2.1 (a)

$$\begin{aligned}
k &= 8.617 \times 10^{-5} \text{ eV/K} \\
n_i(T = 300 \text{ K}) &= 1.66 \times 10^{15} (300 \text{ K})^{3/2} \exp\left[-\frac{0.66 \text{ eV}}{2\,(8.617 \times 10^{-5} \text{ eV/K})\,(300 \text{ K})}\right] \text{ cm}^{-3} \\
&= \boxed{2.465 \times 10^{13} \text{ cm}^{-3}} \\
n_i(T = 600 \text{ K}) &= 1.66 \times 10^{15} (600 \text{ K})^{3/2} \exp\left[-\frac{0.66 \text{ eV}}{2\,(8.617 \times 10^{-5} \text{ eV/K})\,(600 \text{ K})}\right] \text{ cm}^{-3} \\
&= \boxed{4.124 \times 10^{16} \text{ cm}^{-3}}
\end{aligned}$$

Compared to the values obtained in Example 2.1, we can see that the intrinsic carrier concentration in Ge at $T = 300$ K is $\frac{2.465\times10^{13}}{1.08\times10^{10}} = 2282$ times higher than the intrinsic carrier concentration in Si at $T = 300$ K. Similarly, at $T = 600$ K, the intrinsic carrier concentration in Ge is $\frac{4.124\times10^{16}}{1.54\times10^{15}} = 26.8$ times higher than that in Si.

(b) Since phosphorus is a Group V element, it is a donor, meaning $N_D = 5 \times 10^{16}$ cm^{-3}. For an n-type material, we have:

$$\begin{aligned}
n = N_D &= \boxed{5 \times 10^{16} \text{ cm}^{-3}} \\
p(T = 300 \text{ K}) &= \frac{[n_i(T = 300 \text{ K})]^2}{n} = \boxed{1.215 \times 10^{10} \text{ cm}^{-3}} \\
p(T = 600 \text{ K}) &= \frac{[n_i(T = 600 \text{ K})]^2}{n} = \boxed{3.401 \times 10^{16} \text{ cm}^{-3}}
\end{aligned}$$

2. (a) Mobility of electrons in Si $= 1350\ cm^2/V\text{-}s$
Mobility of holes in Si $= 480\ cm^2/V\text{-}s$

$\Rightarrow$ velocity of electrons $= \mu_n E = \left(1350 \frac{cm^2}{V\text{-}s}\right)\left(0.1 \frac{V}{\mu m}\right)$

$$= 1.35 \cdot 10^4\ m/s$$

velocity of holes $= \mu_p E = \left(480 \frac{cm^2}{V\text{-}s}\right)\left(0.1 \frac{V}{\mu m}\right)$

$$= 4.8 \cdot 10^3\ m/s$$

(b) Given $E = 0.1\ V/\mu m$ hole current negligible
$\mu_n = 1350\ cm^2/V\text{-}s$ $\mu_p = 480\ cm^2/V\text{-}s$

$$J_{tot} = 1\ mA/\mu m^2 = q[\mu_n n E + \mu_p p E] \approx q\mu_n n E$$

$$\therefore n = \frac{J_{tot}}{q\mu_n E} = \frac{1\ mA/\mu m^2}{(1.6\cdot 10^{-19} C)(1350\ cm^2/V\text{-}s)(0.1\ V/\mu m)}$$

$$= 4.6 \cdot 10^{17}\ cm^{-3}$$

2.3 (a) Since the doping is uniform, we have no diffusion current. Thus, the total current is due only to the drift component.

$$\begin{aligned}
I_{tot} &= I_{drift} \\
&= q(n\mu_n + p\mu_p)AE \\
n &= 10^{17}\ \text{cm}^{-3} \\
p &= n_i^2/n = (1.08 \times 10^{10})^2/10^{17} = 1.17 \times 10^3\ \text{cm}^{-3} \\
\mu_n &= 1350\ \text{cm}^2/\text{V}\cdot\text{s} \\
\mu_p &= 480\ \text{cm}^2/\text{V}\cdot\text{s} \\
E &= V/d = \frac{1\ \text{V}}{0.1\ \mu\text{m}} \\
&= 10^5\ \text{V/cm} \\
A &= 0.05\ \mu\text{m} \times 0.05\ \mu\text{m} \\
&= 2.5 \times 10^{-11}\ \text{cm}^2
\end{aligned}$$

Since $n\mu_n \gg p\mu_p$, we can write

$$\begin{aligned}
I_{tot} &\approx qn\mu_n AE \\
&= \boxed{54.1\ \mu\text{A}}
\end{aligned}$$

(b) All of the parameters are the same except n_i, which means we must re-calculate p.

$$\begin{aligned}
n_i(T = 400\ \text{K}) &= 3.657 \times 10^{12}\ \text{cm}^{-3} \\
p = n_i^2/n &= 1.337 \times 10^8\ \text{cm}^{-3}
\end{aligned}$$

Since $n\mu_n \gg p\mu_p$ still holds (note that n is 9 orders of magnitude larger than p), the hole concentration once again drops out of the equation and we have

$$\begin{aligned}
I_{tot} &\approx qn\mu_n AE \\
&= \boxed{54.1\ \mu\text{A}}
\end{aligned}$$

2.4 (a) From Problem 1, we can calculate n_i for Ge.

$$\begin{aligned}
n_i(T = 300\ \text{K}) &= 2.465 \times 10^{13}\ \text{cm}^{-3} \\
I_{tot} &= q(n\mu_n + p\mu_p)AE \\
n &= 10^{17}\ \text{cm}^{-3} \\
p &= n_i^2/n = 6.076 \times 10^{9}\ \text{cm}^{-3} \\
\mu_n &= 3900\ \text{cm}^2/\text{V}\cdot\text{s} \\
\mu_p &= 1900\ \text{cm}^2/\text{V}\cdot\text{s} \\
E &= V/d = \frac{1\ \text{V}}{0.1\ \mu\text{m}} \\
&= 10^{5}\ \text{V/cm} \\
A &= 0.05\ \mu\text{m} \times 0.05\ \mu\text{m} \\
&= 2.5 \times 10^{-11}\ \text{cm}^2
\end{aligned}$$

Since $n\mu_n \gg p\mu_p$, we can write

$$\begin{aligned}
I_{tot} &\approx qn\mu_n AE \\
&= \boxed{156\ \mu\text{A}}
\end{aligned}$$

(b) All of the parameters are the same except n_i, which means we must re-calculate p.

$$\begin{aligned}
n_i(T = 400\ \text{K}) &= 9.230 \times 10^{14}\ \text{cm}^{-3} \\
p &= n_i^2/n = 8.520 \times 10^{12}\ \text{cm}^{-3}
\end{aligned}$$

Since $n\mu_n \gg p\mu_p$ still holds (note that n is 5 orders of magnitude larger than p), the hole concentration once again drops out of the equation and we have

$$\begin{aligned}
I_{tot} &\approx qn\mu_n AE \\
&= \boxed{156\ \mu\text{A}}
\end{aligned}$$

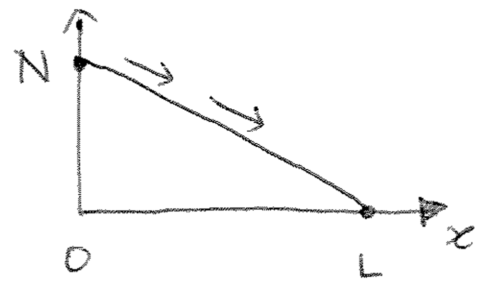
2.5 Since there's no electric field, the current is due entirely to diffusion. If we define the current as positive when flowing in the positive x direction, we can write

$$\begin{aligned}
I_{tot} &= I_{diff} = AJ_{diff} = Aq\left[D_n \frac{dn}{dx} - D_p \frac{dp}{dx}\right] \\
A &= 1\ \mu\text{m} \times 1\ \mu\text{m} = 10^{-8}\ \text{cm}^2 \\
D_n &= 34\ \text{cm}^2/\text{s} \\
D_p &= 12\ \text{cm}^2/\text{s} \\
\frac{dn}{dx} &= -\frac{5 \times 10^{16}\ \text{cm}^{-3}}{2 \times 10^{-4}\ \text{cm}} = -2.5 \times 10^{20}\ \text{cm}^{-4} \\
\frac{dp}{dx} &= \frac{2 \times 10^{16}\ \text{cm}^{-3}}{2 \times 10^{-4}\ \text{cm}} = 10^{20}\ \text{cm}^{-4} \\
I_{tot} &= \left(10^{-8}\ \text{cm}^2\right)\left(1.602 \times 10^{-19}\ \text{C}\right)\left[\left(34\ \text{cm}^2/\text{s}\right)\left(-2.5 \times 10^{20}\ \text{cm}^{-4}\right) - \left(12\ \text{cm}^2/\text{s}\right)\left(10^{20}\ \text{cm}^{-4}\right)\right] \\
&= \boxed{-15.54\ \mu\text{A}}
\end{aligned}$$

6. Given Area = a

find total e ectrons stored.

$$n(x) = -\frac{N}{L}x + N$$



$\therefore$ total electrons stored

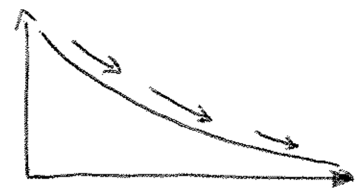
$$= \int a \cdot n(x)\, dx = \int_0^L a\left(-\frac{N}{L}x + N\right)dx$$

$$= aN\left(-\frac{x^2}{2L} + x\right)\Bigg|_0^L = \underline{\frac{aNL}{2}}$$

7. Given Area $= a$

find total electrons stored.

$$n(x) = N \cdot \exp\left(\frac{-x}{L_d}\right)$$



$\therefore$ total electrons stored

$$= \int a\, n(x)\, dx = \int_0^\infty a \cdot N \cdot \exp\left(\frac{-x}{L_d}\right) dx$$

$$= aN\left(-L_d \cdot \exp\frac{-x}{L_d}\right)\Big|_0^\infty = aNL_d.$$

For the linear profile, the result depends on the length, L.

For the exponential profile, the result is constant (since L_d is constant.)

2.8 Assume the diffusion lengths L_n and L_p are associated with the electrons and holes, respectively, in this material and that $L_n, L_p \ll 2$ µm. We can express the electron and hole concentrations as functions of x as follows:

$$\begin{aligned}
n(x) &= Ne^{-x/L_n} \\
p(x) &= Pe^{(x-2)/L_p} \\
\text{\# of electrons} &= \int_0^2 an(x)dx \\
&= \int_0^2 aNe^{-x/L_n}dx \\
&= -aNL_n \left(e^{-x/L_n}\right)\Big|_0^2 \\
&= -aNL_n \left(e^{-2/L_n} - 1\right) \\
\text{\# of holes} &= \int_0^2 ap(x)dx \\
&= \int_0^2 aPe^{(x-2)/L_p}dx \\
&= aPL_p \left(e^{(x-2)/L_p}\right)\Big|_0^2 \\
&= aPL_p \left(1 - e^{-2/L_p}\right)
\end{aligned}$$

Due to our assumption that $L_n, L_p \ll 2$ µm, we can write

$$\begin{aligned}
e^{-2/L_n} &\approx 0 \\
e^{-2/L_p} &\approx 0 \\
\text{\# of electrons} &\approx \boxed{aNL_n} \\
\text{\# of holes} &\approx \boxed{aPL_p}
\end{aligned}$$

9. Drift is analogous to water flow in a river.

Water flows from top of mountain to bottom because of gravitational field; electron flows from one terminal to the other because of electric field.

<u>DRIFT</u>		<u>WATER FLOW</u>
electrons	$\longleftrightarrow$	water
electric field	$\longleftrightarrow$	gravitational field.
drift/current	$\longleftrightarrow$	water flow

2.10 (a)

$$n_n = N_D = \boxed{5 \times 10^{17}\ \mathrm{cm}^{-3}}$$
$$p_n = n_i^2/n_n = \boxed{233\ \mathrm{cm}^{-3}}$$
$$p_p = N_A = \boxed{4 \times 10^{16}\ \mathrm{cm}^{-3}}$$
$$n_p = n_i^2/p_p = \boxed{2916\ \mathrm{cm}^{-3}}$$

(b) We can express the formula for V_0 in its full form, showing its temperature dependence:

$$V_0(T) = \frac{kT}{q} \ln \left[\frac{N_A N_D}{\left(5.2 \times 10^{15}\right)^2 T^3 e^{-E_g/kT}} \right]$$
$$V_0(T = 250\ \mathrm{K}) = \boxed{906\ \mathrm{mV}}$$
$$V_0(T = 300\ \mathrm{K}) = \boxed{849\ \mathrm{mV}}$$
$$V_0(T = 350\ \mathrm{K}) = \boxed{789\ \mathrm{mV}}$$

Looking at the expression for $V_0(T)$, we can expand it as follows:

$$V_0(T) = \frac{kT}{q} \left[\ln(N_A) + \ln(N_D) - 2\ln\left(5.2 \times 10^{15}\right) - 3\ln(T) + E_g/kT \right]$$

Let's take the derivative of this expression to get a better idea of how V_0 varies with temperature.

$$\frac{dV_0(T)}{dT} = \frac{k}{q} \left[\ln(N_A) + \ln(N_D) - 2\ln\left(5.2 \times 10^{15}\right) - 3\ln(T) - 3 \right]$$

From this expression, we can see that if $\ln(N_A) + \ln(N_D) < 2\ln\left(5.2 \times 10^{15}\right) + 3\ln(T) + 3$, or equivalently, if $\ln(N_A N_D) < \ln\left[\left(5.2 \times 10^{15}\right)^2 T^3\right] - 3$, then V_0 will decrease with temperature, which we observe in this case. In order for this not to be true (i.e., in order for V_0 to increase with temperature), we must have either very high doping concentrations or very low temperatures.

2.11 Since the p-type side of the junction is undoped, its electron and hole concentrations are equal to the intrinsic carrier concentration.

$$
\begin{aligned}
n_n &= N_D = 3 \times 10^{16}\ \mathrm{cm}^{-3} \\
p_p &= n_i = 1.08 \times 10^{10}\ \mathrm{cm}^{-3} \\
V_0 &= V_T \ln\left(\frac{N_D n_i}{n_i^2}\right) \\
&= (26\ \mathrm{mV}) \ln\left(\frac{N_D}{n_i}\right) \\
&= \boxed{386\ \mathrm{mV}}
\end{aligned}
$$

2.12 (a)

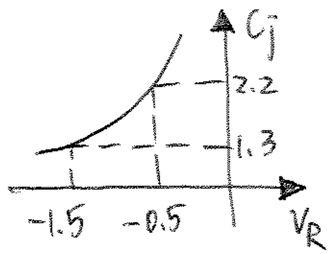
$$
\begin{aligned}
C_{j0} &= \sqrt{\frac{q\epsilon_{\mathrm{Si}}}{2}\frac{N_A N_D}{N_A+N_D}\frac{1}{V_0}} \\
C_j &= \frac{C_{j0}}{\sqrt{1-V_R/V_0}} \\
N_A &= 2\times 10^{15}\ \mathrm{cm}^{-3} \\
N_D &= 3\times 10^{16}\ \mathrm{cm}^{-3} \\
V_R &= -1.6\ \mathrm{V} \\
V_0 &= V_T \ln\left(\frac{N_A N_D}{n_i^2}\right) = 701\ \mathrm{mV} \\
C_{j0} &= 14.9\ \mathrm{nF/cm^2} \\
C_j &= 8.22\ \mathrm{nF/cm^2} \\
&= \boxed{0.082\ \mathrm{fF/cm^2}}
\end{aligned}
$$

(b) Let's write an equation for C_j' in terms of C_j assuming that C_j' has an acceptor doping of N_A'.

$$
\begin{aligned}
C_j' &= 2C_j \\
\sqrt{\frac{q\epsilon_{\mathrm{Si}}}{2}\frac{N_A' N_D}{N_A'+N_D}\frac{1}{V_T\ln(N_A'N_D/n_i^2)-V_R}} &= 2C_j \\
\frac{q\epsilon_{\mathrm{Si}}}{2}\frac{N_A' N_D}{N_A'+N_D}\frac{1}{V_T\ln(N_A'N_D/n_i^2)-V_R} &= 4C_j^2 \\
q\epsilon_{\mathrm{Si}} N_A' N_D &= 8C_j^2(N_A'+N_D)(V_T\ln(N_A'N_D/n_i^2)-V_R) \\
N_A'\left[q\epsilon_{\mathrm{Si}}N_D - 8C_j^2(V_T\ln(N_A'N_D/n_i^2)-V_R)\right] &= 8C_j^2 N_D(V_T\ln(N_A'N_D/n_i^2)-V_R) \\
N_A' &= \frac{8C_j^2 N_D(V_T\ln(N_A'N_D/n_i^2)-V_R)}{q\epsilon_{\mathrm{Si}}N_D - 8C_j^2(V_T\ln(N_A'N_D/n_i^2)-V_R)}
\end{aligned}
$$

We can solve this by iteration (you could use a numerical solver if you have one available). Starting with an initial guess of $N_A' = 2\times 10^{15}\ \mathrm{cm}^{-3}$, we plug this into the right hand side and solve to find a new value of $N_A' = 9.9976\times 10^{15}\ \mathrm{cm}^{-3}$. Iterating twice more, the solution converges to $N_A' = 1.025\times 10^{16}\ \mathrm{cm}^{-3}$. Thus, we must increase the N_A by a factor of $N_A'/N_A = 5.125 \approx \boxed{5}$.

13.



$$\frac{C_{j0}}{\sqrt{1+\frac{0.5}{V_0}}} = 2.2 \quad \text{——} \quad ①$$

$$\frac{C_{j0}}{\sqrt{1+\frac{1.5}{V_0}}} = 1.3 \quad \text{——} \quad ②$$

① ÷ ② :

$$\frac{1+{}^{1.5}/_{V_0}}{1+{}^{0.5}/_{V_0}} = \left(\frac{2.2}{1.3}\right)^2 \Rightarrow V_0 = 0.0365\ V$$

Substitute V_0 into ①:

$$C_{j0} = 2.2\sqrt{1+\frac{0.5}{V_0}} \approx 8.43\ fF/\mu m^2$$

$$\Rightarrow \frac{N_A N_D}{N_A+N_D} = (C_{j0})^2 \cdot V_0 \cdot \frac{2}{\varepsilon_{si} q}$$

$$= \left(8.43\ \frac{fF}{\mu m^2}\right)^2 \times (0.0365V) \cdot \frac{2}{\varepsilon_{si} q} \approx 3.13 \cdot 10^{17}\ cm^{-3}$$

Fix a value for $N_A > \frac{N_A N_D}{N_A + N_D} \triangleq y$

$$N_A = 2 \cdot 10^{18}\,cm^{-3} \Rightarrow N_D = \frac{y N_A}{N_A - y}$$

$$= \frac{(3.13 \cdot 10^{17}\,cm^{-3})(2 \cdot 10^{18}\,cm^{-3})}{(2 \cdot 10^{18} - 3.13 \cdot 10^{17})\,cm^{-3}}$$

$$\approx 3.71 \cdot 10^{17}\,cm^{-3}$$

14 (a) In forward bias, $I_D = 1mA$, $V_D = 750mV$

$$\therefore I_S \approx I_D e^{-\frac{V_D}{V_T}} = (1mA)\exp[-750mV/26mV]$$

$$= 2.97 \cdot 10^{-16} A$$

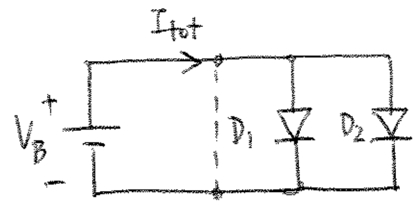
(b) Since $I_S \propto$ Area, doubling area implies doubling I_S. From (a),

$$I_D = 1\ mA = 2 \times I_S\, e^{V_D/V_T}$$

$$\therefore V_D = V_T \ln\left(\frac{I_D}{2I_S}\right) = (26mV)\ln\left(\frac{1mA}{2 \cdot 2.97 \times 10^{-16} A}\right)$$

$$= 0.732\ V$$

15 (a)



$$I_{tot} = I_{D_1} + I_{D_2} = I_{S_1}(e^{V_B/V_T}-1) + I_{S_2}(e^{V_B/V_T}-1)$$

$$= (I_{S_1} + I_{S_2})(e^{V_B/V_T}-1)$$

Therefore, the parallel combination operates as an exponential device, with an equivalent saturation current of $I_{S_1} + I_{S_2}$.

(b) By KVL, $V_{D_1} = V_{D_2}$

$$\Rightarrow V_T \ln\left(\frac{I_{D_1}}{I_{S_1}}\right) = V_T \ln\left(\frac{I_{D_2}}{I_{S_2}}\right)$$

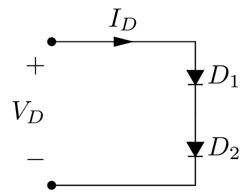
Also, $I_{tot} = I_{D_1} + I_{D_2} \Rightarrow I_{D_2} = I_{tot} - I_{D_1}$

$$\therefore V_T \ln\left(\frac{I_{D_1}}{I_{S_1}}\right) = V_T \ln\left(\frac{I_{tot} - I_{D_1}}{I_{S_2}}\right)$$

$$\Rightarrow I_{D_1} = I_{tot}\left(\frac{I_{S_1}}{I_{S_1} + I_{S_2}}\right)$$

$$\Rightarrow I_{D_2} = I_{tot}\left(\frac{I_{S_2}}{I_{S_1} + I_{S_2}}\right)$$

2.16 (a) The following figure shows the series diodes.



Let V_{D1} be the voltage drop across D_1 and V_{D2} be the voltage drop across D_2. Let $I_{S1} = I_{S2} = I_S$, since the diodes are identical.

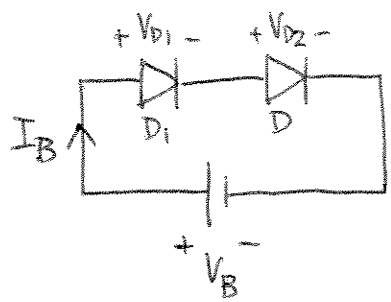
$$\begin{aligned}
V_D &= V_{D1} + V_{D2} \\
&= V_T \ln\left(\frac{I_D}{I_S}\right) + V_T \ln\left(\frac{I_D}{I_S}\right) \\
&= 2V_T \ln\left(\frac{I_D}{I_S}\right) \\
I_D &= I_S e^{V_D/2V_T}
\end{aligned}$$

Thus, the diodes in series act like a single device with an exponential characteristic described by $I_D = I_S e^{V_D/2V_T}$.

(b) Let V_D be the amount of voltage required to get a current I_D and V_D' the amount of voltage required to get a current $10I_D$.

$$\begin{aligned}
V_D &= 2V_T \ln\left(\frac{I_D}{I_S}\right) \\
V_D' &= 2V_T \ln\left(\frac{10I_D}{I_S}\right) \\
V_D' - V_D &= 2V_T \left[\ln\left(\frac{10I_D}{I_S}\right) - \ln\left(\frac{I_D}{I_S}\right)\right] \\
&= 2V_T \ln(10) \\
&= \boxed{120 \text{ mV}}
\end{aligned}$$

17.



Find I_B, V_{D_1}, V_{D2} in terms of V_B, I,, I_{S2}

By KVL, $V_B = V_{D_1} + V_{D2} = V_T \ln\left(\frac{I_B}{I_{S_1}}\right) + V_T \ln\left(\frac{I_B}{I_{S2}}\right)$

$$\Rightarrow \quad V_B = V_T \ln\left(\frac{I_B^2}{I_{S_1} I_{S2}}\right)$$

$$\therefore \; I_B = \sqrt{I_{S_1} I_{S2} \cdot \exp V_B/V_T} = \sqrt{I_{S_1} I_{S2}} \cdot \exp\left(\frac{V_B}{2V_T}\right).$$

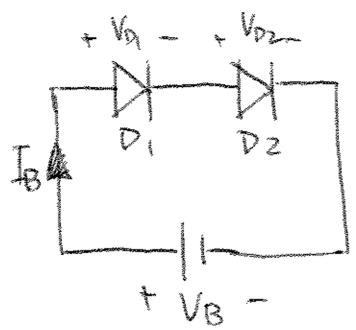
$$V_{D_1} = V_T \ln\left(\frac{I_B}{I_{S_1}}\right) = V_T \ln\left(\frac{\sqrt{I_{S_1} I_{S2}}}{I_{S_1}} \cdot \exp \frac{V_B}{2V_T}\right)$$

$$= V_T \ln \sqrt{\frac{I_{S2}}{I_{S_1}}} + \frac{V_B}{2}$$

$$V_{D2} = V_T \ln\left(\frac{I_B}{I_{S2}}\right) = V_T \ln\left(\frac{\sqrt{I_{S_1} I_{S2}}}{I_{S2}} \cdot \exp \frac{V_B}{2V_T}\right)$$

$$= V_T \ln \sqrt{\frac{I_{S1}}{I_{S2}}} + \frac{V_B}{2}$$

18.



$$V_B = V_T \ln \frac{I_B}{I_{S1}} + V_T \ln \frac{I_B}{I_{S2}} = V_T \ln\left(\frac{I_B^2}{I_{S1} I_{S2}}\right)$$

$$\Rightarrow I_B = \sqrt{I_{S1} I_{S2}} \cdot \exp \frac{V_B}{2V_T}$$

Increase I_B by 10 times:

$$I_{B,new} = 10 I_B$$

$$\Rightarrow V_{B,new} = V_T \ln\left(\frac{I_{B,new}^2}{I_{S1} I_{S2}}\right) = V_T \ln\left[\frac{(10 I_B)^2}{I_{S1} I_{S2}}\right]$$

$$= V_T \ln\left(\frac{I_B^2}{I_{S1} I_{S2}}\right) + V_T \ln 100$$

$$= V_B + V_T \ln 100 \approx V_B + 0.120\text{ V}$$

$\therefore$ V_B increases by 0.120 V.

2.19

$$\begin{aligned}
V_X &= I_X R_1 + V_{D1} \\
&= I_X R_1 + V_T \ln\left(\frac{I_X}{I_S}\right) \\
I_X &= \frac{V_X}{R_1} - \frac{V_T}{R_1}\ln\left(\frac{I_X}{I_S}\right)
\end{aligned}$$

For each value of V_X, we can solve this equation for I_X by iteration. Doing so, we find

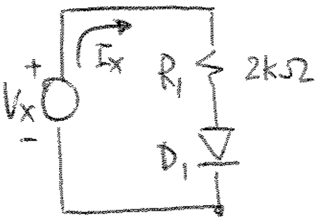
$$\begin{aligned}
I_X(V_X = 0.5\text{ V}) &= 0.435\ \mu\text{A} \\
I_X(V_X = 0.8\text{ V}) &= 82.3\ \mu\text{A} \\
I_X(V_X = 1\text{ V}) &= 173\ \mu\text{A} \\
I_X(V_X = 1.2\text{ V}) &= 267\ \mu\text{A}
\end{aligned}$$

Once we have I_X, we can compute V_D via the equation $V_D = V_T \ln(I_X/I_S)$. Doing so, we find

$$\begin{aligned}
V_D(V_X = 0.5\text{ V}) &= \boxed{499\text{ mV}} \\
V_D(V_X = 0.8\text{ V}) &= \boxed{635\text{ mV}} \\
V_D(V_X = 1\text{ V}) &= \boxed{655\text{ mV}} \\
V_D(V_X = 1.2\text{ V}) &= \boxed{666\text{ mV}}
\end{aligned}$$

As expected, V_D varies very little despite rather large changes in I_D (in particular, as I_D experiences an increase by a factor of over 3, V_D changes by about 5 %). This is due to the exponential behavior of the diode. As a result, a diode can allow very large currents to flow once it turns on, up until it begins to overheat.

20.



Since $I_{S_1} \propto$ Area, I_{D_1} becomes:

$$I_{D_1} = \underbrace{10 \times (2 \cdot 10^{-15} A)}_{I_{S_1}'} \left(e^{\frac{V_{D1}}{V_T}} - 1\right)$$

$\boxed{V_X = 0.8\ V}$ Suppose D_1 is on. Assume $V_{D_1} = 0.7V$

$$V_{D_1} = 0.7\ V \Rightarrow I_X = \frac{V_X - V_{D_1}}{R_1} = \frac{0.1\ V}{2\ k\Omega} = 0.05\ mA$$

$$\Rightarrow V_{D_1} = V_T \ln\left(I_X / I_{S_1}'\right) = (0.026V) \ln\left(\frac{0.05\ mA}{20 \cdot 10^{-15} A}\right)$$

$$= 0.563\ V$$

$$V_{D_1} = 0.563V \Rightarrow I_X = \frac{(0.8 - 0.563)\ V}{2\ k\Omega} = 0.12\ mA$$

$$\Rightarrow V_{D_1} = (0.026\ V) \ln\left(\frac{0.12\ mA}{20 \cdot 10^{-15} A}\right) \approx 0.585\ V$$

$$V_{D_1} = 0.585V \Rightarrow I_X = \frac{(0.8 - 0.585)V}{2\ k\Omega} = 0.11\ mA$$

$$\Rightarrow V_{D_1} = (0.026\ V) \ln\left(\frac{0.11\ mA}{20 \cdot 10^{-15} A}\right) \approx 0.583\ V$$

$$V_{D_1} = 0.583\ V \Rightarrow I_X = \frac{(0.8 - 0.583)V}{2\ k\Omega} = 0.11\ mA$$

$$\Rightarrow V_{D_1} = 0.583\ V$$

$\therefore$ $V_{D_1} \approx 0.583\ V$

$I_X \approx 0.11\ mA.$

$\boxed{V_X = 1.2\text{ V}}$ Suppose D_1 is on. Use results from previous calculations as starting point.

$$V_{D_1} = 0.583\text{ V} \Rightarrow I_X = \frac{(1.2 - 0.583)\text{V}}{2\text{ k}\Omega} = 0.31\text{ mA}$$

$$\Rightarrow V_{D_1} = (0.026\text{V}) \ln\left(\frac{0.31\text{mA}}{20\times10^{-15}\text{A}}\right) \approx 0.610\text{ V}$$

$$V_{D_1} = 0.610\text{ V} \Rightarrow I_X = \frac{(1.2 - 0.610)\text{V}}{2\text{ k}\Omega} = 0.30\text{ mA}$$

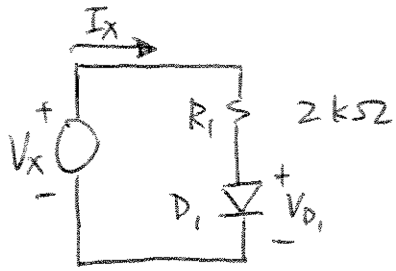
$$\Rightarrow V_{D_1} = (0.026\text{V}) \ln\left(\frac{0.30\text{ mA}}{20\cdot10^{-15}\text{A}}\right) \approx 0.609\text{ V}$$

$$V_{D_1} = 0.609\text{ V} \Rightarrow I_X = \frac{(1.2 - 0.609)\text{V}}{2\text{ k}\Omega} = 0.30\text{mA}$$

$$\Rightarrow V_{D_1} = 0.609\text{ V}$$

$$\therefore V_{D_1} \approx 0.609\text{ V}$$
$$I_X \approx 0.30\text{ mA}.$$

By increasing the cross-section area of D_1, intuitively this means D_1 can conduct same amount of current with less V_{D_1}. The results have shown that in this problem, V_{D_1} is less and I_X is more.

21. 

Given: @ $V_X = 2V$, $V_{D_1} = 850mV$

$$\Rightarrow I_X = \frac{V_X - V_{D_1}}{R_1} = 0.58\ mA$$

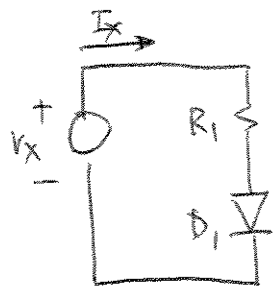
$$\therefore I_S = \frac{I_X}{(e^{V_D/V_T} - 1)} \approx I_X \exp[-V_D/V_T]$$

$$= (0.58\ mA) \exp[-0.85/0.026] \approx 3.64 \cdot 10^{-18} A$$

2.22

$$
\begin{aligned}
V_X/2 = I_X R_1 &= V_{D1} = V_T \ln(I_X/I_S) \\
I_X &= \frac{V_T}{R_1} \ln(I_X/I_S) \\
I_X &= 367\ \mu\text{A (using iteration)} \\
V_X &= 2 I_X R_1 \\
&= \boxed{1.47\ \text{V}}
\end{aligned}
$$

23.



Given $V_x = 1V \Rightarrow I_x = 0.2mA$

$V_x = 2V \Rightarrow I_x = 0.5mA$

Find R_1 and I_s.

By KVL, $V_{D_1} = V_x - I_x R_1 = V_T \ln\left(\frac{I_x}{I_s}\right)$

$$\Rightarrow 1 - (0.2mA)R_1 = (0.026V)\ln\left(\frac{0.2mA}{I_s}\right) \quad \text{—— ①}$$

$$2 - (0.5mA)R_1 = (0.026V)\ln\left(\frac{0.5mA}{I_s}\right) \quad \text{—— ②}$$

② − ① : $1 - (0.3mA)R_1 = (0.026V)\ln\left(\frac{0.5}{0.2}\right)$

$$\Rightarrow R_1 = \frac{1-(0.026)\ \ V}{0.3m\ \ A} = 3.25\ k\Omega$$

Substitute R_1 into ① :

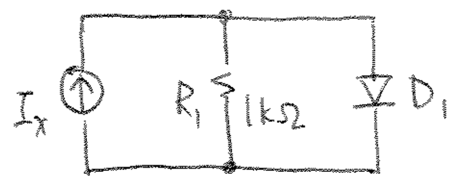
$$I_s = I_x \cdot \exp\left[-\frac{V_x - I_x R_1}{V_T}\right]$$

$$= (0.2mA)\exp\left[-\frac{1-(0.2m)(3.25k)}{0.026}\right] \approx 2.94 \cdot 10^{-10} A$$

$\therefore\ R_1 \approx 3.25\ k\Omega$

$I_s \approx 2.94 \cdot 10^{-10} A.$

24.



Given $I_S = 3 \cdot 10^{-16} A$, find V_{D_1}.

By KCL, $I_x = \frac{V_{D_1}}{R_1} + I_{D_1} = \frac{V_T}{R_1} \ln\left(\frac{I_{D_1}}{I_S}\right) + I_{D_1}$

Since I_x, V_T, R_1 and I_S are known, this can be solved directly with special programs or graphing calculators. However, this can be also solved by iterations: Assume a V_{D_1}, calculate I_{D_1}, and re-iterate on V_{D_1}.

Assume $V_{D_1} = 0.7\ V$ as starting point.

$\boxed{I_x = 1\,mA}$

$V_{D_1} = 0.7\ V \Rightarrow I_{D_1} = I_x - V_{D_1}/R_1 = 1\,mA - \frac{0.7V}{1k\Omega} = 0.3\,mA$

$$\Rightarrow V_{D_1} = V_T \ln\left(I_x/I_S\right)$$
$$= (0.026V)\ln\left(\frac{0.3mA}{3 \cdot 10^{-16}A}\right) \approx 0.718\ V$$

$V_{D_1} = 0.718V \Rightarrow I_{D_1} = 1\,mA - \frac{0.718\ V}{1\ k\Omega} = 0.28\ mA$

$$\Rightarrow V_{D_1} = (0.026V)\ln\left(\frac{0.28mA}{3 \cdot 10^{-16}A}\right) \approx 0.717\ V$$

$V_{D_1} = 0.717\,V \Rightarrow I_{D_1} = 1mA - \dfrac{0.717V}{1\,k\Omega} = 0.28\,mA$

$\Rightarrow V_{D_1} = 0.717\,V$

$\therefore V_{D_1} \approx 0.717\,V.$

$\boxed{I_X = 2mA}$ Assume $V_{D_1} = 0.717\,V$ from previous result.

$V_{D_1} = 0.717\,V \Rightarrow I_{D_1} = 2mA - \dfrac{0.717V}{1\,k\Omega} = 1.28\,mA$

$\Rightarrow V_{D_1} = (0.026V)\ln\left(\dfrac{1.28\,mA}{3\cdot 10^{-16}A}\right) \approx 0.756V$

$V_{D_1} = 0.756\,V \Rightarrow I_{D_1} = 2mA - \dfrac{0.756V}{1\,k\Omega} = 1.24\,mA$

$\Rightarrow V_{D_1} = (0.026\,V)\ln\left(\dfrac{1.24mA}{3\cdot 10^{-16}A}\right) \approx 0.755V$

$V_{D_1} = 0.755V \Rightarrow I_{D_1} = 2mA - \dfrac{0.755\,V}{1\,k\Omega} = 1.24\,mA$

$\Rightarrow V_{D_1} = 0.755V$

$\therefore V_{D_1} = 0.755\,V$

$\boxed{I_X = 4\text{ mA}}$ Assume $V_{D_1} = 0.755\text{ V}$ from previous result.

$$V_{D_1} = 0.755\text{V} \Rightarrow I_{D_1} = 4\text{mA} - \frac{0.755\text{V}}{1\text{k}\Omega} = 3.25\text{ mA}$$

$$\Rightarrow V_{D_1} = (0.026)\text{V} \ln\left(\frac{3.25\text{mA}}{3\cdot 10^{-16}\text{A}}\right) \approx 0.780\text{ V}$$

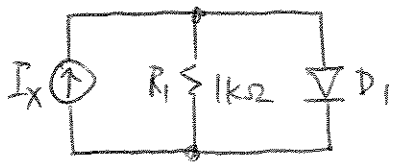
$$V_{D_1} = 0.780\text{ V} \Rightarrow I_{D_1} = 4\text{mA} - \frac{0.780\text{V}}{1\text{k}\Omega} = 3.22\text{ mA}$$

$$\Rightarrow V_{D_1} = (0.026\text{ V})\ln\left(\frac{3.22\text{mA}}{3\cdot 10^{-16}\text{ A}}\right) \approx 0.780\text{V}$$

$\therefore V_{D_1} \approx 0.780\text{ V}.$

Note: As I_X increases, I_{D_1} increases, while (V_{D_1}/R_1) stays relatively the same. Because of the exponential characteristic, the diode, once on, will <u>absorb</u> as much current as necessary to satisfy KCL.

25.



Given $I_{D_1} = 0.5mA$ when $I_X = 1.3mA$, find I_S.

This means $V_{D_1} = (I_X - I_{D_1})R_1$

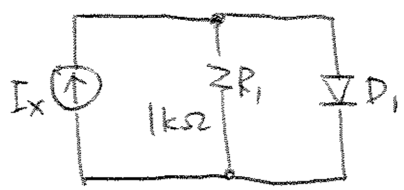
$$= (0.8mA)1k\Omega = 0.8V$$

$$\Rightarrow I_S = I_{D_1} \cdot \exp\left[-V_{D_1}/V_T\right]$$

$$= (0.5mA)\exp\left[-0.8V/0.026V\right]$$

$$\approx 2.17 \cdot 10^{-17} A$$

2b



Given $I_{R_1} = I_x/2$

$I_S = 3 \cdot 10^{-16}\,A$

find I_x.

$$V_{D_1} = \frac{I_x}{2} \cdot R_1 = V_T \ln\left(\frac{I_x/2}{I_S}\right)$$

This can be solved directly with special programs or graphing calculators. Alternatively, one can solve this iteratively by hand.

Assume $V_D = 0.8\ V$.

$$V_D = 0.8\,V \Rightarrow (I_x/2) = \frac{V_D}{R_1} = \frac{0.8\,V}{1\,k\Omega} = 0.8\,mA$$

$$\Rightarrow V_D = V_T \ln\left(\frac{I_x/2}{I_S}\right) = (0.026V)\ln\left(\frac{0.8\,mA}{3 \cdot 10^{-16}A}\right)$$

$$\approx 0.744\ V$$

$$V_D = 0.744\,V \Rightarrow I_x/2 = \frac{0.744V}{1k\Omega} = 0.744\ mA$$

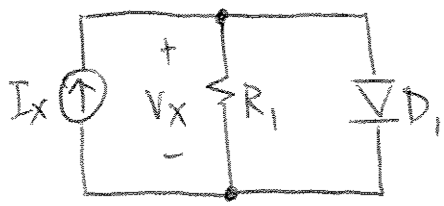
$$\Rightarrow V_D = (0.026V)\ln\left(\frac{0.744\,mA}{3 \cdot 10^{-16}A}\right) \approx 0.742\,V$$

$V_D = 0.742V \Rightarrow I_X/2 = \dfrac{0.742V}{1k\Omega} = 0.742\ mA$

$\Rightarrow V_D = (0.026V)\ln\left(\dfrac{0.742mA}{3\cdot 10^{-16}A}\right) \approx 0.742V$

$\therefore I_X = 2(0.742mA) = 1.48\ mA$

27.



Given $I_X = 1mA \rightarrow V_X = 1.2V$

$I_X = 2mA \rightarrow V_X = 1.8V$

find R_1 and I_S.

$I_{D_1} = I_X - V_X/R_1$ (KCL)

By KVL, $V_X = V_T \ln\left(\frac{I_{D_1}}{I_S}\right) = V_T \ln\left(\frac{I_X - V_X/R_1}{I_S}\right)$

$$\Rightarrow (1.2\,V) = (0.026\,V)\ln\left[\frac{(1mA) - (1.2V)/R_1}{I_S}\right] \quad \text{—— ①}$$

$$(1.8\,V) = (0.026\,V)\ln\left[\frac{(2mA) - (1.8\,V)/R_1}{I_S}\right] \quad \text{—— ②}$$

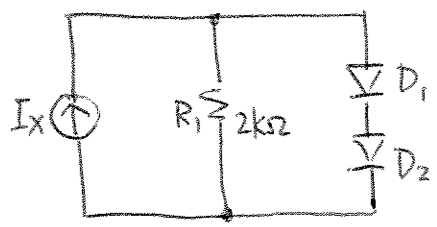
② − ①: $0.6V = (0.026V)\ln\left(\frac{2mA - 1.8V/R_1}{1mA - 1.2V/R_1}\right)$

$$\Rightarrow R_1 = \frac{1.2 \cdot \exp[0.6/0.026] - 1.8}{1\,mA \cdot \exp[0.6/0.026] - 2\,mA} \approx 1.2\,k\Omega$$

$$I_S = I_D \exp\left[-V_X/V_T\right] = \left(2mA - \frac{1.8V}{1.2k\Omega}\right)\exp\left[\frac{-1.8V}{0.026V}\right]$$

$$\approx 4.29 \cdot 10^{-34}\,A.$$

28.



Given $D_1 = D_2$ with $I_S = 5 \cdot 10^{-16} A$

Find V_{R_1} for $I_X = 2mA$.

Current through the diodes $= I_D$
$= I_X - \frac{V_{R_1}}{R_1}$ where V_{R_1} = voltage across R_1

$$\Rightarrow V_{R_1} = 2 \cdot V_T \ln\left(\frac{I_D}{I_S}\right) = 2\left[V_T \ln\left(\frac{I_X}{I_S} - \frac{V_{R_1}}{I_S R_1}\right)\right]$$

This can be solved directly with special programs or graphing calculators or by hand iteratively.

Assume a V_{R_1}, calculate I_D, and re-iterate on new $V_{R_1} = (2 \times V_{D_1})$. From experience, most diodes conduct at $V_D \approx 0.7V$. Assume $V_{R_1} = 1.4V$.

$$V_{R_1} = 1.4V \Rightarrow I_D = I_X - \frac{V_{R_1}}{R_1} = 2mA - \frac{1.4V}{2k\Omega} = 1.3mA$$

$$\Rightarrow V_{R_1} = 2V_T \ln\left(\frac{I_D}{I_S}\right)$$

$$= 2(0.026V)\ln\left(\frac{1.3mA}{5 \times 10^{-16}A}\right) \approx 1.49V$$

$$V_{R_1} = 1.49V \Rightarrow I_D = 2mA - \frac{1.49}{2k\Omega} = 1.26\ mA$$

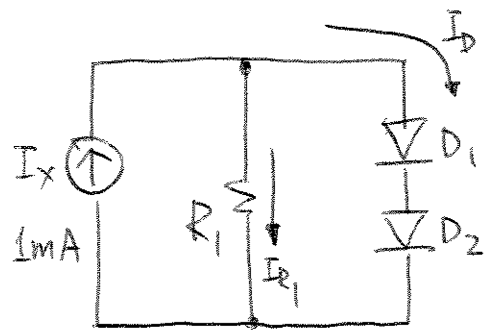
$$\Rightarrow V_{R_1} = 2(0.026V)\ln\left(\frac{1.26\,mA}{5\cdot 10^{-16}A}\right) \approx 1.48V$$

$$V_{R_1} = 1.48V \Rightarrow I_D = 2mA - \frac{1.48V}{2k\Omega} = 1.26\ mA$$

$$\Rightarrow V_{R_1} = 1.48V$$

$\therefore$ voltage across $R_1 = 1.48\,V$

29.



Given $I_{R_1} = 0.5\,mA$,
$I_S = 5 \cdot 10^{-16}\,A$ for each diode.

Find R_1.

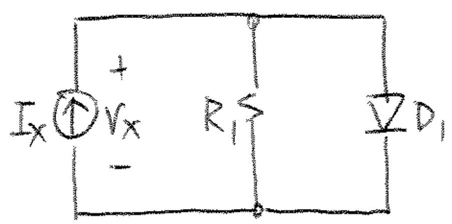
By KCL, $I_D = I_X - I_{R_1} = 0.5\,mA$

$$\Rightarrow V_{D_1} = V_{D_2} = V_T \ln\left(\frac{I_D}{I_S}\right) = 0.026 \ln\left(\frac{0.5mA}{5 \cdot 10^{-16}A}\right)$$

$$\approx 0.718\ V$$

$$\therefore R_1 = \frac{V_{R_1}}{I_{R_1}} = \frac{2V_{D_1}}{I_{R_1}} = \frac{2(0.718V)}{0.5\,mA} = 2.87\,k\Omega$$

30.

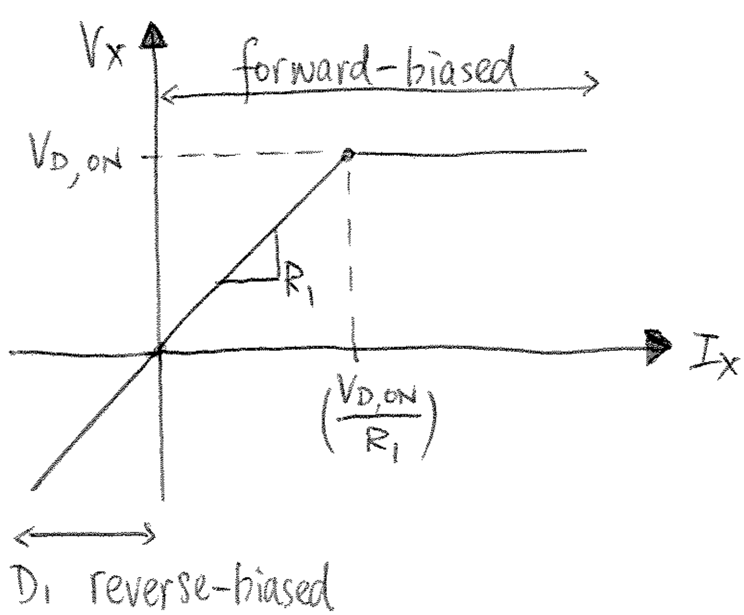


(a) Constant-voltage model:

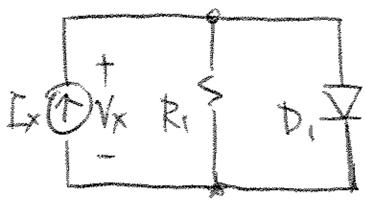
Consider, first, the extreme cases: when D_1 is off, we have the following:

This implies V_X is linearly proportional to I_X

When D_1 is on, V_X is <u>fixed</u> (by KVL) by D_1 ($= V_{D,ON}$). This implies that any additional current from I_X cannot flow through R_1, which means D_1 will absorb all the currents to satisfy KVL.



(b) exponential model :



Assume I_S negligible.

When D_1 is off, most of I_X flows through R_1. When D_1 is on, V_{D_1} ($= V_X$) follows this relationship:

$$V_{D_1} = V_X = V_T \ln\left(\frac{I_{D_1}}{I_S}\right) = V_T \ln\left(\frac{I_X}{I_S} - \frac{V_X}{I_X R_1}\right)$$

$$\Rightarrow I_X = I_S \exp(V_X/V_T) + V_X/R_1$$

$\approx I_S \exp(V_X/V_T)$ when D_1 is forward-biased ($V_X > V_T$)

i.e. $V_X \approx V_T \ln(I_X/I_S)$