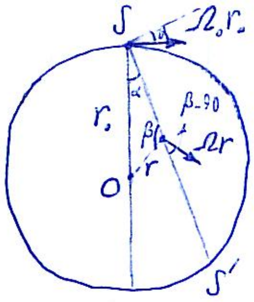
باسمه تعالی

دوره ی 20 - تابستان 86 - امتحان اول



1) الف)

$$\begin{cases} V_{\parallel} = \Omega r \cos(\beta - 90) - \Omega_\circ r_\circ \sin\alpha \\ \dfrac{\sin\alpha}{r} = \dfrac{\sin\beta}{r_\circ} \rightarrow r\sin\beta = r_\circ \sin\alpha \\ \cos(\beta - 90) = \sin\beta \end{cases} \longrightarrow \boxed{V_{\parallel} = (\Omega - \Omega_\circ) r_\circ \sin\alpha}$$

ب)

$$V_{\parallel Max} = (\Omega_{Max} - \Omega_\circ) r_\circ \sin\alpha = (\Omega_{(r_{min})} - \Omega_\circ) r_\circ \sin\alpha \longrightarrow \beta = \frac{\pi}{2} \longrightarrow \boxed{r_{min} = r_\circ \sin\alpha}$$

ج)

$$V_{\parallel Max} = \Omega r_\circ \sin\alpha - \Omega_\circ r_\circ \sin\alpha \Longrightarrow$$

$$V = \Omega r = \Omega r_\circ \sin\alpha \longrightarrow \boxed{V = V_{\parallel Max} + \Omega_\circ r_\circ \sin\alpha}$$

2) الف)

$$\boxed{\vec{r} = \left(-\frac{g t^2}{2} + v_\circ \sin\theta\, t\right)\hat{j} + (v_\circ \cos\theta\, t)\hat{i}}$$

ب)

$$\frac{y}{x} = \tan\alpha \longrightarrow \frac{-\frac{gt}{2} + v_\circ \sin\theta}{v_\circ \cos\theta} = \tan\alpha \longrightarrow \boxed{t = \frac{2v_\circ}{g}(\sin\theta - \cos\theta \tan\alpha)}$$

پ)

$$\vec{V} = (-gt + v_\circ \sin\theta)\hat{j} + (v_\circ \cos\theta)\hat{i} \longrightarrow \boxed{\vec{V} = v_\circ\left[(2\cos\theta \tan\alpha - \sin\theta)\hat{j} + (\cos\theta)\hat{i}\right]}$$

ت)

$$\boxed{\frac{2\cos\theta \tan\alpha - \sin\theta}{\cos\theta} = \tan(\alpha - \varphi)}$$

ث)

$$\boxed{\varphi = \alpha \rightarrow 2\tan\alpha = \tan\theta} \quad , \quad \boxed{\varphi = \frac{\pi}{2} \rightarrow \frac{\sin\theta - 2\cos\theta \tan\alpha}{\cos\theta} = \cot\alpha}$$

ج)

$$\frac{-\frac{g x^2}{2v_\circ^2 \cos^2\theta} + x\tan\theta}{x} = \tan\alpha \longrightarrow \frac{-gH\cot\alpha}{2v_\circ^2 \cos^2\theta} + \tan\theta = \tan\alpha \longrightarrow$$

$$v_\circ^2 = \left(\frac{gH\cot\alpha}{2}\right)\left(\frac{1}{\cos^2\theta(\tan\theta - \tan\alpha)}\right) \quad , \quad \frac{dv_\circ}{d\theta} = 0 \rightarrow \frac{dv_\circ^2}{d\theta} = 0 \rightarrow$$

$$\frac{1}{\left[\cos^2\theta(\tan\theta - \tan\alpha)\right]^2} \cdot (2\sin\theta\cos\theta\tan\alpha + \cos^2\theta - \sin^2\theta) = 0 \rightarrow \tan(2\theta) = -\cot\alpha \rightarrow$$

$$-\cot\alpha = \tan\left(\alpha - \frac{\pi}{2}\right) \rightarrow 2\theta = \alpha + \frac{\pi}{2} \rightarrow \boxed{\theta = \frac{\alpha}{2} + \frac{\pi}{4}} \rightarrow$$

$$\begin{cases} \cos\theta = \frac{1}{\sqrt{2}}\left(\cos\left(\frac{\alpha}{2}\right) - \sin\left(\frac{\alpha}{2}\right)\right) \\ \sin\theta = \frac{1}{\sqrt{2}}\left(\sin\left(\frac{\alpha}{2}\right) + \cos\left(\frac{\alpha}{2}\right)\right) \end{cases} \rightarrow v_\circ^2 = \left(\frac{gH\cot\alpha}{2}\right)\left(\frac{1}{\frac{\sin(2\theta)}{2} - \tan\alpha\cos^2\theta}\right) \rightarrow$$

$$v_\circ^2 = \left(\frac{gH\cot\alpha}{2}\right)\left(\frac{1}{\frac{\cos\alpha}{2} - \frac{\tan\alpha}{2}(1 - \sin\alpha)}\right) \rightarrow \boxed{v_\circ = \sqrt{\frac{gH\cot\alpha}{\tan\alpha + \cos\alpha + \tan\alpha\sin\alpha}}} \rightarrow \boxed{v = \sqrt{\frac{gH\cot\alpha\cos\alpha}{1 - \sin\alpha}}}$$

با بسط تیلور

$$\vec{E}=\frac{kQ(\vec{r}-\vec{a})}{(r^2+a^2-2ar\cos\alpha)^{3/2}}\longrightarrow \vec{E}_{(2)}=\frac{kQ}{r^3}\left(1-\frac{3}{2}\frac{a^2}{r^2}+\frac{3a}{r}\cos\alpha+4\frac{a^2}{r^2}\cos^2\alpha\right)(\vec{r}-\vec{a})$$ (a (3

$$\boxed{\vec{E}=\frac{kQ}{r^3}\left(1-\frac{3}{2}\frac{a^2}{r^2}+\frac{3\vec{a}\cdot\vec{r}}{r^2}+\frac{4(\vec{a}\cdot\vec{r})}{r^4}\right)(\vec{r}-\vec{a})}$$

$$\begin{cases}\cos\alpha=\sin\vartheta\\ \vec{r}=\vec{z}+\vec{\rho}\\ \vec{a}=0\end{cases}\longrightarrow \vec{E}=\frac{kQ}{r^3}\left(\vec{r}+\left(\frac{a}{r}\right)(3\cos\alpha)(\vec{\rho}+\vec{z})+\left(\frac{a}{r}\right)^2\left(4\cos^2\alpha-\frac{3}{2}\right)(\vec{\rho}+\vec{z})\right)\longrightarrow$$ (b

(c

$$\vec{E}=\frac{kQ}{r^2}\left\{\hat{r}+\left(\frac{a}{r}\right)\left[\hat{z}(3\sin\vartheta\cos\vartheta)+\hat{\rho}(3\sin^2\vartheta)\right]+\left(\frac{a}{r}\right)^2\left[\hat{z}\left(4\sin^2\vartheta-\frac{3}{2}\right)\cos\vartheta+\hat{\rho}\left(4\sin^2\vartheta-\frac{3}{2}\right)\sin\vartheta\right]\right\}$$ (d (e

$$\longrightarrow \boxed{f_{1(\vartheta)}=3\sin\vartheta\cos\vartheta}\ ,\ \boxed{f_{2(\vartheta)}=3\sin^2\vartheta}\ ,\ \boxed{f_{3(\vartheta)}=\left(4\sin^2\vartheta-\frac{3}{2}\right)\cos\vartheta}$$

$$\boxed{f_{4(\vartheta)}=\left(4\sin^2\vartheta-\frac{3}{2}\right)\sin\vartheta}$$

غلط است.

$$\boxed{\vec{E}=\frac{kQ}{r^2}\left\{\hat{r}+\left(\frac{a}{r}\right)\left[\hat{z}(3\sin\vartheta\cos\vartheta)+\hat{\rho}(3\sin^2\vartheta)\right]+\left(\frac{a}{r}\right)^2\left[\hat{z}\left(4\sin^2\vartheta-\frac{3}{2}\right)\cos\vartheta+\hat{\rho}\left(4\sin^2\vartheta-\frac{3}{2}\right)\sin\vartheta\right]\right\}}$$

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1) الف)

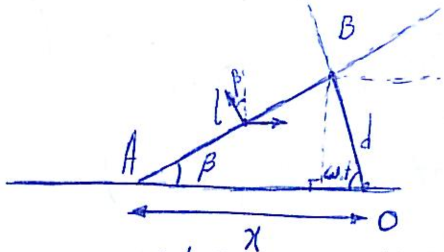
$$\frac{\sin\beta}{d}=\frac{\sin(\omega_0 t)}{l}\longrightarrow \cos\beta\,\dot{\beta}=\frac{\omega_0 d}{l}\cos(\omega_0 t)\longrightarrow \boxed{\dot{\beta}=\frac{\omega_0 d\cos(\omega_0 t)}{l\sqrt{1-\left(\frac{d}{l}\right)^2\sin^2(\omega_0 t)}}}$$

ب)

$$\ddot{\beta}=\frac{\omega_0 d}{l}\left(\frac{-\omega_0\sqrt{1-\left(\frac{d}{l}\right)^2\sin^2(\omega_0 t)}\ \sin(\omega_0 t)}{1-\left(\frac{d}{l}\right)^2\sin^2(\omega_0 t)}-\frac{\frac{1}{2\sqrt{1-\left(\frac{d}{l}\right)^2\sin^2(\omega_0 t)}}\cdot\omega_0\cos(\omega_0 t)\cdot -2\left(\frac{d}{l}\right)^2\sin(\omega_0 t)\cos(\omega_0 t)}{1-\left(\frac{d}{l}\right)^2\sin^2(\omega_0 t)}\right)$$

$$\longrightarrow \ddot{\beta}=\frac{\omega_0^2 d\sin(\omega_0 t)}{l}\cdot\frac{\left(\frac{d}{l}\right)^2\cos^2(\omega_0 t)-1+\left(\frac{d}{l}\right)^2\sin^2(\omega_0 t)}{\left(1-\left(\frac{d}{l}\right)^2\sin^2(\omega_0 t)\right)^{3/2}}\longrightarrow$$

$$\boxed{\ddot{\beta}=\frac{\omega_0^2 d\left(\left(\frac{d}{l}\right)^2-1\right)\sin(\omega_0 t)}{l\left(1-\left(\frac{d}{l}\right)^2\sin^2(\omega_0 t)\right)^{3/2}}}$$



پ)

$$x=d\cos(\omega_0 t)+l\sqrt{1-\left(\frac{d}{l}\right)^2\sin^2(\omega_0 t)}\longrightarrow \dot{x}=-\omega_0 d\sin(\omega_0 t)+\frac{l\cdot -2\left(\frac{d}{l}\right)^2\omega_0\sin(\omega_0 t)\cos(\omega_0 t)}{2\sqrt{1-\left(\frac{d}{l}\right)^2\sin^2(\omega_0 t)}}$$

$$\longrightarrow \dot{x}=-\omega_0 d\sin(\omega_0 t)\left(1+\frac{\frac{d}{l}\cos(\omega_0 t)}{\sqrt{1-\left(\frac{d}{l}\right)^2\sin^2(\omega_0 t)}}\right)$$

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$$\ddot{x} = -\omega_0 d\left(\omega_0 \cos(\omega_0 t) + \frac{d}{l}\cdot\frac{\omega_0\sqrt{1-\left(\frac{d}{l}\right)^2\sin^2(\omega_0 t)}\left(\cos^2(\omega_0 t)-\sin^2(\omega_0 t)\right)+\dfrac{2\left(\frac{d}{l}\right)^2\sin^2(\omega_0 t)\cos^2(\omega_0 t)\,\omega_0}{2\sqrt{1-\left(\frac{d}{l}\right)^2\sin^2(\omega_0 t)}}}{1-\left(\frac{d}{l}\right)^2\sin^2(\omega_0 t)}\right)$$ ت)

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$$\boxed{\ddot{x} = -\omega_0^2 d\left(\cos(\omega_0 t) + \frac{\cos(2\omega_0 t)+\left(\frac{d}{l}\right)^2\sin^4(\omega_0 t)}{\left(1-\left(\frac{d}{l}\right)^2\sin^2(\omega_0 t)\right)^{3/2}}\cdot\frac{d}{l}\right)}$$

ث)

$$\vec{v} = \left(\omega_0 d\sin(\omega_0 t) + \frac{\omega_0\frac{d^2}{l}\sin(\omega_0 t)\cos(\omega_0 t)}{\sqrt{1-\left(\frac{d}{l}\right)^2\sin^2(\omega_0 t)}} - \frac{\omega_0\frac{d^2}{l}\sin(\omega_0 t)\cos(\omega_0 t)}{2\sqrt{1-\left(\frac{d}{l}\right)^2\sin^2(\omega_0 t)}}\right)\hat{i} + \left(\frac{\omega_0 d\cos(\omega_0 t)}{2}\right)\hat{j}$$

y, x

$$\rightarrow \boxed{\vec{v} = \omega_0 d\sin(\omega_0 t)\left(1+\frac{\frac{d}{l}\cos(\omega_0 t)}{2\sqrt{1-\left(\frac{d}{l}\right)^2\sin^2(\omega_0 t)}}\right)\hat{i} + \left(\frac{\omega_0 d\cos(\omega_0 t)}{2}\right)\hat{j}} \rightarrow$$

$$\vec{v} = \frac{1}{2}\cdot 45\cdot\sin\left(\tfrac{1}{20}\right)\left(1+\frac{\frac{45}{75}\cos\left(\frac{1}{20}\right)}{2\sqrt{1-\left(\frac{45}{75}\right)^2\sin^2\left(\frac{1}{20}\right)}}\right)\hat{i} + \left(\frac{1}{4}\cdot 45\cdot\cos\left(\tfrac{1}{20}\right)\right)\hat{j} \rightarrow \boxed{\vec{v} \simeq (1.46)\hat{i} + (11.23)\hat{j}} \rightarrow$$

$$\boxed{|\vec{v}| \simeq 11.32\ \frac{cm}{s}}$$

اشتباه کردیم

اشتباه کردیم

$$\vec{a} = \omega_0^2 d\left(\cos(\omega_0 t) + \frac{d}{2l}\cdot\frac{-\sqrt{1-\left(\frac{d}{l}\right)^2\sin^2(\omega_0 t)}\,\sin(\omega_0 t) + \dfrac{\left(\frac{d}{l}\right)^2\sin(\omega_0 t)\cos^2(\omega_0 t)}{\sqrt{1-\left(\frac{d}{l}\right)^2\sin^2(\omega_0 t)}}}{1-\left(\frac{d}{l}\right)^2\sin^2(\omega_0 t)}\right)\hat{i} + \left(\frac{-\omega_0^2 d\sin(\omega_0 t)}{2}\right)\hat{j}$$

$$\rightarrow \vec{a} = \omega_0^2 d\left(\cos(\omega_0 t) + \frac{d}{2l}\cdot\frac{-\sin(\omega_0 t)+\left(\frac{d}{l}\right)^2\sin(\omega_0 t)}{\left(1-\left(\frac{d}{l}\right)^2\sin^2(\omega_0 t)\right)^{3/2}}\right)\hat{i} + \left(\frac{-\omega_0^2 d\sin(\omega_0 t)}{2}\right)\hat{j} \rightarrow$$

$$\boxed{\vec{a} = \omega_0^2 d\left(\cos(\omega_0 t) + \frac{d}{2l}\frac{\left(\left(\frac{d}{l}\right)^2-1\right)\sin(\omega_0 t)}{\left(1-\left(\frac{d}{l}\right)^2\sin^2(\omega_0 t)\right)^{3/2}}\right)\hat{i} + \left(\frac{-\omega_0^2 d\sin(\omega_0 t)}{2}\right)\hat{j}} \rightarrow$$

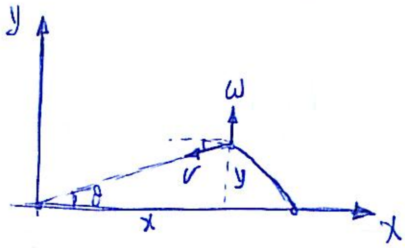
$$\vec{a} = \frac{1}{4}\cdot 45\left(\cos\left(\tfrac{1}{20}\right) + \frac{45}{2\times 75}\cdot\frac{\left(\left(\frac{45}{75}\right)^2-1\right)\sin\left(\frac{1}{20}\right)}{\left(1-\left(\frac{45}{75}\right)^2\sin^2\left(\frac{1}{20}\right)\right)^{3/2}}\right)\hat{i} + \left(-\frac{1}{4}\cdot\frac{45}{2}\cdot\sin\left(\tfrac{1}{20}\right)\right)\hat{j} \rightarrow \boxed{\vec{a} \simeq (11.12)\hat{i} + (-0.28)\hat{j}}$$

$$\rightarrow \boxed{|\vec{a}| \simeq 11.12\ \frac{cm}{s^2}}$$ اشتباه است

2) الف

$$\dot{\vec{r}} = (-v\cos\theta)\hat{i} + (w - v\sin\theta)\hat{j} \rightarrow \boxed{\dot{\vec{r}} = \left(-v\frac{x}{\sqrt{x^2+y^2}}\right)\hat{i} + \left(w - v\frac{y}{\sqrt{x^2+y^2}}\right)\hat{j}}$$

$$\frac{\dot{x}}{\dot{y}} = \frac{v\frac{x}{\sqrt{x^2+y^2}}}{v\frac{y}{\sqrt{x^2+y^2}} - \omega} \longrightarrow \boxed{\frac{dy}{dx} = \frac{vy - \omega\sqrt{x^2+y^2}}{vx}}$$



ب)

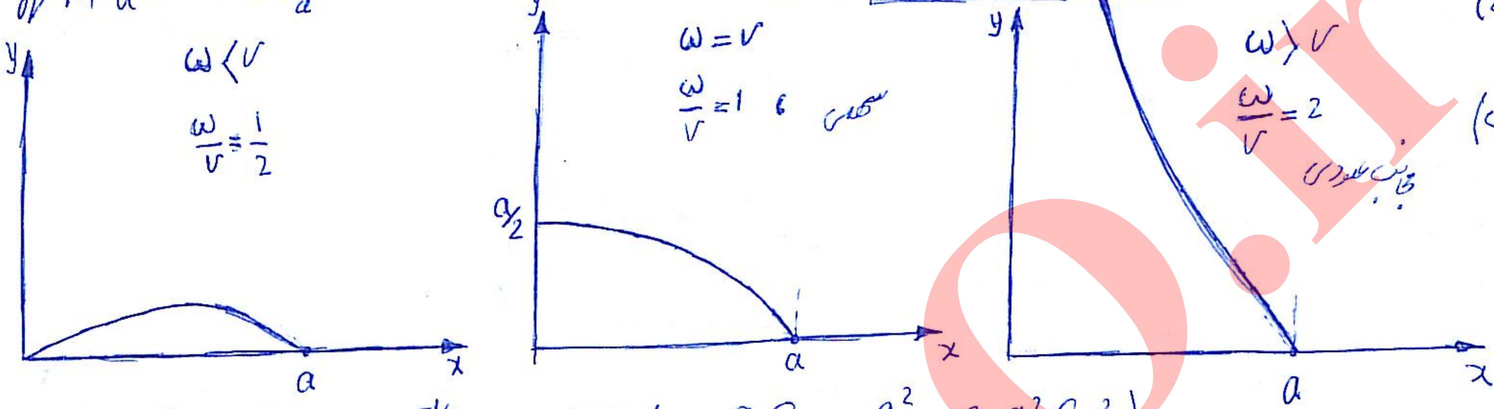
$$u + x\frac{du}{dx} = u - \frac{\omega}{v}\sqrt{1+u^2} \longrightarrow \boxed{\frac{du}{\sqrt{1+u^2}} = -\frac{\omega}{v}\cdot\frac{dx}{x}}$$

پ)

$$\int_0^u \frac{du}{\sqrt{1+u^2}} = -\frac{\omega}{v}\int_a^x \frac{dx}{x} \longrightarrow \cancel{\tan^{-1}(u) = -\frac{\omega}{v}\ln\left(\frac{x}{a}\right)} \longrightarrow \cancel{\boxed{y = x\tan\left(\frac{\omega}{v}\ln\left(\frac{a}{x}\right)\right)}}$$

ادامه پس از تست و سرق

ت)



ث)

$$V = \frac{kQ}{r}\left(1 + \frac{a^2}{r^2} - 2\frac{a}{r}\cos\alpha\right)^{-1/2} \longrightarrow V = \frac{kQ}{r}\left(1 + \frac{a}{r}\cos\alpha - \frac{a^2}{2r^2} + \frac{3}{2}\frac{a^2}{r^2}\cos^2\alpha\right) \longrightarrow$$ (a (4

$$\boxed{V = \frac{kQ}{r}\left(1 + \frac{\vec{a}\cdot\vec{r}}{r^2} - \frac{a^2}{2r^2} + \frac{3}{2}\frac{(\vec{a}\cdot\vec{r})^2}{r^4}\right)}$$

$$V' = kQ\left(\frac{1}{\sqrt{r^2+a^2+2ra_x}} + \frac{1}{\sqrt{r^2+a^2-2a_x r}} + \frac{1}{\sqrt{r^2+a^2-2a_y r}} + \frac{1}{\sqrt{r^2+a^2+2a_y r}} + \frac{1}{\sqrt{r^2+a^2-2a_z r}} + \frac{1}{\sqrt{r^2+a^2+2ra_z}}\right)$$ (b

$$\longrightarrow V' = \frac{kQ}{r}\left(\left(1+\frac{a^2}{r^2}+\frac{2a_x}{r}\right)^{-1/2} + \cdots + \left(1+\frac{a^2}{r^2}+\frac{2a_z}{r}\right)^{-1/2}\right) \longrightarrow V' = \frac{kQ}{r}\left(6 - \frac{3a^2}{r^2} + \frac{3}{r^2}(a_x^2+a_y^2+a_z^2)\right)$$

$$\longrightarrow \boxed{V' = \frac{6kQ}{r}}$$

$$\varphi = \frac{4kQ}{\sqrt{r^2+\frac{b^2}{2}}} \longrightarrow \varphi = \frac{4kQ}{r}\left(1+\frac{b^2}{2r^2}\right)^{-1/2} \longrightarrow \boxed{\varphi = \frac{4kQ}{r}\left(1 - \frac{b^2}{4r^2}\right)}$$ (c

$$\varphi' = kQ\left(\frac{1}{\sqrt{z^2+(x-\frac{b}{2})^2+(y-\frac{b}{2})^2}} + \frac{1}{\sqrt{z^2+(x+\frac{b}{2})^2+(y-\frac{b}{2})^2}} + \frac{1}{\sqrt{z^2+(x-\frac{b}{2})^2+(y+\frac{b}{2})^2}} + \frac{1}{\sqrt{z^2+(x+\frac{b}{2})^2+(y+\frac{b}{2})^2}}\right)$$ (d

$$\longrightarrow \varphi' = \frac{kQ}{\sqrt{x^2+y^2+z^2}}\left(\frac{1}{\sqrt{1+\frac{b^2}{2(x^2+y^2+z^2)} - \frac{b(x+y)}{x^2+y^2+z^2}}} + \frac{1}{\sqrt{1+\frac{b^2}{2(x^2+y^2+z^2)} + \frac{b(x-y)}{x^2+y^2+z^2}}} + \right.$$

$$\left.\frac{1}{\sqrt{1+\frac{b^2}{2(x^2+y^2+z^2)} + \frac{b(y-x)}{x^2+y^2+z^2}}} + \frac{1}{\sqrt{1+\frac{b^2}{2(x^2+y^2+z^2)} + \frac{b(x+y)}{x^2+y^2+z^2}}}\right) \longrightarrow$$

پاسخنامه نظری

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4) ادامه d)

$$\rightarrow \varphi' = \frac{kQ}{\sqrt{x^2+y^2+z^2}}\left(4 - \frac{b^2}{x^2+y^2+z^2} - \frac{b(x+y-x-y+y-x+x-y)}{2(x^2+y^2+z^2)}\right.$$

$$\left. + \frac{3}{8}\cdot\frac{b^2}{(x^2+y^2+z^2)^2}\left(x^2+y^2+2xy+x^2+y^2-2xy+x^2+y^2-2xy+x^2+y^2+2xy\right)\right) \rightarrow$$

$$\rightarrow \boxed{\varphi' = \frac{kQ}{\sqrt{x^2+y^2+z^2}}\left(4 - \frac{b^2}{x^2+y^2+z^2} + \frac{3}{2}\cdot\frac{b^2(x^2+y^2)}{(x^2+y^2+z^2)^2}\right)} \quad \text{یا} \quad \boxed{\varphi' = \frac{kQ}{r}\left(4 - \frac{b^2}{r^2} + \frac{3}{2}\cdot\frac{b^2|\vec{r}\times\hat{z}|^2}{r^4}\right)}$$

3) a)

$$Y^\alpha \times R^\beta \times \rho^\eta \times g^\theta$$

$$\begin{cases} Y = \frac{kg}{m\cdot s^2}, & g = \frac{m}{s^2} \\ R = m, & \rho = \frac{kg}{m^3} \end{cases} \rightarrow kg^{\alpha+\eta} \times m^{\theta-\alpha-3\eta+\beta} \times s^{-2\alpha-2\theta} = 1 \rightarrow \begin{cases} \alpha = -\eta \rightarrow \theta = \eta \\ \alpha = -\theta \\ \theta-\alpha-3\eta+\beta = 0 \rightarrow \theta+\theta-3\theta+\beta = 0 \\ \rightarrow \theta = \beta \end{cases}$$

$$\left(Y^{-\beta} \times R^\beta \times \rho^\beta \times g^\beta\right) \rightarrow \boxed{\left(\frac{\rho R g}{Y}\right), \left(\frac{R}{h}\right)}$$

b)

$$x = Y^\alpha \times R^\beta \times \rho^\eta \times g^\theta \times h^\gamma \times c,\ x = 1 \rightarrow kg^{\alpha+\eta} \times m^{\theta-\alpha-3\eta+\beta+\gamma} \times s^{-2\alpha-2\theta} = 1 \rightarrow$$

$$\begin{cases} \alpha = -\eta \rightarrow \eta = \theta \\ \alpha = -\theta \\ \theta+\theta-3\theta+\beta+\gamma = 0 \rightarrow \beta = \theta - \gamma \end{cases} \rightarrow x = (c)\, Y^{-\theta} \times R^{\theta-\gamma} \times \rho^\theta \times g^\theta \times h^\gamma \rightarrow \boxed{x = (c)\left(\frac{\rho g}{Y}\right)^\theta \times h^\gamma \times R^{\theta-\gamma}}$$

$$\rightarrow \boxed{x = (c)\left(\frac{\rho g R}{Y}\right)^\theta \times \left(\frac{h}{R}\right)^\gamma}$$

c) $\boxed{x \text{ نسبت به } \left(\frac{h}{R}\right) \text{ و } \left(\frac{\rho g R}{Y}\right) \text{ صعودی است.}}$

با افزایش g جسم با سرعت بیشتری به زمین خورده پس در نتیجه توان g باید مثبت باشد و تغییر شکل بیشتر است. با افزایش Y طبق رابطه‌ی مدول یانگ Δl کاهش یافته. پس توان Y نیز باید منفی باشد پس در نتیجه x نسبت به $\left(\frac{\rho g R}{Y}\right)$ صعودی است.
با افزایش h جسم با سرعت بیشتری برخورد کرده پس در نتیجه تغییر شکل بیشتر بوده، پس توان آن باید مثبت باشد. بنابراین باید x نسبت به $\left(\frac{h}{R}\right)$ صعودی باشد.

d) $\boxed{x \text{ نسبت به } (R) \text{ صعودی است.}}$

می‌دانیم: $x = (c)\left(\frac{\rho g R}{Y}\right)^\theta$

در نتیجه توان R، θ است. از طرفی $\theta > 0$. پس x نسبت به R صعودی است.

e)

$$\gamma = \theta \rightarrow \boxed{x = (c)\left(\frac{\rho g h}{Y}\right)^\theta}$$

f)

$$\boxed{x = (c)\left(\frac{\rho g R}{Y}\right)^{-\beta} \times \left(\frac{h}{R}\right)^\alpha}$$

پاسخنامه تشریحی

$x \sim R^{2-\alpha-\beta} \longrightarrow 2-\alpha-\beta > 0 \longrightarrow \boxed{\beta+\alpha < 2}$ (9

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$$\ln\left(\frac{u+\sqrt{1+u^2}}{1}\right) = -\frac{\omega}{v}\ln\left(\frac{x}{a}\right) \longrightarrow$$

$$\frac{y}{x}+\sqrt{1+\frac{y^2}{x^2}} = \left(\frac{a}{x}\right)^{\omega/v} \longrightarrow \cancel{1}+\cancel{\frac{y^2}{x^2}} = \cancel{\frac{y^2}{x^2}}+\left(\frac{a}{x}\right)^{2\omega/v} - 2\left(\frac{y}{x}\right)\left(\frac{a}{x}\right)^{\omega/v} \longrightarrow 2\left(\frac{y}{x}\right) = \left(\frac{a}{x}\right)^{\omega/v} - \left(\frac{x}{a}\right)^{\omega/v}$$

$$\longrightarrow \boxed{y = \frac{x}{2}\left(\left(\frac{a}{x}\right)^{\omega/v} - \left(\frac{x}{a}\right)^{\omega/v}\right)}$$

ادامه قسمت ث - تابستان 86 - امتحان دوم - سؤال -

$$\vec{a} = \omega_0^2 d\left(\cos(\omega_0 t) + \frac{d}{2l}\cdot\frac{\left(\cos^2(\omega_0 t)-\sin^2(\omega_0 t)\right)\sqrt{1-\left(\frac{d}{l}\right)^2\sin^2(\omega_0 t)} + \frac{\left(\frac{d}{l}\right)^2\sin^2(\omega_0 t)\cos^2(\omega_0 t)}{\sqrt{1-\left(\frac{d}{l}\right)^2\sin^2(\omega_0 t)}}}{1-\left(\frac{d}{l}\right)^2\sin^2(\omega_0 t)}\right)\hat{i} + \left(\frac{-\omega_0^2 d\sin(\omega_0 t)}{2}\right)\hat{j}$$

$$\longrightarrow \vec{a} = \omega_0^2 d\left(\cos(\omega_0 t) + \frac{d}{2l}\cdot\frac{\cos^2(\omega_0 t)-\sin^2(\omega_0 t)-\left(\frac{d}{l}\right)^2\sin^2(\omega_0 t)\cos^2(\omega_0 t)+\left(\frac{d}{l}\right)^2\sin^2(\omega_0 t)\cos^2(\omega_0 t)+\left(\frac{d}{l}\right)^2\sin^4(\omega_0 t)}{\left[1-\left(\frac{d}{l}\right)^2\sin^2(\omega_0 t)\right]^{3/2}}\right)\hat{i}$$

$$+\left(\frac{-\omega_0^2 d\sin(\omega_0 t)}{2}\right)\hat{j} \longrightarrow \boxed{\vec{a} = \omega_0^2 d\left(\cos(\omega_0 t) + \frac{d}{2l}\cdot\frac{\cos(2\omega_0 t)+\left(\frac{d}{l}\right)^2\sin^4(\omega_0 t)}{\left[1-\left(\frac{d}{l}\right)^2\sin^2(\omega_0 t)\right]^{3/2}}\right)\hat{i} + \left(\frac{-\omega_0^2 d\sin(\omega_0 t)}{2}\right)\hat{j}} \longrightarrow$$

$$\vec{a} = \frac{1}{4}\cdot 45\left(\cos\left(\frac{1}{20}\right) + \frac{45}{2\times 75}\cdot\frac{\cos\left(\frac{1}{10}\right)+\left(\frac{45}{75}\right)^2\sin^4\left(\frac{1}{20}\right)}{\left[1-\left(\frac{45}{75}\right)^2\sin^2\left(\frac{1}{20}\right)\right]^{3/2}}\right)\hat{i} + \left(-\frac{1}{4}\cdot\frac{45}{2}\cdot\sin\left(\frac{1}{20}\right)\right)\hat{j} \longrightarrow$$

$$\vec{a} \simeq (14.59)\hat{i} + (-0.28)\hat{j} \longrightarrow \boxed{|\vec{a}| \simeq 14.59}\ \mathrm{cm/s^2}$$

$$\vec{a} \simeq \frac{45}{4}\left(1+\frac{3}{2\times 5}\cdot\frac{1+\frac{9}{25}\times\frac{1}{20^4}}{\left(1-\frac{9}{25}\times\frac{1}{20^2}\right)^{3/2}}\right)\hat{i} + \left(-\frac{45}{160}\right)\hat{j} \longrightarrow \vec{a} \simeq \frac{45}{4}\left(1+\frac{3}{10}\right)\hat{i} + \left(-\frac{9\times 5^4}{10^5}\right)\hat{j} \longrightarrow$$

$$\vec{a} = \left(\frac{45\times 13}{40}\right)\hat{i} - (0.28)\hat{j} \longrightarrow \vec{a} \simeq (14.7)\hat{i} - (0.28)\hat{j} \longrightarrow \boxed{|\vec{a}| \simeq 14.7}\ \mathrm{cm/s^2}$$

$$\vec{r} \simeq \frac{45}{40}\left(1+\frac{3}{2\times 5}\right)\hat{i} + \left(\frac{45}{4}\right)\hat{j} \longrightarrow \vec{v} = \left(\frac{13\times 45\times 5^2}{10000}\right)\hat{i} + \left(\frac{45\times 25}{100}\right)\hat{j} = \left(\frac{13\times 1125}{10^4}\right)\hat{i} + (11.25)\hat{j} \simeq (1.47)\hat{i} + (11.25)\hat{j}$$

$$\longrightarrow \boxed{|\vec{v}| = 11.34}\ \mathrm{cm/s}$$

بسمه تعالی

تابستان 86 - امتحان سوم

1) الف)

$$\begin{cases} t = \frac{L}{v} = \frac{\pi(r^2 - r_0^2)}{vd} \\ T = \frac{\pi(R_0^2 - r_0^2)}{vd} \end{cases} \to \frac{t}{T} = \frac{r^2 - r_0^2}{R_0^2 - r_0^2} \to r^2 = r_0^2 + \frac{t}{T}(R_0^2 - r_0^2) \to \boxed{r = \sqrt{r_0^2 + \frac{t}{T}(R_0^2 - r_0^2)}}$$

ب)

$$\dot\varphi = \frac{v}{r} \to \varphi = v\int_0^t \frac{dt}{\sqrt{r_0^2 + \frac{t}{T}(R_0^2 - r_0^2)}} = v\frac{2T}{R_0^2 - r_0^2}\sqrt{r_0^2 + \frac{t}{T}(R_0^2 - r_0^2)}\Bigg|_0^t \to \boxed{\varphi = \frac{2vT}{R_0^2 - r_0^2}\left(\sqrt{r_0^2 + \frac{t}{T}(R_0^2 - r_0^2)} - r_0\right)}$$

پ)

$$\begin{cases} \theta = \varphi + \psi \\ \tan\psi = \frac{L_0 - vt}{r} \end{cases} \to \boxed{\theta = \frac{2vT}{R_0^2 - r_0^2}\left(\sqrt{r_0^2 + \frac{t}{T}(R_0^2 - r_0^2)} - r_0\right) + \tan^{-1}\left(\frac{L_0 - vt}{\sqrt{r_0^2 + \frac{t}{T}(R_0^2 - r_0^2)}}\right)}$$

ت)

$$\dot\theta = \dot\varphi + \dot\psi = \frac{v}{r} + \dot\psi$$

$$\dot\psi = \frac{-v\sqrt{r_0^2 + \frac{t}{T}(R_0^2 - r_0^2)} - \frac{(L_0 - vt)\cdot\frac{R_0^2 - r_0^2}{T}}{\sqrt{r_0^2 + \frac{t}{T}(R_0^2 - r_0^2)}\times 2}}{\left(r_0^2 + \frac{t}{T}(R_0^2 - r_0^2)\right)} \times \frac{r_0^2 + \frac{t}{T}(R_0^2 - r_0^2)}{r_0^2 + \frac{t}{T}(R_0^2 - r_0^2) + (L_0 - vt)^2} = \frac{-\frac{1}{2T}\left(2vTr_0^2 + 2vt(R_0^2 - r_0^2) + (L_0 - vt)(R_0^2 - r_0^2)\right)}{\left(r_0^2 + \frac{t}{T}(R_0^2 - r_0^2) + (L_0 - vt)^2\right)\sqrt{r_0^2 + \frac{t}{T}(R_0^2 - r_0^2)}}$$

$$\to \dot\psi = -\frac{2vTr_0^2 + (L_0 + vt)(R_0^2 - r_0^2)}{\left(r_0^2 + \frac{t}{T}(R_0^2 - r_0^2) + (L_0 - vt)^2\right)\sqrt{r_0^2 + \frac{t}{T}(R_0^2 - r_0^2)}} \times \frac{1}{2T} \to$$

$$\boxed{\dot\theta = \frac{1}{\sqrt{r_0^2 + \frac{t}{T}(R_0^2 - r_0^2)}}\left(v - \frac{2vTr_0^2 + (L_0 + vt)(R_0^2 - r_0^2)}{2T\left(r_0^2 + \frac{t}{T}(R_0^2 - r_0^2) + (L_0 - vt)^2\right)}\right)}$$

ث)

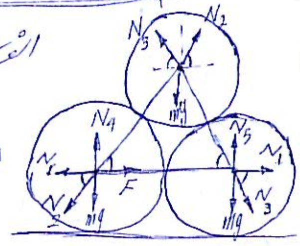
$$\dot r' = \left(\sqrt{(L_0 - vt)^2 + r^2}\right)' = \frac{r\dot r + (L_0 - vt)\times(-v)}{\sqrt{(L_0 - vt)^2 + r^2}} = \frac{\frac{(R_0^2 - r_0^2)}{2T} - v(L_0 - vt)}{\sqrt{(L_0 - vt)^2 + r_0^2 + \frac{t}{T}(R_0^2 - r_0^2)}} \to$$

$$\boxed{\vec{v}_A = \left(\frac{(R_0^2 - r_0^2) - 2vT(L_0 - vt)}{2T\sqrt{r_0^2 + (L_0 - vt)^2 + \frac{t}{T}(R_0^2 - r_0^2)}}\right)\hat r + \sqrt{\frac{r_0^2 + (L_0 - vt)^2 + \frac{t}{T}(R_0^2 - r_0^2)}{r_0^2 + \frac{t}{T}(R_0^2 - r_0^2)}}\left(v - \frac{2vTr_0^2 + (L_0 + vt)(R_0^2 - r_0^2)}{2T\left(r_0^2 + \frac{t}{T}(R_0^2 - r_0^2) + (L_0 - vt)^2\right)}\right)\hat\theta}$$

2) الف)

$$\begin{cases} (N_2 - N_3)\times\frac{1}{2} = ma \\ F - N_1 - \frac{N_2}{2} = ma \\ (N_2 + N_3)\frac{\sqrt3}{2} = mg \\ N_1 + N_3\times\frac{1}{2} = ma \end{cases}$$

$$N_3 = 0 \to N_2 = \frac{2mg}{\sqrt3} = 2ma = 2N_1 \to F_{Max} = 3ma \to \boxed{F_{Max} = \frac{3mg}{\sqrt3}} \to \boxed{F_{Max} = \sqrt3 mg}$$



ب)

$$N_1 = 0 \to N_3 = 2ma, \quad N_2 = 4ma \to ma = \frac{mg}{3\sqrt3} \to$$

$$F = 3ma \to \boxed{F_{min} = \frac{mg}{\sqrt3}}$$

پ)

$$\cancel{N_3=0} \quad F + \frac{N_3 - N_2}{2} = N_2 - N_3 \to \begin{cases} N_2 + N_3 = \frac{2mg}{\sqrt3} \\ N_2 - N_3 = \frac{2F}{3} \end{cases} \to N_2 = \frac{mg}{\sqrt3} + \frac{F}{3}, \quad N_3 = \frac{mg}{\sqrt3} - \frac{F}{3}$$

$$\begin{cases} N_4 = mg + \frac{\sqrt3}{2}N_2 \\ N_5 = mg + \frac{\sqrt3}{2}N_3 \end{cases} \to \begin{cases} N_4 = mg + \frac{mg}{2} + \frac{mg}{3} \\ N_5 = mg + \frac{mg}{2} - \frac{mg}{3} \end{cases} \to \begin{cases} \boxed{N_4 = \frac{11}{6}mg} \\ \boxed{N_5 = \frac{7}{6}mg} \end{cases}$$

$$F = \frac{mg}{2}\left(\frac{1}{\sqrt3} + \sqrt3\right) \to F = \frac{2mg}{\sqrt3}$$

امتحان اول ـ تابستان 86 ـ درس 20

3) الف)

$$\vec{E}=\frac{kQ}{|\vec{r}-\vec{a}|^3}(\vec{r}-\vec{a})=\frac{kQ(\vec{r}-\vec{a})}{\sqrt{a^2+r^2-2ar\cos\theta}}=\frac{kQ(\vec{r}-\vec{a})}{r^3}\left(1+\frac{a^2}{r^2}-2\frac{a}{r}\cos\theta\right)^{-3/2}\longrightarrow$$

$$\vec{E}=\frac{kQ(\vec{r}-\vec{a})}{r^3}\left(1-\frac{3}{2}\frac{a^2}{r^2}+\frac{3a}{r}\cos\theta+\frac{15}{8}\cdot\frac{4a^2}{r^2}\cos^2\theta\right)\longrightarrow\boxed{\vec{E}=\frac{kQ}{r^3}\left[\vec{r}\left(1+\frac{3a}{r}\cos\theta+\frac{3}{2}\frac{a^2}{r^2}(5\cos^2\theta-1)\right)-\left(1+\frac{3a}{r}\cos\theta\right)\vec{a}\right]}$$

ب)

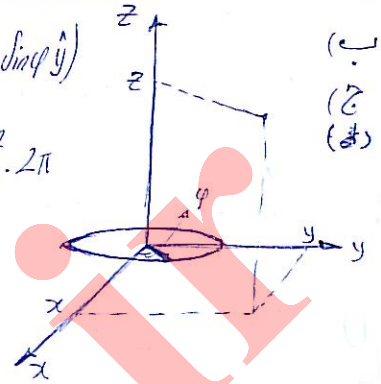
ج)

د)

$$d\vec{E}=\frac{k\lambda a\,d\varphi}{r^3}\left[\vec{r}\left(1+\frac{3a}{r^2}(x\cos\varphi+y\sin\varphi)+\frac{15}{2}\cdot\frac{a^2}{r^4}(x\cos\varphi+y\sin\varphi)^2-\frac{3}{2}\frac{a^2}{r^2}\right)-a(\cos\varphi\,\hat{x}+\sin\varphi\,\hat{y})\right.$$

$$\left.-\frac{3a^2}{r^2}(x\cos\varphi+y\sin\varphi)(\sin\varphi\,\hat{y}+\cos\varphi\,\hat{x})\right]\Rightarrow\vec{E}=\frac{k\lambda a}{r^3}\left[\vec{r}\left(2\pi+\frac{15}{2}\frac{a^2}{r^4}(x^2+y^2)\cdot\frac{2\pi}{2}-\frac{3}{2}\frac{a^2}{r^2}\cdot2\pi\right)\right.$$

$$\left.-\frac{3a^2}{r^2}\left(x\cdot\frac{2\pi}{2}\hat{x}+y\cdot\frac{2\pi}{2}\hat{y}\right)\right]\longrightarrow$$



$$\vec{E}=\frac{k.Q}{2\pi r^3}\left[2\pi\vec{r}+\frac{15a^2\pi}{2r^4}\cdot r^2\sin^2\theta\cdot\vec{r}-\frac{3\pi a^2}{r^2}\vec{r}-\frac{3\pi a^2}{r^2}\vec{\rho}\right]=\frac{kQ}{2\pi r^3}\left(2\pi\vec{r}+\frac{\pi a^2}{r^2}\left(\frac{15}{2}\sin^2\theta\,(r\cos\theta\,\hat{z}+r\sin\theta\,\hat{\rho})-3r(\cos\theta\,\hat{z}+\sin\theta\,\hat{\rho})\right.\right.$$

$$\left.\left.-3.r\sin\theta\,\hat{\rho}\right)\right)\longrightarrow\vec{E}=\frac{kQ}{r^2}\left(\hat{r}+\left(\frac{a}{r}\right)^2\left[\left(\frac{15}{4}\sin^2\theta\cos\theta-\frac{3}{2}\cos\theta\right)\hat{z}+\left(\frac{15}{4}\sin^3\theta-\frac{3}{2}\sin\theta-\frac{3}{2}\sin\theta\right)\hat{\rho}\right]\right)\longrightarrow$$

$$\vec{E}=\frac{kQ}{r^2}\left\{\hat{r}+\left(\frac{a}{r}\right)^2\left[\frac{3}{2}\cos\theta\left(\frac{5}{2}\sin^2\theta-1\right)\hat{z}+3\sin\theta\left(\frac{5}{4}\sin^2\theta-1\right)\hat{\rho}\right]\right\}\longrightarrow\boxed{f_{1(\theta)}=f_{2(\theta)}=0}$$

$$\boxed{f_{3(\theta)}=\frac{3}{2}\left(\frac{5}{2}\sin^2\theta-1\right)\cos\theta}\quad,\quad\boxed{f_{4(\theta)}=3\sin\theta\left(\frac{5}{4}\sin^2\theta-1\right)}$$

امتحان چهارم 86

1) آ) ب)

$$\begin{cases}-kv^2-g\sin\theta=\frac{dv}{dt}\\-g\cos\theta=-\frac{v^2}{r}=v\dot{\theta}=v\frac{d\theta}{dt}\end{cases}\longrightarrow\frac{dv}{v\,d\theta}=\frac{kv^2+g\sin\theta}{g\cos\theta}\rightarrow\frac{dv}{d\theta}=\frac{kv^2+g\sin\theta}{g\cos\theta}v\rightarrow\boxed{f_{(v,\theta)}=\frac{kv^2+g\sin\theta}{g\cos\theta}v}$$

$$\begin{cases}-k\dot{x}^2=\ddot{x}\rightarrow-k\dot{x}=\frac{d\dot{x}}{dx}\rightarrow-kx=\ln\left(\frac{v\cos\theta}{v_0\cos\theta_0}\right)\longrightarrow\boxed{x_{(\theta)}=\frac{1}{k}\ln\left(\frac{v_0\cos\theta_0}{v_{(\theta)}\cos\theta}\right)}\\-k\dot{y}^2-g=\ddot{y}\longrightarrow-dy=\frac{1}{2k}\frac{d(g+k\dot{y}^2)}{g+k\dot{y}^2}\longrightarrow-2k(y-y_0)=\ln\left(\frac{g+k\dot{y}^2}{g+k\dot{y}_0^2}\right)\longrightarrow y=y_0+\frac{1}{2k}\ln\left(\frac{g+k\dot{y}_0^2}{g+k\dot{y}^2}\right)\longrightarrow\boxed{y=\frac{1}{2k}\ln\left(\frac{g+kv_0^2\sin^2\theta_0}{g+kv^2\sin^2\theta}\right)}\end{cases}$$

پ) بالا رفتن

$$\dot{y}=0\longrightarrow y_0'=\frac{1}{2k}\ln\left(1+\frac{kv_0^2\sin^2\theta_0}{g}\right)\longrightarrow y=y_0'-\frac{1}{2k}\ln\left(\frac{g-k\dot{y}^2}{g-k\dot{y}_0^2}\right)\longrightarrow\boxed{y=\frac{1}{2k}\ln\left(\left(\frac{g+kv_0^2\sin^2\theta_0}{g^2}\right)(g-kv^2\sin^2\theta)\right)}$$

پایین آمدن

ت)

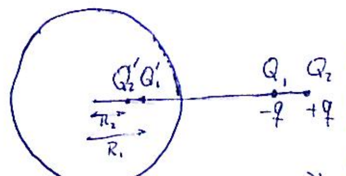
$$\frac{dv}{d\theta}=\frac{kv^3}{g}\longrightarrow-\frac{1}{2}\left(\frac{1}{v^2}-\frac{1}{v_0^2}\right)=\frac{k}{g}(\theta-\theta_0)\longrightarrow\frac{1}{v^2}=\frac{1}{v_0^2}+\frac{k}{g}(\theta_0-\theta)\longrightarrow\boxed{v=\sqrt{\frac{v_0^2g}{g+kv_0^2(\theta_0-\theta)}}}$$

$$\boxed{x=\frac{1}{k}\ln\left(\sqrt{1+\frac{kv_0^2}{g}(\theta_0-\theta)}\right)}\quad,\quad\boxed{y=0}\quad,\quad\boxed{h_{max}=0}$$

ث)

ج)

$$\boxed{Q_1'=+\frac{R}{D}q}\quad,\quad Q_2'=-\frac{R}{D+\frac{P}{q}}q\longrightarrow\boxed{Q_2'=-\frac{R}{D}q+\frac{RP}{qD^2}}\quad,\quad\boxed{R_1=\frac{R^2}{D}}\quad,\quad R_2=\frac{R^2}{D+\frac{P}{q}}\longrightarrow$$



2) (a

(b

(c

(d تنها گزینه صحیح

$$\boxed{R_2=\frac{R^2}{D}-\frac{R^2P}{qD^2}}\longrightarrow\boxed{Q'=\frac{RP}{qD^2}}\longrightarrow P'=\frac{Rq}{D}\times\frac{R^2P}{qD}\longrightarrow\boxed{P'=\frac{R^3}{D^3}P}$$

1)

$$\vec{F}_{(r)}=\vec{F}_{\text{واقعی}}-m\left(\ddot{\vec{R}}+\dot{\vec{\omega}}\times\vec{r}+\vec{\omega}\times(\vec{\omega}\times\vec{r})-2\vec{\omega}\times\vec{v}_{(r)}\right)\longrightarrow\vec{F}_{(r)}=(-T)\hat{r}+(\cos\theta\,\hat{r}-\sin\theta\,\hat{\theta})gm-m\left(\vec{\omega}\times(\vec{\omega}\times\vec{r})-2\vec{\omega}\times\vec{v}\right)=$$

$$(mg\cos\theta-T)\hat{r}-mg\sin\theta\,\hat{\theta}-m\left(\omega^2\ell(-\hat{\rho})-2\omega\ell(\hat{z})\times(\dot{\theta}\hat{\theta})\right)\longrightarrow$$

ب)

$$mg\cos\theta-T+m\omega^2\ell\sin^2\theta=-m\ell(\dot{\theta}^2)\quad\text{(آ}$$

ب) $\boxed{T=mg}$ ، $\cancel{T=\frac{mg^2}{g}}$

$$-g\sin\theta+\omega^2\ell\sin\theta\cos\theta=\ell\ddot{\theta}\longrightarrow\boxed{\theta=0}\ ,\ \boxed{\cos\theta=\frac{g}{\omega^2\ell}}\qquad T=m\left(\frac{g^2}{\omega^2\ell}+\omega^2\ell-\frac{g^2}{\omega^2\ell}\right)\longrightarrow\boxed{T=m\omega^2\ell}$$

$$\theta=0:\ \ell\ddot{\theta}+(g-\omega^2\ell)\theta=0\longrightarrow g>\omega^2\ell\longrightarrow\boxed{\omega<\sqrt{\frac{g}{\ell}}}\qquad,\ \ell\ddot{\theta}+\left(g-\omega^2\ell(\cos\theta_0-\theta\sin\theta_0)\right)(\sin\theta_0+\theta\cos\theta_0)=0\longrightarrow$$

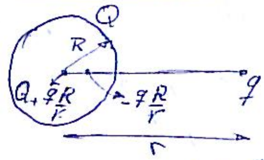
ت)

باسمه تعالی

$$\boxed{\omega < \sqrt{\frac{g}{L}}}$$

ش) $L\ddot{\theta} + (\omega^2 L \sin^2\theta_0)\theta = 0 \longrightarrow \cos\theta_0 = \frac{g}{\omega^2 L} \;/\; \boxed{\Omega_1 = \sqrt{\frac{g}{L} - \omega^2}},\ \theta = 0 \;,\; \boxed{\Omega_2 = \omega\sqrt{1-\left(\frac{g}{\omega^2 L}\right)^2}}$

2) a) $\boxed{F_{(r)} = kq\left(\frac{Q+q\frac{R}{r}}{r^2} - \frac{q\frac{R}{r}}{\left(r-\frac{R^2}{r}\right)^2}\right)}$ $\quad \cancel{U = \frac{Kq}{2}\left(\frac{Q+q\frac{R}{r}}{r^2} - \frac{q\frac{R}{r}}{r-\frac{R^2}{r}}\right) + \frac{KQ}{2}}$



b) روش اول:

$$U = \frac{1}{2}\sum q_i V_i = \frac{1}{2}\left(\frac{Kq}{r}\cdot Q + kq\left(\frac{K(Q+\frac{R}{r}q)}{r} - \frac{Kq\,R/r}{r-\frac{R^2}{r}}\right)\right) =$$

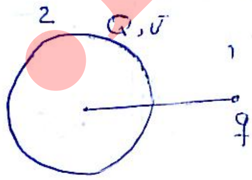
$$\frac{1}{2}\left(\frac{2KqQ}{r} + \frac{Kq^2R}{r}\left(\frac{1}{r} - \frac{1}{r-\frac{R^2}{r}}\right)\right) = \frac{KqQ}{r} + \frac{Kq^2R}{2r}\cdot\frac{-R^2}{(r^2-R^2)r} \longrightarrow \boxed{U = \frac{KqQ}{r} - \frac{Kq^2R^3}{2r^2(r^2-R^2)}}$$

روش دوم:

$$U = -\int_\infty^r F_{(r)}\,dr = -Kq\left(-\frac{Q}{r} - \frac{qR}{2r^2} - \frac{qR}{2}\int_\infty^r \frac{d(r^2-R^2)}{(r^2-R^2)^2}\right) = +Kq\left(\frac{Q}{r} + \frac{qR}{2r^2} - \frac{qR}{2(r^2-R^2)}\right) \longrightarrow$$

$$\boxed{U = \frac{KqQ}{r} - \frac{Kq^2R^3}{2r^2(r^2-R^2)}}$$

c)



$$\begin{cases} V_1 = P_{11}q_1 + P_{12}q_2 \\ V_2 = P_{12}q_1 + P_{22}q_2 \end{cases} ,\ q_1 = 0 \longrightarrow \begin{cases} V_1 = P_{12}q_2 = \frac{Kq_2}{r} \\ V_2 = P_{22}q_2 = \frac{Kq_2}{R} \end{cases} \longrightarrow \begin{cases} P_{12} = \frac{K}{r} \\ P_{22} = \frac{K}{R} \end{cases}$$

به این دلیل در جمله V_2 ، $P_{22}q_2$ را در بالا قرار ندادم / که پتانسیل حاصل از $F_{(r)}$ یعنی نیروی متقابل بین دو بار و نه نیروی بین هر مجموعه بار روی خود

$$V = \frac{Kq}{r} + \frac{KQ}{R} \rightarrow Q = \frac{RV}{K} - q\frac{R}{r} \longrightarrow \boxed{F_{(r)} = Kq\left(\frac{RV}{Kr^2} - \frac{qR/r}{\left(r-\frac{R^2}{r}\right)^2}\right)}$$

d) روش اول:

$$U = -\int_\infty^r F_{(r)}\,dr = -Kq\left(\frac{RV}{K}\cdot\frac{1}{r} + \frac{qR}{2(r^2-R^2)}\right) \longrightarrow \boxed{U = \frac{R}{r}qV - \frac{Kq^2R}{2(r^2-R^2)}}$$

روش دوم:

$$\cancel{U_{کل} = \frac{1}{2}\sum q_i V_i = \frac{1}{2}\left(\frac{2Kq}{r}\cdot\frac{RV}{K} + \frac{2Kq}{r}\cdot -q\frac{R}{r} + \frac{Kq^2R}{r}\left(\frac{1}{r} - \frac{1}{r-\frac{R^2}{r}}\right)\right) + \frac{1}{2}\frac{KQ^2}{R}}$$

$$\cancel{U_{کل} + \frac{KQ^2}{2R} - W_{باتری} = U_{F_{(r)}} \rightarrow U_{F_{(r)}} =}$$

$$U = \frac{1}{2}\sum q_i V_i = \frac{1}{2}\left(\frac{RV^2}{K} - \frac{qRV}{r} + q\left(\frac{RV}{r} - \frac{KqR}{r^2-R^2}\right)\right)$$

بسمه تعالی

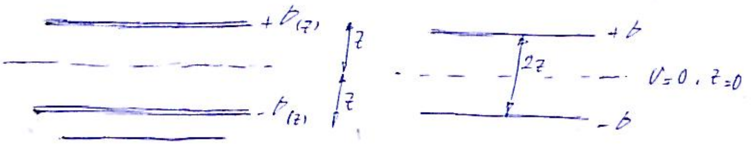
(3 a) $\boxed{\varphi_{(z)} = -\varphi_{(-z)}}$

(b) $\boxed{\vec{E} = -\frac{\partial \varphi_{(z)}}{\partial z}\hat{k}}$

(c) $\nabla^2\varphi_{(z)} = 0 \to \boxed{\frac{\partial^2\varphi_{(z)}}{\partial z^2} = 0}$

(d) $\boxed{\varphi_{(z)} = az + b}$

(e) $dV = \frac{\rho_{(z)}}{\varepsilon_0} z \to dV = \frac{\rho_z z\, dz}{\varepsilon_0} \Rightarrow V_{(z)} = \frac{\int_0^a \rho_{(z)} z\, dz}{\varepsilon_0} \to$



$\boxed{V_{(z)} = \left(\frac{A}{2\varepsilon_0}\right)\frac{z}{|z|}}$, $|z| \geq a$

(f) $V = \begin{cases} \frac{\rho a}{\varepsilon_0}\cdot\frac{z}{|z|} & |z| \geq a \\ \frac{\rho z}{\varepsilon_0} & |z| \leq a \end{cases} \to V = \begin{cases} \left(\frac{\alpha}{2\varepsilon_0}\right)\frac{z}{|z|} & |z| > a \\ \left(\frac{\alpha}{2\varepsilon_0}\right)\frac{z}{a} & |z| < a \end{cases}$

(4 a) $\vec{E} = -\vec{\nabla} V \to \boxed{\vec{E} = -\left(V_{(z)}\frac{\partial f_{(x,y)}}{\partial x}\hat{i} + V_z\frac{\partial f_{(x,y)}}{\partial y}\hat{j} + f_{(x,y)}\frac{\partial V_{(z)}}{\partial z}\hat{k}\right)}$

(b) $\boxed{V_{(z)} = V_{(-z)}}$

(c) $\frac{\sigma_{(x,y)}}{2\varepsilon_0} = -f_{(x,y)} V'_{(0)} \to \boxed{f_{(x,y)} = -\frac{\sigma_{(x,y)}}{2V'_{(0)}\varepsilon_0}}$

(d

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(1 ~~الف~~ (ب

$\hat{T} = \frac{\vec{v}}{|\vec{v}|} \Rightarrow \boxed{\frac{a\cos\varphi\,\hat{x} - b\sin\varphi\,\hat{y}}{\sqrt{a^2\cos^2\varphi + b^2\sin^2\varphi}} = \hat{T}}$

(آ $\hat{N} = \frac{\vec{\nabla} f}{|\vec{\nabla} f|} = \frac{\frac{y}{b^2}\hat{y} + \frac{x}{a^2}\hat{x}}{\sqrt{\left(\frac{y}{b^2}\right)^2 + \left(\frac{x}{a^2}\right)^2}} = \frac{\frac{\cos\varphi}{b}\hat{y} + \frac{\sin\varphi}{a}\hat{x}}{\sqrt{\frac{\cos^2\varphi}{b^2} + \frac{\sin^2\varphi}{a^2}}} \to \boxed{\hat{N} = \frac{b\sin\varphi\,\hat{x} + a\cos\varphi\,\hat{y}}{\sqrt{b^2\sin^2\varphi + a^2\cos^2\varphi}}}$

(پ

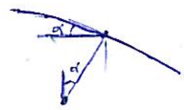
$R = \frac{(1+f'^2)^{3/2}}{|f''|} = \frac{\left(1 + \left(\frac{df}{d\varphi}\times\frac{d\varphi}{dx}\right)^2\right)^{3/2}}{\left|\frac{df'}{d\varphi}\times\frac{d\varphi}{dx}\right|} = \frac{\left(1 + \left(-b\sin\varphi \times \frac{1}{a\cos\varphi}\right)^2\right)^{3/2}}{\left|\frac{-b}{a\cos^2\varphi}\times\frac{1}{a\cos\varphi}\right|} \Rightarrow \boxed{R = \frac{(a^2\cos^2\varphi + b^2\sin^2\varphi)^{3/2}}{ab}}$

$f = b\sqrt{1 - \frac{x^2}{a^2}}$

$f = b\cos\varphi$, $x = a\sin\varphi \to \frac{d\varphi}{dx} = \frac{1}{a\cos\varphi}$

(ت

$N - mg\cos\alpha = \frac{mv^2}{R}$ ~~$\to N = mag\cos\varphi$~~ $\quad N = \frac{mag\cos\varphi}{\sqrt{a^2\cos^2\varphi + b^2\sin^2\varphi}} - \frac{2mgab^2(1-\cos\varphi)}{(a^2\cos^2\varphi + b^2\sin^2\varphi)^{3/2}} \to$



$\tan\alpha = -\frac{dy}{dx} = \frac{b\sin\varphi}{a\cos\varphi} \to \cos\alpha = \frac{a\cos\varphi}{\sqrt{a^2\cos^2\varphi + b^2\sin^2\varphi}}$, $\frac{1}{2}mv^2 + mgy = mgb \to mv^2 = 2mgb(1-\cos\varphi)$

$\boxed{N = \frac{mag}{\sqrt{a^2\cos^2\varphi + b^2\sin^2\varphi}}\left(\cos\varphi - \frac{2b^2(1-\cos\varphi)}{(a^2\cos^2\varphi + b^2\sin^2\varphi)}\right)}$

(ث $\boxed{a^2\cos^3\varphi = b^2\left(2(1-\cos\varphi) - \cos\varphi\sin^2\varphi\right)}$

$\frac{b}{a} = 1 - \frac{b}{a} \to \frac{b^2}{a^2} = 1 - 2\epsilon \to (\cos\varphi_{(0)} - \varphi_{(1)}\sin\varphi_{(0)})^3 = (1-2\epsilon)\left(2 - 2\cos\varphi_0 + 2\varphi_{(1)}\sin\varphi_0 - (\cos\varphi_0 - \varphi_{(1)}\sin\varphi_0)(\sin^2\varphi_0 + 2\varphi_{(1)}\sin\varphi_0\cos\varphi_0)\right) \to$

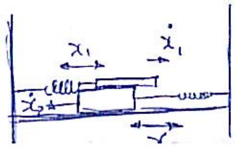
$\cos\varphi_0 = \frac{2}{3} \to \sin\varphi_0 = \frac{\sqrt{5}}{3} \to$ ~~$\frac{8}{27}$~~ $\cos^3\varphi_0 - 3\sin\varphi_0\cos^2\varphi_0\,\varphi_{(1)} = \left[(2 - 2\cos\varphi_0 - \cos\varphi_0\sin^2\varphi_0) + (2\varphi_{(1)}\sin\varphi_0 + \varphi_{(1)}\sin^3\varphi_0 - 2\varphi_{(1)}\sin\varphi_0\cos^2\varphi_0)\right](1-2\epsilon)$

$3\times\frac{\sqrt{5}}{3}\times\frac{4}{9}\varphi_{(1)} = -2\epsilon\left(2 - \frac{4}{3} - \frac{10}{27}\right) +$ ~~$\frac{2\sqrt{5}}{3}$~~ $\frac{\sqrt{5}}{3}\varphi_{(1)}\left(2 + \frac{5}{9} - \frac{8}{9}\right) \to \varphi_{(1)}\frac{\sqrt{5}}{3}\left(\frac{18 - 3 + 12}{9}\right) = 2\epsilon\left(\frac{54 - 36 - 10}{27}\right) \to$

$\varphi_{(1)} = 2\epsilon\times\frac{8}{27\sqrt{5}} \to \varphi_{(1)} = \frac{16}{27\sqrt{5}}\epsilon \to \boxed{\varphi^{(1)} = \cos^{-1}\left(\frac{2}{3}\right) + \frac{16}{27\sqrt{5}}\epsilon}$, $\epsilon = 1 - \frac{b}{a}$

(2

$-kx_2 - b(\dot{x}_2 + \dot{x}_1) = m\ddot{x}_2$
$-kx_1 - b(\dot{x}_1 + \dot{x}_2) = m\ddot{x}_1$
$\to \begin{cases} m(\ddot{x}_2 - \ddot{x}_1) = k(x_1 - x_2) \\ -k(x_2 + x_1) - 2b(\dot{x}_1 + \dot{x}_2) = m(\ddot{x}_1 + \ddot{x}_2) \end{cases} \to \begin{cases} y_2\left(\frac{k}{m}\right) + \ddot{y}_2 = 0 \\ \ddot{y}_1 + \left(\frac{2b}{m}\right)\dot{y}_1 + \left(\frac{k}{m}\right)y_1 = 0 \end{cases} \to$



باسمه تعالی

امتحان فاینال 86

2) ادامه:

ب)

$$\begin{cases} y_2 = A_2 \sin(\omega_0 t) + B_2 \cos(\omega_0 t) \\ y_1 = e^{-\gamma t}\left(A_1 \sin\omega t + B_1 \cos(\omega t)\right) \end{cases} \longrightarrow \begin{cases} \dot y_{2(0)} = 0 \to A_2 = 0 \\ y_{2(0)} = x_0 \to B_2 = x_0 \end{cases} , \begin{cases} \dot y_{1(0)} = 0 \to \omega(A_1\cos(\omega t) - B_1 \sin(\omega t)) - \gamma(A_1 \sin\omega t + B_1\cos\omega t) = 0 \\ y_{1(0)} = 3x_0 \to B_1 = 3x_0 \end{cases}$$

$$\longrightarrow A_1\omega - \gamma B_1 = 0 \to A_1 = \frac{3\gamma x_0}{\omega} \quad , \omega = \sqrt{\frac{k}{m} - \left(\frac{b}{m}\right)^2} \longrightarrow \text{می‌دانیم} \quad , \gamma = \frac{b}{m} \to A_1 = \frac{3 b x_0}{m\sqrt{\frac{k}{m} - \left(\frac{b}{m}\right)^2}} \quad , \omega_0 = \sqrt{\frac{k}{m}}$$

$$\longrightarrow \begin{cases} y_2 = x_0 \cos(\omega_0 t) \\ y_1 = e^{-\frac{b}{m}t}\left(\left(\frac{3bx_0}{\sqrt{mk - b^2}}\right)\sin(\omega t) + 3x_0 \cos(\omega t)\right) \end{cases} \longrightarrow \begin{cases} x_1 = \frac{x_0}{2}\left\{\cos(\omega_0 t) + 3e^{-\left(\frac{b}{m}\right)t}\left(\left(\frac{1}{\sqrt{\frac{mk}{b^2} - 1}}\right)\sin(\omega t) + \cos(\omega t)\right)\right\} \\ x_2 = \frac{x_0}{2}\left\{-\cos(\omega_0 t) + 3e^{-\left(\frac{b}{m}\right)t}\left(\left(\frac{1}{\sqrt{\frac{mk}{b^2} - 1}}\right)\sin(\omega t) + \cos(\omega t)\right)\right\} \end{cases}$$

3)

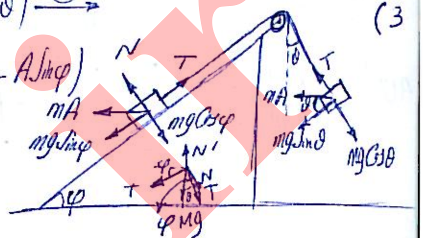
$$\begin{cases} mg\cos\theta - T - mA\sin\theta = m(\ddot r - r\dot\theta^2) \\ -g\sin\theta - A\cos\theta = 2\dot r\dot\theta + r\ddot\theta \\ N + mA\sin\varphi = mg\cos\varphi \\ T - mg\sin\varphi - mA\cos\varphi = m\ddot r \\ N\sin\varphi + T\sin\theta - T\cos\varphi = MA \end{cases}$$

$$③,⑤ \to mg\cos\varphi\sin\varphi - mA\sin^2\varphi = MA + T(\cos\varphi - \sin\theta) \xrightarrow{④}$$

$$m(\ddot r + A\cos\varphi + g\sin\varphi)(\cos\varphi - \sin\theta) + MA = m\sin\varphi(g\cos\varphi - A\sin\varphi)$$

الف)

$$\longrightarrow \boxed{A = \frac{\ddot r(\sin\theta - \cos\varphi) + g\sin\theta\sin\varphi}{M + m(1 - \sin\theta\cos\varphi)}\, m}$$



ب)

$$mg\cos\varphi\sin\varphi + T(\sin\theta - \cos\varphi) = \frac{(M + m\sin^2\varphi)}{m\cos\varphi}(T - mg\sin\varphi - m\ddot r) \longrightarrow$$

$$m^2 g\cos^2\varphi\sin\varphi + m(\ddot r + g\sin\varphi)(M + m\sin^2\varphi) = T\left(M + m\sin^2\varphi - m\sin\theta\cos\varphi + m\cos^2\varphi\right) \longrightarrow \boxed{T = \frac{(M+m)\, g\sin\varphi + \ddot r(M + m\sin^2\varphi)}{M + m(1 - \sin\theta\cos\varphi)}\, m}$$

پ) ت)

$$-\left(g\sin\theta + \frac{\ddot r(\sin\theta - \cos\varphi) + g\sin\theta\sin\varphi}{M + m(1 - \sin\theta\cos\varphi)}\, m\cos\theta\right) = 2\dot r\dot\theta + r\ddot\theta$$

$$g(\cos\theta - \sin\varphi) - A(\sin\theta + \cos\varphi) = 2\ddot r - r\dot\theta^2 \longrightarrow g(\cos\theta - \sin\varphi) - \frac{\ddot r(\sin\theta - \cos\varphi) + g\sin\theta\sin\varphi}{M + m(1 - \sin\theta\cos\varphi)}\, m(\sin\theta + \cos\varphi) = 2\ddot r - r\dot\theta^2$$

ث)

$$\ddot r_{(0)} = g\frac{(1 - \sin\varphi)}{2} \quad , \quad \cancel{g(1-\sin\varphi)} + \frac{m}{M}\cos^2\varphi\, \ddot r_{(0)} = 2\ddot r_{(1)} \longrightarrow \ddot r_{(1)} = \frac{m}{2M}\cos^2\varphi \times \frac{g}{2}(1 - \sin\varphi) \longrightarrow \ddot r^{(1)} = \frac{g(1 - \sin\varphi)}{2}\left(1 + \frac{m}{M}\cos^2\varphi\right)$$

$$\longrightarrow \dot r^{(1)} = \frac{g(1 - \sin\varphi)}{2}\left(1 + \frac{m}{M}\cos^2\varphi\right)t \longrightarrow \boxed{r^{(1)} = \frac{g(1 - \sin\varphi)}{4}\left(1 + \frac{m}{M}\cos^2\varphi\right)t^2 + l_0}$$

ج)

$$-g\theta_{(1)} + \frac{m}{M}\cos\varphi\, \ddot r_{(0)} = 2\dot r_{(1)}\dot\theta_{(1)} + r_{(0)}\ddot\theta_{(1)} \longrightarrow$$

$$\theta_{(1)} = a_0 + a_1 t + a_2 t^2 \to \dot\theta_{(1)} = a_1 + 2a_2 t \to \ddot\theta_{(1)} = 2a_2$$

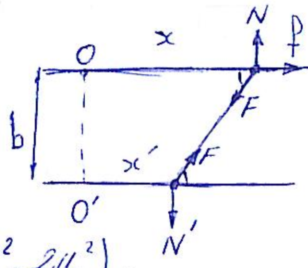
$$-g(a_0 + a_1 t + a_2 t^2) + \frac{m}{M} g\frac{(1 - \sin\varphi)}{2}\cos\varphi = g(1 - \sin\varphi)t\,(a_1 + 2a_2 t) + \left(\frac{g(1 - \sin\varphi)t^2}{4} + l_0\right)(2a_2)$$

$$\theta_{(1)}\big|_{t=0} = 0 \to a_0 = 0 \quad , \quad \dot\theta\big|_{t=0} = 0 \to a_1 = 0 \quad \Big\} \to a_2 = \frac{g(1 - \sin\varphi)\cos\varphi}{4 l_0}\cdot\frac{m}{M} \longrightarrow$$

$$\boxed{\theta^{(1)} = \frac{mg(1 - \sin\varphi)\cos\varphi}{4Ml_0}\, t^2}$$

4) a)

$$\begin{cases} f - \frac{kq^2(x - x')}{((x - x')^2 + b^2)^{3/2}} = m\ddot x \\ \frac{kq^2(x - x')}{((x - x')^2 + b^2)^{3/2}} = m\ddot x' \end{cases} \longrightarrow \boxed{\begin{cases} f = 2m\ddot X \\ f - \frac{2kq^2 u}{(u^2 + b^2)^{3/2}} = m\ddot u \end{cases}} \longrightarrow \boxed{\omega = \sqrt{\frac{2kq^2}{mb^3}}}$$



c)

$$f - \frac{2kq^2(u_{(0)} + u_{(1)})}{(u_{(0)}^2 + b^2)^{3/2}}\left(1 - \frac{3u_{(0)}u_{(1)}}{u_{(0)}^2 + b^2}\right) = m\ddot u_{(0)} + m\ddot u_{(1)} \longrightarrow \begin{cases} m\ddot u_{(1)} = f - \frac{2kq^2(b^2 - 2u_{(0)}^2)}{(u_0^2 + b^2)^{5/2}}\, u_{(1)} \\ m\ddot u_{(0)} + \frac{2kq^2 u_{(0)}}{(b^2 + u_{(0)}^2)^{3/2}} = 0 \end{cases}$$

d)

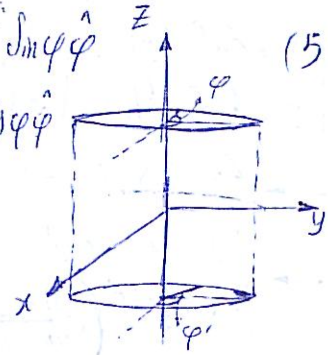
$$\boxed{u_{(0)} = 0} \to \ddot u_{(1)} + \frac{2kq^2}{mb^3}u_{(1)} = \frac{f}{m} \longrightarrow u_{(1)} = A\sin(\omega t) + B\cos(\omega t) + \frac{f}{m\omega^2}$$

$$\dot u_{(1)}\big|_{t=0} = 0 \quad , u_{(1)}\big|_{t=0} = 0 \quad \Big\} \to B = -\frac{f}{m\omega^2} \quad , A = 0 \quad \Big\} \to \boxed{u^{(1)}_{(t)} = \frac{fb^3}{2kq^2}\left(1 - \cos(\omega t)\right)}$$

باسمه تعالی

$$\vec F_{12}=\frac{-kq^2\left(2R\hat z+R(\cos\varphi-\cos\varphi')\hat i+R(\sin\varphi-\sin\varphi')\hat j\right)}{R^3\left(4+2-2\cos(\varphi-\varphi')\right)^{3/2}}$$

$$\begin{cases}\hat i=\cos\varphi\,\hat r-\sin\varphi\,\hat\varphi\\ \hat j=\sin\varphi\,\hat r+\cos\varphi\,\hat\varphi\end{cases}$$ (5



$$\vec F_{12}=-\frac{kq^2\left(2\hat z+(1-\cos(\varphi-\varphi'))\hat r+\sin(\varphi-\varphi')\hat\varphi\right)}{R^2\left(2(3-\cos(\varphi-\varphi'))\right)^{3/2}}\longrightarrow$$

$$\frac{kq^2\sin(\omega t-\varphi)}{R^2\left[2(3-\cos(\omega t-\varphi))\right]^{3/2}}=mR\ddot\varphi\longrightarrow\boxed{\ddot\varphi=\left(\frac{kq^2}{mR^3}\right)\frac{\sin(\omega t-\varphi)}{(6-2\cos(\omega t-\varphi))^{3/2}}}$$

$\boxed{v_1=\sqrt{2gH}}$ a (6

$2ft_1=2mv_1\rightarrow\boxed{t_1=\frac{m\sqrt{2gH}}{f}}$ (b

(c $v_1^2=2\left(\frac{2f}{m}\right)x_1\rightarrow\boxed{x_1=\frac{2mgH}{f}}$

$2mgt_1=2mv\rightarrow\boxed{v=\frac{mg\sqrt{2gH}}{f}}$ (d

(e
$$v^2-v_1^2=-2\left(g+\frac{f}{m}\right)h\rightarrow h=\frac{2gH\left(1-\frac{m^2g^2}{f^2}\right)}{\frac{2}{m}(mg+f)}\rightarrow h=\frac{mgH(f-mg)}{f^2}\rightarrow\boxed{h=\frac{mgH}{f}\left(1-\frac{mg}{f}\right)}$$

(f
$$x_2=\frac{mv_2^2}{f}\rightarrow\cancel{x_2=\frac{2m^3g^3H}{f^3}}\quad\boxed{x_2=\frac{2m^2g^2H}{f^2}}$$

$$v_2^2-v^2=2gh\rightarrow v_2^2=\frac{2m^2g^3H}{f^2}+\frac{2mg^2H}{f}-\frac{2m^2g^3H}{f^2}\rightarrow v_2^2=\frac{2mg^2H}{f}$$

(g
$$x_n=\frac{mv_n^2}{f}\rightarrow x_n=\frac{2mgH}{f}\left(\frac{mg}{f}\right)^{n-1}$$

$$v_n^2=\frac{mg}{f}v_{n-1}^2\rightarrow v_n=v_1\left(\frac{mg}{f}\right)^{\frac{n-1}{2}}\rightarrow v_n^2=v_1^2\left(\frac{mg}{f}\right)^{n-1}=2gH\left(\frac{mg}{f}\right)^{n-1}$$

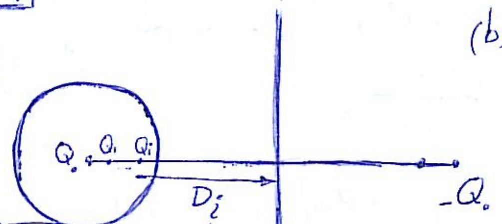
$$X_n=\sum_{n=1}^{n}x_n=\frac{2mgH}{f}\left(1+\frac{mg}{f}+\frac{m^2g^2}{f^2}+\dots+\left(\frac{mg}{f}\right)^{n-1}\right)\rightarrow X_n=\frac{2mgH}{f}\cdot\frac{1-\left(\frac{mg}{f}\right)^n}{1-\frac{mg}{f}}\rightarrow X_n=\frac{2mgH}{f-mg}\left(1-\left(\frac{mg}{f}\right)^n\right)$$

$$\cancel{X_N>L\rightarrow1-\left(\frac{mg}{f}\right)^N>\frac{L(f-mg)}{2mgH}}$$

$$X_N>L\rightarrow\boxed{\frac{2mgH}{f-mg}\left(1-\left(\frac{mg}{f}\right)^N\right)>L}\ ,\ \boxed{\frac{1-\left(\frac{mg}{f}\right)^N}{1-\frac{mg}{f}}>\frac{fL}{2mgH}}$$

(b, (a (8
$$Q_{i+1}=-\frac{R}{D+D_i}(-Q_i)\rightarrow\boxed{Q_{i+1}=\frac{R}{D+D_i}Q_i}\ ,\ \boxed{D_{i+1}=\frac{R^2}{D+D_i}}$$

$$\boxed{x_{i+1}=\frac{1}{\frac{D}{R}+x_i}}$$ (c



$$x=\frac{1}{\frac{D}{R}+\frac{1}{\frac{D}{R}+\frac{1}{\dots}}}\rightarrow\frac{1}{x}=\frac{D}{R}+\frac{1}{\frac{D}{R}+\frac{1}{\dots}}\rightarrow x^2+\left(\frac{D}{R}\right)x-1=0\rightarrow\boxed{x=\frac{\sqrt{4+\left(\frac{D}{R}\right)^2}-\frac{D}{R}}{2}}$$ (d

$$z_i=\frac{1}{x_i-x}\rightarrow x_i=x+\frac{1}{z_i}\rightarrow x+\frac{1}{z_{i+1}}=\frac{1}{\frac{1}{x}-x+x+\frac{1}{z_i}}\rightarrow z_{i+1}=\frac{1}{x\left(\frac{z_i}{x+z_i}-1\right)}\rightarrow z_{i+1}=$$ (e