[redacted]

[redacted] The $ABCD$ constants of a three-phase transmission line are

4-1

$$\begin{aligned} A = D &= 0.936 + j0.016 = 0.936\angle 0.98^\circ \\ B &= 33.5 + j138 = 142\angle 76.4^\circ\ \Omega \\ C &= (-5.18 + j914) \times 10^{-6}\ \text{S} \end{aligned}$$

The load at the receiving end is 50 MW at 220 kV with a power factor of 0.9 lagging. Find the magnitude of the sending-end voltage and the voltage regulation. Assume the magnitude of the sending-end voltage remains constant.

Solution:

$$\begin{aligned} I_R &= \frac{50,000}{\sqrt{3} \times 220 \times 0.9}\angle -25.84^\circ = 145.8\angle -25.84^\circ\ \text{A} \\ V_R &= \frac{220,000}{\sqrt{3}} = 127,000\angle 0^\circ\ \text{V} \\ V_S &= 0.936\angle 0.98^\circ \times 127,000\angle 0^\circ + 142\angle 76.4^\circ \times 145.8\angle -25.84^\circ \\ &= 118,855 + j2033 + 13,153 + j15,990 = 133.23\angle 7.77^\circ\ \text{kV} \end{aligned}$$

With line-to-line sending-end voltage $|V_S| = \sqrt{3} \times 133.23 = 230.8$ kV,

$$\begin{aligned} |V_{R,NL}| &= \frac{230.8}{0.936} = 246.5\ \text{kV} \\ \%\ \text{Reg.} &= \frac{246.5 - 220}{220} \times 100 = 12.0\,\% \end{aligned}$$

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[redacted] A 200-mi transmission line has the following parameters at 60 Hz

4-2

$$\text{resistance } r = 0.21 \ \ \Omega/\text{mi per phase}$$
$$\text{series reactance } x = 0.78 \ \ \Omega/\text{mi per phase}$$
$$\text{shunt susceptance } b = 5.42 \times 10^{-6} \ \ \text{S/mi per phase}$$

(*a*) Determine the attenuation constant α, wavelength λ and the velocity of propagation of the line at 60 Hz.

(*b*) If the line is open circuited at the receiving end and the receiving-end voltage is maintained at 100 kV line-to-line use Eqs. (6.26) and (6.27) to determine the incident and reflected components of the sending-end voltage and current.

(*c*) Hence determine the sending-end voltage and current of the line.

Solution:

(*a*)

$$r = 0.21\ \Omega/\text{mi} \qquad x_l = 0.78\ \Omega/\text{mi}$$
$$z = (0.21 + j0.78)\ \Omega/\text{mi} = 0.808\underline{/77.31^\circ}\ \Omega/\text{mi}$$
$$y = 5.42 \times 10^{-6}\underline{/77.31^\circ}\ \text{S/mi}$$
$$\gamma = \sqrt{zy} = 2.092 \times 10^{-3}\underline{/82.47^\circ}\ \text{mi}^{-1}$$
$$= \alpha + j\beta = (2.744 \times 10^{-4} + j2.074 \times 10^{-3})\ \text{mi}^{-1}$$
$$\text{Attenuation-constant } \alpha = 2.744 \times 10^{-4}\ \text{nepers/mi}$$
$$\text{Wavelength } \lambda = \frac{2\pi}{\beta} = \frac{2\pi \times 10^3}{2.074}\ \text{mi} = 3030\ \text{mi}$$
$$\text{Velocity of propagation } \lambda f = \frac{2\pi f}{\beta} = \frac{120\pi \times 10^3}{2.074}\ \text{mi/s} = 181770\ \text{mi/s}$$

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[redacted] A 60 Hz three-phase transmission is 175 mi long. It has a total series impedance
4-3 of $35 + j140\ \Omega$ and a shunt admittance of $930 \times 10^{-6}\underline{/90^\circ}$ S. It delivers 40 MW at 220 kV, with 90% power factor lagging. Find the voltage at the sending end by (*a*) the short-line approximation, (*b*) the nomial-π approximation and (*c*) the long-line equation.

Solution:

$$\begin{aligned} l &= 175 \text{ mi} \\ Z &= 35 + j40 = 144.3\underline{/75.96^\circ}\ \Omega \\ Y &= 930 \times 10^{-6} \text{ S} \\ I_R &= \frac{40,000}{\sqrt{3} \times 220 \times 0.9} = 116.6\underline{/-25.84^\circ} \text{ A} \end{aligned}$$

(*a*) Using the short-line approximation,

$$\begin{aligned} V_S &= 127,017 + 116.6\underline{/-25.84^\circ} \times 144.3\underline{/75.96^\circ} = 127,017 + 10,788 + j12,912 \\ &= 138,408\underline{/5.35^\circ} \text{ V} \\ |V_S| &= \sqrt{3} \times 138,408 = 239.73 \text{ kV} \end{aligned}$$

(*b*) Using the nominal-π approximation and Eq. (6.5),

$$\begin{aligned} V_S &= 127,017\left(\frac{0.1342}{2}\underline{/165.96^\circ} + 1\right) + 144.3\underline{/75.96^\circ} \times 116.6\underline{/-25.84^\circ} \\ &= 127,017\,(0.935 + j0.0163) + 10,788 + j12,912 = 129,549 + j14,982 \\ &= 130,412\underline{/6.6^\circ} \\ |V_S| &= \sqrt{3} \times 130,412 = 225.88 \text{ kV} \end{aligned}$$

(*c*) Using the long-line equation,

$$\begin{aligned} Z_c &= \left(\frac{144.3\underline{/75.96^\circ}}{930 \times 10^{-6}\underline{/90^\circ}}\right)^{\frac{1}{2}} = 394\underline{/-7.02^\circ} \\ \gamma l &= \sqrt{144.3 \times 930 \times 10^{-6}\underline{/165.96^\circ}} = 0.3663\underline{/83.0^\circ} = 0.0448 + j0.364 \\ \epsilon^{0.0448}\epsilon^{j0.364} &= 1.0458\underline{/20.86^\circ} = 0.9773 + j0.3724 \end{aligned}$$

[redacted]

$$
\begin{aligned}
\epsilon^{-0.0448}\epsilon^{-j0.364} &= 0.9562\underline{/-20.86^\circ} = 0.8935 - j0.3405 \\
\cosh\gamma l &= (0.9773 + j0.3724 + 0.8935 - j0.3405)/2 = 0.9354 + j0.0160 \\
\sinh\gamma l &= (0.9773 + j0.3724 - 0.8935 + j0.3405)/2 = 0.0419 + j0.3565 \\
V_S &= 127,017\,(0.9354 + j0.0160) + 116.6\underline{/-25.84^\circ} \times 394\underline{/-7.02^\circ}\,(0.0419 + j0.3565) \\
&= 118,812 + j2,032 + 10,563 + j12,715 = 129,315 + j14,747 \\
&= 130,153\underline{/6.5^\circ}\ \text{V} \\
|V_S| &= \sqrt{3} \times 130,153 = 225.4\ \text{kV}
\end{aligned}
$$

[redacted]

$$V_S = 130.15\ \text{kV} \qquad V_R = 127.02\ \text{kV}$$

For $I_R = 0$, $V_S = V_R \cosh\gamma l$,

$$
\begin{aligned}
|V_{R,\,NL}| &= \frac{130.15}{|0.9354 + j0.0161|} = 139.12\ \text{kV} \\
\%\ \text{Reg.} &= \frac{139.12 - 127.02}{127.02} \times 100 = 9.53\,\%
\end{aligned}
$$

[redacted] A three-phase 60-Hz transmission line is 250 mi long. The voltage at the sending
4-4 end is 220 kV. The parameters of the line are $R = 0.2\ \Omega/\text{mi}$, $X = 0.8\ \Omega/\text{mi}$ and $Y = 5.3\ \mu\text{S/mi}$. Find the sending-end current when there is no load on the line.

Solution:

$$
\begin{aligned}
Z &= (0.2 + j0.8) \times 250 = 206.1\underline{/75.96^\circ} \\
Y &= 250 \times 5.3 \times 10^{-6} = 1.325 \times 10^{-3}\underline{/90^\circ} \\
\gamma l &= \sqrt{ZY} = \sqrt{206.1 \times 1.325 \times 10^{-3}\underline{/165.96^\circ}} = 0.5226\underline{/82.98^\circ} \\
&= 0.0639 + j0.5187 \\
Z_c &= \sqrt{Z/Y} = \sqrt{\frac{206.1\underline{/75.96^\circ}}{1.325 \times 10^{-3}\underline{/90^\circ}}} = 394\underline{/-7.02^\circ}\ \Omega
\end{aligned}
$$

By Eq. (6.39) for $I_R = 0$,

$$
\begin{aligned}
I_S &= (V_S/Z_c)\,\frac{\sinh\gamma l}{\cosh\gamma l} \\
\beta l &= 0.5187\ \text{rad} = 29.72^\circ \\
\epsilon^{\alpha l}\epsilon^{j\beta l} &= 0.9258 + j0.5285 \\
\epsilon^{-\alpha l}\epsilon^{-j\beta l} &= 0.8147 - j0.4651 \\
\cosh\gamma l &= \frac{1}{2}(0.9258 + 0.8147 + j0.5285 - j0.4651) = 0.8709\underline{/2.086^\circ}
\end{aligned}
$$

[redacted]

$$\sinh \gamma l = \frac{1}{2}\left[0.9258 - 0.8147 + j\,(0.5285 + 0.4651)\right] = 0.4999\underline{\angle 83.61^\circ}$$

$$I_S = \frac{220,000/\sqrt{3}}{394\underline{\angle -7.02^\circ}} \times \frac{0.4999\underline{\angle 83.61^\circ}}{0.8709\underline{\angle 2.086^\circ}} = 185.0\underline{\angle 88.54^\circ}\ \mathrm{A}$$

[redacted]

[redacted]