In The Name Of God

Power Electronics

Chopper Converters

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Example 7.4. *Show that for a basic dc to dc converter, the critical inductance of the filter circuit is given by*

$$L = \frac{V_0^2 (V_s - V_0)}{2f\, V_s\, P_0}$$

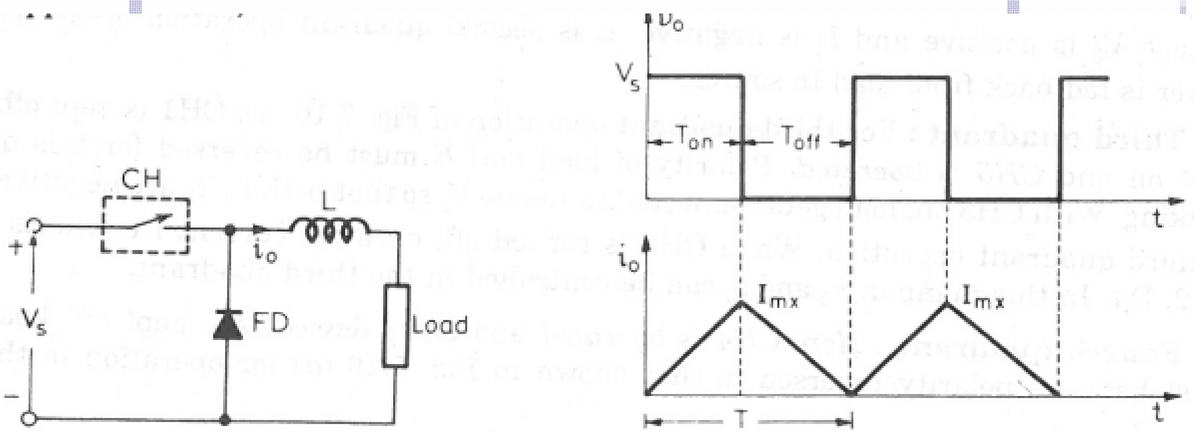
where V_0, V_s, P_0 and f are load voltage, source voltage, load power and chopping frequency respectively.

Solution. The critical inductance L is that value of inductance for which the output current falls to zero at $t = T$ during the turn-off period of the chopper. A typical waveform of output current, with critical inductance in the load circuit, is shown in Fig. 7.11 (*b*). If current variation, from zero to I_{mx} during T_{on} and from I_{mx} to zero during T_{off}, is assumed linear, then average value of output current I_0 is given by

$$I_0 \cdot T = \frac{1}{2} I_{mx}\, T_{on} + \frac{1}{2} I_{mx}\, T_{off}$$

$$= \frac{1}{2} I_{mx} \left(T_{on} + T_{off}\right) = \frac{1}{2} I_{mx}\, T$$

or $I_{mx} = 2\, I_0$ = maximum value of chopper current at $t = T_{on}$. It is seen from Fig. 7.11 (*a*) that when chopper CH is on,



(*a*) (*b*)

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$$V_0 + L\frac{di}{dt} = V_s \quad \text{or} \quad V_0 + L\frac{I_{mx}}{T_{on}} = V_s$$

or

$$L\frac{2\,I_0}{T_{on}} = V_s - V_0$$

$\therefore$

$$L = \frac{(V_s - V_0)\,T_{on}}{2\,I_0} \qquad \text{...}(i)$$

But average value of output voltage $V_0 = f\,T_{on}\,V_s$ and output, or load, power $P_0 = V_0\,I_0$. This gives

$$T_{on} = \frac{V_0}{f \cdot V_s} \quad \text{and} \quad I_0 = \frac{P_0}{V_0}$$

Substituting these values of T_{on} and I_0 in Eq (i), we get

$$L = \frac{(V_s - V_0)\,V_0^2}{2\,f\,V_s\,P_0}$$

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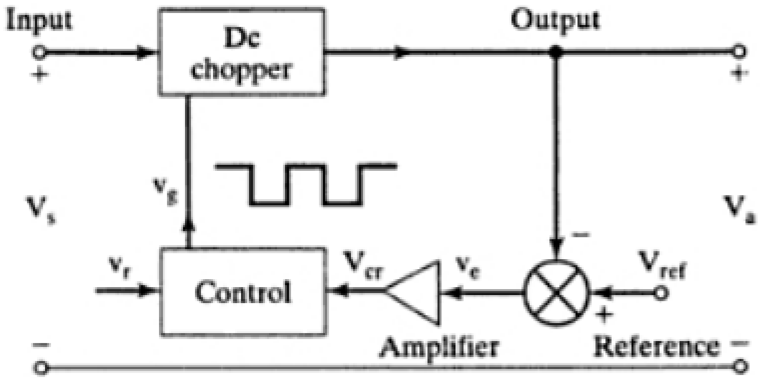
DC Chopper

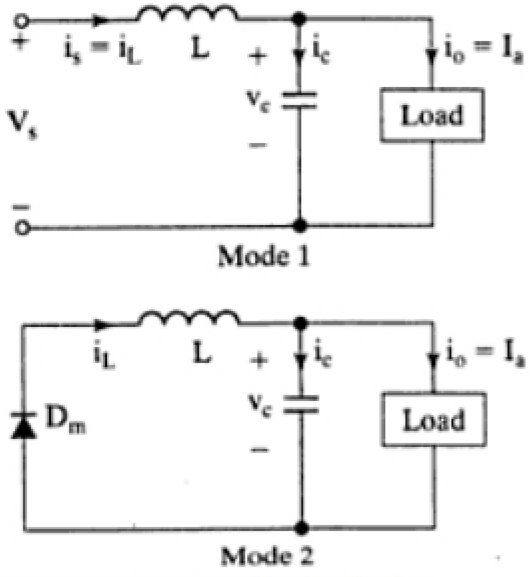
Switching regulators are commercially available as integrated circuits. The designer can select the switching frequency by choosing the values of R and C of frequency oscillator. As a rule of thumb, to maximize efficiency, the minimum oscillator period should be about 100 times longer than the transistor switching time; for example, if a transistor has a switching time of 0.5 μs, the oscillator period would be 50 μs, which gives the maximum oscillator frequency of 20 kHz. This limitation is due to a switching loss in the transistor. The transistor switching loss increases with the switching frequency and as a result the efficiency decreases. In addition, the core loss of inductors limits the high-frequency operation. Control voltage v_c is obtained by comparing the output voltage with its desired value. The v_{cr} can be compared with a sawtooth voltage v_r to generate the PWM control signal for the dc converter. There are four basic topologies of switching regulators [33, 34]:

1. Buck regulators
2. Boost regulators
3. Buck–boost regulators
4. Cúk regulators



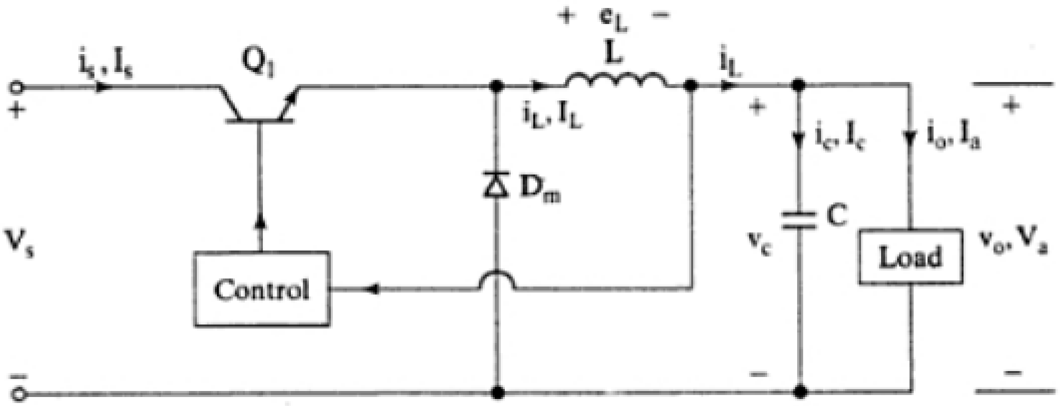
Buck Regulators

In a buck regulator, the average output voltage V_a, is less than the input voltage, V_s—hence the name "buck," a very popular regulator [6, 7]. The circuit diagram of a buck regulator using a power BJT is shown in Figure 5.16a, and this is like a step-down converter. The circuit operation can be divided into two modes. Mode 1 begins when transistor Q_1 is switched on at $t = 0$. The input current, which rises, flows through filter inductor L, filter capacitor C, and load resistor R. Mode 2 begins when transistor Q_1 is switched off at $t = t_1$. The freewheeling diode D_m conducts due to energy stored in the inductor; and the inductor current continues to flow through L, C, load, and diode D_m. The inductor current falls until transistor Q_1 is switched on again in the next cycle. The equivalent circuits for the modes of operation are shown in Figure 5.16b. The waveforms for the voltages and currents are shown in Figure 5.16c for a continuous current flow in the inductor L. It is assumed that the current rises and falls linearly. In practical circuits, the switch has a finite, nonlinear resistance. Its effect can generally be negligible in most applications. Depending on the switching frequency, filter inductance, and capacitance, the inductor current could be discontinuous.



The voltage across the inductor L is, in general,

$$e_L = L\frac{di}{dt}$$



(a) Circuit diagram

Assuming that the inductor current rises linearly from I_1 to I_2 in time t_1,

$$V_s - V_a = L\frac{I_2 - I_1}{t_1} = L\frac{\Delta I}{t_1} \tag{5.44}$$

or

$$t_1 = \frac{\Delta I\, L}{V_s - V_a} \tag{5.45}$$

and the inductor current falls linearly from I_2 to I_1 in time t_2,

$$-V_a = -L\frac{\Delta I}{t_2} \tag{5.46}$$

or

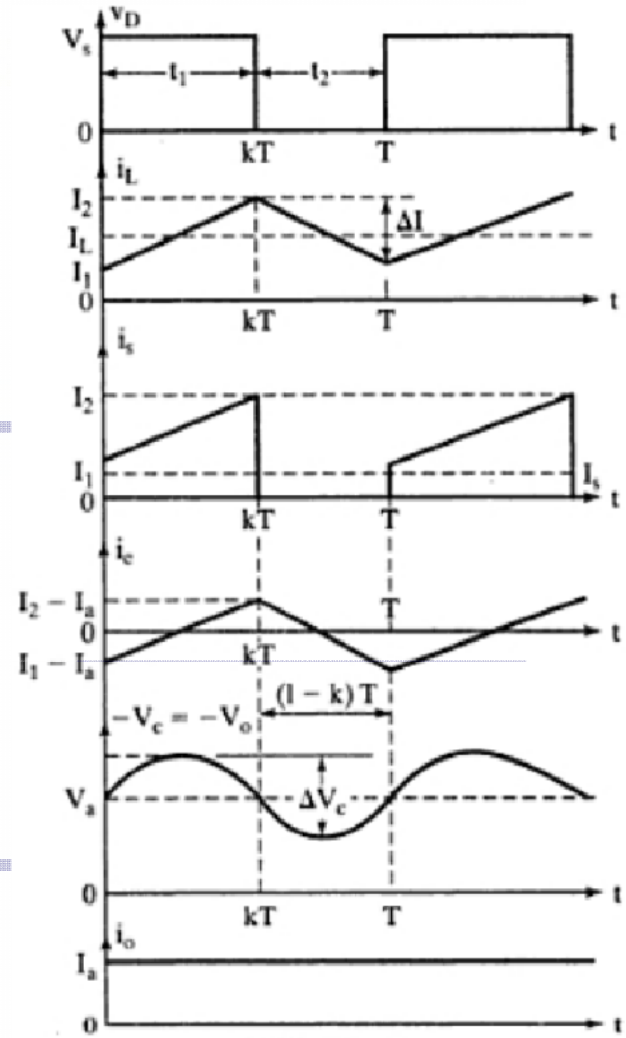
$$t_2 = \frac{\Delta I\, L}{V_a} \tag{5.47}$$

where $\Delta I = I_2 - I_1$ is the peak-to-peak ripple current of the inductor L. Equating the value of ΔI in Eqs. (5.44) and (5.46) gives

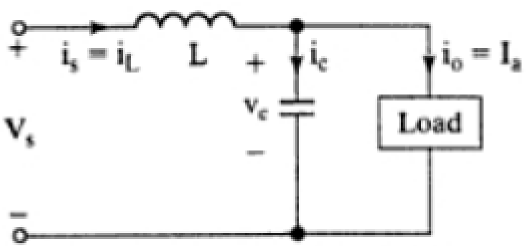
$$\Delta I = \frac{(V_s - V_a)t_1}{L} = \frac{V_a t_2}{L}$$

Substituting $t_1 = kT$ and $t_2 = (1 - k)T$ yields the average output voltage as

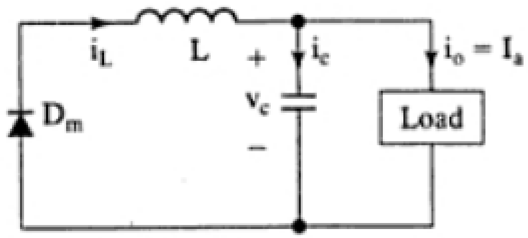
$$V_a = V_s\frac{t_1}{T} = kV_s \tag{5.48}$$



(c) Waveforms



Mode 1



Mode 2

Assuming a lossless circuit, $V_sI_s = V_aI_a = kV_sI_a$ and the average input current

$$I_s = kI_a \tag{5.49}$$

The switching period T can be expressed as

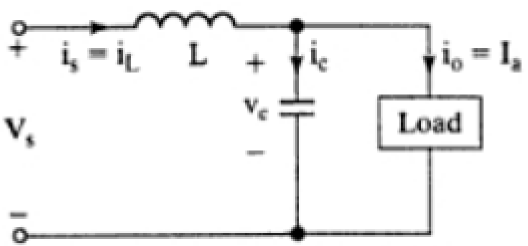
$$T = \frac{1}{f} = t_1 + t_2 = \frac{\Delta I\, L}{V_s - V_a} + \frac{\Delta I\, L}{V_a} = \frac{\Delta I\, LV_s}{V_a(V_s - V_a)} \tag{5.50}$$

which gives the peak-to-peak ripple current as

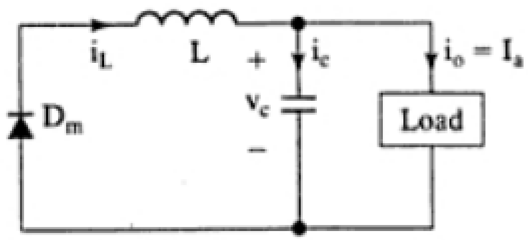
$$\Delta I = \frac{V_a(V_s - V_a)}{fLV_s} \tag{5.51}$$

or

$$\Delta I = \frac{V_s k(1 - k)}{fL} \tag{5.52}$$



Mode 1



Mode 2

Using Kirchhoff's current law, we can write the inductor current i_L as

$$i_L = i_c + i_o$$

If we assume that the load ripple current Δi_o is very small and negligible, $\Delta i_L = \Delta i_c$. The average capacitor current, which flows into for $t_1/2 + t_2/2 = T/2$, is

$$I_c = \frac{\Delta I}{4}$$

The capacitor voltage is expressed as

$$v_c = \frac{1}{C}\int i_c\, dt + v_c(t = 0)$$

and the peak-to-peak ripple voltage of the capacitor is

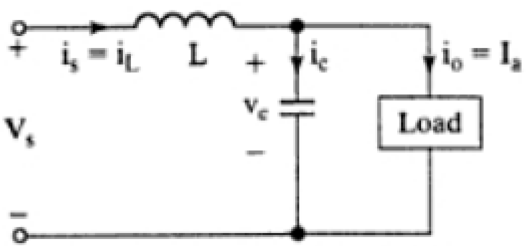
$$\Delta V_c = v_c - v_c(t = 0) = \frac{1}{C}\int_0^{T/2} \frac{\Delta I}{4}\, dt = \frac{\Delta I\, T}{8C} = \frac{\Delta I}{8fC} \tag{5.53}$$

Substituting the value of ΔI from Eq. (5.51) or (5.52) in Eq. (5.53) yields

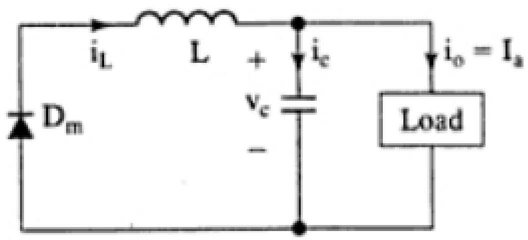
$$\Delta V_c = \frac{V_a(V_s - V_a)}{8LCf^2 V_s} \tag{5.54}$$

or

$$\Delta V_c = \frac{V_s k(1 - k)}{8LCf^2} \tag{5.55}$$



Mode 1



Mode 2

Condition for continuous inductor current and capacitor voltage. If I_L is the average inductor current, the inductor ripple current $\Delta I = 2I_L$.

Using Eqs. (5.48) and (5.52), we get

$$\frac{V_S(1-k)k}{fL} = 2I_L = 2I_a = \frac{2kV_S}{R}$$

which gives the critical value of the inductor L_c as

$$L_c = L = \frac{(1-k)R}{2f} \tag{5.56}$$

If V_c is the average capacitor voltage, the capacitor ripple voltage $\Delta V_c = 2V_a$. Using Eqs. (5.48) and (5.55), we get

$$\frac{V_S(1-k)k}{8LCf^2} = 2V_a = 2kV_S$$

which gives the critical value of the capacitor C_c as

$$C_c = C = \frac{1-k}{16Lf^2} \tag{5.57}$$

The buck regulator requires only one transistor, is simple, and has high efficiency greater than 90%. The *di/dt* of the load current is limited by inductor L. However, the input current is discontinuous and a smoothing input filter is normally required. It provides one polarity of output voltage and unidirectional output current. It requires a protection circuit in case of possible short circuit across the diode path.

Example 5.5 Finding the Values of *LC* Filter for the Buck Regulator

The buck regulator in Figure 5.16a has an input voltage of $V_s = 12$ V. The required average output voltage is $V_a = 5$ V at $R = 500\ \Omega$ and the peak-to-peak output ripple voltage is 20 mV. The switching frequency is 25 kHz. If the peak-to-peak ripple current of inductor is limited to 0.8 A, determine (a) the duty cycle k, (b) the filter inductance L, and (c) the filter capacitor C, and (d) the critical values of L and C.

Solution

$V_s = 12$ V, $\Delta V_c = 20$ mV, $\Delta I - 0.8$ A, $f = 25$ kHz, and $V_a = 5$ V.

a. From Eq. (5.48), $V_a = kV_s$ and $k = V_a/V_s = 5/12 = 0.4167 = 41.67\%$.

b. From Eq. (5.51),

$$L = \frac{5(12 - 5)}{0.8 \times 25{,}000 \times 12} = 145.83\ \mu\text{H}$$

c. From Eq. (5.53),

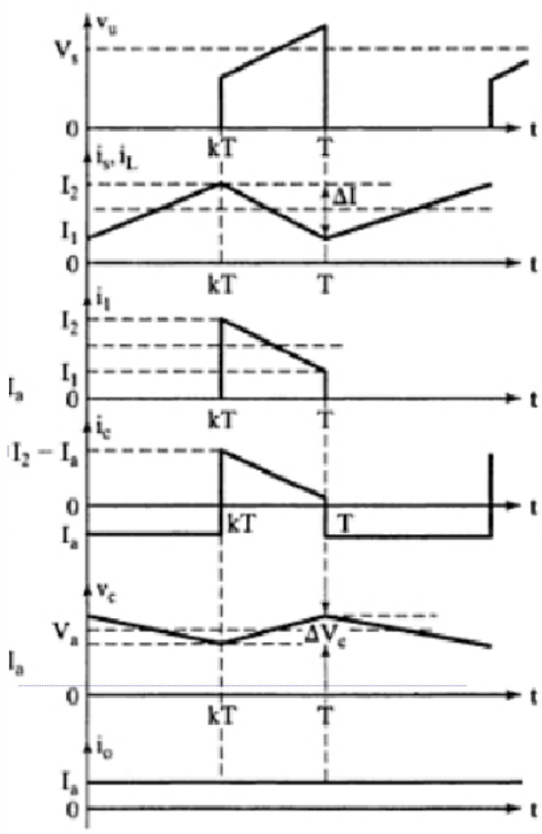
$$C = \frac{0.8}{8 \times 20 \times 10^{-3} \times 25{,}000} = 200\ \mu\text{F}$$

d. From Eq. (5.56), we get $L_c = \dfrac{(1 - k)R}{2f} = \dfrac{(1 - 0.4167) \times 500}{2 \times 25 \times 10^3} = 5.83$ mH

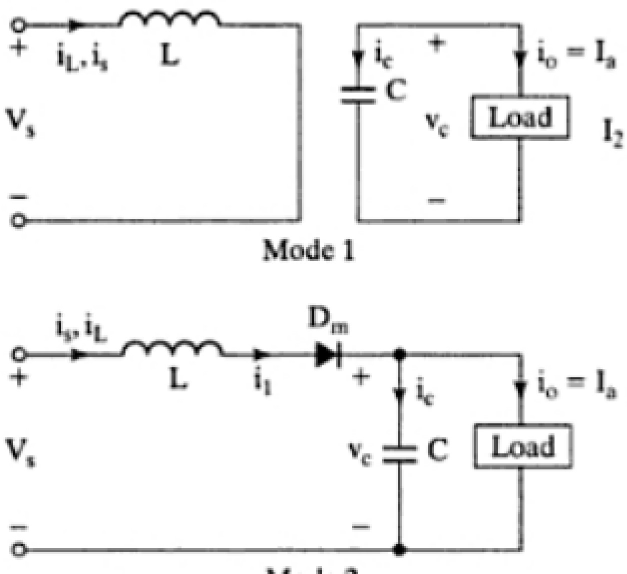
From Eq. (5.57), we get $C_c = \dfrac{1 - k}{16Lf^2} = \dfrac{1 - 0{,}4167}{16 \times 5.83 \times 10^2 \times (25 \times 10^3)^2} = 0.4\ \mu\text{F}$

Boost Regulators

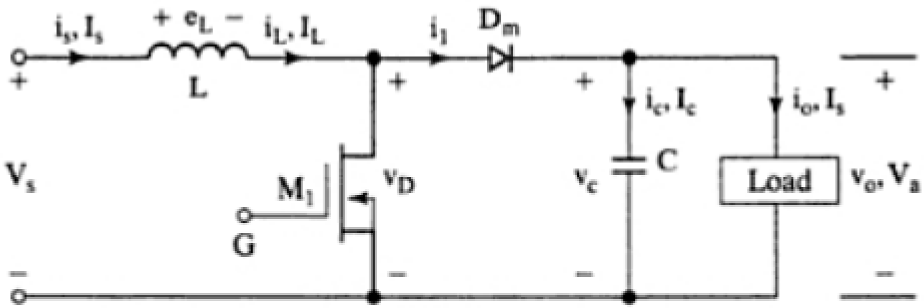
In a boost regulator [8, 9] the output voltage is greater than the input voltage—hence the name "boost." A boost regulator using a power MOSFET is shown in Figure 5.17a. The circuit operation can be divided into two modes. Mode 1 begins when transistor M_1 is switched on at $t = 0$. The input current, which rises, flows through inductor L and transistor Q_1. Mode 2 begins when transistor M_1 is switched off at $t = t_1$. The current that was flowing through the transistor would now flow through L, C, load, and diode D_m. The inductor current falls until transistor M_1 is turned on again in the next cycle. The energy stored in inductor L is transferred to the load. The equivalent circuits for the modes of operation are shown in Figure 5.17b. The waveforms for voltages and currents are shown in Figure 5.17c for continuous load current, assuming that the current rises or falls linearly.



(c) Waveforms



(b) Equivalent circuits



(a) Circuit diagram

Assuming that the inductor current rises linearly from I_1 to I_2 in time t_1,

$$V_s = L\frac{I_2 - I_1}{t_1} = L\frac{\Delta I}{t_1} \tag{5.58}$$

or

$$t_1 = \frac{\Delta I L}{V_s} \tag{5.59}$$

and the inductor current falls linearly from I_2 to I_1 in time t_2,

$$V_s - V_a = -L\frac{\Delta I}{t_2} \tag{5.60}$$

or

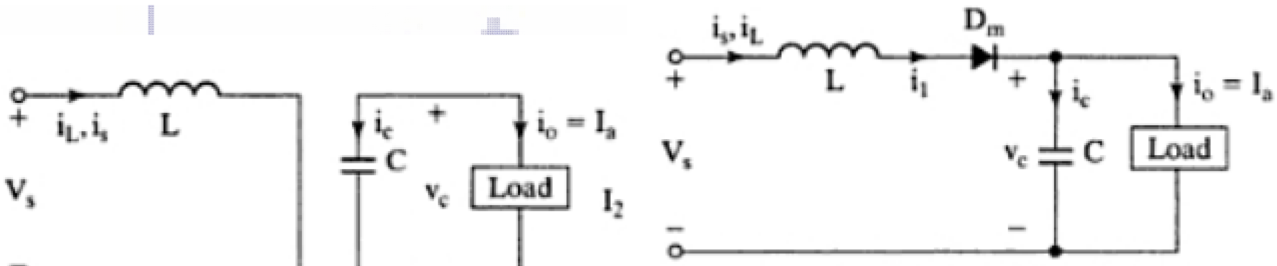
$$t_2 = \frac{\Delta I L}{V_a - V_s} \tag{5.61}$$

where $\Delta I = I_2 - I_1$ is the peak-to-peak ripple current of inductor L. From Eqs. (5.58) and (5.60),

$$\Delta I = \frac{V_s t_1}{L} = \frac{(V_a - V_s)t_2}{L}$$

Substituting $t_1 = kT$ and $t_2 = (1 - k)T$ yields the average output voltage,

$$V_a = V_s\frac{T}{t_2} = \frac{V_s}{1 - k} \tag{5.62}$$



Mode 1

Mode 2

(b) Equivalent circuits

which gives

$$(1 - k) = \frac{V_S}{V_a} \tag{5.63}$$

Substituting $k = t_1/T = t_1 f$ into Eq. (5.63) yields

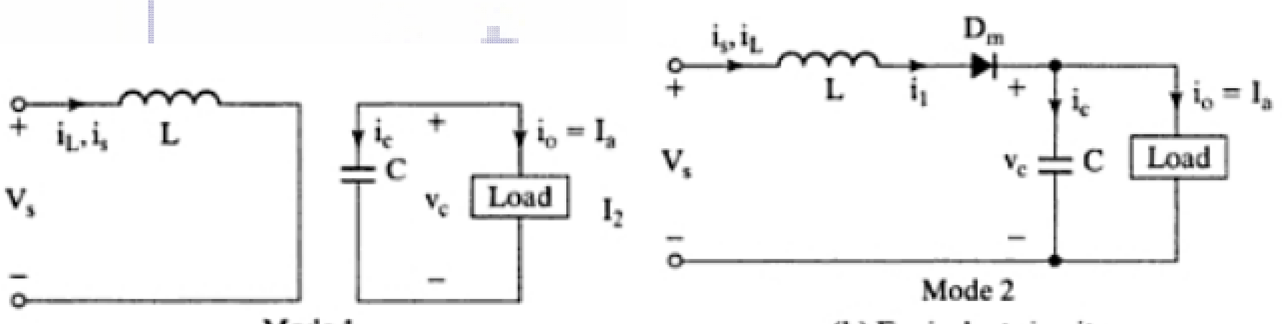
$$t_1 = \frac{V_a - V_S}{V_a f} \tag{5.64}$$

Assuming a lossless circuit, $V_s I_s = V_a I_a = V_s I_a/(1 - k)$ and the average input current is

$$I_s = \frac{I_a}{1 - k} \tag{5.65}$$

The switching period T can be found from

$$T = \frac{1}{f} = t_1 + t_2 = \frac{\Delta I L}{V_s} + \frac{\Delta I L}{V_a - V_s} = \frac{\Delta I L V_a}{V_s(V_a - V_s)} \tag{5.66}$$



(b) Equivalent circuits

and this gives the peak-to-peak ripple current:

$$\Delta I = \frac{V_s(V_a - V_s)}{fLV_a} \tag{5.67}$$

or

$$\Delta I = \frac{V_s k}{fL} \tag{5.68}$$

When the transistor is on, the capacitor supplies the load current for $t = t_1$. The average capacitor current during time t_1 is $I_c = I_a$ and the peak-to-peak ripple voltage of the capacitor is

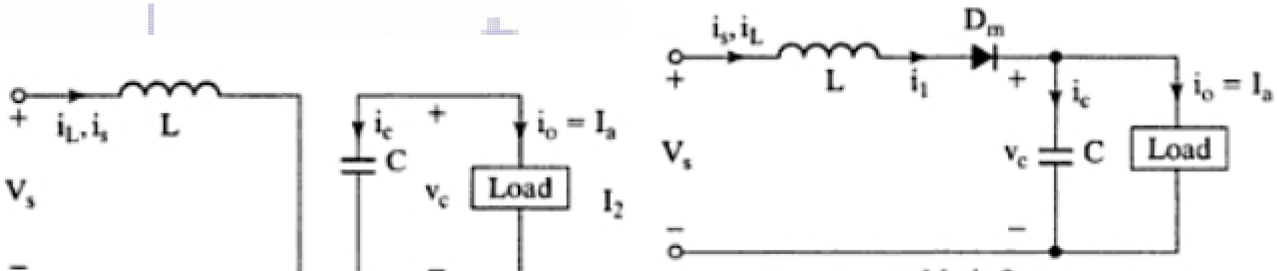
$$\Delta V_c = v_c - v_c(t = 0) = \frac{1}{C}\int_0^{t_1} I_c\,dt = \frac{1}{C}\int_0^{t_1} I_a = \frac{I_a t_1}{C} \tag{5.69}$$

Substituting $t_1 = (V_a - V_s)/(V_a f)$ from Eq. (5.64) gives

$$\Delta V_c = \frac{I_a(V_a - V_s)}{V_a f C} \tag{5.70}$$

or

$$\Delta V_c = \frac{I_a k}{fC} \tag{5.71}$$



Mode 1

Mode 2

(b) Equivalent circuits

Condition for continuous inductor current and capacitor voltage. If I_L is the average inductor current, the inductor ripple current $\Delta I = 2I_L$.

Using Eqs. (5.62) and (5.68), we get

$$\frac{kV_S}{fL} = 2I_L = 2I_a = \frac{2V_S}{(1-k)R}$$

which gives the critical value of the inductor L_c as

$$L_c = L = \frac{k(1-k)R}{2f} \tag{5.72}$$

If V_c is the average capacitor voltage, the capacitor ripple voltage $\Delta V_c = 2V_a$. Using Eq. (5.71), we get

$$\frac{I_a k}{Cf} = 2V_a = 2I_a R$$

which gives the critical value of the capacitor C_c as

$$C_c = C = \frac{k}{2fR} \tag{5.73}$$

A boost regulator can step up the output voltage without a transformer. Due to a single transistor, it has a high efficiency. The input current is continuous. However, a high-peak current has to flow through the power transistor. The output voltage is very sensitive to changes in duty cycle k and it might be difficult to stabilize the regulator. The average output current is less than the average inductor current by a factor of $(1 - k)$, and a much higher rms current would flow through the filter capacitor, resulting in the use of a larger filter capacitor and a larger inductor than those of a buck regulator.

Example 5.6 Finding the Currents and Voltage in the Boost Regulator

A boost regulator in Figure 5.17a has an input voltage of $V_s = 5$ V. The average output voltage $V_a = 15$ V and the average load current $I_a = 0.5$ A. The switching frequency is 25 kHz. If $L = 150$ μH and $C = 220$ μF, determine (a) the duty cycle k, (b) the ripple current of inductor ΔI, (c) the peak current of inductor I_2, (d) the ripple voltage of filter capacitor ΔV_c, and (e) the critical values of L and C.

Solution

$V_s = 5$ V, $V_a = 15$ V, $f = 25$ kHz, $L = 150$ μH, and $C = 220$ μF.

a. From Eq. (5.62), $15 = 5/(1 - k)$ or $k = 2/3 = 0.6667 = 66.67\%$.

b. From Eq. (5.67),

$$\Delta I = \frac{5 \times (15 - 5)}{25{,}000 \times 150 \times 10^{-6} \times 15} = 0.89 \text{ A}$$

c. From Eq. (5.65), $I_s = 0.5/(1 - 0.667) = 1.5$ A and peak inductor current,

$$I_2 = I_s + \frac{\Delta I}{2} = 1.5 + \frac{0.89}{2} = 1.945 \text{ A}$$

d. From Eq. (5.71),

$$\Delta V_c = \frac{0.5 \times 0.6667}{25{,}000 \times 220 \times 10^{-6}} = 60.61 \text{ mV}$$

e. $R = \frac{V_a}{I_a} = \frac{15}{0.5} = 30\ \Omega$

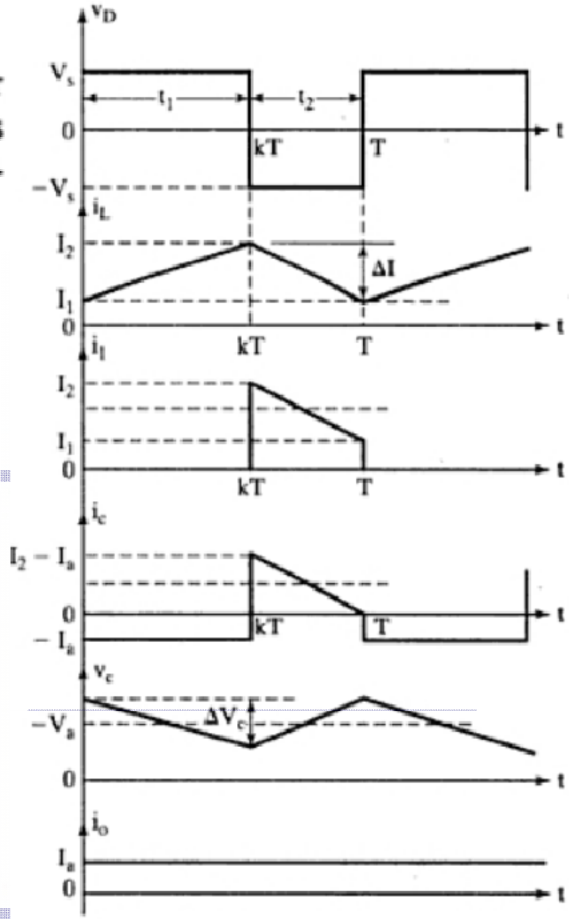
From Eq. (5.72), we get $L_c = \frac{(1 - k)kR}{2f} = \frac{(1 - 0.6667) \times 0.6667 \times 30}{2 \times 25 \times 10^3} = 133$ μH

From Eq. (5.73), we get $C_c = \frac{k}{2fR} = \frac{0.6667}{2 \times 25 \times 10^3 \times 30} = 0.44$ μF

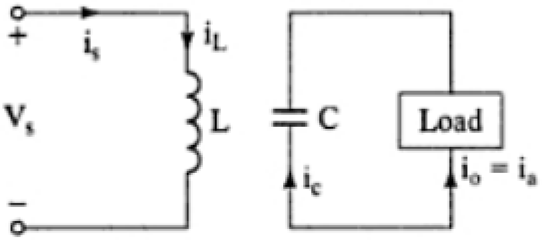
Buck–Boost Regulators

A buck–boost regulator provides an output voltage that may be less than or greater than the input voltage—hence the name "buck–boost"; the output voltage polarity is opposite to that of the input voltage. This regulator is also known as an *inverting regulator*. The circuit arrangement of a buck–boost regulator is shown in Figure 5:18a.

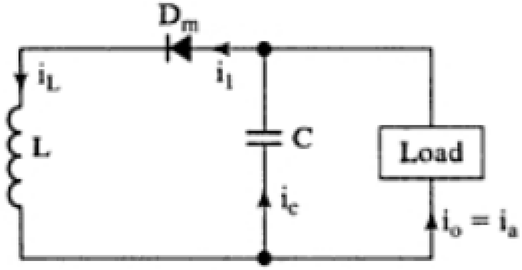
The circuit operation can be divided into two modes. During mode 1, transistor Q_1 is turned on and diode D_m is reversed biased. The input current, which rises, flows through inductor L and transistor Q_1. During mode 2, transistor Q_1 is switched off and the current, which was flowing through inductor L, would flow through L, C, D_m, and the load. The energy stored in inductor L would be transferred to the load and the inductor current would fall until transistor Q_1 is switched on again in the next cycle. The equivalent circuits for the modes are shown in Figure 5.18b. The waveforms for steady-state voltages and currents of the buck–boost regulator are shown in Figure 5.18c for a continuous load current.



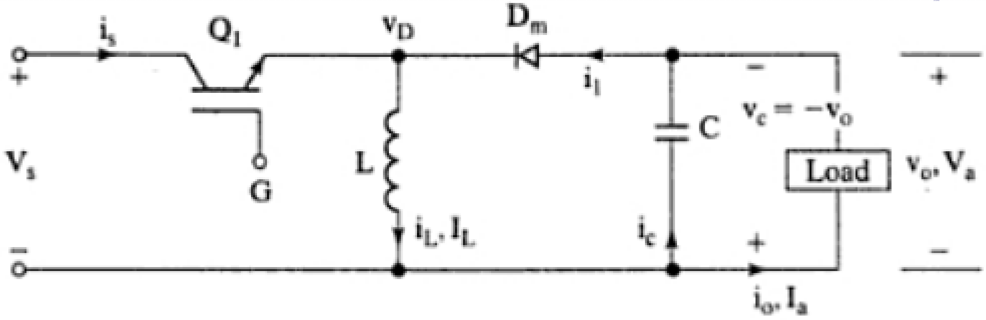
Chopper



Mode 1



Mode 2



(a) Circuit diagram

Assuming that the inductor current rises linearly from I_1 to I_2 in time t_1,

$$V_s = L\frac{I_2 - I_1}{t_1} = L\frac{\Delta I}{t_1} \tag{5.74}$$

or

$$t_1 = \frac{\Delta I L}{V_s} \tag{5.75}$$

and the inductor current falls linearly from I_2 to I_1 in time t_2,

$$V_a = -L\frac{\Delta I}{t_2} \tag{5.76}$$

or

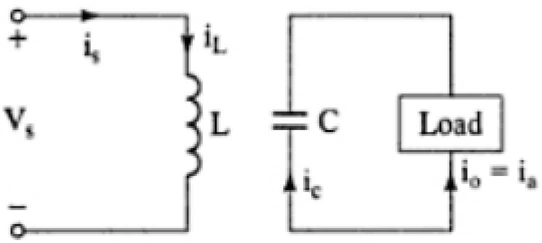
$$t_2 = \frac{-\Delta I L}{V_a} \tag{5.77}$$

where $\Delta I = I_2 - I_1$ is the peak-to-peak ripple current of inductor L. From Eqs. (5.74) and (5.76),

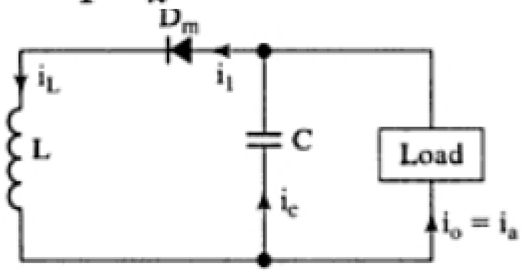
$$\Delta I = \frac{V_s t_1}{L} = \frac{-V_a t_2}{L}$$

Substituting $t_1 = kT$ and $t_2 = (1 - k)T$, the average output voltage is

$$V_a = -\frac{V_s k}{1 - k} \tag{5.78}$$



Mode 1



Mode 2

Substituting $t_1 = kT$ and $t_2 = (1 - k)T$ into Eq. (5.78) yields

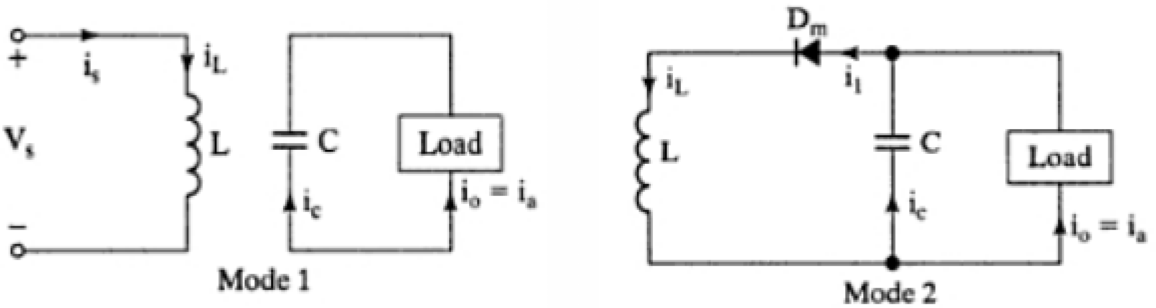
$$(1 - k) = \frac{-V_S}{V_a - V_S} \tag{5.79}$$

Substituting $t_2 = (1 - k)T$, and $(1 - k)$ from Eq. (5.79) into Eq. (5.78) yields

$$t_1 = \frac{V_a}{(V_a - V_S)f} \tag{5.80}$$

Assuming a lossless circuit, $V_s I_s = -V_a I_a = V_s I_a k/(1 - k)$ and the average input current I_s is related to the average output current I_a by

$$I_s = \frac{I_a k}{1 - k} \tag{5.81}$$



The switching period T can be found from

$$T = \frac{1}{f} = t_1 + t_2 = \frac{\Delta I L}{V_s} + \frac{\Delta I L}{V_a} = \frac{\Delta I L(V_a - V_s)}{V_s V_a} \tag{5.82}$$

and this gives the peak-to-peak ripple current,

$$\Delta I = \frac{V_s V_a}{f L(V_a - V_s)} \tag{5.83}$$

or

$$\Delta I = \frac{V_s k}{f L} \tag{5.84}$$

When transistor Q_1 is on, the filter capacitor supplies the load current for $t = t_1$. The average discharging current of the capacitor is $I_c = I_a$ and the peak-to-peak ripple voltage of the capacitor is

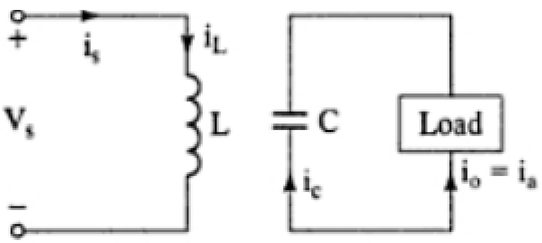
$$\Delta V_c = \frac{1}{C}\int_0^{t_1} I_c \, dt = \frac{1}{C}\int_0^{t_1} I_a \, dt = \frac{I_a t_1}{C} \tag{5.85}$$

Substituting $t_1 = V_a/[(V_a - V_s)f]$ from Eq. (5.80) becomes

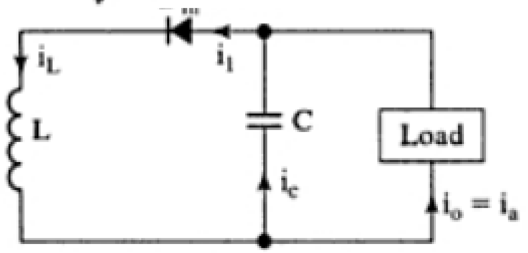
$$\Delta V_c = \frac{I_a V_a}{(V_a - V_s) f C} \tag{5.86}$$

or

$$\Delta V_c = \frac{I_a k}{f C} \tag{5.87}$$



Mode 1



Mode 2

Condition for continuous inductor current and capacitor voltage. If I_L is the average inductor current, the inductor ripple current $\Delta I = 2I_L$. Using Eqs. (5.78) and (5.84), we get

$$\frac{kV_s}{fL} = 2I_L = 2I_a = \frac{2kV_s}{(1-k)R}$$

which gives the critical value of the inductor L_c as

$$L_c = L = \frac{(1-k)R}{2f} \tag{5.88}$$

If V_c is the average capacitor voltage, the capacitor ripple voltage $\Delta V_c = 2V_a$. Using Eq. (5.87), we get

$$\frac{I_a k}{Cf} = 2V_a = 2I_a R$$

which gives the critical value of the capacitor C_c as

$$C_c = C = \frac{k}{2fR} \tag{5.89}$$

A buck–boost regulator provides output voltage polarity reversal without a transformer. It has high efficiency. Under a fault condition of the transistor, the *di/dt* of the fault current is limited by the inductor L and will be V_s/L. Output short-circuit protection would be easy to implement. However, the input current is discontinuous and a high peak current flows through transistor Q_1.

Example 5.7 Finding the Currents and Voltage in the Buck–Boost Regulator

The buck–boost regulator in Figure 5.18a has an input voltage of $V_s = 12$ V. The duty cycle $k = 0.25$ and the switching frequency is 25 kHz. The inductance $L = 150$ μH and filter capacitance $C = 220$ μF. The average load current $I_a = 1.25$ A. Determine (a) the average output voltage, V_a; (b) the peak-to-peak output voltage ripple, ΔV_c; (c) the peak-to-peak ripple current of inductor, ΔI; (d) the peak current of the transistor, I_p; and (e) the critical values of L and C.

Solution

$V_s = 12$ V, $k = 0.25$, $I_a = 1.25$ A, $f = 25$ kHz, $L = 150$ μH, and $C = 220$ μF.

a. From Eq. (5.78), $V_a = -12 \times 0.25/(1 - 0.25) = -4$ V.

b. From Eq. (5.87), the peak-to-peak output ripple voltage is

$$\Delta V_c = \frac{1.25 \times 0.25}{25{,}000 \times 220 \times 10^{-6}} = 56.8 \text{ mV}$$

c. From Eq. (5.84), the peak-to-peak inductor ripple is

$$\Delta I = \frac{12 \times 0.25}{25{,}000 \times 150 \times 10^{-6}} = 0.8 \text{ A}$$

d. From Eq. (5.81), $I_s = 1.25 \times 0.25/(1 - 0.25) = 0.4167$ A. Because I_s is the average of duration kT, the peak-to-peak current of the transistor,

$$I_p = \frac{I_s}{k} + \frac{\Delta I}{2} = \frac{0.4167}{0.25} + \frac{0.8}{2} = 2.067 \text{ A}$$

e. $R = \dfrac{-V_a}{I_a} = \dfrac{4}{1.25} = 3.2\ \Omega$

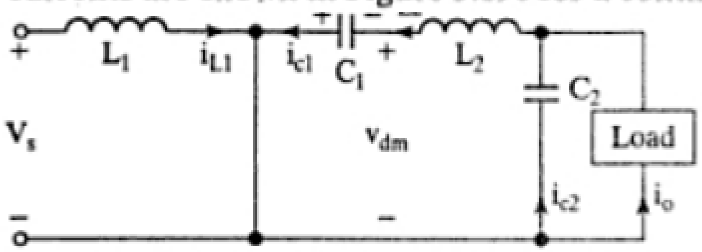
From Eq. (5.88), we get $L_c = \dfrac{(1-k)R}{2f} = \dfrac{(1 - 0.25) \times 3.2}{2 \times 25 \times 10^3} = 450$ μH.

From Eq. (5.89), we get $C_c = \dfrac{k}{2fR} = \dfrac{0.25}{2 \times 25 \times 10^3 \times 3.2} = 1.56$ μF.

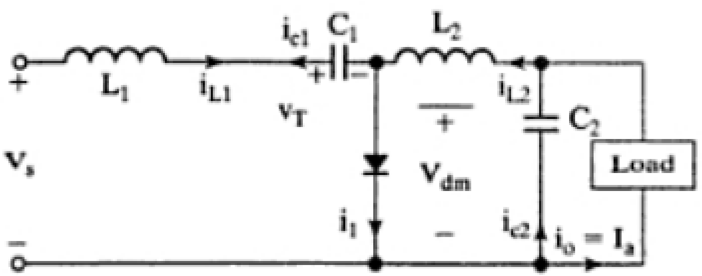
Cúk Regulators

The circuit arrangement of the Cúk regulator [10] using a power bipolar junction transistor (BJT) is shown in Figure 5.19a. Similar to the buck–boost regulator, the Cúk regulator provides an output voltage that is less than or greater than the input voltage, but the output voltage polarity is opposite to that of the input voltage. It is named after its inventor [1]. When the input voltage is turned on and transistor Q_1 is switched off, diode D_m is forward biased and capacitor C_1 is charged through L_1, D_m, and the input supply V_s.

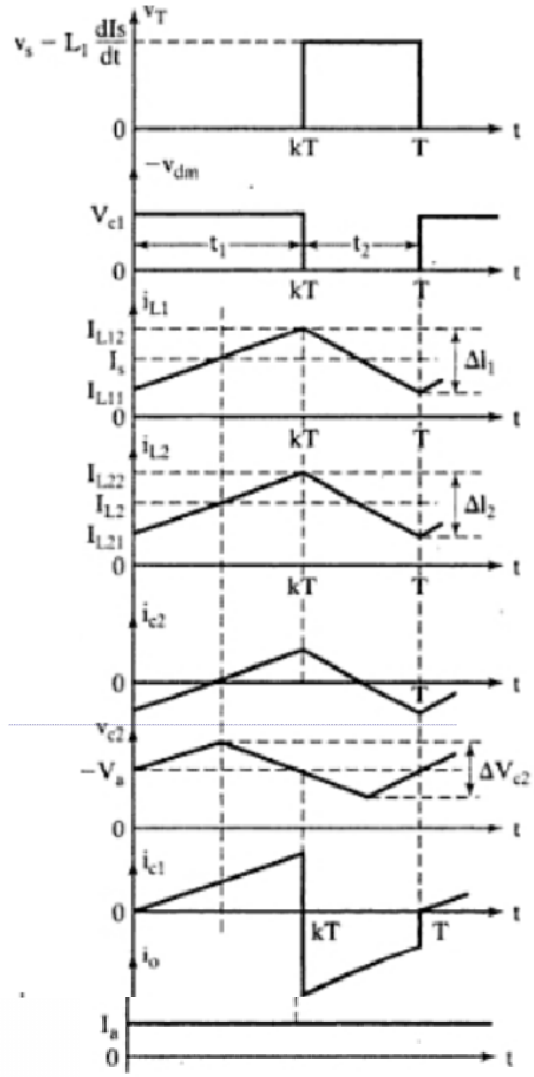
The circuit operation can be divided into two modes. Mode 1 begins when transistor Q_1 is turned on at $t = 0$. The current through inductor L_1 rises. At the same time, the voltage of capacitor C_1 reverse biases diode D_m and turns it off. The capacitor C_1 discharges its energy to the circuit formed by C_1, C_2, the load, and L_2. Mode 2 begins when transistor Q_1 is turned off at $t = t_1$. The capacitor C_1 is charged from the input supply and the energy stored in the inductor L_2 is transferred to the load. The diode D_m and transistor Q_1 provide a synchronous switching action. The capacitor C_1 is the medium for transferring energy from the source to the load. The equivalent circuits for the modes are shown in Figure 5.19b and the waveforms for steady-state voltages and currents are shown in Figure 5.19c for a continuous load current.

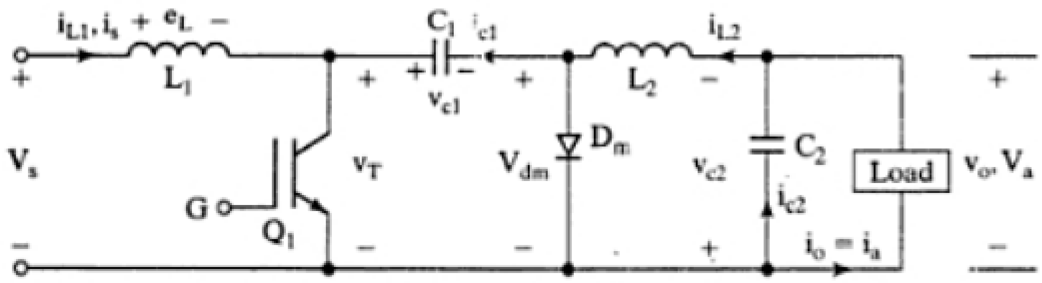


Mode 1



Mode 2





(a) Circuit diagram

Assuming that the current of inductor L_1 rises linearly from I_{L11} to I_{L12} in time t_1,

$$V_s = L_1 \frac{I_{L12} - I_{L11}}{t_1} = L_1 \frac{\Delta I_1}{t_1} \tag{5.90}$$

or

$$t_1 = \frac{\Delta I_1 L_1}{V_s} \tag{5.91}$$

and due to the charged capacitor C_1, the current of inductor L_1 falls linearly from I_{L12} to I_{L11} in time t_2,

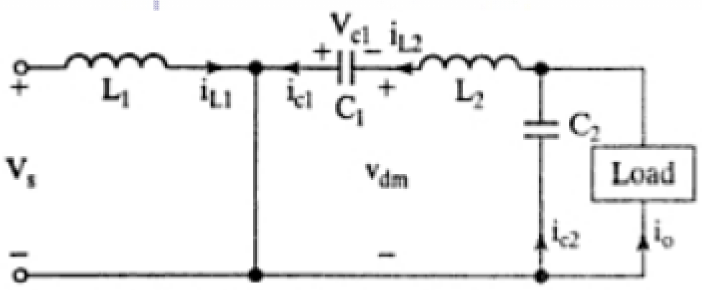
$$V_s - V_{c1} = -L_1 \frac{\Delta I_1}{t_2} \tag{5.92}$$

or

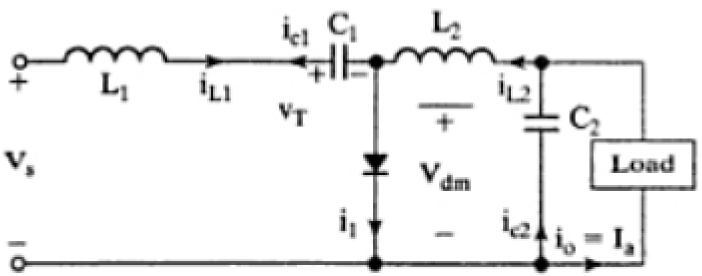
$$t_2 = \frac{-\Delta I_1 L_1}{V_s - V_{c1}} \tag{5.93}$$

where V_{c1} is the average voltage of capacitor C_1, and $\Delta I_1 = I_{L12} - I_{L11}$. From Eqs. (5.90) and (5.92).

$$\Delta I_1 = \frac{V_s t_1}{L_1} = \frac{-(V_s - V_{c1}) t_2}{L_1}$$



Mode 1



Mode 2

Substituting $t_1 = kT$ and $t_2 = (1 - k)T$, the average voltage of capacitor C_1 is

$$V_{c1} = \frac{V_s}{1 - k} \tag{5.94}$$

Assuming that the current of filter inductor L_2 rises linearly from I_{L21} to I_{L22} in time t_1,

$$V_{c1} + V_a = L_2 \frac{I_{L22} - I_{L21}}{t_1} = L_2 \frac{\Delta I_2}{t_1} \tag{5.95}$$

or

$$t_1 = \frac{\Delta I_2 L_2}{V_{c1} + V_a} \tag{5.96}$$

and the current of inductor L_2 falls linearly from I_{L22} to I_{L21} in time t_2,

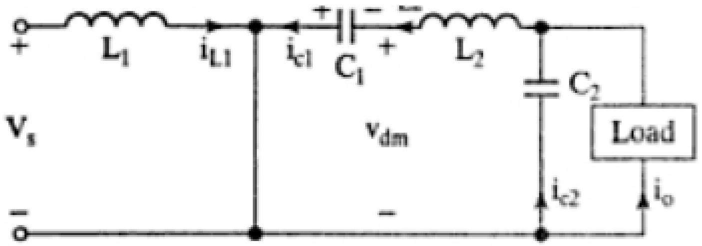
$$V_a = -L_2 \frac{\Delta I_2}{t_2} \tag{5.97}$$

or

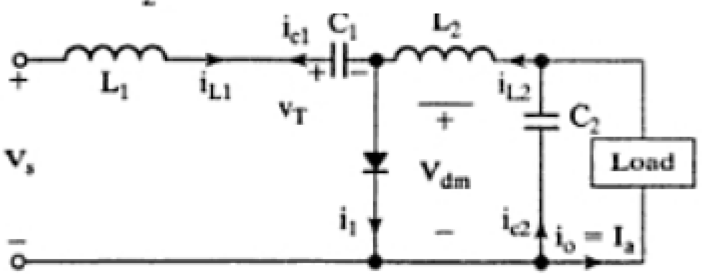
$$t_2 = -\frac{\Delta I_2 L_2}{V_a} \tag{5.98}$$

where $\Delta I_2 = I_{L22} - I_{L21}$. From Eqs. (5.95) and (5.97),

$$\Delta I_2 = \frac{(V_{c1} + V_a)t_1}{L_2} = -\frac{V_a t_2}{L_2}$$



Mode 1



Mode 2

Substituting $t_1 = kT$ and $t_2 = (1 - k)T$, the average voltage of capacitor C_1 is

$$V_{c1} = -\frac{V_a}{k} \tag{5.99}$$

Equating Eq. (5.94) to Eq. (5.99), we can find the average output voltage as

$$V_a = -\frac{kV_s}{1 - k} \tag{5.100}$$

which gives

$$k = \frac{V_a}{V_a - V_S} \tag{5.101}$$

$$1 - k = \frac{V_S}{V_S - V_a} \tag{5.102}$$

Assuming a lossless circuit, $V_sI_s = -V_aI_a = V_sI_ak/(1 - k)$ and the average input current,

$$I_s = \frac{kI_a}{1 - k} \tag{5.103}$$

The switching period T can be found from Eqs. (5.91) and (5.93):

$$T = \frac{1}{f} = t_1 + t_2 = \frac{\Delta I_1 L_1}{V_s} - \frac{\Delta I_1 L_1}{V_s - V_{c1}} = \frac{-\Delta I_1 L_1 V_{c1}}{V_s(V_s - V_{c1})} \tag{5.104}$$

which gives the peak-to-peak ripple current of inductor L_1 as

$$\Delta I_1 = \frac{-V_s(V_s - V_{c1})}{fL_1V_{c1}} \tag{5.105}$$

or

$$\Delta I_1 = \frac{V_s k}{f L_1} \tag{5.106}$$

The switching period T can also be found from Eqs. (5.96) and (5.98):

$$T = \frac{1}{f} = t_1 + t_2 = \frac{\Delta I_2 L_2}{V_{c1} + V_a} - \frac{\Delta I_2 L_2}{V_a} = \frac{-\Delta I_2 L_2 V_{c1}}{V_a(V_{c1} + V_a)} \tag{5.107}$$

$$\Delta I_2 = \frac{-V_a(V_{c1} + V_a)}{f L_2 V_{c1}} \tag{5.108}$$

or

$$\Delta I_2 = -\frac{V_a(1-k)}{f L_2} = \frac{k V_s}{f L_2} \tag{5.109}$$

When transistor Q_1 is off, the energy transfer capacitor C_1 is charged by the input current for time $t = t_2$. The average charging current for C_1 is $I_{c1} = I_s$ and the peak-to-peak ripple voltage of the capacitor C_1 is

$$\Delta V_{c1} = \frac{1}{C_1}\int_0^{t_2} I_{c1}\, dt = \frac{1}{C_1}\int_0^{t_2} I_s\, dt = \frac{I_s t_2}{C_1} \tag{5.110}$$

Equation (5.102) gives $t_2 = V_s/[(V_s - V_a)f]$ and Eq. (5.110) becomes

$$\Delta V_{c1} = \frac{I_s V_s}{(V_s - V_a) f C_1} \tag{5.111}$$

or

$$\Delta V_{c1} = \frac{I_s(1-k)}{f C_1} \tag{5.112}$$

If we assume that the load current ripple Δi_o is negligible, $\Delta i_{L2} = \Delta i_{c2}$. The average charging current of C_2, which flows for time $T/2$, is $I_{c2} = \Delta I_2/4$ and the peak-to-peak ripple voltage of capacitor C_2 is

$$\Delta V_{c2} = \frac{1}{C_2}\int_0^{T/2} I_{c2}\,dt = \frac{1}{C_2}\int_0^{T/2} \frac{\Delta I_2}{4}\,dt = \frac{\Delta I_2}{8fC_2} \tag{5.113}$$

or

$$\Delta V_{c2} = \frac{V_a(1-k)}{8C_2L_2f^2} = \frac{kV_s}{8C_2L_2f^2} \tag{5.114}$$

Condition for continuous inductor current and capacitor voltage. If I_{L1} is the average current of inductor L_1, the inductor ripple current $\Delta I_1 = 2I_{L1}$. Using Eqs. (5.103) and (5.106), we get

$$\frac{kV_S}{fL_1} = 2I_{L1} = 2I_S = \frac{2kI_a}{1-k} = 2\left(\frac{k}{1-k}\right)^2\frac{V_S}{R}$$

which gives the critical value of the inductor L_{c1} as

$$L_{c1} = L_1 = \frac{(1-k)^2R}{2kf} \tag{5.115}$$

If I_{L2} is the average current of inductor L_2, the inductor ripple current $\Delta I_1 = 2I_{L2}$. Using Eqs. (5.100) and (5.109), we get

$$\frac{kV_S}{fL_2} = 2I_{L2} = 2I_a = \frac{2V_a}{R} = \frac{2kV_S}{(1-k)R}$$

which gives the critical value of the inductor L_{c2} as

$$L_{c2} = L_2 = \frac{(1-k)R}{2f} \tag{5.116}$$

If V_{c1} is the average capacitor voltage, the capacitor ripple voltage $\Delta V_{c1} = 2V_a$. Using $\Delta V_{c1} = 2\,V_a$ into Eq. (5.112), we get

$$\frac{I_S(1-k)}{fC_1} = 2V_a = 2I_aR$$

which, after substituting for I_S, gives the critical value of the capacitor C_{c1} as

$$C_{c1} = C_1 = \frac{k}{2fR} \tag{5.117}$$

If V_{c2} is the average capacitor voltage, the capacitor ripple voltage $\Delta V_{c2} = 2V_a$. Using Eq. (5.100) and (5.114), we get

$$\frac{kV_S}{8C_2L_2f^2} = 2V_a = \frac{2kV_S}{1-k}$$

which, after substituting for L_2 from Eq. (5.116), gives the critical value of the capacitor C_{c2} as

$$C_{c2} = C_2 = \frac{1}{8fR} \tag{5.118}$$

The Cúk regulator is based on the capacitor energy transfer. As a result, the input current is continuous. The circuit has low switching losses and has high efficiency. When transistor Q_1 is turned on, it has to carry the currents of inductors L_1 and L_2. As a result a high peak current flows through transistor Q_1. Because the capacitor provides the energy transfer, the ripple current of the capacitor C_1 is also high. This circuit also requires an additional capacitor and inductor.

Example 5.8 Finding the Currents and Voltages in the Cúk Regulator

The input voltage of a Cúk converter in Figure 5.19a, $V_s = 12$ V. The duty cycle $k = 0.25$ and the switching frequency is 25 kHz. The filter inductance is $L_2 = 150\ \mu$H and filter capacitance is $C_2 = 220\ \mu$F. The energy transfer capacitance is $C_1 = 200\ \mu$F and inductance $L_1 = 180\ \mu$H. The average load current is $I_a = 1.25$ A. Determine (a) the average output voltage V_a; (b) the average input current I_s; (c) the peak-to-peak ripple current of inductor L_1, ΔI_1; (d) the peak-to-peak ripple voltage of capacitor C_1, ΔV_{c1}; (e) the peak-to-peak ripple current of inductor L_2, ΔI_2; (f) the peak-to-peak ripple voltage of capacitor C_2, ΔV_{c2}; and (g) the peak current of the transistor I_p.

Solution

$V_s = 12$ V, $k = 0.25$, $I_a = 1.25$ A, $f = 25$ kHz, $L_1 = 180\ \mu$H, $C_1 = 200\ \mu$F, $L_2 = 150\ \mu$H, and $C_2 = 220\ \mu$F.

a. From Eq. (5.100), $V_a = -0.25 \times 12/(1 - 0.25) = -4$ V.
b. From Eq. (5.103), $I_s = 1.25 \times 0.25/(1 - 0.25) = 0.42$ A.
c. From Eq. (5.106), $\Delta I_1 = 12 \times 0.25/(25{,}000 \times 180 \times 10^{-6}) = 0.67$ A.
d. From Eq. (5.112), $\Delta V_{c1} = 0.42 \times (1 - 0.25)/(25{,}000 \times 200 \times 10^{-6}) = 63$ mV.
e. From Eq. (5.109), $\Delta I_2 = 0.25 \times 12/(25{,}000 \times 150 \times 10^{-6}) = 0.8$ A.
f. From Eq. (5.113), $\Delta V_{c2} = 0.8/(8 \times 25{,}000 \times 220 \times 10^{-6}) = 18.18$ mV.
g. The average voltage across the diode can be found from

$$V_{dm} = -kV_{c1} = -V_a k \frac{1}{-k} = V_a \qquad (5.119)$$

For a lossless circuit, $I_{L2}V_{dm} = V_a I_a$ and the average value of the current in inductor L_2 is

$$I_{L2} = \frac{I_a V_a}{V_{dm}} = I_a \qquad (5.120)$$
$$= 1.25 \text{ A}$$

Therefore, the peak current of transistor is

$$I_p = I_s + \frac{\Delta I_1}{2} + I_{L2} + \frac{\Delta I_2}{2} = 0.42 + \frac{0.67}{2} + 1.25 + \frac{0.8}{2} = 2.405 \text{ A}$$