

**PROBLEM SOLUTIONS: Chapter 1****Problem 1.1**

Part (a):

$$\mathcal{R}_c = \frac{l_c}{\mu A_c} = \frac{l_c}{\mu_r \mu_0 A_c} = 0 \quad \text{A/Wb}$$

$$\mathcal{R}_g = \frac{g}{\mu_0 A_c} = 1.017 \times 10^6 \quad \text{A/Wb}$$

part (b):

$$\Phi = \frac{NI}{\mathcal{R}_c + \mathcal{R}_g} = 1.224 \times 10^{-4} \quad \text{Wb}$$

part (c):

$$\lambda = N\Phi = 1.016 \times 10^{-2} \quad \text{Wb}$$

part (d):

$$L = \frac{\lambda}{I} = 6.775 \quad \text{mH}$$

**Problem 1.2**

part (a):

$$\mathcal{R}_c = \frac{l_c}{\mu A_c} = \frac{l_c}{\mu_r \mu_0 A_c} = 1.591 \times 10^5 \quad \text{A/Wb}$$

$$\mathcal{R}_g = \frac{g}{\mu_0 A_c} = 1.017 \times 10^6 \quad \text{A/Wb}$$

part (b):

$$\Phi = \frac{NI}{\mathcal{R}_c + \mathcal{R}_g} = 1.059 \times 10^{-4} \quad \text{Wb}$$

part (c):

$$\lambda = N\Phi = 8.787 \times 10^{-3} \quad \text{Wb}$$

part (d):

$$L = \frac{\lambda}{I} = 5.858 \quad \text{mH}$$

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**Problem 1.3**

part (a):

$$N = \sqrt{\frac{Lg}{\mu_0 A_c}} = 110 \text{ turns}$$

part (b):

$$I = \frac{B_{\text{core}}}{\mu_0 N/g} = 16.6 \text{ A}$$

**Problem 1.4**

part (a):

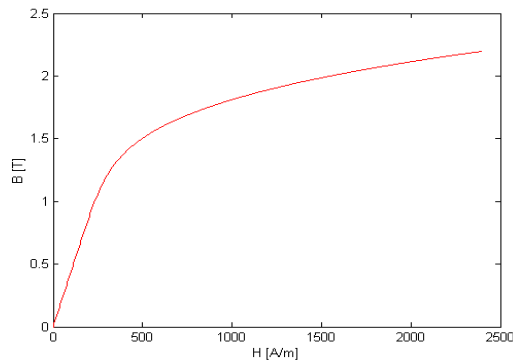
$$N = \sqrt{\frac{L(g + l_c \mu_0/\mu)}{\mu_0 A_c}} = \sqrt{\frac{L(g + l_c \mu_0/(\mu_r \mu_0))}{\mu_0 A_c}} = 121 \text{ turns}$$

part (b):

$$I = \frac{B_{\text{core}}}{\mu_0 N/(g + l_c \mu_0/\mu)} = 18.2 \text{ A}$$

**Problem 1.5**

part (a):

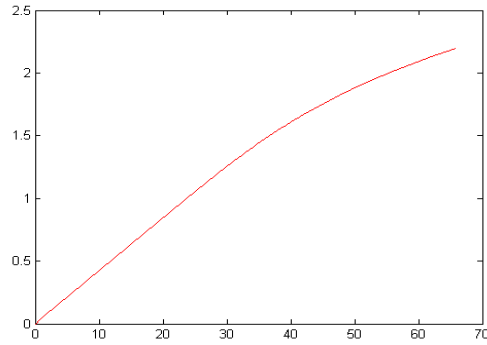


part (b):

$$\mu_r = 1 + \frac{3499}{\sqrt{1 + 0.047(2.2)^{7.8}}} = 730$$

$$I = B \left( \frac{g + \mu_0 l_c/\mu}{\mu_0 N} \right) = 65.8 \text{ A}$$

part (c):



### Problem 1.6

part (a):

$$H_g = \frac{NI}{2g}; \quad B_c = \left(\frac{A_g}{A_c}\right) B_g = B_g \left(1 - \frac{x}{X_0}\right)$$

part (b): Equations

$$2gH_g + H_c l_c = NI; \quad B_g A_g = B_c A_c$$

and

$$B_g = \mu_0 H_g; \quad B_c = \mu H_c$$

can be combined to give

$$B_g = \left(\frac{NI}{2g + \left(\frac{\mu_0}{\mu}\right) \left(\frac{A_g}{A_c}\right) (l_c + l_p)}\right) = \left(\frac{NI}{2g + \left(\frac{\mu_0}{\mu}\right) \left(1 - \frac{x}{X_0}\right) (l_c + l_p)}\right)$$

### Problem 1.7

part (a):

$$I = B \left(\frac{g + \left(\frac{\mu_0}{\mu}\right) (l_c + l_p)}{\mu_0 N}\right) = 2.15 \text{ A}$$

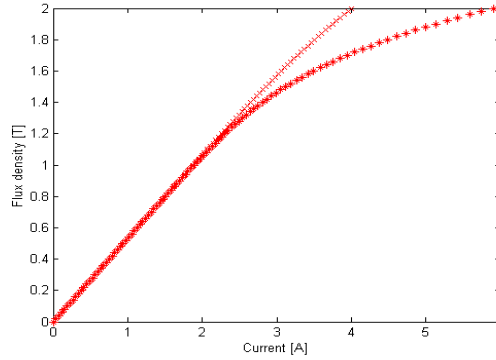
part (b):

$$\mu = \mu_0 \left(1 + \frac{1199}{\sqrt{1 + 0.05B^8}}\right) = 1012 \mu_0$$

$$I = B \left(\frac{g + \left(\frac{\mu_0}{\mu}\right) (l_c + l_p)}{\mu_0 N}\right) = 3.02 \text{ A}$$

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part (c):

**Problem 1.8**

$$g = \left( \frac{\mu_0 N^2 A_c}{L} \right) - \left( \frac{\mu_0}{\mu} \right) l_c = 0.353 \text{ mm}$$

**Problem 1.9**

part (a):

$$l_c = 2\pi(R_o - R_i) - g = 3.57 \text{ cm}; \quad A_c = (R_o - R_i)h = 1.2 \text{ cm}^2$$

part (b):

$$\mathcal{R}_g = \frac{g}{\mu_0 A_c} = 1.33 \times 10^7 \text{ A/Wb}; \quad \mathcal{R}_c = 0 \text{ A/Wb};$$

part (c):

$$L = \frac{N^2}{\mathcal{R}_g + \mathcal{R}_c} = 0.319 \text{ mH}$$

part (d):

$$I = \frac{B_g(\mathcal{R}_c + \mathcal{R}_g)A_c}{N} = 33.1 \text{ A}$$

part (e):

$$\lambda = NB_g A_c = 10.5 \text{ mWb}$$

**Problem 1.10**

part (a): Same as Problem 1.9

part (b):

$$\mathcal{R}_g = \frac{g}{\mu_0 A_c} = 1.33 \times 10^7 \text{ A/Wb}; \quad \mathcal{R}_c = \frac{l_c}{\mu A_c} = 3.16 \times 10^5 \text{ A/Wb}$$

part (c):

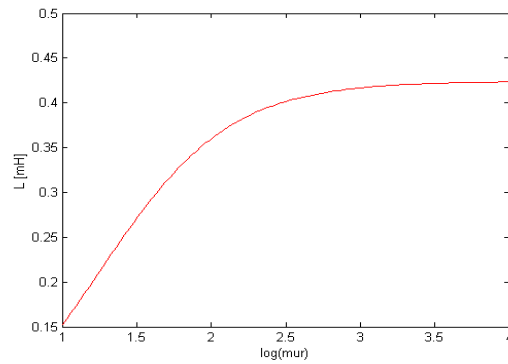
$$L = \frac{N^2}{\mathcal{R}_g + \mathcal{R}_g} = 0.311 \text{ mH}$$

part (d):

$$I = \frac{B_g(\mathcal{R}_c + \mathcal{R}_g)A_c}{N} = 33.8 \text{ A}$$

part (e): Same as Problem 1.9.

### Problem 1.11



Minimum  $\mu_r = 340$ .

### Problem 1.12

$$L = \frac{\mu_0 N^2 A_c}{g + l_c / \mu_r}$$

### Problem 1.13

$$L = \frac{\mu_0 N^2 A_c}{g + l_c / \mu_r} = 30.5 \text{ mH}$$

### Problem 1.14

part (a):

$$V_{\text{rms}} = \frac{\omega N A_c B_{\text{peak}}}{\sqrt{2}} = 19.2 \text{ V rms}$$

part (b):

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{\omega L} = 1.67 \text{ A rms}; \quad W_{\text{peak}} = 0.5L(\sqrt{2} I_{\text{rms}})^2 = 8.50 \text{ mJ}$$

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**Problem 1.15**

part (a):

$$R_3 = \sqrt{R_1^2 + R_2^2} = 4.27 \text{ cm}$$

part (b):

$$L = \frac{\mu_0 A_g N^2}{g + \left(\frac{\mu_0}{\mu}\right) l_c} = 251 \text{ mH}$$

part (c): For  $\omega = 2\pi 60$  rad/sec and  $\lambda_{\text{peak}} = N A_g B_{\text{peak}} = 0.452$  Wb:

$$(i) \quad V_{\text{rms}} = \omega \lambda_{\text{peak}} = 171 \text{ V rms}$$

$$(ii) \quad I_{\text{rms}} = \frac{V_{\text{rms}}}{\omega L} = 1.81 \text{ A rms}$$

$$(iii) \quad W_{\text{peak}} = 0.5L(\sqrt{2}I_{\text{rms}})^2 = 0.817 \text{ J}$$

part (d): For  $\omega = 2\pi 50$  rad/sec and  $\lambda_{\text{peak}} = N A_g B_{\text{peak}} = 0.452$  Wb:

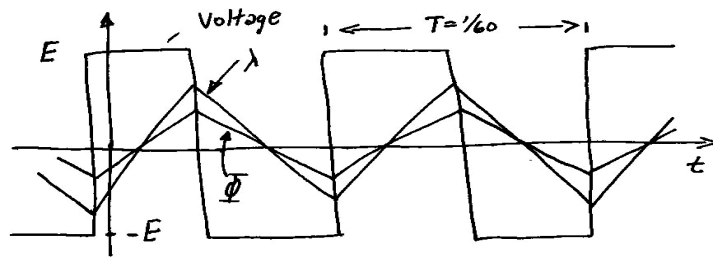
$$(i) \quad V_{\text{rms}} = \omega \lambda_{\text{peak}} = 142 \text{ V rms}$$

$$(ii) \quad I_{\text{rms}} = \frac{V_{\text{rms}}}{\omega L} = 1.81 \text{ A rms}$$

$$(iii) \quad W_{\text{peak}} = 0.5L(\sqrt{2}I_{\text{rms}})^2 = 0.817 \text{ J}$$

**Problem 1.16**

part (a):



part (b):

$$E_{\max} = 4fNA_cB_{\text{peak}} = 345 \text{ V}$$

**Problem 1.17**

part (a):

$$N = \frac{LI}{A_c B_{\text{sat}}} = 99 \text{ turns}; \quad g = \frac{\mu_0 NI}{B_{\text{sat}}} - \frac{\mu_0 l_c}{\mu} = 0.36 \text{ mm}$$

part (b): From Eq.3.21

$$W_{\text{gap}} = \frac{A_c g B_{\text{sat}}^2}{2\mu_0} = 0.207 \text{ J}; \quad W_{\text{core}} = \frac{A_c l_c B_{\text{sat}}^2}{2\mu} = 0.045 \text{ J}$$

Thus  $W_{\text{tot}} = W_{\text{gap}} + W_{\text{core}} = 0.252 \text{ J}$ . From Eq. 1.47,  $(1/2)LI^2 = 0.252 \text{ J}$ .  
Q.E.D.

**Problem 1.18**

part (a): Minimum inductance = 4 mH, for which  $g = 0.0627 \text{ mm}$ ,  $N = 20$  turns and  $V_{\text{rms}} = 6.78 \text{ V}$

part (b): Maximum inductance = 144 mH, for which  $g = 4.99 \text{ mm}$ ,  $N = 1078$  turns and  $V_{\text{rms}} = 224 \text{ V}$

**Problem 1.19**

part (a):

$$L = \frac{\mu_0 \pi a^2 N^2}{2\pi r} = 56.0 \text{ mH}$$

part (b): Core volume  $V_{\text{core}} \approx (2\pi r)\pi a^2 = 40.0 \text{ m}^3$ . Thus

$$W = V_{\text{core}} \left( \frac{B^2}{2\mu_0} \right) = 4.87 \text{ J}$$

part (c): For  $T = 30 \text{ sec}$ ,

$$\frac{di}{dt} = \frac{(2\pi r B)/(\mu_0 N)}{T} = 2.92 \times 10^3 \text{ A/sec}$$

$$v = L \frac{di}{dt} = 163 \text{ V}$$

**Problem 1.20**

part (a):

$$A_{\text{cu}} = f_w ab; \quad \text{Vol}_{\text{cu}} = 2ab(w + h + 2a)$$

part (b):

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$$B = \mu_0 \left( \frac{J_{\text{cu}} A_{\text{cu}}}{g} \right)$$

part (c):

$$J_{\text{cu}} = \frac{NI}{A_{\text{cu}}}$$

part (d):

$$P_{\text{diss}} = \text{Vol}_{\text{cu}} (\rho J_{\text{cu}}^2)$$

part (e):

$$W_{\text{mag}} = \text{Vol}_{\text{gap}} \left( \frac{B^2}{2\mu_0} \right) = gwh \left( \frac{B^2}{2\mu_0} \right)$$

part (f):

$$\frac{L}{R} = \frac{\left(\frac{1}{2}\right) LI^2}{\left(\frac{1}{2}\right) RI^2} = \frac{W_{\text{mag}}}{P_{\text{diss}}} = \frac{2W_{\text{mag}}}{P_{\text{diss}}} = \frac{\mu_0 wh A_{\text{cu}}^2}{\rho g \text{Vol}_{\text{cu}}}$$

**Problem 1.21**

Using the equations of Problem 1.20

$$P_{\text{diss}} = 115 \text{ W}$$

$$I = 3.24 \text{ A}$$

$$N = 687 \text{ turns}$$

$$R = 10.8 \Omega$$

$$\tau = 6.18 \text{ msec}$$

$$\text{Wire size} = 23 \text{ AWG}$$

**Problem 1.22**

part (a):

$$(i) \quad B_1 = \frac{\mu_0 N_1 I_1}{g_1}; \quad B_2 = \frac{\mu_0 N_1 I_1}{g_2}$$

$$(ii) \quad \lambda_1 = N_1 (A_1 B_1 + A_2 B_2) = \mu_0 N_1^2 \left( \frac{A_1}{g_1} + \frac{A_2}{g_2} \right) I_1$$

$$(iii) \quad \lambda_2 = N_2 A_2 B_2 = \mu_0 N_1 N_2 \left( \frac{A_2}{g_2} \right) I_1$$



part (b):

$$(i) \quad B_1 = 0; \quad B_2 = \frac{\mu_0 N_2 I_2}{g_2}$$

$$(ii) \quad \lambda_1 = N_1 A_2 B_2 = \mu_0 N_1 N_2 \left( \frac{A_2}{g_2} \right) I_2$$

$$(iii) \quad \lambda_2 = N_2 A_2 B_2 = \mu_0 N_2^2 \left( \frac{A_2}{g_2} \right) I_2$$

part (c):

$$(i) \quad B_1 = \frac{\mu_0 N_1 I_1}{g_1}; \quad B_2 = \frac{\mu_0 N_1 I_1}{g_2} + \frac{\mu_0 N_2 I_2}{g_2}$$

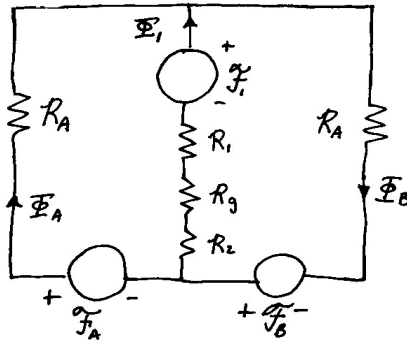
$$(ii) \quad \lambda_1 = N_1 (A_1 B_1 + A_2 B_2) = \mu_0 N_1^2 \left( \frac{A_1}{g_1} + \frac{A_2}{g_2} \right) I_1 + \mu_0 N_1 N_2 \left( \frac{A_2}{g_2} \right) I_2$$

$$(iii) \quad \lambda_2 = N_2 A_2 B_2 = \mu_0 N_1 N_2 \left( \frac{A_2}{g_2} \right) I_1 + \mu_0 N_2^2 \left( \frac{A_2}{g_2} \right) I_2$$

part (d):

$$L_{11} = N_1^2 \left( \frac{A_1}{g_1} + \frac{A_2}{g_2} \right); \quad L_{22} = \mu_0 N_2^2 \left( \frac{A_2}{g_2} \right); \quad L_{12} = \mu_0 N_1 N_2 \left( \frac{A_2}{g_2} \right)$$

### Problem 1.23



$$\mathcal{R}_A = \frac{l_A}{\mu A_c}; \quad \mathcal{R}_1 = \frac{l_1}{\mu A_c}; \quad \mathcal{R}_2 = \frac{l_2}{\mu A_c}; \quad \mathcal{R}_g = \frac{g}{\mu_0 A_c}$$

part (a):

$$L_{11} = \frac{N_1^2}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_g + \mathcal{R}_A/2} = \frac{N_1^2 \mu A_c}{l_1 + l_2 + l_A/2 + g (\mu/\mu_0)}$$

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$$L_{AA} = L_{BB} = \frac{N^2}{\mathcal{R}_A + \mathcal{R}_A || (\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_g)} = \frac{N^2 \mu A_c}{l_A} \left[ \frac{l_A + l_1 + l_2 + g (\mu/\mu_0)}{l_A + 2(l_1 + l_2 + g (\mu/\mu_0))} \right]$$

part (b):

$$L_{AB} = L_{BA} = \frac{N^2 (\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_g)}{\mathcal{R}_A (\mathcal{R}_A + 2(\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_g))} = \frac{N^2 \mu A_c}{l_A} \left[ \frac{l_1 + l_2 + g (\mu/\mu_0)}{l_A + 2(l_1 + l_2 + g (\mu/\mu_0))} \right]$$

$$L_{A1} = L_{1A} = -L_{B1} = -L_{1B} = \frac{-NN_1}{\mathcal{R}_A + 2(\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_g)} = \frac{-NN_1 \mu A_c}{l_A + 2(l_1 + l_2 + g (\mu/\mu_0))}$$

part (c):

$$v_1 = \frac{d}{dt} [L_{A1} i_A + L_{B1} i_B] = L_{A1} \frac{d}{dt} [i_A - i_B]$$

Q.E.D.

**Problem 1.24**

part (a):

$$L_{12} = \frac{\mu_0 N_1 N_2}{2g} [D(w - x)]$$

part (b):

$$\begin{aligned} v_2 &= \frac{d\lambda_2}{dt} = I_0 \frac{dL_{12}}{dt} = - \left( \frac{N_1 N_2 \mu_0 D}{2g} \right) \frac{dx}{dt} \\ &= - \left( \frac{N_1 N_2 \mu_0 D}{2g} \right) \left( \frac{\epsilon \omega w}{2} \right) \cos \omega t \end{aligned}$$

**Problem 1.25**

part (a):

$$H = \frac{N_1 i_1}{2\pi(R_o + R_i)/2} = \frac{N_1 i_1}{\pi(R_o + R_i)}$$

part (b):

$$v_2 = \frac{d}{dt} [N_2 (tn\Delta) B] = N_2 tn\Delta \frac{dB}{dt}$$

part (c):

$$v_o = G \int v_2 dt = GN_2 tn\Delta B$$

**Problem 1.26**

$$\mathcal{R}_g = \frac{g}{\mu_0 A_g} = 4.42 \times 10^5 \text{ A/Wb}; \quad \mathcal{R}_c = \frac{l_c}{\mu A_g} = \frac{333}{\mu} \text{ A/Wb}$$

Want  $\mathcal{R}_g \leq 0.05\mathcal{R}_c \Rightarrow \mu \geq 1.2 \times 10^4 \mu_0$ . By inspection of Fig. 1.10, this will be true for  $B \leq 1.66 \text{ T}$  (approximate since the curve isn't that detailed).

**Problem 1.27**

part (a):

$$N_1 = \frac{V_{\text{peak}}}{\omega t (R_o - R_i) B_{\text{peak}}} = 57 \text{ turns}$$

part (b):

$$(i) \quad B_{\text{peak}} = \frac{V_{o,\text{peak}}}{GN_2 t (R_o - R_i)} = 0.833 \text{ T}$$

$$(ii) \quad V_1 = N_1 t (R_o - R_i) \omega B_{\text{peak}} = 6.25 \text{ V, peak}$$

**Problem 1.28**

part (a): From the M-5 magnetization curve, for  $B = 1.2 \text{ T}$ ,  $H_m = 14 \text{ A/m}$ . Similarly,  $H_g = B/\mu_0 = 9.54 \times 10^5 \text{ A/m}$ . Thus, with  $I_1 = I_2 = I$

$$I = \frac{H_m(l_A + l_C - g) + H_g g}{N_1} = 38.2 \text{ A}$$

part (b):

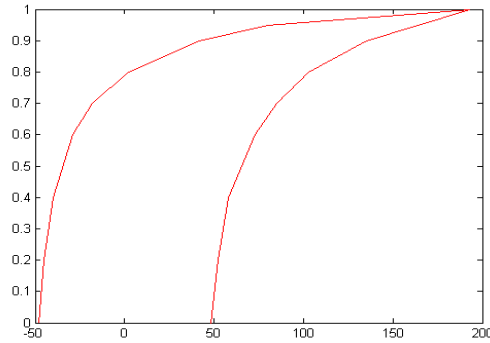
$$W_{\text{gap}} = \frac{g A_{\text{gap}} B^2}{2\mu_0} = 3.21 \text{ Joules}$$

part (c):

$$\lambda = 2N_1 A_A B = 0.168 \text{ Wb}; \quad L = \frac{\lambda}{I} = 4.39 \text{ mH}$$

**Problem 1.29**

part (a):



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- part (b): Area = 191 Joules  
 part (c): Core loss = 1.50 W/kg.

**Problem 1.30**

$B_{\text{rms}} = 1.1 \text{ T}$  and  $f = 60 \text{ Hz}$ ,

$$V_{\text{rms}} = \omega N A_c B_{\text{rms}} = 46.7 \text{ V}$$

Core volume =  $A_c l_c = 1.05 \times 10^{-3} \text{ m}^3$ . Mass density =  $7.65 \times 10^3 \text{ kg/m}^3$ . Thus, the core mass =  $(1.05 \times 10^{-3})(7.65 \times 10^3) = 8.03 \text{ kg}$ .

At  $B = 1.1 \text{ T rms} = 1.56 \text{ T peak}$ , core loss density =  $1.3 \text{ W/kg}$  and rms VA density is  $2.0 \text{ VA/kg}$ . Thus, the core loss =  $1.3 \times 8.03 = 10.4 \text{ W}$ . The total exciting VA for the core is  $2.0 \times 8.03 = 16.0 \text{ VA}$ . Thus, its reactive component is given by  $\sqrt{16.0^2 - 10.4^2} = 12.2 \text{ VAR}$ .

The rms energy storage in the air gap is

$$W_{\text{gap}} = \frac{g A_c B_{\text{rms}}^2}{\mu_0} = 3.61 \text{ Joules}$$

corresponding to an rms reactive power of

$$\text{VAR}_{\text{gap}} = \omega W_{\text{gap}} = 1361 \text{ Joules}$$

Thus, the total rms exciting VA for the magnetic circuit is

$$\text{VA}_{\text{rms}} = \text{sqrt}10.4^2 + (1361 + 12.2)^2 = 1373 \text{ VA}$$

and the rms current is  $I_{\text{rms}} = \text{VA}_{\text{rms}}/V_{\text{rms}} = 29.4 \text{ A}$ .

**Problem 1.31**

part(a): Area increases by a factor of 4. Thus the voltage increases by a factor of 4 to  $e = 1096 \cos 377t$ .

part (b):  $l_c$  doubles therefore so does the current. Thus  $I = 0.26 \text{ A}$ .

part (c): Volume increases by a factor of 8 and voltage increases by a factor of 4. There  $I_{\phi, \text{rms}}$  doubles to  $0.20 \text{ A}$ .

part (d): Volume increases by a factor of 8 as does the core loss. Thus  $P_c = 128 \text{ W}$ .

**Problem 1.32**

From Fig. 1.19, the maximum energy product for samarium-cobalt occurs at (approximately)  $B = 0.47 \text{ T}$  and  $H = -360 \text{ kA/m}$ . Thus the maximum energy product is  $1.69 \times 10^5 \text{ J/m}^3$ .

Thus,

$$A_m = \left( \frac{0.8}{0.47} \right) 2 \text{ cm}^2 = 3.40 \text{ cm}^2$$

and

$$l_m = -0.2 \text{ cm} \left( \frac{0.8}{\mu_0(-3.60 \times 10^5)} \right) = 0.35 \text{ cm}$$

Thus the volume is  $3.40 \times 0.35 = 1.20 \text{ cm}^3$ , which is a reduction by a factor of  $5.09/1.21 = 4.9$ .

**Problem 1.33**

From Fig. 1.19, the maximum energy product for neodymium-iron-boron occurs at (approximately)  $B = 0.63 \text{ T}$  and  $H = -470 \text{ kA/m}$ . Thus the maximum energy product is  $2.90 \times 10^5 \text{ J/m}^3$ .

Thus,

$$A_m = \left( \frac{0.8}{0.63} \right) 2 \text{ cm}^2 = 2.54 \text{ cm}^2$$

and

$$l_m = -0.2 \text{ cm} \left( \frac{0.8}{\mu_0(-4.70 \times 10^5)} \right) = 0.27 \text{ cm}$$

Thus the volume is  $2.54 \times 0.25 = 0.688 \text{ cm}^3$ , which is a reduction by a factor of  $5.09/0.688 = 7.4$ .

**Problem 1.34**

From Fig. 1.19, the maximum energy product for samarium-cobalt occurs at (approximately)  $B = 0.47 \text{ T}$  and  $H = -360 \text{ kA/m}$ . Thus the maximum energy product is  $1.69 \times 10^5 \text{ J/m}^3$ . Thus, we want  $B_g = 1.2 \text{ T}$ ,  $B_m = 0.47 \text{ T}$  and  $H_m = -360 \text{ kA/m}$ .

$$h_m = -g \left( \frac{H_g}{H_m} \right) = -g \left( \frac{B_g}{\mu_0 H_m} \right) = 2.65 \text{ mm}$$

$$A_m = A_g \left( \frac{B_g}{B_m} \right) = 2\pi R h \left( \frac{B_g}{B_m} \right) = 26.0 \text{ cm}^2$$

$$R_m = \sqrt{\frac{A_m}{\pi}} = 2.87 \text{ cm}$$

**Problem 1.35**

From Fig. 1.19, the maximum energy product for neodymium-iron-boron occurs at (approximately)  $B_m = 0.63 \text{ T}$  and  $H_m = -470 \text{ kA/m}$ . The magnetization curve for neodymium-iron-boron can be represented as

$$B_m = \mu_R H_m + B_r$$

where  $B_r = 1.26 \text{ T}$  and  $\mu_R = 1.067\mu_0$ . The magnetic circuit must satisfy

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$$H_m d + H_g g = Ni; \quad B_m A_m = B_g A_g$$

part (a): For  $i = 0$  and  $B_g = 0.5$  T, the minimum magnet volume will occur when the magnet is operating at the maximum energy point.

$$A_m = \left( \frac{B_g}{B_m} \right) A_g = 4.76 \text{ cm}^2$$

$$d = - \left( \frac{H_g}{H_m} \right) g = 1.69 \text{ mm}$$

part (b):

$$i = \frac{\left[ B_g \left( \frac{d A_g}{\mu_R A_m} + \frac{g}{\mu_0} \right) - \frac{B_g d}{\mu_R} \right]}{N}$$

For  $B_g = 0.75$ ,  $i = 17.9$  A.

For  $B_g = 0.25$ ,  $i = 6.0$  A.

Because the neodymium-iron-boron magnet is essentially linear over the operating range of this problem, the system is linear and hence a sinusoidal flux variation will correspond to a sinusoidal current variation.

## PROBLEM SOLUTIONS: Chapter 2

### Problem 2.1

At 60 Hz,  $\omega = 120\pi$ .

$$\text{primary: } (V_{\text{rms}})_{\text{max}} = N_1\omega A_c (B_{\text{rms}})_{\text{max}} = 2755 \text{ V, rms}$$

$$\text{secondary: } (V_{\text{rms}})_{\text{max}} = N_2\omega A_c (B_{\text{rms}})_{\text{max}} = 172 \text{ V, rms}$$

At 50 Hz,  $\omega = 100\pi$ . Primary voltage is 2295 V, rms and secondary voltage is 143 V, rms.

### Problem 2.2

$$N = \frac{\sqrt{2}V_{\text{rms}}}{\omega A_c B_{\text{peak}}} = 167 \text{ turns}$$

### Problem 2.3

$$N = \sqrt{\frac{75}{8}} = 3 \text{ turns}$$

### Problem 2.4

Resistance seen at primary is  $R_1 = (N_1/N_2)^2 R_2 = 6.25\Omega$ . Thus

$$I_1 = \frac{V_1}{R_1} = 1.6 \text{ A}$$

and

$$V_2 = \left(\frac{N_2}{N_1}\right) V_1 = 40 \text{ V}$$

### Problem 2.5

The maximum power will be supplied to the load resistor when its impedance, as reflected to the primary of the ideal transformer, equals that of the source (2 k $\Omega$ ). Thus the transformer turns ratio  $N$  to give maximum power must be

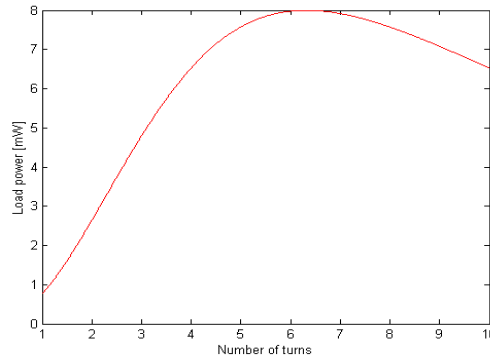
$$N = \sqrt{\frac{R_s}{R_{\text{load}}}} = 6.32$$

Under these conditions, the source voltage will see a total resistance of  $R_{\text{tot}} = 4 \text{ k}\Omega$  and the current will thus equal  $I = V_s/R_{\text{tot}} = 2 \text{ mA}$ . Thus, the power delivered to the load will equal

$$P_{\text{load}} = I^2(N^2 R_{\text{load}}) = 8 \text{ mW}$$

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Here is the desired MATLAB plot:



### Problem 2.6

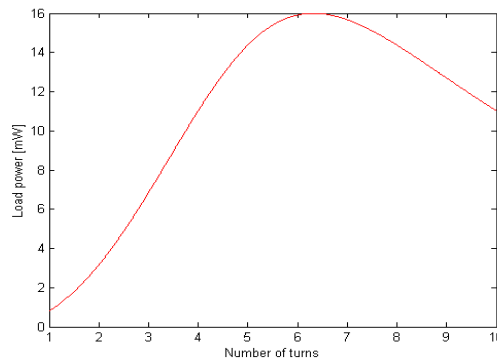
The maximum power will be supplied to the load resistor when its impedance, as reflected to the primary of the ideal transformer, equals that of the source ( $2 \text{ k}\Omega$ ). Thus the transformer turns ratio  $N$  to give maximum power must be

$$N = \sqrt{\frac{R_s}{R_{\text{load}}}} = 6.32$$

Under these conditions, the source voltage will see a total impedance of  $Z_{\text{tot}} = 2 + j2 \text{ k}\Omega$  whose magnitude is  $2\sqrt{2} \text{ k}\Omega$ . The current will thus equal  $I = V_s/|Z_{\text{tot}}| = 2\sqrt{2} \text{ mA}$ . Thus, the power delivered to the load will equal

$$P_{\text{load}} = I^2(N^2 R_{\text{load}}) = 16 \text{ mW}$$

Here is the desired MATLAB plot:





**Problem 2.7**

$$V_2 = V_1 \left( \frac{X_m}{X_{l_1} + X_m} \right) = 266 \text{ V}$$

**Problem 2.8**

part (a): Referred to the secondary

$$L_{m,2} = \frac{L_{m,1}}{N^2} = 150 \text{ mH}$$

part(b): Referred to the secondary,  $X_m = \omega L_{m,2} = 56.7 \Omega$ ,  $X_{l_2} = 84.8 \text{ m}\Omega$  and  $X_{l_1} = 69.3 \text{ m}\Omega$ . Thus,

$$(i) \quad V_1 = N \left( \frac{X_m}{X_m + X_{l_2}} \right) V_2 = 7960 \text{ V}$$

and

$$(ii) \quad I_{sc} = \frac{V_2}{X_{sc}} = \frac{V_2}{X_{l_2} + X_m || X_{l_1}} = 1730 \text{ A}$$

**Problem 2.9**

part (a):

$$I_1 = \frac{V_1}{X_{l_1} + X_m} = 3.47 \text{ A}; \quad V_2 = NV_1 \left( \frac{X_m}{X_{l_1} + X_m} \right) = 2398 \text{ V}$$

part (b): Let  $X'_{l_2} = X_{l_2}/N^2$  and  $X_{sc} = X_{l_1} + X_m || (X_m + X'_{l_2})$ . For  $I_{rated} = 50 \text{ kVA}/120 \text{ V} = 417 \text{ A}$

$$V_1 = I_{rated} X_{sc} = 23.1 \text{ V}$$

$$I_2 = \frac{1}{N} \left( \frac{X_m}{X_m + X_{l_2}} \right) I_{rated} = 15.7 \text{ A}$$

**Problem 2.10**

$$I_L = \frac{P_{load}}{V_L} = 55.5 \text{ A}$$

and thus

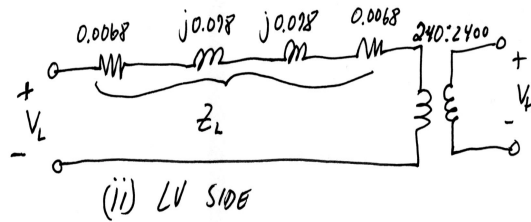
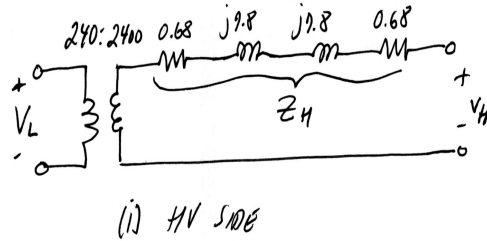
$$I_H = \frac{I_L}{N} = 10.6 \text{ A}; \quad V_H = NV_L + jX_H I_H = 2381 \angle 9.6^\circ \text{ V}$$

The power factor is  $\cos(9.6^\circ) = 0.986$  lagging.

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**Problem 2.11**

part (a):



part (b):

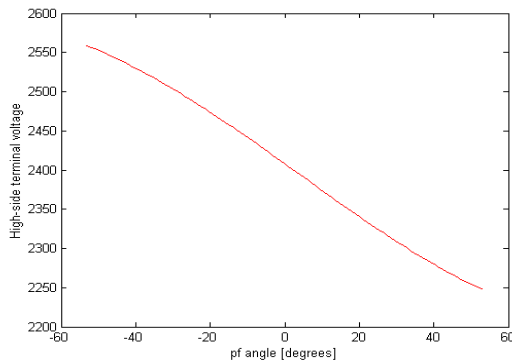
$$\hat{I}_{\text{load}} = \frac{30 \text{ kW}}{230 \text{ V}} e^{j\phi} = 93.8 e^{j\phi} \text{ A}$$

where  $\phi$  is the power-factor angle. Referred to the high voltage side,  $\hat{I}_H = 9.38 e^{j\phi} \text{ A}$ .

$$\hat{V}_H = Z_H \hat{I}_H$$

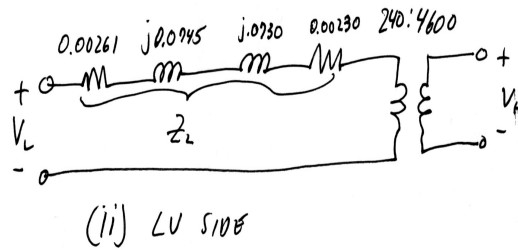
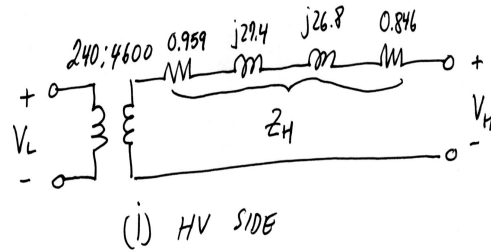
Thus, (i) for a power factor of 0.85 lagging,  $V_H = 2413 \text{ V}$  and (ii) for a power factor of 0.85 leading,  $V_H = 2199 \text{ V}$ .

part (c):



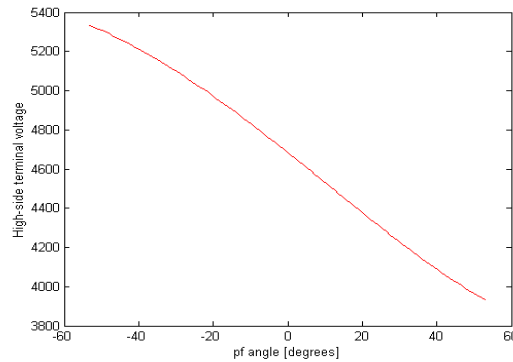
**Problem 2.12**

part (a):



part (b): Following methodology of Problem 2.11, (i) for a power factor of 0.85 lagging,  $V_H = 4956$  V and (ii) for a power factor of 0.85 leading,  $V_H = 4000$  V.

part (c):



**Problem 2.13**

part (a):  $I_{load} = 160 \text{ kW} / 2340 \text{ V} = 68.4 \text{ A}$  at  $\angle = \cos^{-1}(0.89) = 27.1^\circ$

$$\hat{V}_{t,H} = N(\hat{V}_L + Z_t I_L)$$

which gives  $V_H = 33.7 \text{ kV}$ .

part (b):

$$\hat{V}_{send} = N(\hat{V}_L + (Z_t + Z_f) I_L)$$

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which gives  $V_{\text{send}} = 33.4$  kV.

part (c):

$$S_{\text{send}} = P_{\text{send}} + jQ_{\text{send}} = \hat{V}_{\text{send}} \hat{I}_{\text{send}}^* = 164 \text{ kW} - j64.5 \text{ kVAR}$$

Thus  $P_{\text{send}} = 164$  kW and  $Q_{\text{send}} = -64.5$  kVAR.

**Problem 2.14**

Following the methodology of Example 2.6, efficiency = 98.4 percent and regulation = 1.25 percent.

**Problem 2.15**

part (a):

$$|Z_{\text{eq,L}}| = \frac{V_{\text{sc,L}}}{I_{\text{sc,L}}} = 107.8 \text{ m}\Omega$$

$$R_{\text{eq,L}} = \frac{P_{\text{sc,L}}}{I_{\text{sc,L}}^2} = 4.78 \text{ m}\Omega$$

$$X_{\text{eq,L}} = \sqrt{|Z_{\text{eq,L}}|^2 - R_{\text{eq,L}}^2} = 107.7 \text{ m}\Omega$$

and thus

$$Z_{\text{eq,L}} = 4.8 + j108 \text{ m}\Omega$$

part (b):

$$R_{\text{eq,H}} = N^2 R_{\text{eq,L}} = 0.455 \text{ }\Omega$$

$$X_{\text{eq,H}} = N^2 X_{\text{eq,L}} = 10.24 \text{ }\Omega$$

$$Z_{\text{eq,H}} = 10.3 + j0.46 \text{ m}\Omega$$

part (c): From the open-circuit test, the core-loss resistance and the magnetizing reactance as referred to the low-voltage side can be found:

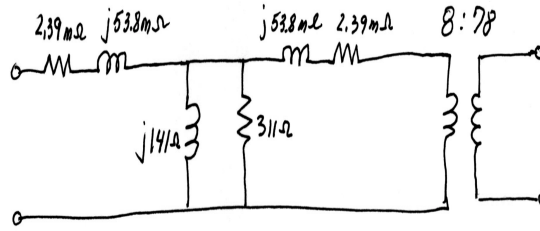
$$R_{\text{c,L}} = \frac{V_{\text{oc,L}}^2}{P_{\text{oc,L}}} = 311 \text{ }\Omega$$

$$S_{\text{oc,L}} = V_{\text{oc,L}} I_{\text{oc,L}} = 497 \text{ kVA}; \quad Q_{\text{oc,L}} = \sqrt{S_{\text{oc,L}}^2 - P_{\text{oc,L}}^2} = 45.2 \text{ kVAR}$$

and thus

$$X_{m,L} = \frac{V_{oc,L}^2}{Q_{oc,L}} = 141 \Omega$$

The equivalent-T circuit for the transformer from the low-voltage side is thus:



part (d): We will solve this problem with the load connected to the high-voltage side but referred to the low-voltage side. The rated low-voltage current is  $I_L = 50 \text{ MVA}/8 \text{ kV} = 6.25 \text{ kA}$ . Assume the load is at rated voltage. Thus the low-voltage terminal voltage is

$$V_L = |V_{load} + Z_{eq,L}I_L| = 8.058 \text{ kV}$$

and thus the regulation is given by  $(8.053-8)/8 = 0.0072 = 0.72$  percent.

The total loss is approximately equal to the sum of the open-circuit loss and the short-circuit loss (393 kW). Thus the efficiency is given by

$$\eta = \frac{P_{load}}{P_{in}} = \frac{50.0}{50.39} = 0.992 = 99.2 \text{ percent}$$

part (e): We will again solve this problem with the load connected to the high-voltage side but referred to the low-voltage side. Now,  $\hat{I}_L = 6.25 \angle 25.8^\circ \text{ kA}$ . Assume the load is at rated voltage. Thus the low-voltage terminal voltage is

$$V_L = |V_{load} + Z_{eq,L}\hat{I}_L| = 7.758 \text{ kV}$$

and thus the regulation is given by  $(7.758-8)/8 = -0.0302 = -3.02$  percent. The efficiency is the same as that found in part (d),  $\eta = 99.2$  percent.

### Problem 2.16

The core length of the second transformer is  $\sqrt{2}$  times that of the first, its core area of the second transformer is twice that of the first, and its volume is  $2\sqrt{2}$  times that of the first. Since the voltage applied to the second transformer is twice that of the first, the flux densities will be the same. Hence, the core loss will be proportional to the volume and

$$\text{Coreloss} = 2\sqrt{2}3420 = 9.67 \text{ kW}$$

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The magnetizing inductance is proportional to the area and inversely proportional to the core length and hence is  $\sqrt{2}$  times larger. Thus the no-load magnetizing current will be  $\sqrt{2}$  times larger in the second transformer or

$$I_{\text{no-load}} = \sqrt{2} 4.93 = 6.97 \text{ A}$$

**Problem 2.17**

part (a): Rated current at the high-voltage side is  $20 \text{ kVA}/2.4 \text{ kV} = 8.33 \text{ A}$ . Thus the total loss will be  $P_{\text{loss}} = 122 + 257 = 379 \text{ W}$ . The load power is equal to  $0.8 \times 20 = 16 \text{ kW}$ . Thus the efficiency is

$$\eta = \frac{16}{16.379} = 0.977 = 97.7 \text{ percent}$$

part (b): First calculate the series impedance ( $Z_{\text{eq,H}} = R_{\text{eq,H}} + jX_{\text{eq,H}}$ ) of the transformer from the short-circuit test data.

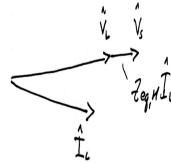
$$R_{\text{eq,H}} = \frac{P_{\text{sc,H}}}{I_{\text{sc,H}}^2} = 3.69 \Omega$$

$$S_{\text{sc,H}} = V_{\text{sc,H}} I_{\text{sc,H}} = 61.3 \times 8.33 = 511 \text{ kVA}$$

Thus  $Q_{\text{sc,H}} = \sqrt{S_{\text{sc,H}}^2 - P_{\text{sc,H}}^2} = 442 \text{ VAR}$  and hence

$$X_{\text{eq,H}} = \frac{Q_{\text{sc,H}}}{I_{\text{sc,H}}^2} = 6.35 \Omega$$

The regulation will be greatest when the primary and secondary voltages of the transformer are in phase as shown in the following phasor diagram



Thus the voltage drop across the transformer will be equal to  $\Delta V = |I_{\text{load}}| |Z_{\text{eq,H}}| = 61.2 \text{ V}$  and the regulation will equal  $61.2 \text{ V}/2.4 \text{ kV} = 0.026 = 2.6 \text{ percent}$ .

**Problem 2.18**

For a power factor of 0.87 leading, the efficiency is 98.4 percent and the regulation will equal -3.48 percent.

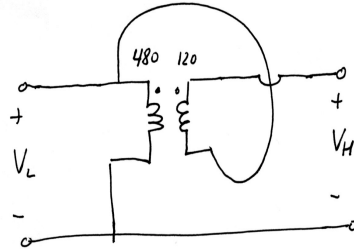
**Problem 2.19**

part (a): The voltage rating is 2400 V:2640 V.

part (b): The rated current of the high voltage terminal is equal to that of the 240-V winding,  $I_{\text{rated}} = 30 \times 10^3/240 = 125 \text{ A}$ . Hence the kVA rating of the transformer is  $2640 \times 125 = 330 \text{ kVA}$ .

**Problem 2.20**

part (a):



part (b): The rated current of the high voltage terminal is equal to that of the 120-V winding,  $I_{\text{rated}} = 10^4/120 = 83.3$  A. Hence the kVA rating of the transformer is  $600 \times 83.3 = 50$  kVA.

part (c): The full load loss is equal to that of the transformer in the conventional connection,  $P_{\text{loss}} = (1 - 0.979) 10 \text{ kW} = 210$  W. Hence as an auto-transformer operating with a load at 0.85 power factor ( $P_{\text{load}} = 0.85 \times 50 \text{ kW} = 42.5 \text{ kW}$ ), the efficiency will be

$$\eta = \frac{42.5 \text{ kW}}{42.71 \text{ kW}} = 0.995 = 99.5 \text{ percent}$$

**Problem 2.21**

part (a): The voltage rating is 78 kV:86 kV. The rated current of the high voltage terminal is equal to that of the 8-kV winding,  $I_{\text{rated}} = 50 \times 10^6/8000 = 6.25$  kA. Hence the kVA rating of the transformer is  $86 \text{ kV} \times 6.25 \text{ kA} = 537.5$  MVA.

part (b): The loss at rated voltage and current is equal to 393 kW and hence the efficiency will be

$$\eta = \frac{537.5 \text{ MW}}{538.1 \text{ MW}} = 0.9993 = 99.93 \text{ percent}$$

**Problem 2.22**

No numerical result required for this problem.

**Problem 2.23**

- part (a): 7.97 kV:2.3 kV; 191 A:651 A; 1500 kVA  
 part (b): 13.8 kV:1.33 kV; 109 A:1130 A; 1500 kVA  
 part (c): 7.97 kV:1.33 kV; 191 A:1130 A; 1500 kVA  
 part (d): 13.8 kV:2.3 kV; 109 A:651 A; 1500 kVA

**Problem 2.24**

part (a):

- (i) 23.9 kV:115 kV, 300 MVA  
 (ii)  $Z_{\text{eq}} = 0.0045 + j0.19 \Omega$   
 (iii)  $Z_{\text{eq}} = 0.104 + j4.30 \Omega$

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part (b):

- (i) 23.9 kV:66.4 kV, 300 MVA
- (ii)  $Z_{eq} = 0.0045 + j0.19 \Omega$
- (iii)  $Z_{eq} = 0.0347 + j1.47 \Omega$

**Problem 2.25**

Following the methodology of Example 2.8,  $V_{load} = 236$  V, line-to-line.

**Problem 2.26**

The total series impedance is  $Z_{tot} = Z_f + Z_t = j11.7 + 0.11 + j2.2 \Omega = 0.11 + j13.9 \Omega$ . The transformer turns ratio is  $N = 9.375$ . The load current, as referred to the transformer high-voltage side will be

$$I_{load} = N^2 \left( \frac{325 \text{ MVA}}{\sqrt{3} 24 \text{ kV}} \right) e^{j\phi} = 7.81 e^{j\phi} \text{ kA}$$

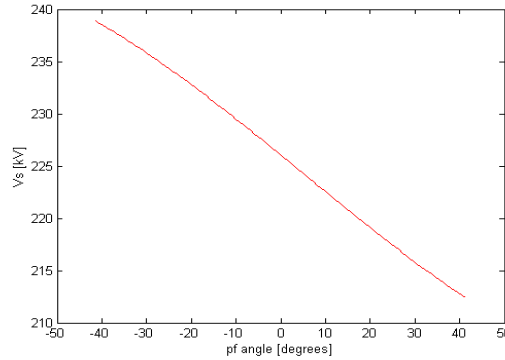
where  $\phi = -\cos^{-1} 0.93 = -21.6^\circ$ . The line-to-neutral load voltage is  $V_{load} = 24\sqrt{3}$  kV.

part (a): At the transformer high-voltage terminal

$$V = \sqrt{3} |NV_{load} + I_{load}Z_t| = 231.7 \text{ kV, line-to-line}$$

part (b): At the sending end

$$V = \sqrt{3} |NV_{load} + I_{load}Z_{tot}| = 233.3 \text{ kV, line-to-line}$$

**Problem 2.27****Problem 2.28**

First calculate the series impedance ( $Z_{eq,H} = R_{eq,H} + jX_{eq,H}$ ) of the transformer from the short-circuit test data.

$$Z_{eq,H} = 0.48 = j1.18 \Omega$$



The total impedance between the load and the sending end of the feeder is  $Z_{\text{tot}} = Z_f + Z_{\text{eq,H}} = 0.544 + j2.058 \Omega$ . The transformer turns ratio is  $N = 2400:120\sqrt{3} = 11.6$ .

part (a): The referred load voltage  $V_{\text{load}}$  and current  $I_{\text{load}}$  will be in phase and can be assumed to be the phase reference. Thus we can write the phasor equation for the sending-end voltage as:

$$\hat{V}_s = V_{\text{load}} + I_{\text{load}} Z_{\text{tot}}$$

We know that  $V_s = 2400/\sqrt{3} = 1386$  V and that  $I_{\text{load}} = 100 \text{ kVA}/(\sqrt{3} \cdot 2.4 \text{ kV})$ . Taking the magnitude of both sides of the above equation gives a quadratic equation in  $V_{\text{load}}$

$$V_{\text{load}}^2 + 2R_{\text{tot}}I_{\text{load}}V_{\text{load}} + |Z_{\text{tot}}|^2I_{\text{load}}^2 - V_s^2 = 0$$

which can be solved for  $V_{\text{load}}$

$$V_{\text{load}} = -R_{\text{tot}}I_{\text{load}} + \sqrt{V_s^2 - (X_{\text{tot}}I_{\text{load}})^2} = 1.338 \text{ kV}$$

Referred to the low-voltage side, this corresponds to a load voltage of  $1.338 \text{ kV}/N = 116$  V, line-to-neutral or 201 V, line-to-line.

part (b):

$$\text{Feeder current} = \left| \frac{2400}{\sqrt{3}Z_{\text{tot}}} \right| = 651 \text{ A}$$

$$\text{HV winding current} = \frac{651}{\sqrt{3}} = 376 \text{ A}$$

$$\text{LV winding current} = 651N = 7.52 \text{ kA}$$

### Problem 2.29

part (a): The transformer turns ratio is  $N = 7970/120 = 66.4$ . The secondary voltage will thus be

$$\hat{V}_2 = \frac{V_1}{N} \left( \frac{jX_m}{R_1 + jX_1 + jX_m} \right) = 119.74 \angle 0.101^\circ$$

part (b): Defining  $R'_L = N^2 R_L = N^2 1 \text{ k}\Omega = 4.41 \text{ M}\Omega$  and

$$Z_{\text{eq}} = jX_m || (R'_2 + R'_L + jX'_2) = 134.3 + j758.1 \text{ k}\Omega$$

the primary current will equal

$$\hat{I}_1 = \frac{7970}{R_1 + jX_1 + Z_{\text{eq}}} = 10.3 \angle -79.87^\circ \text{ mA}$$

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The secondary current will be equal to

$$\hat{I}_2 = N\hat{I}_1 \left( \frac{jX_m}{R'_2 + R'_L + j(X_m + X_2)} \right) = 119.7\angle 0.054^\circ \text{ mA}$$

and thus

$$\hat{V}_2 = R_L\hat{I}_2 = 119.7\angle 0.054^\circ \text{ V}$$

part (c): Following the methodology of part (b)

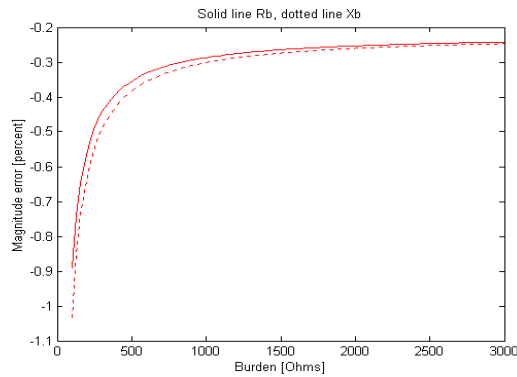
$$\hat{V}_2 = 119.6\angle 0.139^\circ \text{ V}$$

### Problem 2.30

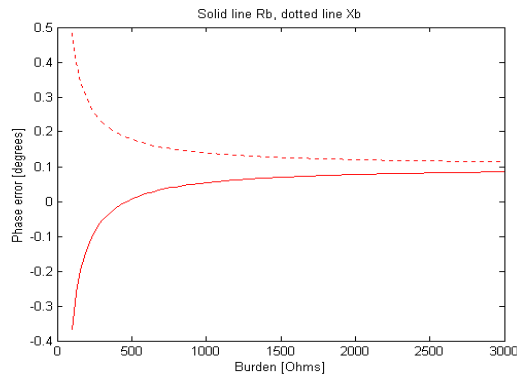
This problem can be solved iteratively using MATLAB. The minimum reactance is  $291 \Omega$ .

### Problem 2.31

part (a):



part (b):



**Problem 2.32**

part (a): The transformer turns ratio  $N = 200/5 = 40$ . For  $I_1 = 200$  A

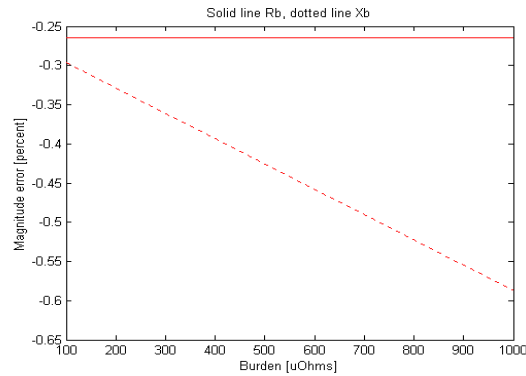
$$I_2 = \frac{I_1}{N} \left( \frac{jX_m}{R'_2 + j(X_m + X'_2)} \right) = 4.987 \angle 0.024^\circ$$

part (b): Defining  $R'_L = N^2 250 \mu\Omega = 0.4 \Omega$

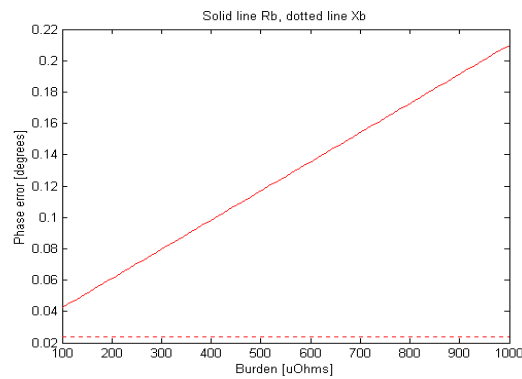
$$I_2 = \frac{I_1}{N} \left( \frac{jX_m}{R'_2 + R'_L + j(X_m + X'_2)} \right) = 4.987 \angle 0.210^\circ$$

**Problem 2.33**

part (a):



part (b):

**Problem 2.34**

$$Z_{\text{base,L}} = \frac{V_{\text{base,L}}^2}{P_{\text{base}}} = 1.80 \Omega$$

$$Z_{\text{base,H}} = \frac{V_{\text{base,H}}^2}{P_{\text{base}}} = 245 \Omega$$

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Thus

$$R_1 = 0.0095 Z_{\text{base,L}} = 17.1 \text{ m}\Omega; \quad X_1 = 0.063 Z_{\text{base,L}} = 113 \text{ m}\Omega$$

$$X_m = 148 Z_{\text{base,L}} = 266 \text{ }\Omega$$

$$R_2 = 0.0095 Z_{\text{base,H}} = 2.33 \text{ }\Omega; \quad X_2 = 0.063 Z_{\text{base,H}} = 15.4 \text{ }\Omega$$

**Problem 2.35**

part (a):

$$(i) Z_{\text{base,L}} = \frac{(7.97 \times 10^3)^2}{75 \times 10^3} = 0.940 \text{ }\Omega; \quad X_L = 0.12 Z_{\text{base,L}} = 0.113 \text{ }\Omega$$

$$(ii) Z_{\text{base,H}} = \frac{(7970)^2}{75 \times 10^3} = 847 \text{ }\Omega; \quad X_H = 0.12 Z_{\text{base,H}} = 102 \text{ }\Omega$$

part (b):

$$(i) 797 \text{ V}:13.8 \text{ kV}, 225 \text{ kVA}$$

$$(ii) X_{\text{pu}} = 0.12$$

$$(iii) X_H = 102 \text{ }\Omega$$

$$(iv) X_L = 0.339 \text{ }\Omega$$

part (c):

$$(i) 460 \text{ V}:13.8 \text{ kV}, 225 \text{ kVA}$$

$$(ii) X_{\text{pu}} = 0.12$$

$$(iii) X_H = 102 \text{ }\Omega$$

$$(iv) X_L = 0.113 \text{ }\Omega$$

**Problem 2.36**part (a): In each case,  $I_{\text{pu}} = 1/0.12 = 8.33 \text{ pu}$ .

$$(i) I_{\text{base,L}} = P_{\text{base}}/(\sqrt{3} V_{\text{base,L}}) = 225 \text{ kVA}/(\sqrt{3} 797 \text{ V}) = 163 \text{ A}$$

$$I_L = I_{\text{pu}} I_{\text{base,L}} = 1359 \text{ A}$$

$$(ii) I_{\text{base,H}} = P_{\text{base}}/(\sqrt{3} V_{\text{base,H}}) = 225 \text{ kVA}/(\sqrt{3} 13.8 \text{ kV}) = 9.4 \text{ A}$$

$$I_H = I_{\text{pu}} I_{\text{base,H}} = 78.4 \text{ A}$$

part (b): In each case,  $I_{\text{pu}} = 1/0.12 = 8.33 \text{ pu}$ .

$$(i) I_{\text{base,L}} = P_{\text{base}}/(\sqrt{3} V_{\text{base,L}}) = 225 \text{ kVA}/(\sqrt{3} 460 \text{ V}) = 282 \text{ A}$$

$$I_L = I_{\text{pu}} I_{\text{base,L}} = 2353 \text{ A}$$

$$(ii) I_{\text{base,H}} = P_{\text{base}}/(\sqrt{3} V_{\text{base,H}}) = 225 \text{ kVA}/(\sqrt{3} 13.8 \text{ kV}) = 9.4 \text{ A}$$

$$I_H = I_{\text{pu}} I_{\text{base,H}} = 78.4 \text{ A}$$

**Problem 2.37**

part (a): On the transformer base

$$X_{\text{gen}} = \left( \frac{P_{\text{base,t}}}{P_{\text{base,g}}} \right) 1.57 = \left( \frac{800 \text{ MVA}}{850 \text{ MVA}} \right) 1.57 = 1.27 \text{ pu}$$

part (b): On the transformer base, the power supplied to the system is  $P_{\text{out}} = 700/850 = 0.824$  pu and the total power is  $S_{\text{out}} = P_{\text{out}}/pf = 0.825/0.95 = 0.868$  pu. Thus, the per unit current is  $\hat{I} = 0.868 \angle \phi$ , where  $\phi = -\cos^{-1} 0.95 = -18.2^\circ$ .

(i) The generator terminal voltage is thus

$$\hat{V}_t = 1.0 + \hat{I}Z_t = 1.03 \angle 3.94^\circ \text{ pu} = 26.8 \angle 3.94^\circ \text{ kV}$$

and the generator internal voltage is

$$\hat{V}_{\text{gen}} = 1.0 + \hat{I}(Z_t + Z_{\text{gen}}) = 2.07 \angle 44.3^\circ \text{ pu} = 53.7 \angle 44.3^\circ \text{ kV}$$

(ii) The total output of the generator is given by  $S_{\text{gen}} = \hat{V}_t \hat{I}^* = 0.8262 + 0.3361j$ . Thus, the generator output power is  $P_{\text{gen}} = 0.8262 \times 850 = 702.2$  MW. The corresponding power factor is  $P_{\text{gen}}/|S_{\text{gen}}| = 0.926$  lagging.

## PROBLEM SOLUTIONS: Chapter 3

### Problem 3.1

By analogy to Example 3.1,

$$T = 2B_0 Rl [I_1 \sin \alpha + I_2 \cos \alpha] = 6.63 \times 10^{-2} [I_1 \sin \alpha + I_2 \cos \alpha] \quad \text{N}\cdot\text{m}$$

Thus

$$\text{part (a): } T = 0.530 \cos \alpha \text{ N}\cdot\text{m}$$

$$\text{part (b): } T = 0.530 \sin \alpha \text{ N}\cdot\text{m}$$

$$\text{part (c): } T = 0.530 [I_1 \sin \alpha + I_2 \cos \alpha] \text{ N}\cdot\text{m}$$

### Problem 3.2

$$T = 0.5304 \text{ N}\cdot\text{m}$$

### Problem 3.3

Can calculate the inductance as

$$L = \frac{N\phi}{I} = \frac{1000 \times 0.13}{10} = 13 \quad \text{H}$$

Thus

$$W_{\text{fld}} = \frac{1}{2} LI^2 = 650 \quad \text{Joules}$$

### Problem 3.4

part (a): For  $x = 0.9$  mm,  $L = 29.5$  mH and thus, for  $I = 6$  A,  $W_{\text{fld}} = 0.531$  Joules.

part (b): For  $x = 0.9$  mm,  $L = 19.6$  mH and thus, for  $I = 6$  A,  $W_{\text{fld}} = 0.352$  Joules. Hence,  $\Delta W_{\text{fld}} = -0.179$  Joules.

### Problem 3.5

For a coil voltage of 0.4 V, the coil current will equal  $I = 0.4/0.11 = 3.7$  A. Under the assumption that all electrical transients have died out, the solution will be the same as that for Problem 3.4, with a current of 3.7 A instead of 6.0 A.

$$\text{part (a): } W_{\text{fld}} = 0.202 \text{ Joules}$$

$$\text{part (b): } \Delta W_{\text{fld}} = -0.068 \text{ Joules.}$$

### Problem 3.6

For  $x = x_0$ ,  $L = L_0 = 30$  mH. The rms current is equal to  $I_{\text{rms}} = I_0/\sqrt{2}$  and thus

part (a):

$$\langle W_{\text{fld}} \rangle = \frac{1}{2} LI_{\text{rms}}^2 = 0.227 \quad \text{Joules}$$

part (b):

$$\langle P_{\text{diss}} \rangle = I_{\text{rms}}^2 R = 1.63 \text{ W}$$

### Problem 3.7

part (a):

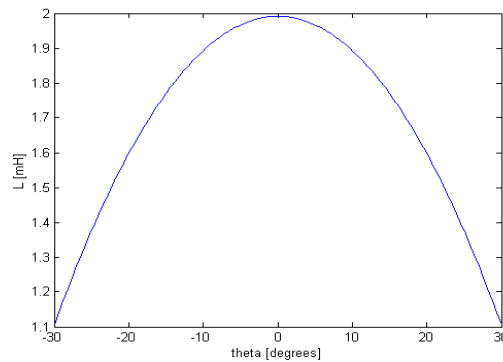
$$B_g = \frac{\mu_0 N i}{2g}$$

$$\begin{aligned} W_{\text{fld}} &= \left( \frac{B_g^2}{2\mu_0} \right) \times \text{Air-gap volume} = \left( \frac{B_g^2}{2\mu_0} \right) \times 2gA_g \\ &= \frac{\mu_0 N^2 A_0}{4g} \left( 1 - \left( \frac{4\theta}{\pi} \right)^2 \right) i^2 \end{aligned}$$

part (b):

$$L = \frac{2W_{\text{fld}}}{i^2} = \frac{\mu_0 N^2 A_0}{2g} \left( 1 - \left( \frac{4\theta}{\pi} \right)^2 \right)$$

Here is the MATLAB plot:



### Problem 3.8

part (a):

$$v_C(t) = V_0 e^{-t/\tau}; \quad \tau = RC$$

part (b):  $W_{\text{fld}} = q^2/(2C) = C v_C^2/2$ . Thus

$$W_{\text{fld}}(0) = \frac{C V_0^2}{2}; \quad W_{\text{fld}}(\infty) = 0$$

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part (c)

$$i_R(t) = \frac{v_C(t)}{R}; \quad P_{\text{diss}}(t) = i_R^2(t)R = \frac{V_0^2 e^{-2t/\tau}}{R}$$

$$W_{\text{diss}} = \int_0^{\infty} P_{\text{diss}}(t) dt = \frac{CV_0^2}{2}$$

**Problem 3.9**

part (a):

$$i_L(t) = \frac{V_0}{R} e^{-t/\tau}; \quad \tau = \frac{L}{R}$$

part (b):

$$W_{\text{fld}}(0) = \frac{V_0^2 L}{2R^2}; \quad W_{\text{fld}}(\infty) = 0$$

part (c)

$$P_{\text{diss}}(t) = i_L^2(t)R = \frac{V_0^2 e^{-2t/\tau}}{R}$$

$$W_{\text{diss}} = \int_0^{\infty} P_{\text{diss}}(t) dt = \frac{V_0^2 L}{2R^2}$$

**Problem 3.10**

Given:

$$\tau = \frac{L}{R} = 4.8 \text{ sec}; \quad I^2 R = 1.3 \text{ MW}$$

Thus

$$\frac{1}{2} L i^2 = \frac{1}{2} L \left( \frac{i^2 R}{R} \right) = \left( \frac{\tau}{2} \right) i^2 R = 6.24 \text{ MJoules}$$

**Problem 3.11**

part (a): Four poles

part (b):

$$T_{\text{fld}} = \frac{\partial W'_{\text{fld}}}{\partial \theta_m} = \frac{d}{d\theta_m} \left[ \frac{I_0^2}{2} (L_0 + L_2 \cos 2\theta_m) \right] = -I_0^2 L_2 \sin 2\theta_m$$

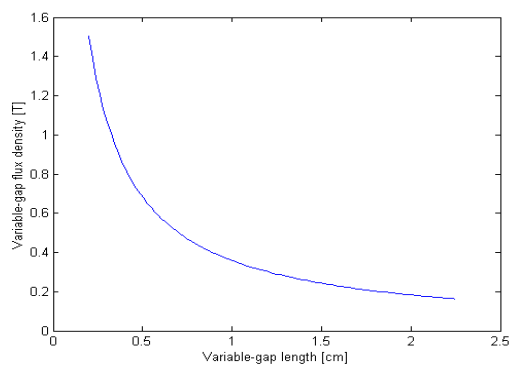


**Problem 3.12**

part (a):

$$B_g = \frac{\mu_0 Ni}{g + g_1 R / (2h)}$$

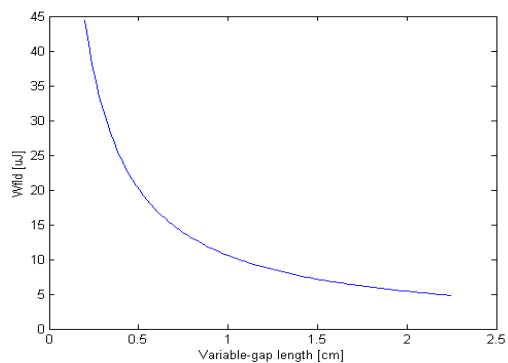
where  $g_1$  is the length of the fixed gap,  $l$  is its length and  $R$  is the radius of the solenoid. Here is the MATLAB plot:



part (b):

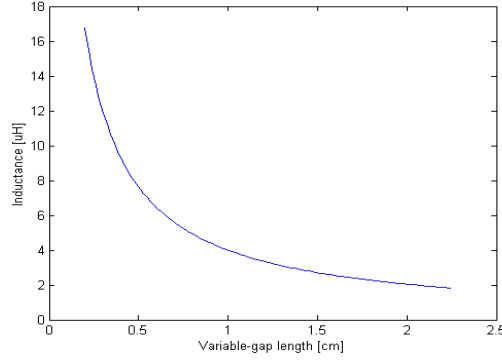
$$W_{fld} = \pi R^2 g \left( \frac{B_g^2}{2\mu_0} \right)$$

Here is the MATLAB plot:



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part (c):  $L = 2W_{fd}/i^2$ . Here is the MATLAB plot:



### Problem 3.13

If the plunger is moved very slowly (i.e.  $idL/dt \ll Ldi/dt$ , the current will be essentially constant and all of the change in stored energy will come from the mechanical work applied to the plunger. Thus,

part (a):

$$\text{Work} = W_{fd}(g = 0.2 \text{ cm}) - W_{fd}(g = 2.25 \text{ cm}) = 46.7 \text{ } \mu\text{Joules}$$

part (b): The battery will supply only the energy dissipated in the coil.

### Problem 3.14

The coil inductance is equal to  $L = \mu_0 N^2 A_c / (2g)$  and hence the lifting force is equal to

$$f_{fd} = \frac{i^2}{2} \frac{dL}{dg} = - \left( \frac{\mu_0 N^2 A_c}{4g^2} \right) i^2$$

where the minus sign simply indicates that the force acts in the direction to reduce the gap (and hence lift the mass). The required force is equal to 931 N (the mass of the slab times the acceleration due to gravity, 9.8 m/sec<sup>2</sup>). Hence, setting  $g = g_{\min}$  and solving for  $i$  gives

$$i_{\min} = \left( \frac{2g_{\min}}{N} \right) \sqrt{\frac{f_{fd}}{\mu_0 A_c}} = 385 \text{ mA}$$

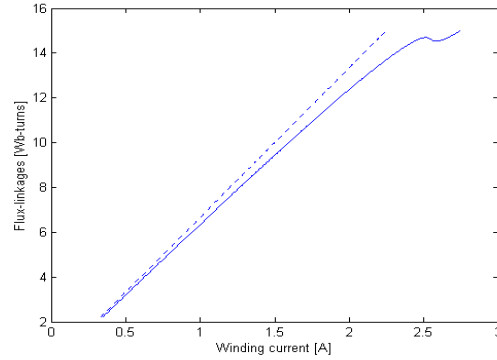
and  $v_{\min} = i_{\min} R = 1.08 \text{ V}$ .

### Problem 3.15

part (a):

$$\begin{aligned} a_1 &= -9.13071 \times 10^{-5} & a_2 &= 0.124209 & a_3 &= 28.1089 \\ & & a_4 &= 10558.2 & & \\ b_1 &= 9.68319 \times 10^{-11} & b_2 &= -1.37037 \times 10^{-7} & & \\ & & b_3 &= 6.32831 \times 10^{-5} & b_4 &= 1.71793 \times 10^{-3} \end{aligned}$$

part (b): (i) Here is the MATLAB plot:

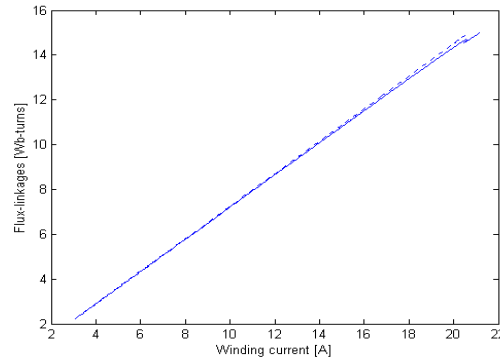


(ii)

$$W_{\text{fld}} = 13.0 \text{ Joules and } W'_{\text{fld}} = 13.7 \text{ Joules}$$

$$\text{Assuming no core reluctance, } W_{\text{fld}} = 11.8 \text{ Joules and } W'_{\text{fld}} = 13.0 \text{ Joules}$$

part (c): (i) Here is the MATLAB plot:



(ii)

$$W_{\text{fld}} = 142 \text{ Joules and } W'_{\text{fld}} = 148 \text{ Joules}$$

$$\text{Assuming no core reluctance, } W_{\text{fld}} = 139 \text{ Joules and } W'_{\text{fld}} = 147 \text{ Joules}$$

### Problem 3.16

$$L = \frac{\mu_0 N^2 A_c}{g}; \quad f_{\text{fld}} = \left(\frac{i^2}{2}\right) \frac{dL}{dg} = -\frac{i^2 L}{2g}$$

The time-averaged force can be found by setting  $i = I_{\text{rms}}$  where  $I_{\text{rms}} = V_{\text{rms}}/(\omega L)$ . Thus

$$\langle f_{\text{fld}} \rangle = -\frac{I_{\text{rms}}}{2g\omega^2 L} = -\frac{V_{\text{rms}}^2}{2\omega^2 \mu_0 N^2 A_c} = -115 \text{ N}$$

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Because the inductor is being driven by a voltage source, the gap flux density remains constant independent of the air-gap length and hence the force also remains constant.

**Problem 3.17**

part (a):

$$B_s = \frac{\mu_0 i}{s}$$

part (b):

$$\phi_s = B_s x l = \frac{\mu_0 x l i}{s}$$

part (c): Note that as the coil moves upward in the slot, the energy associated with the leakage flux associated withing the coil itself remains constant while the energy in the leakage flux above the coil changes. Hence to use the energy method to calculate the force on the coil it is necessary only to consider the energy in the leakage flux above the slot.

$$W_{fld} = \int \frac{B_s^2}{2\mu_0} dV = \frac{\mu_0 x l i^2}{2s}$$

Because this expression is explicitly in terms of the coil current  $i$  and because the magnetic energy is stored in air which is magnetically linear, we know that  $W'_{fld} = W_{fld}$ . We can therefore find the force from

$$f_{fld} = \frac{dW'_{fld}}{dx} = \frac{\mu_0 l i^2}{2s}$$

This force is positive, acting to increase  $x$  and hence force the coil further into the slot.

part (d):  $f_{fld} = 18.1$  N/m.**Problem 3.18**

$$W'_{fld} = \left( \frac{\mu_0 H^2}{2} \right) \times \text{coil volume} = \left( \frac{\mu_0 \pi r_0^2 N^2}{2h} \right) i^2$$

Thus

$$f = \frac{dW'_{fld}}{dr_0} = \left( \frac{\mu_0 \pi r_0 N^2}{h} \right) I_0^2$$

and hence the pressure is

$$P = \frac{f}{2\pi r_0 h} = \left( \frac{\mu_0 N^2}{2h^2} \right) I_0^2$$

The pressure is positive and hence acts in such a direction as to increase the coil radius  $r_0$ .

**Problem 3.19**

part (a):

$$W_{\text{fld}}(q, x) = \int_0^q v(q', x) dq'$$

part (b):

$$f_{\text{fld}} = - \left. \frac{\partial W_{\text{fld}}}{\partial x} \right|_q$$

part (c):

$$W'_{\text{fld}} = vq - dW_{\text{fld}} \Rightarrow dW'_{\text{fld}} = qdv + f_{\text{fld}}dx$$

Thus

$$W'_{\text{fld}} = \int_0^v q(v', x) dv'; \quad f_{\text{fld}} = \left. \frac{\partial W'_{\text{fld}}}{\partial x} \right|_v$$

**Problem 3.20**

part (a):

$$W_{\text{fld}} = \int_0^q v(q', x) dq' = \frac{q^2}{2C} = \frac{xq^2}{2\epsilon_0 A}$$

$$W'_{\text{fld}} = \int_0^v q(v', x) dv' = \frac{Cv^2}{2} = \frac{\epsilon_0 Av^2}{2x}$$

part (b):

$$f_{\text{fld}} = \left. \frac{\partial W'_{\text{fld}}}{\partial x} \right|_v = \frac{Cv^2}{2} = \frac{\epsilon_0 Av^2}{2x^2}$$

and thus

$$f_{\text{fld}}(V_0, \delta) = \frac{\epsilon_0 AV_0^2}{2\delta^2}$$

**Problem 3.21**

part (a):

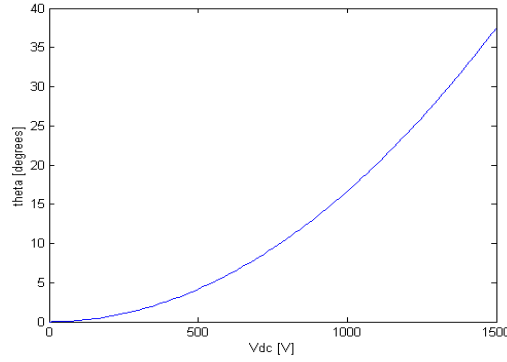
$$T_{\text{fld}} = \left( \frac{V_{\text{dc}}^2}{2} \right) \frac{dC}{d\theta} = \left( \frac{Rd}{2g} \right) V_{\text{dc}}^2$$

part (b): In equilibrium,  $T_{\text{fld}} + T_{\text{spring}} = 0$  and thus

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$$\theta = \theta_0 + \left( \frac{Rd}{2gK} \right) V_{dc}^2$$

Here is the plot:

**Problem 3.22**

part (a):

$$L_{11} = \frac{\mu_0 N_1^2 A}{2g_0}; \quad L_{22} = \frac{\mu_0 N_2^2 A}{2g_0}$$

part (b):

$$L_{12} = \frac{\mu_0 N_1 N_2 A}{2g_0};$$

part (c):

$$W'_{fld} = \frac{1}{2}L_{11}i_1^2 + \frac{1}{2}L_{22}i_2^2 + L_{12}i_1i_2 = \frac{\mu_0 A}{4g_0} (N_1i_1 + N_2i_2)^2$$

part (d):

$$f_{fld} = \left. \frac{\partial W'_{fld}}{\partial g_0} \right|_{i_1, i_2} = -\frac{\mu_0 A}{4g_0^2} (N_1i_1 + N_2i_2)^2$$

**Problem 3.23**

part (a):

$$W'_{fld} = \frac{1}{2}L_{11}i_1^2 + \frac{1}{2}L_{22}i_2^2 + L_{12}i_1i_2 = I^2 (L_{11} + L_{22} + 2L_{12}) \sin^2 \omega t$$

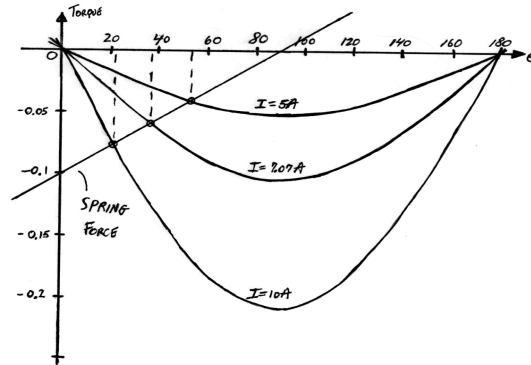
$$T_{fld} = \left. \frac{\partial W'_{fld}}{\partial \theta} \right|_{i_1, i_2} = -4.2 \times 10^{-3} I^2 \sin \theta \sin^2 \omega t \quad \text{N}\cdot\text{m}$$

part (b):

$$T_{fld} = -2.1 \times 10^{-3} I^2 \sin \theta \quad \text{N}\cdot\text{m}$$

part (c):  $T_{fld} = -0.21 \text{ N}\cdot\text{m}$ .

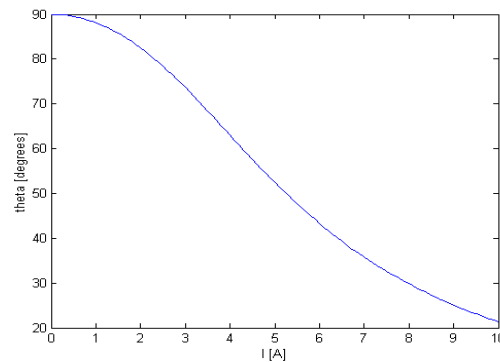
part (d):



part (e): The curve of spring force versus angle is plotted as a straight line on the plot of part (d). The intersection with each curve of magnetic force versus angle gives the equilibrium angle for that value of current. For greater accuracy, MATLAB can be used to search for the equilibrium points. The results of a MATLAB analysis give:

$I$	$\theta$
5	52.5°
7.07	35.3°
10	21.3°

part (f):



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**Problem 3.24**

part (a):

$$T_{\text{fld}} = i_1 i_2 \frac{dL_{12}}{d\theta} = -2.8 i_1 i_2 \sin \theta \quad \text{N}\cdot\text{m}$$

$$\lambda_2 = 0 \Rightarrow i_2 = - \left( \frac{L_{12}}{L_{22}} \right) i_1 = -1.12 i_1 \cos \theta$$

Therefore, for  $i_1 = 10 \sin \omega t$ ,

$$\begin{aligned} T_{\text{fld}} &= -3.14 i_1^2 \sin \theta \cos \theta = -314 \sin^2(\omega t) \sin \theta \cos \theta \\ &= -78.5 (1 - \cos(2\omega t)) \sin(2\theta) \quad \text{N}\cdot\text{m} \end{aligned}$$

part (b):

$$\langle T_{\text{fld}} \rangle = -78.5 \quad \text{N}\cdot\text{m}$$

part (c): It will not rotate. It will come to rest at angular positions where

$$\langle T_{\text{fld}} \rangle = 0 \quad \text{and} \quad \frac{d \langle T_{\text{fld}} \rangle}{d\theta} = 0$$

i.e. at  $\theta = 90^\circ$  or  $\theta = 270^\circ$ .**Problem 3.25**part (a): Winding 1 produces a radial magnetic which, under the assumption that  $g \ll r_0$ ,

$$B_{r,1} = \frac{\mu_0 N_1}{g} i_1$$

The z-directed Lorentz force acting on coil 2 will be equal to the current in coil 2 multiplied by the radial field  $B_{r,1}$  and the length of coil 2.

$$f_z = 2\pi r_0 N_2 B_{r,1} i_2 = \frac{2\pi r_0 \mu_0 N_1 N_2}{g} i_1 i_2$$

part (b): The self inductance of winding 1 can be easily written based upon the winding-1 flux density found in part (a)

$$L_{11} = \frac{2\pi r_0 l \mu_0 N_1^2}{g}$$

The radial magnetic flux produced by winding 2 can be found using Ampere's law and is a function of  $z$ .

$$B_z = \begin{cases} 0 & 0 \leq z \leq x \\ -\frac{\mu_0 N_2 i_2 (z-x)}{gh} & x \leq z \leq x+h \\ -\frac{\mu_0 N_2 i_2}{g} & x+h \leq z \leq l \end{cases}$$



Based upon this flux distribution, one can show that the self inductance of coil 2 is

$$L_{22} = \frac{2\pi r_0 \mu_0 N_2^2}{g} \left( l - x - \frac{2h}{3} \right)$$

part (c): Based upon the flux distribution found in part (b), the mutual inductance can be shown to be

$$L_{12} = \frac{2\pi r_0 \mu_0 N_1 N_2}{g} \left( x + \frac{h}{2} - l \right)$$

part (d):

$$f_{\text{fld}} = \frac{d}{dx} \left[ \frac{1}{2} L_{11} i_1^2 + \frac{1}{2} L_{22} i_2^2 + L_{12} i_1 i_2 \right] = -\frac{\pi r_0 \mu_0 N_2^2}{g} i_2^2 + \frac{2\pi r_0 \mu_0 N_1 N_2}{g} i_1 i_2$$

Note that this force expression includes the Lorentz force of part (a) as well as a reluctance force due to the fact that the self inductance of coil 2 varies with position  $x$ . Substituting the given expressions for the coil currents gives:

$$f_{\text{fld}} = -\frac{\pi r_0 \mu_0 N_2^2}{g} I_2^2 \cos^2 \omega t + \frac{2\pi r_0 \mu_0 N_1 N_2}{g} I_1 I_2 \cos \omega t$$

### Problem 3.26

The solution follows that of Example 3.8 with the exception of the magnet properties of samarium-cobalt replaced by those of neodymium-boron-iron for which  $\mu_R = 1.06\mu_0$ ,  $H'_c = -940$  kA/m and  $B_r = 1.25$  T. The result is

$$f_{\text{fld}} = \begin{cases} -203 \text{ N} & \text{at } x = 0 \text{ cm} \\ -151 \text{ N} & \text{at } x = 0.5 \text{ cm} \end{cases}$$

### Problem 3.27

part (a): Because there is a winding, we don't need to employ a "fictitious" winding. Solving

$$H_m d + H_g g_0 = Ni; \quad B_m w D = B_g (h - x) D$$

in combination with the constitutive laws

$$B_m = \mu_R (H_m - H_c); \quad B_g = \mu_0 H_g$$

gives

$$B_m = \frac{\mu_0 (Ni + H_c d)}{\frac{d\mu_0}{\mu_R} + \frac{wg_0}{(h-x)}}$$

Note that the flux in the magnetic circuit will be zero when the winding current is equal to  $I_0 = -H_c d/N$ . Hence the coenergy can be found from

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integrating the flux linkage of the winding from an initial state where it is zero (i.e. with  $i = I_0$ ) to a final state where the current is equal to  $i$ . The flux linkages are given by  $\lambda = NwDB_m$  and hence

$$W'_{\text{fld}}(i, x) = \int_{I_0}^i \lambda(i', x) di' = \frac{\mu_0 w D N}{\frac{d\mu_0}{\mu_R} + \frac{wg_0}{(h-x)}} \left[ \frac{Ni^2}{2} + H_c \left( i + \frac{H_c d}{2N} \right) \right]$$

The force is then

$$f_{\text{fld}} = \frac{dW'_{\text{fld}}}{dx} = \frac{-\mu_0 w^2 D N g_0}{\left( \frac{\mu_0 d(h-x)}{\mu_R} + wg_0 \right)^2} \left[ \frac{Ni^2}{2} + H_c \left( i + \frac{H_c d}{2N} \right) \right]$$

(i) for  $i = 0$ ,

$$f_{\text{fld}} = \frac{dW'_{\text{fld}}}{dx} = \frac{-\mu_0 w^2 D g_0 (H_c d)^2}{2 \left( \frac{\mu_0 d(h-x)}{\mu_R} + wg_0 \right)^2}$$

where the minus sign indicates that the force is acting upwards to support the mass against gravity.

(ii) The maximum force occurs when  $x = h$

$$f_{\text{max}} = -\frac{\mu_0 w D (H_c d)^2}{2} = -M_{\text{max}} a$$

where  $a$  is the acceleration due to gravity. Thus

$$M_{\text{max}} = \frac{\mu_0 w D (H_c d)^2}{2a}$$

part (b): Want

$$f(I_{\text{min}, x=h} = -a \frac{M_{\text{max}}}{2} = -\frac{\mu_0 w D (H_c d)^2}{4}$$

Substitution into the force expression of part (a) gives

$$I_{\text{min}} = (2 - \sqrt{2})(-H_c d) = -0.59 H_c d$$

### Problem 3.28

part (a): Combining

$$H_m d + H_g g = 0; \quad \pi r_0^2 B_m = 2\pi r_0 l B_g$$

$$B_g = \mu_0 H_g; \quad B_m = \mu_R (H_m - H_{rmc})$$

gives

$$B_g = \frac{-H_c d \mu_0}{g + \left(\frac{\mu_0}{\mu_R}\right) \left(\frac{2ld}{r_0}\right)}$$

part (b): The flux linkages of the voice coil can be calculate in two steps. First calculate the differential flux linkages of a differential section of the voice coil of  $dN_2$  turns at height  $z'$  above the bottom of the voice coil (which is at  $z = x$ ).

$$d\lambda_2 = dN_2 \int_{z'}^l B_g (2\pi r_0) dz = \left[ \frac{(-H_c d \mu_0)(2\pi r_0)(l - z')}{g + \left(\frac{\mu_0}{\mu_R}\right) \left(\frac{2ld}{r_0}\right)} \right] dN_2$$

Recognizing that  $dN_2 = (N_2/h)dz'$  we can now integrate over the coil to find the total flux linkages

$$\lambda_2 = \int_x^{x+h} d\lambda_2 = \frac{N_2(-H_c d \mu_0)(2\pi r_0)(l - x - h/2)}{g + \left(\frac{\mu_0}{\mu_R}\right) \left(\frac{2ld}{r_0}\right)}$$

part (c): Note from part (a) that the magnet in this case can be replaced by a winding of  $N_1 i_1 = -H_c d$  ampere-turns along with a region of length  $d$  and permeability  $\mu_R$ . Making this replacement from part (a), the self inductance of the winding can be found

$$\lambda_{11} = N_1 \Phi_{11} = 2\pi r_0 h N_1 B_g = \frac{2\pi r_0 h N_1^2 d \mu_0}{g + \left(\frac{\mu_0}{\mu_R}\right) \left(\frac{2ld}{r_0}\right)} i_1$$

and thus

$$L_{11} = \frac{2\pi r_0 h N_1^2 d \mu_0}{g + \left(\frac{\mu_0}{\mu_R}\right) \left(\frac{2ld}{r_0}\right)}$$

Similar, the mutual inductance with the voice coil can be found from part (b) as

$$L_{12} = \frac{\lambda_2}{i_1} = \frac{N_1 \lambda_2}{-H_c d} = \frac{N_2 N_2 \mu_0 (2\pi r_0)(l - x - h/2)}{g + \left(\frac{\mu_0}{\mu_R}\right) \left(\frac{2ld}{r_0}\right)}$$

We can now find the coenergy (ignoring the term  $L_{22}i_2^2/2$ )

$$\begin{aligned} W'_{fld} &= \frac{1}{2} L_{11} i_1^2 + L_{12} i_1 i_2 \\ &= \frac{\mu_0 (H_c d)^2 \pi r_0 h}{g + \left(\frac{\mu_0}{\mu_R}\right) \left(\frac{2ld}{r_0}\right)} + \frac{\mu_0 N_2 (-H_c d)(2\pi r_0 d)(l - x - \frac{h}{2})}{g + \left(\frac{\mu_0}{\mu_R}\right) \left(\frac{2ld}{r_0}\right)} i_2 \end{aligned}$$

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part (d):

$$f_{\text{fld}} = \frac{dW'_{\text{fld}}}{dx} = -\frac{\mu_0 N_2 (-H_c d)(2\pi r_0 d)}{g + \left(\frac{\mu_0}{\mu_R}\right) \left(\frac{2ld}{r_0}\right)}$$

**Problem 3.29**

part (a):

$$H_m t_m + H_x x + H_g g = 0; \quad \pi(R_3^2 - R_2^2)B_m = \pi R_1^2 B_x = 2\pi R_1 h B_g$$

$$B_g = \mu_0 H_g; \quad B_x = \mu_0 H_x; \quad B_m = \mu_R (H_m - H_c)$$

where  $\mu_R = 1.05\mu_0$  and  $H'_c = -712$  kA/m.

Solving gives

$$B_g = \left( \frac{\mu_0 R_1 (-H_c t_m)}{2hx + gR_1 + \frac{2\mu_0 R_1^2 h t_m}{\mu_R (R_3^2 - R_2^2)}} \right) = 0.562 \text{ T}$$

and

$$B_x = \left( \frac{2h}{R_1} \right) B_g = 0.535 \text{ T}$$

part (b): We can replace the magnet by an equivalent winding of  $Ni = -H_c t_m$ . The flux linkages of this equivalent winding can then be found to be

$$\lambda = N(2\pi R_1 h)B_g = \left( \frac{2\pi\mu_0 h R_1^2 N^2}{2hx + gR_1 + \frac{2\mu_0 R_1^2 h t_m}{\mu_R (R_3^2 - R_2^2)}} \right) i = Li$$

The force can then be found as

$$\begin{aligned} f_{\text{fld}} &= \frac{i^2}{2} \frac{dL}{dx} = \frac{-2\pi\mu_0 (hR_1)^2 (Ni)^2}{\left(2hx + gR_1 + \frac{2\mu_0 R_1^2 h t_m}{\mu_R (R_3^2 - R_2^2)}\right)^2} \\ &= \frac{-2\pi\mu_0 (hR_1)^2 (-H_c t_m)^2}{\left(2hx + gR_1 + \frac{2\mu_0 R_1^2 h t_m}{\mu_R (R_3^2 - R_2^2)}\right)^2} = -0.0158 \text{ N} \end{aligned}$$

part (c):

$$X_0 = x - \frac{f}{K} = 4.0 \text{ mm}$$

**Problem 3.30**

part (a): If the plunger is stationary at  $x = 0.9a$ , the inductance will be constant at  $L = 0.1L_0$ . Thus

$$i(t) = \frac{V_0}{R} e^{-t/\tau}$$

where  $\tau = L/R$ .

The force will thus be

$$f_{\text{fld}} = \frac{i^2}{2} \frac{dL}{dx} = -\frac{L_0}{2a} \left( \frac{V_0}{R} \right)^2 e^{-2t/\tau}$$

part (b):

$$X_0 = 0.9a + \frac{f}{K_0} = 0.9a - \frac{L_0}{2aK_0} \left( \frac{V_0}{R} \right)^2$$

**Problem 3.31**

part (a): Since the current is fixed at  $i = I_0 = 4$  A, the force will be constant at  $f = -I_0^2 L_0 / (2a) = -1.45$  N. Thus

$$X_0 = 0.9 * a + \frac{f}{K_0} = 1.56 \text{ cm}$$

part (b):

$$M \frac{d^2 x}{dt^2} = f + K_0(0.9a - x) \Rightarrow 0.2 \frac{d^2 x}{dt^2} = 5.48 - 350x \text{ N}$$

$$v = I_0 R + I_0 \frac{dL}{dt} = I_0 R - \frac{L_0}{a} \frac{dx}{dt} \Rightarrow v = 6 - 0.182 \frac{dx}{dt}$$

part (c): The equations can be linearized by letting  $x = X_0 + x'(t)$  and  $v = V_0 + v'(t)$ . The result is

$$\frac{d^2 x'}{dt^2} = -1750x'$$

and

$$v' = -0.182 \frac{dx'}{dt}$$

part (d) For  $\epsilon$  in meters,

$$x'(t) = \epsilon \cos \omega t \text{ m}$$

where  $\epsilon = \sqrt{1750} = 41.8$  rad/sec and

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$$v'(t) = 7.61\epsilon \sin \omega t \quad \text{V}$$

**Problem 3.32**

part (a): For a dc voltage of  $V_0 = 6$  V, the corresponding dc current will be  $I_0 = V_0/R = 4$  A, the same as Problem 3.31. Hence the equilibrium position will be the same;  $X_0 = 1.56$  cm.

part (b): For a fixed voltage, the dynamic equations become:

$$V_0 = iR + \frac{d}{dt}(Li) = iR + L_0 \left(1 - \frac{x}{a}\right) \frac{di}{dt} - \left(\frac{L_0}{a}\right) i \frac{dx}{dt}$$

or

$$6 = 1.5i + 4 \times 10^{-3}(1 - 40x) \frac{di}{dt} - (0.182)i \frac{dx}{dt}$$

and

$$M \frac{d^2x}{dt^2} = f - K_0(0.9a - x) = - \left(\frac{i^2}{2}\right) \left(\frac{L_0}{a}\right) + K_0(0.9a - x)$$

or

$$0.2 \frac{d^2x}{dt^2} = -0.0909i^2 + 6.93 - 350x$$

part (c): The equations can be linearized by letting  $x = X_0 + x'(t)$  and  $i = I_0 + i'(t)$ . The result is

$$0 = i'R + L_0 \left(1 - \frac{X_0}{a}\right) \frac{di'}{dt} - \left(\frac{L_0}{a}\right) I_0 \frac{dx}{dt}$$

or

$$0 = 1.5i' + 1.5 \times 10^{-3} \frac{di'}{dt} - 0.728 \frac{dx}{dt}$$

and

$$M \frac{d^2x'}{dt^2} = - \left(\frac{I_0 L_0}{a}\right) i' - K_0 x'$$

or

$$0.2 \frac{d^2x'}{dt^2} = -0.727i'^2 - 350x'$$

**Problem 3.33**

part (a): Following the derivation of Example 3.1, for a rotor current of 8 A, the torque will be give by  $T = T_0 \sin \alpha$  where  $T_0 = -0.0048$  N·m. The stable equilibrium position will be at  $\alpha = 0$ .

part (b):

$$J \frac{d^2 \alpha}{dt^2} = T_0 \sin \alpha$$

part (c): The incremental equation of motion is

$$J \frac{d^2 \alpha}{dt^2} = T_0 \alpha$$

and the natural frequency is

$$\omega = \sqrt{\frac{T_0}{J}} = 0.62 \text{ rad/sec}$$

corresponding to a frequency of 0.099 Hz.

**Problem 3.34**

As long as the plunger remains within the core, the inductance is equal to

$$L = \frac{\mu_0 d \pi N^2}{ag} \left( \left( \frac{a}{2} \right)^2 - x^2 \right)$$

where  $x$  is the distance from the center of the solenoid to the center of the core. Hence the force is equal to

$$f_{\text{fld}} = \frac{i^2}{2} \frac{dL}{dx} = -\frac{\mu_0 d \pi N^2 i^2 x}{ag}$$

Analogous to Example 3.10, the equations of motor are

$$f_t = -M \frac{d^2 x}{dt^2} - B \frac{dx}{dt} - K(x - l_0) - \frac{\mu_0 d \pi N^2 i^2 x}{ag}$$

The voltage equation for the electric system is

$$v_t = iR + \frac{\mu_0 d \pi N^2}{ag} \left( \left( \frac{a}{2} \right)^2 - x^2 \right) \frac{di}{dt} - \frac{2\mu_0 d \pi N^2 x}{ag} \frac{dx}{dt}$$

These equations are valid only as long as the motion of the plunger is limited so that the plunger does not extend out of the core, i.e. ring, say, between the limits  $-a/2 < x < a/2$ .

## PROBLEM SOLUTIONS: Chapter 4

### Problem 4.1

part (a):  $\omega_m = 1200 \times \pi/30 = 40\pi$  rad/sec

part (b): 60 Hz;  $120\pi$  rad/sec

part (c):  $1200 \times 5/6 = 1000$  r/min

### Problem 4.2

The voltages in the remaining two phases can be expressed as  $V_0 \cos(\omega t - 2\pi/3)$  and  $V_0 \cos(\omega t + 2\pi/3)$ .

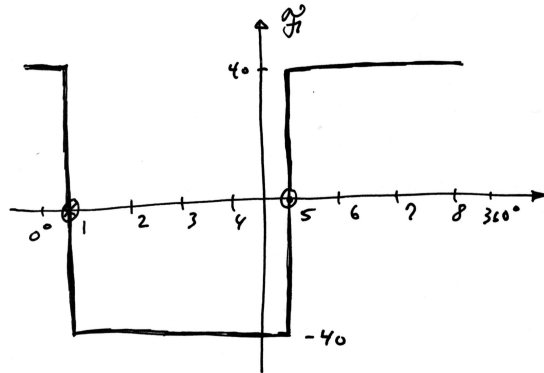
### Problem 4.3

part (a): It is an induction motor.

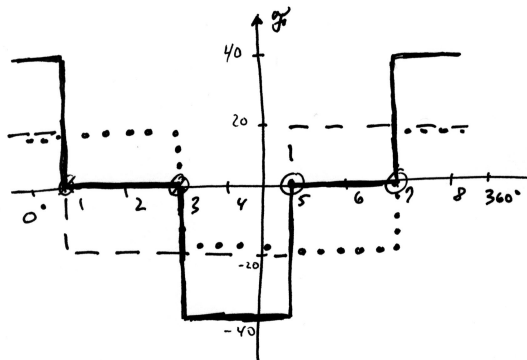
parts (b) and (c): It sounds like an 8-pole motor supplied by 60 Hz.

### Problem 4.4

part (a):

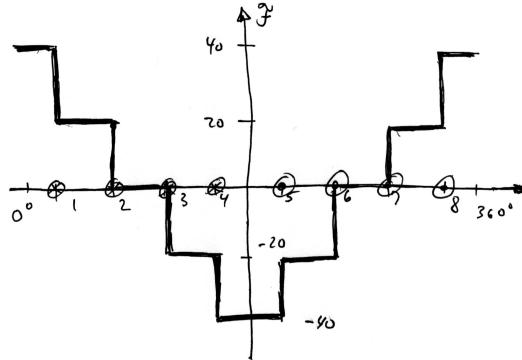


part (b):

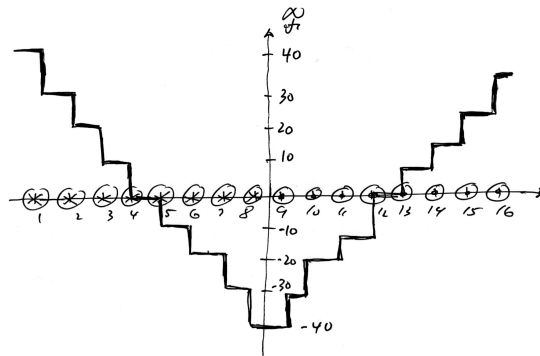




part (c):



part (d):



#### Problem 4.5

Under this condition, the mmf wave is equivalent to that of a single-phase motor and hence the positive- and negative-traveling mmf waves will be of equal magnitude.

#### Problem 4.6

The mmf and flux waves will reverse direction. Reversing two phases is the procedure for reversing the direction of a three-phase induction motor.

#### Problem 4.7

$$\mathcal{F}_1 = F_{\max} \cos \theta_{ae} \cos \omega_e t = \frac{F_{\max}}{2} (\cos (\theta_{ae} - \omega_t) + \cos (\theta_{ae} + \omega_t))$$

$$\mathcal{F}_2 = F_{\max} \sin \theta_{ae} \sin \omega_e t = \frac{F_{\max}}{2} (\cos (\theta_{ae} - \omega_t) - \cos (\theta_{ae} + \omega_t))$$

and thus

$$\mathcal{F}_{\text{total}} = \mathcal{F}_1 + \mathcal{F}_2 = F_{\max} \cos (\theta_{ae} - \omega_t)$$

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**Problem 4.8**For  $n$  odd

$$\left| \frac{\int_{-\beta/2}^{\beta/2} \cos(n\theta) d\theta}{\int_{-\pi/2}^{\pi/2} \cos(n\theta) d\theta} \right| = \sin\left(\frac{n\theta}{2}\right)$$

For  $\beta = 5\pi/6$ ,

$$\sin\left(\frac{n\theta}{2}\right) = \begin{cases} 0.97 & n = 1 \\ 0 & n = 3 \\ 0.26 & n = 5 \end{cases}$$

**Problem 4.9**

part (a): Rated speed = 1200 r/min

part (b):

$$I_r = \frac{\pi g B_{\text{ag1,peak}}(\text{poles})}{4\mu_0 k_r N_r} = 113 \text{ A}$$

part (c):

$$\Phi_P = \left(\frac{2}{3}\right) l R B_{\text{ag1,peak}} = 0.937 \text{ Wb}$$

**Problem 4.10**From the solution to Problem 4.9,  $\Phi_P = 0.937$  Wb.

$$V_{\text{rms}} = \frac{\omega N \Phi}{\sqrt{2}} = 8.24 \text{ kV}$$

**Problem 4.11**From the solution to Problem 4.9,  $\Phi_P = 0.937$  Wb.

$$V_{\text{rms}} = \frac{\omega k_w N_a \Phi}{\sqrt{2}} = 10.4 \text{ kV}$$

**Problem 4.12**The required rms line-to-line voltage is  $V_{\text{rms}} = 13.0/\sqrt{3} = 7.51$  kV. Thus

$$N_a = \frac{\sqrt{2} V_{\text{rms}}}{\omega k_w \Phi} = 39 \text{ turns}$$

**Problem 4.13**

part (a): The flux per pole is

$$\Phi = 2l R B_{\text{ag1,peak}} = 0.0159 \text{ Wb}$$

The electrical frequency of the generated voltage will be 50 Hz. The peak voltage will be

$$V_{\text{peak}} = \omega N \Phi = 388 \text{ V}$$

Because the space-fundamental winding flux linkage is at its peak at time  $t = 0$  and because the voltage is equal to the time derivative of the flux linkage, we can write

$$v(t) = \pm V_{\text{peak}} \sin \omega t$$

where the sign of the voltage depends upon the polarities defined for the flux and the stator coil and  $\omega = 120\pi$  rad/sec.

part (b): In this case,  $\Phi$  will be of the form

$$\Phi(t) = \Phi_0 \cos^2 \omega t$$

where  $\Phi_0 = 0.0159$  Wb as found in part (a). The stator coil flux linkages will thus be

$$\lambda(t) = \pm N \Phi(t) = N \Phi_0 \cos^2 \omega t = \pm \frac{1}{2} N \Phi_0 (1 + \cos 2\omega t)$$

and the generated voltage will be

$$v(t) = \mp \omega \Phi_0 \sin 2\omega t$$

This scheme will not work since the dc-component of the coil flux will produce no voltage.

#### Problem 4.14

$$\begin{aligned} \mathcal{F}_a &= i_a [A_1 \cos \theta_a + A_3 \cos 3\theta_a + A_5 \cos 5\theta_a] \\ &= I_a \cos \omega t [A_1 \cos \theta_a + A_3 \cos 3\theta_a + A_5 \cos 5\theta_a] \end{aligned}$$

Similarly, we can write

$$\begin{aligned} \mathcal{F}_b &= i_b [A_1 \cos (\theta_a - 120^\circ) + A_3 \cos 3(\theta_a - 120^\circ) + A_5 \cos 5(\theta_a - 120^\circ)] \\ &= I_a \cos (\omega t - 120^\circ) [A_1 \cos (\theta_a - 120^\circ) + A_3 \cos 3\theta_a + A_5 \cos (5\theta_a + 120^\circ)] \end{aligned}$$

and

$$\begin{aligned} \mathcal{F}_c &= i_c [A_1 \cos (\theta_a + 120^\circ) + A_3 \cos 3(\theta_a + 120^\circ) + A_5 \cos 5(\theta_a + 120^\circ)] \\ &= I_a \cos (\omega t + 120^\circ) [A_1 \cos (\theta_a + 120^\circ) + A_3 \cos 3\theta_a + A_5 \cos (5\theta_a - 120^\circ)] \end{aligned}$$

The total mmf will be

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$$\begin{aligned}
\mathcal{F}_{\text{tot}} &= \mathcal{F}_a + \mathcal{F}_b + \mathcal{F}_c \\
&= \frac{3}{2} I_a [A_1 \cos(\theta_a - \omega t) A_5 \cos(5\theta_a + \omega t)] \\
&= \frac{3}{2} I_a [A_1 \cos(\theta_a - \omega t) A_5 \cos 5 \left( \theta_a + \left( \frac{\omega t}{5} \right) \right)]
\end{aligned}$$

We see that the combined mmf contains only a fundamental space-harmonic component that rotates in the forward direction at angular velocity  $\omega$  and a 5'th space-harmonic that rotates in the negative direction at angular velocity  $\omega/5$ .

**Problem 4.15**

The turns must be modified by a factor of

$$\left( \frac{18}{24} \right) \left( \frac{1200}{1400} \right) = \frac{9}{14} = 0.64$$

**Problem 4.16**

$$\Phi_p = \frac{30E_a}{N(\text{poles})n} = 6.25 \text{ mWb}$$

**Problem 4.17**

part (a):

$$\Phi_p = \left( \frac{2}{\text{poles}} \right) 2B_{\text{peak}}lr = \left( \frac{2}{4} \right) \times 2 \times 1.25 \times 0.21 \times (.0952/2) = 12.5 \text{ mWb}$$

$$N_{\text{ph}} = \frac{V_{\text{rms}} \times \text{poles}}{\sqrt{2} \pi f_{\text{me}} k_w \Phi_p} = \frac{(230/\sqrt{3}) \times 4}{\sqrt{2} \pi \times 60 \times 0.925 \times 0.0125} = 43 \text{ turns}$$

part (b): From Eq. B.27

$$L = \frac{16\mu_0lr}{\pi g} \left( \frac{k_w N_{\text{ph}}}{\text{poles}} \right)^2 = 21.2 \text{ mH}$$

**Problem 4.18**

part (a):

$$\Phi_p = \frac{V_{\text{rms}}}{\sqrt{2} \pi N_{\text{ph}}} = 10.8 \text{ mWb}$$

$$B_{\text{peak}} = \frac{\Phi_p}{2lr} = 0.523 \text{ T}$$

part (b):

$$I_f = \frac{\pi B_{\text{peak}} g}{2\mu_0 k_r N_r} = 0.65 \text{ A}$$

part (c):

$$L_{\text{af}} = \frac{\lambda_{\text{a,peak}}}{I_f} = \frac{\sqrt{2} V_{\text{rms}}/\omega}{I_f} = 0.69 \text{ H}$$

**Problem 4.19**

No numerical solution required.

**Problem 4.20**

$$\Phi_{\text{peak}} = \left( \frac{2Dl}{\text{poles}} \right) B_{\text{peak}}$$

$$F_{r,\text{peak}} = \frac{4k_r N_r I_{r,\text{max}}}{\pi \times \text{poles}}$$

$$T_{\text{peak}} = \frac{\pi}{2} \left( \frac{\text{poles}}{2} \right)^2 \Phi_{\text{peak}} F_{r,\text{peak}} = 4.39 \times 10^6 \text{ N}\cdot\text{m}$$

$$P_{\text{peak}} = T_{\text{peak}} \omega_m = 828 \text{ MW}$$

**Problem 4.21**

$$\Phi_{\text{peak}} = \left( \frac{2Dl}{\text{poles}} \right) B_{\text{peak}}$$

$$F_{r,\text{peak}} = \frac{4k_r N_r I_{r,\text{max}}}{\pi \times \text{poles}}$$

$$T_{\text{peak}} = \frac{\pi}{2} \left( \frac{\text{poles}}{2} \right)^2 \Phi_{\text{peak}} F_{r,\text{peak}} = 16.1 \text{ N}\cdot\text{m}$$

$$P_{\text{peak}} = T_{\text{peak}} \omega_m = 6.06 \text{ kW}$$

**Problem 4.22**

part (a):

$$\begin{aligned} T &= i_a i_f \frac{dM_{\text{af}}}{d\theta_0} + i_b i_f \frac{dM_{\text{bf}}}{d\theta_0} \\ &= M i_f (i_b \cos \theta_0 - i_a \sin \theta_0) \end{aligned}$$

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This expression applies under all operating conditions.

part (b):

$$T = 2MI_0^2(\cos \theta_0 - \sin \theta_0) = 2\sqrt{2} MI_0^2 \sin(\theta_0 - \pi/4)$$

Provided there are any losses at all, the rotor will come to rest at  $\theta_0 = \pi/4$  for which  $T = 0$  and  $dt/d\theta_0 < 0$ .

part (c):

$$\begin{aligned} T &= \sqrt{2} MI_a I_f (\sin \omega t \cos \theta_0 - \cos \omega t \sin \theta_0) \\ &= \sqrt{2} MI_a I_f \sin(\omega t - \theta_0) = \sqrt{2} MI_a I_f \sin \delta \end{aligned}$$

part (d):

$$\begin{aligned} v_a &= R_a i_a + \frac{d}{dt} (L_{aa} i_a + M_{af} i_f) \\ &= \sqrt{2} I_a (R_a \cos \omega t - \omega L_{aa} \sin \omega t) - \omega M I_f \sin(\omega t - \delta) \end{aligned}$$

$$\begin{aligned} v_b &= R_a i_b + \frac{d}{dt} (L_{aa} i_b + M_{bf} i_f) \\ &= \sqrt{2} I_a (R_a \sin \omega t + \omega L_{aa} \cos \omega t) + \omega M I_f \cos(\omega t - \delta) \end{aligned}$$

#### Problem 4.23

$$\begin{aligned} T &= MI_f (i_b \cos \theta_0 - i_a \sin \theta_0) \\ &= \sqrt{2} MI_f [(I_a + I'/2) \sin \delta + (I'/2) \sin(2\omega t + \delta)] \end{aligned}$$

The time-averaged torque is thus

$$\langle T \rangle = \sqrt{2} MI_f (I_a + I'/2) \sin \delta$$

#### Problem 4.24

part (a):

$$\begin{aligned} T &= \frac{i_a^2}{2} \frac{dL_{aa}}{d\theta_0} + \frac{i_b^2}{2} \frac{dL_{bb}}{d\theta_0} + i_a i_b \frac{dL_{ab}}{d\theta_0} + i_a i_f \frac{dM_{af}}{d\theta_0} + i_b i_f \frac{dM_{bf}}{d\theta_0} \\ &= \sqrt{2} I_a I_f M \sin \delta + 2I_a^2 L_2 \sin 2\delta \end{aligned}$$

part (b): Motor if  $T > 0$ ,  $\delta > 0$ . Generator if  $T < 0$ ,  $\delta < 0$ .

part (c): For  $I_f = 0$ , there will still be a reluctance torque  $T = 2I_a^2 L_2 \sin 2\delta$  and the machine can still operate.

**Problem 4.25**

part (a):

$$v = \frac{f}{\lambda} = 25 \text{ m/sec}$$

part (b): The synchronous rotor velocity is 25 m/sec.

part (c): For a slip of 0.045, the rotor velocity will be  $(1 - 0.045) \times 25 = 23.9$  m/sec.**Problem 4.26**

$$\begin{aligned} I_{\text{rms}} &= \frac{B_{\text{peak}}}{\sqrt{2}} \left( \frac{g}{\mu_0} \right) \left( \frac{2}{3} \right) \left( \frac{\pi}{4} \right) \left( \frac{2p}{k_w N_{\text{ph}}} \right) \\ &= \frac{1.45}{\sqrt{2}} \left( \frac{9.3 \times 10^{-3}}{\mu_0} \right) \left( \frac{2}{3} \right) \left( \frac{\pi}{4} \right) \left( \frac{2 \times 7}{0.91 \times 280} \right) = 218 \text{ A} \end{aligned}$$

**Problem 4.27**part (a): Defining  $\beta = 2\pi/\text{wavelength}$ 

$$\Phi_p = w \int_0^{\pi/\beta} B_{\text{peak}} \cos \beta x dx = \frac{2wB_{\text{peak}}}{\beta} = 1.48 \text{ mWb}$$

part (b): Since the rotor is 5 wavelengths long, the armature winding will link 10 poles of flux with 10 turns per pole. Thus,  $\lambda_{\text{peak}} = 10\Phi_p = 0.148$  Wb.part (c):  $\omega = \beta v$  and thus

$$V_{\text{rms}} = \frac{\omega \lambda_{\text{peak}}}{\sqrt{2}} = 34.6 \text{ V, rms}$$

## PROBLEM SOLUTIONS: Chapter 5

### Problem 5.1

Basic equations are  $T \propto \Phi_R F_f \sin \delta_{RF}$ . Since the field current is constant,  $F_f$  is constant, Note also that the resultant flux is proportional to the terminal voltage and inversely to the frequency  $\Phi_R \propto V_t/f$ . Thus we can write

$$T \propto \frac{V_t \sin \delta_{RF}}{f}$$

$$P = \omega_f T \propto V_t \sin \delta_{RF}$$

- part (a): Reduced to  $31.1^\circ$
- part (b): Unchanged
- part (c): Unchanged
- part (d): Increased to  $39.6^\circ$

### Problem 5.2

part (a): The windings are orthogonal and hence the mutual inductance is zero.

part (b): Since the two windings are orthogonal, the phases are uncoupled and hence the flux linkage under balanced two-phase operation is unchanged by currents in the other phase. Thus, the equivalent inductance is simply equal to the phase self-inductance.

### Problem 5.3

$$L_{ab} = -\frac{1}{2} (L_{aa} - L_{al}) = -2.25 \text{ mH}$$

$$L_s = \frac{3}{2} (L_{aa} - L_{al}) + L_{al} = 7.08 \text{ mH}$$

### Problem 5.4

part (a):

$$L_{af} = \frac{\sqrt{2} V_{1-1,\text{rms}}}{\sqrt{3} \omega I_f} = 79.4 \text{ mH}$$

part (b): Voltage =  $(50/60) 15.4 \text{ kV} = 12.8 \text{ kV}$ .

### Problem 5.5

part (a): The magnitude of the phase current is equal to

$$I_a = \frac{40 \times 10^3}{0.85 \times \sqrt{3} 460} = 59.1 \text{ A}$$

and its phase angle is  $-\cos^{-1} 0.85 = -31.8^\circ$ . Thus



$$\hat{I}_a = 59.1e^{-j31.8^\circ}$$

Then

$$\hat{E}_{af} = V_a - jX_s \hat{I}_a = \frac{460}{\sqrt{3}} - j4.15 \times 59.1e^{-j31.8^\circ} = 136 \angle -56.8^\circ \text{ V}$$

The field current can be calculated from the magnitude of the generator voltage

$$I_f = \frac{\sqrt{2}E_{af}}{\omega L_{af}} = 11.3 \text{ A}$$

part (b):

$$\hat{E}_{af} = 266 \angle -38.1^\circ \text{ V}; \quad I_f = 15.3 \text{ A}$$

part (c):

$$\hat{E}_{af} = 395 \angle -27.8^\circ \text{ V}; \quad I_f = 20.2 \text{ A}$$

### Problem 5.6

The solution is similar to that of Problem 5.5 with the exception that the synchronous impedance  $jX_s$  is replaced by the impedance  $Z_f + jX_s$ .

part (a):

$$\hat{E}_{af} = 106 \angle -66.6^\circ \text{ V}; \quad I_f = 12.2 \text{ A}$$

part (b):

$$\hat{E}_{af} = 261 \angle -43.7^\circ \text{ V}; \quad I_f = 16.3 \text{ A}$$

part (c):

$$\hat{E}_{af} = 416 \angle -31.2^\circ \text{ V}; \quad I_f = 22.0 \text{ A}$$

### Problem 5.7

part (a):

$$L_{af} = \frac{\sqrt{2}V_{1-l,rms}}{\sqrt{3}\omega I_f} = 49.8 \text{ mH}$$

part (b):

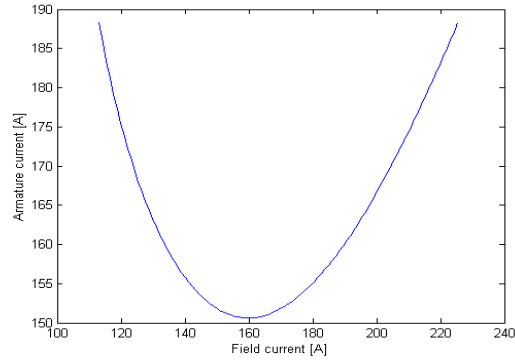
$$\hat{I}_a = \frac{600 \times 10^3}{\sqrt{3} 2300} = 151 \text{ A}$$

$$\hat{E}_{af} = V_a - jX_s \hat{I}_a = 1.77 \angle -41.3^\circ \text{ V}$$

$$I_f = \frac{\sqrt{2}E_{af}}{\omega L_{af}} = 160 \text{ A}$$

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part (c): See plot below. Minimum current will when the motor is operating at unity power factor. From the plot, this occurs at a field current of 160 A.

**Problem 5.8**

part (a):

$$Z_{\text{base}} = \frac{V_{\text{base}}^2}{P_{\text{base}}} = \frac{(26 \times 10^3)^2}{750 \times 10^6} = 0.901 \, \Omega$$

$$L_s = \frac{X_{s,\text{pu}} Z_{\text{base}}}{\omega} = 4.88 \, \text{mH}$$

part (b):

$$L_{\text{al}} = \frac{X_{\text{al},\text{pu}} Z_{\text{base}}}{\omega} = 0.43 \, \text{mH}$$

part (c):

$$L_{\text{aa}} = \frac{2}{3}(L_s - L_{\text{al}}) + L_{\text{al}} = 3.40 \, \text{mH}$$

**Problem 5.9**

part (a):

$$\text{SCR} = \frac{\text{AFNL}}{\text{AFSC}} = 0.520$$

part (b):

$$Z_{\text{base}} = (26 \times 10^3)^2 / (800 \times 10^6) = 0.845 \, \Omega$$

$$X_s = \frac{1}{\text{SCR}} = 2.19 \, \text{pu} = 1.85 \, \Omega$$

part (c):

$$X_{s,u} = \frac{\text{AFSC}}{\text{AFNL, ag}} = 1.92 \, \text{pu} = 1.62 \, \Omega$$

**Problem 5.10**

part (a):

$$\text{SCR} = \frac{\text{AFNL}}{\text{AFSC}} = 1.14$$

part (b):

$$Z_{\text{base}} = 4160^2 / (5000 \times 10^3) = 3.46 \Omega$$

$$X_s = \frac{1}{\text{SCR}} = 1.11 \text{ pu} = 3.86 \Omega$$

part (c):

$$X_{s,u} = \frac{\text{AFSC}}{\text{AFNL}_{ag}} = 0.88 \text{ pu} = 3.05 \Omega$$

**Problem 5.11**

No numerical solution required.

**Problem 5.12**

part (a): The total power is equal to  $S = P/\text{pf} = 4200 \text{ kW}/0.87 = 4828 \text{ kVA}$ .  
The armature current is thus

$$\hat{I}_a = \frac{4828 \times 10^3}{\sqrt{3} \cdot 4160} \angle(\cos^{-1} 0.87) = 670 \angle 29.5^\circ \text{ A}$$

Defining  $Z_s = R_a + jX_s = 0.038 + j4.81 \Omega$

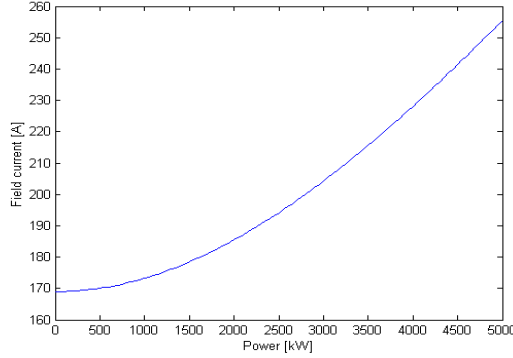
$$|E_{af}| = |V_a - Z_s I_a| = \left| \frac{4160}{\sqrt{3}} - Z_s I_a \right| = 4349 \text{ V, line-to-neutral}$$

Thus

$$I_f = \text{AFNL} \left( \frac{4349}{4160/\sqrt{3}} \right) = 306 \text{ A}$$

part (b): If the machine speed remains constant and the field current is not reduced, the terminal voltage will increase to the value corresponding to 306 A of field current on the open-circuit saturation characteristic. Interpolating the given data shows that this corresponds to a value of around 4850 V line-to-line.

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**Problem 5.13****Problem 5.14**

At rated power, unity power factor, the armature current will be  $I_a = 5000 \text{ kW}/(\sqrt{3} \cdot 4160 \text{ V}) = 694 \text{ A}$ . The power dissipated in the armature winding will then equal  $P_{\text{arm}} = 3 \times 694^2 \times 0.011 = 15.9 \text{ kW}$ .

The field current can be found from

$$|E_{\text{af}}| = |V_a - Z_s I_a| = \left| \frac{4160}{\sqrt{3}} - Z_s I_a \right| = 3194 \text{ V, line-to-neutral}$$

and thus

$$I_f = \text{AFNL} \left( \frac{3194}{4160/\sqrt{3}} \right) = 319 \text{ A}$$

At  $125^\circ\text{C}$ , the field-winding resistance will be

$$R_f = 0.279 \left( \frac{234.5 + 125}{234.5 + 75} \right) = 0.324 \Omega$$

and hence the field-winding power dissipation will be  $P_{\text{field}} = I_f^2 R_f = 21.1 \text{ kW}$ .

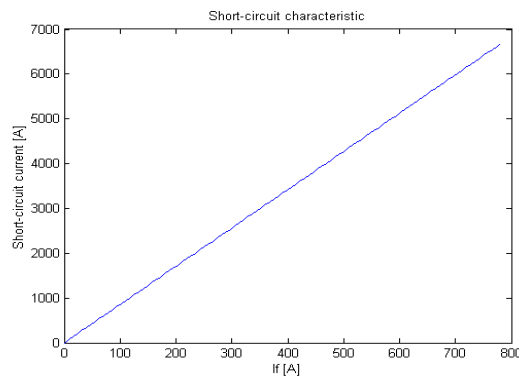
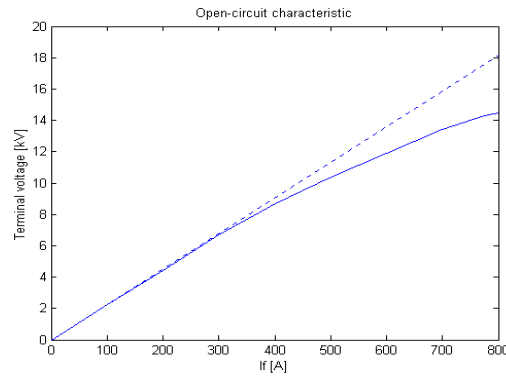
The total loss will then be

$$P_{\text{tot}} = P_{\text{core}} + P_{\text{arm}} + P_{\text{friction/windage}} + P_{\text{field}} = 120 \text{ kW}$$

Hence the output power will equal  $4880 \text{ kW}$  and the efficiency will equal  $4880/5000 = 0.976 = 97.6\%$ .

**Problem 5.15**

part (a):



part (b): AFNL = 736 A. AFSC = 710 A.

part (c): (i) SCR = 10.4, (ii)  $X_s = 0.964$  per unit and (iii)  $X_{s,u} = 1.17$  per unit.**Problem 5.16**For  $V_a = 1.0$  per unit,  $E_{af,max} = 2.4$  per unit and  $X_s = 1.6$  per unit

$$Q_{\max} = \frac{E_{af,max} - V_a}{X_s} = 0.875 \text{ per unit}$$

**Problem 5.17**

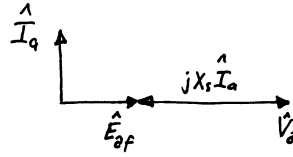
part (a):

$$Z_{\text{base}} = \frac{V_{\text{base}}^2}{P_{\text{base}}} = 5.29 \Omega$$

$$X_s = \frac{1}{\text{SCR}} = 0.595 \text{ per-unit} = 3.15 \Omega$$

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part (b): Using generator convention for current



part (c):

$$E_{af} = \frac{150}{420} = 0.357 \text{ per-unit}$$

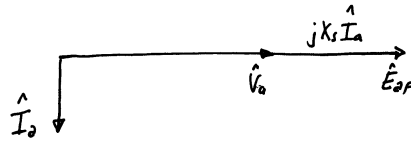
For  $V_a = 1.0$  per-unit,

$$\hat{I}_a = \frac{E_{af} - V_a}{jX_s} = 1.08 \angle 90^\circ \text{ per-unit} = 1.36 \angle 90^\circ \text{ kA}$$

using  $I_{\text{base}} = 1255 \text{ A}$ .

part (d): It looks like an inductor.

part (e):

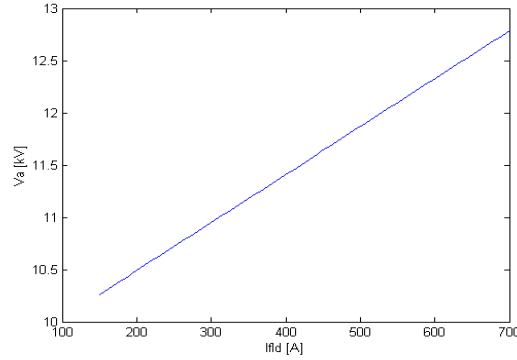


$$E_{af} = \frac{700}{420} = 1.67 \text{ per-unit}$$

For  $V_a = 1.0$  per-unit,

$$\hat{I}_a = \frac{E_{af} - V_a}{jX_s} = 1.12 \angle -90^\circ \text{ per-unit} = 1.41 \angle -90^\circ \text{ kA}$$

In this case, it looks like a capacitor.

**Problem 5.18****Problem 5.19**

- part (a): It was underexcited, absorbing reactive power.  
 part (b): It increased.  
 part (c): The answers are the same.

**Problem 5.20**

part (a):

$$X_s = \frac{226}{842} = 0.268 \text{ per-unit}$$

part (b):  $P = 0.875$  and  $S = P/0.9 = 0.972$ , both in per unit. The power-factor angle is  $-\cos^{-1} 0.9 = -25.8^\circ$  and thus  $\hat{I}_a = 0.875 \angle -25.8^\circ$ .

$$\hat{E}_{af} = V_a + jX_s \hat{I}_a = 1.15 \angle 11.6^\circ \text{ per-unit}$$

The field current is  $I_f = \text{AFNL} |\hat{E}_{af}| = 958$  A. The rotor angle is  $11.6^\circ$  and the reactive power is

$$Q = \sqrt{S^2 - P^2} = 4.24 \text{ MVA}$$

part (c): Now  $|E_{af}| = 1.0$  per unit.

$$\delta = \sin^{-1} \left( \frac{PX_s}{V_a} |E_{af}| \right) = 13.6^\circ$$

and thus  $\hat{E}_{af} = 1.0 \angle 13.6^\circ$ .

$$\hat{I}_a = \frac{\hat{E}_{af} - V_a}{jX_s} = 0.881 \angle 6.79^\circ$$

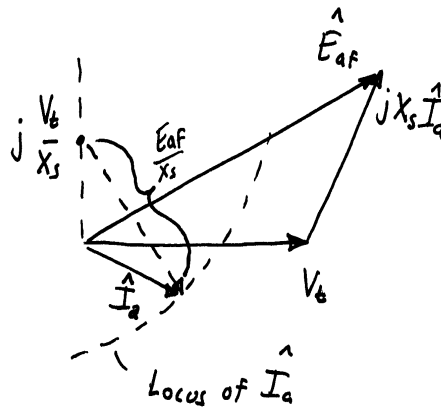
$$Q = \text{Imag}[V_a \hat{I}_a^*] = -0.104 \text{ per-unit} = -1.04 \text{ MVAR}$$

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**Problem 5.21**

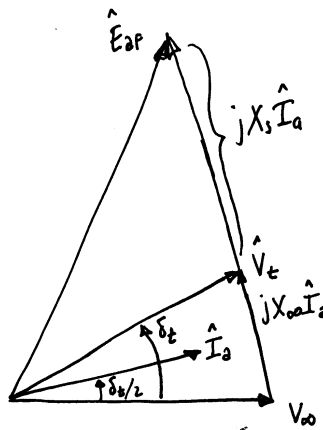
$$\hat{I}_a = \frac{\hat{E}_{af} - V_a}{jX_s} = j\frac{V_a}{X_s} + \frac{E_{af}}{X_s}(\sin \delta - j \cos \delta)$$

The first term is a constant and is the center of the circle. The second term is a circle of radius  $E_{af}/X_s$ .

**Problem 5.22**

part (a):

(i)



(ii)  $V_t = V_\infty = 1.0$  per unit.  $P = 375/650 = 0.577$  per unit. Thus

$$\delta_t = \sin^{-1} \left( \frac{PX_\infty}{V_t V_\infty} \right) = 12.6^\circ$$



and

$$\hat{I}_a = \frac{V_t e^{j\delta_t} - V_\infty}{jX_\infty} = 0.578 \angle 3.93^\circ \text{ per-unit}$$

$I_{\text{base}} = P_{\text{base}} / (\sqrt{3} V_{\text{base}}) = 15.64 \text{ kA}$  and thus  $I_a = 9.04 \text{ kA}$ .

(iii) The generator terminal current lags the terminal voltage by  $\delta_t/2$  and thus the power factor is

$$\text{pf} = \cos^{-1} \delta_t/2 = 0.998 \text{ lagging}$$

(iv)

$$|\hat{E}_{af}| = |V_\infty + j(X_\infty + X_s)\hat{I}_a| = 1.50 \text{ per-unit} = 36.0 \text{ kV, line-to-line}$$

part (b):

(i) Same phasor diagram

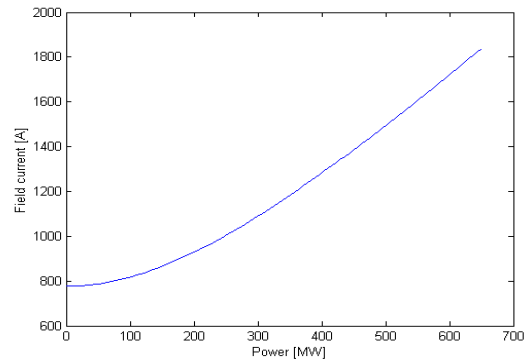
(ii)  $\hat{I}_a = 0.928 \angle 6.32^\circ$  per-unit.  $I_a = 14.5 \text{ kA}$ .

(iii) pf = 0.994 lagging

(iv)  $E_{af} = 2.06$  per unit = 49.4 kV, line-to-line.

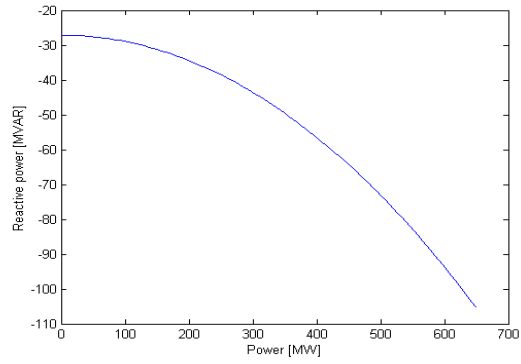
### Problem 5.23

part (a):

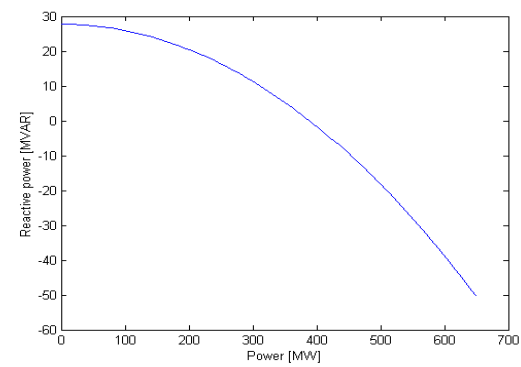
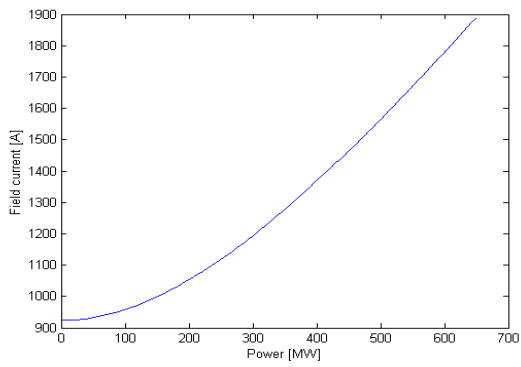


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part (b):



part (c):

**Problem 5.24**

part (a): From the solution to Problem 5.15,  $X_s = 0.964$  per unit. Thus, with  $V_\infty = E_{af} = 1.0$  per unit

$$P_{\max} = \frac{V_{\infty} E_{af}}{X_s} = 1.04 \text{ per-unit}$$

noindent Hence, full load can be achieved. This will occur at

$$\delta = \sin^{-1} \left( \frac{X_s}{E_{af} V_{\infty}} \right) = 74.6^{\circ}$$

part (b): The generator base impedance is  $1.31 \Omega$ . Thus,  $X_{\infty} = 0.14/1.31 = 0.107$  per unit. Now

$$P_{\max} = \frac{V_{\infty} E_{af}}{(X_{\infty} + X_s)} = 1.04 \text{ per-unit} = 0.934 \text{ per-unit} = 135 \text{ MW}$$

### Problem 5.25

Following the calculation steps of Example 5.9,  $E_{af} = 1.35$  per unit.

### Problem 5.26

Now  $X_d = .964$  per unit and  $X_q = 0.723$  per unit. Thus

part (a):

$$P = \frac{V_{\infty} E_{af}}{X_d} \sin \delta + \frac{V_{\infty}^2}{2} \left( \frac{1}{X_q} - \frac{1}{X_d} \right) \sin 2\delta = 1.037 \sin \delta + 0.173 \sin 2\delta$$

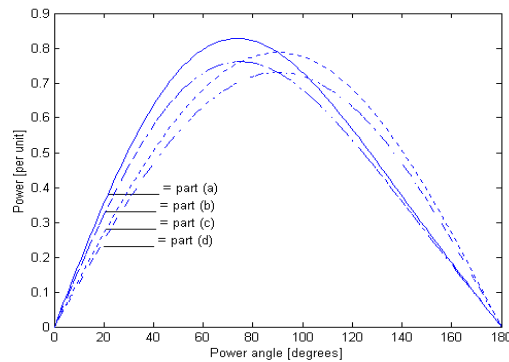
An iterative solution with MATLAB shows that maximum power can be achieved at  $\delta = 53.6^{\circ}$ .

part (b): Letting  $X_D = X_d + X_{\infty}$  and  $X_Q = X_q + X_{\infty}$

$$P = \frac{V_{\infty} E_{af}}{X} \sin \delta + \frac{V_{\infty}^2}{2} \left( \frac{1}{X_Q} - \frac{1}{X_D} \right) \sin 2\delta = 0.934 \sin \delta + 0.136 \sin 2\delta$$

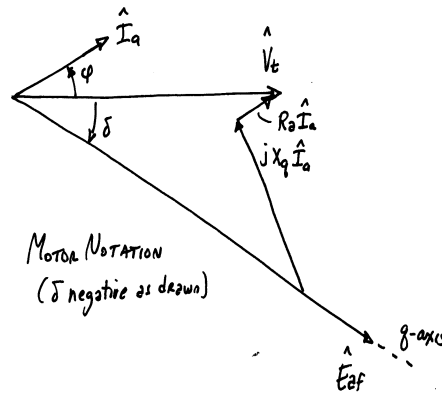
An iterative solution with MATLAB shows that maximum power that can be achieved is 141 Mw, which occurs at a power angle of  $75^{\circ}$ .

### Problem 5.27

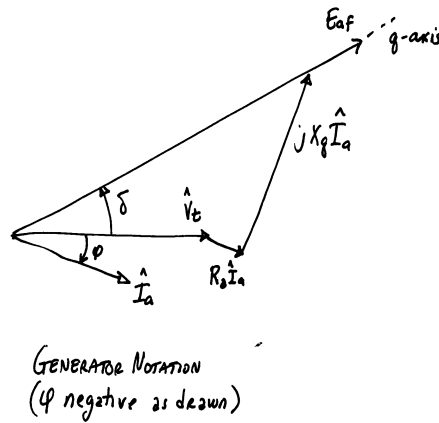


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## Problem 5.28



## Problem 5.29



## Problem 5.30

For  $E_{af} = 0$ ,

$$P_{\max} = \frac{V_t^2}{2} \left( \frac{1}{X_q} - \frac{1}{X_d} \right) = 0.21 = 21\%$$

This maximum power occurs for  $\delta = 45^\circ$ .

$$I_d = \frac{V_t \cos \delta}{X_d} = 0.786 \text{ per-unit}$$

$$I_q = \frac{V_t \sin \delta}{X_q} = 1.09 \text{ per-unit}$$

and thus  $I_a = \sqrt{I_d^2 + I_q^2} = 1.34$  per unit.

$$S = V_t I_a = 1.34 \text{ per-unit}$$

Hence

$$Q = \sqrt{S^2 - P^2} = 1.32 \text{ per-unit}$$

### Problem 5.30

$$P = \frac{V_\infty E_{af}}{X_d} \sin \delta + \frac{V_\infty^2}{2} \left( \frac{1}{X_q} - \frac{1}{X_d} \right) \sin 2\delta$$

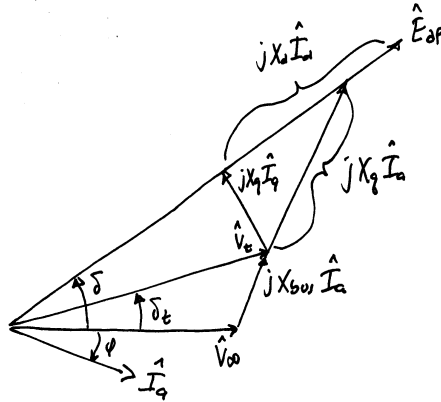
The generator will remain synchronized as long as  $P_{\max} > P$ . An iterative search with MATLAB can easily be used to find the minimum excitation that satisfies this condition for any particular loading.

part (a): For  $P = 0.5$ , must have  $E_{af} \geq 0.327$  per unit.

part (b): For  $P = 1.0$ , must have  $E_{af} \geq 0.827$  per unit.

### Problem 5.32

part (a):



part (b): We know that  $P = 0.95$  per unit and that

$$P = \frac{V_\infty V_t}{X_{bus}} \sin \delta_t$$

and that

$$\hat{I}_a = \frac{\hat{V}_t - V_\infty}{jX_t}$$

It is necessary to solve these two equations simultaneously for  $\hat{V}_t = V_t \angle \delta_t$  so that both the required power is achieved as well as the specified power factor

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angle with respect to the generator terminal voltage. This is most easily done iteratively with MATLAB. Once this is done, it is straightforward to calculate

$$V_t = 1.02 \text{ per-unit}; \quad E_{af} = 2.05 \text{ per-unit}; \quad \delta = 46.6^\circ$$

**Problem 5.33**

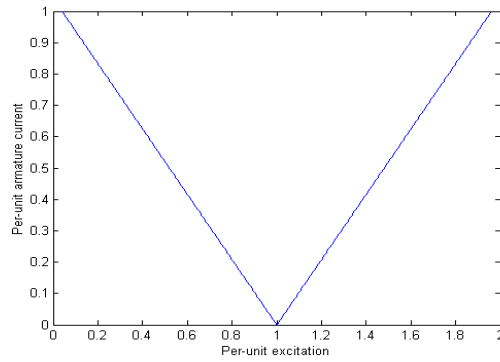
part (a): Define  $X_D = X_d + X_{bus}$  and  $X_Q = X_q + X_{bus}$ .

(i)

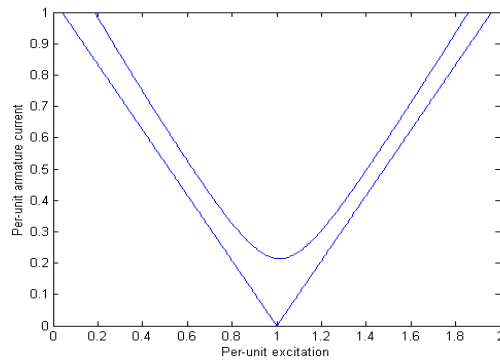
$$E_{af,\min} = V_{bus} - X_D = 0.04 \text{ per-unit}$$

$$E_{af,\max} = V_{bus} + X_D = 1.96 \text{ per-unit}$$

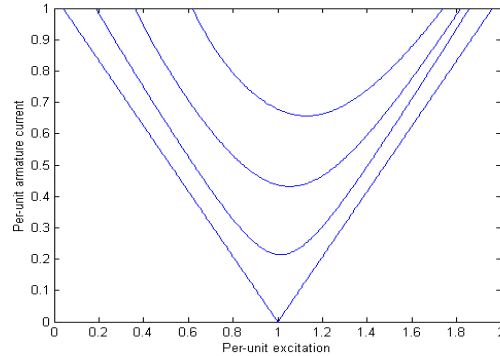
(ii)



part (b):



part (c):



### Problem 5.34

$$f = \frac{n \times \text{poles}}{120} = \frac{3000 \times 6}{120} = 150 \text{ Hz}$$

### Problem 5.35

part (a): Because the load is resistive, we know that

$$I_a = \frac{P}{3V_a} = \frac{4500}{\sqrt{3}192} = 13.5 \text{ A}$$

part (b): We know that  $E_{af} = 208/\sqrt{3} = 120 \text{ V}$ . Solving

$$E_{af} = \sqrt{V_a^2 + (X_s I_a)^2}$$

for  $X_s$  gives

$$X_s = \frac{\sqrt{E_{af}^2 - V_a^2}}{I_a} = 3.41 \Omega$$

part (c): The easiest way to solve this is to use MATLAB to iterate to find the required load resistance. If this is done, the solution is  $V_a = 108 \text{ V}$  (line-to-neutral) = 187 V (line-to-line).

### Problem 5.36

$$\hat{I}_a = \frac{E_a}{R_a + R_b + j\omega L_a} = \frac{\omega K_a}{R_a + R_b + j\omega L_a}$$

Thus

$$|\hat{I}_a| = \frac{\omega K_a}{\sqrt{(R_a + R_b)^2 + (\omega L_a)^2}} = \frac{K_a}{L_a \sqrt{1 + \left(\frac{R_a + R_b}{\omega L_a}\right)^2}}$$

Clearly,  $I_a$  will remain constant with speed as long as the speed is sufficient to insure that  $\omega \gg (R_a + R_b)/L_a$

## PROBLEM SOLUTIONS: Chapter 6

### Problem 6.1

part (a): Synchronous speed is 1800 r/min. Therefore,

$$s = \frac{1800 - 1755}{1800} = 0.025 = 2.5\%$$

part (b): Rotor currents are at slip frequency,  $f_r = s60 = 1.5$  Hz.

part (c): The stator flux wave rotates at synchronous speed with respect to the stator (1800 r/min). It rotates at slip speed ahead of the rotor ( $s1800 = 45$  r/min).

part (d): The rotor flux wave is synchronous with that of the stator. Thus it rotates at synchronous speed with respect to the stator (1800 r/min). It rotates at slip speed ahead of the rotor ( $s1800 = 45$  r/min).

### Problem 6.2

part (a): The slip is equal to  $s = 0.89/50 = 0.0178$ . The synchronous speed for a 6-pole, 50-Hz motor is 1000 r/min. Thus the rotor speed is

$$n = (1 - s)1000 = 982 \text{ r/min}$$

part (b): The slip of a 4-pole, 60-Hz motor operating at 1740 r/min is

$$s = \frac{1800 - 1740}{1800} = 0.0333 = 3.33\%$$

The rotor currents will therefore be at slip frequency  $f_r = 60 * 0.0333 = 2$  Hz.

### Problem 6.3

part (a): The synchronous speed is clearly 1200 r/min. Therefore the motor has 6 poles.

part (b): The full-load slip is

$$s = \frac{1200 - 1112}{1200} = 0.0733 = 7.33\%$$

part (c): The rotor currents will be at slip frequency  $f_r = 60 * 0.0733 = 4.4$  Hz.

part (d): The rotor field rotates at synchronous speed. Thus it rotates at 1200 r/min with respect to the stator and  $(1200 - 1112) = 88$  r/min with respect to the rotor.

### Problem 6.4

part (a): The wavelength of the fundamental flux wave is equal to the span of two poles or  $\lambda = 4.5/12 = 0.375$  m. The period of the applied excitation is  $T = 1/75 = 13.33$  msec. Thus the synchronous speed is



$$v_s = \frac{\lambda}{T} = 28.1 \text{ m/sec} = 101.3 \text{ km/hr}$$

part (b): Because this is an induction machine, the car in this case) will never reach synchronous speed.

part (c):

$$s = \frac{101.3 - 95}{101.3} = 0.0622 = 6.22\%$$

The induced track currents will be a slip frequency,  $f = s75 = 4.66 \text{ Hz}$ .

part (d): For a slip of 6.22% and a car velocity of 75 km/hr, the synchronous velocity must be

$$v_s = \frac{75}{1 - s} = 80.0 \text{ km/hr}$$

Thus the electrical frequency must be

$$f = 80 \left( \frac{75}{101.3} \right) = 59.2 \text{ Hz}$$

and the track currents will be at a frequency of  $sf = 3.68 \text{ Hz}$ .

### Problem 6.5

part (a): For operation at constant slip frequency  $f_r$ , the applied electrical frequency  $f_e$  is related to the motor speed in r/min  $n$  as

$$f_e = n \left( \frac{\text{poles}}{120} \right) + f_r$$

and thus, since the slip frequency  $f_r$  remains constant, we see that the applied electrical frequency will vary linearly with the desired speed.

Neglecting the voltage drop across the armature leakage inductance and winding resistance, the magnitude of the armature voltage is proportional to the air-gap flux density and the frequency. Hence the magnitude of the armature voltage must vary linearly with electrical frequency and hence the desired speed.

part (b): The electrical frequency of the rotor currents is equal to the slip frequency and hence will remain constant. Since the rotor will be operating in a constant flux which varies at a constant frequency, the magnitude of the rotor currents will be unchanged.

part (c): Because the rotor air-gap flux density and the rotor currents are unchanged, the torque will remain constant.

### Problem 6.6

part (a): Since the torque is proportional to the square of the voltage, the torque-speed characteristic will simply be reduced by a factor of 4.

part (b): Neglecting the effects of stator resistance and leakage reactance, having both the voltage and frequency maintains constant air-gap flux. Hence

the torque-speed characteristic looks the same as the original but the synchronous speed is halved.

**Problem 6.7**

- part (a): Four poles.
- part (b): Counterclockwise
- part (c): 1800 r/min
- part (d): Four poles
- part (e): No. There will be dc flux linking the induction-motor rotor windings but there will be no resultant voltage at the slip rings.

**Problem 6.8**

- part (a): 1500 r/min
- part (b): The induction motor rotor is rotating at 1500 r/min in the clockwise direction. Its stator flux wave is rotating at  $3000 \times (2/\text{poles}) = 1000$  r/min in the counterclockwise direction. Thus, the rotor sees a flux wave rotating at 2500 r/min. Noting that a flux wave rotating at 1000 r/min would produce 50-Hz voltages at the slip rings, we see that in this case the rotor frequency will be  $f_r = 50 \times (2500/1000) = 125$  Hz.
- part (c): Now the stator flux wave will rotate at 1000 r/min in the clockwise direction and the rotor will see a flux wave rotating at 500 r/min. The induced voltage will therefore be at a frequency of 25 Hz.

**Problem 6.9**

- part (a):  $R_1$  will decrease by a factor of 1.04 to  $0.212 \Omega$ .
- part (b):  $X_m$  will increase by a factor of 1/.85 to  $53.8 \Omega$ .
- part (c):  $R_2$  will decrease by a factor of 3.5/5.8 to  $0.125 \Omega$ .
- part (d): All values will decrease by a factor of 3.

**Problem 6.10**

This problem can be solved by direct substitution into the equations in chapter 6, which can in-turn be easily implemented in MATLAB. The following table of results was obtained from a MATLAB script which implemented the equivalent-circuit equations assuming the core-loss resistance  $R_c$  is in parallel with the magnetizing reactance.  $R_c$  was calculated as

$$R_c = \frac{460^2}{220} = 962 \Omega$$

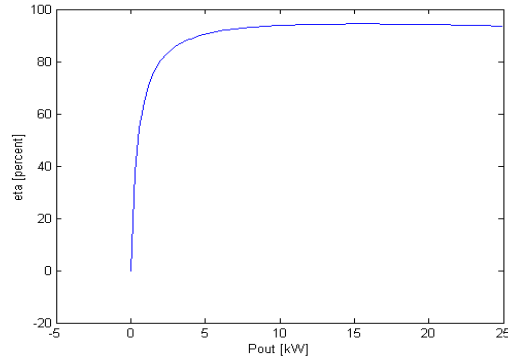
slip [%]	1.0	2.0	3.0
speed [r/min]	1782	1764	1746
$T_{\text{out}}$ [N·m]	8.5	16.5	23.4
$P_{\text{out}}$ [kW]	8.5	16.5	23.4
$P_{\text{in}}$ [kW]	45.8	89.6	128
power factor	0.81	0.87	0.85
efficiency [%]	93.3	94.4	93.8

**Problem 6.11**

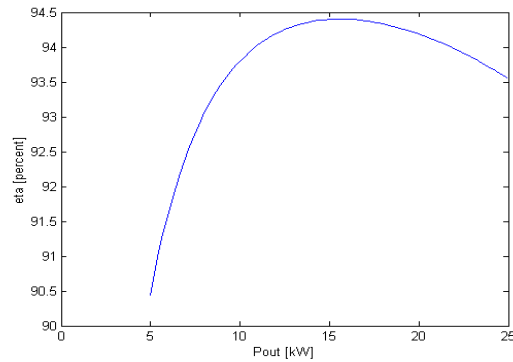
- part (a): 1741 r/min

part (b): 1799 r/min

part (c):



part (d):



### Problem 6.12

For a speed of 1725 r/min,  $P_{\text{out}} = 426$ , kW,  $P_{\text{in}} = 457$  kW,  $\text{pf} = 0.751$  lagging and  $\eta = 93.3$  %.

### Problem 6.13

It is necessary to find the value of  $R_2$ . This can be easily done by writing a MATLAB script to iteratively find that value of  $R_2$  which full-load internal torque at a slip of 3.5%. The result is  $R_2 = 0.0953 \Omega$ . Once this is done, the same MATLAB script can be used to substitute the machine parameters into the equations of chapter 6 to find  $T_{\text{max}} = 177$  N·m at a slip of 18.2% and  $T_{\text{start}} = 71.6$  N·m.

### Problem 6.14

This problem is readily solved once the value of  $R_2$  has been found as discussed in the solution to Problem 6.13. The impedance of the feeder must be added in series with the armature resistance  $R_1$  and leakage reactance  $X_1$ . A MATLAB script can then be written to find the desired operating point. The

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result is that the motor achieves rated output at a slip of 3.67% and a terminal voltage of 221.6 V, line-to-line.

**Problem 6.15**

part (a): For  $R_1 = 0$ ,  $R_{1,eq} = 0$  and thus from Eq. 6.34

$$\frac{R_2}{s_{\max T}} = (X_{1,eq} + X_2)$$

From Eq. 6.36,

$$T_{\max} = \frac{0.5n_{ph}V_{1,eq}^2}{\omega_s(X_{1,eq} + X_2)}$$

and from Eq.6.33 with  $s = 1$

$$T_{\text{start}} = \frac{n_{ph}V_{1,eq}^2 R_2}{\omega_s[R_2^2 + (X_{1,eq} + X_2)^2]}$$

Noting that

$$\frac{T_{\max}}{T_{\text{start}}} = \frac{2.20}{1.35} = 1.63$$

we can take the ratio of the above equations

$$\frac{T_{\max}}{T_{\text{start}}} = 1.63 = \frac{R_2^2 + (X_{1,eq} + X_2)^2}{R_2(X_{1,eq} + X_2)} = \frac{\left(\frac{R_2}{X_{1,eq} + X_2}\right)^2 + 1}{\frac{R_2}{X_{1,eq} + X_2}}$$

From Eq.6.34, with  $R_{eq,1} = 0$ ,  $s_{\max T} = R_2/(X_{1,eq} + X_2)$ . Hence

$$\frac{0.5(s_{\max T}^2 + 1)}{s_{\max T}} = 1.63$$

which can be solved to give  $s_{\max T} = 0.343 = 34.3\%$ .

part (b): From Eq. 6.33 with  $R_{eq,1} = 0$  and with  $s = s_{\text{rated}}$ ,

$$T_{\text{rated}} = \frac{n_{ph}V_{1,eq}^2 (R_2/s_{\text{rated}})}{\omega_s[(R_2/s_{\text{rated}})^2 + (X_{1,eq} + X_2)^2]}$$

and thus

$$\begin{aligned} \frac{T_{\max}}{T_{\text{rated}}} = 2.1 &= \frac{0.5[(R_2/s_{\text{rated}})^2 + (X_{eq,1} + X_2)^2]}{(R_2/s_{\text{rated}})(X_{eq,1} + X_2)} \\ &= \frac{0.5[1 + (s_{\max T}/s_{\text{rated}})^2]}{s_{\max T}/s_{\text{rated}}} \end{aligned}$$

This can be solved to give

$$s_{\text{rated}} = 0.240s_{\text{maxT}} = 0.0824 = 8.24\%$$

part (c):

$$\hat{I}_{2,\text{start}} = \frac{\hat{V}_{1,\text{eq}}}{R_2 + j(X_{\text{eq},1} + X_2)} = \frac{\hat{V}_{1,\text{eq}}}{(X_{\text{eq},1} + X_2)(s_{\text{maxT}} + j)}$$

$$\hat{I}_{2,\text{rated}} = \frac{\hat{V}_{1,\text{eq}}}{R_2/s_{\text{rated}} + j(X_{\text{eq},1} + X_2)} = \frac{\hat{V}_{1,\text{eq}}}{(X_{\text{eq},1} + X_2)(s_{\text{maxT}}/s_{\text{rated}} + j)}$$

Thus

$$\frac{|\hat{I}_{2,\text{start}}|}{|\hat{I}_{2,\text{rated}}|} = \frac{|s_{\text{maxT}}/s_{\text{rated}} + j|}{|s_{\text{maxT}} + j|} = \frac{|4.16 + j|}{|.343 + j|} = 4.05 = 405\%$$

### Problem 6.16

Given  $T_{\text{max}} = 2.3T_{\text{fl}}$ ,  $s_{\text{maxT}} = 0.55$  and  $s_{\text{fl}} = 0.087$ , start by taking the ratio of Eqs. 6.36 and 6.33

$$\frac{T_{\text{max}}}{T_{\text{fl}}} = \frac{0.5[(R_{1,\text{eq}} + R_2/s_{\text{fl}})^2 + (X_{1,\text{eq}} + X_2)^2]}{(R_{1,\text{eq}} + \sqrt{R_{1,\text{eq}}^2 + (X_{1,\text{eq}} + X_2)^2})(R_2/s_{\text{fl}})}$$

Substituting Eq. 6.34 gives

$$\frac{T_{\text{max}}}{T_{\text{fl}}} = \frac{0.5s_{\text{fl}} \left[ \left( \frac{2}{s_{\text{fl}}} \right) \left( \frac{R_{1,\text{eq}}}{R_2} \right) + \left( \frac{1}{s_{\text{fl}}} \right)^2 + \left( \frac{1}{s_{\text{maxT}}} \right)^2 \right]}{\frac{R_{1,\text{eq}}}{R_2} + \frac{1}{s_{\text{maxT}}}}$$

Substituting given values and solving gives

$$\frac{R_{\text{eq},1}}{R_2} = 1.315$$

From Eq. 6.33 we can write

$$\frac{T_{\text{start}}}{T_{\text{fl}}} = s_{\text{fl}} \left[ \frac{\left( \frac{R_{\text{eq},1}}{R_2} + \frac{1}{s_{\text{fl}}} \right)^2 + \left( \frac{X_{\text{eq},1} + X_2}{R_2} \right)^2}{\left( \frac{R_{\text{eq},1}}{R_2} + \frac{1}{s_{\text{maxT}}} \right)^2 + \left( \frac{X_{\text{eq},1} + X_2}{R_2} \right)^2} \right]$$

From Eq. 6.34,

$$\left( \frac{X_{1,\text{eq}} + X_2}{R_2} \right)^2 = \left( \frac{1}{s_{\text{maxT}}} \right)^2 - \left( \frac{R_{1,\text{eq}}}{R_2} \right)^2$$

and thus we can solve for

$$T_{\text{start}} = 1.26T_{\text{fl}}$$

### Problem 6.17

part (a): Using MATLAB to solve the equivalent-circuit equations, from the equivalent-circuit at a slip of 3.35%, the power applied to the shaft can be calculated to be 503.2 kW. Thus the rotational loss is 3.2 kW. Similarly, the input power to the equivalent circuit is 528.0 kW. Based upon an efficiency of 94%, the actual motor input power is  $500 \text{ kW}/0.94 = 531.9 \text{ kW}$ . Thus, the core losses are equal to  $531.9 - 528.0 = 3.9 \text{ kW}$ .

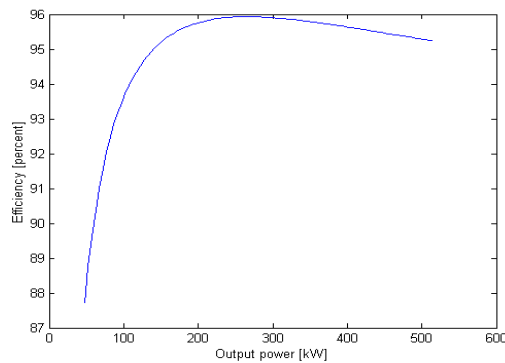
part (b): The equivalent circuit is solved in the normal fashion. For ease of calculation, the core loss can be accounted for by a resistor connected at the equivalent-circuit terminals (based upon the results of part (a), this corresponds to a resistance of  $1.47 \text{ k}\Omega$ ). The shaft input power is equal to the negative of the shaft power calculated from the equivalent circuit plus the rotational loss power. The electrical output power is equal to the negative of the input power to the equivalent circuit. The result is (using MATLAB):

- (i) Generator output power = 512 kW
- (ii) efficiency = 91.6%
- (iii) power factor = 0.89

part (c): Basically the same calculation as part (b). The impedance of the feeder must be added to armature impedance of the induction motor. The result is (using MATLAB):

- (i) Power supplied to the bus = 498 kW
- (ii) Generator output power = 508 kW

### Problem 6.18



### Problem 6.19

part (a): Given  $I_{2,\text{maxT}}^2 R_2 = 9.0I_{2,\text{fl}}^2 R_2$ . Thus  $I_{2,\text{maxT}} = 3.0I_{2,\text{fl}}$ . Ignoring  $R_1$ ,  $R_{1,\text{eq}} = 0$  and we can write

$$\hat{I}_2 = \frac{\hat{V}_{\text{eq}}}{R_2/s + j(X_{\text{eq}} + X_2)}$$

and thus

$$\frac{\hat{I}_{2,\text{fl}}}{\hat{I}_{2,\text{maxT}}} = \frac{j(X_{\text{eq}} + X_2) + R_2/s_{\text{maxT}}}{j(X_{\text{eq}} + X_2) + R_2/s_{\text{fl}}}$$

Substitution from Eq. 6.34

$$(X_{1,\text{eq}} + X_2) = \frac{R_2}{s_{\text{maxT}}}$$

gives

$$\frac{\hat{I}_{2,\text{fl}}}{\hat{I}_{2,\text{maxT}}} = \frac{j + 1}{j + s_{\text{maxT}}/s_{\text{fl}}}$$

and thus

$$\frac{I_{2,\text{fl}}}{I_{2,\text{maxT}}} = \frac{|j + 1|}{|j + s_{\text{maxT}}/s_{\text{fl}}|} = \frac{\sqrt{2}}{\sqrt{1 + (s_{\text{maxT}}/s_{\text{fl}})^2}}$$

Finally, we can solve for  $s_{\text{maxT}}$

$$s_{\text{maxT}} = 4.12s_{\text{fl}} = 0.0948 = 9.48\%$$

part (b): Taking the ratio of Eqs. 6.36 and 6.33 with  $R_{1,\text{eq}} = 0$  and substitution of Eq. 6.34 gives

$$\frac{T_{\text{max}}}{T_{\text{fl}}} = \frac{0.5[(R_2/s_{\text{fl}})^2 + (X_{1,\text{eq}} + X_2)^2]}{(X_{1,\text{eq}} + X_2)(R_2/s_{\text{fl}})} = \frac{0.5[1 + (s_{\text{maxT}}/s_{\text{fl}})^2]}{(s_{\text{maxT}}/s_{\text{fl}})} = 2.18$$

In other words,  $T_{\text{max}} = 2.18$  per unit.

part (c): In a similar fashion, one can show that

$$\frac{T_{\text{start}}}{T_{\text{fl}}} = s_{\text{fl}} \left( \frac{1 + (s_{\text{maxT}}/s_{\text{fl}})^2}{1 + s_{\text{maxT}}^2} \right) = 0.41$$

In other words,  $T_{\text{start}} = 0.41$  per unit.

### Problem 6.20

part (a):  $T \propto I_2^2 R_2/s$ . Thus

$$\frac{T_{\text{start}}}{T_{\text{fl}}} = s_{\text{fl}} \left( \frac{I_{2,\text{start}}}{I_{2,\text{fl}}} \right)^2 = 1.32$$

and thus  $T_{\text{start}} = 1.32$  per unit.

part (b): As in the solution to Problem 6.15, neglecting the effects of  $R_1$

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$$\frac{|\hat{I}_{2,\text{start}}|}{|\hat{I}_{2,\text{rated}}|} = \frac{|s_{\text{maxT}}/s_{\text{fl}} + j|}{|s_{\text{maxT}} + j|}$$

This can be solved for  $s_{\text{maxT}}$

$$s_{\text{maxT}} = s_{\text{fl}} \sqrt{\frac{1 - (I_{2,\text{start}}/I_{\text{fl}})^2}{(s_{\text{fl}} I_{2,\text{start}}/I_{\text{fl}})^2 - 1}} = 0.224 = 22.4\%$$

Again, from the solution to Problem 6.15,

$$\frac{T_{\text{max}}}{T_{\text{rated}}} = \frac{0.5[1 + (s_{\text{maxT}}/s_{\text{fl}})^2]}{s_{\text{maxT}}/s_{\text{fl}}} = 3.12$$

and thus  $T_{\text{max}} = 3.12$  per unit.

**Problem 6.21**

part (a): Solving the equations of chapter 6, with  $s = 1$  for starting, with MATLAB yields

$$\begin{aligned} I_{\text{start}} &= 233 \text{ A} \\ T_{\text{start}} &= 79.1 \text{ N}\cdot\text{m} \end{aligned}$$

part (b): (i) When the motor is connected in Y, the equivalent-circuit parameters will be three times those of the normal  $\Delta$  connection. Thus

$$\begin{aligned} R_1 &= 0.135 \ \Omega \\ R_2 &= 0.162 \ \Omega \\ X_1 &= 0.87 \ \Omega \\ X_2 &= 0.84 \ \Omega \\ X_m &= 28.8 \ \Omega \end{aligned}$$

(ii)

$$\begin{aligned} I_{\text{start}} &= 77.6 \text{ A} \\ T_{\text{start}} &= 26.3 \text{ N}\cdot\text{m} \end{aligned}$$

**Problem 6.22**

part (a):

$$P_{\text{rot}} = P_{\text{nl}} - 3I_{\text{nl}}^2 R_1 = 2672 \text{ W}$$

part (b): The parameters are calculated following exactly the procedure found in Example 6.5. The results are:

$$\begin{aligned} R_1 &= 1.11 \ \Omega \\ X_1 &= 3.90 \ \Omega \\ R_2 &= 1.34 \ \Omega \\ X_2 &= 3.90 \ \Omega \\ X_m &= 168 \ \Omega \end{aligned}$$



part (c): Solving the equations of chapter 6 using the equivalent-circuit parameters of part (b)

$$\begin{aligned} I_a &= 29.1 \text{ A} \\ P_{\text{in}} &= 106 \text{ kW} \\ \text{power factor} &= 0.91 \text{ lagging} \\ P_{\text{out}} &= 100 \text{ kW} \\ \text{efficiency} &= 94.5\% \end{aligned}$$

### Problem 6.23

Because this is a blocked-rotor test, one can ignore the magnetizing reactance  $X_m$ . As a result, the motor input impedance can be approximated as

$$Z_{\text{in}} \approx R_1 + R_2 + j(X_1 + X_2)$$

$R_2$  can be calculated from the blocked-rotor power and current

$$R_2 = \frac{B_{\text{bl}}}{3I_{\text{bl}}^2} - R_1$$

which gives

$$\begin{aligned} \text{Motor 1: } R_2 &= 0.174 \ \Omega \\ \text{Motor 2: } R_2 &= 0.626 \ \Omega \end{aligned}$$

The motor starting torque is proportional to  $I_{\text{bl}}^2 R_2$  and thus the torque ratio is given by

$$\frac{T_{\text{motor2}}}{T_{\text{motor1}}} = \frac{(I_2^2)_{\text{motor2}}(R_2)_{\text{motor2}}}{(I_2^2)_{\text{motor1}}(R_2)_{\text{motor1}}} = \left( \frac{(R_2)_{\text{motor2}}}{(R_2)_{\text{motor1}}} \right) \left( \frac{(I_2^2)_{\text{motor2}}}{(I_2^2)_{\text{motor1}}} \right)$$

Thus, for the same currents, the torque will be simply proportional to the resistance ratio and hence

$$\frac{T_{\text{motor2}}}{T_{\text{motor1}}} = 0.278$$

From the given data, we see that for the same voltage, the current ratio will be  $(I_2)_{\text{motor2}}/(I_2)_{\text{motor1}} = 99.4/74.7 = 1.39$  and hence

$$\frac{T_{\text{motor2}}}{T_{\text{motor1}}} = 0.492$$

### Problem 6.24

Rotational loss = 3120 W

$$\begin{aligned} R_1 &= 0.318 \ \Omega & R_2 &= 0.605 \ \Omega \\ X_1 &= 2.28 \ \Omega & X_2 &= 3.42 \ \Omega & X_m &= 63.4 \ \Omega \end{aligned}$$

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**Problem 6.25**Neglecting  $R_1$  and hence  $R_{eq,1}$  gives from Eq. 6.35

$$s_{\max T} = \frac{R_2}{X_{1,eq} + X_2}$$

and from Eq. 6.36

$$T_{\max} = \frac{0.5n_{ph}V_{1,eq}^2}{\omega_s(X_{1,eq} + X_2)} = \frac{0.5n_{ph}V_{1,eq}^2 s_{\max T}}{\omega_s R_2}$$

If the frequency is reduced from 60 to 50 Hz,  $X_{1,eq} + X_2$  will drop by a factor of 5/6 and hence  $s_{\max T}$  will increase by a factor of 6/5 to  $s_{\max T} = 18\%$ , corresponding to a speed of  $1000(1 - 0.18) = 820$  r/min.

 $T_{\max}$  will increase as

$$\frac{(T_{\max})_{50}}{(T_{\max})_{60}} = \frac{(190/230)^2(6/5)}{5/6} = 0.983$$

or  $(T_{\max})_{50} = 283\%$ **Problem 6.26** $s_{\max T} \propto R_2$ . Therefore

$$R_2 = \frac{1.1}{[(s_{\max T})_{R_{ext}=1.1}/(s_{\max T})_{R_{ext}=0}] - 1} = 2.07 \Omega$$

**Problem 6.27**

part (a): From the solution to Problem 6.15

$$\frac{T_{\max}}{T_{fl}} = \frac{0.5[1 + (s_{\max T}/s_{fl})^2]}{s_{\max T}/s_{fl}}$$

Given that  $T_{\max}/T_{fl} = 2.25$  and  $s_{\max T} = 0.16$ , this can be solved for  $s_{fl} = 0.0375 = 3.75\%$ .

part (b): The rotor rotor power dissipation at rated load is given by

$$P_{rotor} = P_{rated} \left( \frac{s_{fl}}{1 - s_{fl}} \right) = 2.9 \text{ kW}$$

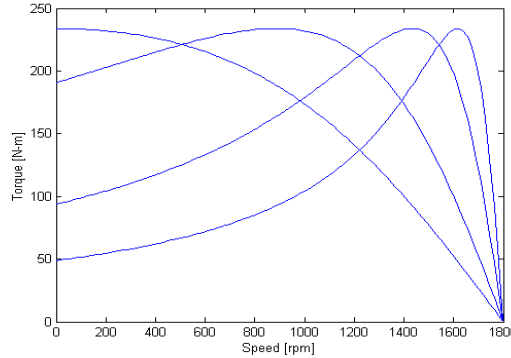
part (c): From the solution to Problem 6.19

$$\frac{T_{start}}{T_{fl}} = s_{fl} \left( \frac{1 + (s_{\max T}/s_{fl})^2}{1 + s_{\max T}^2} \right) = 0.70$$

Rated torque is equal to  $75 \text{ kW}/\omega_{m,fl}$  where  $\omega_{m,fl} = 60\pi(1 - s_{fl}) = 181.4$  rad/sec. Thus  $T_{rated} = 413 \text{ N}\cdot\text{m}$  and  $T_{start} = 0.70$  per unit =  $290 \text{ N}\cdot\text{m}$ .

part (d): If the rotor resistance is doubled, the motor impedance will be the same if the slip is also doubled. Thus, the slip will be equal to  $s = 2s_{fl} = 7.50\%$ .

part (e): The torque will equal to full-load torque. Thus  $T = 413 \text{ N}\cdot\text{m}$ .

**Problem 6.28****Problem 6.29**

part (a):

$$P_{\text{rotor}} = P_{\text{rated}} \left( \frac{s_{\text{fl}}}{1 - s_{\text{fl}}} \right) = 3.63 \text{ kW}$$

part (b): From the solution to Problem 6.15

$$\frac{T_{\text{max}}}{T_{\text{fl}}} = \frac{0.5[1 + (s_{\text{maxT}}/s_{\text{fl}})^2]}{s_{\text{maxT}}/s_{\text{fl}}}$$

Given that  $T_{\text{max}}/T_{\text{fl}} = 3.10$  and  $s_{\text{fl}} = (1200 - 1158)/1200 = 0.035$ , this can be solved for  $s_{\text{maxT}} = 0.211 = 21.1\%$ . This corresponds to a speed of  $1200(1 - 0.211) = 947 \text{ r/min}$ .

part (c): Sufficient resistance must be inserted to increase  $s_{\text{maxT}}$  from 0.211 to 1.0. Thus  $R_{2,\text{tot}} = 0.17/0.211 = 0.806 \Omega$  and hence the added resistance must be  $R_{\text{ext}} = 0.806 - 0.211 = 0.635 \Omega$ .

part (d): The applied voltage must be reduced by a factor of 5/6 to 383 V, line-to-line.

part (e): From Eq. 6.35,  $s_{\text{maxT}} = R_2/(X_{1,\text{eq}} + X_2)$ . If the frequency decreases by a factor of 5/6, the reactances will also decrease by a factor of 5/6 and hence  $s_{\text{maxT}}$  will increase by a factor of 6/5 to 0.042. Hence, the corresponding speed will be  $1000(1 - 0.042) = 958 \text{ r/min}$ .

**Problem 6.30**

If the impedance of the motor at starting is made equal to that of the motor at a slip of 5.6% the starting current will be equal to 200% of its rated value. This can be done by increasing the rotor resistance for  $90/2 = 45 \text{ m}\Omega$  to

$$R_{2,\text{tot}} = \frac{0.045}{0.056} = 804 \text{ m}\Omega$$

and hence the required added resistance will be  $R_{\text{ext}} = 804 - 45 = 759 \text{ m}\Omega$ . The starting torque under this condition will be 190% of the full-load torque.

**Problem 6.31**

The synchronous speed of this motor is  $3600/8 = 450$  r/min (47.12 rad/sec). Thus its full-load speed is  $450(1 - 0.041) = 431.6$  r/min. The corresponding torque will be  $(250 \times 10^3)[47.12(1 - 0.041)] = 5.53 \times 10^3$  N·m. At a speed of 400 r/min, the torque will be  $5.53 \times 10^3(400/431.6)^2 = 4.75 \times 10^3$  N·m.

With no external resistance, the slope of the torque-speed characteristic is thus  $5.53 \times 10^3/431.6 = 12.81$ . The slope of the desired torque-speed characteristic is  $4.75 \times 10^3/400 = 11.88$ . Since the initial slope of the torque-speed characteristic is inversely proportional to the rotor resistance, the total rotor resistance must be

$$R_{\text{tot}} = \left( \frac{12.81}{11.88} \right) 24.5 = 26.4 \text{ m}\Omega$$

Therefore the required added resistance is  $26.4 - 24.5 = 1.9$  m $\Omega$ .

## PROBLEM SOLUTIONS: Chapter 7

### Problem 7.1

- part (a):  $\omega_m \propto V_a$ .  
 part (b):  $\omega_m \propto \frac{1}{I_f}$   
 part (c):  $\omega_m$  will be constant.

### Problem 7.2

part (a): For constant terminal voltage, the product  $nI_f$  (where  $n$  is the motor speed) is constant. Hence, since  $I_f \propto 1/R_f$

$$\frac{R_f}{1180} = \frac{R_f + 5}{1250}$$

and hence  $R_f = 84.2 \Omega$ .

part (b): 1380 r/min

### Problem 7.3

Check this

- part (a):  $\omega_m$  halved;  $I_a$  constant  
 part (b):  $\omega_m$  halved;  $I_a$  doubled  
 part (c):  $\omega_m$  halved;  $I_a$  halved  
 part (d):  $\omega_m$  constant;  $I_a$  doubled  
 part (e):  $\omega_m$  halved;  $I_a$  reduced by a factor of 4.

### Problem 7.4

part (a): Rated armature current = 25 kW/250-V = 100 A.

part (b): At 1200 r/min,  $E_a$  can be determined directly from the magnetization curve of Fig. 7.27. The armature voltage can be calculated as

$$V_a = E_a + I_a R_a$$

and the power output as  $P_{\text{out}} = V_a I_a$ . With  $I_a = 100$  A

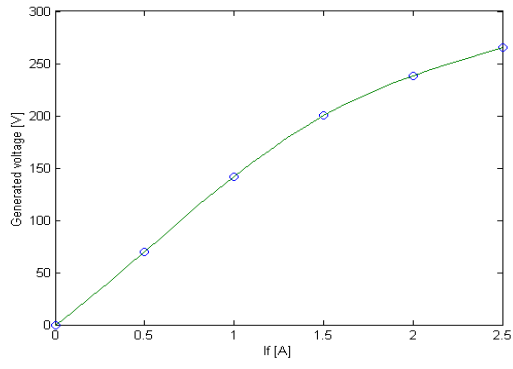
$I_f$ [A]	$E_a$ [V]	$V_a$ [V]	$P_{\text{out}}$ [kW]
1.0	150	164	16.4
2.0	240	254	25.4
2.5	270	284	28.4

part (c): The solution proceeds as in part (b) but with the generated voltage equal to  $900/1200 = 0.75$  times that of part (b)

$I_f$ [A]	$E_a$ [V]	$V_a$ [V]	$P_{\text{out}}$ [kW]
1.0	112	126	12.6
2.0	180	194	19.4
2.5	202	216	21.6

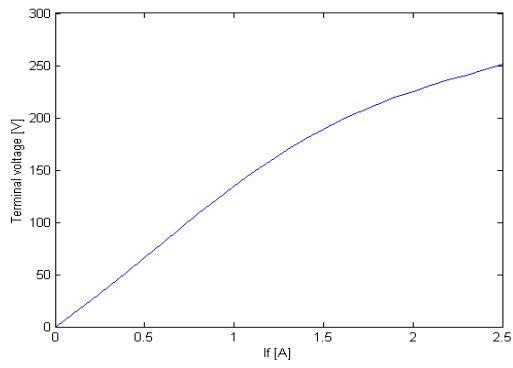
**Problem 7.5**

part (a):

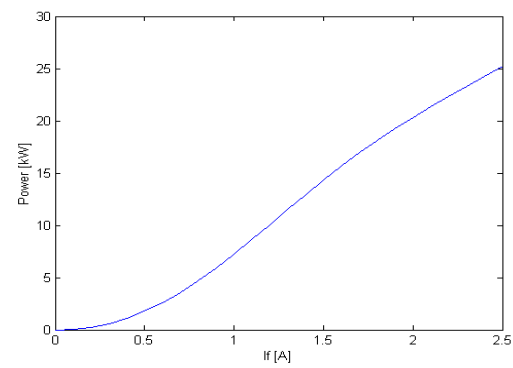


part (b):

(i)



(ii)



**Problem 7.6**

part (a):

$$P_{\text{shaft}} = E_a \left( \frac{V_a - E_a}{R_a} \right)$$

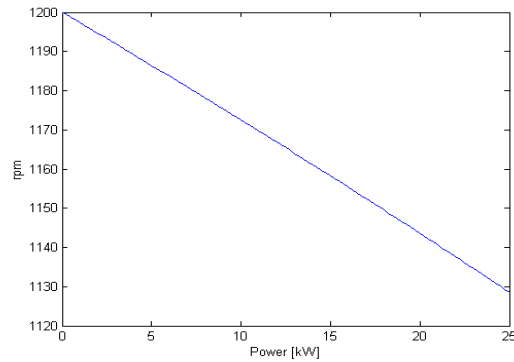
and thus

$$E_a = \frac{V_a + \sqrt{V_a^2 - 4P_{\text{shaft}}R_a}}{2}$$

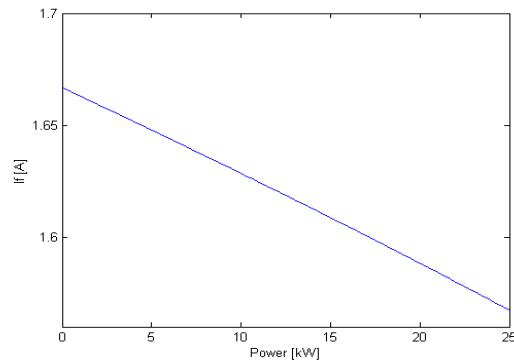
The motor speed  $n$  can then be found from

$$n = 1200 \left( \frac{E_a}{1.67 \times 1200} \right) \text{ r/min}$$

Here is the desired plot, produced by MATLAB

part (b): The solution for  $E_a$  proceeds as in part (a). With the speed constant at 1200 r/min (and hence constant  $\omega_m$ ), solve for  $I_f$  as

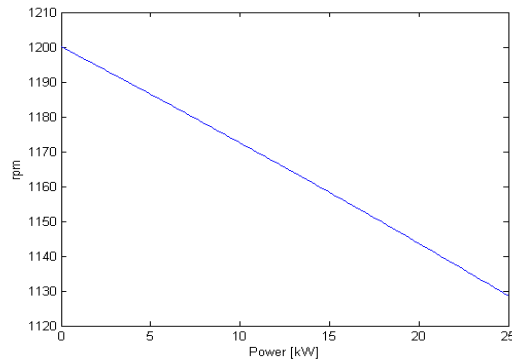
$$I_f = \frac{E_a}{K_f}$$

where  $K_f = 150$  D V/A. Here is the desired plot, produced by MATLAB.

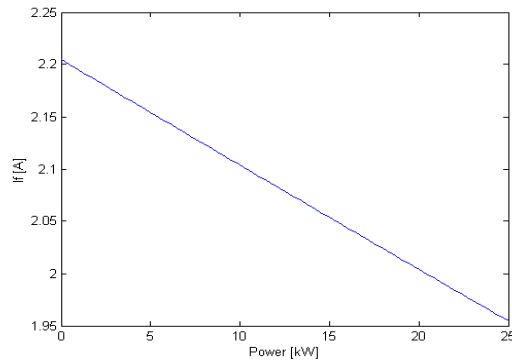
**Problem 7.7**

The solution is similar to that of Problem 7.6 with the exception that the assumed straight-line magnetization characteristic is replaced by the non-linear characteristic of Fig. 7.27. MATLAB, with the 'spline()' function used to represent the non-linear characteristic of Fig. 7.27, then produces the following plots.

part (a):



part (b):

**Problem 7.8**

part (a): From the load data, the generated voltage is equal to  $254 + 62.7 \times 0.175 = 265$  V. From the magnetizing curve (using the 'spline()' function of MATLAB), the corresponding field current is 1.54 A. Hence the demagnetizing effect of this armature current is equal to  $(1.95 - 1.54)500 = 204$  A-turns/pole.

part (b): At the desired operating point, the generator output power will be  $250 \text{ V} \times 61.5 \text{ A} = 15.4 \text{ kW}$ . Therefore, the motor speed will be

$$n = 1195 - 55 \left( \frac{15.4}{15} \right) = 1139 \text{ r/min}$$

Because the machine terminal voltage at no load must equal 230 V, from the magnetizing curve we see that the shunt field under this operating condition



must equal 1.05 A and hence the shunt field resistance must be 219  $\Omega$ . Hence, under this loading condition, with a terminal voltage of 250 V, the armature voltage will be  $250 + 61.5 \times 0.065 = 250.8$  V, the shunt field current will equal  $250.8/219 = 1.15$  A and thus the armature current will equal  $61.5 + 1.15 = 62.7$  A.

The generated voltage can now be calculated to be  $250.8 + 62.7(0.175) = 286$  V. The corresponding voltage on the 1195 r/min mag curve will be  $E_a = 286(1195/1139) = 285$  V and hence the required net field ampere-turns is (using the MATLAB 'spline()' function) 1042 A-turns. The shunt-field ampere-turns is  $1.15 \times 500 = 575$  A-turns, the demagnetizing armature amp-turns are 204 A-turns and hence the required series turns are

$$N_s = \frac{1042 - (575 - 204)}{61.5} = 10.6 \approx 11 \text{ turns}$$

### Problem 7.9

From the given data, the generated voltage at  $I_a = 90$  A and  $n(90) = 975$  r/min is

$$E_a(90) = V_a - I_a(R_a + R_s) = 230 - 90(0.11 + 0.08) = 212.9 \text{ V}$$

Similarly, the generated voltage at  $I_a = 30$  A is

$$E_a(30) = 230 - 30(0.11 + 0.08) = 224.3 \text{ V}$$

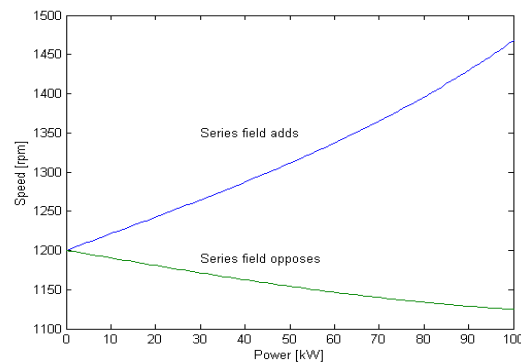
Since  $E_a \propto n\Phi$

$$\frac{E_a(30)}{E_a(90)} = \left( \frac{n(30)}{n(90)} \right) \left( \frac{\Phi(30)}{\Phi(90)} \right)$$

Making use of the fact that  $\Phi(30)/\Phi(90) = 0.48$ , we can solve for  $n(30)$

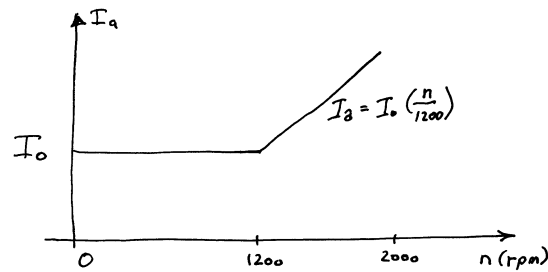
$$n(30) = n(90) \left( \frac{E_a(30)}{E_a(90)} \right) \left( \frac{\Phi(90)}{\Phi(30)} \right) = 2140 \text{ r/min}$$

### Problem 7.10

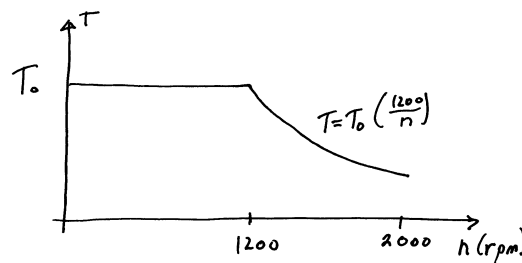


**Problem 7.11**

part (a): For constant field current, and hence constant field flux, constant torque corresponds to constant armature current. Thus for speeds up to 1200 r/min, the armature current will remain constant. For speeds above 1200 r/min, ignoring the voltage drop across the armature resistance, the motor speed will be inversely proportional to the field current (and hence the field flux). Thus the armature current will increase linearly with speed from its value at 1200 r/min. Note that as a practical matter, the armature current should be limited to its rated value, but that limitation is not considered in the plot below.



part (b): In this case, the torque will remain constant as the speed is increased to 1200 r/min. However, as the field flux drops to increase the speed above 1200 r/min, it is not possible to increase the armature current as the field flux is reduced to increase the speed above 1200 r/min and hence the torque track the field flux and will decrease in inverse proportion to the change in speed above 1200 r/min.

**Problem 7.12**

part (a): With constant terminal voltage and speed variation obtained by field current control, the field current (and hence the field flux) will be inversely proportional to the speed. Constant power operation (motor A) will then require

constant armature current. Constant torque (motor B) will require that the armature current variation be proportional to the motor speed. Thus

$$\begin{aligned} \text{motor A: } I_a &= 125 \text{ A} \\ \text{motor B: } I_a &= 125(500/1800) = 34.7 \text{ A} \end{aligned}$$

part (b):

$$\begin{aligned} \text{motor A: } I_a &= 125 \text{ A} \\ \text{motor B: } I_a &= 125(1800/125) = 450 \text{ A} \end{aligned}$$

part (c): Under armature voltage control and with constant field current, the speed will be proportional to the armature voltage. The generated voltage will be proportional to the speed. Constant-power operation (motor A) will require armature current that increases inversely with speed while constant torque operation (motor B) will require constant armature current.

For the conditions of part (a):

$$\begin{aligned} \text{motor A: } I_a &= 125(1800/125) = 450 \text{ A} \\ \text{motor B: } I_a &= 125 \text{ A} \end{aligned}$$

For the conditions of part (b):

$$\begin{aligned} \text{motor A: } I_a &= 125(500/1800) = 34.7 \text{ A} \\ \text{motor B: } I_a &= 125 \text{ A} \end{aligned}$$

### Problem 7.13

$$\omega_m = \frac{E_a}{K_a \Phi_d} = \frac{V_a - I_a R_a}{K_a \Phi_d}$$

$$I_a = \frac{T}{K_a \Phi_d}$$

Thus

$$\omega_m = \frac{1}{K_a \Phi_d} \left( V_a - \frac{T R_a}{K_a \Phi_d} \right)$$

The desired result can be obtained by taking the derivative of  $\omega_m$  with  $\Phi_d$

$$\begin{aligned} \frac{d\omega_m}{d\Phi_d} &= \frac{1}{K_a \Phi_d^2} \left( \frac{2T R_a}{K_a \Phi_d} - V_a \right) \\ &= \frac{1}{K_a \Phi_d^2} (2I_a R_a - V_a) \\ &= \frac{1}{K_a \Phi_d^2} (V_a - 2E_a) \end{aligned}$$

From this we see that for  $E_a > 0.5V_a$ ,  $d\omega_a/d\Phi_d < 0$  and for  $E_a < 0.5V_a$ ,  $d\omega_a/d\Phi_d > 0$ . Q.E.D.

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**Problem 7.14**

part (a):

Synchronous machine:

$$I_{a,ac} = \frac{30 \times 10^3}{\sqrt{3} 460} = 37.7 \text{ A}$$

$$E_{af} = |V_{a,ac} + jX_s I_{a,ac}| = |460/\sqrt{3} + j5.13 \times 37.7| = 328.4 \text{ V, 1-n}$$

DC machine:

$$P = E_a I_{a,dc} = 30 \text{ kW}$$

$$E_a = V_{a,dc} - I_{a,dc} R_a$$

Thus,

$$E_a^2 = \frac{V_{a,dc} + \sqrt{V_{a,dc}^2 - 4PR_a}}{2} = 226 \text{ V}$$

part (b): Increase the dc-motor field excitation until  $E_a = V_{a,dc} = 230 \text{ V}$ , in which case the dc motor input current will equal zero and it will produce no shaft power. The ac machine will operate at a power angle of zero and hence its terminal current will be

$$I_{a,ac} = \frac{E_{af} - V_{a,ac}}{X_s} = 12.2 \text{ A}$$

part (c): If one further increases the dc-machine field excitation the dc machine will act as a generator. In this case, defining the dc generator current as positive out of the machine,

$$P = E_a I_{a,dc} = 30 \text{ kW}$$

$$E_a = V_{a,dc} + I_{a,dc} R_a$$

Thus,

$$E_a^2 = \frac{V_{a,dc} + \sqrt{V_{a,dc}^2 + 4PR_a}}{2} = 226 \text{ V}$$

and

$$I_{a,dc} = \frac{E_a - V_{a,dc}}{R_a} = 128 \text{ A}$$

The ac machine will now be operating as a motor. The armature current will be the negative of that of part (a) and hence the power factor will be unity. Its magnitude will be 37.7 A.

**Problem 7.15**

First find the demagnetizing mmf. At rated load,

$$E_a = V_a - I_a R_{\text{tot}} = 600 - 250 \times 0.125 = 568.8 \text{ V}$$

Using the MATLAB 'spline' function, the corresponding field current on the 400 r/min magnetizing curve is

$$I_f = 232 \text{ A}$$

Thus, the demagnetizing mmf at a current of 250 A is equal to  $250 - 232 = 18$  A and in general, the effective series-field current will be equal to

$$I_{s,\text{eff}} = I_a - 18 \left( \frac{I_a}{250} \right)^2$$

For a starting current of 460 A, the effective series field current will thus equal 399 A. Using the MATLAB 'spline()' function, this corresponds to a generated voltage of 474 V from the 400 r/min magnetization curve. The corresponding torque (which will be the same as the starting torque due to the same flux and armature current) can then be calculated as

$$T = \frac{E_a I_a}{\omega_m} = \frac{474 \times 560}{400(\pi/30)} = 5200 \text{ N} \cdot \text{m}$$

**Problem 7.16**

At no load,  $E_{a,\text{nl}} = 230 - 6.35 \times 0.11 = 229.3$  V. At full load,  $E_{a,\text{fl}} = 230 - 115 * 0.11 = 217.4$  V. But,  $E_a \propto n\Phi$ , thus

$$n_{\text{fl}} = n_{\text{nl}} \left( \frac{E_{a,\text{fl}}}{E_{a,\text{nl}}} \right) \left( \frac{\Phi_{\text{nl}}}{\Phi_{\text{fl}}} \right) = 2150 \left( \frac{217.4}{229.3} \right) \left( \frac{1}{0.94} \right) = 2168 \text{ r/min}$$

**Problem 7.17**

The motor power is given by  $P = E_a I_a$ , where

$$E_a = K_a \Phi_d \omega_m$$

and where, from Eq. 7.3

$$K_a = \frac{\text{poles } C_a}{2\pi m} = \frac{4 \times 666}{2\pi \times 2} = 212$$

Thus, for  $\Phi_d = 0.01$ ,  $E_a = K_a \Phi_d \omega_m = 2.12 \omega_m$ .

$$I_a = \frac{V_t - E_a}{R_a}$$

Using MATLAB and its 'spline()' function to represent the fan characteristics, an iterative routine can be written to solve for the operating point (the intersection of the motor and fan characteristics). The result is that the motor will operate at a speed of 999 r/min and an output power of 8.39 kW.

**Problem 7.18**

part (a): Assuming negligible voltage drop across the armature resistance at no load, the field current can be found from the 1300 r/min magnetization curve by setting  $E_a = 230$  V. This can be most easily done using the MATLAB 'spline()' function. The result is  $I_f = 1.67$  A. This corresponds to  $N_f I_f = 2500$  a·turns of mmf.

part (b): At rated load,  $E_a = V_a - I_a R_a = 230 - 46.5 \times 0.17 = 222.1$  V. From the no-load, 1300 r/min magnetization curve, the corresponding field current is 1.50 A (again obtained using the MATLAB 'spline()' function). Thus, the effective armature reaction is

$$\begin{aligned} \text{Armature reaction} &= (1.67 - 1.5) \text{ A} \times 1500 \text{ turns/pole} \\ &= 251 \text{ A} \cdot \text{turns/pole} \end{aligned}$$

part (c): With the series field winding,  $R_{\text{tot}} = R_a + R_s = 0.208 \Omega$ . Thus, under this condition,  $E_a = V_a - I_a R_a = 220.3$ . This corresponds to a 1300 r/min generated voltage of 236.7 V and a corresponding field current (determined from the magnetization curve using the MATLAB 'spline()' function) of 1.84 A, corresponding to a total of 2755 A·turns. Thus, the required series field turns will be

$$N_s = \frac{2755 - (2500 - 251)}{46.5} = 10.8$$

or, rounding to the nearest integer,  $N_s = 11$  turns/pole.

part (d): Now the effective field current will be

$$I_{\text{eff}} = \frac{2500 - 251 + 20 \times 46.5}{1500} = 2.12 \text{ A}$$

From the 1300 r/min magnetization curve,  $E_a = 246.1$  V while the actual  $E_a = V_a - R_{\text{tot}} I_a = 220.3$  V. Hence the new speed is

$$n = 1300 \left( \frac{220.3}{246.1} \right) = 1164 \text{ r/min}$$

**Problem 7.19**

part (a): At full load, 1185 r/min, with a field current of 0.554 A

$$E_a = V_a - I_a R_{\text{tot}} = 221.4 \text{ V}$$

where  $R_{\text{tot}} = 0.21 + 0.035 = 0.245 \Omega$ .

An 1825 r/min magnetization curve can be obtained by multiplying 230 V by the ratio of 1185 r/min divided by the given speed for each of the points in the data table. A MATLAB 'spline()' fit can then be used to determine that this generated voltage corresponds to a field-current of 0.527 A. Thus, the armature reaction is  $(0.554 - 0.527)2000 = 53.4$  A·turns/pole.

part (b): The full-load torque is

$$T = \frac{E_a I_a}{\omega_m} = \frac{221.4 \times 34.2}{1185(\pi/30)} = 62.8 \text{ N} \cdot \text{m}$$

part (c): The maximum field current is  $230/310 = 0.742 \Omega$ . The effective field current under this condition will therefore be

$$I_{\text{eff}} = \frac{2000 \times 0.742 - 160}{2000} = 0.662 \text{ A}$$

From the 1185 r/min magnetization curve found in part (b), this corresponds to a generated voltage of 245 V. Thus, the corresponding torque will be

$$T = \frac{E_a I_a}{\omega_m} = \frac{245 \times 65}{1185(\pi/30)} = 128 \text{ N} \cdot \text{m}$$

part (d): With the addition of  $0.05 \Omega$ , the total resistance in the armature circuit will now be  $R_{\text{tot}} = 0.295 \Omega$ . The required generated voltage will thus be

$$E_a = V_a - I_a R_{\text{tot}} = 219.6 \text{ V}$$

This corresponds to  $219.6(1185/1050) = 247.8 \text{ V}$  on the 1185 r/min magnetization curve and a corresponding effective field current of 0.701 A.

As can be seen from the data table, a no-load speed of 1200 r/min corresponds to a field current of 0.554 A. Thus the series-field A·turns must make up for the difference between that required and the actual field current as reduced by armature reaction.

$$\begin{aligned} N_s &= \frac{N_f(I_{f,\text{eff}} - I_f) + \text{Armature reaction}}{I_a} \\ &= \frac{2000(0.701 - 0.554) + 53.4}{35.2} = 9.8 \text{ turns} \end{aligned}$$

### Problem 7.20

part (a): From the demagnetization curve, we see that the shunt field current is 0.55 A since the no-load generated voltage must equal 230 V. The full-load generated voltage is

$$E_a = V_a - I_a R_a = 219.4 \text{ V}$$

and the corresponding field current (from the demagnetization curve obtained using the MATLAB 'spline()' function) is 0.487 A. Thus the demagnetization is equal to  $2000(0.55 - 0.487) = 127$  A·turns.

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part (b): The total effective armature resistance is now  $R_{\text{tot}} = 0.15 + 0.028 = 0.178 \Omega$ . Thus, the full-load generated voltage will be

$$E_a = V_a - I_a R_{\text{tot}} = 217.4 \text{ V}$$

The net effective field current is now equal to  $0.487 + 70.8(4/2000) = 0.628 \text{ A}$ . The corresponding voltage at 1750 r/min (found from the magnetization curve using the MATLAB 'spline()' function) is 242.7 V and hence the full-load speed is

$$n = 1750 \left( \frac{217.4}{242.7} \right) = 1567 \text{ r/min}$$

part (c): The effective field current under this condition will be

$$I_{\text{eff}} = 0.55 + 125(4/2000) - 230/2000 = 0.685 \text{ A}$$

From the 1750 r/min magnetization curve (using the MATLAB 'spline()' function), this corresponds to a generated voltage of 249 V. Thus, the corresponding torque will be

$$T = \frac{E_a I_a}{\omega_m} = \frac{249 \times 125}{1750(\pi/30)} = 170 \text{ N} \cdot \text{m}$$

### Problem 7.21

part (a): For a constant torque load, changing the armature resistance will not change the armature current and hence  $I_a = 60 \text{ A}$ .

part(b):

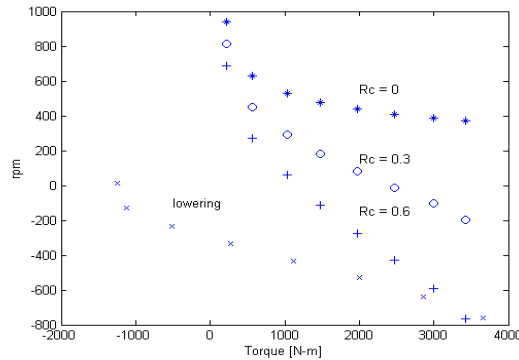
$$E_a = V_a - R_a I_a$$

Thus, without the added  $1.0 \Omega$  resistor,  $E_a = 216 \text{ V}$  and with it  $E_a = 156 \text{ V}$ . Thus,

$$\text{Speed ratio} = \frac{156}{216} = 0.72$$

### Problem 7.22

parts (a) and (b):





Operation in the fourth quadrant means that the motor torque is positive while the speed is negative. In this case the motor is acting as a generator and absorbing energy from the lowering load, which would otherwise accelerate due to the effects of gravity.

part (c): -473 r/min

### Problem 7.23

part (a): At rated load,  $E_a = 230 - 122 \times 0.064 = 222$  V. Thus, rated-load speed is

$$n = 1150 \left( \frac{222}{230} \right) = 1133 \text{ r/min}$$

part (b): The maximum value of the starting resistance will be required at starting.

$$\frac{230}{R_a + R_{\max}} = 2 \times 122 = 244$$

and thus  $R_{\max} = 0.878 \Omega$ .

part (c): For each value of  $R_{\text{tot}} = R_a + R_{\text{ext}}$ , the armature current will reach its rated value when the motor reaches a speed such that

$$E_a = 230 - 122R_{\text{tot,old}}$$

At this point  $R_{\text{tot}}$  will be reduced such that the armature current again reaches 122 A. Based upon this algorithm, the external resistance can be controlled as shown in the following table:

Step number	$R_{\text{ext}} [\Omega]$	$E_{a,\text{min}} [\text{V}]$	$n_{\text{min}} [\text{r/min}] [\text{V}]$	$E_{a,\text{max}} [\text{V}]$	$n_{\text{max}} [\text{r/min}]$
1	0.878	0	0	115	587
2	0.407	115	587	173	882
3	0.170	173	882	202	1030
4	0.051	202	1030	216	1101
5	0	216	1101	-	-

### Problem 7.24

part (a): At no load,  $E_{a,\text{nl}} = K_m \omega_{m,\text{nl}} = V_a$ . Thus

$$\omega_{m,\text{nl}} = \frac{V_a}{K_m} = \frac{85}{0.21} = 405 \text{ rad/sec}$$

Hence, the full-load speed is  $\omega_{m,\text{nl}}(30/\pi) = 3865 \text{ r/min}$ .

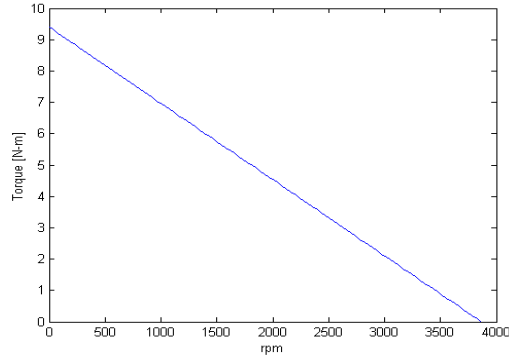
part (b): At zero speed, the current will be  $I_a = V_a/R_a = 44.7$  A and the corresponding torque will be  $T = K_m I_a = 9.4 \text{ N}\cdot\text{m}$ .

part (c):

$$T = K_m I_a = \frac{K_m(V_a - E_a)}{R_a} = \frac{K_m(V_a - K_m \omega_m)}{R_a}$$

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Here is the desired plot, obtained using MATLAB:



### Problem 7.25

part (a): At no load,  $\omega_{m,nl} = 11,210(\pi/30) = 1174$  rad/sec and  $E_{a,nl} = V_a - I_{a,nl}R_a = 4.94$  V. Thus

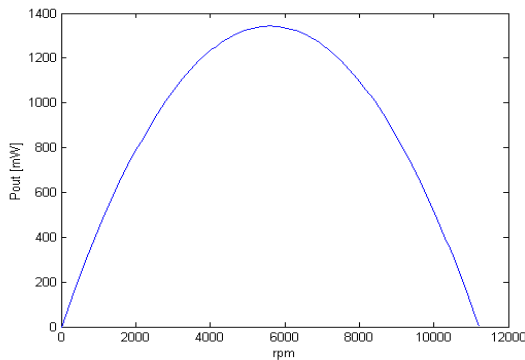
$$K_m = \frac{E_{a,nl}}{\omega_{m,nl}} = 4.21 \times 10^{-3} \text{ V}/(\text{rad}/\text{sec})$$

part (b): The no load rotational losses are

$$P_{rot,nl} = E_{a,nl}I_{a,nl} = 62 \text{ mW}$$

part (c): At zero speed, the current will be  $I_a = V_a/R_a = 1.09$  A and the corresponding torque will be  $T = K_m I_a = 4.6$  mN·m.

part (d): The output power versus speed characteristic is parabolic as shown below.



An iterative MATLAB scripts can easily find the two desired operating points: 2761 r/min for which the efficiency is 24.3% and 8473 r/min for which the efficiency is 72.8%.

**Problem 7.26**

No numerical solution required for this problem.

**Problem 7.27**

Based upon the calculations of Problem 7.25, at 8750 r/min, the rotational losses will be 29.4 mW. Thus, the total required electromechanical power will be  $P = 779$  mW. The generated voltage will be  $E_a = K_m \omega_m = 3.86$  V and the armature current will thus be  $I_a = P/E_a = 0.202$  A.

Thus the desired armature voltage will be

$$V_a = E_a + R_a I_a = 4.79 \text{ V}$$

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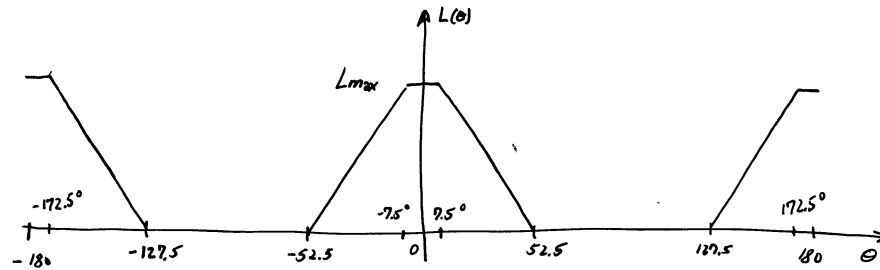
## PROBLEM SOLUTIONS: Chapter 8

### Problem 8.1

part (a): In this case,  $\beta = 45^\circ = \pi/4$  rad and

$$L_{\max} = \frac{N^2 \mu_0 \beta R D}{2g} = 96 \text{ mH}$$

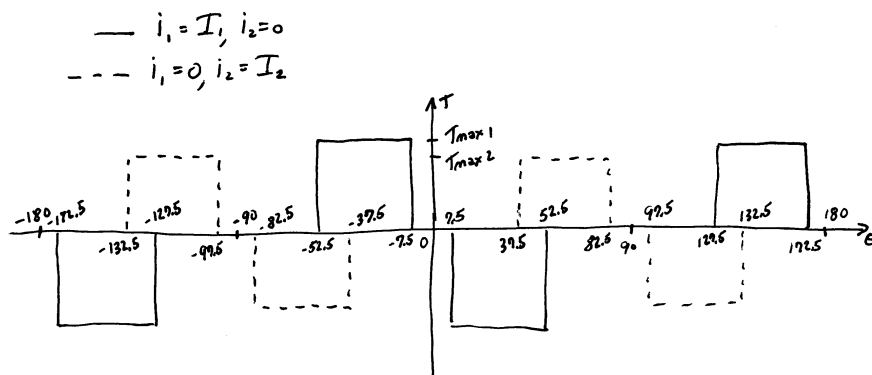
and there is a  $15^\circ$  overlap region of constant inductance.



part (b):

$$T_{\max,1} = \frac{L_{\max} I_1^2}{2\beta} = 6.1 \times 10^{-2} I_1^2 \text{ N} \cdot \text{m}$$

$$T_{\max,1} = \frac{L_{\max} I_2^2}{2\beta} = 6.1 \times 10^{-2} I_2^2 \text{ N} \cdot \text{m}$$



part (c):  $i_1 = i_2 = 5$  A;

(i)  $\omega = 0$   $T_{\text{net}} = 0$

(ii)  $\omega = 45^\circ$   $T_{\text{net}} = 0$  (iii)  $\omega = 75^\circ$   $T_{\text{net}} = 1.53$  N·m

### Problem 8.2

When a single phase is excited, magnetic circuit analysis can show that all the mmf drop occurs across the two air gaps associated with that phase. Thus, there is no additional mmf available to drive flux through the second phase.

### Problem 8.3

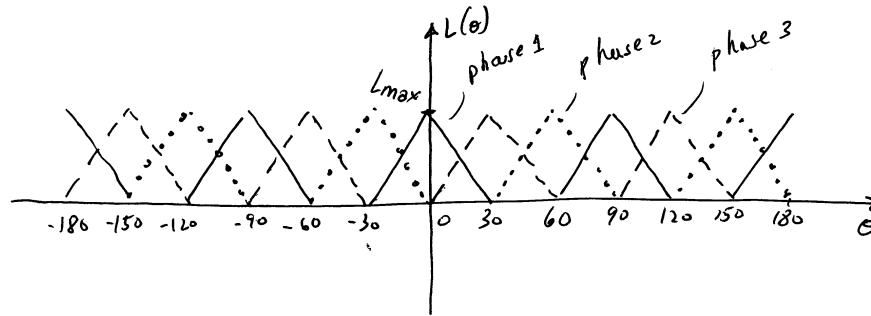
Same argument as in the solution of Problem 8.3.

### Problem 8.4

part (a) and (b):

$$L_{\text{max}} = \frac{DR\alpha\mu_0 N^2}{2g} = 21.5 \text{ mH}$$

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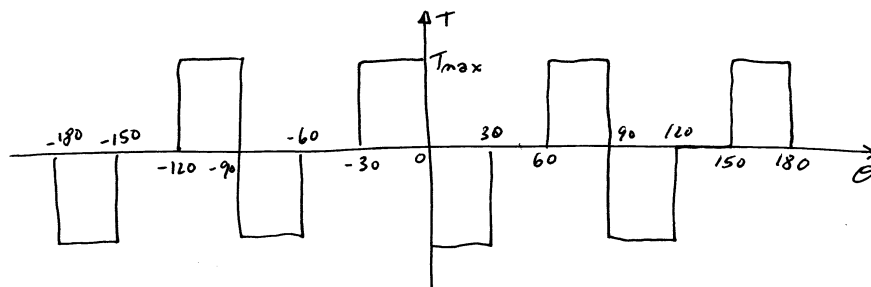


part (c):

$$I_0 = \frac{2gB}{\mu_0 N} = 6.96 \text{ A}$$

part (d):

$$T_{\max} = \frac{I_0^2}{2} \left( \frac{L_{\max}}{\alpha/2} \right) = 1.99 \text{ N} \cdot \text{m}$$



part (e):

phase 1 ON:

$$-120^\circ \leq \theta \leq -90^\circ, -30^\circ \leq \theta \leq 0^\circ, 60^\circ \leq \theta \leq 90^\circ, 150^\circ \leq \theta \leq 180^\circ$$

phase 2 ON:

$$-150^\circ \leq \theta \leq -120^\circ, -60^\circ \leq \theta \leq -30^\circ, 30^\circ \leq \theta \leq 60^\circ, 120^\circ \leq \theta \leq 150^\circ$$

phase 3 ON:

$$-180^\circ \leq \theta \leq -150^\circ, -90^\circ \leq \theta \leq -60^\circ, 0^\circ \leq \theta \leq 30^\circ, 90^\circ \leq \theta \leq 120^\circ$$

part (f): The rotor will rotate  $90^\circ$  in 30 msec.

$$n = \frac{(1/4) \text{ r}}{35 \text{ msec}} = 7.14 \text{ r/sec} = 429 \text{ r/min}$$

The rotor will rotate in the clockwise direction if the phase sequence is 1 - 2 - 3 - 1 ....

### Problem 8.5

When the rotor is aligned with any given pole pair, it is clearly med-way between the other two pole pairs. Thus rotation in one direction will increase the inductance of one set of poles and decrease the inductance of the remaining set. Thus, depending on which of the remaining poles is excited, it is possible to get torque in either direction.

### Problem 8.6

The rotor will rotate  $15^\circ$  as each consecutive phase is excited. Thus, the rotor will rotate 1 revolution in 24 sequences of phase excitation or 8 complete sets of phase excitation. Thus, the rotor will rotate 1 revolution in  $8 \times 15 = 120!$  msec. Thus it will rotate at  $1/0.12 = 8.33 \text{ r/sec} = 500 \text{ r/min}$ .

### Problem 8.7

part (a): If phase 1 is shut off and phase 2 is turned on, the rotor will move to the left by  $2\beta/3 \approx 4.29^\circ$ . Similarly, turning off phase 2 and turning on phase 3 will cause the rotor to move yet another  $4.29^\circ$ . Thus, starting with phase 1 on, to move  $21.4^\circ/4.29^\circ \approx 5$  steps, the sequence will be:

1 ON  
1 OFF & 2 ON  
2 OFF & 3 ON  
3 OFF & 1 ON  
1 OFF & 2 ON  
2 OFF & 3 ON

part (b): Clockwise is equivalent to rotor rotation to the right. The required phase sequence will be ... 1 - 3 - 2 - 1 - 3 - 2 .... The rotor will rotate  $\approx 4.29^\circ/\text{step}$  and hence the rotor speed will be

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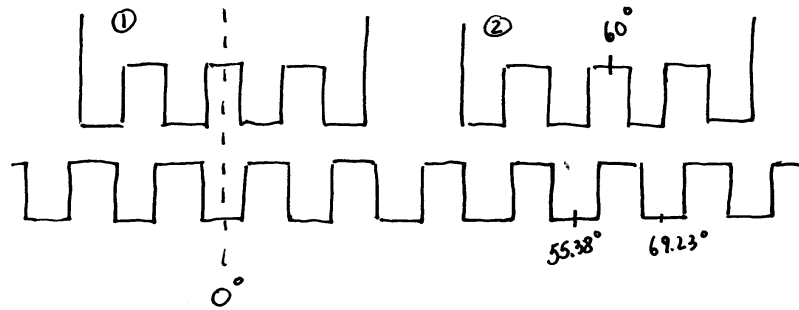
$$125 \text{ r/min} \times \frac{360^\circ}{r} \times \frac{1 \text{ step}}{4.29^\circ} = 1.05 \times 10^4 \text{ steps/min}$$

Thus the required step time is

$$\frac{\text{time}}{\text{step}} = \frac{1 \text{ min}}{8400 \text{ step}} \times \frac{60 \text{ sec}}{\text{min}} = 5.72 \text{ msec/step}$$

### Problem 8.8

part (a): When phase 1 is energized, the rotor will be aligned as shown in the following figure:



From the figure, we see that if phase 1 is turned off and phase 2 is energized, the rotor will rotate  $4.61^\circ$  degrees to the right (clockwise) to align with the phase-2 pole. Similarly, if phase 3 is excited after phase 1 is turned off, the rotor will rotate  $4.61^\circ$  degrees to the left (counterclockwise).

part (b):

$$80 \text{ r/min} \times \frac{360^\circ}{r} \times \frac{1 \text{ step}}{4.61^\circ} = 6.25 \times 10^3 \text{ steps/min}$$

$$\frac{\text{time}}{\text{step}} = \frac{1 \text{ min}}{6.25 \times 10^3 \text{ step}} \times \frac{60 \text{ sec}}{\text{min}} = 9.6 \text{ msec/step}$$

The required phase sequence will thus be ... 1 - 3 - 2 - 1 - 3 - 2 - 1 ...

### Problem 8.9

part (a): For time in which the current is building up

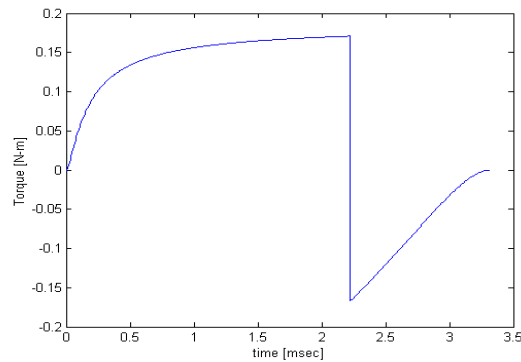
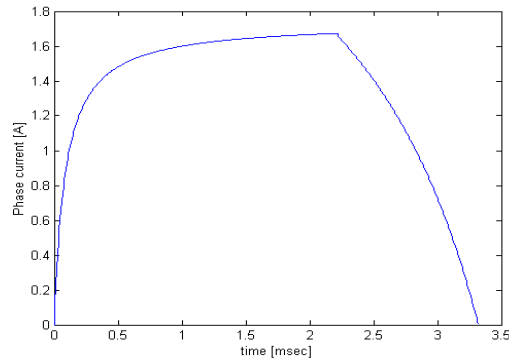
$$i_1(t) = \frac{100t}{0.005 + 57.5t} \text{ A}$$



This expression is valid until time  $t = 2.22$  msec, at which point  $i_1(t) = 1.67$  A.  
part (b):

$$i_1(t) = \frac{0.22 - 200(t - 2.22 \times 10^{-3})}{0.05 + 57.5(4.44 \times 10^{-3}) - 7}$$

This expression is valid until time  $t = 3.32$  msec.  
part (c): Here are the desired plots



The integral under the torque curve is  $2.38 \times 10^{-4}$  N·m·sec while the positive torque integral is 3.29 N·m·sec. Thus there is a 25.7% reduction in torque due to negative torque production during the current-decay period.

### Problem 8.10

part (a): For time in which the current is building up

$$i_1(t) = \frac{100t}{0.005 + 57.5t} \text{ A}$$

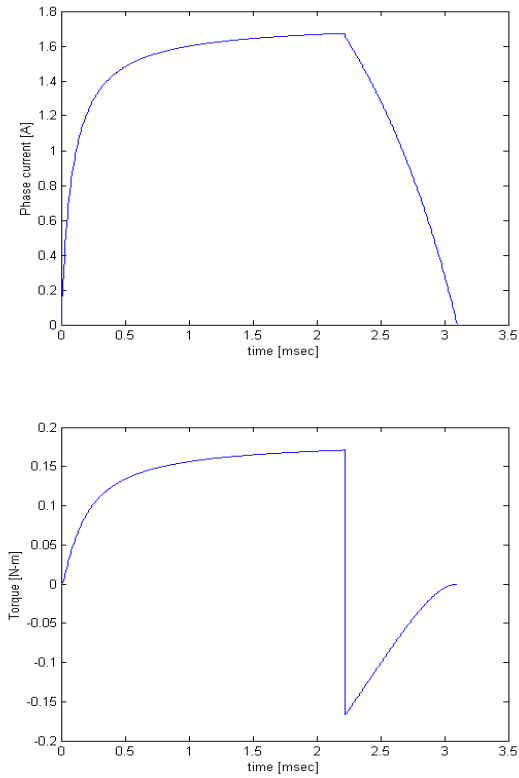
This expression is valid until time  $t = 2.22$  msec, at which point  $i_1(t) = 1.67$  A.  
part (b):

$$i_1(t) = \frac{0.22 - 250(t - 2.22 \times 10^{-3})}{0.05 + 57.5(4.44 \times 10^{-3}) - 7}$$

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This expression is valid until time  $t = 3.10$  msec.

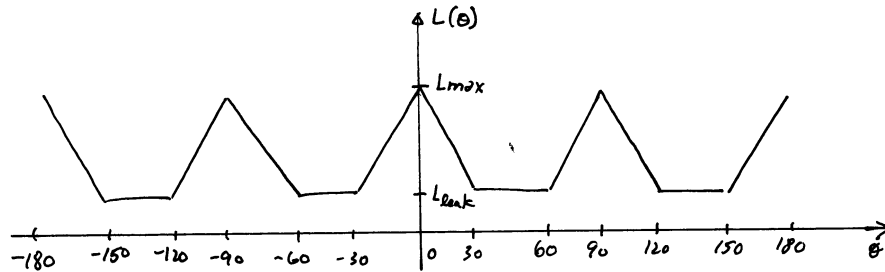
part (c): Here are the desired plots



The integral under the torque curve is  $2.59 \times 10^{-4}$  N·m·sec while the positive torque integral is 3.20 N·m·sec. Thus there is a 19.0% reduction in torque due to negative torque production during the current-decay period.

### Problem 8.11

part (a): The phase inductance looks like the plot of Problem 8.4, part (a), with the addition of the  $L_{\text{leak}} = 4.5$  mH leakage inductance. Now  $L_{\text{max}} = 21.5 + 4.5 = 26.0$  mH.



part (b): For  $-30^\circ \leq \theta \leq 0^\circ$

$$\frac{dL}{d\theta} = \frac{21.5 \text{ mH}}{\pi/3 \text{ rad}} = 20.5 \text{ mH/rad}$$

$$\omega_m = \frac{d\theta}{dt} = \frac{1750 \text{ r}}{\text{min}} \times \frac{2\pi \text{ rad}}{\text{r}} \times \frac{1 \text{ min}}{60 \text{ sec}} = 183 \text{ rad/sec}$$

$$\frac{dL}{dt} = \omega_m \frac{dL}{d\theta} = 3.76 \Omega$$

The governing equation is

$$v = iR + L \frac{di}{dt} + i \frac{dL}{dt}$$

Noting that  $dL/dt \gg R$ , we can approximate this equation as

$$v \approx \frac{d(Li)}{dt}$$

and thus

$$i(t) = \frac{\int v(t) dt}{L(t)}$$

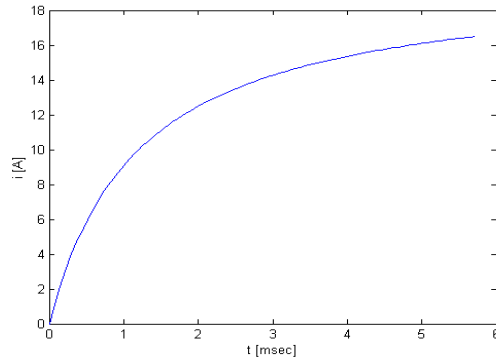
Substituting  $v(t) = 75 \text{ V}$  and  $L(t) = 4.5 \times 10^{-3} + 3.76t$  then gives

$$i(t) = \frac{75 t}{4.5 \times 10^{-3} + 3.76t}$$

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which is valid over the range  $0 \leq t \leq 5.71$  msec, at which time  $i(t) = 16.5$  A.

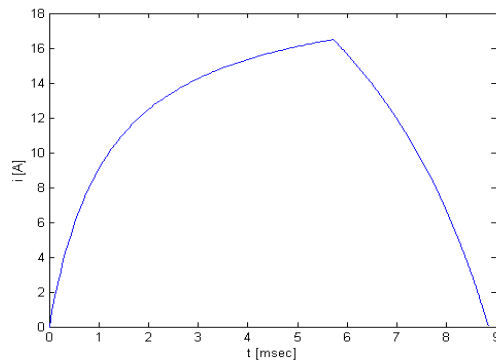
Here is the desired plot:



part (c): During this time, starting at time  $t = 5.71$  msec,  $v(t) = -75$  V and  $L(t) = 26.0 \times 10^{-3} - 3.76(t - 5.71 \times 10^{-3})$ . Thus

$$i(t) = 16.5 + \frac{-75(t - 5.71 \times 10^{-3})}{26.0 \times 10^{-3} - 3.76(t - 5.71 \times 10^{-3})}$$

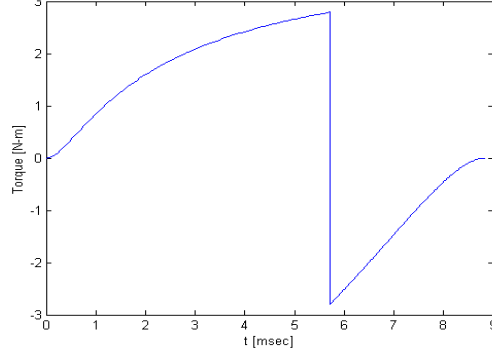
which reaches zero at  $t = 8.84$  msec. Here is the plot of the total current transient.



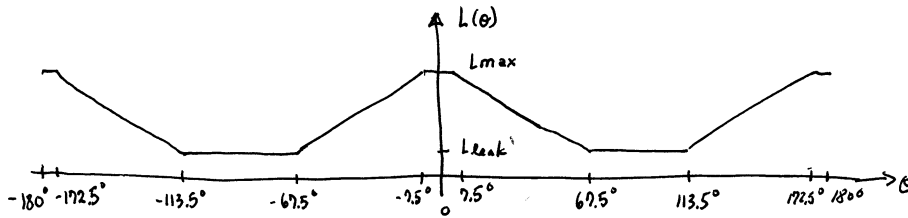
part (d): The torque is given by

$$T = \frac{i^2}{2} \frac{dL}{d\theta}$$

Here is the plot:



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**Problem 8.12**part (a): The plot of  $L_\theta$  is shown belowHere, from Examples 8.1 and 8.3,  $L_{\text{leak}} = 5 \text{ mH}$  and  $L_{\text{max}} = 133 \text{ mH}$ .part (b): The solution for  $-67.5^\circ \leq \theta \leq -7.5^\circ$  ( $0 \leq t \leq 2.5 \text{ msec}$ ) is exactly the same as part (a) of Example 8.3

$$i(t) = \frac{100t}{0.005 + 51.2t} \text{ A}$$

For  $-7.5^\circ \leq \theta \leq 7.5^\circ$  ( $2.5 \text{ msec} \leq t \leq 3.13 \text{ msec}$ ),  $dL/dt = 0$  and thus

$$v = iR + L \frac{di}{dt} \Rightarrow -100 = 1.5i + 0.133 \frac{di}{dt}$$

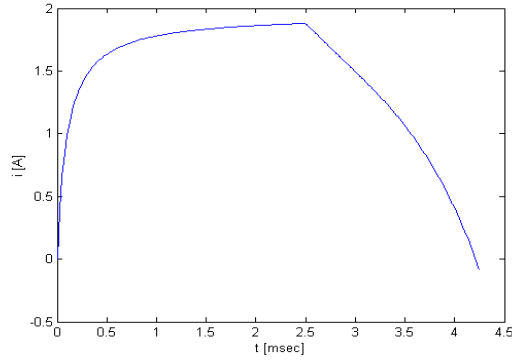
This equation has an exponential solution with time constant  $\tau = L/R = 88.7 \text{ msec}$ .

$$i = -66.7 + 68.6e^{-(t-0.0025)/0.0887}$$

At  $t = 3.13 \text{ msec}$ ,  $i(t) = 1.39 \text{ A}$ .Following time  $t = 3.13 \text{ msec}$ , the solution proceeds as in Example 8.3. Thus

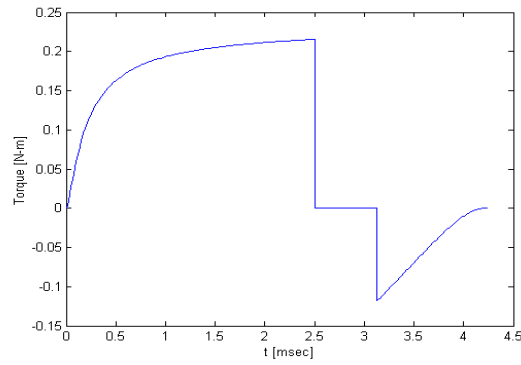
$$i(t) = 1.468 - \frac{100 - 3.13 \times 10^{-3}}{0.005 - 51.2(t - 5.63 \times 10^{-3})}$$

The current reaches zero at  $t = 4.25 \text{ msec}$ . Here is the corresponding plot, produced by MATLAB



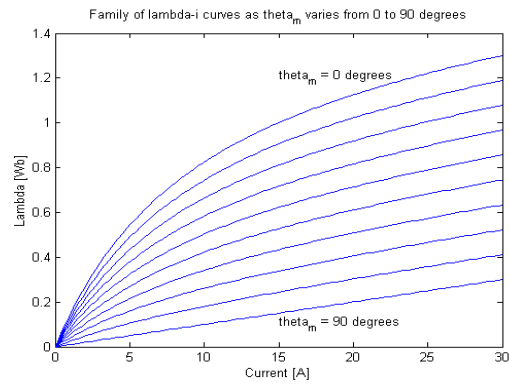
part (c):

$$T = \frac{i^2}{2} \frac{dL}{d\theta}$$



**Problem 8.13**

part (a):



part (b):

$$\frac{\text{Inverter volt - ampere rating}}{\text{Net output power}} = 1.55$$

part (c):  $P_{\text{mech}} = 2968 \text{ W}$ .

### Problem 8.14

part (a): Based upon the discussion in the text associated with Fig. 8.18, the following table can be produced:

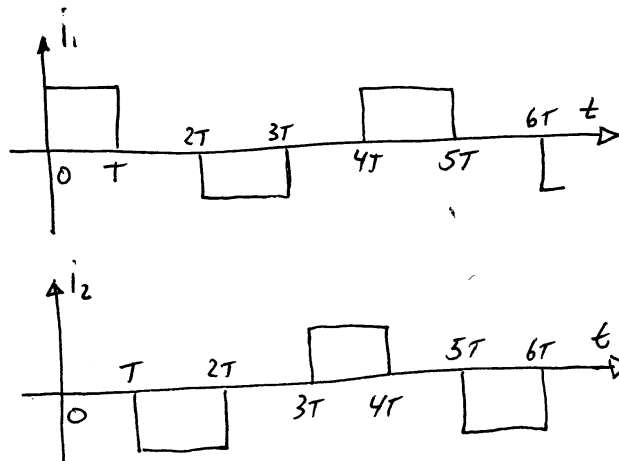
$\theta_m$	bit pattern	$i_1$	$i_2$
$0^\circ$	1000	$I_0$	0
$45^\circ$	1010	$I_0$	$I_0$
$90^\circ$	0010	0	$I_0$
$135^\circ$	0110	$-I_0$	$I_0$
$180^\circ$	0100	$-I_0$	0
$225^\circ$	0101	$-I_0$	$-I_0$
$270^\circ$	0001	0	$-I_0$
$315^\circ$	1001	$I_0$	$-I_0$

part (b): There will be 8 pattern changes per revolution. At 1200 r/min = 20 r/sec, there must be 160 pattern changes per second, corresponding to a time of 6.25 msec between pattern changes.

### Problem 8.15

part (a): The rotor will rotate  $2^\circ$  counter clockwise.

part (b): The phase excitation will look like (with  $T = 41.7 \text{ msec}$ ):



part (c):

$$\frac{8\text{r}}{\text{min}} = \frac{2880^\circ}{\text{min}} = \frac{48^\circ}{\text{sec}} = \frac{2^\circ}{41.7 \text{ msec}}$$

The frequency will be

$$f = \frac{1}{4T} = 6 \text{ Hz}$$

**Problem 8.16**

part (a): The displacement will be  $360^\circ/(3 \times 14) = 8.571^\circ$ .

part (b): There will be one revolution of the motor for every 14 cycles of the phase currents. Hence

$$f = \left( \frac{900 \text{ r}}{\text{min}} \right) \left( \frac{1 \text{ min}}{60 \text{ sec}} \right) \left( \frac{14 \text{ cycles}}{\text{r}} \right) = 210 \text{ Hz}$$



## PROBLEM SOLUTIONS: Chapter 9

### Problem 9.1

part (a):

$$\hat{I}_{\text{main}} = \frac{\hat{V}}{Z_{\text{main}}} = 13.8\angle -56.4^\circ \text{ A}$$

$$\hat{I}_{\text{aux}} = \frac{\hat{V}}{Z_{\text{aux}}} = 9.9\angle -49.2^\circ \text{ A}$$

part (b): We want the angle of the auxiliary-winding current to lead that of the main winding by  $90^\circ$  ( $\pi/2$  rad). Thus, defining  $Z'_{\text{aux}} = Z_{\text{aux}} + jX_C$  ( $X_C = 1/\omega C$ ), we want

$$\angle Z'_{\text{aux}} = \tan^{-1} \frac{\text{Im}[Z_{\text{aux}}] - X_C}{\text{Re}[Z_{\text{aux}}]} = \angle Z_{\text{main}} + \frac{\pi}{2}$$

Thus  $X_C = 14.5 \Omega$  and  $C = 183 \mu\text{F}$ .

part (c):

$$\hat{I}_{\text{main}} = \frac{\hat{V}}{Z_{\text{main}}} = 13.8\angle -56.4^\circ \text{ A}$$

$$\hat{I}_{\text{aux}} = \frac{\hat{V}}{Z'_{\text{aux}}} = 12.6\angle 33.6^\circ \text{ A}$$

### Problem 9.2

The solution is basically the same as for Problem 9.1, but now with  $Z_{\text{main}} = 4.82 + j6.04 \Omega$  and  $Z_{\text{aux}} = 7.95 + j7.68 \Omega$  and  $\omega = 100\pi$ .

part (a):

$$\hat{I}_{\text{main}} = 15.5\angle -51.4^\circ \text{ A}$$

$$\hat{I}_{\text{aux}} = 10.9\angle -44.0^\circ \text{ A}$$

part (b):  $C = 227 \mu\text{F}$ .

part (c):

$$\hat{I}_{\text{main}} = 15.5\angle -51.4^\circ \text{ A}$$

$$\hat{I}_{\text{aux}} = 11.8\angle 38.6^\circ \text{ A}$$

### Problem 9.3

No numerical solution required.

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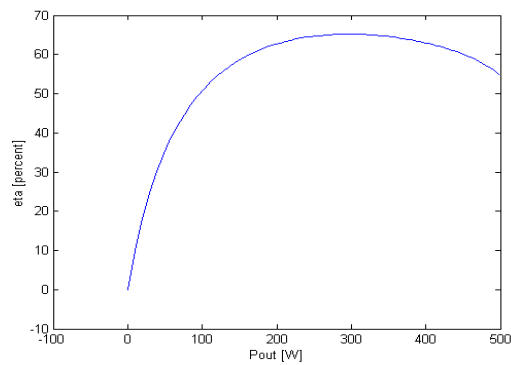
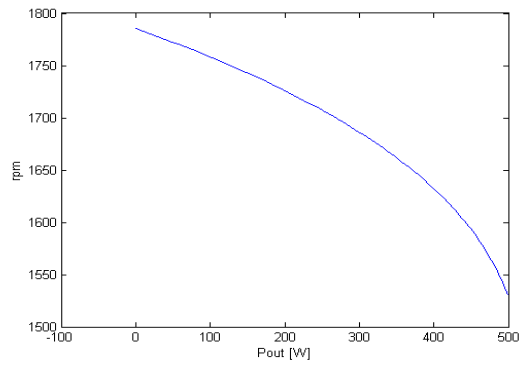
**Problem 9.4**

$I_{\text{main}} = 3.48 \text{ A}$   
power factor = 0.59  
 $P_{\text{out}} = 132 \text{ W}$   
speed = 1719 r/min  
Torque = 0.732 N·m  
efficiency = 58.5 %

**Problem 9.5**

The solution follows that of Example 9.2.

$I_{\text{main}} = 4.38 \text{ A}$   
power factor = 0.65  
 $P_{\text{out}} = 204 \text{ W}$   
speed = 1724 r/min  
Torque = 1.13 N·m  
efficiency = 63.0 %

**Problem 9.6**

**Problem 9.7**

part (a): From Eq. 4.6, the peak amplitude, in space and time, of the mmf waves are given by

$$F_{\text{peak}} = \frac{4}{\pi} \left( \frac{k_w N_{\text{ph}}}{\text{poles}} \right) I_{\text{peak}}$$

Thus

$$F_{\text{main,peak}} = \frac{4}{\pi} \left( \frac{42}{4} \right) (20.7 \sqrt{2}) = 391 \text{ A} \cdot \text{turns}$$

and

$$F_{\text{aux,peak}} = \frac{4}{\pi} \left( \frac{68}{4} \right) (11.1 \sqrt{2}) = 340 \text{ A} \cdot \text{turns}$$

part (b): The auxiliary winding current must be phase shifted by  $90^\circ$  from that of the main winding and the mmf amplitudes must be equal. Hence,  $I_{\text{aux}}$  should be increased to

$$I_{\text{aux}} = I_{\text{main}} \left( \frac{N_{\text{main}}}{N_{\text{aux}}} \right) = 12.8 \text{ A}$$

**Problem 9.8**

The internal torque is proportional to  $R_{r\text{mf}} - R_{\text{b}}$  and thus is equal to zero when  $R_{\text{f}} = R_{\text{b}}$ . From Example 9.2,

$$R_{\text{f}} = \left( \frac{X_{\text{m,main}}^2}{X_{22}} \right) \frac{1}{sQ_{2,\text{main}} + 1/(sQ_{2,\text{main}})}$$

and

$$R_{\text{b}} = \left( \frac{X_{\text{m,main}}^2}{X_{22}} \right) \frac{1}{(2-s)Q_{2,\text{main}} + 1/((2-s)Q_{2,\text{main}})}$$

We see that  $R_{\text{f}} = R_{\text{b}}$  if  $(2-s)Q_{2,\text{main}} = 1/(sQ_{2,\text{main}})$  or

$$s = 1 \pm \sqrt{1 + \frac{1}{Q_{2,\text{main}}}}$$

and thus

$$n = n_{\text{s}}(1-s) = \pm n_{\text{s}} \sqrt{1 + \frac{1}{Q_{2,\text{main}}}}$$

where  $n_{\text{s}}$  is the synchronous speed in r/min.

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**Problem 9.9**

The solution follows that of Example 9.3.

part (a):  $I_\alpha = 34.5\angle -32.3^\circ$  A,  $I_\beta = 7.1\angle 30.0^\circ$  A

part (b):  $P_{\text{mech}} = 6700$  W

**Problem 9.10**

part (a): Following the calculations of Example 9.3 with  $s = 1$ ,  $T_{\text{mech}} = 14.8$  N·m.

part (b): Setting

$$V_\alpha = 230 \text{ V} \quad V_\beta = 230e^{j90^\circ} \text{ V}$$

gives  $T_{\text{mech}} = 16.4$  N·m.

part (c): Letting  $\hat{V}_\alpha = V_\alpha$  and  $\hat{V}_\beta = jV_\beta$  gives

$$V_f = \frac{V_\alpha + jV_\beta}{2}; \quad V_b = \frac{V_\alpha - jV_\beta}{2}$$

Let  $Z = R_1 + jX_1 + Z_f$ . Thus

$$I_f = \frac{V_f}{Z} = \frac{V_\alpha + jV_\beta}{2Z}; \quad I_b = \frac{V_b}{Z} = \frac{V_\alpha - jV_\beta}{2Z}$$

$$T = \frac{P_{\text{gap},f} - P_{\text{gap},b}}{\omega_s} = \frac{R_f(I_f^2 - I_b^2)}{\omega_s} = \left( \frac{R_f}{|Z|^2} \right) V_\alpha V_\beta$$

Clearly, the same torque would be achieved if the phase voltages were each equal in magnitude to  $\sqrt{V_\alpha V_\beta}$ .

**Problem 9.11**

The impedance  $Z$  must be added to the impedances of the motor of Problem 9.9. The solution then proceeds as in Example 9.3. The terminal voltage can be found as

$$\hat{V}_{t,\alpha} = \hat{V}_\alpha - \hat{I}_\alpha Z$$

$$\hat{V}_{t,\beta} = \hat{V}_\beta - \hat{I}_\beta Z$$

For

$$\hat{V}_\alpha = 235\angle 0^\circ; \quad \hat{V}_\beta = 212\angle 78^\circ$$

a MATLAB analysis gives

$$\hat{V}_{t,\alpha} = 205\angle -8.0^\circ; \quad \hat{V}_{t,\beta} = 194\angle 73^\circ$$

which is clearly more balanced than the applied voltage.

**Problem 9.12**part (a): slip  $s = 0.035$ 

$$\hat{V}_f = \frac{1}{2}(\hat{V}_\alpha - j\hat{V}_\beta) = 214.8\angle -12.5^\circ \text{ V}$$

$$\hat{V}_b = \frac{1}{2}(\hat{V}_\alpha + j\hat{V}_\beta) = 47.6\angle 77.5^\circ \text{ V}$$

 $Z_f$  and  $Z_b$  can be calculated from Eqs. 9.4 and 9.5 with  $s = 0.035$ 

$$\hat{I}_f = \frac{\hat{V}_f}{R_1 + jX_1 + Z_f} = 2.99\angle -64.0^\circ$$

$$\hat{I}_b = \frac{\hat{V}_b}{R_1 + jX_1 + Z_b} = 4.48\angle 0.7^\circ$$

$$P_{\text{gap},f} = 2I_f^2 R_f = 784 \text{ W} \quad P_{\text{gap},b} = 2I_b^2 R_b = 65.9 \text{ W}$$

$$T_{\text{gap}} = \frac{(P_{\text{gf}} - P_{\text{gb}})}{\omega_s} = 3.81 \text{ N} \cdot \text{m}$$

part (b): Repeating the analysis of part (a) with  $s = 1$  gives  $T_{\text{start}} = 12.0 \text{ n} \cdot \text{m}$ .

part (c): Now we have a two-phase machine operating under balanced two-phase conditions. We can now apply the analysis of Section 6.5.

$$V_{1,\text{eq}} = \left| V_1 \left( \frac{jX_m(R_1 + jX_1)}{R_1 + j(X_1 + X_m)} \right) \right| = 208 \text{ V}$$

and  $R_{1,\text{eq}} + jX_{1,\text{eq}} = jX_m$  in parallel with  $R_1 + jX_1 = 0.698 + j5.02 \Omega$ .

Thus

$$s_{\text{max},T} = \frac{R_2}{\sqrt{R_{1,\text{eq}}^2 + (X_{1,\text{eq}} + X_2)^2}} = 0.348$$

and

$$T_{\text{max}} = \frac{1}{\omega_s} \left[ \frac{0.5n_{\text{ph}}V_{1,\text{eq}}^2}{R_{1,\text{eq}} + \sqrt{R_{1,\text{eq}}^2 + (X_{1,\text{eq}} + X_2)^2}} \right] = 20.8 \text{ N} \cdot \text{m}$$

part (d): Now we have a single-phase machine operating with  $V_\alpha = 220 \text{ V}$  and  $s = 0.04$ 

$$\hat{I}_\alpha = \frac{V_\alpha}{(R_1 + jX_1)0.5(Z_f + Z_b)} = 5.73\angle -52.3^\circ \text{ A}$$

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$$P_{\text{gap}} = 0.5I_{\alpha}^2(R_f - R_b) = 691 \text{ W}$$

$$P_{\text{mech}} = P_{\text{gap}}(1 - s) = 664 \text{ W}$$

and finally

$$P_{\text{out}} = P_{\text{mech}} - P_{\text{rot}} = 599 \text{ W}$$

part (e):

$$\hat{I}_f = \hat{I}_b = 0.5\hat{I}_{\alpha} = 2.82\angle -52.3^{\circ}$$

$$\hat{V}_f = (R_1 + jX_1 + Z_f)\hat{I}_f; \quad \hat{V}_b = (R_1 + jX_1 + Z_b)\hat{I}_b$$

and thus

$$\hat{V}_{\beta} = j(V_f - V_b) = 0.5j\hat{I}_{\alpha}(Z_f - Z_b) = 167\angle 81.3^{\circ} \text{ V}$$

In other words, the open-circuit voltage across the  $\beta$  winding will be 167 V.**Problem 9.13**

This problem can be solved using a MATLAB script similar to that written for Example 9.5.

part (a):  $T_{\text{start}} = 0.28 \text{ N}\cdot\text{m}$ .

part (b):  $I_{\text{main}} = 19.3 \text{ A}$ ;  $I_{\text{aux}} = 3.2 \text{ A}$

part (c):  $I = 21.3 \text{ A}$  and the power factor is 0.99 lagging

part (d):  $P_{\text{out}} = 2205 \text{ W}$

part (e):  $P_{\text{in}} = 2551 \text{ W}$  and  $\eta = 86.4\%$

**Problem 9.14**

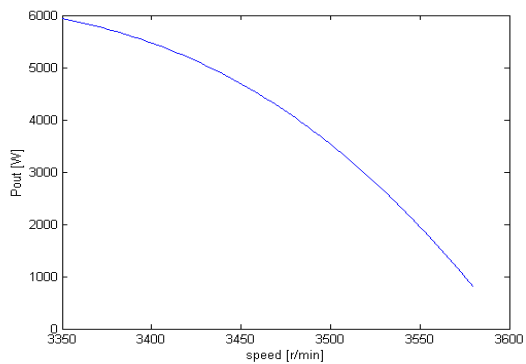
This problem can be solved using a MATLAB script similar to that written for Example 9.5. An iterative search gives  $C = 70.4 \mu\text{F}$  and an efficiency of 87.1%.

**Problem 9.15**

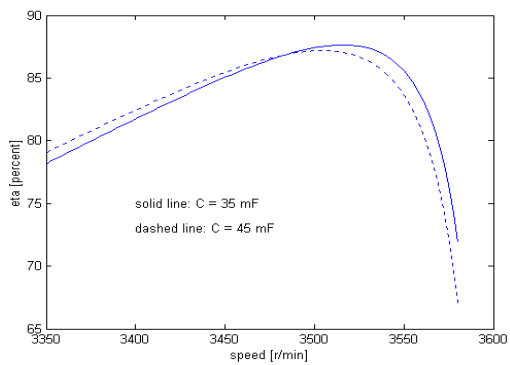
This problem can be solved using a MATLAB script similar to that written for Example 9.5. An iterative search shows that the minimum capacitance is  $80.9 \mu\text{F}$ .

**Problem 9.16**

part (a):



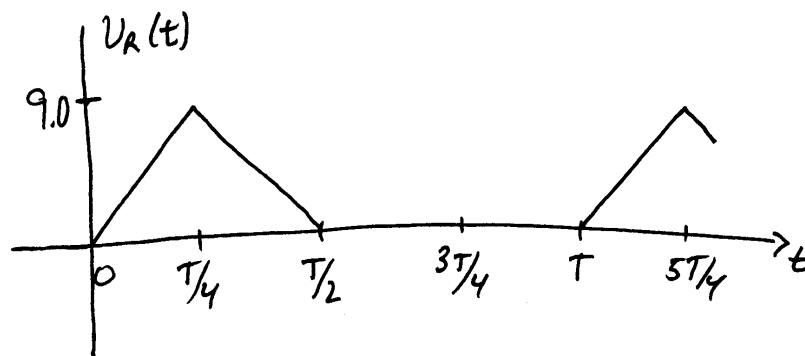
parts (b) and (c):



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**PROBLEM SOLUTIONS: Chapter 10****Problem 10.1**

part (a):



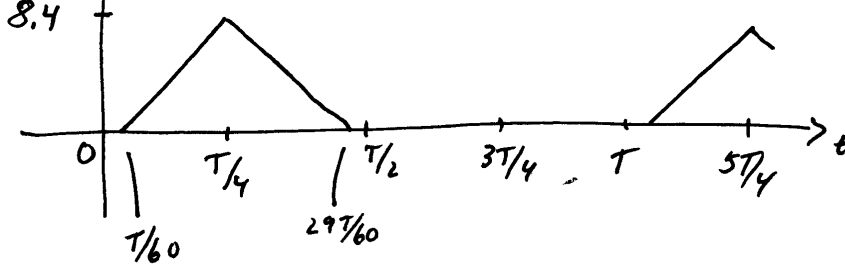
part (b):

$$V_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T v_{\text{R}}^2(t) dt} = \sqrt{\frac{2}{T} \int_0^{T/4} \left(\frac{4V_0 t}{T}\right)^2 dt} = \frac{V_0}{\sqrt{6}} = 3.67 \text{ V}$$

part (c):  $\langle p_{\text{diss}} \rangle = V_{\text{rms}}^2/R = 9 \text{ mW}$ .**Problem 10.2**

part (a): The diode does not turn on until the source voltage reaches 0.6 V, which occurs at time  $t = (0.6/4V_0)T = T/60$ .





part (b):

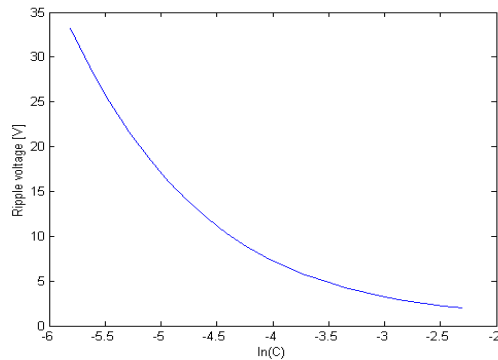
$$V_{\text{rms}} = \sqrt{\frac{2}{T} \int_{T/60}^{T/4} \left( \frac{4 \times 8.4t}{T} \right)^2 dt} = 3.43 \text{ V}$$

part (c):  $\langle p_{\text{diss}} \rangle = V_{\text{rms}}^2 / R = 7.8 \text{ mW}$ .

### Problem 10.3

$$V_{\text{rms}} = \sqrt{\frac{2}{T} \int_{t_d}^{T/4} \left( \frac{4V_0 t}{T} \right)^2 dt} = \frac{32V_0^2}{3} \left( \frac{1}{4^3} - \left( \frac{t_d}{T} \right)^3 \right)$$

### Problem 10.4



### Problem 10.5

part (a): Peak  $V_R = 310 \text{ V}$ .

part (b): Ripple voltage = 25.7 V.

part (c): Time-averaged power dissipated in the load resistor = 177 W.

part (d): Time-averaged power dissipated in the diode bridge = 0.41 W.

### Problem 10.6

If  $v_s(t) \geq 0$ , diode D1 is ON, diode D2 is off and the inductor current is governed by the following differential equation:

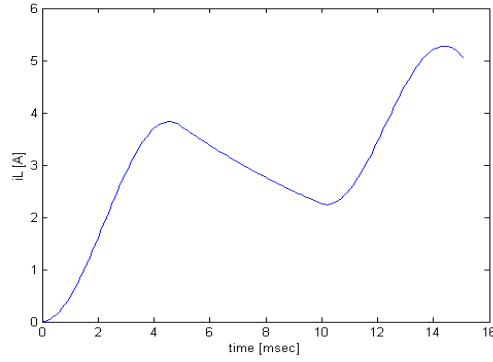
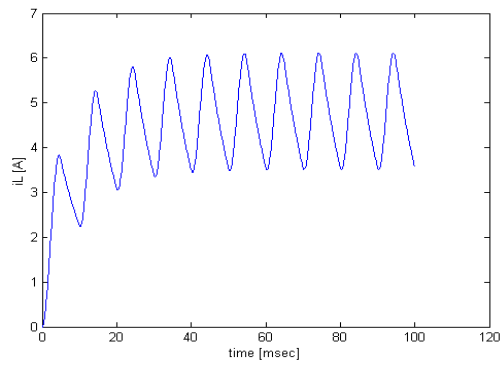
$$L \frac{di}{dt} + Ri = v_s(t)$$

of  $v_s(t) < 0$ , diode D1 is off and diode D2 is on and the inductor current is governed by the differential equation:

$$L \frac{di}{dt} + Ri = 0$$

A simple integration implemented in MATLAB produces the following plot:

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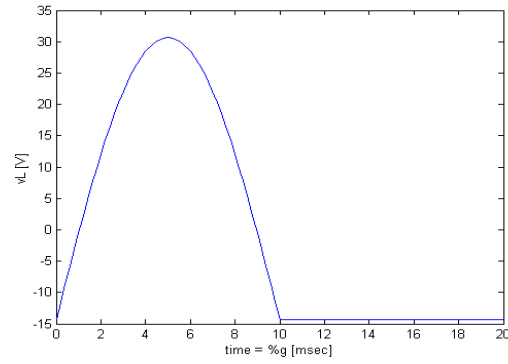
**Problem 10.7****Problem 10.8**part (a): Letting  $T = 2\pi/\omega$ 

$$V_{dc} = \frac{1}{T} \int_0^{T/2} V_0 \sin \omega t = \frac{V_0}{\pi} = 14.3 \text{ V}$$

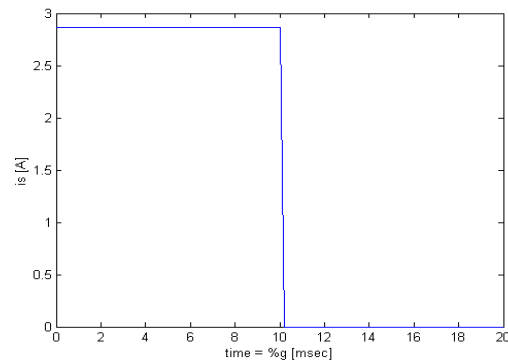
part (b):

$$I_{dc} = \frac{V_{dc}}{R} = 2.9 \text{ A}$$

part (c):



part (d):



### Problem 10.9

part (a): Letting  $T = 2\pi/\omega$

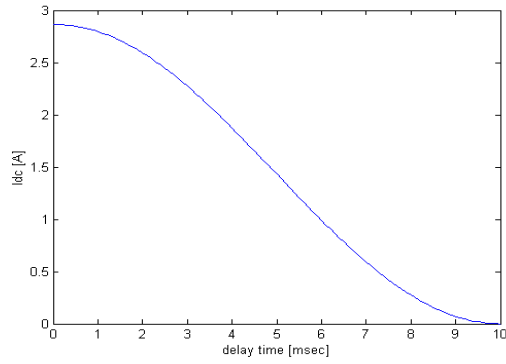
$$V_{dc} = \frac{1}{T} \int_{t_d}^{T/2} V_0 \sin \omega t = \frac{V_0}{2\pi} (1 + \cos \omega t_d)$$

part (b):

$$I_{dc} = \frac{V_{dc}}{R} = \frac{V_0}{2\pi R} (1 + \cos \omega t_d)$$

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part (c):

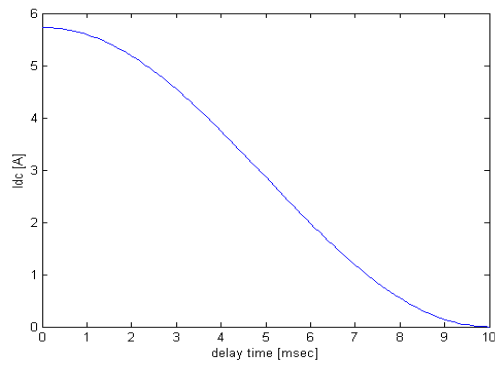
**Problem 10.10**part (a): Letting  $T = 2\pi/\omega$ 

$$V_{dc} = \frac{2}{T} \int_{t_d}^{T/2} V_0 \sin \omega t = \frac{V_0}{\pi} (1 + \cos \omega t_d)$$

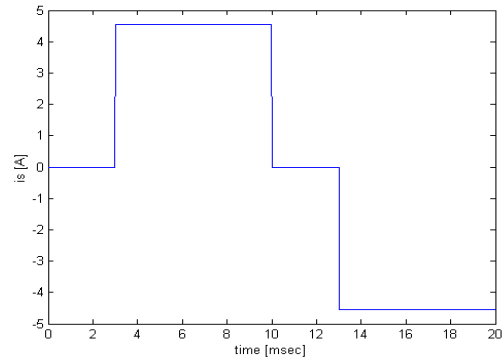
part (b):

$$I_{dc} = \frac{V_{dc}}{R} = \frac{V_0}{\pi R} (1 + \cos \omega t_d)$$

part (c):



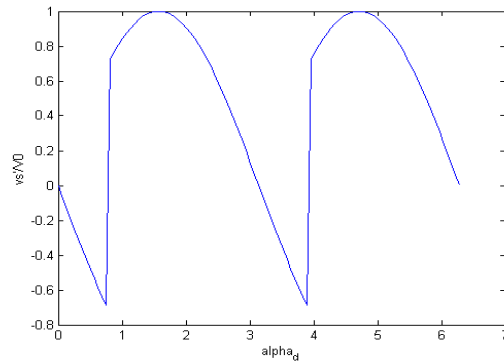
part (d):



### Problem 10.11

part (a):

(i)



(ii)

$$V_{dc} = \frac{1}{\pi} \int_{\pi/4}^{5\pi/4} V_0 \sin \theta d\theta = \frac{V_0}{\pi} \cos \alpha_2 = \frac{V_0}{\pi\sqrt{2}}$$

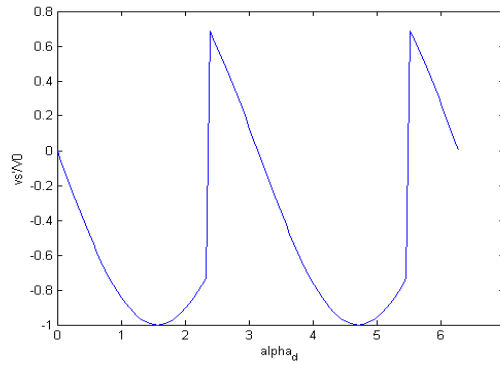
(iii)

$$P_{load} = V_{dc} I_{dc} = \frac{V_0 I_{dc}}{\pi\sqrt{2}}$$

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part (b):

(i)



(ii)

$$V_{dc} = \frac{1}{\pi} \int_{3\pi/4}^{7\pi/4} V_0 \sin \theta \, d\theta = \frac{V_0}{\pi} \cos \alpha_2 = -\frac{V_0}{\pi\sqrt{2}}$$

(iii)

$$P_{load} = V_{dc} I_{dc} = -\frac{V_0 I_{dc}}{\pi\sqrt{2}}$$

The power is negative, hence energy is being extracted from the load.

**Problem 10.12**

part (a) From Eq. 10.11

$$I_{dc} = \frac{2V_0}{\pi R + 2\omega L_s} = 18.3 \text{ A}$$

and from Eq. 10.8

$$t_c = \frac{1}{\omega} \cos^{-1} \left[ 1 - \frac{2I_{dc}\omega L_s}{V_0} \right] = 3.12 \text{ msec}$$

part (b): For  $L_s = 0$ 

$$I_{dc} = \frac{2V_0}{\pi R} = 23.6 \text{ A}$$

**Problem 10.13**

part (a): At 1650 r/min, the generated voltage of the dc motor is equal to

$$E_a = 85 \left( \frac{1650}{1725} \right) = 81.3 \text{ V}$$

The motor input power will then be

$$P_{\text{in}} = I_a(E_a + I_a R_a) = 665 \text{ W}$$

part (b):

$$V_{\text{dc}} = \left( \frac{2V_0}{\pi} \right) \cos \alpha_d = \left( \frac{2\sqrt{2} \times 115}{\pi} \right) \cos \alpha_d = 103.5 \cos \alpha_d \text{ V}$$

Thus for  $V_{\text{dc}} = E_a + I_a(R_a + R_L) = 90.5 \text{ V}$ ,  $\alpha_d = 29.1^\circ$ .

#### Problem 10.14

The rated current of this motor is

$$I_{\text{rated}} = \frac{P_{\text{rated}}}{V_{\text{rated}}} = \frac{1000}{85} = 11.8 \text{ A}$$

The controller must limit  $I_{\text{dc}}$  to twice  $I_{\text{rated}}$  or 23.6 A. Under this condition,  $V_{\text{dc}} = I_a(R_a + R_L) = 28.5 \text{ V}$ .

From part (b) of the solution to Problem 10.13,

$$V_{\text{dc}} = 103.5 \cos \alpha_d \text{ V}$$

and thus the controller must set  $\alpha_d = 74.0^\circ$ .

#### Problem 10.15

The required dc voltage is  $V_f = I_f R_f = 277 \text{ V}$ . From Eq. 10.19,

$$V_{1-1,\text{rms}} = \frac{\pi V_f}{3\sqrt{2}} = 204 \text{ V, rms}$$

#### Problem 10.16

The required dc voltage is  $V_f = I_f R_f = 231 \text{ V}$ . From Eq. 10.20,

$$\alpha_d = \cos^{-1} \left( \frac{\pi V_f}{3\sqrt{2} V_{1-1,\text{rms}}} \right) = 39.0^\circ$$

#### Problem 10.17

part (a): The magnet resistance is sufficiently small that its voltage drop can be ignored while the magnet is being charged. The desired charge rate is

$$\frac{di}{dt} = 80 \text{ A/25 sec} = 3.2 \text{ A/sec}$$

Thus the required dc voltage will be

$$V_{\text{dc}} = L \frac{di}{dt} = 15.7 \text{ V}$$

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Thus

$$\alpha_d = \cos^{-1} \left( \frac{\pi V_{dc}}{3\sqrt{2} V_{1-1,rms}} \right) = \cos^{-1} \left( \frac{\pi 15.7}{3\sqrt{2} \times 15} \right) = 39.3^\circ$$

part (b): Constant current simply requires a dc voltage of  $V_{dc} = RI_{dc} = 3.6 \times 10^{-3} \times 80 = 0.29$  V. Thus

$$\alpha_d = \cos^{-1} \left( \frac{\pi V_{dc}}{3\sqrt{2} V_{1-1,rms}} \right) = \cos^{-1} \left( \frac{\pi 15.7}{3\sqrt{2} \times 0.288} \right) = 89.2^\circ$$

**Problem 10.18**

part (a):

$$\begin{aligned} V_1 &= \frac{2}{T} \int_0^T v(t) \cos \left( \frac{2\pi t}{T} \right) dt = \frac{8}{T} \int_0^{DT/2} V_0 \cos \left( \frac{2\pi t}{T} \right) dt \\ &= \frac{V_0}{4\pi} \sin \pi D = 51.5 \text{ V} \end{aligned}$$

part (b):

Harmonic number	Peak amplitude [V]
1	51.5
2	0
3	6.6
4	0
5	12.7
6	0
7	2.8
8	0
9	5.7
10	0

**Problem 10.19**

part (a):

$$V_3 = \frac{3}{T} \int_0^T v(t) \cos \left( \frac{6\pi t}{T} \right) dt = \frac{4V_0}{3\pi} \sin(3\pi D)$$

$A_3 = 0$  for  $D = 1/3$



part (b):

Harmonic number	Peak amplitude [V]
1	55.1
2	0
3	0.007
4	0
5	11.0
6	0
7	7.9
8	0
9	0.007
10	0

### Problem 10.20

part (a):

Time period	S1	S2	S3	S4
$0 \leq \omega t < \alpha_d + DT/2$	ON	OFF	ON	OFF
$\alpha_d + DT/2 \leq \omega t < \pi + \alpha_d - DT/2$	ON	ON	OFF	OFF
$\pi + \alpha_d - DT/2 \leq \omega t < \pi + \alpha_d + DT/2$	OFF	ON	OFF	ON
$\pi + \alpha_d + DT/2 \leq \omega t < 2\pi + \alpha_d - DT/2$	OFF	OFF	ON	ON
$2\pi + \alpha_d - DT/2 \leq \omega t < 2\pi$	ON	OFF	ON	OFF

part (b): By analogy to the solution of Problem 10.18, part (a)

$$I_1 = \frac{I_0}{4\pi} \sin \pi D$$

and by inspection  $\phi_1 = \alpha_d$ .

part (c):

$$p(t) = \langle i_1(t)v_L(t) \rangle = \frac{V_a I_1}{2} \cos \phi_1 = \frac{V_a I_0}{8\pi} \cos \alpha_d$$

### Problem 10.21

From Eq. 10.34

$$(i_L)_{\text{avg}} = \frac{[2D - 1]V_0}{R} = 17.65 \text{ A}$$

From Eq. 10.29

$$(i_L)_{\text{min}} = -\left(\frac{V_0}{R}\right) \frac{[1 - 2e^{-\frac{T(1-D)}{\tau}} + e^{-\frac{T}{\tau}}]}{(1 - e^{-\frac{T}{\tau}})} = 17.45 \text{ A}$$

and from Eq. 10.20

$$(i_L)_{\text{max}} = \left(\frac{V_0}{R}\right) \frac{[1 - 2e^{-\frac{DT}{\tau}} + e^{-\frac{T}{\tau}}]}{(1 - e^{-\frac{T}{\tau}})} = 17.84 \text{ A}$$

Finally

$$\text{Ripple} = (i_L)_{\text{max}} - (i_L)_{\text{min}} = 0.39 \text{ A}$$

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## PROBLEM SOLUTIONS: Chapter 11

### Problem 11.1

part (a): From the no-load data

$$K_f = \frac{E_{a,nl}}{\omega_{m,nl} I_{f,nl}} = \frac{120}{(1718\pi/30) \times 0.7} = 0.953$$

Combining

$$T = \frac{E_a I_a}{\omega_m}$$

and

$$V_a = E_a + I_a R_a$$

gives

$$\begin{aligned} E_a &= 0.5 \left( V_a + \sqrt{V_a^2 - 4\omega_m T R_a} \right) \\ &= 0.5 \left( 120 + \sqrt{120^2 - 4(1800\pi/30) \times 15.2 \times 0.145} \right) = 116.4 \text{ V} \end{aligned}$$

Thus

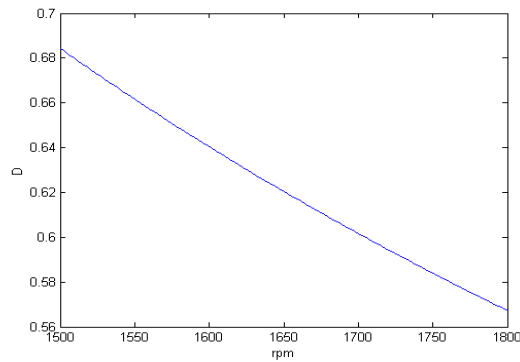
$$I_f = \frac{E_a}{\omega_m K_f} = 0.648 \text{ A}$$

and, defining  $I_{f,\max} = 120/104 = 1.14 \text{ A}$ ,

$$D = \frac{I_f}{I_{f,\max}} = 0.567$$

part (b):  $I_f = 0.782 \text{ A}$  and  $D = 0.684$ .

part (c):

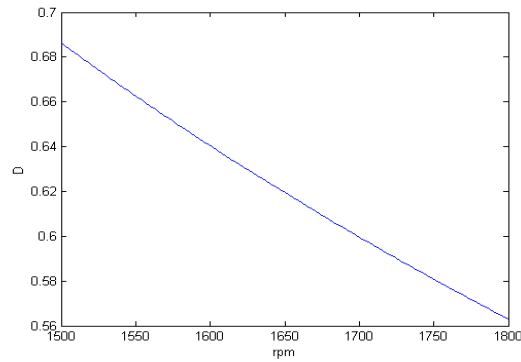


**Problem 11.2**

part (a):  $I_f = 0.645$  A and  $D = 0.565$ .

part (b):  $I_f = 0.784$  A and  $D = 0.686$ .

part (c):

**Problem 11.3**

part (a): From part (a) of Problem 11.1,  $k_f = 0.953$ . For  $E_a = 120$  V and  $n = 1300$  r/min ( $\omega_m = 1300\pi/30$ ) and thus

$$I_{f,1} = \frac{E_a}{\omega_m K_f} = 0.925 \text{ A}$$

and

$$D = \frac{I_f}{I_{f,\max}} = 0.809$$

where  $I_{f,\max} = 1.14$  A as found in Problem 11.1.

part (b):  $I_{f,2} = DI_{f,\max} = 0.686$  A.

$$\omega_m = \frac{E_a}{I_f K_f} = 183.7 \text{ r/min}$$

and thus  $n = 30\omega_m/\pi$ .

part (c):

$$i_f(t) = I_{f,2} + (I_{f,1} - I_{f,2})e^{-t/\tau} = 0.686 + 0.239e^{-t/\tau}$$

where  $\tau = L_f/R_f = 35.2$  msec.

part (d):

$$J \frac{d\omega_m}{dt} = K_f i_f(t) E_a = K_f i_f(t) \left( \frac{V_a - K_f i_f(t) \omega_m}{R_a} \right)$$

where  $i_f(t)$  is as given in part (c).

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**Problem 11.4**

part (a):

$$E_{a,nl} = V_a - R_a I_{nl} = 239.7 \text{ V}$$

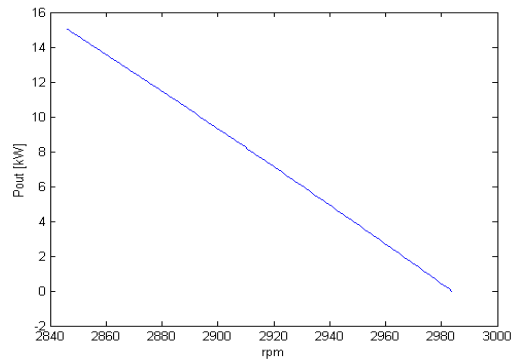
The rotational loss is given by  $P_{\text{rot}} = E_{a,nl} I_{a,nl} = 374 \text{ W}$ .

Based upon  $I_f = V_a / R_f = 1.81 \text{ A}$ ,

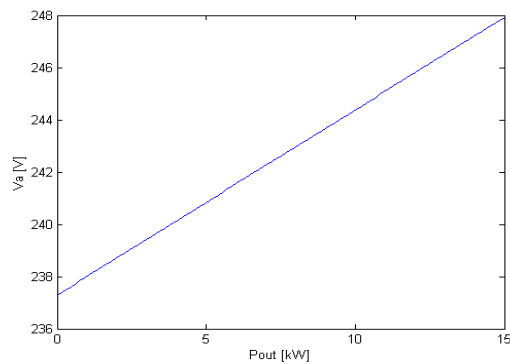
$$\omega_{m,nl} = \frac{E_{a,nl}}{I_f K_f} = 312.5 \text{ r/min}$$

and thus  $n_{nl} = 30 \omega_{m,nl} / \pi = 2984 \text{ r/min}$ .

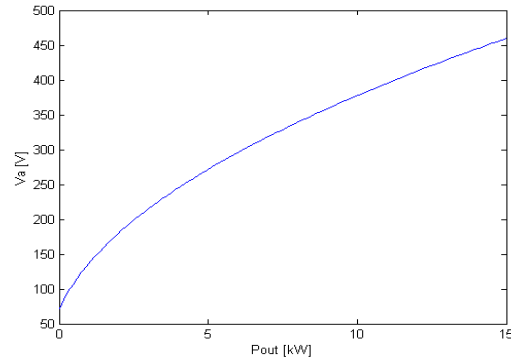
part (b):



part (c):



part (d):



### Problem 11.5

part (a): First convert  $K_m$  to SI units.

$$K_m = 2.18 \times 10^{-4} \frac{\text{V}}{\text{r/min}} \frac{1 \text{r/min}}{\pi/30 \text{ rad/sec}} = 2.08 \text{ mV}/(\text{rad/sec})$$

$T_{\text{stall}} = 0.094 \text{ oz}\cdot\text{cm} = 6.64 \times 10^{-4} \text{ N}\cdot\text{m}$ . At stall,  $I_{a,\text{stall}} = T_{\text{stall}}/K_m = 0.319 \text{ A}$ . Thus

$$R_a = \frac{V_a}{I_a} = \frac{3}{0.319} = 9.41 \Omega$$

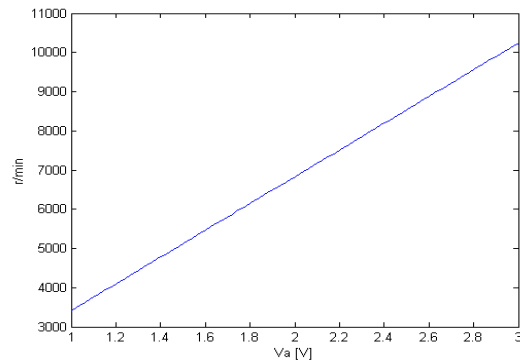
part (b):  $\omega_{m,\text{nl}} = n_{\text{nl}} \left(\frac{\pi}{30}\right) = 1299 \text{ rad/sec}$ . Thus  $E_{a,\text{nl}} = \omega_{m,\text{nl}} K_{m,\text{nl}} = 2.70 \text{ V}$ .

$$I_{a,\text{nl}} = \frac{V_a - E_{a,\text{nl}}}{R_a} = 31.5 \text{ mA}$$

and thus the no-load rotational loss is

$$P_{\text{rot}} = E_{a,\text{nl}} I_{a,\text{nl}} = 85.3 \text{ mW}$$

part (c):



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**Problem 11.6**part (a):  $\omega_{m,nl} = n_{nl} \left( \frac{\pi}{30} \right) = 374.9 \text{ rad/sec.}$ 

$$E_{a,nl} = V_a - I_{a,nl}R_a = 24.0 \text{ V}$$

$$K_m = \frac{E_{a,nl}}{\omega_{m,nl}} = 63.9 \text{ mV/(rad/sec)}$$

part (b):  $P_{\text{rot}} = E_{a,nl}I_{a,nl} = 11.3 \text{ W}$ 

part (c):

$D$	$I_a$ [A]	r/min	$P_{\text{load}}$ [W]
0.80	13.35	3393	293
0.75	12.70	3179	261
0.70	12.05	2964	231
0.65	11.40	2749	203
0.60	10.70	2535	176
0.55	10.05	2320	151
0.50	9.30	2107	127

**Problem 11.7**

The rotor acceleration is governed by the differential equation:

$$J \frac{d\omega_m}{dt} = T = K_m I_a$$

Converted to  $\text{kg}\cdot\text{m}^2$ , the moment of inertia is  $4.52 \times 10^{-9} \text{ kg}\cdot\text{m}^2$ . Thus, to get to the final speed  $\omega_m = 1.2 \times 10^4 \pi/30 = 1257 \text{ rad/sec}$ ,

$$t = \frac{J\omega_m}{K_m I_a} = \frac{(4.5 \times 10^{-9}) \times 1257}{(2.08 \times 10^{-3}) \times 0.1} = 27.3 \text{ msec}$$

**Problem 11.8**

part (a):

$$I_{a,\text{rated}} = \frac{P_{\text{rated}} + P_{\text{rot}}}{\omega_{m,\text{rated}} K_m} = \frac{1187}{(3000\pi/30) \times 0.465} = 8.12 \text{ A}$$

$$T_{\text{rated}} = K_m I_{a,\text{rated}} = 3.78 \text{ N}\cdot\text{m}$$

part (b):  $T_{\text{load}} = K_m I_a - T_{\text{rot}}$ . Here  $T_{\text{rot}} = 87/(3000\pi/30) = 0.27 \text{ N}\cdot\text{m}$ . Thus  $T_{\text{load}} = 2.61 \text{ N}\cdot\text{m}$  and  $P_{\text{load}} = T_{\text{load}}\omega_m = 729 \text{ W}$ .

part (c): The differential equation governing the motor speed is

$$J \frac{d\omega_m}{dt} = \left( \frac{\pi J}{30} \right) \frac{dn}{dt} = T_{\text{mech}} - T_{\text{rot}} - T_{\text{load}}$$

Here,  $n$  is the motor speed in r/min,  $T_{\text{mech}} = K_m I_a = 0.465 \times 7.0 = 3.26 \text{ N}\cdot\text{m}$  and, from part (b),

$$T_{\text{load}} = 729 \left( \frac{n}{2670} \right) = 0.273 n \text{ N}\cdot\text{m}$$

Thus, the differential equation is

$$\frac{dn}{dt} + 4.094 n - 1.252 \times 10^4 = 0$$

and the solution is

$$n = 3051 - 381e^{-t/\tau} \text{ r/min}$$

where  $\tau = 0.255 \text{ sec}$ .

### Problem 11.9

From the solution to Problem 11.8,  $I_{a,\text{rated}} = 8.12 \text{ A}$ . Neglecting rotational losses, the motor speed can be calculated from the differential equation

$$J \frac{d\omega_m}{dt} = T_{\text{mech}} = -K_m I_{a,\text{rated}}$$

and thus

$$\omega_m = \omega_{m,0} - \left( \frac{K_m I_{a,\text{rated}}}{J} \right) t$$

and thus the motor will reach zero speed at time

$$t = \frac{J\omega_{m,0}}{K_m I_{a,\text{rated}}} = \frac{(1.86 \times 10^{-3}) \times (3000\pi/30)}{0.465 \times 8.12} = 0.155 \text{ sec}$$

### Problem 11.10

part (a): Rated speed =  $120f/\text{poles} = 1800 \text{ r/min}$ .

part (b):

$$I_{\text{rated}} = \frac{P_{\text{rated}}}{\sqrt{3} V_{\text{rated}}} = 138.0 \text{ A}$$

part (c): In per unit,  $V_a = 1.0$  and  $P = 1000/1100 = 0.909$ . Thus,  $I_a = 0.909$  and

$$E_{af} = V_a - jX_s I_a = 1.55 \angle -49.7^\circ \text{ per unit}$$

Thus  $I_f = 1.55 \times 85 = 131 \text{ A}$ .

part (d): The inverter frequency will equal  $f = 60(1300/1800) = 43.3 \text{ Hz}$  and the motor power input will be  $P = 1000(1300/1800)^{2.5} = 443 \text{ W}$ . If one scales the base voltage and base power with frequency then the base impedance

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will scale with frequency. Hence, under this new operating condition, the per-unit terminal voltage, generated voltage and synchronous reactance will be unchanged while the per-unit power will be  $P = 443/(1100 \times 43.3/60) = 0.558$  per unit. Thus

$$\delta = -\sin^{-1}\left(\frac{PX_s}{V_a E_{af}}\right) = -27.9^{\text{circ}}$$

and  $\hat{E}_{af} = 1.55\angle^{-27.9^\circ}$

$$\hat{I}_a = \frac{V_a - \hat{E}_{af}}{jX_s} = 0.688\angle 26.8^\circ$$

Thus the power factor is  $\cos^{-1}(26.8^\circ) = 0.89$  leading.

part (e): Continuing with the base quantities of part (d),  $I_a = 0.558$  per unit and

$$E_{af} = V_a - jX_s I_a = 1.20\angle -47.0^\circ \text{ per unit}$$

and thus  $I_f = 1.20 \times 85 = 102$  A.

#### Problem 11.11

part (a): No numerical calculation required.

part (b):

1500 r/min:

$$V_a = 3.83 \text{ kV, 1-l}$$

$$P_{\max} = 833 \text{ kW}$$

$$I_f = 131 \text{ A}$$

2000 r/min:

$$V_a = 4.60 \text{ kV, 1-l}$$

$$P_{\max} = 1000 \text{ kW}$$

$$I_f = 126 \text{ A}$$

#### Problem 11.12

$$L_s = 5.23 \text{ mH}$$

$$L_{af} = 63.1 \text{ mH}$$

$$T_{\text{rated}} = 531 \text{ N} \cdot \text{m}$$

#### Problem 11.13

part (a):

$$L_{af} = \frac{\sqrt{2} V_{\text{base}}}{\sqrt{3} \omega_{\text{base}} \text{AFNL}} = 63.1 \text{ mH}$$

$L_s$  can be calculated from the per-unit value of  $X_s$ .



$$Z_{\text{base}} = \frac{V_{\text{base}}^2}{P_{\text{base}}} = 2.12 \Omega$$

and  $L_{\text{base}} = Z_{\text{base}}/\omega_{\text{base}} = 5.6 \text{ mH}$ . Thus,

$$X_s = \frac{X_s}{Z_{\text{base}}} = 1.97 \Omega$$

$$L_s = \frac{X_s}{L_{\text{base}}} = 5.23 \text{ mH}$$

part (b):  $\omega_{\text{m,base}} = \omega_{\text{base}}(2/\text{poles}) = 60\pi$  and  $T_{\text{base}} = P_{\text{base}}/\omega_{\text{m,base}} = 531 \text{ N}\cdot\text{m}$ . Thus,  $T = 0.5 T_{\text{base}} = 265 \text{ N}\cdot\text{m}$ .

$$i_Q = \left(\frac{2}{3}\right) \left(\frac{2}{\text{poles}}\right) \left(\frac{T}{L_{\text{af}}I_f}\right) = 100 \text{ A}$$

$$I_a = \frac{i_Q}{\sqrt{2}} = 70.8 \text{ A, rms}$$

part (c):

$$E_{\text{af}} = \frac{\omega_{\text{base}}L_{\text{af}}I_f}{\sqrt{2}} = 235 \text{ V}$$

Because  $i_D = 0$ ,  $\hat{I}_a$  and  $\hat{E}_{\text{af}}$  both lie along the quadrature axis. Thus, the terminal voltage magnitude will be given by

$$V_a = |E_{\text{af}} + jX_sI_a| = 274 \text{ V, l-n} = 474 \text{ V, l-l}$$

#### Problem 11.14

part (a): The various machine parameters were calculated in the solution to Problem 11.13.  $T = 0.75T_{\text{base}} = 398 \text{ N}\cdot\text{m}$  and  $\omega_{\text{m}} = 1475\pi/30 = 154.5 \text{ rad/sec}$ . Thus,  $P = \omega_{\text{m}}T = 61.5 \text{ kW}$ .

part (b):

$$i_Q = \left(\frac{2}{3}\right) \left(\frac{2}{\text{poles}}\right) \left(\frac{T}{L_{\text{af}}I_f}\right) = 145.1 \text{ A}$$

$$I_a = \frac{i_Q}{\sqrt{2}} = 102.6 \text{ A, rms}$$

part (c):  $f_e = 60(1475/1800) = 49.2 \text{ Hz}$ .

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part (d):

$$E_{af} = \frac{\omega_{base} L_{af} I_f}{\sqrt{2}} = 200 \text{ V}$$

Because  $i_D = 0$ ,  $\hat{I}_a$  and  $\hat{E}_{af}$  both lie along the quadrature axis. Thus, the terminal voltage magnitude will be given by

$$V_a = |E_{af} + jX_s I_a| = 260 \text{ V, l-n} = 450 \text{ V, l-l}$$

**Problem 11.15**

part (a): The various machine parameters were calculated in the solution to Problem 11.13. With  $T_{ref}$  increased to  $0.8T_{base} = 424 \text{ N}\cdot\text{m}$

$$i_Q = \left(\frac{2}{3}\right) \left(\frac{2}{\text{poles}}\right) \left(\frac{T_{ref}}{L_{af} I_f}\right) = 160.3 \text{ A}$$

$$I_a = \frac{i_Q}{\sqrt{2}} = 113.3 \text{ A, rms}$$

$$E_{af} = \frac{\omega_{base} L_{af} I_f}{\sqrt{2}} = 235.3 \text{ V}$$

Because  $i_D = 0$ ,  $\hat{I}_a$  and  $\hat{E}_{af}$  both lie along the quadrature axis. Thus, the terminal voltage magnitude will be given by

$$V_a = |E_{af} + jX_s I_a| = 324.5 \text{ V, l-n} = 562.1 \text{ V, l-l} = 1.22 \text{ per unit}$$

part (b): The required calculations follow those of Example 11.9.

(i) The terminal voltage will be set equal to 460 V, l-l (1.0 per unit).

(ii) The line-to-neutral terminal voltage is  $V_{a,l-n} = 460/\sqrt{3} = 265.6 \text{ V}$ .

Thus

$$I_a = \frac{(\omega_m T_{ref})}{3V_{a,l-n}} = 100.4 \text{ A, rms}$$

(iii)

$$\delta = \tan^{-1} \left( \frac{\omega_m L_s I_a}{V_{a,l-n}} \right) = -26.7^\circ$$

Thus

$$i_{rmQ} = \sqrt{2} I_a \sin \delta = 113.8 \text{ A}$$

and

$$i_{rmD} = \sqrt{2} I_a \sin \delta = -84.9 \text{ A}$$

(iv)

$$I_f = \left(\frac{2}{3}\right) \left(\frac{2}{\text{poles}}\right) \left(\frac{T_{\text{ref}}}{L_{\text{af}} i_Q}\right) = 19.7 \text{ A}$$

**Problem 11.16**

$$\begin{aligned} T &= 4431 \text{ N}\cdot\text{m} \\ I_f &= 121.7 \text{ A} \\ i_D &= -78.4 \text{ A} \\ i_Q &= 95.5 \text{ A} \\ I_a &= 87.4 \text{ A, rms} \\ V_a &= 1993 \text{ V, rms, l-l} \end{aligned}$$

**Problem 11.17**

part (a):

$$\Lambda_{\text{PM}} = \frac{\sqrt{2} V_{\text{a,rated, l-n}}}{\omega_{\text{m,rated}}} = \frac{\sqrt{2} (230/\sqrt{3})}{3500\pi/30} = 0.512 \text{ Wb}$$

part (b): The frequency will be 60 Hz and hence  $X_s = \omega_e L_s = 3.24 \Omega$ .  
 $E_{\text{af}} = (3600/3500)(230/\sqrt{3}) = 136.6 \text{ V}$ .  $I_{\text{a,rated}} = 2000/(\text{sqrt}3 \cdot 230) = 5.02 \text{ A}$ .  
 The armature current is equal to

$$I_a = \frac{(V_{\text{a,rated, l-n}} - \hat{E}_{\text{af}})}{jX_s}$$

Although the magnitude of  $\hat{E}_{\text{af}} = E_{\text{af}} \angle \delta$  is known, its angle  $\delta$  (required to give  $I_a = I_{\text{a,rated}}$ ) is not. A MATLAB script can be used to easily iterate to find that  $\delta = -6.73^\circ$ . The motor power is then given by

$$P = - \left( \frac{3E_{\text{af}} V_{\text{a,rated, l-n}}}{X_s} \right) \sin \delta = 1.96 \text{ kW}$$

Then,

$$T = \frac{P}{\omega_m} = 5.22 \text{ N}\cdot\text{m}$$

and

$$i_Q = \left(\frac{2}{3}\right) \left(\frac{2}{\text{poles}}\right) \left(\frac{T}{\Lambda_{\text{PM}}}\right) = 6.80 \text{ A}$$

$$i_D = \sqrt{2I_{\text{a,rated}}^2 + i_Q^2} = 2.05 \text{ A}$$

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**Problem 11.18**

As calculated in the solution to Problem 11.17,  $\Lambda_{PM} = 0.512$  Wb. At 4000 r/min,  $f_e = (4000/3600)60 = 66.7$  Hz and thus  $\omega_e = 2\pi f_e = 418.9$  rad/sec.

The rms line-to-neutral armature flux linkages under this operating condition will be

$$\lambda_a = \frac{\sqrt{2} V_{a,\text{rated, l-n}}}{\omega_e} = 0.448 \text{ Wb}$$

and maximum torque will correspond to operating the motor at its rated armature current  $I_a = I_{a,\text{rated}} = 5.02$  A.

Solving

$$\begin{aligned} \lambda_a &= \sqrt{\frac{\lambda_D^2 + \lambda_Q^2}{2}} \\ &= \sqrt{\frac{(L_s + \Lambda_{PM})^2 + (L_s i_Q)^2}{2}} \\ &= \sqrt{\frac{2(L_s I_a)^2 + 2L_s i_D \Lambda_{PM} + \Lambda_{PM}^2}{2}} \end{aligned}$$

for  $i_D$  gives

$$i_D = \frac{2\lambda_a^2 - 2(L_s I_a)^2 - \Lambda_{PM}^2}{2L_s \Lambda_{PM}} = -4.38 \text{ A.}$$

Thus

$$i_Q = \sqrt{2I_a^2 - i_D^2} = 5.62 \text{ A}$$

Thus, the maximum torque will be given by

$$T_{\max} = \left(\frac{3}{2}\right) \left(\frac{\text{poles}}{2}\right) \Lambda_{PM} i_Q = 4.32 \text{ N} \cdot \text{m}$$

and, for a speed of 4000 r/min ( $\omega_m = 418.9$  rad/sec)

$$P_{\max} = \omega_m T_{\max} = 1810 \text{ W}$$

**Problem 11.19**

The rated current of this motor is

$$I_{a,\text{rated}} = \frac{P_{\text{rated}}}{\sqrt{3} (V_{a,\text{rated, l-l}})} = 37.7 \text{ A}$$

$$\Lambda_{PM} = \frac{\sqrt{2} (V_{a,\text{rated, l-n}})}{\omega_e} = \frac{\sqrt{2}(230/\text{sqrt}3)}{7620\pi/30} = 0.235 \text{ Wb}$$

part (a): The torque will be maximized when  $i_Q$  is maximized. In this case, this will occur when  $i_D = 0$  and thus

$$i_{Q,\max} = \sqrt{2} I_{a,\text{rated}} = 53.2 \text{ A}$$

and thus

$$T_{\max} = \left(\frac{3}{2}\right) \left(\frac{\text{poles}}{2}\right) \Lambda_{\text{PM}} i_{Q,\max} = 18.8 \text{ N} \cdot \text{m}$$

part (b): The peak line-to-neutral flux linkages are

$$\lambda_a = \sqrt{\Lambda_{\text{PM}}^2 + \lambda_Q^2} = \sqrt{\Lambda_{\text{PM}}^2 + (L_s i_{Q,\max})^2} = 0.257 \text{ Wb}$$

Thus, to avoid exceeding rated terminal voltage, the electrical frequency of the motor must be limited to

$$\omega_{e,\max} = \frac{\sqrt{2} (V_{a,\text{rated, l-n}})}{\lambda_a} = 731.9 \text{ rad/sec}$$

and the corresponding motor speed will be

$$n = \omega_{e,\max} \left(\frac{30}{\pi}\right) = 6989 \text{ r/min}$$

part (c): At 10,000 r/min,  $\omega_e = 10000\pi/30 = 1047 \text{ rad/sec}$ . In order to maintain rated terminal voltage, the peak line-to-neutral armature flux linkages must now be limited to

$$\lambda_{a,\max} = \frac{\sqrt{2} (V_{a,\text{rated, l-n}})}{\omega_e} = 0.179 \text{ Wb}$$

Thus, solving the peak line-to-neutral armature flux linkages

$$\begin{aligned} \lambda_{a,\max} &= \sqrt{\lambda_D^2 + \lambda_Q^2} \\ &= \sqrt{(L_s + \Lambda_{\text{PM}})^2 + (L_s i_Q)^2} \\ &= \sqrt{\frac{2(L_s I_{a,\text{rated}})^2 + 2L_s i_D \Lambda_{\text{PM}} + \Lambda_{\text{PM}}^2}{2}} \end{aligned}$$

for  $i_D$  gives

$$i_D = \frac{\lambda_{a,\max}^2 - 2(L_s I_{a,\text{rated}})^2 - \Lambda_{\text{PM}}^2}{2L_s \Lambda_{\text{PM}}} = -37.3 \text{ A.}$$

and thus

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$$i_Q = \sqrt{2I_{a,\text{rated}}^2 - i_D^2} = 38.0 \text{ A}$$

The motor torque is then given by

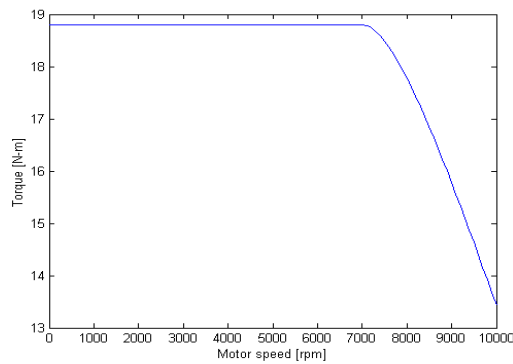
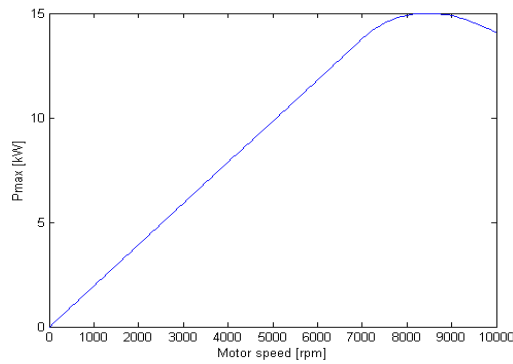
$$T = \left(\frac{3}{2}\right) \left(\frac{\text{poles}}{2}\right) \Lambda_{PM} i_Q = 13.4 \text{ N} \cdot \text{m}$$

Since this is a two-pole machine and  $\omega_m = \omega_e$ , the corresponding power will be  $P = \omega_m T = 14.1 \text{ kW}$ .

The motor power factor will be

$$\text{power factor} = \frac{P}{\sqrt{3} (V_{a,\text{rated}} - I_{a,\text{rated}})} = 0.937$$

### Problem 11.20



### Problem 11.21

part (a): Following the analysis of Chapter 6

$$Z_{1,\text{eq}} = R_{1,\text{eq}} + jX_{1,\text{eq}} = \frac{jX_m(R_1 + jX_1)}{R_1 + j(X_1 + X_m)} = 0.099 + j1.08\Omega$$

$$T_{\max} = \frac{0.5n_{\text{ph}}V_{1,\text{eq}}^2}{\omega_s(X_{1,\text{eq}} + X_2)} = 234 \text{ N} \cdot \text{m}$$

$$s_{\max T} = \frac{R_2}{\sqrt{R_{1,\text{eq}}^2 + (X_{1,\text{eq}} + X_2)^2}} = 0.102 = 10.2\%$$

part (b):  $\omega_s = 2\pi f(2/\text{poles}) = 60\pi = 188.5$  rad/sec. At  $s = 0.029$ ,  $\omega_m = (1 - s)\omega_s = 183.0$  rad/sec. The torque is given by

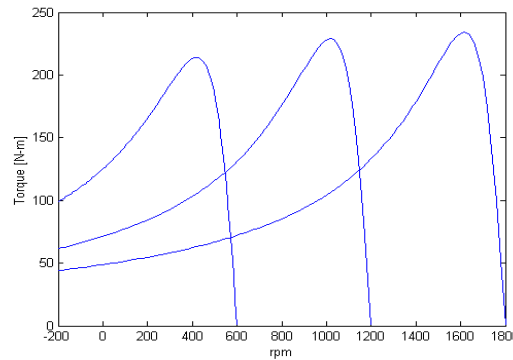
$$T = \frac{1}{\omega_s} \left[ \frac{n_{\text{ph}} V_{1,\text{eq}}^2 (R_2/s)}{(R_{1,\text{eq}} + (R_2/s))^2 + (X_{1,\text{eq}} + X_2)^2} \right] = 126 \text{ N} \cdot \text{m}$$

and the power is  $P = \omega_m T = 23.1$  kW.

part (c): With the frequency reduced from 60 Hz to 30 Hz,  $\omega_s$ , the terminal voltage and each reactance must be scaled by the factor (35/60). The torque expression can be solved for the slip. This can most easily be done iteratively with a MATLAB script. The result is  $s = 0.051 = 5.1\%$ , the speed is 997 r/min and the output power is 13.1 kW.

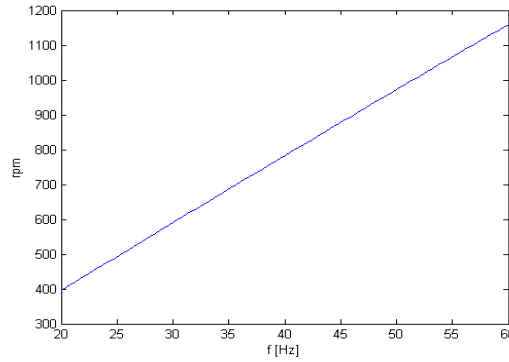
### Problem 11.22

part (a):



part (b): The same MATLAB script can be used to iteratively find the drive frequency for which  $s_{\max T} = 1.0$ . The result is a drive frequency of 5.44 Hz and a torque of 151 N·m.

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**Problem 11.23****Problem 11.24**

The motor torque is a function of the ratio  $R_2/s$ . The slip with  $R_{2,\text{ext}} = 0$  is

$$s_0 = \frac{1200 - 1157}{1200} = 0.0358$$

and that with  $R_{2,\text{ext}} = 0.87 \Omega$  is

$$s_1 = \frac{1200 - 1072}{1200} = 0.1067$$

Thus, solving

$$\frac{R_2}{s_0} = \frac{R_2 + 0.87}{s_1}$$

for  $R_2$  gives  $R_2 = 0.44 \Omega$ .

**Problem 11.25**

The motor torque is a function of the ratio  $R_2/s$ . The slip with  $R_{2,\text{ext}} = 0$  is

$$s_0 = \frac{1200 - 1157}{1200} = 0.0358$$

The desired operating speed corresponds to a slip of

$$s_1 = \frac{1200 - 850}{1200} = 0.2917$$

Thus substituting the value of  $R_2$  found in the solution to Problem 11.24 and solving for  $R_{2,\text{ext}}$

$$\frac{R_2}{s_0} = \frac{R_2 + R_{2,\text{ext}}}{s_1}$$



for  $R_{2,\text{ext}}$  gives  $R_{2,\text{ext}} = 3.14 \Omega$ .

**Problem 11.26**

part (a): If  $R_1$  is assumed negligible, the torque expression becomes

$$T = \frac{1}{\omega_s} \left[ \frac{n_{\text{ph}} V_{1,\text{eq}}^2 (R_2/s)}{(R_2/s)^2 + (X_{1,\text{eq}} + X_2)^2} \right]$$

Substituting the corresponding expressions for  $T_{\text{max}}$

$$T_{\text{max}} = \frac{1}{\omega_s} \left[ \frac{0.5 n_{\text{ph}} V_{1,\text{eq}}^2}{X_{1,\text{eq}} + X_2} \right]$$

$$s_{\text{maxT}} = \frac{R_2}{X_{1,\text{eq}} + X_2}$$

gives

$$T = T_{\text{max}} \left( \frac{2}{\frac{s}{s_{\text{maxT}}} + \frac{s_{\text{maxT}}}{s}} \right)$$

Defining the ratio of full-load torque to maximum torque as

$$k \equiv \frac{T_{\text{fl}}}{T_{\text{max}}} = \frac{1}{2.12} = 0.472$$

the full-load slip can then be found as

$$s_{\text{fl}} = s_{\text{maxT}} \left( \frac{k}{1 + \sqrt{1 - k^2}} \right) = 0.0414 = 4.14\%$$

part (b): The full load rotor power dissipation is given by

$$P_{\text{rotor}} = P_{\text{fl}} \left( \frac{s_{\text{fl}}}{1 - s_{\text{fl}}} \right) = 3240 \text{ W}$$

part (c): At rated load,  $\omega_{\text{m,rated}} = (1 - s_{\text{fl}})\omega_s = 180.7 \text{ rad/sec}$ . The rated torque is  $T_{\text{rated}} = P_{\text{rated}}/\omega_{\text{m,rated}} = 415 \text{ N}\cdot\text{m}$ .

Setting  $s = 1$  gives

$$T_{\text{start}} = T_{\text{max}} \left( \frac{2}{\frac{1}{s_{\text{maxT}}} + s_{\text{maxT}}} \right) = 68.1\% = 283 \text{ N}\cdot\text{m}$$

part (d): If the stator current is at its full load value, this means that  $R_2/s$  is equal to its full load value and hence the torque will be equal to the full-load torque, 415 N·m.

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part (e): The slip will be twice the original full load slip or 8.26%.

**Problem 11.27**

part (a):

$$s_{fl} = \frac{1200 - 1169}{1200} = 0.0258$$

and thus the full-load rotor power dissipation is equal to

$$P_{rotor} = P_{fl} \left( \frac{s_{fl}}{1 - s_{fl}} \right) = 928 \text{ W}$$

part (b): Defining

$$k \equiv \frac{T_{max}}{T_{fl}} = 2.45$$

and using the derivation found in the solution to Problem 11.26 gives

$$s_{maxT} = s_{fl} \left( k + \sqrt{k^2 - 1} \right) = 0.1211$$

Thus the motor speed at maximum torque is  $n_{max} = 1200(1 - s_{maxT}) = 1055 \text{ r/min}$ .

part (c): We want  $s_{maxT}$  to increase by a factor of  $1/0.1211 = 8.26$ . Thus the rotor resistance must increase by this factor. In other words

$$R_2 + R_{2,ext} = 8.26R_2$$

which gives  $R_{2,ext} = 1.67 \Omega$ .

part (d): The 50-Hz voltage will be (5/6) that of 60-Hz. Thus the applied voltage will be 367 V, line-to-line.

part (e): If the frequency and voltage are scaled from their rated value by a factor  $k_f$ , the torque expression becomes

$$T = \left( \frac{1}{k_f \omega_{s0}} \right) \left[ \frac{n_{ph} (k_f V_{1,eq})^2 (R_2/s)}{(R_2/s)^2 + (k_f (X_{1,eq} + X_2))^2} \right]$$

where  $\omega_{s0}$  is the rated-frequency synchronous speed of the motor. Clearly, the torque expression will remain constant if the slip scales inversely with  $k_f$ . Thus

$$s_{fl,50} = \left( \frac{60}{50} \right) s_{fl,60} = 0.031$$

The synchronous speed at 50 Hz is 1000 r/min and thus

$$n_{fl,50} = 1000(1 - s_{fl,50}) = 969 \text{ r/min}$$

**Problem 11.28**

part (a): From the data given in Problem 11.23, the motor inductances are:

$$L_1 = 3.13 \text{ mH}; \quad L_2 = 3.50 \text{ mH}; \quad L_m = 128 \text{ mH};$$

and thus

$$L_S = L_1 + L_m = 131.5 \text{ mH}$$

and

$$L_R = L_2 + L_m = 131.9 \text{ mH}$$

$R_a = R_1 = 108 \text{ m}\Omega$  and  $R_{aR} = R_2 = 296 \text{ m}\Omega$ . Finally, the rated motor torque  
The peak flux linkages corresponding to rated voltage line-to-neutral voltage  
are given by

$$\lambda_{\text{rated}} = \frac{\sqrt{2} V_{\text{base}}}{\sqrt{3} \omega_{\text{base}}} = \frac{\sqrt{2} 2400}{\sqrt{3} (120\pi)} = 5.19 \text{ Wb}$$

The required torque can be determined from the given power and speed as

$$T_{\text{mech}} = \frac{P_{\text{mech}}}{\omega_m} = \frac{400 \times 10^3}{1148\pi/30} = 3327 \text{ N} \cdot \text{m}$$

Setting  $\lambda_{\text{DR}} = \lambda_{\text{rated}}$  gives

$$i_Q = \left(\frac{2}{3}\right) \left(\frac{2}{\text{poles}}\right) \left(\frac{L_R}{L_m}\right) \left(\frac{T_{\text{mech}}}{\lambda_{\text{DR}}}\right) = 146.1 \text{ A}$$

and

$$i_D = \frac{\lambda_{\text{DR}}}{L_m} = 40.5 \text{ A}$$

part (b):

$$I_a = \sqrt{\frac{i_D^2 + i_Q^2}{2}} = 107.2 \text{ A}$$

part (c):

$$\omega_{\text{me}} = \omega_m \left(\frac{\text{poles}}{2}\right) = 360.6 \text{ rad/sec}$$

$$\omega_e = \omega_{\text{me}} + \left(\frac{R_{aR}}{L_R}\right) \left(\frac{i_Q}{i_D}\right) = 368.7 \text{ rad/sec}$$

and thus

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$$f_e = \frac{\omega_2}{2\pi} = 58.7 \text{ Hz}$$

part (d):

$$\begin{aligned} V_a &= \sqrt{\frac{(R_a i_D - \omega_e (L_S - \frac{L_m^2}{L_R}) i_Q)^2 + (R_a i_Q + \omega_e L_S i_D)^2}{2}} \\ &= 1421 \text{ V, l-n} = 2461 \text{ V, l-l} \end{aligned}$$

**Problem 11.29**

part (a): From the given data, the motor inductances are:

$$L_1 = 0.915 \text{ mH}; \quad L_2 = 0.936 \text{ mH}; \quad L_m = 49.34 \text{ mH};$$

and thus

$$L_S = L_1 + L_m = 50.25 \text{ mH}$$

and

$$L_R = L_2 + L_m = 50.27 \text{ mH}$$

 $R_a = R_1 = 32.2 \text{ m}\Omega$  and  $R_{aR} = R_2 = 70.3 \text{ m}\Omega$ . Finally, the rated motor torque

The peak flux linkages corresponding to rated voltage line-to-neutral voltage are given by

$$\lambda_{\text{rated}} = \frac{\sqrt{2} V_{\text{base}}}{\sqrt{3} \omega_{\text{base}}} = \frac{\sqrt{2} 230}{\sqrt{3} (120\pi)} = 0.498 \text{ Wb}$$

The motor torque is  $T_{\text{mech}} = 85(1300/1800) = 61.4 \text{ N}\cdot\text{m}$ . Setting  $\lambda_{\text{DR}} = \lambda_{\text{rated}}$ , we can solve for  $i_Q$  and  $i_D$ 

$$i_Q = \left(\frac{2}{3}\right) \left(\frac{2}{\text{poles}}\right) \left(\frac{L_R}{L_m}\right) \left(\frac{T_{\text{mech}}}{\lambda_{\text{DR}}}\right) = 41.9 \text{ A}$$

and

$$i_D = \frac{\lambda_{\text{DR}}}{L_m} = 10.1 \text{ A}$$

The motor mechanical velocity in electrical rad/sec is

$$\omega_{\text{me}} = \omega_m \left(\frac{\text{poles}}{2}\right) = 272.3 \text{ rad/sec}$$

and thus

$$\omega_e = \omega_{me} + \left( \frac{R_{aR}}{L_R} \right) \left( \frac{i_Q}{i_D} \right) = 278.1 \text{ rad/sec}$$

and

$$f_e = \frac{\omega_e}{2\pi} = 44.3 \text{ Hz}$$

part (b):

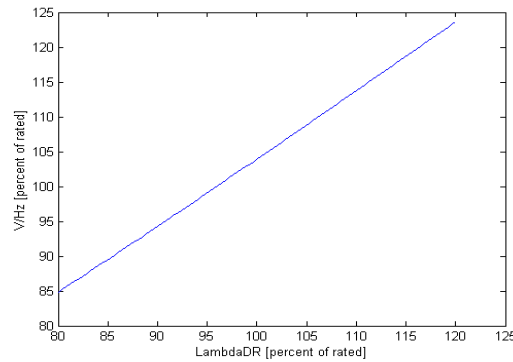
$$I_a = \sqrt{\frac{i_D^2 + i_Q^2}{2}} = 30.4 \text{ A}$$

$$\begin{aligned} V_a &= \sqrt{\frac{(R_a i_D - \omega_e (L_S - \frac{L_m^2}{L_R}) i_Q)^2 + (R_a i_Q + \omega_e L_S i_D)^2}{2}} \\ &= 101.8 \text{ V, l-n} = 176.3 \text{ V, l-l} \end{aligned}$$

part (c):

$$S_{in} = \sqrt{3} V_a I_a = 9.30 \text{ kVA}$$

part (d):



### Problem 11.30

The motor parameters are calculated in the solution to Problem 11.29.

part (a): The motor torque is  $T_{mech} = 85(1450/1800) = 68.5 \text{ N}\cdot\text{m}$ . Setting  $\lambda_{DR} = \lambda_{rated}$ , we can solve for  $i_Q$  and  $i_D$

$$i_Q = \left( \frac{2}{3} \right) \left( \frac{2}{\text{poles}} \right) \left( \frac{L_R}{L_m} \right) \left( \frac{T_{mech}}{\lambda_{DR}} \right) = 46.7 \text{ A}$$

$$i_D = \frac{\lambda_{DR}}{L_m} = 10.1 \text{ A}$$

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and

$$I_a = \sqrt{\frac{i_D^2 + i_Q^2}{2}} = 33.8 \text{ A}$$

part (b):

The motor mechanical velocity in electrical rad/sec is

$$\omega_{me} = \omega_m \left( \frac{\text{poles}}{2} \right) = 303.7 \text{ rad/sec}$$

and thus

$$\omega_e = \omega_{me} + \left( \frac{R_{aR}}{L_R} \right) \left( \frac{i_Q}{i_D} \right) = 310.1 \text{ rad/sec}$$

and

$$f_e = \frac{\omega_e}{2\pi} = 49.4 \text{ Hz}$$

part (c):  $i_Q$  is now increased to 51.4 A and hence, with  $I_{DR}$  and hence  $\lambda_{DR}$  unchanged

$$T_{mech} = \left( \frac{3}{2} \right) \left( \frac{\text{poles}}{2} \right) \left( \frac{L_m}{L_R} \right) \lambda_{DR} i_Q = 75.3 \text{ N} \cdot \text{m}$$

Thus the motor speed is

$$n = 1800 \left( \frac{T_{mech}}{85} \right) = 1595 \text{ r/min}$$

and  $\omega_m = n\pi/30 = 167.0 \text{ rad/sec}$ .

$$P_{mech} = T_{mech}\omega_m = 11.4 \text{ kW}$$

part (d): The terminal voltage is

$$\begin{aligned} V_a &= \sqrt{\frac{(R_a i_D - \omega_e (L_S - \frac{L_m^2}{L_R}) i_Q)^2 + (R_a i_Q + \omega_e L_S i_D)^2}{2}} \\ &= 125.6 \text{ V, l-n} = 217.5 \text{ V, l-l} \end{aligned}$$

The drive frequency can be found from

$$\omega_{me} = \omega_m \left( \frac{\text{poles}}{2} \right) = 334.1 \text{ rad/sec}$$

$$\omega_e = \omega_{me} + \left( \frac{R_{aR}}{L_R} \right) \left( \frac{i_Q}{i_D} \right) = 341.2 \text{ rad/sec}$$

and

$$f_e = \frac{\omega_2}{2\pi} = 54.3 \text{ Hz}$$

part (e):

$$S_{in} = \sqrt{3} V_a I_a = 13.9 \text{ kVA}$$

part (f): Iteration with a MATLAB script gives  $\lambda_{DR} = 95.3\%$  of  $\lambda_{rated}$ .

**Problem 11.31**

part (a): From the given data, the motor inductances are:

$$L_1 = 4.96 \text{ mH}; \quad L_2 = 6.02 \text{ mH}; \quad L_m = 118.3 \text{ mH};$$

and thus

$$L_S = L_1 + L_m = 123.3 \text{ mH}$$

and

$$L_R = L_2 + L_m = 124.3 \text{ mH}$$

$R_a = R_1 = 212 \text{ m}\Omega$  and  $R_{aR} = R_2 = 348 \text{ m}\Omega$ . Finally, the rated motor torque

The peak flux linkages corresponding to rated voltage line-to-neutral voltage are given by

$$\lambda_{rated} = \frac{\sqrt{2} V_{base}}{\sqrt{3} \omega_{base}} = \frac{\sqrt{2} 4160}{\sqrt{3} (120\pi)} = 9.01 \text{ Wb}$$

At a power output of 1050 kW and a speed of 828 r/min,  $\omega_m = 84.3 \text{ rad/sec}$ ,  $T_{mech} = 1.25 \times 10^4 \text{ N}\cdot\text{m}$ . Setting  $\lambda_{DR} = \lambda_{rated}$  gives

$$i_Q = \left(\frac{2}{3}\right) \left(\frac{2}{\text{poles}}\right) \left(\frac{T}{\Lambda_{PM}}\right) = 242.1 \text{ A}$$

$$i_D = \sqrt{2I_{a,rated}^2 + i_Q^2} = 76.2 \text{ A}$$

$$I_a = \sqrt{\frac{i_D^2 + i_Q^2}{2}} = 179.5 \text{ A}$$

The terminal voltage is

$$\begin{aligned} V_a &= \sqrt{\frac{(R_a i_D - \omega_e (L_S - \frac{L_m^2}{L_R}) i_Q)^2 + (R_a i_Q + \omega_e L_S i_D)^2}{2}} \\ &= 2415 \text{ V, l-n} = 4183 \text{ V, l-l} \end{aligned}$$

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The drive frequency can be found from

$$\omega_{me} = \omega_m \left( \frac{\text{poles}}{2} \right) = 337.2 \text{ rad/sec}$$

$$\omega_e = \omega_{me} + \left( \frac{R_{aR}}{L_R} \right) \left( \frac{i_Q}{i_D} \right) = 346.1 \text{ rad/sec}$$

and

$$f_e = \frac{\omega_e}{2\pi} = 55.1 \text{ Hz}$$

part (b): The equivalent-circuit of Chapter 6 can be analyzed readily using MATLAB as follows:

- All the reactances must be scaled from their 60-Hz values to 55.1 Hz.
- The rms input voltage must be set equal to 2415 V, line-to-neutral.
- The slip must be calculated based upon a synchronous speed of  $n_s = 60f_e(2/\text{poles}) = 826 \text{ r/min}$ .

If this is done, the equivalent circuit will give exactly the same results as those of part (a).