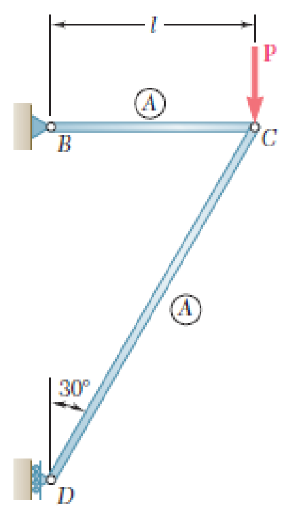
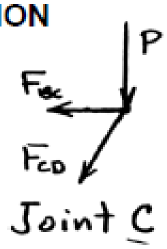
In the Name of GOD

Problem 8-1 In the truss shown, all members are made of the same material and have the uniform cross-sectional area indicated. Determine the strain energy of the truss when the load P is applied. **Answer:** $1.5\dfrac{P^2 l}{EA}$



SOLUTION



$$+\uparrow \Sigma F_y = 0: \quad -\frac{\sqrt{3}}{2}F_{CD} - P = 0 \qquad F_{CD} = -\frac{2}{\sqrt{3}}P$$

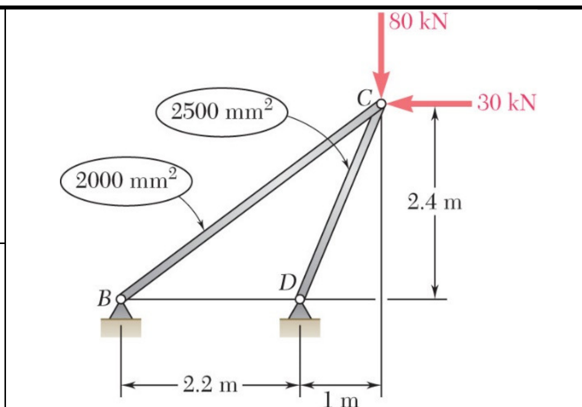
$$\xrightarrow{+} \Sigma F_x = 0: \quad -F_{BC} - \frac{1}{2}F_{CD} = 0 \qquad F_{BC} = \frac{1}{\sqrt{3}}P$$

$$U = \Sigma\frac{F^2L}{2EA} = \frac{1}{2E}\Sigma\frac{F^2L}{A}$$

Member	F	L	A	F^2L/A
BC	$\frac{1}{\sqrt{3}}P$	l	A	$\frac{1}{3}P^2l/A$
CD	$-\frac{2}{\sqrt{3}}P$	$2l$	A	$\frac{8}{3}P^2l/A$
Σ				$3P^2l/A$

$$U = \frac{1}{2E}\left(3\frac{P^2l}{A}\right) \qquad U = 1.5\frac{P^2l}{EA} \blacktriangleleft$$

Problem 8-2 Each member of the truss shown is made of aluminum and has the cross-sectional area shown. Using $E = 72$ GPa, determine the strain energy of the truss for the loading shown. **Answer:** 59.8 J



SOLUTION

$$L_{BC} = (3.2^2 + 2.4^2)^{1/2} = 4 \text{ m}$$

$$L_{CD} = (1^2 + 2.4^2)^{1/2} = 2.6 \text{ m}$$

$$E = 72 \text{ GPa} = 72 \times 10^9 \text{ Pa}$$

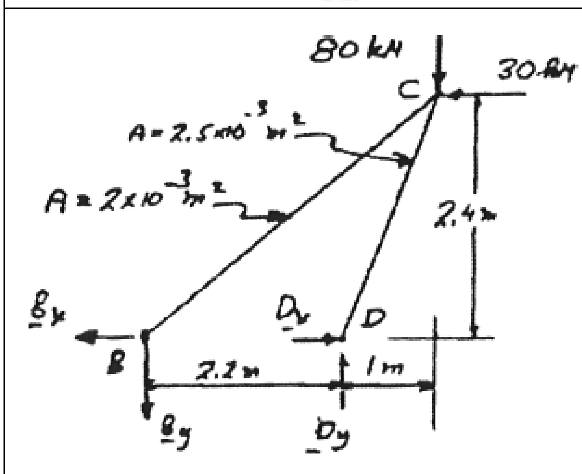
Equilibrium of truss.

$$+\circlearrowleft \sum M_B = 0: (30)(2.4) - (80)(3.2) + D_y(2.2) = 0$$

$$D_y = 83.636 \text{ kN} \uparrow$$

$$+\uparrow \sum F_y = 0: \quad D_y - B_y - 80 = 0$$

$$83.636 - B_y - 80 = 0 \qquad B_y = 3.636 \text{ kN} \downarrow$$



Member forces.

$$F_{BC} = B_y \frac{4\text{ m}}{2.4\text{ m}} = (3.636\text{ kN})\left(\frac{4}{2.4}\right) = 6.061\text{ kN}$$

$$F_{CD} = -D_y \frac{2.6\text{ m}}{2.4\text{ m}} = -(83.636\text{ kN})\left(\frac{2.6}{2.4}\right) = -90.606\text{ kN}$$

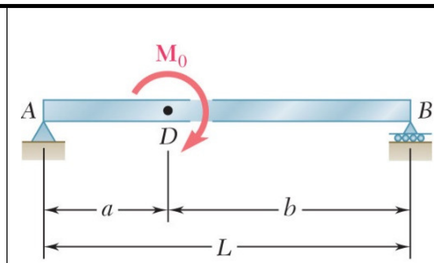
Strain energy. $U = U_{BC} + U_{CD} = \sum \frac{F_i L_i}{2AE}$

$$U = \frac{F_{BC}^2 L_{BC}}{2EA_{BC}} + \frac{F_{CD}^2 L_{CD}}{2EA_{CD}} = \frac{(6.061\times 10^3)^2(4)}{(2)(72\times 10^9)(2\times 10^{-3})} + \frac{(90.606\times 10^3)^2(2.6)}{(2)(72\times 10^9)(2.5\times 10^{-3})}$$

$$= 0.510\text{ J} + 59.290\text{ J} \qquad U = 59.8\text{ J} \blacktriangleleft$$

Problem 8-3 Taking into account only the effect of normal stresses, determine the strain energy of the prismatic beam *AB* for the loading shown.

Answer: $\frac{M_0^2\left(a^3+b^3\right)}{6EIL^2}$

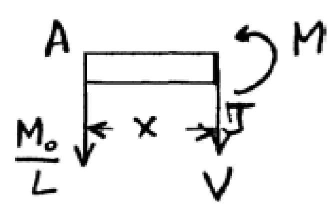


SOLUTION

$$+\circlearrowleft \Sigma M_B = 0: \quad -R_A L - M_0 = 0 \qquad R_A = \frac{M_0}{L}\downarrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad R_B L - M_0 = 0 \qquad R_B = \frac{M_0}{L}\uparrow \qquad +\circlearrowleft \Sigma M_J = 0: \quad \frac{M_0 x}{L} + M = 0$$

A to D: $\quad M = -\frac{M_0 x}{L}$

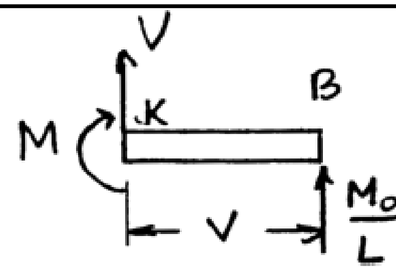


$$U_{AD} = \int_0^a \frac{M^2 dx}{2EI} = \frac{M_0^2}{2EIL^2}\int_0^a x^2 dx = \frac{M_0^2 a^3}{6EIL^2}$$

D to B: $\quad +\circlearrowleft \Sigma M_K = 0: \quad -M + \frac{M_0 v}{L} = 0 \qquad M = \frac{M_0 v}{L}$

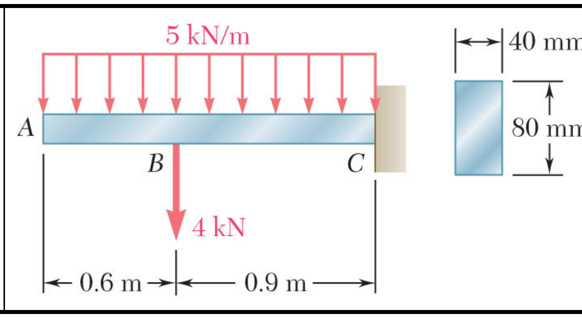
$$U_{DB} = \int_0^b \frac{M^2 dv}{2EI} = \frac{M_0^2}{2EIL^2}\int_0^b v^2 dv = \frac{M_0^2 b^3}{6EIL^2}$$

Total: $U = U_{AD} + U_{DB}$ $\quad U = \dfrac{M_0^2(a^3+b^3)}{6EIL^2}$ ◀



Problem 8-4 For the beam and loading shown, determine the deflection at point B. Use $E = 200$ GPa.

Answer: 7.25 mm ↓

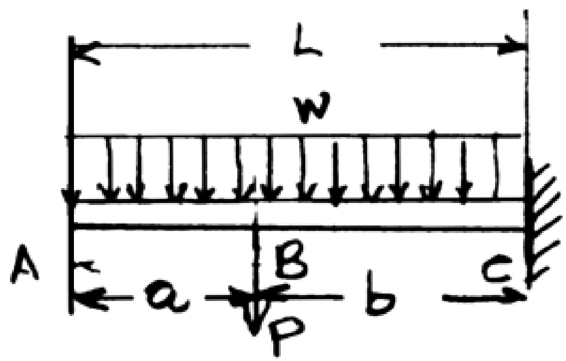


SOLUTION

$$U = \int_0^a \frac{M^2}{2EI}dx + \int_a^L \frac{M^2}{2EI}dx \qquad \delta_B = \frac{\partial U}{\partial P} = \int_0^a \frac{M}{EI}\frac{\partial M}{\partial P}dx + \int_a^L \frac{M}{EI}\frac{\partial M}{\partial P}dx$$

Portion AB: $\quad (0 \le x \le a) \qquad M = -\frac{1}{2}wx^2 \qquad \frac{\partial M}{\partial P} = 0$

$$\int_0^a \frac{M}{EI}\frac{\partial M}{\partial P}dx = 0$$



Portion BC: $\quad (a < x \le L)$

$$M = -\frac{1}{2}wx^2 - P(x-a)$$

$$\frac{\partial M}{\partial P} = -(x-a)$$

$$\int_a^L \frac{M}{EI}\frac{\partial M}{\partial P}dx = \frac{w}{2EI}\int_a^L x^2(x-a)dx + \frac{P}{EI}\int_a^L (x-a)^2 dx$$

$$= \frac{w}{2EI}\int_a^L (x^3 - ax^2)dx + \frac{P}{EI}\int_0^b v^2 dv = \frac{w}{2EI}\left(\frac{L^4}{4} - \frac{aL^3}{3} - \frac{a^4}{4} + \frac{a^4}{3}\right) + \frac{Pb^3}{3EI}$$

$$\delta_B = 0 + \frac{w}{2EI}\left(\frac{L^4}{4} - \frac{aL^3}{3} + \frac{a^4}{12}\right) + \frac{Pb^3}{3EI}$$

<u>Data</u>: $a = 0.6$ m, $b = 0.9$ m, $L = a + b = 1.5$ m, $w = 5\times10^3$ N/m

$P = 4\times10^3$ N $\quad I = \frac{1}{12}(40)(80)^3 = 1.70667\times10^6\ \text{mm}^4 = 1.70667\times10^{-6}\ \text{m}^4$

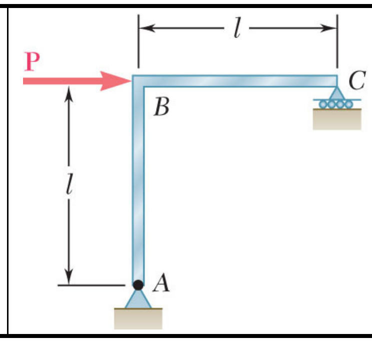
$EI = (200\times10^9)(1.70667\times10^{-6}) = 341{,}333\ \text{N}\cdot\text{m}^2$

$$\delta_B = 0 + \frac{5\times10^3}{(2)(341{,}333)}\left[\frac{(1.5)^4}{4} - \frac{(0.6)(1.5)^3}{3} + \frac{(0.6)^3}{12}\right] + \frac{(4\times10^3)(0.9)^3}{(3)(341{,}333)}$$

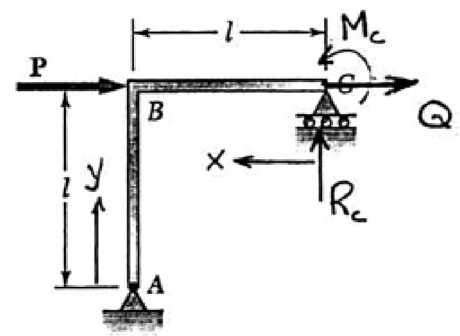
$= 7.25\times10^{-3}$ m $\qquad \delta_B = 7.25$ mm $\downarrow$ ◀

Problem 8-5 Two rods *AB* and *BC* of the same flexural rigidity *EI* are welded together at *B*. For the loading shown, determine (a) the deflection of point *C*, (b) the slope of member *BC* at point *C*.

Answer: $(a)\ \dfrac{2PL^3}{3EI}\rightarrow$, $(b)\ \dfrac{PL^2}{6EI}$



SOLUTION



Add horizontal force Q and couple M_C at C.

$$+\circlearrowleft \Sigma M_A = 0:\quad R_C l + M_C - (P+Q)l = 0$$

$$R_C = P + Q + \frac{M_C}{l}$$

$$+\!\longrightarrow \Sigma F_x = 0:\quad P + Q + R_{Ax} = 0 \qquad R_{Ax} = P + Q \leftarrow$$

<u>Member *AB*</u>:

$$M = R_{Ax}y = (P+Q)y,\qquad \frac{\partial M}{\partial Q} = y,\qquad \frac{\partial M}{\partial M_C} = 0$$

$$U_{AB} = \int_0^l \frac{M^2}{2EI}dy$$

Set $Q = 0$ and $M_C = 0$.

$$\frac{\partial U_{AB}}{\partial Q} = \frac{1}{EI}\int_0^l M\frac{\partial M}{\partial Q}dy = \frac{1}{EI}\int_0^l (Py)(y)dy = \frac{1}{3}\frac{Pl^3}{EI}\qquad \frac{\partial U_{AB}}{\partial M_C} = \frac{1}{EI}\int_0^l M\frac{\partial M}{\partial M_A}dx = 0$$

<u>Member *BC*</u>: $\quad M = M_C + R_C x = M_C + \left(P + Q + \frac{M_C}{l}\right)x$

$$\frac{\partial M}{\partial Q} = x,\qquad \frac{\partial M}{\partial M_C} = 1 - \frac{x}{l}\qquad U_{BC} = \int_0^l \frac{M^2}{2EI}dx$$

Set $Q = 0$ and $M_C = 0$.

$$\frac{\partial U_{BC}}{\partial Q} = \frac{1}{EI}\int_0^l M\frac{\partial M}{\partial Q}dx = \frac{1}{EI}\int_0^l (Px)x\,dx = \frac{1}{3}\frac{Pl^3}{EI}$$

$$\frac{\partial U}{\partial M_A} = \frac{1}{EI}\int_0^l M\frac{\partial M}{\partial M_A}dx = \frac{1}{EI}\int_0^l (Px)\left(1-\frac{x}{l}\right)dx = \frac{P}{EI}\int_0^l\left(x-\frac{x^2}{l}\right)dx$$

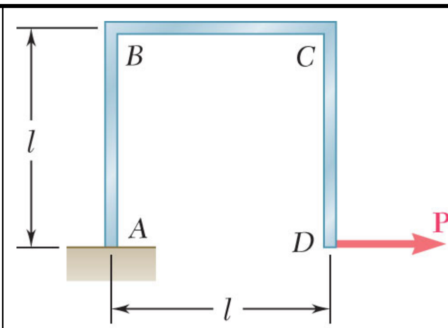
$$= \frac{P}{EI}\left(\frac{1}{2}l^2 - \frac{1}{3}l^2\right) = \frac{1}{6}\frac{Pl^2}{EI}$$

(a) Deflection at C. $\delta_C = \frac{\partial U_{AB}}{\partial Q} + \frac{\partial U_{BC}}{\partial Q}$ $\delta_C = \frac{2Pl^3}{3EI} \rightarrow$ ◀

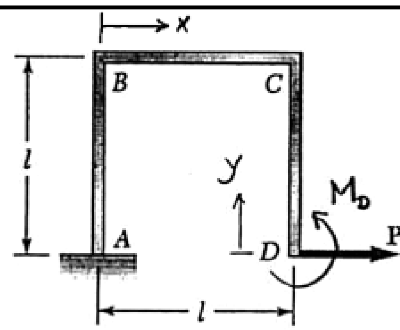
(b) Slope at C. $\circlearrowright \theta_C = \frac{\partial U_{AB}}{\partial M_A} + \frac{\partial U_{BC}}{\partial M_C}$ $\theta_C = \frac{Pl^2}{6EI}$ ◀

Problem 8-6 A uniform rod of flexural rigidity EI is bent and loaded as shown. Determine (a) the horizontal deflection of point D, (b) the slope at point D. (c) the vertical deflection of point D, (d) the slope of BC at point C. **Answer:**

$(a)\ \frac{5PL^3}{3EI}\rightarrow$, $(b)\ \frac{2PL^2}{EI}$, $(c)\ \frac{PL^3}{EI}\uparrow$, $(d)\ \frac{3PL^2}{EI}$



SOLUTION



Add couple M_D at point D.

Reactions at A: $R_{Ay} = 0$, $R_{Ax} = P \leftarrow$, $M_A = M_0 \circlearrowleft$

Member AB:

$$M = M_A + R_A y = M_D + Py \qquad \frac{\partial M}{\partial P} = y, \qquad \frac{\partial M}{\partial M_D} = 1$$

$$U_{AB} = \int_0^l \frac{M^2}{2EI}dy \qquad \text{Set } M_D = 0.$$

$$\frac{\partial U_{AB}}{\partial P} = \frac{1}{EI}\int_0^l M\frac{\partial M}{\partial P}dy = \frac{1}{EI}\int_0^l (Py)y\,dy = \frac{Pl^3}{3EI}$$

$$\frac{\partial U_{AB}}{\partial M_0} = \frac{1}{EI}\int_0^l M\frac{\partial M}{\partial M_0}dy = \frac{1}{EI}\int_0^l (Py)(1)\,dy = \frac{Pl^2}{2EI}$$

Member BC: $M = M_A + R_A l = M_D + Pl$ $\quad \frac{\partial M}{\partial P} = l, \quad \frac{\partial M}{\partial M_D} = 1$

$U_{BC} = \int_0^l \frac{M^2}{2EI} dx$ Set $M_D = 0$.

$$\frac{\partial U_{BC}}{\partial P} = \frac{1}{EI}\int_0^l M \frac{\partial M}{\partial P} dx = \frac{1}{EI}\int_0^l (Pl)(l)\, dx = \frac{Pl^3}{EI}$$

$$\frac{\partial U_{BC}}{\partial M_D} = \frac{1}{EI}\int_0^l M \frac{\partial M}{\partial M_D} dx = \frac{1}{EI}\int_0^l (Pl)(1)\, dx = \frac{Pl^2}{EI}$$

Member CD: $M = M_D + Py$ $\quad \frac{\partial M}{\partial P} = y \quad \frac{\partial M}{\partial M_D} = 1$

$U_{CD} = \int_0^l \frac{M^2}{2EI} dy$ Set $M_D = 0$.

$$\frac{\partial U_{CD}}{\partial P} = \frac{1}{EI}\int_0^l M \frac{\partial M}{\partial P} dy = \frac{1}{EI}\int_0^l (Py)(y)\, dy = \frac{Pl^3}{3EI}$$

$$\frac{\partial U_{CD}}{\partial M_D} = \frac{1}{EI}\int_0^l M \frac{\partial M}{\partial M_D} dy = \frac{1}{EI}\int_0^l (Py)(1)\, dy = \frac{Pl^2}{2EI}$$

(a) Horizontal deflection of point D.

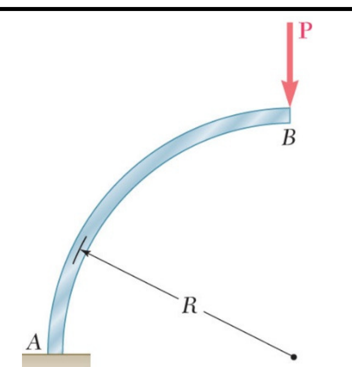
$$\delta_P = \frac{\partial U_{AB}}{\partial P} + \frac{\partial U_{BC}}{\partial P} + \frac{\partial U_{CD}}{\partial P} = \left(\frac{1}{3} + 1 + \frac{1}{3}\right)\frac{Pl^3}{EI} \qquad \delta_P = \frac{5Pl^3}{3EI} \rightarrow \blacktriangleleft$$

(b) Slope at point D.

$$\theta_D = \frac{\partial U_{AB}}{\partial M_D} + \frac{\partial U_{BC}}{\partial M_D} + \frac{\partial U_{CD}}{\partial M_D} = \left(\frac{1}{2} + 1 + \frac{1}{2}\right)\frac{Pl^2}{EI} \qquad \theta_D = \frac{2Pl^2}{EI} \circlearrowright \blacktriangleleft$$

Problem 8-7 For the beam and loading shown and using Castigliano's theorem, determine (a) the horizontal deflection of point B, (b) the vertical deflection of point B.

Answer: (a) $\frac{PR^3}{2EI} \rightarrow$, (b) $\frac{\pi PR^3}{4EI} \downarrow$.

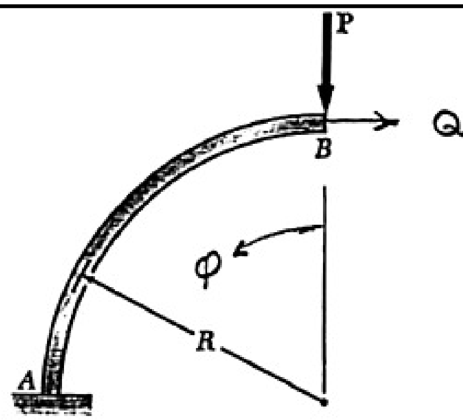


SOLUTION

Add horizontal force Q at point B.

Use polar coordinate φ.

$$U = \int_0^{\pi/2} \frac{M^2}{2EI} R\,d\varphi$$



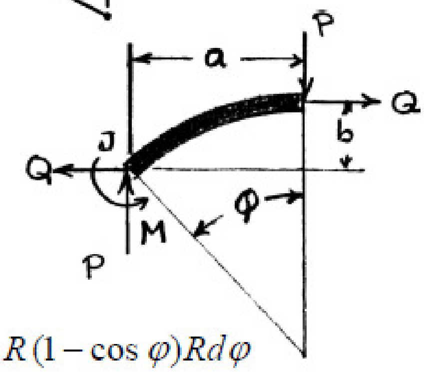
Bending moment.

$$+\circlearrowleft \Sigma M_J = 0: \quad M - Pa - Qb = 0$$

$$M = Pa + Qb$$

$$= PR\sin\varphi + QR(1-\cos\varphi)$$

$$\frac{\partial M}{\partial P} = R\sin\varphi \qquad \frac{\partial M}{\partial Q} = R(1-\cos\varphi) \qquad \text{Set } Q = 0.$$



$$(a) \qquad \delta_Q = \frac{\partial U}{\partial Q} = \frac{1}{EI}\int_0^{\pi/2} M\frac{\partial M}{\partial Q} R\,d\varphi = \frac{1}{EI}\int_0^{\pi/2} PR\sin\varphi\, R(1-\cos\varphi) R\,d\varphi$$

$$= \frac{PR^3}{EI}\int_0^{\pi/2}(\sin\varphi - \sin\varphi\cos\varphi)\,d\varphi = \frac{PR^3}{EI}\left(-\cos\varphi - \frac{1}{2}\sin^2\varphi\right)\Bigg|_0^{\pi/2}$$

$$= \frac{PR^3}{EI}\left(-\cos\frac{\pi}{2} + \cos 0 - \frac{1}{2}\sin^2\frac{\pi}{2} + \frac{1}{2}\sin^2 0\right) = \frac{PR^3}{EI}\left(0 + 1 - \frac{1}{2} + 0\right) \qquad \delta_Q = \frac{PR^3}{2EI} \rightarrow \blacktriangleleft$$

$$(b) \qquad \delta_P = \frac{\partial U}{\partial P} = \frac{1}{EI}\int_0^{\pi/2} M\frac{\partial M}{\partial P} R\,d\varphi = \frac{1}{EI}\int_0^{\pi/2} PR\sin\varphi\, R\sin\varphi\, R\,d\varphi$$

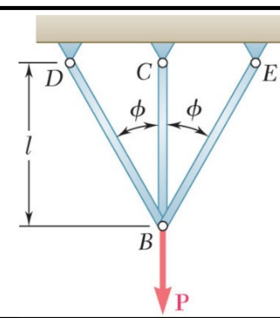
$$= \frac{PR^3}{EI}\int_0^{\pi/2}\sin^2\varphi\,d\varphi = \frac{PR^3}{EI}\int_0^{\pi/2}\frac{1}{2}(1-\cos 2\varphi)\,d\varphi$$

$$= \frac{PR^3}{EI}\left(\frac{1}{2}\varphi - \frac{1}{2}\sin 2\varphi\right)\Bigg|_0^{\pi/2} = \frac{PR^3}{EI}\left(\frac{1}{2}\cdot\frac{\pi}{2} - \frac{1}{2}\cdot 0 - \frac{1}{2}\sin\pi + \frac{1}{2}\cdot\sin 0\right)$$

$$= \frac{PR^3}{EI}\left(\frac{\pi}{4} - 0 - 0 + 0\right) \qquad \delta_P = \frac{\pi PR^3}{4EI} \downarrow \blacktriangleleft$$

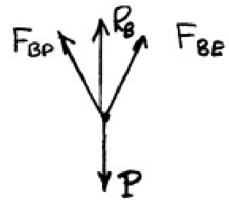
Problem 8-8 Three members of the same material and same cross-sectional area are used to support the loading P. Determine the force in member BC.

Answer: $F_{BC} = \dfrac{P}{\left(1 + 2\cos^3\theta\right)}$



SOLUTION

Detach member BC at support C. Add reaction R_C as a load.



$$U = \Sigma \frac{F^2 L}{2EA} \qquad y_C = \frac{\partial U}{\partial R_C} = \Sigma \frac{FL}{EA}\frac{\partial F}{\partial R_C} = 0$$

Joint C: $\quad F_{BC} = R_C$

Joint B: $\quad \overset{+}{\rightarrow} \Sigma F_x = 0: \quad F_{BE} \sin\varphi - F_{BD} \sin\varphi = 0 \qquad F_{BE} = F_{BD}$

$$+\uparrow \Sigma F_y = 0: \quad F_{BD} \cos\varphi + F_{BE} \cos\varphi + R_B - P \qquad F_{BD} = F_{BE} = \frac{P - R_B}{2\cos\varphi}$$

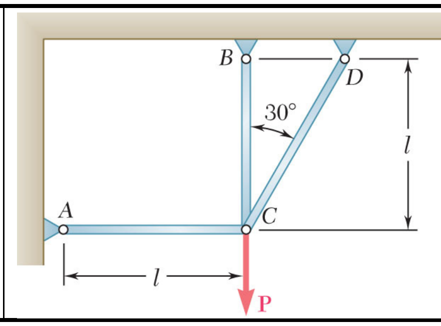
Member	F	$\partial F/\partial R_B$	L	$(FL/EA)\,(\partial F/\partial R_B)$
BD	$(P-R_B)/2\cos\varphi$	$-1/2\cos\varphi$	$l/\cos\varphi$	$(R_B - P)l/4EA\cos^3\varphi$
BE	$(P-R_B)/2\cos\varphi$	$-1/2\cos\varphi$	$l/\cos\varphi$	$(R_B - P)l/4EA\cos^3\varphi$
BC	R_B	1	l	$R_B l/EA$

$$y_B = -Pl/2EA\cos^3\varphi + R_B l/2EA\cos^3\varphi + R_B l/EA = 0$$

$$R_B = \frac{P}{1 + 2\cos^3\varphi} \qquad F_{BC} = R_B \qquad F_{BC} = \frac{P}{1+2\cos^3\varphi} \blacktriangleleft$$

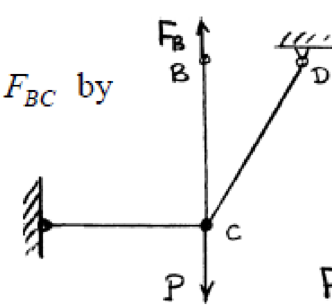
Problem 8-9 Three members of the same material and same cross-sectional area are used to support the loading P. Determine the force in member BC.

Answer: $F_{BC} = 0.652P$

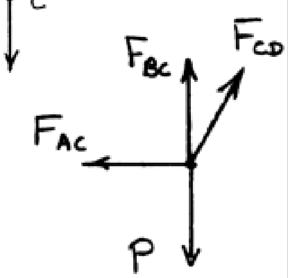


SOLUTION

Cut member BC at end B and replace member force F_{BC} by load F_B acting on member BC at B.



$$\delta_B = \frac{\partial U}{\partial F_B} = \frac{\partial}{\partial F_B}\Sigma \frac{F^2 L}{EA} = \frac{1}{EA}\Sigma F \frac{\partial F}{\partial F_B} L = 0$$



Joint C: $\quad +\uparrow \Sigma F_y = 0: \quad \frac{\sqrt{3}}{2} F_{CD} + F_{BC} - P = 0 \qquad F_{CD} = \frac{2}{\sqrt{3}} P - \frac{2}{\sqrt{3}} F_B$

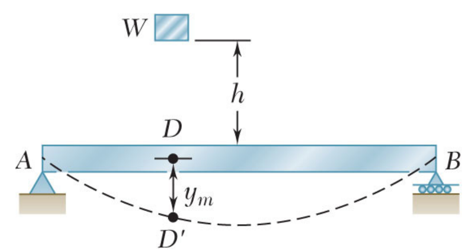
$$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad F_{AC} - \frac{1}{2} F_{CD} = 0 \qquad F_{AC} = \frac{1}{\sqrt{3}} P - \frac{1}{\sqrt{3}} F_B$$

Member	F	$\partial F/\partial F_B$	L	$F(\partial F/\partial F_B)L$
AC	F_B	1	l	$F_B l$
BC	$\frac{1}{\sqrt{3}}P-\frac{1}{\sqrt{3}}F_B$	$-\frac{1}{\sqrt{3}}$	l	$-\frac{1}{3}Pl+\frac{1}{3}F_B l$
CD	$\frac{2}{\sqrt{3}}P-\frac{2}{\sqrt{3}}F_B$	$-\frac{2}{\sqrt{3}}$	$\frac{2}{\sqrt{3}}l$	$-\frac{8}{\sqrt{3}}Pl+\frac{8}{\sqrt{3}}F_B l$
Σ				$-\left(\frac{1}{3}+\frac{8}{\sqrt{3}}\right)Pl+\left(\frac{4}{3}+\frac{8}{\sqrt{3}}\right)F_B l$

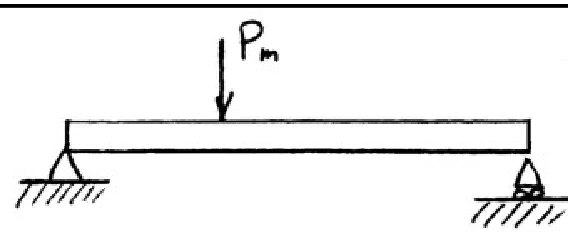
$$\delta_B=-\left(\frac{1}{3}+\frac{8}{\sqrt{3}}\right)\frac{Pl}{EA}+\left(\frac{4}{3}+\frac{8}{\sqrt{3}}\right)\frac{F_B l}{EA}=0 \qquad F_B=\frac{\frac{1}{3}+\frac{8}{\sqrt{3}}}{\frac{4}{3}+\frac{8}{\sqrt{3}}}P=\frac{8+\sqrt{3}}{8+4\sqrt{3}}P=0.652P$$

$F_{BC}=F_B \qquad F_{BC}=0.652P$ ◀

Problem 8-10 A block of weight W is dropped from a height h onto the horizontal beam AB and hits point D. Denoting by y_m the exact value of the maximum deflection at D and by y'_m the value obtained by neglecting the effect of this deflection on the change in potential energy of the block, show that the absolute value of the relative error is $(y'_m-y_m)/y_m$, never exceeding $y'_m/2h$.



SOLUTION



$$U=\frac{1}{2}P_m y_m=\frac{1}{2}ky_m^2$$

where k is the spring constant for a load at point D.

Work of falling weight: exact: Work $=W(h+y_m)$ approximate: Work $\approx Wh$

Equating work and energy, $\frac{1}{2}ky_m^2=W(h+y_m)$ (1) exact

$\frac{1}{2}ky'^2_m=Wh$ (2) approximate

where y'_m is the approximate value for y_m.

Subtracting, $\frac{1}{2}k\left(y_m^2-y'^2_m\right)=Wy_m \qquad y_m^2-y'^2_m=(y_m-y'_m)(y_m+y'_m)=\frac{2W}{k}y_m$

Relative error: $\dfrac{y_m - y'_m}{y_m} = \dfrac{2W}{k(y_m + \tilde{y}_m)}$ But $\dfrac{2W}{k} = \dfrac{y'^2_m}{h}$ from Eq. (2).

Relative error $= \dfrac{y_m - y'_m}{y_m} = \dfrac{y'^2_m}{h(y_m + y'_m)} < \dfrac{y'_m}{2h}$

Problem 9-1 A 2-m-long pin-ended column of square cross section is to be made of wood. Assuming $E = 13$ GPa, $\sigma_{all} = 12$ MPa, and using a factor of safety of 2.5 in computing Euler's critical load for buckling, determine the size of the cross section if the column is to safely support (*a*) a 100-kN load, (*b*) a 200-kN load.

(a) For the 100-kN Load. Using the given factor of safety, we make

$$P_{cr} = 2.5(100 \text{ kN}) = 250 \text{ kN} \qquad L = 2 \text{ m} \qquad E = 13 \text{ GPa}$$

in Euler's formula and solve for I. We have

$$I = \frac{P_{cr} L^2}{\pi^2 E} = \frac{(250 \times 10^3 \text{ N})(2 \text{ m})^2}{\pi^2 (13 \times 10^9 \text{ Pa})} = 7.794 \times 10^{-6} \text{ m}^4$$

Recalling that, for a square of side a, we have $I = a^4/12$, we write

$$\frac{a^4}{12} = 7.794 \times 10^{-6} \text{ m}^4 \qquad a = 98.3 \text{ mm} \approx 100 \text{ mm}$$

We check the value of the normal stress in the column:

$$\sigma = \frac{P}{A} = \frac{100 \text{ kN}}{(0.100 \text{ m})^2} = 10 \text{ MPa}$$

Since σ is smaller than the allowable stress, a 100 × 100-mm cross section is acceptable.

(b) For the 200-kN Load. Solving again for I, but making now $P_{cr} = 2.5(200) = 500$ kN, we have

$$I = 15.588 \times 10^{-6} \text{ m}^4$$

$$\frac{a^4}{12} = 15.588 \times 10^{-6} \qquad a = 116.95 \text{ mm}$$

The value of the normal stress is

$$\sigma = \frac{P}{A} = \frac{200 \text{ kN}}{(0.11695 \text{ m})^2} = 14.62 \text{ MPa}$$

Since this value is larger than the allowable stress, the dimension obtained is not acceptable, and we must select the cross section on the basis of its resistance to compression. We write

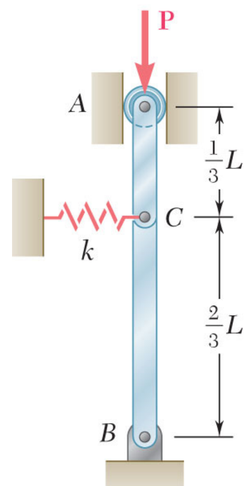
$$A = \frac{P}{\sigma_{all}} = \frac{200\text{ kN}}{12\text{ MPa}} = 16.67 \times 10^{-3}\text{ m}^2$$

$$a^2 = 16.67 \times 10^{-3}\text{ m}^2 \qquad a = 129.1\text{ mm}$$

A 130 × 130-mm cross section is acceptable.

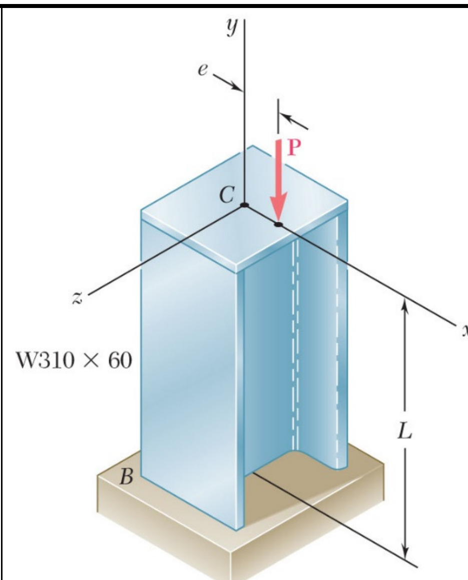
Problem 9-2 Two rigid bars *AC* and *BC* are connected as shown to a spring of constant *k*. Knowing that the spring can act in either tension or compression, determine the critical load P_{cr} for the system.

Answer: $P_{cr} = \dfrac{2KL}{9}$



Problem 9-3 An axial load *P* is applied at a point located on the *x* axis at a distance $e = 12$ mm from the geometric axis of the rolled-steel column *BC*. Assuming that $L = 3.5$ m and using $E = 200$ GPa, determine (a) the load P for which the horizontal deflection at end *C* is 15 mm, (b) the corresponding maximum stress in the column. (Help: $A = 7590 \times 10^{-6}\text{m}^2$, $I_x = 125 \times 10^{-6}\text{ m}^4$, $I_z = 18.4 \times 10^{-6}\text{ m}^4$).

Answer: (a) 368.28 kN , (b) 103.8 MPa



$L = 3.5\text{ m} \qquad L_e = 2L = 7.0\text{ m}$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2(200\times10^9)(18.3\times10^{-6})}{(7.0)^2} = 737.2\times10^3\text{ N} = 737.2\text{ kN}$$

$$y_{max} = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right] \qquad \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{y_{max}+e}{e} \qquad \cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{e}{y_{max}+e}$$

$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi}\arccos\frac{e}{y_{max}+e}\right]^2 = \left[\frac{2}{\pi}\arccos\frac{12}{15+12}\right]^2 = 0.49957$$

(a) $P = 0.49957\,P_{cr} = 368.28\text{ kN}$

$$M_{max} = P(e+y_{max}) = (368.28\times10^3)(12+15)(10^{-3}) = 9944\text{ N}\cdot\text{m}$$

(b) $\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{M}{S_y} = \frac{368.28\times10^3}{7590\times10^{-6}} + \frac{9944}{180\times10^{-6}} = 103.8\times10^6\text{ Pa}$

$= 103.8\text{ MPa}$