Implementation and Validation of the Johnson-Holmquist Ceramic Material Model in LS-Dyna

Authors:

Duane S. Cronin*, Khahn Bui**, Christian Kaufmann* Grant McIntosh***, Todd Berstad**

> *University of Waterloo, Ontario, Canada ** LSTC, California, United States *** DRDC Valcartier, Quebec, Canada

Correspondence:

Duane Cronin Department of Mechanical Engineering University of Waterloo, Waterloo, Ontario N2L 3G1 Canada

> Tel: 519-888-4567 x2682 Fax: 519-888-6197 e-mail: dscronin@uwaterloo.ca

Keywords: Ceramic, Constitutive model, Ballistic impact

ABSTRACT

Ceramic materials are commonly used in protective armour applications and may be subject to high-energy ballistic impacts in these situations. Under simple loading conditions, ceramics may be regarded as elastic-brittle materials. However, when considering ballistic impacts, the post-yield response of the ceramic becomes significant. One of the most widely used constitutive models for simulating the postyield response of ceramic materials is the JH-2 ceramic model. This constitutive equation was developed by Johnson and Holmquist and incorporates the effect of damage on residual material strength and the resulting bulking during the compressive failure of a ceramic material.

The relevant equations describing the response of the material are described. In particular, the model parameters currently available for common ballistic ceramic materials are presented.

The JH-2 constitutive model has been implemented in LS-Dyna as material 110 (*MAT_JOHNSON_HOLMQUIST_CERAMICS). Validation against the available test cases in the literature is discussed, and a sample calculation of a sphere impacting a ceramic material is presented. The JH-2 model in LS-Dyna has also been used by Kaufman et al. to successfully simulate the ballistic impact of 12.7 mm armourpiercing projectiles on supported alumina tiles.

NOMENCLATURE

$A, N, C, B, M, D_1, D_2, \beta$	
JH-2 Material Constants	

D	Material damage (total)
$\Delta arepsilon_{ m P}$	Plastic strain increment
$arepsilon_{ m f}$	Plastic fracture strain
G	Elastic shear modulus
HEL	Hugoniot elastic limit
K_1, K_2, K_3	Equation of state constants
Р	Current pressure
P _{HEL}	Pressure at HEL
P*	Normalized pressure
ΔP	Bulking pressure
$ ho_{\scriptscriptstyle o}$	Initial density
ρ	Current density
ΔU	Incremental energy loss due to damage increment
σ^{*}	Current normalized strength
σ	Effective stress
$\sigma^*_{_f}$	Fractured material strength
$\sigma_{_{ m HEL}}$	Equivalent stress at the HEL
σ_i^*	Intact material strength

INTRODUCTION

The analysis of ceramics or other brittle materials subject to impact is of significant interest due to their extensive use in personnel (body), light (vehicle) and heavy (tank) amour applications. Ceramics are an important component for these systems due to their low density and high hardness [1], key parameters in the performance of any protective armor system. Other areas of interest include mechanical components, such as turbine blades, where the prediction of impact response is essential.

Although experimental testing is always necessary, there is a considerable motivation for the development and validation of numerical models in this area. With respect to ballistic impacts on ceramics, the response of the ceramic is dependant on: projectile size, projectile velocity, projectile construction and material, material supporting the ceramic (backing) and, of course, the mechanical properties of the ceramic [2]. It can be appreciated that to understand these dependencies through experimental testing, given the considerable degree of scatter in this type of data can be very costly and time-consuming.

While several constitutive models exist to describe the response of œramics to various types of loading, the Johnson-Holmquist (JH-2) model [3] has been found to provide good results while capturing the essential components of ceramic response to ballistic impacts [2]. It should be noted that any constitutive equation embodies assumptions, some of which are tied to the scale of the model. For example, the initiation of failure in ceramic materials is, to a great extent, tied to the presence of microscopic defects [1]. Thus, a representative constitutive model must embody this effect to some degree. The key to any model is achieving the correct balance between accurate representations of the physical phenomenon while maintaining some degree of computational efficiency. The JH-2 model achieves this through representation of the initiation and propagation of failure via a damage variable.

This study is focused on the implementation and validation of the J-H2 model within LS-Dyna (Version 970).

IMPACT RESPONSE OF CERAMIC MATERIALS

The impact response of ceramics is unique due to the brittle nature of these materials. Of particular importance is the low/negligible ductility exhibited under both quasi-static and dynamic loading, and the influence of hydrostatic pressure on the strength of the material [3]. When a ceramic material is subjected to a dynamic impact, two distinct responses can be identified. The first phase, which occurs over time scales measured at the microsecond level, begins upon impact. A compressive stress wave initiates at the impact site and travels radially outwards from this point [4,5]. The compressive stress wave velocity, determined by the shock response of the material, may greatly exceed the elastic wave velocity for a given impact. If the magnitude of the compressive stress wave exceeds the local dynamic strength of the material, damage begins to accumulate through the formation of cracks. This fracture front travels at the elastic wave speed in the material and forms a conoid of comminuted or pulverized material under the impact location. When the compressive stress wave reaches a free surface of the ceramic it reflects as a tensile wave and may lead to the formation of spall (tensile cracking) damage if the dynamic tensile strength of the ceramic material is exceeded.

The second phase of impact corresponds to large scale deformation and erosion of the ceramic and/or projectile [5]. This phase occurs over much larger time scales (typically milliseconds) and terminates when the projectile penetrates or is arrested by the ceramic. At low impact velocities, the projectile may undergo deformation and fracture, and may be defeated with only moderate damage to the œramic. For intermediate and high impact velocities, and harder projectiles, erosion of the œramic may occur as the projectile penetrates into the ceramic material. It can be appreciated that this material description includes several material properties such as the dynamic uniaxial yield strength or Hugoniot Elastic Limit (HEL) and spall strengths, and that these parameters should be included in the constitutive model. In addition, a œscription for the initiation and evolution of damage (fracture) within the material must be included.

JOHNSON-HOLMQUIST CERAMIC CONSTITUTIVE MODEL

The Johnson-Holmquist ceramic constitutive model was first proposed to describe the response of brittle materials to large deformations. The first version of the model [6] (JH-1) did not allow for progressive damage of the material. In addition, the description of material strength was represented with multiple linear segments over the representative pressure and damage regimes. The second version of this model, known as JH-2 [3] addressed these issues, expressing the material strength and damage as functions of the representative variables. More importantly, the evolution of damage within the material was considered. The JH-2 model also included normalization of the strength parameters by the HEL to allow for a more direct comparison of various materials. The JH-2 version of the model is the focus of this study and implementation.

The goal of any constitutive model is to adequately represent the response of a material to various loading conditions. As indicated previously, this results in a trade-off between modeling the material response at a suitable scale and comput ational efficiency. Specifically, it is well known that damage in ceramics initiates in the form of small cracks, which grow and coalesce to form fractur ed or comminuted material [1]. However, modeling the initiation and progression of damage on this level is recognized to be numerically impractical and somewhat unnecessary. Damage in the JH-2 model is represented by a state (damage) variable corresponding to the average damage within a specific volume of material, a finite element. This damage evolves as the material is subjected to deformation and results in a corresponding decrease in strength. Material strength, and thus damage, are both functions of the pressure at a particular location in the material.

Within the LS-Dyna hydrocode [7], material response is calculated in an terative step-wise fashion where increments in element deformation (strain) lead to changes in stress via the material constitutive equation. The step size corresponds to the specific value of the time step for a given iteration within LS-Dyna. Of particular importance is the fact that time or path dependant constitutive equations rely on history variables (plastic strain, damage etc.) from the previous time step to determine the current state of the material. For the following constitutive model discussion, variables with no subscript correspond to the current values (i.e. at time *t* or step *n*). Variables with the subscript n+1 correspond to the new values after a time increment ?t and variables with the subscript n-1 refer to values from the previous time step. The goal of the constitutive relationship is to calculate the new material stress state at step n+1, after time increment ?t.

The JH-2 constitutive model [3] requires several material constants to completely describe the response of a particular material. Initially the material response is considered to be elastic, with the stress state completely described by the elastic material properties (shear modulus) and equation of state. Based on the current material deformation, μ (equation 1) and corresponding pressure (equations 2a and 2b) can be calculated. This is the equation of state for the material.

$$\mu = \frac{\rho}{\rho_0} - 1 \tag{1}$$

$$P = K_1 \mu + K_2 \mu^2 + K_3 \mu^3 + \Delta P_{n-1}$$
 (Compression) (2a)

$$P = K_1 \mu$$
 (Tension) (2b)

In equation (2a), ?P corresponds to the bulking pressure of the material and is determined by the amount of accumulated damage.

Under compressive loading, damage begins to accumulate within the material when the deviator stress exceeds a critical value. This damage accumulation is tracked via a damage parameter (ranging from 0 to 1.0), and the corresponding nonrecoverable or plastic strain. Thus, the current material strength is determined by the damaged and undamaged strength curves as well as the current material damage. Figure 1 shows these curves for a ceramic material. Both the strength and pressure are normalized by the equivalent stress at the HEL and the pressure at the HEL respectively. When subjected to tensile pressure, the material responds elastically until brittle failure at a specified effective stress value. This corresponds to complete instantaneous damage.



Figure 1 Strength versus pressure for intact (undamaged) and damaged material

The intact material strength is defined as:

$$\sigma_{i}^{*} = A(P^{*} + T^{*})^{N} (1 + C \ln \dot{\varepsilon})$$
(3)

and the fractured material strength as:

D – I - 51

$$\sigma_f^* = B(P^*)^M (1 + C \ln \dot{\varepsilon}) \tag{4}$$

Although the JH-2 model does account for strain rate effects, it has been noted that these effects are typically secondary compared the pressure effects [8]. This has been noted experimentally, and is reflected in the typical values for the constants in the JH-2 model. The current material strength is then determined from equation 5.

$$\sigma^* = \sigma_i^* - D(\sigma_i^* - \sigma_f^*) \tag{5}$$

Based on the current strain and time increments, the current effective strain rate and total strains can be calculated. The current strength (equation 5) can then be used with the radial return method, a common approach to address plasticity in LS-Dyna [7], can be used to determine the current increment in plastic strain ($?e_P$). From this, the current increment in damage can be determined as shown in equation 6.

$$\Delta D = \frac{\Delta \varepsilon_P}{\varepsilon_f}, \quad D = \sum \frac{\Delta \varepsilon_P}{\varepsilon_f} \tag{6}$$

where the plastic strain to fracture under a constant pressure is defined as:

$$\varepsilon_f = D_1 (P^* + T^*)^{D_2}$$
 (7)

As previously indicated, an increment in damage leads to material bulking. This can be described physically as the larger volume a fractured material occupies compared to the intact material. Constraint or confinement from surrounding material results in a local increase in pressure. The bulking pressure is zero for undamaged material, and the bulking pressure at the next time increment is:

$$\Delta P_{n+1} = -K_1 \mu + \sqrt{\left(K_1 \mu + \Delta P\right)^2 + 2\beta K_1 \Delta U}$$
(8)

The new bulking pressure depends on the amount of incremental energy loss that is converted to potential or hydrostatic energy through bulking. The parameter ß describes the amount of energy converted, and is typically set to 1.0. The energy loss corresponding to increased bulking pressure and reduced deviator stresses is defined as:

$$\Delta U = U(D) - U(D_{n+1}), U(D) = \frac{\sigma}{6G}$$
(9)

In this case, the energies are calculated using the current material damage curves, but with the current and updated value of damage for each location in the material.

This summarizes the operating equations for the JH-2 constitutive model. When using LS-Post to view results, the total stress, strain, pressure and permanent or plastic strain may be found in the usual locations. The current bulking pressure for an element is recorded as History Variable 1 and the material damage as History Variable 2. Note that the *DATABASE_EXTENT_BINARY command is required to specify the storage of history variables in the output file. The JH-2 model is currently implemented for solid elements only; however, axisymmetric elements may also be used as they utilize the same material model formulation.

MATERIAL MODEL CONSTANTS

At present, several common ballistic ceramic materials have been characterized and presented in the literature. The properties are summarized in table 1. These materials include: 99.5 % Alumina (Al_2O_3) [8], Boron Carbide (B_4C) [9], Silicon Carbide (SiC) [13] and Aluminum Nitride (AlN) [10]. Another common ceramic material, silica-based glass, has also been characterized [11].

					Silica
	B4C	SiC	AIN	AI2O3	Float Glass
Reference	[9]	[12]	[10]	[8]	[11]
Density (kg/m^3)	2510	3163	3226	3700	2530
Shear Modulus (GPa)	197	183	127	90.16	30.4
Strength Constants					
A	0.927	0.96	0.85	0.93	0.93
В	0.7	0.35	0.31	0.31	0.088
С	0.005	0.0	0.013	0.0	0.003
М	0.85	1.0	0.21	0.6	0.35
N	0.67	0.65	0.29	0.6	0.77
Ref Strain Rate (EPSI)	1.0	1.0	1.0	1.0	1.0
Tensile Strength (GPa)	0.26	0.37	0.32	0.2	0.15
Normalized Fracture Strength	0.2	0.8	NA	NA	0.5
HEL (GPa)	19	14.567	9	2.79	5.95
HEL Pressure (GPa)	8.71	5.9	5	1.46	2.92
HEL Vol. Strain	0.0408		0.0242	0.01117	
HEL Strength (GPa)	15.4	13.0	6.0	2.0	4.5
Damage Constants					
D1	0.001	0.48	0.02	0.005	0.053
D2	0.5	0.48	1.85	1.0	0.85
Equation of State					
K1 (GPa) (Bulk Modulus)	233	204.785	201	130.95	45.4
K2 (GPa)	-593	0	260	0	-138
K3 (GPa)	2800	0	0	0	290
Beta	1.0	1.0	1.0	1.0	1.0

 Table 1
 Constitutive constants for ceramic materials

The actual determination of these constitutive constants is complicated since many cannot be determined directly, and must be inferred. In addition, the types of testing necessary to determine these constants are varied.

The elastic constants for a particular material are commonly available from the manufacturer or in published data. The pressure-volume relationship may be determined from flyer plate impact experiments or quasi-static approaches, such as the diamond-anvil technique [10]. It should be noted that materials which undergo a phase change at elevated pressures may not be adequately described by this model. It is also necessary to determine the strength relationships for both the intact and damaged material as a function of pressure. This is dependent on constants such as the HEL and tensile hydrostatic pressure strength, and also must include strain rate effects. The determination of these relationships is outlined by Johnson and Holmquist [10]. However, it should be noted that in the absence of appropriate experimental data, some of the constants may be determined by the calibration of numerical simulations to more convenient experiments, such as ballistic impact tests.

In addition to the material constants listed above, the implementation of the JH-2 model in LS-Dyna also includes a criterion for material erosion. The first option is based on a critical value of effective plastic strain, and the second on tensile failure. In the current formulation, these criteria are mutually exclusive (i.e. only one of the criteria may be used in a simulation). It must be emphasized that, in general, erosion criteria should be used with caution as early or premature erosion of material can lead to incorrect model predictions, and significantly increase the mesh-size dependency of the calculation. In the current form, the erosion criteria are meant to approximate material erosion due to projectile penetration or failure due to excessive tensile pressure [12]. Damage initiates when the stress of the intact material exceeds the material strength at the current pressure. This leads to an increment of plastic strain and corresponding increase in damage. The erosion criterion is based on the total plastic strain, where the element is eroded when the plastic strain exceeds the specified plastic strain (FS). If any negative value is entered for the failure strain (FS), element erosion occurs if the tensile pressure exceeds the specified maximum tensile pressure. It should be noted that the failure strain (FS) is only an erosion criterion and is not included in the material/damage calculations.

VALIDATION CASES

Johnson and Holmquist [3] present three validation cases for this constitutive model. All cases involve the confined compression and release of a ceramic material with variation of the damage representation to demonstrate the response of the model. A schematic of the boundary conditions for these test cases is shown in Figure 2. The three-dimensional test element was a cube with sides 1.0 m in length. It was constrained from displacing on five sides, with the external load applied via displacement conditions to the sixth side (top). For each test, the material was displaced vertically downwards by 0.05 m and then released until a zero stress-state was reached. Due to bulking, the final volume of the material was larger than the original volume resulting in a non-zero displacement corresponding to zero stress.



Figure 2 Test case boundary conditions

The material properties for the three validation cases are outlined in Table 2. For Case A, the material was defined as having no fractured strength ($s_f=0$), and was not allowed to accumulate plastic strain ($e_f=0$). As such, the material damaged completely once the material strength was exceeded (Figure 3, Case A). This led to an instantaneous increase in bulking pressure of 0.56 GPa. It should be noted that the final bulking pressures are not apparent in Figure 3, but can be found in the reference by Johnson and Holmquist [3]. In Case B, the material was defined as having no fractured strength ($s_f=0$), but was allowed to accumulate plastic strain so that complete damage did not occur instantaneously. In this case, the bulking pressure increased with damage to a maximum value of 0.72 GPa when the material was completely damaged. Case C incorporated both fractured material strength and the accumulation of plastic strain. The results of the validation cases compared to the data published by Johnson and Holmquist are presented in Figure 3 in terms of effective stress versus pressure. It can be seen that, in addition to predicting the correct final bulking pressure, the stress-pressure histories are also in agreement.

	Case A	Case B	Case C
Density (kg/m^3)	3700	3700	3700
Shear Modulus (GPa)	90.16	90.16	90.16
Strength Constants			
A	0.93	0.93	0.93
В	0	0	0.31
С	0	0	0
Μ	0	0	0.6
N	0.6	0.6	0.6
Ref Strain Rate (EPSI)	1.0	1.0	1.0
Tensile Strength (GPa)	0.2	0.2	0.2
Normalized Fracture Strength	0	NA	NA
HEL (GPa)	2.79	2.79	2.79
HEL Pressure (GPa)	1.46	1.46	1.46
HEL Vol. Strain	0.01117	0.01117	0.01117
HEL Strength (GPa)	2.0	2.0	2.0
Damage Constants			
D1	0	0.005	0.005
D2	0	1	1.0
Equation of State			
K1 (GPa) (Bulk Modulus)	130.95	130.95	130.95
K2 (GPa)	0	0	0
K3 (GPa)	0	0	0
Beta	1.0	1.0	1.0

 Table 2
 Constitutive constants for validation cases



Figure 3 Stress versus pressure histories for single element validation cases

SAMPLE CALCULATION

A sample calculation consisting of a 3.0 mm diameter steel sphere impacting a 5.0 mm thick flat plate of silica-based glass was developed to demonstrate the various aspects of the JH-2 model in LS-Dyna. The numerical model (Figure 4) was constructed using axisymmetric elements approximately 0.1 mm in size.



Figure 4 Sample calculation finite element model

D – I - 56

4th European LS-DYNA Users Conference

The resulting contours of damage and pressure are shown in Figure 5 at various times during the impact. The damage values range from 0.0 (undamaged) to 1.0 (completely damaged). The contours of pressure range from 0.0 to 1.0×10^8 Pa, with positive values of pressure corresponding to compression. It can be seen that damage initiates at the impact location, with the fracture front expanding outwards in the radial direction. It can readily be seen that the compression wave generated by the impact (right side of Figure 5) travels significantly faster than the damage front, as observed experimentally. Of particular interest are the pressure and damage plots at 1.2 µs (Figure 5). At this time, the compression wave has traveled through the thickness of the glass (ceramic) material and reflected at the free surface resulting in a tensile wave. In this case, the reflected wave was of sufficient pressure to initiate tensile (spall) failure of the material as indicated in the damage plot. Although not compared directly here, similar experimental tests [1] demonstrate similar results in terms of damage evolution and spall.



Figure 5 Sample calculation finite element model

It is evident that the finite element mesh undergoes significant deformation due to the damage, erosion and subsequent ejection of material (evident at 4 μ s in Figure 5).

At this point, much of the ejected material is not longer in contact with the projectile and is completely damaged so that it no longer contributes significantly to the response of the projectile and glass. However, this highly deformed material can significantly reduce the time-step in LS-Dyna. In this case, appropriate implement ation of the failure strain erosion criterion (FS = 3.0) allowed for this unnecessary material to be eroded (removed from the calculation) while not adversely affecting the simulation predictions. The tensile failure criterion was also considered by setting FS to a value less than zero. However, this criterion seemed to be moderately numerically unstable in this simulation and is only recommended in cases where it is necessary to remove material which has already failed in tension. It should be noted that erosion of this material may lead to incorrect predictions, particularly when the damaged material is later re-loaded in compression.

Kaufmann et al. [2] have used the JH-2 material model in LS-Dyna to simulate ballistic impacts on armour-grade alumina with an aluminum backing. The projectile was a 12.7mm armour-piercing round, impacting the ceramic at velocities of 750, 850 and 915 m/s. Both the alumina and the armour-piercing projectile core were modeled with the JH-2 constitutive model. The numerical results, based on the depth of penetration of the projectile in the aluminum backing, were in good agreement with the experimentally measured values.

SUMMARY AND CONCLUSIONS

The implementation and validation of the Johnson-Holmquist (JH-2) constitutive model in LS-Dyna (Version 970), based on the constitutive equations proposed by Johnson and Holmquist, has been presented. Within this model, the initiation and progression of cracking through the material is tracked with a representative damage variable. The constitutive model expresses the material strength as a function of pressure and damage, while incorporating material bulking due to damage.

The current implementation of JH-2 model in LS-Dyna accurately reproduces the published data for simple single element validation cases. A simple sample calculation of a steel sphere impacting a silica-based glass plate was presented to demonstrate application of this constitutive model to an impact simulation. These results demonstrate the experimentally observed phenomena of damage evolution, fracture propagation, and spall damage resulting from reflected compression waves at a free surface. The JH-2 model in LS-Dyna has also been used to successfully simulate the ballistic impact of 12.7 mm armour-piercing projectiles on supported alumina tiles.

REFERENCES

- 1. MEYERS, M.A., (1994), <u>Dynamic Behavior of Materials</u>, John Wiley & Sons Inc., Canada.
- KAUFMANN C., CRONIN D.S., WORSWICK M.J., BOURGET D., "Numerical modeling of the ballistic response of ceramic tiles supported by semi-infinite metallic backings", Personal Armour Systems Symposium – Pass, The Hague, The Netherlands, November 18-22, 2002
- JOHNSON, G.R. AND HOLMQUIST, T.J., (1994) "An improved computational constitutive model for brittle materials", High-Pressure Science and Technology – 1994, American Institute of Physics.
- 4. DEN REIJER, P.C. (1991) <u>Impact on Ceramic Faced Armor</u>, Technische Univ. Delft (Netherlands), PhD Thesis.

4th European LS-DYNA Users Conference

- KAUFMANN, C., CRONIN D.S., WORSWICK M.J., PAGEAU G., BETH A. (2003), "Influence of Material Properties on the Ballistic Performance of Ceramics for Personal Body Armour", Shock and Vibration Journal, SAV 10(1), February 2003, pp. 51-59.
- JOHNSÓN, G.R. AND HOLMQUIST, T.J. (1992), "A Computional Constitutive Model for Brittle Materials Subjected to Large Strains", Shock-wave and Highstrain-rate Phenomena in Materials, edited by M.A. Meyers, L.E. Murr and K.P. Staudhammer, Marcel Dekker Inc., New York, pp. 1075-1081.
- 7. HALLQUIST, J.O. (1998) "LS-DYNA Theory Manual", Livermore Software Technology Corp., Livermore.
- ANDERSON, C.E., JOHNSON, G.R., HOLMQUIST, T.J. (1995) "Ballistic experiments and computations of confined 99.5% Al₂O₃ ceramic tiles", Proceedings of Fifteenth International Symposium on Ballistics, Jerusalem, Israel, May 1995.
- of Fifteenth International Symposium on Ballistics, Jerusalem, Israel, May 1995.
 JOHNSON, G.R., HOLMQUIST, T.J. (1999), "Response of boron cabide subjected to large strains, high strain rates, and high pressures", Journal of Applied Physics, Vol. 85, Number 12, June 15, 1999.
- HOLMQUIST, T.J.M TEMPLETON, D.W., BISHNOI, K.D. (2001), "Consitiutive Modeling of Aluminum Nitride for Large Strain, High-Strain Rate, and High-Pressure Applications" Int. J. of Impact Engineering, 2001, Vol. 25, pp. 211-231.
- 11. HOLMQUIST, T.J., JOHNSON,G.R., GRADY, D.E., LOPATIN, C.M. and HERTEL, E.S. (1995), "High strain rate properties and constitutive modeling of glass", Proceedings of Fifteenth International Symposium on Ballistics, Jerus a-lem, Israel, May 1995.
- 12. HALLQUIST, J.O. (2001) "LS-DYNA Keyword User's Manual Volume II (Version 960)", Livermore Software Technology Corp., Livermore.
- LEGALLIC, C., CAURET, M., TRANCHET, J., CHARTAGNAC, P., GIL, F., JAMES, B., PICKUP, M., MILTON, A.L. and CARSON, W.A. (1996), "A consideration of damage in the interaction between tungsten rod penetrators and ceramic materials", 16th International Symposium on Ballistics, San Francisco, CA, USA, 23-27 September, 1996.

Material I

4th European LS-DYNA Users Conference