a\cos\alpha)}{\left((x+a\cos\alpha)^2+a^2\sin^2\alpha\right)^{3/2}}} \longrightarrow$$

ب)

$$E_x = \frac{KQ}{x^3}\left[(x-a\cos\alpha)\left(1+\frac{3a\cos\alpha}{x}\right) - (x+a\cos\alpha)\left(1-\frac{3a\cos\alpha}{x}\right)\right] \longrightarrow$$

$$E_x = \frac{KQ}{x^3}\left[-2a\cos\alpha + 6a\cos\alpha\right] \longrightarrow \boxed{E_x = \frac{aQ\sin(\omega t)}{\pi\varepsilon_0 x^3}}$$

$$F_{(x)} = m\ddot{x} \longrightarrow \boxed{\ddot{x} = \frac{aQ}{m\pi\varepsilon_0}\cdot\frac{\sin(\omega t)}{x^3}}$$

9)

$$\begin{cases} x = \rho\cos\alpha \\ y = \rho\sin\alpha \end{cases} \longrightarrow \begin{cases} \dot{x} = \dot\rho\cos\alpha - \rho\sin\alpha\,\dot\alpha \\ \dot{y} = \dot\rho\sin\alpha + \rho\cos\alpha\,\dot\alpha \end{cases} \longrightarrow \begin{cases} \ddot{x} = \ddot\rho\cos\alpha - \dot\rho\dot\alpha\sin\alpha - \rho\ddot\alpha\sin\alpha - \dot\rho\dot\alpha\sin\alpha - \rho\cos\alpha\,\dot\alpha^2 \\ \ddot{y} = \ddot\rho\sin\alpha + \dot\rho\cos\alpha\,\dot\alpha + \rho\ddot\alpha\cos\alpha + \dot\rho\dot\alpha\cos\alpha - \rho\sin\alpha\,\dot\alpha^2 \end{cases}$$

$$\begin{cases} -N\sin\alpha = m\ddot{x} \\ N\cos\alpha - mg = m\ddot{y} \end{cases} \longrightarrow \cancel{N = m(g\cos\alpha + \ddot\rho - \rho\dot\alpha^2)} \quad \begin{cases} N = m(g\cos\alpha + 2\dot\rho\dot\alpha + \rho\ddot\alpha) \\ -g\sin\alpha = \ddot\rho - \rho\dot\alpha^2 \end{cases}$$

(i

$$\ddot\rho = \rho\omega^2 - g\sin(\omega t) \longrightarrow A\omega^2 e^{\omega t} + B\omega^2 e^{-\omega t} - D\omega^2\cos(\omega t) - C\omega^2\sin(\omega t) = A\omega^2 e^{\omega t} + B\omega^2 e^{-\omega t}$$

$$+ C\omega^2\sin(\omega t) + D\omega^2\cos(\omega t) - g\sin(\omega t) \longrightarrow \boxed{D=0}\ ,\ 2C\omega^2 = g \longrightarrow \boxed{C = \frac{g}{2\omega^2}}$$

$$\begin{cases} A+B = a \\ C\omega + A\omega - B\omega = 0 \end{cases} \longrightarrow A = \frac{a-c}{2} \longrightarrow A = \frac{a - \frac{g}{2\omega}}{2} \longrightarrow \boxed{A = \frac{a}{2} - \frac{g}{4\omega^2}}\ ,\ \boxed{B = \frac{a}{2} + \frac{g}{4\omega^2}}$$

10) (a

$$\vec{V} = \dot{x}\,\hat{i} + \dot{y}\,\hat{j} \longrightarrow \vec{V} = a\cos\alpha\,\dot\alpha\,\hat{i} - b\sin\alpha\,\dot\alpha\,\hat{j} \longrightarrow \boxed{\vec{V} = \dot\alpha\left(a\cos\alpha\,\hat{i} - b\sin\alpha\,\hat{j}\right)}$$

(b

$$\dot{\vec{V}} = \ddot{x}\,\hat{i} + \ddot{y}\,\hat{j} \longrightarrow \dot{\vec{V}} = \left(-a\sin\alpha\,\dot\alpha^2 + a\cos\alpha\,\ddot\alpha\right)\hat{i} - \left(b\cos\alpha\,\dot\alpha^2 + b\sin\alpha\,\ddot\alpha\right)\hat{j} \longrightarrow$$

$$\boxed{\dot{\vec{V}} = a\left(\cos\alpha\,\ddot\alpha - \sin\alpha\,\dot\alpha^2\right)\hat{i} - b\left(\cos\alpha\,\dot\alpha^2 + \sin\alpha\,\ddot\alpha\right)\hat{j}}$$

(c

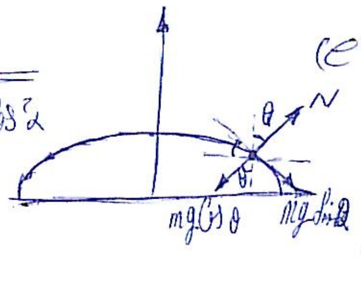
$$mgb = mgb\cos\alpha + \frac{1}{2}m\left(a^2\cos^2\alpha + b^2\sin^2\alpha\right)\dot\alpha^2 \longrightarrow \boxed{\dot\alpha^2 = \frac{2gb(1-\cos\alpha)}{a^2\cos^2\alpha + b^2\sin^2\alpha}}$$

(d

$$\hat{n} = \frac{\vec\nabla f}{|\vec\nabla f|} = \frac{\frac{y}{b^2}\hat{j} + \frac{x}{a^2}\hat{i}}{\sqrt{\left(\frac{y}{b^2}\right)^2 + \left(\frac{x}{a^2}\right)^2}} = \frac{\frac{\sin\alpha}{a}\hat{i} + \frac{\cos\alpha}{b}\hat{j}}{\sqrt{\frac{\sin^2\alpha}{a^2} + \frac{\cos^2\alpha}{b^2}}} \longrightarrow \boxed{\hat{n} = \frac{b\sin\alpha\,\hat{i} + a\cos\alpha\,\hat{j}}{\sqrt{b^2\sin^2\alpha + a^2\cos^2\alpha}}}$$

(e

$$(mg)\,\hat{j}\cdot\hat{n} + N = \frac{mV^2}{r} \longrightarrow f = -\frac{2gb(1-\cos\alpha)ab}{a^2\cos^2\alpha + b^2\sin^2\alpha} \times \frac{(a^2\cos^2\alpha + b^2\sin^2\alpha)}{(a^2\cos^2\alpha + b^2\sin^2\alpha)^{3/2}} + \frac{ag\cos\alpha}{\sqrt{b^2\sin^2\alpha + a^2\cos^2\alpha}}$$



$$\longrightarrow \boxed{f = \frac{ag}{\sqrt{a^2\cos^2\alpha + b^2\sin^2\alpha}}\left(\cos\alpha - \frac{2b^2(1-\cos\alpha)}{a^2\cos^2\alpha + b^2\sin^2\alpha}\right)}$$

$$r = \frac{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{3/2}}{\frac{d^2y}{dx^2}} = \frac{\left(1 + \frac{b^2\sin^2\alpha}{a^2\cos^2\alpha}\right)^{3/2}}{\frac{b}{a}\times\frac{1}{\cos^2\alpha}\times\frac{1}{a\cos\alpha}} = \frac{\left(a^2\cos^2\alpha + b^2\sin^2\alpha\right)^{3/2}}{ab}$$

پاسخ تشریحی

$f=0 \to \boxed{(a^2\cos^2\alpha_0 + b^2\sin^2\alpha_0)\cos\alpha_0 = 2b^2(1-\cos\alpha_0)}$ (f

$V_1 = V_2 \to \frac{4}{3}\pi R^3 = \frac{4\pi}{3}a^2 b \to R^2 = a^2(1-\epsilon) \to a = \frac{R}{\sqrt{1-\epsilon}} \to \boxed{a = R\left(1+\frac{\epsilon}{2}\right)}$ (g

$\left((1+\epsilon)\cos^2\alpha_0 + (1-2\epsilon)\sin^2\alpha_0\right)\cos\alpha_0 = 2(1-2\epsilon)(1-\cos\alpha_0) \to$ (h

$\cos\alpha_0 + \epsilon\cos\alpha_0(\cos^2\alpha_0 + 2\cos^2\alpha_0 - 2) = 2(1-\cos\alpha_0) - 4\epsilon(1-\cos\alpha_0) \to \alpha_0 = \alpha_{(0)} + \alpha_{(1)} \to$

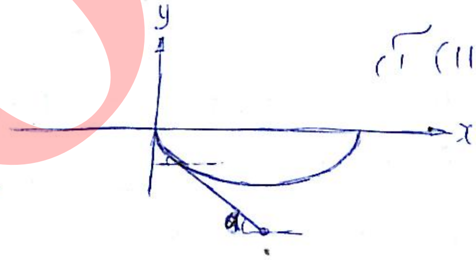
$\cos\alpha_{(0)} - \alpha_{(1)}\sin\alpha_{(0)} + \epsilon\cos\alpha_{(0)}(3\cos^2\alpha_{(0)} - 2) = 2(1-\cos\alpha_{(0)}) + 2\alpha_{(1)}\sin\alpha_{(0)} - 4\epsilon + 4\epsilon\cos\alpha_0$

$\cos\alpha_{(0)} = \frac{2}{3}$, $-\alpha_{(1)}\times\frac{\sqrt5}{3} + \frac{2}{3}\epsilon\left(\frac{4}{3}-2\right) = \frac{2\sqrt5}{3}\alpha_{(1)} - 4\epsilon + \frac{8}{3}\epsilon$

$3\frac{\sqrt5}{3}\alpha_{(1)} = \frac{4\epsilon}{3}\left(-\frac{1}{3}+3-2\right) = \epsilon\left(+\frac{8}{9}\right) \to \boxed{\alpha_{(1)} = +\frac{8}{9\sqrt5}\epsilon} \to \boxed{\alpha_0 = \cos^{-1}\left(\frac{2}{3}\right) + \frac{8}{9\sqrt5}\epsilon}$

$\to \cos\alpha_0 = \frac{2}{3} - \frac{8}{9\sqrt5}\times\frac{\sqrt5}{3}\epsilon \to \boxed{\cos\alpha_0 = \frac{2}{3}\left(1+\frac{4\epsilon}{9}\right)} \to \cancel{\boxed{\cos\alpha_0 = \frac{10}{27}}}$

(11 آ)



$y' = y - (4l - L)\sin\alpha$

$x' = x + (4l - L)\cos\alpha$

$L = \sqrt{dx^2 + dy^2} = l\sqrt{(1-\cos\theta)^2 + \sin^2\theta}\,d\theta \to L = l\sqrt2\int_0^\theta \sqrt2\sin\left(\frac{\theta}{2}\right)d\theta \to$

$L = -4l\left(\cos\left(\frac{\theta}{2}\right) - 1\right) \to L = 4l\left(1-\cos\left(\frac{\theta}{2}\right)\right)$

$\tan\alpha = -\frac{dy}{dx} = +\frac{\sin\theta}{1-\cos\theta} = \cot\left(\frac{\theta}{2}\right) \to \alpha + \frac{\theta}{2} = \frac{\pi}{2} \to \alpha = \frac{\pi}{2} - \frac{\theta}{2}$

$$\to \begin{cases} y' = l(\cos\theta - 1) - \left(4l\cos\frac{\theta}{2}\right)\cos\frac{\theta}{2} \\ x' = l(\theta - \sin\theta) + \left(4l\cos\left(\frac{\theta}{2}\right)\right)\sin\left(\frac{\theta}{2}\right) \end{cases} \to \begin{cases} y' = l(\cos\theta - 1 - 2 - 2\cos\theta) \\ x' = l(\theta + \sin\theta) \end{cases} \to \boxed{\begin{cases} y' = -l(+3+\cos\theta) \\ x' = l(\theta+\sin\theta) \end{cases}}$$

(ب

$ds' = v\,dt = \sqrt{2g(2l + y')}\,dt = \sqrt{2gl(1+\cos\theta)}\,dt = \cancel{2l\sin\left(\frac{\theta}{2}\right)d\theta \to \sqrt{\frac{g}{l}}\,dt = 2\frac{\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)}\frac{d\theta}{2}} \to$

$\frac{ds'}{d\theta} = \frac{\sqrt{dx'^2 + dy'^2}}{d\theta} = l\sqrt{(1+\cos\theta)^2 + \sin^2\theta} = \sqrt2\, l\sqrt{1+\cos\theta} \to T = \sqrt{\frac{l}{g}}(\pi + \pi) \to \boxed{T = 2\pi\sqrt{\frac{l}{g}}}$