سام احد ۱

سالار یوسف زاده

بسمه تعالی
سورن ریاضی

سری دوم

① الف) $\cot^2\theta + 1 = \dfrac{\cos^2\theta}{\sin^2\theta} + \dfrac{\sin^2\theta}{\sin^2\theta} = \dfrac{\cos^2\theta + \sin^2\theta}{\sin^2\theta} = \dfrac{1}{\sin^2\theta} \Rightarrow \cot^2\theta = \dfrac{1}{\sin^2\theta} - 1$

$$\boxed{\cot^2\theta = \dfrac{1 - \sin^2\theta}{\sin^2\theta}}$$

ب) $\cot\theta \times \cos\theta = \dfrac{\cos\theta}{\sin\theta} \times \dfrac{\cos\theta}{1} = \dfrac{\cos^2\theta}{\sin\theta} = \dfrac{1 - \sin^2\theta}{\sin\theta} = \boxed{\dfrac{1}{\sin\theta} - \sin\theta}$

ج) $\tan^2\theta \times \cos^2\theta = \dfrac{\sin^2\theta}{\cancel{\cos^2\theta}} \times \cancel{\cos^2\theta} = \boxed{\sin^2\theta}$

د) $\sin^4\theta - \cos^4\theta = \underbrace{(\sin^2\theta + \cos^2\theta)}_{1}(\sin^2\theta - \cos^2\theta) = \sin^2\theta - (1 - \sin^2\theta) = \boxed{2\sin^2\theta - 1}$

② الف) $\sin^2\theta - \cos^2\theta = (1 - \cos^2\theta) - \cos^2\theta = 1 - \cos^2\theta - \cos^2\theta = \boxed{1 - 2\cos^2\theta}$

ب) $(1 - \cot^2\theta)(1 + \cot^2\theta) = \left(1 - \dfrac{\cos^2\theta}{\sin^2\theta}\right)\left(1 + \dfrac{\cos^2\theta}{\sin^2\theta}\right) = \left(1 - \dfrac{\cos^2\theta}{1 - \cos^2\theta}\right)\left(1 + \dfrac{\cos^2\theta}{1 - \cos^2\theta}\right)$

$= 1 - \dfrac{\cos^4\theta}{(1 - \cos^2\theta)^2} = 1 - \dfrac{\cos^4\theta}{\cos^4\theta - 2\cos^2\theta + 1} = \dfrac{\cancel{\cos^4\theta} - 2\cos^2\theta + 1 - \cancel{\cos^4\theta}}{\cos^4\theta - 2\cos^2\theta + 1} = \boxed{\dfrac{1 - 2\cos^2\theta}{\cos^4\theta - 2\cos^2\theta + 1}}$

ج) $\dfrac{\sin\theta}{\tan\theta - \cot\theta} = \dfrac{\sin\theta}{\dfrac{\sin\theta}{\cos\theta} - \dfrac{\cos\theta}{\sin\theta}} = \dfrac{\sin\theta}{\dfrac{\sin^2\theta - \cos^2\theta}{\cos\theta \times \sin\theta}} = \sin\theta \times \dfrac{\cos\theta \times \sin\theta}{1 - 2\cos^2\theta}$

$= \dfrac{\cos\theta \times (1 - \cos^2\theta)}{1 - 2\cos^2\theta} = \boxed{\dfrac{\cos\theta - \cos^3\theta}{1 - 2\cos^2\theta}}$

د) $\sin^4\theta + \cos^4\theta = \underbrace{(\sin^2\theta + \cos^2\theta)^2}_{1} - 2\sin^2\theta \times \cos^2\theta = 1 - 2 \times (1 - \cos^2\theta) \times \cos^2\theta$

$1 - 2(\cos^2\theta - \cos^4\theta) = \boxed{1 - 2\cos^2\theta + 2\cos^4\theta}$

③ الف) $\cos^2\theta = \dfrac{1}{\dfrac{1}{\cos^2\theta}} = \dfrac{1}{\dfrac{\sin^2\theta + \cos^2\theta}{\cos^2\theta}} = \boxed{\dfrac{1}{\tan^2 + 1}}$

ب) $\dfrac{1 - \sin^2\theta}{1 - \cos^2\theta} = \dfrac{\cos^2\theta}{\sin^2\theta} = \boxed{\dfrac{1}{\tan^2\theta}}$

ج) $\sin^2\theta = \dfrac{1}{\dfrac{1}{\sin^2\theta}} = \dfrac{1}{\dfrac{\sin^2\theta + \cos^2\theta}{\sin^2\theta}} = \dfrac{1}{1 + \dfrac{\cos^2\theta}{\sin^2\theta}} = \dfrac{1}{1 + \dfrac{1}{\tan^2\theta}} = \dfrac{1}{\dfrac{\tan^2\theta + 1}{\tan^2\theta}} = \boxed{\dfrac{\tan^2\theta}{\tan^2\theta + 1}}$

$$\frac{\sin\theta}{\cos\theta(\tan\theta+\cot\theta)}=\frac{\tan\theta}{\tan\theta+\frac{1}{\tan\theta}}=\frac{\tan\theta}{\frac{\tan^2\theta+1}{\tan\theta}}=\tan\theta\times\frac{\tan\theta}{\tan^2\theta+1}=\boxed{\frac{\tan^2\theta}{\tan^2\theta+1}}$$

الف) $\frac{1+\cos\theta}{\sin^2\theta}=\frac{1+\cos\theta}{1-\cos^2\theta}=\frac{\cancel{1+\cos\theta}}{(\cancel{1+\cos\theta})(1-\cos\theta)}=\frac{1}{1-\cos\theta}$

ب) $2\times\sin\theta\times\cos\theta=\frac{2\sin\theta}{\cos\theta}\times\cos^2\theta=\frac{\frac{2\sin\theta}{\cos\theta}}{\frac{1}{\cos^2\theta}}=\frac{2\tan\theta}{\frac{\sin^2\theta+\cos^2\theta}{\cos^2\theta}}=\frac{2\tan\theta}{1+\tan^2\theta}$

ج) $\frac{\tan^2\theta}{1+\tan^2\theta}+\frac{\cot^2\theta}{1+\cot^2\theta}=\frac{\tan^2\theta}{1+\tan^2\theta}+\frac{\frac{1}{\tan^2\theta}}{1+\frac{1}{\tan^2\theta}}=\frac{\tan^2\theta}{1+\tan^2\theta}+\frac{\frac{1}{\tan^2\theta}}{\frac{\tan^2\theta+1}{\tan^2\theta}}=\frac{\tan^2\theta}{1+\tan^2\theta}+\frac{1}{1+\tan^2\theta}=1$

د) $\tan\theta+\frac{\cos\theta}{1+\sin\theta}=\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{1+\sin\theta}=\frac{\sin\theta+\overbrace{\sin^2\theta+\cos^2\theta}^{1}}{\cos\theta+(\sin\theta\times\cos\theta)}=\frac{\cancel{1+\sin\theta}}{\cos\theta(\cancel{1+\sin\theta})}=\frac{1}{\cos\theta}$

الف) $\sin^4x\times\cos^4x=(\sin^2x+\cos^2x)^2-2\sin^2x\cos^2x=1-2\sin^2x\cos^2x$

ب) $\sin^6x+\cos^6x=(\underbrace{\sin^2x+\cos^2x}_{1})(\sin^4x+\cos^4x-\sin^2x\cos^2x)$

$=(\underbrace{\sin^2x+\cos^2x}_{1})^2-3\sin^2x\cos^2x=1-3\sin^2x\cos^2x$

ج) $\sin^4x-\cos^4x=(\sin^2x-\cos^2x)(\underbrace{\sin^2x+\cos^2x}_{1})=(\sin^2x-1+\sin^2x)$

$=(2\sin^2x-1)=(\sqrt{2}\sin x+1)(\sqrt{2}\sin x-1)$

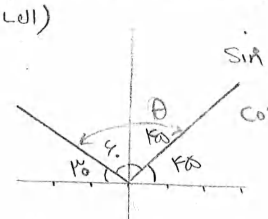
د) $\sin^3x-\cos^3x=(\sin x-\cos x)(\sin^2x+\sin x\cos x+\cos^2x)=(\sin x-\cos x)(1+\sin x\cos x$

الف) $\frac{\sin^2x}{1-\cos x}+\frac{\sin^2x}{1+\cos x}=\frac{1-\cos^2x}{1-\cos x}+\frac{1-\cos^2x}{1+\cos x}=\frac{(\cancel{1-\cos x})(1+\cos x)}{(\cancel{1-\cos x})}+\frac{(1-\cos x)(\cancel{1+\cos x})}{(\cancel{1+\cos x})}$

$=1+\cancel{\cos x}+1-\cancel{\cos x}=2$

ب) $\frac{\sin^4x+\cos^4x-1}{\sin^6x+\cos^6x-1}=\frac{(\sin^2x+\cos^2x)^2-2\sin^2x\cos^2x-1}{(\underbrace{\sin^2x+\cos^2x}_{1})(\sin^4x-\sin^2x\cos^2x+\cos^4x)-1}=\frac{-2\,\cancel{\sin^2x\cos^2x}}{-3\,\cancel{\sin^2x\cos^2x}}=\frac{2}{3}$

الف)



$\sin 45^\circ=\frac{\sqrt{2}}{2}$

$\cos 45^\circ=\frac{\sqrt{2}}{2}$

$\frac{1}{2}\le\sin\theta\le\frac{\sqrt{2}}{2}$

$-\frac{\sqrt{3}}{2}\le\cos\theta\le\frac{\sqrt{2}}{2}$

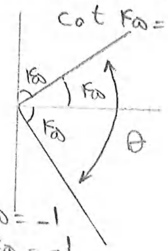
$\sin 150^\circ=\sin 30^\circ=\frac{1}{2}$

$\cos 150^\circ=-\cos 30^\circ=-\frac{\sqrt{3}}{2}$

ب)

$\tan 45^\circ=1$

$\cot 45^\circ=1$



$-1\le\tan\theta\le1$

$-1\le\cot\theta\le1$

$\tan(-45^\circ)=-1$

$\cot(-45^\circ)=-1$

$$\frac{1-\cos x+k\sin x}{\sin x+k(1+\cos x)}=\frac{\dfrac{\sin^2 x}{\cancel{1-\cos^2 x}}\!\ \dfrac{}{1+\cos x}+k\sin x}{\sin x+k(1+\cos x)}=\frac{\dfrac{\sin^2 x+k\sin x(1+\cos x)}{1+\cos x}}{\sin x+k(1+\cos x)}$$

$$=\frac{\sin x\left(\cancel{\sin x+k(1+\cos x)}\right)}{1+\cos x}\times\frac{1}{\cancel{\sin x+k(1+\cos x)}}=\boxed{\frac{\sin x}{1+\cos x}}$$

$$(x\sin\theta-y\cos\theta)^2+(x\cos\theta+y\sin\theta)^2=x^2+y^2$$ -9

$$x^2\sin^2\theta+y^2\cos^2\theta-\cancel{2xy\sin\theta\cos\theta}+x^2\cos^2\theta+y^2\sin^2\theta+2xy\cancel{\sin\theta\cos\theta}=x^2(\underbrace{\sin^2\theta+\cos^2\theta}_{1})+y^2(\underbrace{\sin^2\theta+\cos^2\theta}_{1})$$

$$=x^2+y^2$$

۲۰ حدود

سالار یوسف زاده
تمرین سری سوم

(1)

الف ۱۱:

120	$\sin 120 = \sin 60 = \frac{\sqrt{3}}{2}$	$\cos 120 = -\cos 60 = -\frac{1}{2}$	$\tan 120 = -\sqrt{3}$	$\cot = -\frac{\sqrt{3}}{3}$
210	$\sin 210 = -\sin 30 = -\frac{1}{2}$	$\cos 210 = -\cos 30 = \frac{-\sqrt{3}}{2}$	$\tan 210 = \frac{\sqrt{3}}{3}$	$\cot = \sqrt{3}$
-135	$\sin -135 = -\sin 45 = \frac{-\sqrt{2}}{2}$	$\cos -135 = -\cos 45 = \frac{-\sqrt{2}}{2}$	$\tan -135 = +1$	$\cot -135 = 1$
$1440 = 4\times 360$	$\sin 1440 = \sin 0 = 0$	$\cos 1440 = \cos 0 = 1$	$\tan 1440 = 0$	$\cot 1440 = \infty$
$\frac{-2\pi}{3} = -120^\circ$	$\sin -120 = -\sin 60 = -\frac{\sqrt{3}}{2}$	$\cos -120 = -\cos 60 = \frac{-1}{2}$	$\tan -120 = \sqrt{3}$	$\cot -120 = \frac{+\sqrt{3}}{3}$
$\frac{5\pi}{4} = 225^\circ$	$\sin 225 = -\sin 45 = \frac{-\sqrt{2}}{2}$	$\cos 225 = -\cos 45 = \frac{-\sqrt{2}}{2}$	$\tan 225 = 1$	$\cot 225 = 1$
$\frac{5\pi}{3} = 300^\circ$	$\sin 300 = -\sin 60 = \frac{-\sqrt{3}}{2}$	$\cos 300 = \cos 60 = \frac{1}{2}$	$\tan 300 = -\sqrt{3}$	$\cot 300 = -\frac{\sqrt{3}}{3}$
$\frac{11\pi}{6} = 330^\circ$	$\sin 330 = -\sin 30 = \frac{-1}{2}$	$\cos 330 = \cos 30 = \frac{\sqrt{3}}{2}$	$\tan 330 = \frac{-\sqrt{3}}{3}$	$\cot 330 = -\sqrt{}$

الف) $\boxed{\sin(450 + a) = \sin(90 + a) = \sin(90 - a) = \cos a}$

$\sin(90 + a) = \sin(90 - a)$ ، $\cos(90 + a) = -\cos(90 - a)$

$\tan(90 + a) = -\tan(90 - a)$ ، $\cot(90 + a) = -\cot(90 - a)$

$a < 90$

ب) $\boxed{\cos(540 + a) = \cos(180 + a) = -\cos a}$

$\cos(180 + a) = -\cos a$ ، $\sin(180 + a) = -\sin a$ $\quad \tan(180 + a) = +\tan a$ $\quad \cot(180 + a) = +\cot a$

ج) $\boxed{\tan(540 - a) = \tan(180 - a) = -\tan a}$ $\quad \cot(180 - a) = -\cot a$

$\sin(180 - a) = \sin a$ $\quad \cos(180 - a) = -\cos a$ $\quad \tan(180 - a) = -\tan a$

د) $\boxed{\cot(a - 270) = -\cot(90 - a) = -\tan a}$ $\tan(a - 270) = -\tan(90 - a)$

$\sin(a - 270) = \sin(90 - a)$ $\quad \cos(a - 270) = -\cos(90 - a)$

$\sin a = \frac{8}{17}$ $\quad$ الف) $\sin\left(\frac{3\pi}{2} + a\right) = \sin(270^\circ + a) = \quad = \sin(a - 90) = -\sin(90 - a)$

$-\cos a = \frac{-15}{17}$

$180 \quad a$

ب) $\cos(2\pi - a) = \cos(360 - a) = +\cos a = \frac{+15}{17}$

$\cos a = \frac{15}{17}$

ج) $\tan\left(\frac{\pi}{2} + a\right) = \tan(90 + a) = -\tan(90 - a) = -\cot a = \frac{-15}{8}$

$\tan = \frac{8}{17} \times \frac{17}{15} = \frac{8}{15}$

د) $\cot\left(\frac{3\pi}{2} - a\right) = \cot(270 - a) = \cot(90 - a) = \tan a = \frac{8}{15}$

$\cot = \frac{15}{8}$

$\frac{289}{289} - \frac{64}{289} = \frac{225}{289} = \frac{15}{17}$

(۴)

$$\frac{\sin ۳۷۵ + ۲\sin ۱۰۵}{\cos ۱۶۵ - \cos ۲۵۵} = \frac{۳\sin ۱۵ + ۲\sin ۷۵}{-\cos ۱۵ + \cos ۷۵}$$

$\tan ۷۵ = ۲+\sqrt{۳}$

محاسبات $\sin ، \cos$

$$\tan ۷۵ = \frac{\sin ۷۵}{\cos ۷۵} = ۲+\sqrt{۳} \Rightarrow \sin ۷۵ = (۲+\sqrt{۳})\cos ۷۵$$

$$\sin^۲ ۷۵ + \cos^۲ ۷۵ = (۴+۳+۴\sqrt{۳})\cos^۲ ۷۵ + \cos^۲ ۷۵ = (۸+۴\sqrt{۳})\cos^۲ ۷۵ = ۱ \Rightarrow \cos^۲ ۷۵ = \frac{۱}{۴(۲+\sqrt{۳})}$$

$$\cos ۷۵ = \frac{۱}{۲\sqrt{۲+\sqrt{۳}}} \times \frac{\sqrt{۲+\sqrt{۳}}}{\sqrt{۲+\sqrt{۳}}} = \frac{\sqrt{۲+\sqrt{۳}}}{۲(۲+\sqrt{۳})} = \frac{\sqrt{۲+\sqrt{۳}}}{۴+۲\sqrt{۳}} \Rightarrow \sin ۷۵ = \frac{(۲+\sqrt{۳})}{۲\sqrt{۲+\sqrt{۳}}}$$

$$\sin ۷۵ = \cos ۱۵ = \frac{۲+\sqrt{۳}}{۲\sqrt{۲+\sqrt{۳}}} \quad ، \quad \cos ۷۵ = \sin ۱۵ = \frac{۱}{۲\sqrt{۲+\sqrt{۳}}}$$

$$\Rightarrow \frac{۳\sin ۱۵ + ۲\sin ۷۵}{-\cos ۱۵ + \cos ۷۵} = \frac{\frac{۳}{۲\sqrt{۲+\sqrt{۳}}} + \frac{۴+۲\sqrt{۳}}{۲\sqrt{۲+\sqrt{۳}}}}{\frac{-۲-\sqrt{۳}}{۲\sqrt{۲+\sqrt{۳}}} + \frac{۱}{۲\sqrt{۲+\sqrt{۳}}}} = \frac{\frac{۷+۲\sqrt{۳}}{۲\sqrt{۲+\sqrt{۳}}}}{\frac{-۲-\sqrt{۳}+۱}{۲\sqrt{۲+\sqrt{۳}}}} = \frac{(۷+۲\sqrt{۳})}{\cancel{۲\sqrt{۲+\sqrt{۳}}}} \times \frac{\cancel{۲\sqrt{۲+\sqrt{۳}}}}{-۱-\sqrt{۳}}$$

$$= \boxed{\frac{۷+۲\sqrt{۳}}{-۱-\sqrt{۳}}}$$

(۵)

الف)

$$\frac{\sin ۲۲۵ \times \cos ۳۰ + \cos ۱۳۵ \times \sin ۶۰}{\tan ۲۱۰ \times \cot ۶۰ - \cot ۲۴۰ \tan ۳۳۰} = \frac{(-\sin ۴۵)(\cos ۳۰) + (-\cos ۴۵)(\sin ۶۰)}{(\tan ۳۰)(\cot ۶۰) - (\cot ۶۰)(-\tan ۳۰)}$$

$$= \frac{(-\frac{\sqrt{۲}}{۲})(\frac{\sqrt{۳}}{۲}) + (-\frac{\sqrt{۲}}{۲})(\frac{\sqrt{۳}}{۲})}{(\frac{\sqrt{۳}}{۳})(\frac{\sqrt{۳}}{۳}) - (\frac{\sqrt{۳}}{۳})(-\frac{\sqrt{۳}}{۳})} = \frac{-\frac{\sqrt{۶}}{۴} - \frac{\sqrt{۶}}{۴}}{\frac{۳}{۹} + \frac{۳}{۹}} = \frac{\frac{-۲\sqrt{۶}}{۴}}{\frac{۶}{۹}} = \frac{\frac{-\sqrt{۶}}{۲}}{\frac{۲}{۳}} = \frac{-\sqrt{۶}}{۲} \times \frac{۳}{۲} = \boxed{\frac{-۳\sqrt{۶}}{۴}}$$

ب)

$$\sin^۲ \frac{\pi}{۸} + \sin^۲ \frac{۳\pi}{۸} + \sin^۲ \frac{۵\pi}{۸} + \sin^۲ \frac{۷\pi}{۸} = \sin^۲ (۱۸۰ - \frac{۷\pi}{۸}) + \sin^۲ \frac{۳\pi}{۸} + \sin^۲ (۱۸۰ - \frac{۳\pi}{۸})$$

$$+ \sin^۲ \frac{۷\pi}{۸} = ۲\sin^۲ \frac{۷\pi}{۸} + ۲\sin^۲ \frac{۳\pi}{۸} = ۲(\sin^۲ \frac{۷\pi}{۸} + \sin^۲ \frac{۳\pi}{۸})$$

$$= ۲(\sin^۲ \frac{۷\pi}{۸} + \sin^۲ \frac{۷\pi - ۴\pi}{۸}) = ۲(\sin^۲ \frac{۷\pi}{۸} + \sin^۲ (\frac{۷\pi}{۸} - ۹۰)) = ۲$$

ج)

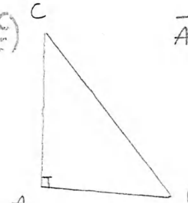
$$\cos^۲ \frac{\pi}{۸} + \cos^۲ \frac{۳\pi}{۸} + \cos^۲ \frac{۵\pi}{۸} + \cos^۲ \frac{۷\pi}{۸} = \cos^۲ (۱۸۰ - \frac{۷\pi}{۸}) + \cos^۲ \frac{۳\pi}{۸} + \cos^۲ (۱۸۰ - \frac{۳\pi}{۸}) + \cos^۲ \frac{۷\pi}{۸}$$

$$- \cancel{\cos^۲ \frac{۷\pi}{۸}} + \cos^۲ \frac{۷\pi}{۸} + \cancel{\cos^۲ \frac{۳\pi}{۸}} - \cos^۲ \frac{۳\pi}{۸} = ۰$$

$$۲(\sin^۲ \frac{۷\pi}{۸} + \underbrace{\sin^۲ -(۹۰ - \frac{۷\pi}{۸})}_{\cos^۲ \frac{۷\pi}{۸}}) = ۲$$

$$\sin^۲ (۹۰ - \frac{۷\pi}{۸}) \qquad \sin (\frac{۷\pi}{۸} - ۹۰) = -\sin (۹۰ - \frac{۷\pi}{۸}) = -\cos \frac{۷\pi}{۸}$$

$\begin{pmatrix} x_C \\ y_C \end{pmatrix}$ C



A $\begin{bmatrix} 2 \\ 5 \end{bmatrix}$ B $\begin{bmatrix} 4 \\ 7 \end{bmatrix}$

$$\overline{AB} = \sqrt{(4-2)^2 + (7-5)^2} = \sqrt{4+4} = \sqrt{4\times 2} = 2\sqrt{2}$$

$$S = \frac{AB \times AC}{2} \Rightarrow 2 = \frac{\cancel{2}\sqrt{2}\times AC}{\cancel{2}} \Rightarrow \sqrt{2}AC = 2 \Rightarrow AC = \frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\cancel{2}\sqrt{2}}{\cancel{2}}$$

$$AC = \sqrt{(x_C-2)^2 + (y_C-5)^2} = \sqrt{2}$$

$$m_{AB} = \frac{y_2-y_1}{x_2-x_1} = \frac{7-5}{4-2} = \frac{2}{2} = 1 \Rightarrow m_{AC} = -1$$

معادله‌ی AC $\quad m_{AC} = -1 \quad A(2,5)$

$$y - y_1 = m(x - x_1) \Rightarrow y-5 = -1(x-2) \Rightarrow y-5 = -x+2 \Rightarrow y = -x+7 \Rightarrow y_C = -x_C+7$$

$$(x_C-2)^2 + (-x_C+2)^2 = 2 \Rightarrow (x_C-2)^2 = 1 \Rightarrow x_C - 2 = 1 \Rightarrow x_C = 3 \quad y_C = 4$$

$$x_C - 2 = -1 \Rightarrow x_C = 1 \quad y_C = 6$$

$$\boxed{C\begin{pmatrix}3\\4\end{pmatrix} \quad C\begin{pmatrix}1\\6\end{pmatrix}}$$

$A = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$

$$y + 4x = 2 \Rightarrow y = -4x+2 \Rightarrow m = -4 \Rightarrow m' = \frac{1}{4}$$

$$m' = \frac{1}{4} \quad (y-y_1) = m(x-x_1) \Rightarrow y-3 = \frac{1}{4}(x-5) \Rightarrow 4y - 12 = x-5 \Rightarrow 4y - x = 7$$

$$\begin{cases} \cancel{4y} + 16x = 8 \\ -\cancel{4y} + x = -7 \end{cases} \Rightarrow 17x = 1 \Rightarrow x = \frac{1}{17}$$

$$4y + \frac{16}{17} = 8 \Rightarrow 4y = \frac{120}{17} \Rightarrow y = \frac{30}{17}$$

$$\Rightarrow (x,y) = \left(\frac{1}{17}, \frac{30}{17}\right)$$

(x,y) ، $(5,3)$

$$\frac{x+5}{2} = \frac{1}{17} \Rightarrow 17x + 85 = 2 \Rightarrow 17x = -83 \Rightarrow x = \frac{-83}{17}$$

$$\frac{y+3}{2} = \frac{30}{17} \Rightarrow 17y + 51 = 60 \Rightarrow 17y = 9 \Rightarrow y = \frac{9}{17}$$

قرینه‌ی نقطه‌ی $A(5,3)$ نسبت به خط $y + 4x - 2 = 0$ برابر $\left(-\frac{83}{17}, \frac{9}{17}\right)$ است.

$a \,|\underline{\,11\,}$ x_1, remainder 2 $a \,|\underline{\,7\,}$ x_2, remainder 1

$a = 11x_1 + 2$

$a = 7(2x_1 - 2) + 1 = 14x_1 - 14 + 1 = 14x_1 - 13$

$$2\left(x_1 = \frac{x_2}{2} + 1\right) \Rightarrow 2x_1 = x_2 + 2 \Rightarrow x_2 = 2x_1 - 2$$

$$11x_1 + 2 = 14x_1 - 13 \Rightarrow 15 = 3x_1 \Rightarrow x_1 = 5 \Rightarrow a = 55 + 2 = 57$$

$$\frac{a-1}{8} = \frac{57-1}{8} = \frac{56}{8} = 7$$

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(9)

سن حمید x

سن برادر y

$$\begin{cases} (x+4) = \frac{3}{2}(y+4) \\ x = 6 + y \end{cases} \Rightarrow (y+10) = \frac{3}{2}(y+4)$$

$$2y + 20 = 3y + 12 \Rightarrow y = 8 \Rightarrow$$ سن حمید $= 6 + 8 = 14$

به نام خداوند بخشنده و مهربان

سالار یوسف زاده (1)

(نمونه‌ی کار ریاضی)

1 - الف)

$\frac{\pi}{10}$: $\frac{D}{360} = \frac{\frac{\pi}{10}}{2\pi} \Rightarrow 2D = 36 \Rightarrow D = \boxed{18^\circ}$ / $\frac{G}{400} = \frac{\frac{\pi}{10}}{2\pi} \Rightarrow G = \boxed{20}$

$\frac{11\pi}{6}$: $\frac{D}{180} = \frac{\frac{11\pi}{6}}{\pi} \Rightarrow D = 180 \times \frac{11}{6} = \boxed{330^\circ}$ / $\frac{G}{200} = \frac{\frac{11\pi}{6}}{\pi} \Rightarrow G = 200 \times \frac{11}{6} = \boxed{\frac{2200}{6}}$

$\frac{4\pi}{3}$: $\frac{D}{180} = \frac{\frac{4\pi}{3}}{\pi} \Rightarrow D = 180 \times \frac{4}{3} = \boxed{240}$ / $\frac{G}{200} = \frac{\frac{4\pi}{3}}{\pi} \Rightarrow G = \frac{4}{3} \times 200 = \boxed{\frac{800}{3}}$

ب)

15 : $\frac{15}{180} = \frac{R}{\pi} \Rightarrow R = \frac{15\pi}{180} = \boxed{\frac{\pi}{12}}$ / 225 : $\frac{225}{180} = \frac{R}{\pi} \Rightarrow R = \frac{225\pi}{180} = \boxed{1{,}25\pi}$

30 : $\frac{30}{180} = \frac{R}{\pi} \Rightarrow R = \frac{3\pi}{18} = \boxed{\frac{\pi}{6}}$ / 375 : $\frac{375}{180} = \frac{R}{\pi} \Rightarrow R = \boxed{\frac{375\pi}{180}}$

2) الف)

$\boxed{\sin a = \frac{3}{5}}$ $\quad \sin^2 a + \cos^2 a = 1 \Rightarrow \frac{9}{25} + \cos^2 a = \frac{25}{25} \Rightarrow \cos^2 a = \frac{16}{25}$

$\Rightarrow \boxed{\cos a = -\frac{4}{5}}$ $\quad \tan a = \frac{\sin a}{\cos a} = \frac{3}{5} \times \frac{-5}{4} = \frac{-3}{4} \Rightarrow \boxed{\tan a = \frac{-3}{4}}$

$\boxed{\cot a = \frac{-4}{3}}$

ب)

$\boxed{\cos a = \frac{5}{13}}$ $\quad \sin^2 a + \cos^2 a = 1 \Rightarrow \sin^2 a + \frac{25}{144} = \frac{144}{144} \Rightarrow \sin^2 a = \frac{119}{144}$

$\boxed{\cot a = \frac{-5}{\sqrt{119}} = \frac{-5\sqrt{119}}{119}}$ $\quad \boxed{\sin a = \frac{-\sqrt{119}}{12}}$ $\quad \tan a = \frac{\sin a}{\cos a} = \frac{-\sqrt{119}}{12} \times \frac{12}{5} = \frac{-\sqrt{119}}{5} \Rightarrow \boxed{\tan a = \frac{-\sqrt{119}}{5}}$

ج)

$\boxed{\tan a = \frac{1}{2}}$ $\quad \tan a = \frac{\sin a}{\cos a} = \frac{1}{2} \Rightarrow \cos a = 2\sin a$

$\sin^2 a + \cos^2 a = 4\sin^2 a + 1\sin^2 a = 5\sin^2 a = 1 \Rightarrow \sin^2 a = \frac{1}{5} \Rightarrow \boxed{\sin a = \frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}}$

$\boxed{\cos a = \frac{2\sqrt{5}}{5}}$ $\quad \boxed{\cot a = 2}$

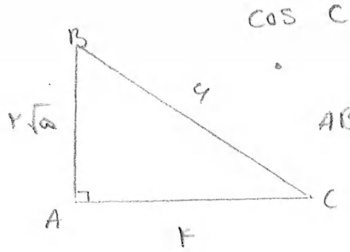
د)

$\boxed{\cot a = \frac{2}{5}} \Rightarrow \boxed{\tan a = \frac{5}{2}}$ $\quad \sin a = \frac{5}{2} \times \frac{-2\sqrt{29}}{29} = \frac{-5\sqrt{29}}{29} \Rightarrow \boxed{\sin a = \frac{-5\sqrt{29}}{29}}$

$\tan a = \frac{\sin a}{\cos a} = \frac{5}{2} \Rightarrow 2\sin a = 5\cos a \Rightarrow \sin a = \frac{5}{2}\cos a$

$\sin^2 a + \cos^2 a = \frac{25}{4}\cos^2 a + \cos^2 a = \frac{29}{4}\cos^2 a = 1 \Rightarrow \cos^2 a = \frac{4}{29} \Rightarrow \boxed{\cos a = \frac{-2}{\sqrt{29}} = \frac{-2\sqrt{29}}{29}}$

3)



$\cos C = \frac{AC}{BC} = \frac{2}{3} \Rightarrow \frac{AC}{6} = \frac{2}{3} \Rightarrow 3AC = 12 \Rightarrow AC = 4$

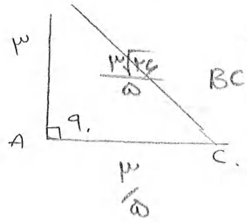
$AB^2 + 16 = 36 \Rightarrow AB^2 = 20 \Rightarrow AB = \sqrt{20} = 2\sqrt{5}$

$\sin C = \cos B = \frac{2\sqrt{5}}{6} = \frac{\sqrt{5}}{3}$, $\cos C = \sin B = \frac{2}{3}$

$\tan C = \cot B = \frac{2\sqrt{5}}{4} = \frac{\sqrt{5}}{2}$ $\quad \cot C = \tan B = \frac{2}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

ب

$\tan C = \frac{AB}{AC} = ۵ \Rightarrow \frac{۳}{AC} = ۵ \Rightarrow ۵AC = ۳ \Rightarrow \boxed{AC = \frac{۳}{۵}}$



$BC^{۲} = ۹ + \frac{۹}{۲۵} = \frac{۲۳۴}{۲۵} \Rightarrow BC = \frac{\sqrt{۹ \times ۲۶}}{۵} = \boxed{\frac{۳\sqrt{۲۶}}{۵}}$

$\sin C = \cos B = \frac{۳}{\frac{۳\sqrt{۲۶}}{۵}} = \cancel{۳} \times \frac{۵}{\cancel{۳}\sqrt{۲۶}} = \frac{۵}{\sqrt{۲۶}} \times \frac{\sqrt{۲۶}}{\sqrt{۲۶}} = \boxed{\frac{۵\sqrt{۲۶}}{۲۶}}$

$\cos C = \sin B = \frac{\cancel{۳}}{\cancel{۵}} \times \frac{\cancel{۵}}{\cancel{۳}\sqrt{۲۶}} = \frac{۱}{\sqrt{۲۶}} \times \frac{\sqrt{۲۶}}{\sqrt{۲۶}} = \boxed{\frac{\sqrt{۲۶}}{۲۶}}$

$\tan C = \cot B = \boxed{۵}$ $\qquad \cot C = \tan B = \boxed{\frac{۱}{۵}}$

۴

1 rad $\quad \frac{D}{\cancel{۳۶۰}\ ۱۸۰} = \frac{۱}{۲\pi} \Rightarrow D\pi = ۱۸۰ \Rightarrow D = \frac{۱۸۰}{\pi} \Rightarrow \sin 1\,\text{rad} = \sin \frac{۱۸۰}{\pi}$

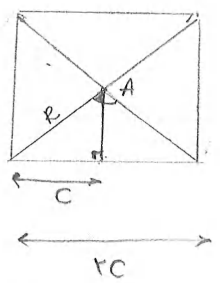
2 rad $\quad \frac{D}{۱۸۰} = \frac{۲}{\pi} \Rightarrow D\pi = ۳۶۰ \Rightarrow D = \frac{۳۶۰}{\pi} \Rightarrow \sin 2\,\text{rad} = \sin \frac{۳۶۰}{\pi}$

6 rad $\quad \frac{D}{۱۸۰} = \frac{۶}{\pi} \Rightarrow D\pi = ۱۰۸۰ \Rightarrow D = \frac{۱۰۸۰}{\pi} \Rightarrow \sin 6\,\text{rad} = \sin \frac{۱۰۸۰}{\pi}$

in ۱ $\sin ۵\,\text{rad} < \sin ۴\,\text{rad} < \sin ۶\,\text{rad} < \sin ۳\,\text{rad} < \sin ۱\,\text{rad} < \sin ۲\,\text{rad}$

۵

$\hat{A} = \frac{۲\pi}{n} \Rightarrow \frac{\hat{A}}{۲} = \frac{\pi}{n}$



$\sin \frac{\hat{A}}{۲} = \frac{c}{R} = \frac{\sin \pi}{n} \Rightarrow cn = \sin \pi \times R$

$\Rightarrow c = R \times \sin\left(\frac{\pi}{n}\right) \Rightarrow \left\{ \text{طول ضلع} \Rightarrow ۲c = ۲R \times \sin\left(\frac{\pi}{n}\right) \right\}$

۶

$x - ۲y = ۵$, $۲x + y = ۵ \Rightarrow \begin{cases} -\cancel{۲x} + ۴y = -۱۰ \\ \cancel{۲x} + y = ۵ \end{cases}$ $\quad ۵y = -۵ \Rightarrow y = -۱$

$x + ۲ = ۵ \Rightarrow x = ۳ \Rightarrow a = ۳$

۷

$y - x = ۳$, $x + ۲y = ۶ \Rightarrow \begin{cases} ۳y - \cancel{۳x} = ۹ \\ ۲y + \cancel{۳x} = ۶ \end{cases} \Rightarrow ۵y = ۱۵ \Rightarrow y = ۳$ $\qquad \boxed{(x, y) = (۰, ۳)}$

$۳ - x = ۳ \Rightarrow x = ۰$

$-۲y = ۱ \Rightarrow -۲y = -x + ۱ \Rightarrow y = \frac{۱}{۲}x - \frac{۱}{۲} \Rightarrow m = \frac{۱}{۲} \Rightarrow \boxed{m' = -۲}$

$(y - y_۱) = m(x - x_۱) \Rightarrow (y - ۳) = -۲(x - ۰) \Rightarrow y - ۳ = -۲x \Rightarrow \boxed{y = -۲x + ۳}$ ✓

$$\begin{cases} x+\cancel{۷y}=۳ \\ x-\cancel{۷y}=-۱ \end{cases} \quad ۲x=۲ \Rightarrow x=۱ \qquad \boxed{(x،y)=(۱،\frac{۲}{۷})}$$

$$۱+۷y=۳ \Rightarrow ۷y=۲ \Rightarrow y=\frac{۲}{۷}$$

$$۴x+y=۵ \Rightarrow y=-۴x+۵ \Rightarrow m'=m=-۴$$

$$(y-y_۱)=m(x-x_۱) \Rightarrow (y-\frac{۲}{۷})=-۴(x-۱) \Rightarrow y-\frac{۲}{۷}=-۴x+۴$$

$$\boxed{y=-۴x+\frac{۳۰}{۷}}$$

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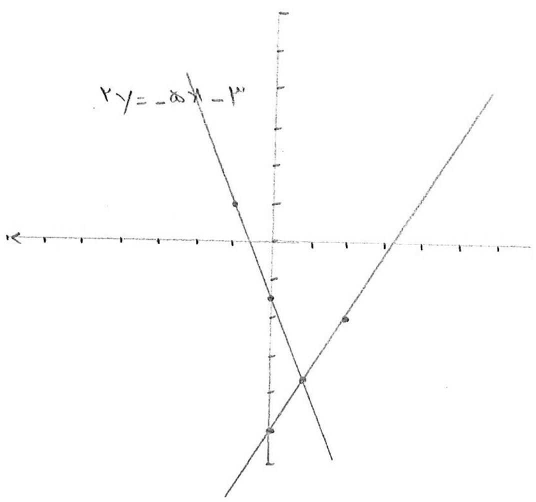
(۹)

$$y=\frac{۳}{۲}x-۵ \Rightarrow \begin{cases} ۲y=۳x-۱۰ \\ ۲y=-۵x-۳ \end{cases} \Leftarrow y=\frac{-۵}{۲}x-\frac{۳}{۲}$$

$$\Rightarrow ۳x-۱۰=-۵x-۳ \Rightarrow ۸x=۷ \Rightarrow \boxed{x=\frac{۷}{۸}}$$

$$۲y=\frac{۲۱}{۸}-\frac{۸۰}{۸}=\frac{-۵۹}{۸} \Rightarrow \boxed{y=\frac{-۵۹}{۱۶}}$$



$y=\frac{۳}{۲}x-۵$

x	۰	۲
y	-۵	-۲

$۲y=-۵x-۳$

x	۰	-۱
y	-۱/۵	۱

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(۱۰)

$A(۲،۳)$

$$mx+۳y=-۳n+۲ \Rightarrow ۲m+۹=-۳n+۲ \Rightarrow \begin{cases} ۲m+۳n=-۷ \\ m+۲n=-۴ \end{cases} \Rightarrow \begin{cases} \cancel{۲m}+۳n=-۷ \\ \cancel{-۲m}-۴n=۸ \end{cases}$$

$$nx+my=۲m-۴ \Rightarrow ۲n+۳m=۲m-۴$$

$m-n$?!

$$\Rightarrow -n=۱ \Rightarrow n=-۱ \qquad ۲m-۳=-۷ \Rightarrow ۲m=-۴ \Rightarrow m=-۲$$

$$\boxed{m-n=-۲+۱=-۱}$$