فصل ۱ – بخش ۳،۴ و ۵

- بخش ۳ – تعریف تنش
- بخش ۴ – تنش نرمال میانگین
- بخش ۵ – تنش برشی میانگین

Figure: 01-01-COC
The bolts used for the connections of this steel framework are subjected to stress.
In this chapter we will discuss how engineers design these connections and their fasteners.

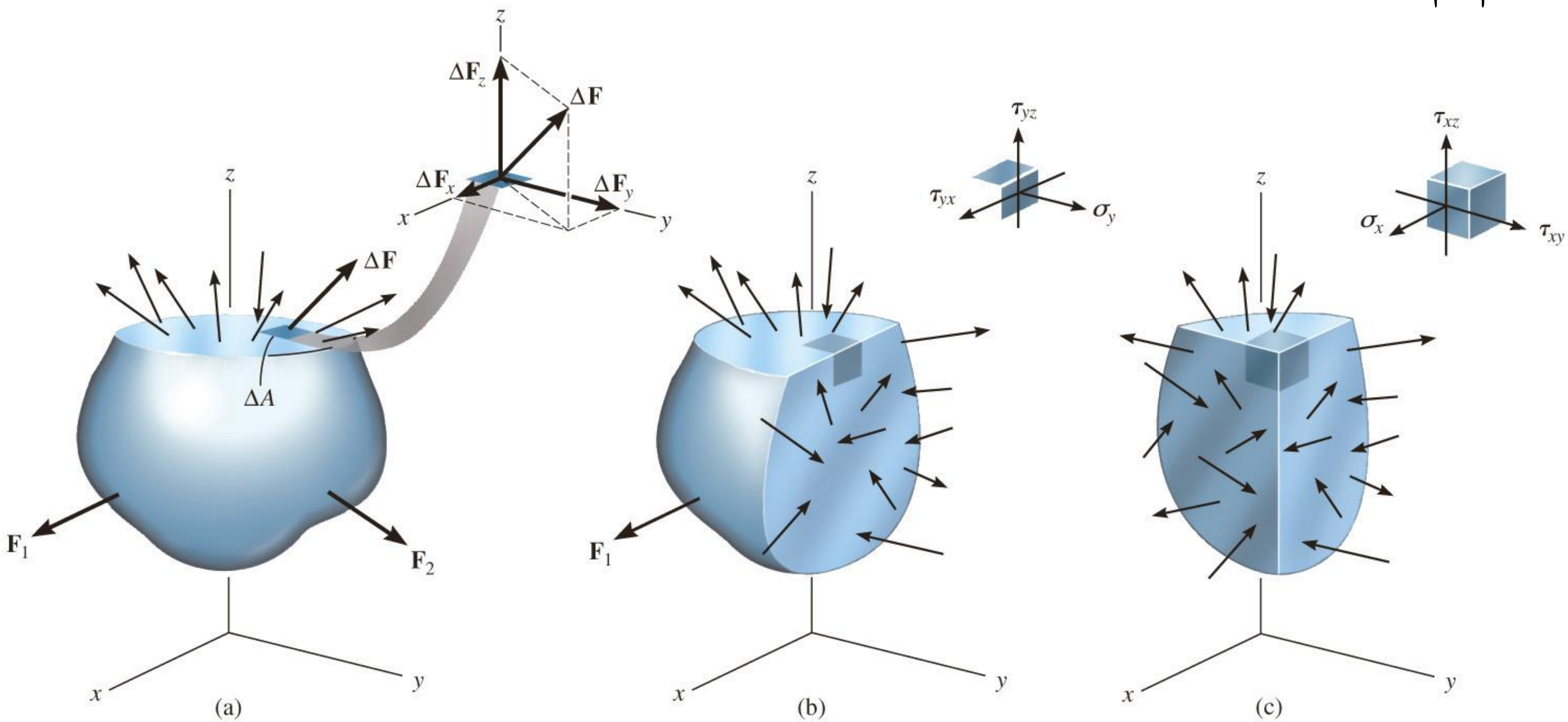
فصل ۱ – بخش ۴،۳ و ۵

- یادآوری: مسائل مقاومت مصالح:
  - یافتن بارهای خارجی مجهول
  - یافتن بارهای داخلی مورد نیاز
  - یافتن خواص هندسی سطح های مورد نیاز

استاتیک

- بارهای داخلی تولید تنش، کرنش و تغییر شکل می کنند.

مقاومت مصالح!!

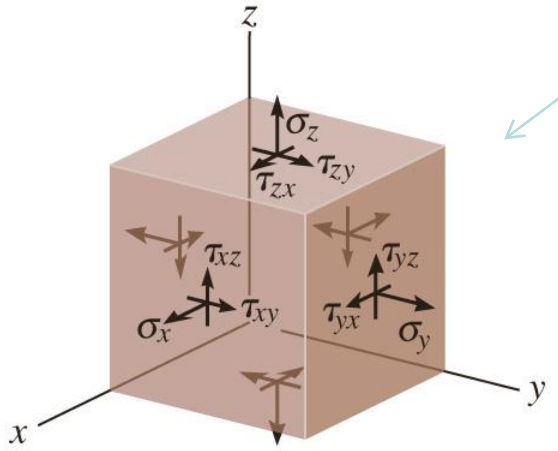


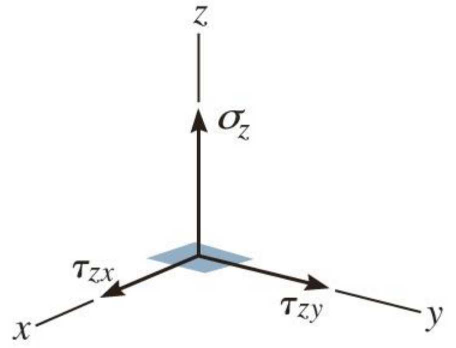
تعریف تنش:

$$\sigma_x = \lim_{\Delta A \to 0} \frac{\Delta F_x}{\Delta A}$$

- نیروی داخلی بر واحد سطح
- شدت نیروی داخلی روی یک صفحه (مساحت) مشخص

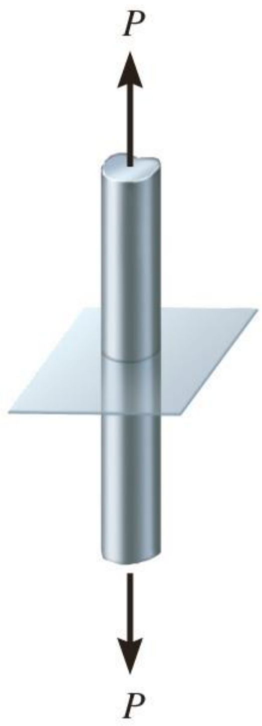
این یک المان تنش سه بعدی است که می تواند تا ۶ تنش مستقل در یک نقطه داشته باشد.





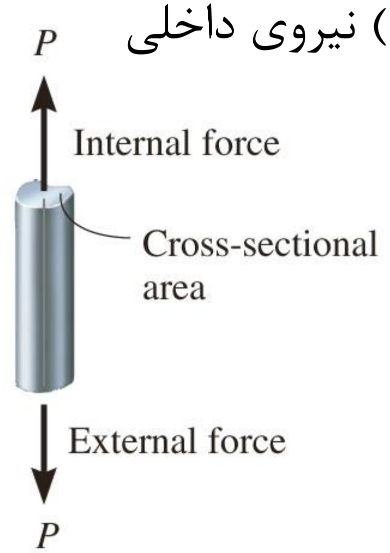
۱-۴:تنش نرمال میانگین

الف) نیروی خارجی

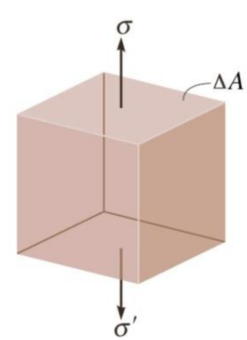


(a)

ب) نیروی داخلی



ج) تنش



۱-۴:تنش نرمال میانگین

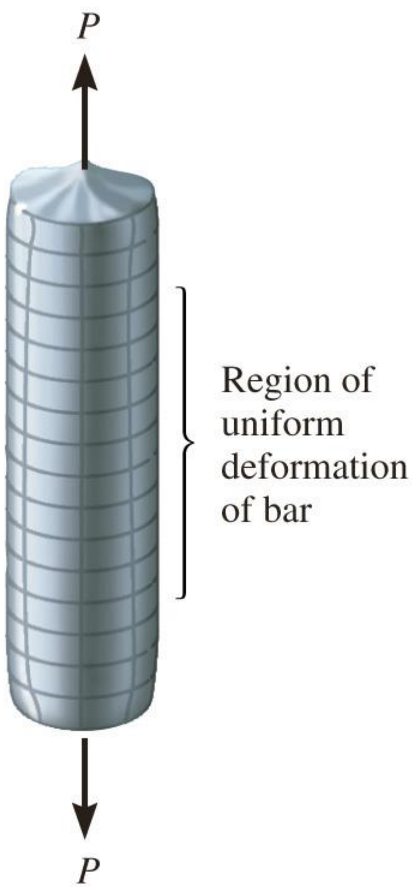
نیروی داخلی
(lbs, N, kN)

$$\sigma = \frac{P}{A}$$

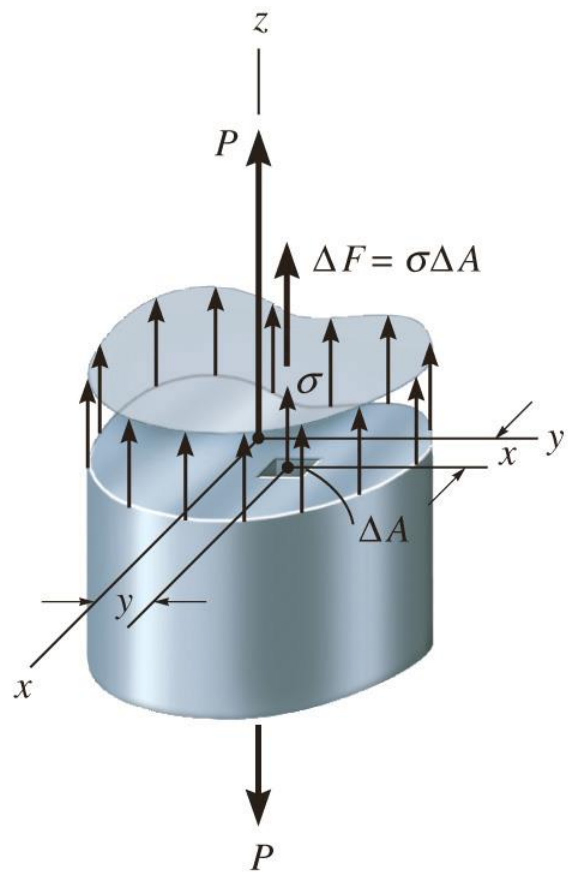
تنش نرمال
(psi, ksi, Mpa)

مساحت سطح مقطع
(in^2, mm^2, m^2)

۱-۴:تنش نرمال میانگین



(c)

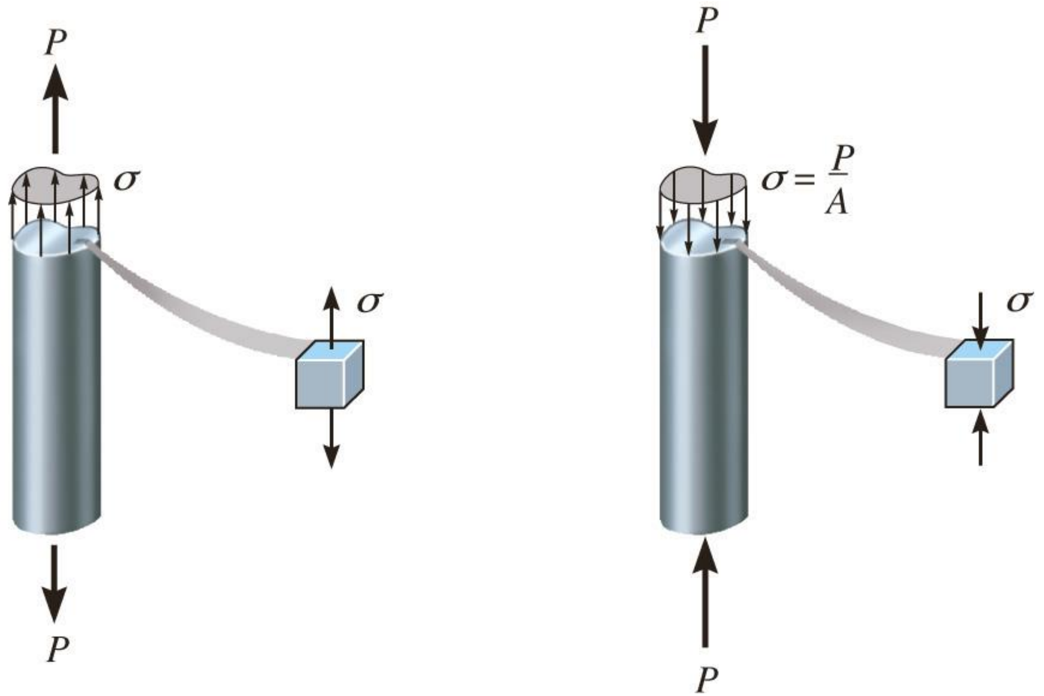


پروفیل تنش (یکنواخت روی سطح مقطع)

توجه: بر اساس تعادل استاتیک:

$$\sigma * A = F$$

۱-۴:تنش نرمال میانگین



کشش Tension

Compression فشار

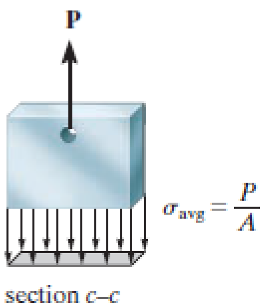
۱-۴:تنش نرمال میانگین

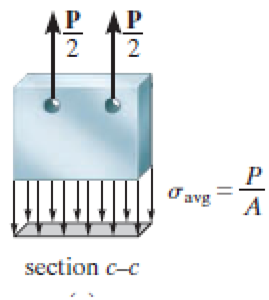
شرایط مورد نیاز برای تنش نرمال میانگین (تنش نرمال یکنواخت) $\sigma = P/A$:

۱. عضو در ابتدا مستقیم بوده و بعد از بارگذاری هم مستقیم باقی می ماند.

۲. همگن و ایزوتروپیک

۳. طبعیت از اصل St. Venant *

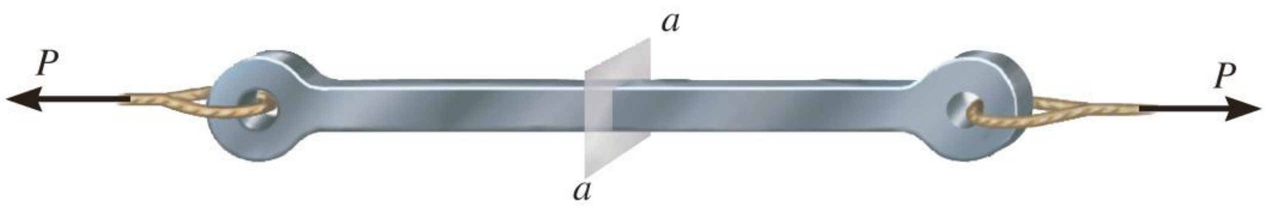




* این تئوری بیان می کند که تنش و کرنش بوجود آمده در نقاطی از جسم که به حد کافی از ناحیه بارگذاری دور هستند، برابر است با تنش و کرنشی که هر بارگذاری معادل استاتیکی دیگر در همان ناحیه ایجاد میکند.

۱-۴:تنش نرمال میانگین

مثال تنش نرمال میانگین



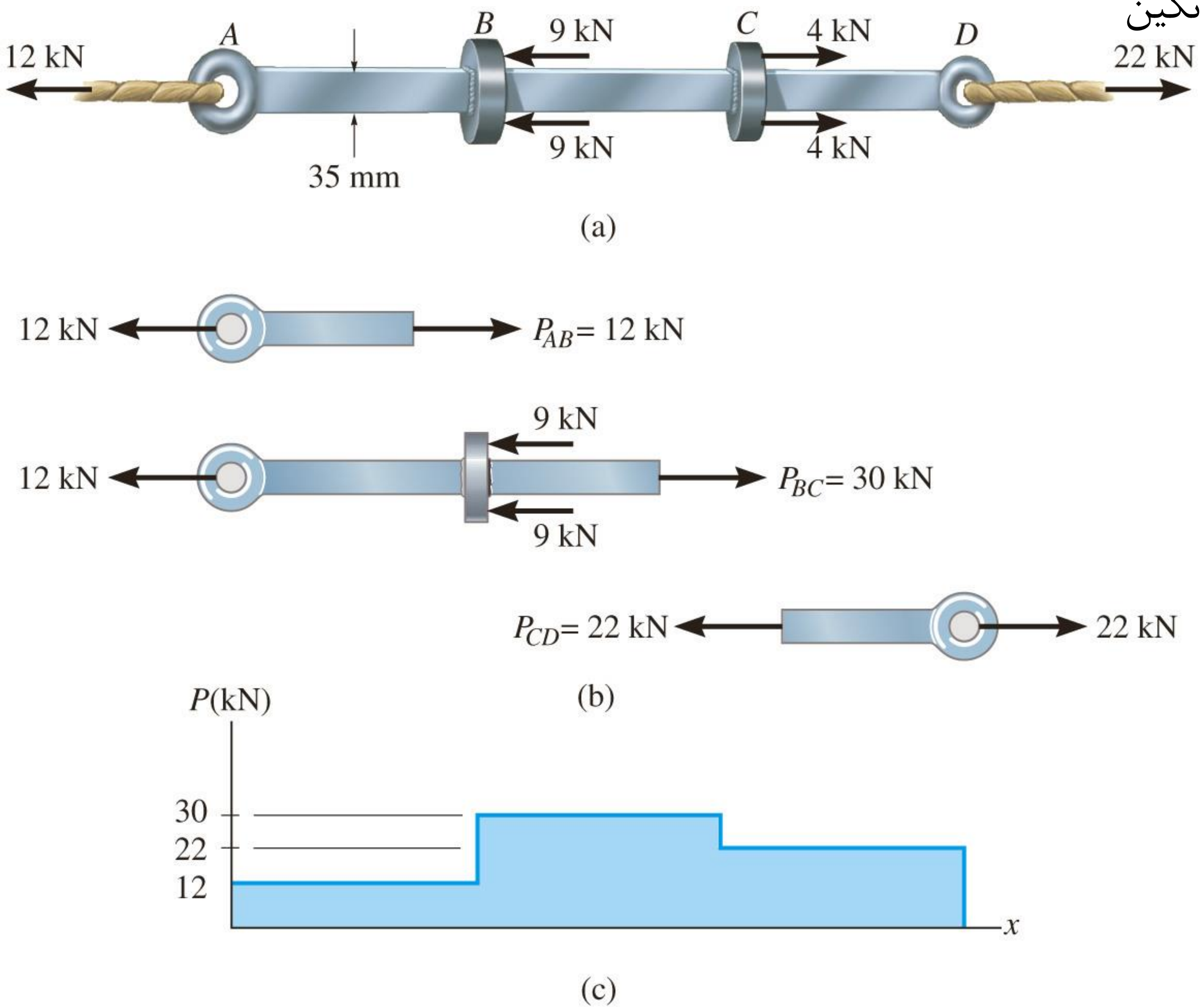
(a)

اگر $P = 20K\ lbs$

و $A = 2\ in^2$

$\sigma = ?$

مثال۲- تنش نرمال میانگین

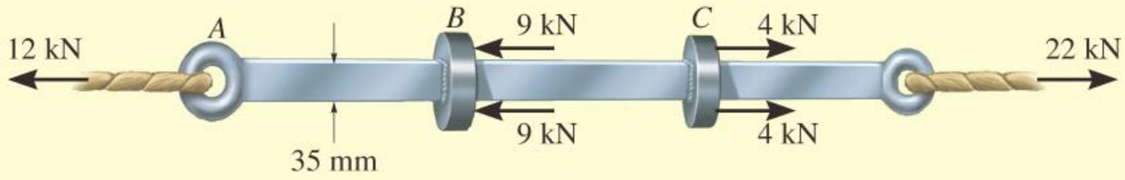


حل استاتیکی انجام شد؛

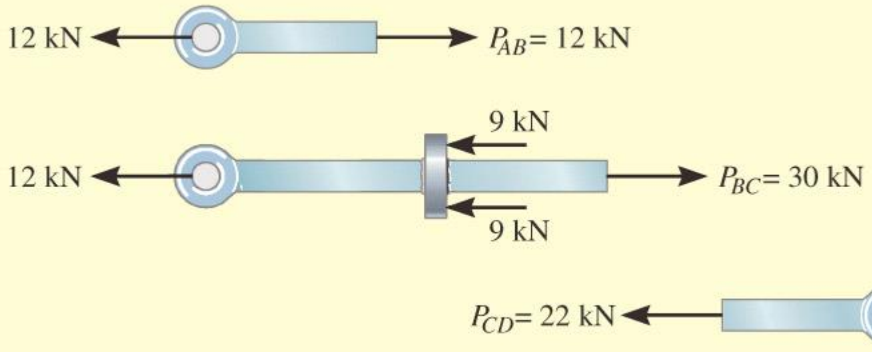
اکنون محاسبه کنید تنش در هر مقطع چقدر است؟؟

EXAMPLE 1.6

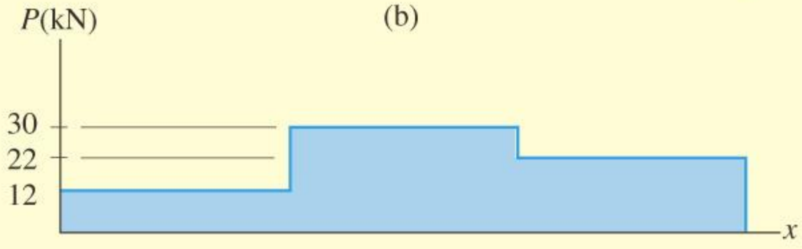
The bar in Fig. 1–16*a* has a constant width of 35 mm and a thickness of 10 mm. Determine the maximum average normal stress in the bar when it is subjected to the loading shown.



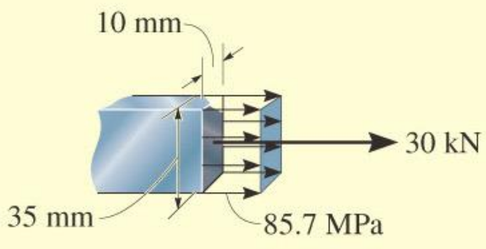
(a)



(b)



(c)



(d)

Fig. 1–16

Solution

Internal Loading. By inspection, the internal axial forces in regions *AB*, *BC*, and *CD* are all constant yet have different magnitudes. Using the method of sections, these loadings are determined in Fig. 1–16*b*; and the normal force diagram which represents these results graphically is shown in Fig. 1–16*c*. By inspection, the largest loading is in region *BC*, where $P_{BC} = 30$ kN. Since the cross-sectional area of the bar is *constant*, the largest average normal stress also occurs within this region of the bar.

Average Normal Stress. Applying Eq. 1–6, we have

$$\sigma_{BC} = \frac{P_{BC}}{A} = \frac{30(10^3)\text{N}}{(0.035\text{ m})(0.010\text{ m})} = 85.7\text{ MPa} \qquad \textit{Ans.}$$

The stress distribution acting on an arbitrary cross section of the bar within region *BC* is shown in Fig. 1–16*d*. Graphically the *volume* (or "block") represented by this distribution of stress is equivalent to the load of 30 kN; that is, 30 kN = (85.7 MPa)(35 mm)(10 mm).

EXAMPLE 1.7

The 80-kg lamp is supported by two rods *AB* and *BC* as shown in Fig. 1–17*a*. If *AB* has a diameter of 10 mm and *BC* has a diameter of 8 mm, determine the average normal stress in each rod.

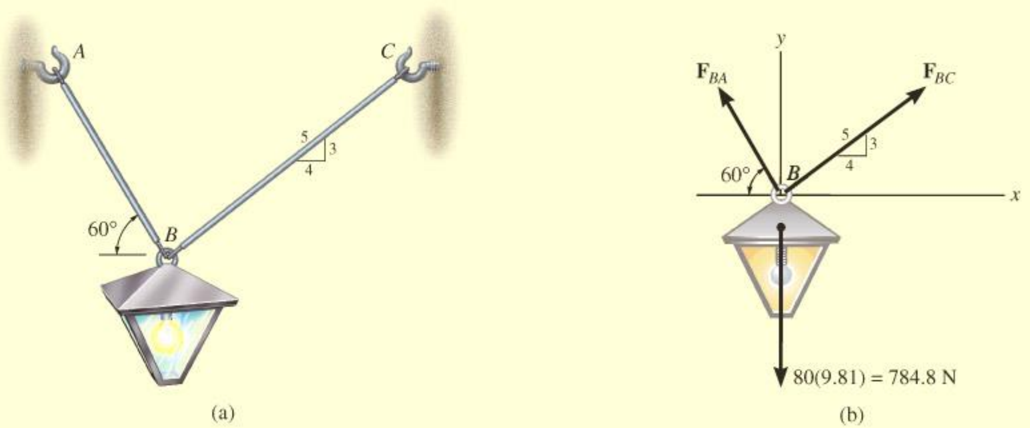


Fig. 1–17

Solution

Internal Loading. We must first determine the axial force in each rod. A free-body diagram of the lamp is shown in Fig. 1–17*b*. Applying the equations of force equilibrium yields

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{BC}\left(\tfrac{4}{5}\right) - F_{BA}\cos 60^\circ = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{BC}\left(\tfrac{3}{5}\right) + F_{BA}\sin 60^\circ - 784.8\text{ N} = 0$$

$$F_{BC} = 395.2\text{ N}, \qquad F_{BA} = 632.4\text{ N}$$

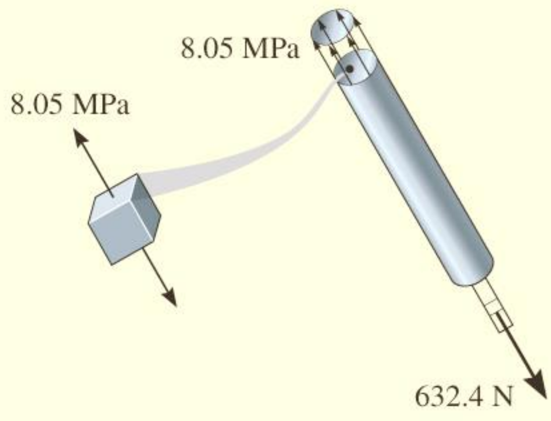
By Newton's third law of action, equal but opposite reaction, these forces subject the rods to tension throughout their length.

Average Normal Stress. Applying Eq. 1–6, we have

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{395.2\text{ N}}{\pi(0.004\text{ m})^2} = 7.86\text{ MPa} \qquad \textit{Ans.}$$

$$\sigma_{BA} = \frac{F_{BA}}{A_{BA}} = \frac{632.4\text{ N}}{\pi(0.005\text{ m})^2} = 8.05\text{ MPa} \qquad \textit{Ans.}$$

The average normal stress distribution acting over a cross section of rod AB is shown in Fig. 1–17*c*, and at a point on this cross section, an element of material is stressed as shown in Fig. 1–17*d*.



(d) (c)

EXAMPLE 1.8

The casting shown in Fig. 1–18*a* is made of steel having a specific weight of $\gamma_{st} = 490 \text{ lb/ft}^3$. Determine the average compressive stress acting at points *A* and *B*.

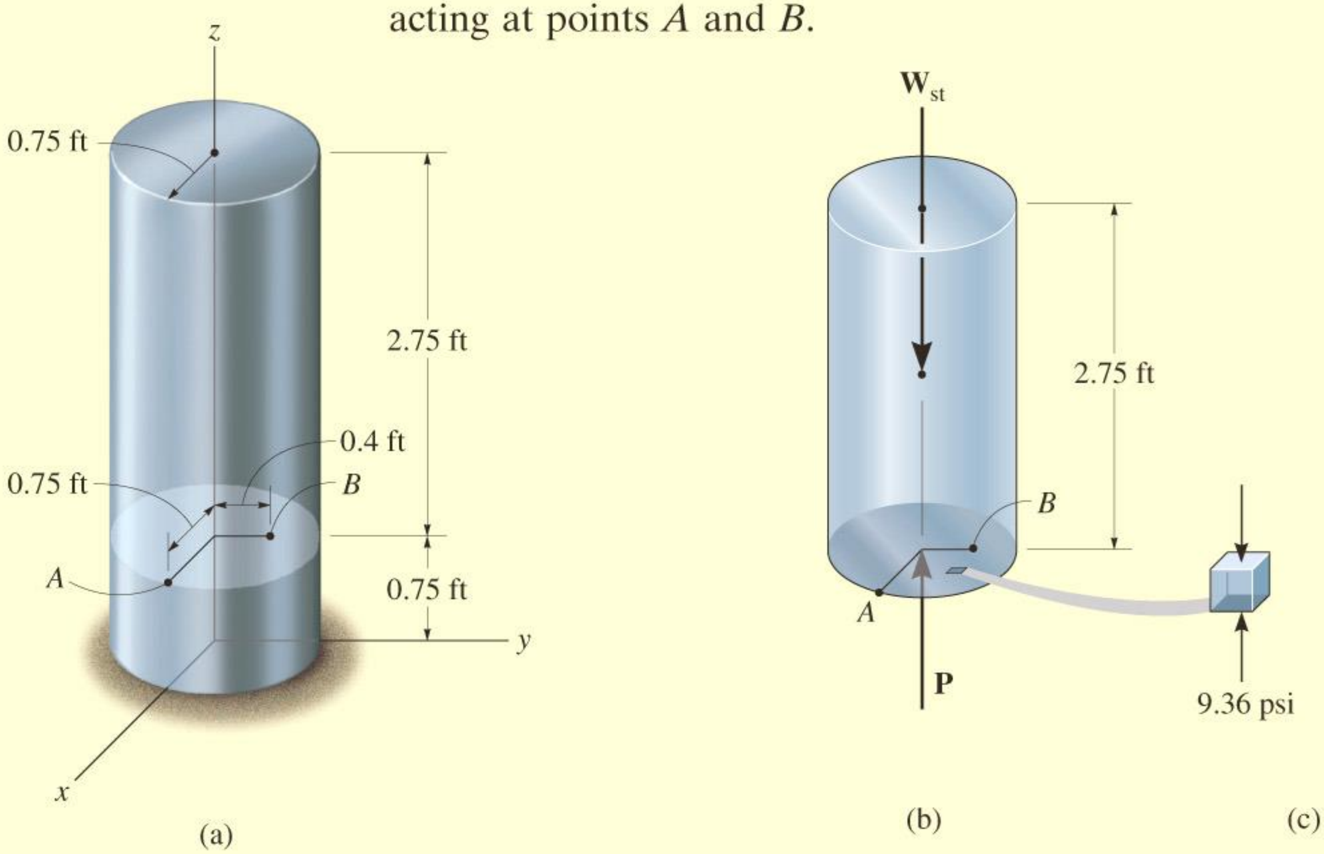


Fig. 1–18

Solution

Internal Loading. A free-body diagram of the top segment of the casting where the section passes through points A and B is shown in Fig. 1–18*b*. The weight of this segment is determined from $W_{st} = \gamma_{st} V_{st}$. Thus the internal axial force P at the section is

$$+\uparrow \Sigma F_z = 0; \qquad P - W_{st} = 0$$

$$P - (490\ \text{lb/ft}^3)(2.75\ \text{ft})\pi(0.75\ \text{ft})^2 = 0$$

$$P = 2381\ \text{lb}$$

Average Compressive Stress. The cross-sectional area at the section is $A = \pi(0.75\ \text{ft})^2$, and so the average compressive stress becomes

$$\sigma = \frac{P}{A} = \frac{2381\ \text{lb}}{\pi(0.75\ \text{ft})^2}$$

$$= 1347.5\ \text{lb/ft}^2 = 1347.5\ \text{lb/ft}^2\ (1\ \text{ft}^2/144\ \text{in}^2)$$

$$= 9.36\ \text{psi} \qquad \textit{Ans.}$$

The stress shown on the volume element of material in Fig. 1–18*c* is representative of the conditions at either point A or B. Notice that this stress acts *upward* on the bottom or shaded face of the element since this face forms part of the bottom surface area of the cut section, and on this surface, the resultant internal force **P** is pushing upward.

EXAMPLE 1.9

Member AC shown in Fig. 1–19a is subjected to a vertical force of 3 kN. Determine the position x of this force so that the average compressive stress at the smooth support C is equal to the average tensile stress in the tie rod AB. The rod has a cross-sectional area of 400 mm^2 and the contact area at C is 650 mm^2.

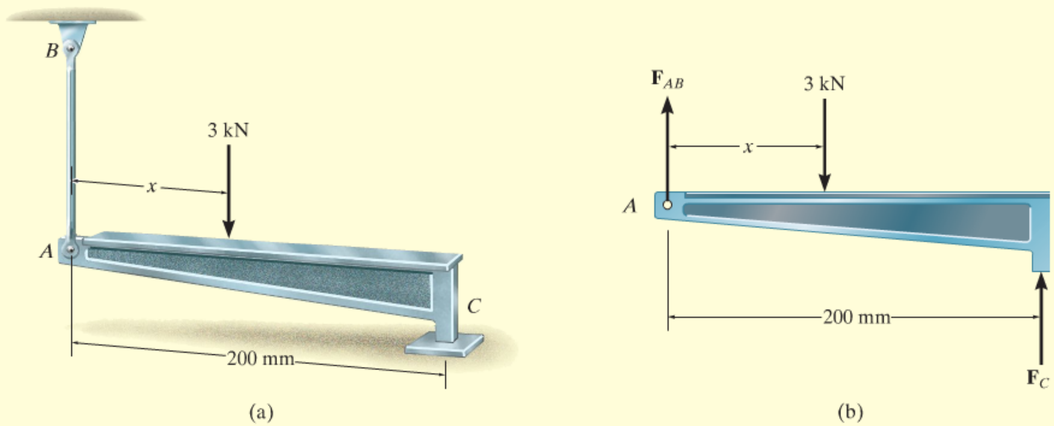


Fig. 1–19

SOLUTION

Internal Loading. The forces at A and C can be related by considering the free-body diagram for member AC, Fig. 1–19b. There are three unknowns, namely, F_{AB}, F_C, and x. To solve this problem we will work in units of newtons and millimeters.

$$+\uparrow \Sigma F_y = 0; \qquad F_{AB} + F_C - 3000 \text{ N} = 0 \qquad (1)$$

$$\circlearrowright + \Sigma M_A = 0; \qquad -3000 \text{ N}(x) + F_C(200 \text{ mm}) = 0 \qquad (2)$$

Average Normal Stress. A necessary third equation can be written that requires the tensile stress in the bar AB and the compressive stress at C to be equivalent, i.e.,

$$\sigma = \frac{F_{AB}}{400 \text{ mm}^2} = \frac{F_C}{650 \text{ mm}^2}$$

$$F_C = 1.625 F_{AB}$$

Substituting this into Eq. 1, solving for F_{AB}, then solving for F_C, we obtain

$$F_{AB} = 1143 \text{ N}$$

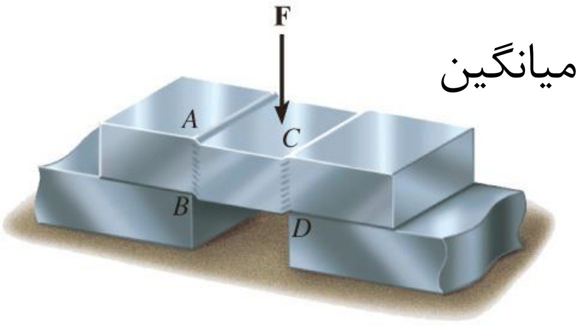
$$F_C = 1857 \text{ N}$$

The position of the applied load is determined from Eq. 2,

$$x = 124 \text{ mm} \qquad \textit{Ans.}$$

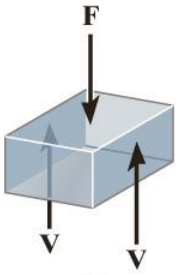
NOTE: $0 < x < 200$ mm, as required.

۱-۵: تنش برشی میانگین



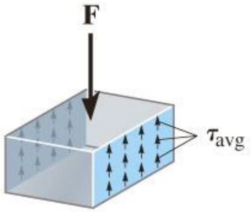
(a)

بار داخلی، V



(b)

تنش، τ



(c)

۱–۵: تنش برشی میانگین

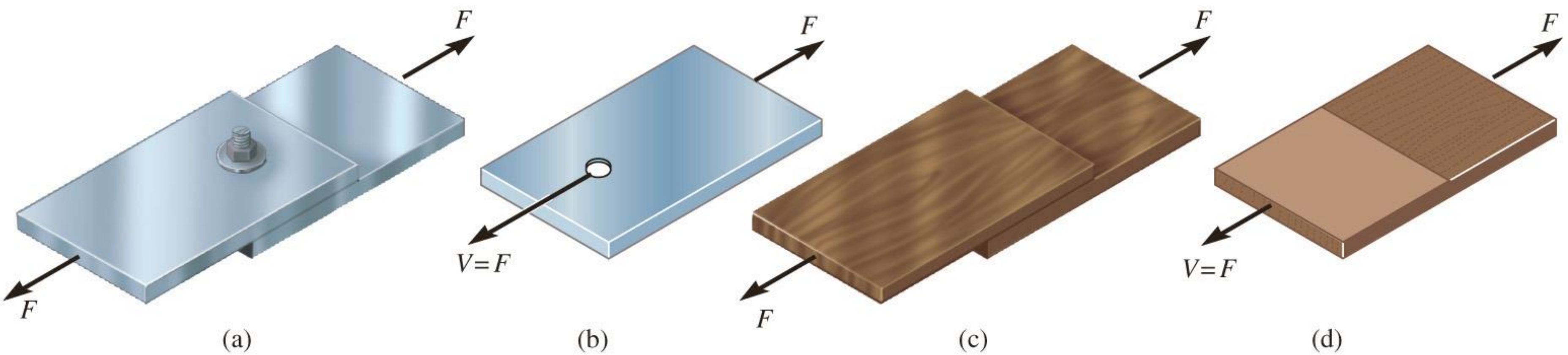
نیروی داخلی
(lbs, K, N, kN)

$$\tau = \frac{P}{A_s}$$

کل سطح مقطع تحت برش
(in^2, mm^2, m^2)

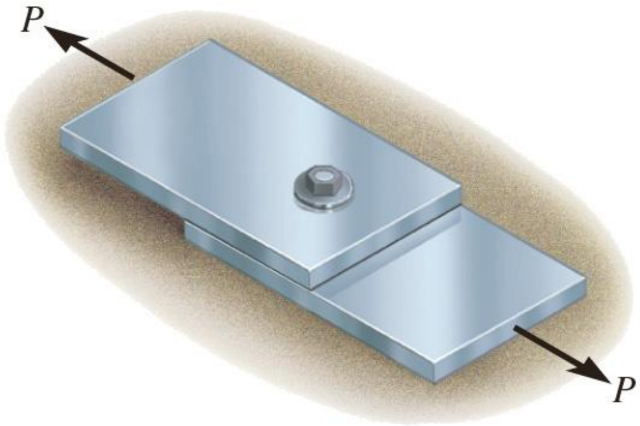
تنش برشی
(psi, ksi, Mpa)

۱-۵: تنش برشی میانگین

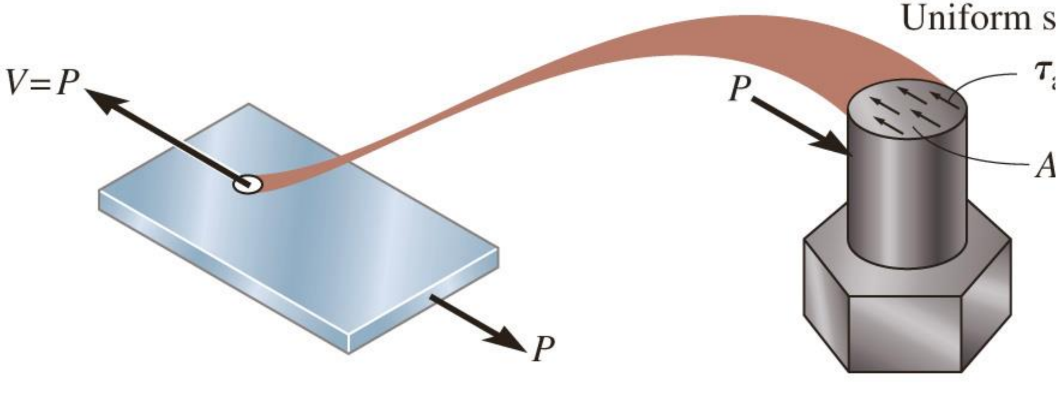


مثال- تنش برشی میانگین

اگر بار ۲۰۰۰ پوند و قطر پیچ ½ اینچ باشد، تنش برشی در پیچ را بیابید.



(a)



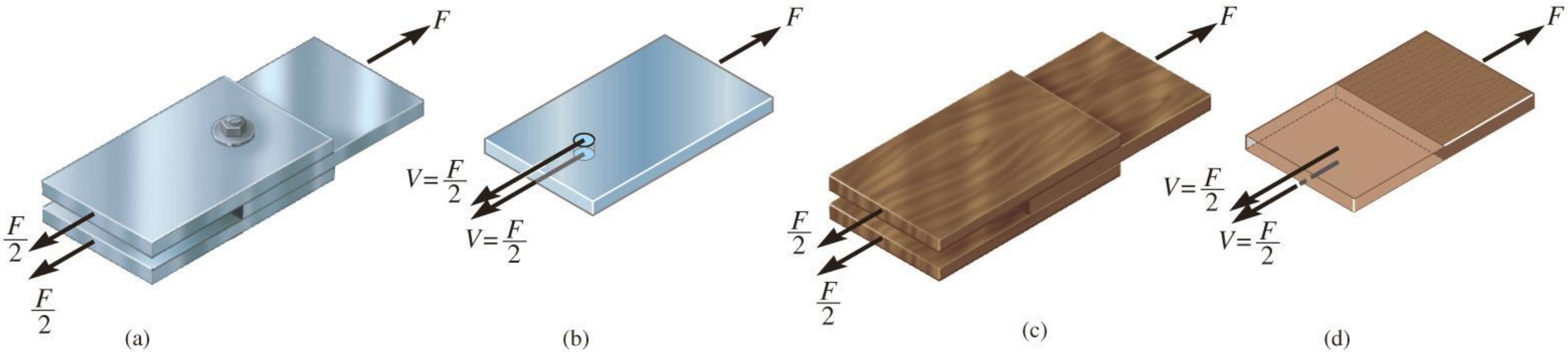
(b) (c)

۱-۵: تنش برشی میانگین

نوع تنش؟؟

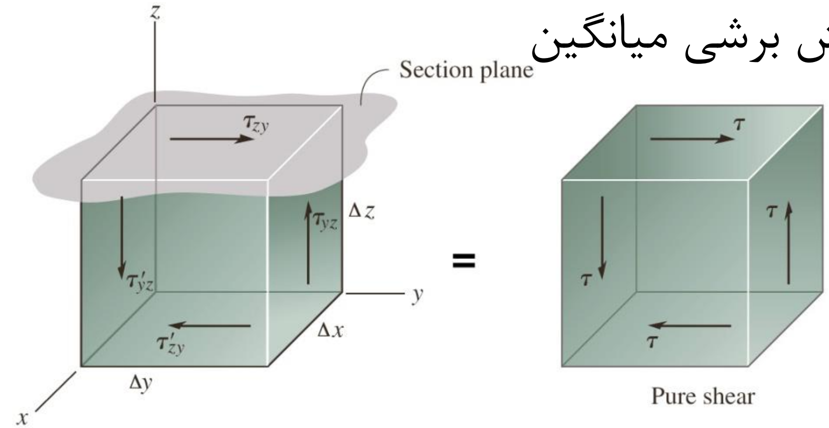
تکی یا دوبل؟؟

۱-۵: تنش برشی میانگین



برش دوبل بهتر است!! نصف تنش در هر بار وجود دارد چونکه دو سطح بار را در برش تحمل می کنند.

۱-۵: تنش برشی میانگین



Shear Stress Equilibrium. Figure 1–21*a* shows a volume element of material taken at a point located on the surface of a sectioned area which is subjected to a shear stress τ_{zy}. Force and moment equilibrium requires the shear stress acting on this face of the element to be accompanied by shear stress acting on three other faces. To show this we will first consider force equilibrium in the *y* direction. Then

$$\Sigma F_y = 0; \qquad \underbrace{\underbrace{\tau_{zy}}_{\text{stress}}\underbrace{(\Delta x\,\Delta y)}_{\text{area}}}_{\text{force}} - \tau'_{zy}\,\Delta x\,\Delta y = 0$$

$$\tau_{zy} = \tau'_{zy}$$

In a similar manner, force equilibrium in the *z* direction yields $\tau_{yz} = \tau'_{yz}$. Finally, taking moments about the *x* axis,

$$\Sigma M_x = 0; \qquad \underbrace{\underbrace{-\underbrace{\tau_{zy}}_{\text{stress}}\underbrace{(\Delta x\,\Delta y)}_{\text{area}}}_{\text{force}}\;\underbrace{\Delta z}_{\text{arm}}}_{\text{moment}} + \tau_{yz}(\Delta x\,\Delta z)\,\Delta y = 0$$

$$\tau_{zy} = \tau_{yz}$$

so that

$$\tau_{zy} = \tau'_{zy} = \tau_{yz} = \tau'_{yz} = \tau$$

In other words, ***all four shear stresses must have equal magnitude and be directed either toward or away from each other at opposite edges of the element***, Fig. 1–21*b*. This is referred to as the *complementary property of shear*, and under the conditions shown in Fig. 1–21, the material is subjected to *pure shear*.

کلید: هر ۴ تنش مقدار برابری دارند.

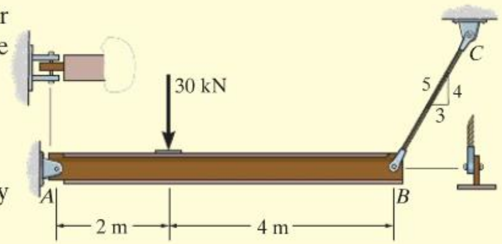
۱-۵: تنش برشی میانگین

(a)

مقطع a-a و مقطع b-b
مفهوم تنش را بیان کنید.

EXAMPLE 1.10

Determine the average shear stress in the 20-mm-diameter pin at A and the 30-mm-diameter pin at B that support the beam in Fig. 1–22a.



(a)

SOLUTION

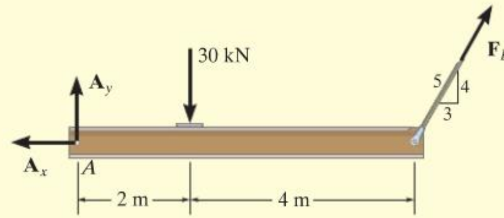
Internal Loadings. The forces on the pins can be obtained by considering the equilibrium of the beam, Fig. 1–22b.

$$\circlearrowleft + \Sigma M_A = 0; \quad F_B\left(\frac{4}{5}\right)(6\text{ m}) - 30\text{ kN}(2\text{ m}) = 0 \quad F_B = 12.5\text{ kN}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad (12.5\text{ kN})\left(\frac{3}{5}\right) - A_x = 0 \quad A_x = 7.50\text{ kN}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y + (12.5\text{ kN})\left(\frac{4}{5}\right) - 30\text{ kN} = 0$$

$$A_y = 20\text{ kN}$$

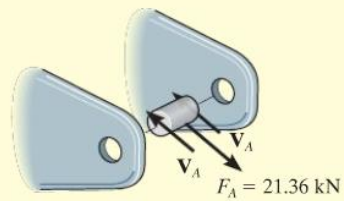


(b)

Thus, the resultant force acting on pin A is

$$F_A = \sqrt{A_x^2 + A_y^2} = \sqrt{(7.50\text{ kN})^2 + (20\text{ kN})^2} = 21.36\text{ kN}$$

The pin at A is supported by two fixed "leaves" and so the free-body diagram of the center segment of the pin shown in Fig. 1–22c has *two* shearing surfaces between the beam and each leaf. The force of the beam (21.36 kN) acting on the pin is therefore supported by shear force on each of these surfaces. This case is called *double shear*. Thus,



(c)

$$V_A = \frac{F_A}{2} = \frac{21.36\text{ kN}}{2} = 10.68\text{ kN}$$

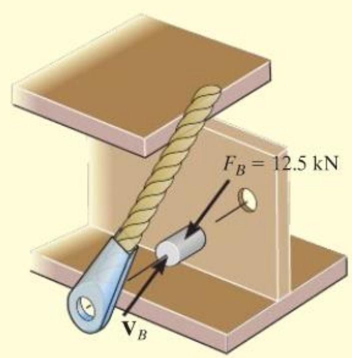
In Fig. 1–22a, note that pin B is subjected to *single shear*, which occurs on the section between the cable and beam, Fig. 1–22d. For this pin segment,

$$V_B = F_B = 12.5\text{ kN}$$

Average Shear Stress.

$$(\tau_A)_{avg} = \frac{V_A}{A_A} = \frac{10.68(10^3)\text{ N}}{\frac{\pi}{4}(0.02\text{ m})^2} = 34.0\text{ MPa} \qquad \textit{Ans.}$$

$$(\tau_B)_{avg} = \frac{V_B}{A_B} = \frac{12.5(10^3)\text{ N}}{\frac{\pi}{4}(0.03\text{ m})^2} = 17.7\text{ MPa} \qquad \textit{Ans.}$$



(d)

Fig. 1–22

EXAMPLE 1.11

If the wood joint in Fig. 1–23*a* has a width of 150 mm, determine the average shear stress developed along shear planes *a–a* and *b–b*. For each plane, represent the state of stress on an element of the material.

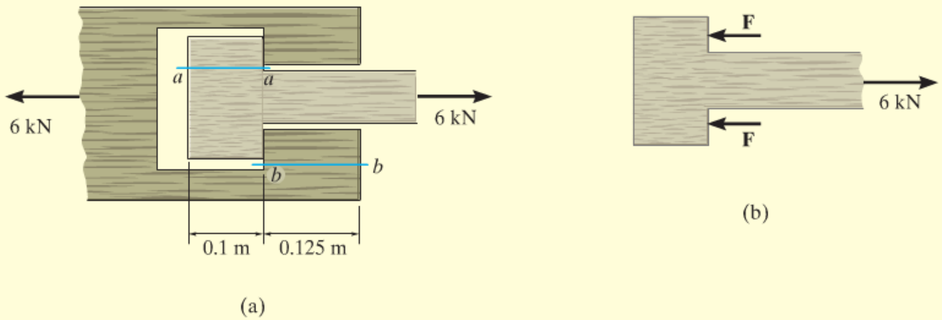


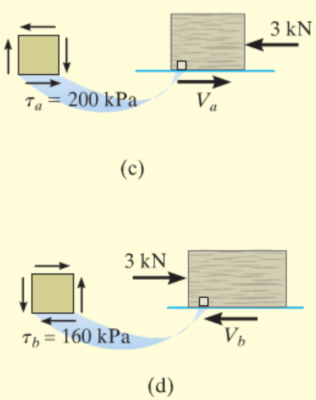
Fig. 1–23

SOLUTION

Internal Loadings. Referring to the free-body diagram of the member, Fig. 1–23*b*,

$$\xrightarrow{+} \Sigma F_x = 0; \quad 6\text{ kN} - F - F = 0 \qquad F = 3\text{ kN}$$

Now consider the equilibrium of segments cut across shear planes *a–a* and *b–b*, shown in Figs. 1–23*c* and 1–23*d*.



$$\xrightarrow{+} \Sigma F_x = 0; \quad V_a - 3\text{ kN} = 0 \qquad V_a = 3\text{ kN}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad 3\text{ kN} - V_b = 0 \qquad V_b = 3\text{ kN}$$

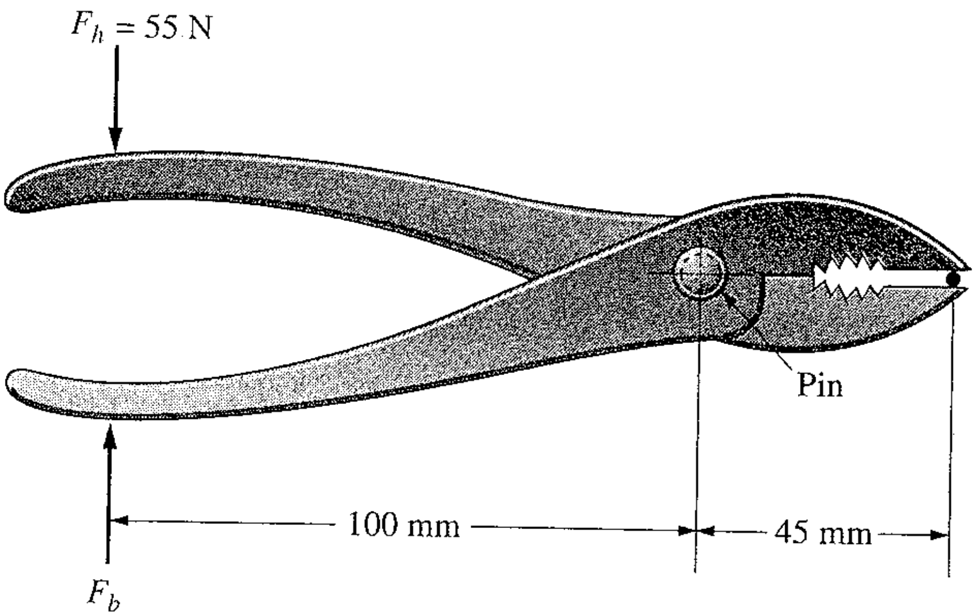
Average Shear Stress.

$$(\tau_a)_{\text{avg}} = \frac{V_a}{A_a} = \frac{3(10^3)\text{ N}}{(0.1\text{ m})(0.15\text{ m})} = 200\text{ kPa} \qquad \textit{Ans.}$$

$$(\tau_b)_{\text{avg}} = \frac{V_b}{A_b} = \frac{3(10^3)\text{ N}}{(0.125\text{ m})(0.15\text{ m})} = 160\text{ kPa} \qquad \textit{Ans.}$$

The state of stress on elements located on sections *a–a* and *b–b* is shown in Figs. 1–23*c* and 1–23*d*, respectively.

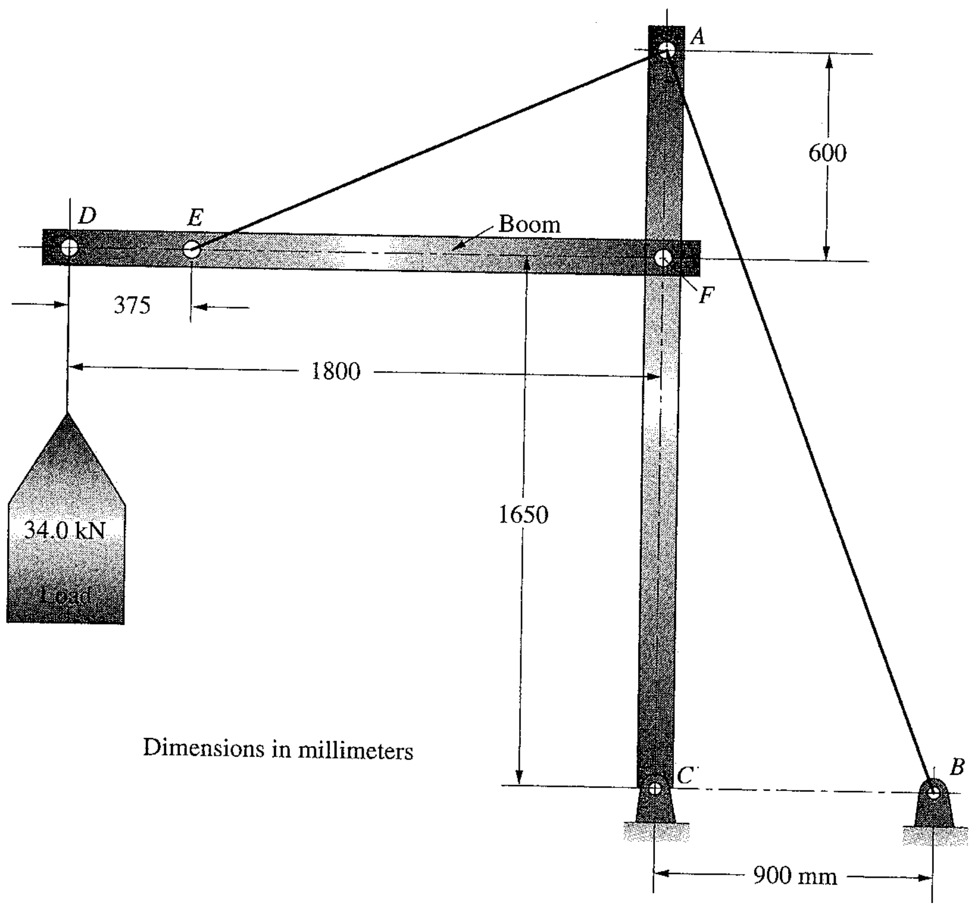
مثال ۱- انبردست



یک انبردست مطابق شکل موجود است.
بیابید: نیروی اعمال شده به سیم و نیرو در پینی که دو بخش انبردست را به هم متصل کرده است.

تنش برشی در پین چقدر است؟

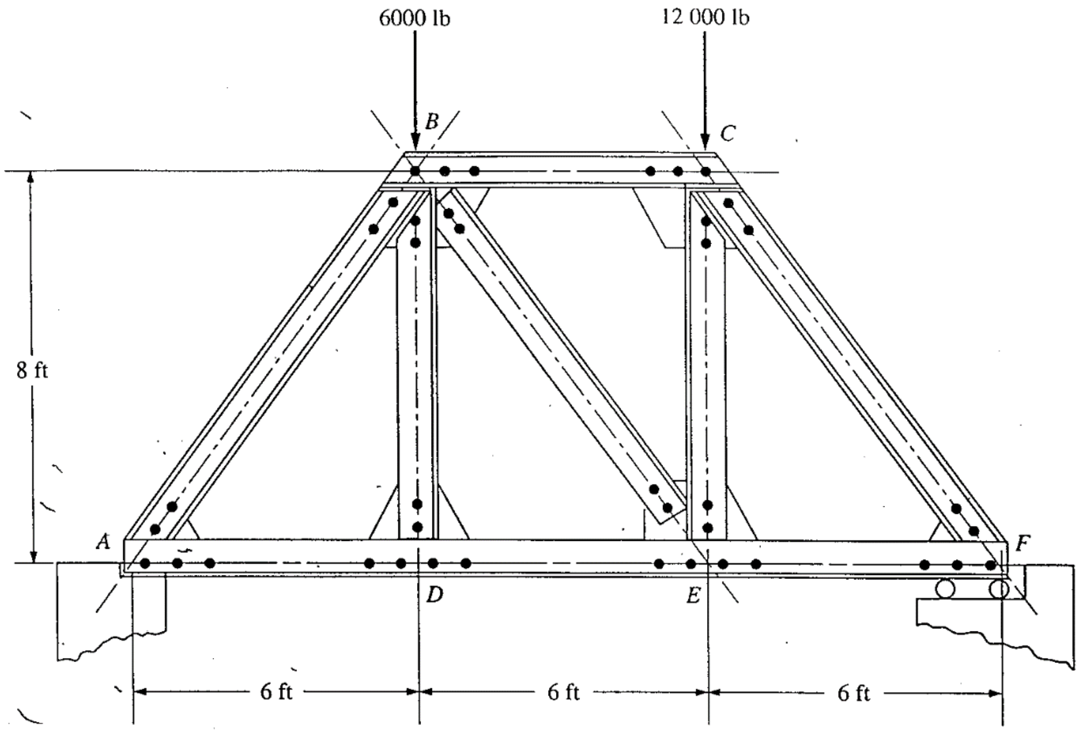
مثال ۲: سازه جرثقیل



سازه جرثقیل مطابق شکل می باشد.
بیابید: نیروها و دیاگرام آزاد برای کابل های AB و AE، تیر DEF و ستون AFC.

علاوه برآن: مطلوبست محاسبه تنش نرمال میانگین در کابلها، تیر و ستون.

مثال ۳: سازه خرپا



Member specifications:

AD, DE, EF L2 × 2 × 1/8 – doubled
BD, CE, BE L2 × 2 × 1/8 – single
AB, BC, CF C3 × 4.1 – doubled

نیروهای اعضای زیر را بیابید
AB, BC, AD, EF, BD, BE, CE, CF

علاوه برآن: تنش نرمال در هر عضو خرپا چقدر است؟