Contents

6. Polyphase Induction Motors

ONE

Transformers

The transformer is a device that transfers electrical energy from one electrical circuit to another electrical circuit through the medium of magnetic field and without a change in the frequency. The electric circuit which receives energy from the supply mains is called *primary* winding and the other circuit which delivers electric energy to the load is called the *secondary* winding.

Actually, the transformer is an electromagnetic energy conversion device, since the energy received by the primary is first converted to magnetic energy and it is then reconverted to useful electrical energy in the other circuits (secondary winding circuit, third winding circuit etc.). Thus primary and secondary windings of a transformer are not connected electrically, but are coupled magnetically. This coupling magnetic field allows the transfer of energy in either direction, from high-voltage to low-voltage circuits or from low-voltage to high-voltage circuits. If the transfer of energy occurs at the same voltage, the purpose of the transformer is merely to isolate the two electric circuits and this use is very rare in power applications. If the secondary winding has more turns than the primary winding, then the secondary voltage is higher than the primary voltage and the transformer is called a *step-up* transformer. In case the secondary winding has less turns than the primary winding, then the secondary voltage is lower than the primary voltage and the transformer is called a *step-down* transformer. Note that a step-up transformer can be used as a step-down transformer, in which case the secondary of step-up transformer becomes the primary of step-down transformer. Actually, a transformer can be termed a step-up or step-down transformer only after it has been put into service. Therefore, when referring to the windings of a particular transformer, the terms high-voltage winding and low-voltage winding should be used instead of primary and secondary windings.

A transformer is the most widely used device in both low and high current circuits. As such, transformers are built in an amazing range of sizes. In electronic, measurement and control circuits, transformer size may be so small that it weighs only a few tens of grams whereas in high voltage power circuits, it may weigh hundreds of tonnes.

In a transformer, the electrical energy transfer from one circuit to another circuit takes place without the use of moving parts—it has, therefore, the highest possible efficiency out of all the electrical machines and requires almost negligible amount of maintenance and supervision.

Insulation considerations limit the generation of alternator (ac generator or synchronous generator) voltages from about 11 to 22 kV. By means of transformers, this voltage is stepped up to higher economical transmission voltage, 400 kV or even higher, in order to reduce the transmission losses. Wherever the electrical energy is required, transformers are installed to step down the voltage suitable for its utilisation for motors, illumination purposes etc. Thus the transformer is the main reason for the widespread popularity of a.c. systems over d.c. systems.

In addition to its use in power systems, transformers are widely used in other prominent areas of electrical engineering. In communication systems, input transformers connect the microphone output to the first stage of an electronic amplifier. Interstage and output transformers are used extensively in radio and television circuits. In electronic and control circuits, transformers are used for impedance matching for maximum power transfer from source to the load. Pulse transformers find wide application in radar, television and digital computers. In power electronics, transformers are extensively used (*i*) for gate-pulse triggering and (*ii*) for synchronizing the pulse gating signals with the ac supply voltage given to the main power circuit. In general, important tasks performed by transformers are :

(*i*) for decreasing or increasing voltage and current levels from one circuit to another circuit (or circuits when there are 2 or more output windings) in low and high current circuits ;

(*ii*) for matching the impedance of a source and its load for maximum power transfer in electronic and control circuits, and

(*iii*) for isolating d.c. while permitting the flow of a.c. between two circuits or for isolating one circuit from another.

Transformer is, therefore, an essential piece of apparatus both for high and low current circuits.

An electromechanical energy conversion device is one which converts energy from electrical to mechanical or from mechanical to electrical. The coupling between the electrical and mechanical systems is through the magnetic field. In a transformer also, the coupling between the primary and secondary windings is by means of the magnetic field. Both in electromechanical energy conversion devices and transformers, the coupling magnetic field behaves in a like manner. Therefore, the fundamental principles involved in the analysis of a transformer are much more common in the analysis of electromechanical energy conversion devices.

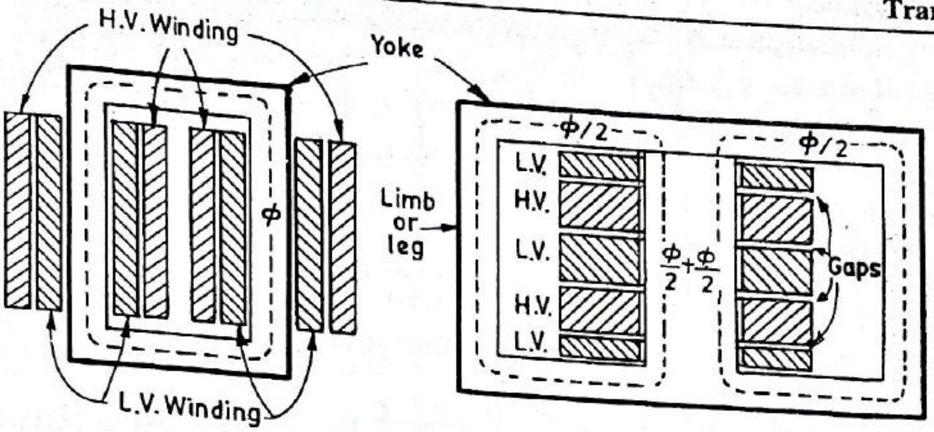
The transformer is a static piece of electric machinery and concepts about its behaviour can be understood in a comparatively simpler manner. In view of the above, the analysis of transformer must serve as a prelude to the study of electromechanical energy conversion devices. At the same time, a transformer is an important energy conversion device and detailed study of its behaviour is justified.

1.1. Transformer Construction

There are two general types of transformers, the core type and the shell type. These two types differ from each other by the manner in which the windings are wound around the magnetic core.

The magnetic core is a stack of thin silicon-steel laminations about 0.35 mm thick for 50 Hz transformers. In order to reduce the eddy current losses, these laminations are insulated from one another by thin layers of varnish. For reducing the core losses, nearly all transformers have their magnetic core made from cold-rolled grain-oriented sheet-steel (C.R.G.O.). This material, when magnetized in the rolling direction, has low core loss and high permeability.

In the core-type, the windings surround a considerable part of steel core as shown in Fig. 1.1 (*a*). In the shell-type, the steel core surrounds a major part of the windings as shown in Fig. 1.1 (*b*). For a given output and voltage rating, core-type transformer requires less iron but more conductor material as compared to a shell-type transformer. The vertical portions of the core are usually called limbs or legs and the tcp and bottom portions are called the yoke. This means that for single-phase transformers, core-type has two-legged core whereas shell-type has three-legged core.



(a) (b)

Fig. 1.1. Constructional details of single-phase (a) core-type transformer (b) shell-type transformer.

In iron-core transformers, most of the flux is confined to high permeability core. There is, however, some flux that leaks through the core legs and non-magnetic material surrounding the core. This flux, called *leakage flux*, links one winding and not the other. A reduction in this leakage flux is desirable as it improves the transformer performance considerably. Consequently, an effort is always made to reduce it. In the core-type transformer, this is achieved by placing half of the low voltage (L.V.) winding over one leg and other half over the second leg or limb. For the high voltage winding also, half of the winding is over one leg and the other half over the second leg, Fig. 1.1 (*a*). L.V. winding is placed adjacent to the steel core and H.V. winding outside, in order to minimise the amount of insulation required.

In the shell type transformer, the L.V. and H.V. windings are wound over the central limb and are interleaved or sandwiched as shown in Fig. 1.1 (*b*). Note that the bottom and top L.V. coils are of half the size of other L.V. coils. Shell-type transformers are preferred for low-voltage low-power levels, whereas core-type construction is used for high-voltage, high-power transformers.

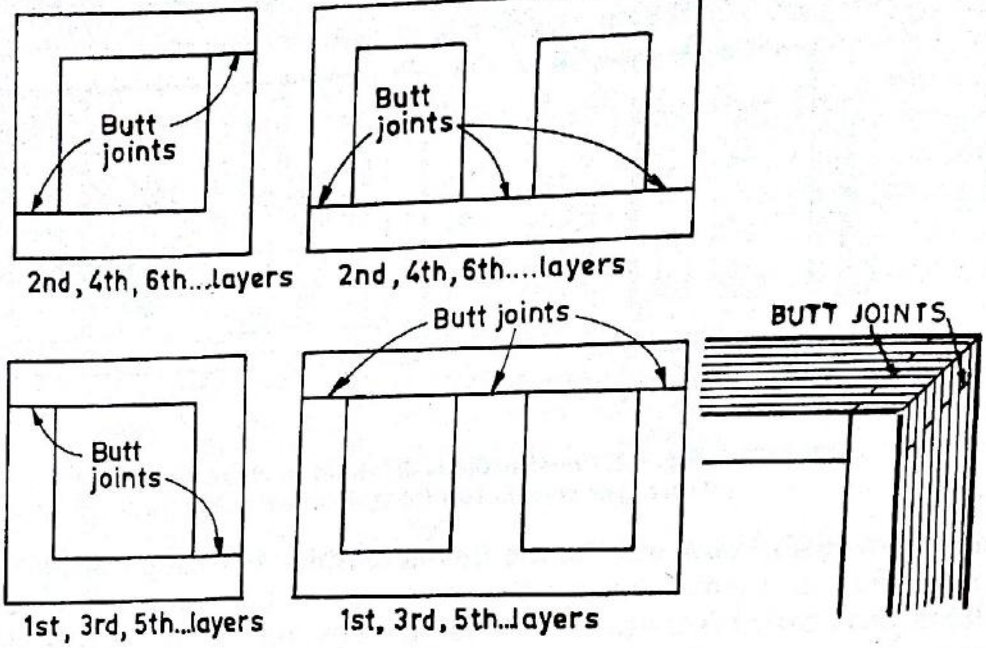
In core-type transformer, the flux has a single path around the legs or yokes, Fig. 1.1 (*a*). In the shell-type transformer, the flux in the central limb divides equally and returns through the outer two legs as shown in Fig. 1.1 (*b*).

There are two types of windings employed for transformers. The concentric coils are used for core-type transformers as shown in Fig. 1.1 (*a*) and interleaved (or sandwiched) coils for shell-type transformers as shown in Fig. 1.1 (*b*).

One type of laminations for the core and shell type of transformers is illustrated in Fig. 1.2 (*a*) and (*b*) respectively. The steel core is assembled in such a manner that the butt joints in adjacent layers are staggered as illustrated in Fig. 1.2 (*c*). The staggering of the butt joints avoids continuous air gap and, therefore, the reluctance of the magnetic circuit is not increased. At the same time, a continuous air gap would reduce the mechanical strength of the core and, therefore, the staggering of the butt joints is essential.

During the transformer construction, first the primary and secondary windings are wound, then the laminations are pushed through the coil openings, layer by layer and the steel core is prepared. The laminations are then tightened by means of clamps and bolts.

Low power transformers are air-cooled whereas large power transformers are immersed in oil for better cooling. In oil-cooled transformers, the oil serves as a coolant and also as an insulating medium.



(*a*) (*b*) (*c*)

Fig. 1.2. Two adjacent layers for (*a*) core and (*b*) shell type of transformers (*c*) arrangement of butt joints in a magnetic core.

For power frequency range of 25 to 400 Hz, transformers are constructed with 0.35 mm thick silicon-steel laminations. For audio-frequency range of 20 to 20,000 Hz, iron core with suitable refinements is used. For high frequencies employed in communication circuits, core is made up of powdered ferromagnetic alloy. In special cases, the magnetic circuit of a transformer may be made of non-magnetic material and in such a case, the transformer is referred to as an *air-core transformer*. The air-core transformer is primarily used in radio devices and in certain types of measuring and testing instruments. Cores made of soft ferrites are also used for pulse transformers as well as for high frequency electronic transformers.

1.2. Principle of Transformer Action

A transformer works on the principle of electromagnetic induction between two (or more) coupled circuits or coils. According to this principle, an e.m.f. is induced in a coil if it links a changing flux.

In core-type transformer, half of the L.V. (and H.V.) winding is on one limb and the other half is on the second limb. In shell-type transformer, the L.V. and H.V. windings are sandwiched. However, for simplifying the drawing and analysis of both these types of transformers, the schematic diagram is as shown in Fig. 1.3. The primary winding P is connected to an alternating voltage source, therefore, an alternating current I_e starts flowing through N_1 turns. The alternating mmf $N_1 I_e$ sets up alternating flux ϕ which is confined to the high permeability iron path as indicated in Fig. 1.3. The alternating flux induces voltage E_1 in the primary P and E_2 in the secondary S. If the load is connected across the secondary, a load current starts flowing.

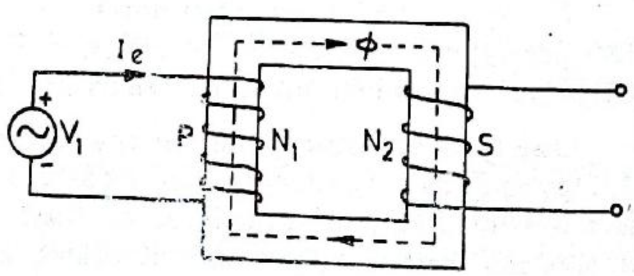


Fig. 1.3. Schematic diagram of a two-winding transformer.

In addition to the secondary winding, there may be a third (or tertiary) winding on the same iron core. The emf induced in the secondary or tertiary winding is usually referred to as the emf due to transformer action. Thus the transformer action requires the existence of alternating mutual flux linking the various windings on a common magnetic core.

A transformer having primary and secondary windings is called a two-winding transformer whereas a transformer having primary, secondary and tertiary windings is known as a three-winding transformer. As stated before, primary is connected to source whereas the secondary and tertiary windings feed the load.

1.3. Ideal Two-winding Transformer

In the beginning, a transformer is assumed to be an ideal one, merely for obtaining an easier explanation of what happens in a transformer. For a transformer to be an ideal one, the various assumptions are as follows :

1. Winding resistances are negligible.
2. All the flux set up by the primary links the secondary windings, *i.e.* all the flux is confined to the magnetic core.
3. The core losses (hysteresis and eddy current losses) are negligible.
4. The core has constant permeability, *i.e.* the magnetization curve for the core is linear.

At a later stage, the effect of these assumptions will be taken up one by one.

It has been stated before that the words primary and secondary should not be used with the two windings of transformer. However, it has been found convenient to use these terms during the transformer analysis. But it should be kept in mind that these are arbitrary terms as explained before. Hence forth, sub-scripts 1 and 2 would be associated respectively with the primary and secondary windings of a transformer.

Let the voltage V_1 applied to the primary of a transformer, with secondary open-circuited, be sinusoidal (or sine wave). Then the current I_e, due to applied voltage V_1, will also be a sine wave. The mmf $N_1 I_e$ and, therefore, the core flux ϕ will follow the variations of I_e very closely. That is, the flux ϕ is in time phase with the current I_e and varies sinusoidally. If I_e is zero, ϕ is zero and if I_e is maximum positive, ϕ is also maximum positive and so on. Therefore, if the applied voltage V_1 has sine waveform, the flux ϕ must also have a sine waveform. Let the sinusoidal variation of flux ϕ be expressed as

$$\phi = \phi_{max} \sin \omega t \qquad \text{...(1.1)}$$

where ϕ_{max} is the maximum value of the magnetic flux in webers and $\omega = 2\pi f$, is the angular frequency in rad/sec and f is the supply frequency in Hz.

The emf e_1 in volts, induced in the primary N_1 turns by the alternating flux ϕ is given by

$$e_1 = -N_1 \frac{d\phi}{dt} \qquad \text{...(1.2)}$$

$$= -N_1 \omega \phi_{max} \cos \omega t$$

$$= N_1 \omega \phi_{max} \sin\left(\omega t - \frac{\pi}{2}\right)$$

Its maximum value, $E_{1\,max}$ occurs when $\sin\left(\omega t - \frac{\pi}{2}\right)$ is equal to 1.

$\therefore$

$$E_{1\,max} = N_1 \omega \phi_{max}$$

and

$$e_1 = E_{1\,max} \sin\left(\omega t - \frac{\pi}{2}\right) \qquad \text{...(1.3)}$$

∴ Rms value of emf E_1 induced in primary winding is given by

$$E_1 = \frac{E_{1\,max}}{\sqrt{2}} = \frac{2\pi}{\sqrt{2}} f N_1 \phi_{max}$$

$$E_1 = \sqrt{2}\,\pi f N_1\,\phi_{max} \qquad ...(1.4)$$

The current I_e in the primary is assumed to flow along the path *abcda*, Fig. 1.4. The e.m.f. e_1 induced in N_1 turns must be in such a direction as to oppose the cause, *i.e.* I_e; as per Lenz's law. Therefore, the direction of e_1 is as shown by the arrows in the primary N_1 turns and it is seen to oppose v_1. Since primary winding resistance is negligible, e_1 at every instant, must be equal and opposite to v_1

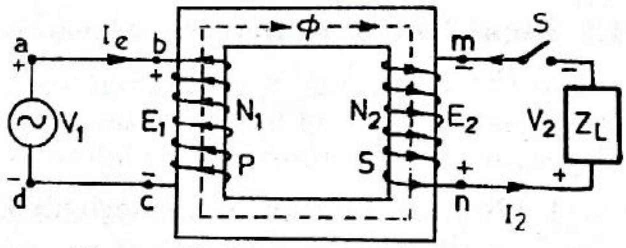


Fig. 1.4. Ideal transformer on load.

i.e. $$v_1 = -e_1 = N_1 \frac{d\phi}{dt}$$

or $$V_1 = -E_1 \qquad ...(1.5)$$

The emf induced in the secondary is

$$e_2 = -N_2 \frac{d\phi}{dt} = -N_2\,\omega\phi_{max}\ \cos\omega t$$

$$= N_2\omega\phi_{max}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= E_{2\,max}\sin\left(\omega t - \frac{\pi}{2}\right) \qquad ...(1.6)$$

∴ Rms value of emf E_2 induced in secondary winding is given by

$$E_2 = \frac{E_{2\,max}}{\sqrt{2}} = \sqrt{2}\,\pi f N_2\phi_{max} \qquad ...(1.7)$$

From Eqs. (1.4) and (1.7),

$$\frac{E_1}{E_2} = \frac{N_1}{N_2}$$

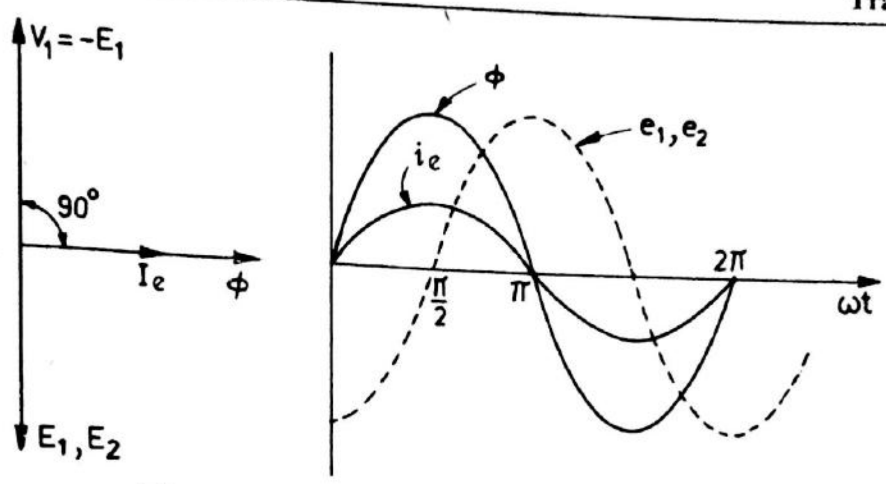
or

$$\frac{E_1}{N_1} = \frac{E_2}{N_2} = \sqrt{2}\pi f\,\phi_{max} \qquad ...(1.8)$$

i.e. emf per turn in primary = emf per turn in the secondary.

The relation expressed by Eq. (1.8) is very significant and must always be kept in mind.

For drawing the phasor diagram of an ideal transformer at no load, the waveforms of ϕ, e_1 and e_2 are drawn in Fig. 1.5 (*b*), with the help of Eqs. (1.1), (1.3) and (1.6) respectively. At time $t = 0$, the flux is zero, therefore, it is drawn horizontally, Fig. 1.5 (*a*). Note that the vertical projection of a phasor in the phasor diagram must be equal to its value in the time diagram. The values of e_1 and e_2 are maximum negative at $t = 0$, these are therefore, drawn downward along the vertical axis. Here N_1 and N_2 are assumed equal for convenience and, therefore, $E_1 = E_2$. The applied voltage V_1 is equal and opposite to E_1 and it is accordingly drawn opposing E_1. It is seen from Fig. 1.5, that e.m.fs. E_1 and E_2 lag by 90° the mutual flux ϕ that induces them. The applied voltage V_1 leads the flux ϕ by 90°.



(a) (b)
Fig. 1.5. Ideal transformer (a) phasor diagram and (b) time diagram.

In Fig. 1.4, if the switch S is closed, a load impedance Z_L gets connected across the secondary terminals. Since the secondary winding resistance is zero, $V_2 = E_2$. According to Lenz's law, the direction of secondary current I_2 should be such that the secondary mmf F_2 $(= I_2N_2)$ is opposite to mutual flux ϕ in the core. For F_2 to be directed against ϕ, the current I_2 must leave the terminal n, pass through the load and enter the terminal m, Fig. 1.4. The secondary winding behaves like a voltage source, therefore, terminal n must be treated as positive and terminal m as negative. This means that when terminal b is positive with respect to c in Fig. 1.4, terminal n is positive with respect to m at the same time. This forms the basis for polarity markings in transformers, Art. 1.11. If secondary winding is wound in a manner opposite to that shown in Fig. 1.4, terminal m would be positive with respect to terminal n. This shows that polarity markings of the windings in transformers depend upon the manner in which the windings are wound around the legs with respect to each other.

In Fig. 1.4, the secondary mmf F_2 being opposite to ϕ, tends to reduce the alternating mutual flux ϕ. Any reduction in ϕ would reduce E_1. For an ideal transformer, $V_1 = -E_1$. If the applied voltage V_1 is constant, E_1 and, therefore, mutual flux ϕ in the core must remain constant, as per Eq. 1.4. This can happen only if the primary draws more current I_1' from the source, in order to neutralise the demagnetizing effect of F_2. In this manner, I_2 causes the primary to take more current, I_1', in addition to I_e such that

$$I_1' N_1 = I_2 N_2$$

or Compensating primary mmf, $F_1 =$ Secondary mmf, F_2 ...(1.9)

Any change in the secondary current is at once reflected by a corresponding automatic change in the primary current so that core flux remains unaltered.

In the above expression, I_1' is called the *load component* of primary current I_1. It is thus seen that core flux in an ideal transformer remains constant and is independent of the load current.

Assuming I_2 to lag behind V_2 by an angle θ_2, the phasor diagram under load for an ideal transformer can be drawn as shown in Fig. 1.6. Since mmfs F_1 and F_2 tend to magnetize the core in opposite directions, they are shown in phase opposition in Fig. 1.6.

The total primary current I_1 is the phasor sum of I_1 and I_e, *i.e.*,

$$\bar{I}_1 = \bar{I}_1' + \bar{I}_e$$

The power factor on the primary side of the ideal transformer is $\cos \theta_1$.

If the magnetizing current I_e is neglected, then Eq. (1.9) becomes

$$I_1 N_1 = I_2 N_2 \quad ...(1.10)$$

i.e. Primary ampere-turns
= Secondary ampere-turns.

Thus for an ideal transformer with $I_e = 0$, we have

$$\frac{V_1}{V_2} = \frac{E_1}{E_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1} \quad ...(1.11)$$

and $\quad E_1 I_1 = E_2 I_2$

or $\quad V_1 I_1 = V_2 I_2 \quad ...(1.12)$

i.e. Primary volt-amperes = Secondary volt-amperes.

In Eqs. (1.11) and (1.12); V_1, V_2, I_1, I_2 have rms values.

From Eq. (1.5),

$$v_1 = N_1 \frac{d\phi}{dt}$$

Similarly $\quad v_2 = N_2 \frac{d\phi}{dt}$

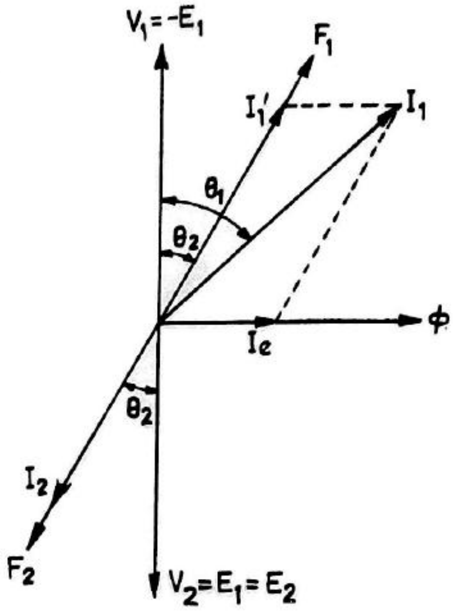


Fig. 1.6. Phasor diagram of an ideal transformer for inductive load.

$$\therefore \quad \frac{v_1}{N_1} = \frac{v_2}{N_2} \quad ...(1.12\ a)$$

Eq. (1.10) in terms of instantaneous value is

$$i_1 N_1 = i_2 N_2 \quad ...(1.12\ b)$$

Multiplication of Eqs. (1.12 *a*) and (1.12 *b*) gives,

$$v_1 i_1 = v_2 i_2 \quad ...(1.13)$$

This means that instantaneous power input into primary equals the instantaneous power output from the secondary. This relation is a consequence of the assumptions (1)–(3) made for an ideal transformer.

If Kirchhoff's voltage law is applied to the primary winding circuit *abcda* of Fig. 1.4, then terminal *b* must be positive with respect to terminal *c*, since current I_e can flow from a high potential to a lower potential only.

∴ For the circuit *abcda*, the Kirchhoff's voltage law gives,

$$v_1 - e_1 = 0$$

$$v_1 = e_1$$

From Eq. (1.5), $\quad v_1 = N_1 \frac{d\phi}{dt}, \quad \therefore \quad e_1 = N_1 \frac{d\phi}{dt} \quad ...(1.14)$

Here the e.m.f. e_1 is treated as a voltage drop in the direction of current I_e and its instantaneous value is given by Eq. 1.14.

In Fig. 1.4, current I_e flows from *b* to *c*, through the primary winding. If I_e and therefore, flux ϕ is increasing, then e.m.f. e_1 induced in the primary should be in such a direction that if e_1 acted alone, it would establish a current opposite to I_e as per Lenz's law. Accordingly, the direction of e_1 is indicated by arrows in the primary winding, terminal *b* is again positive with respect to terminal c and e_1 is seen to be acting opposite to v_1. This physical fact is written in mathematical form as

$$v_1 = -e_1 = N_1 \frac{d\phi}{dt}$$

where e_1 is treated as a reaction e.m.f., counter e.m.f. or generated e.m.f.

The approach which led to the relation $e_1 = N_1 \frac{d\phi}{dt}$ is usually referred to as the circuit view point. The second approach which gave the relation $e_1 = -N_1 \frac{d\phi}{dt}$ is referred to as the field or flux view-point. Any of the two view-points may be followed. Since the field view-point gives better physical concepts of the internal behaviour of a transformer, it is adopted in this book.

Impedance Transformation. A schematic diagram for an ideal transformer is shown in Fig. 1.7 (*a*). A load impedance Z_2 is connected across the secondary side. Dot points marked on the primary and secondary windings indicate terminals of the corresponding polarity at any instant. For example, if dotted terminal of primary winding is positive with respect to undotted terminal at any instant, then dotted terminal of secondary winding is also positive with respect to undotted terminal at the same instant.

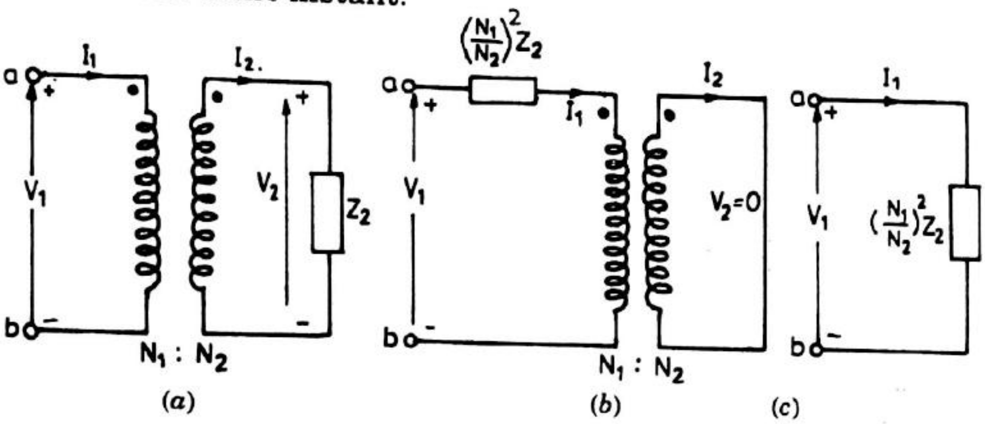


Fig. 1.7. Transfer of impedance in an ideal transformer.

For the secondary circuit, $\frac{V_2}{I_2} = Z_2$, load impedance,

For an ideal transformer, from Eq. (1.11),

$$\frac{V_1}{N_1} = \frac{V_2}{N_2} \text{ and } I_1 N_1 = I_2 N_2$$

or

$$V_1 = \frac{N_1}{N_2} V_2 \qquad \text{....(1.15)}$$

and

$$I_1 = \frac{N_2}{N_1} \cdot I_2 \qquad \text{...(1.16)}$$

Division of Eq. (1.15) by Eq. (1.16) gives effective input impedance at the primary terminals a and b as

$$\frac{V_1}{I_1} = \frac{N_1 \cdot V_2}{N_2} \cdot \frac{N_1}{N_2 I_2}$$

$$= \left(\frac{N_1}{N_2}\right)^2 \cdot \frac{V_2}{I_2} = \left(\frac{N_1}{N_2}\right)^2 Z_2 = Z_2' \qquad \text{...(1.17)}$$

This shows that as far as effect of Z_2 on primary side is concerned, impedance Z_2 in the secondary circuit can be replaced by an equivalent impedance Z_2' in the primary circuit in case

$$Z_2' = \left(\frac{N_1}{N_2}\right)^2 Z_2$$

This is shown in Figs. 1.7 (*b*) and (*c*).

Thus, the three circuits shown in Fig. 1.7 are identical as far as their performance viewed from terminals *a* and *b* is concerned.

Similarly, an impedance Z_1 in the primary circuit can be transferred to (or *referred to*) secondary side as

$$\left(\frac{N_2}{N_1}\right)^2 Z_1 = Z_1'$$

Transferring an impedance from one side of a transformer to the other is called *referring the impedance* to the other side. Similarly, voltages and currents can be referred to either side of transformer by means of Eq. (1.11). In this manner, resultant voltage and current on that side can be evaluated.

For an ideal transformer, it may be summarized that (*i*) voltages are transferred in the direct ratio, (*ii*) currents in the reverse ratio, (*iii*) impedances in the direct ratio squared and (*iv*) power and volt-amperes remain unchanged.

Eq. (1.17) illustrates the impedance-modifying property of a transformer. In practice, this property is exploited for matching a fixed load impedance to the source impedance for the purpose of maximum power transfer from source to load. This is achieved by interposing a transformer of suitable turns ratio between the load and the source.

1.4. Transformer Phasor Diagrams

The purpose of first considering an ideal transformer, *i.e.* a transformer with no core losses, no winding resistances, no magnetic leakage and constant permeability, is merely to highlight the most important aspects of transformer action. Such a transformer never exists and now the phasor diagrams of real transformer with various imperfections will be considered.

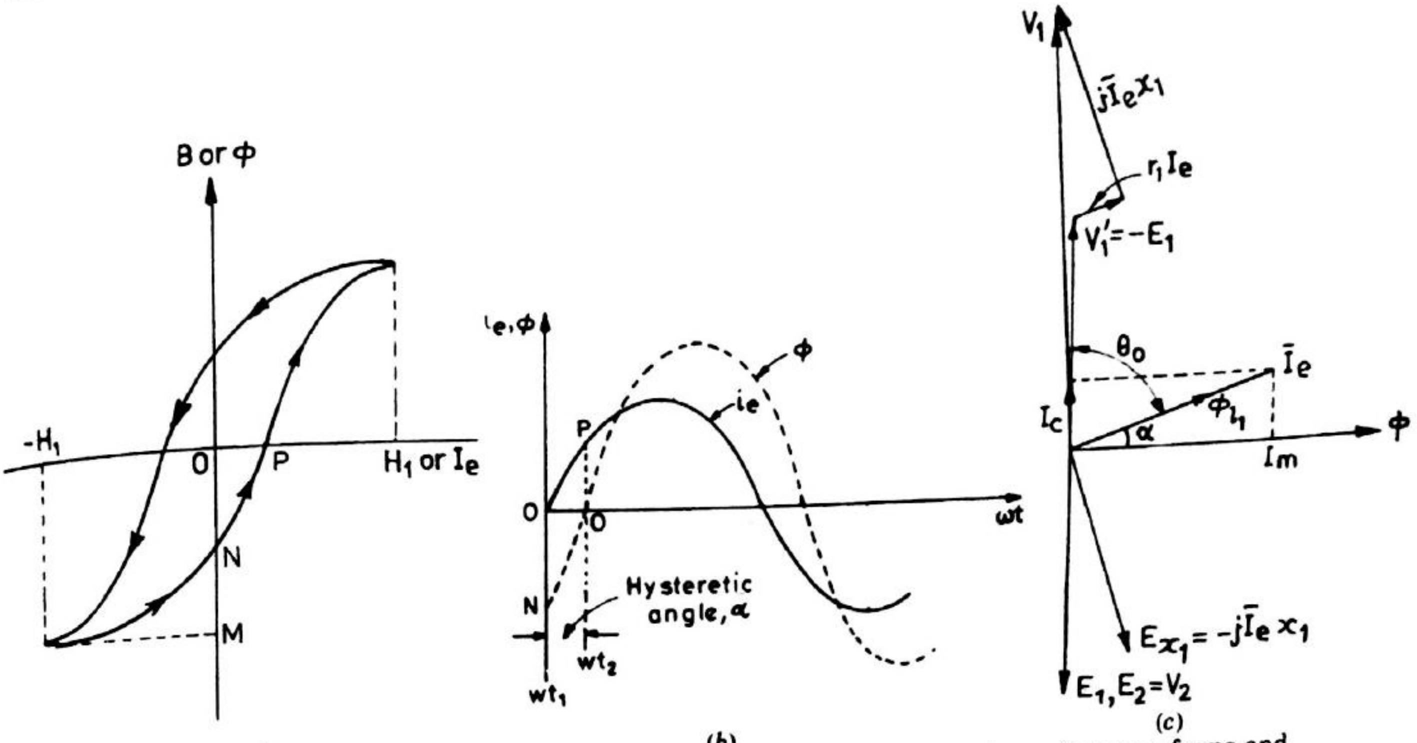
Magnetization curve of the actual transformer core is non-linear and its effect is to introduce higher order harmonics in the magnetizing current. Since all the quantities in a phasor diagram must be of the same frequency, these higher order harmonics (whose frequencies are odd multiples of fundamental frequency) can't be represented in the phasor diagram. So a linear magnetization curve for the transformer core will continue to be assumed.

The phasor diagram of a transformer is now developed, first at no load and then under load.

1.4.1. Transformer phasor diagram at no load. The magnetic flux ϕ being common to both the primary and secondary, is drawn first. The induced emfs E_1 and E_2 lag ϕ by 90° and are shown accordingly in Fig. 1.8 (*b*). The voltage $-E_1$ is being replaced by V_1' just for convenience. Alternatively V_1' may be treated as a voltage drop in the primary, in the direction of flow of primary current. The various imperfections in a real transformer are now considered one by one.

(*a*) *Effect of transformer core loss.* The core loss (or iron loss) consists of hysteresis loss and eddy current loss. These losses are always present in the ferromagnetic core of the transformer, since the transformer is an ac-operated magnetic device. The hysteresis loss in the core is minimised by using high grade material such as cold-rolled-grain oriented (CRGO) steel and the eddy current loss is minimised by using thin laminations for the core.

The current in the primary is alternating, therefore, the magnetizing force H is cyclically varying from one positive value say H_1 to a corresponding negative value $-H_1$, Fig. 1.8 (*a*).



(a) (b) (c)
Fig. 1.8. (a) Hysteresis loop for transformer core (b) exciting current and core flux waveforms and (c) no-load phasor diagram of a transformer.

When the magnetizing force is $-H_1$, the flux density is maximum negative equal to *OM*. As the magnetizing force decreases from $-H_1$, the current I_e decreases and becomes zero for a flux density, or flux, equal to *ON*. When the current I_e becomes positive and equal to *OP*, the flux is reduced to zero but it is going to become positive. The traverse of the loop along the arrows involves time. When i_e is crossing zero positive (passing through zero and becoming positive), the core flux is negative and is equal to *ON* in Fig. 1.8 (*a*). This is shown in Fig. 1.8 (*b*) at instant wt_1, where waveforms are assumed sine waves. When i_e is positive and equal to *OP*, Fig. 1.8 (*a*), the flux is crossing zero and becoming positive ; this is shown in Fig. 1.8 (*b*) at instant wt_2. It is seen from Fig. 1.8 (*b*) that exciting current i_e leads the magnetic flux ϕ (or ϕ lags i_e) by some time angle α. This angle of lead, or lag, being dependent on the hysteresis loop, is called the *hysteretic angle*. In Fig. 1.8 (c), I_e is shown leading ϕ, or ϕ is shown lagging I_e, by hysteretic angle α.

The no-load primary current I_e is called the *exciting current* of the transformer and can be resolved into two components. The component I_m along ϕ is called the *reactive* or *magnetizing current*, since its function is to provide the required magnetic flux ϕ. The second component along V_1' is I_c and this component is called the *core-loss component*, or *power component*, of I_e; since I_c when multiplied by V_1' gives the total core loss P_c.

$$\therefore \quad V_1' I_c = P_c \quad \text{or} \quad I_c = \frac{P_c}{V_1'} \text{ Amp.} \qquad ...(1.18)$$

From Fig. 1.7 (*b*), it is seen that

$$I_e = \sqrt{I_m^2 + I_c^2}$$

Note that in an ideal transformer, core-loss current $I_c = 0$ and therefore exciting current I_e = magnetizing current I_m.

(*b*) *Effect of transformer resistance.* The effect of primary resistance r_1 can be accounted for, by adding to V_1', a voltage drop equal to $r_1 I_e$, as shown in Fig. 1.8 (*b*). Note that $r_1 I_e$ is in phase with I_e and is drawn parallel to I_e in the phasor diagram.

(*c*) *Effect of leakage flux.* The existence of electric potential difference is essential for the establishment of current in an electric circuit. Similarly the magnetic potential difference is necessary for the establishment of flux in a magnetic circuit.

For the direction of current I_e in the primary, Fig. 1.9, the point A is at a higher magnetic potential than point B. This magnetic potential difference establishes : (*i*) the mutual flux ϕ linking both the windings and (*ii*) the primary leakage flux ϕ_{l1}, which links only the primary winding. The distinctive behaviour of the mutual flux ϕ and the primary leakage flux ϕ_{l1} must be carefully understood. The mutual flux ϕ exists entirely in the ferromagnetic core and, therefore, involves hysteresis loop. The current I_e that establishes ϕ must lead it by some hysteretic angle. On the other hand, the primary leakage flux ϕ_{l1} exists largely in air. Although ϕ_{l1} does pass through some iron, the reluctance offered to ϕ_{l1} is mainly due to air. Consequently ϕ_{l1} does not involve any hysteresis loop and it can be taken to be in phase with the current I_e that produces it, Fig. 1.8 (*c*). In the primary winding, ϕ induces an emf E_1 lagging it by 90°; similarly the primary leakage flux ϕ_{l1} induces an e.m.f. E_{x1} in the primary winding and lagging it (*i.e.* ϕ_{l1}) by 90°. Since I_e leads E_{x1} by 90°, it is possible to write $\overline{E}_{x1} = -j\,\overline{I}_e\,x_1$. The primary applied voltage V_1 must have a component $j\,\overline{I}_e\,x_1$, equal and opposite to E_{x1}. Here x_1 has the nature of reactance and is referred to as the primary leakage reactance in ohms. It may be noted that x_1 is a fictitious quantity merely introduced to represent the effects of primary leakage flux.

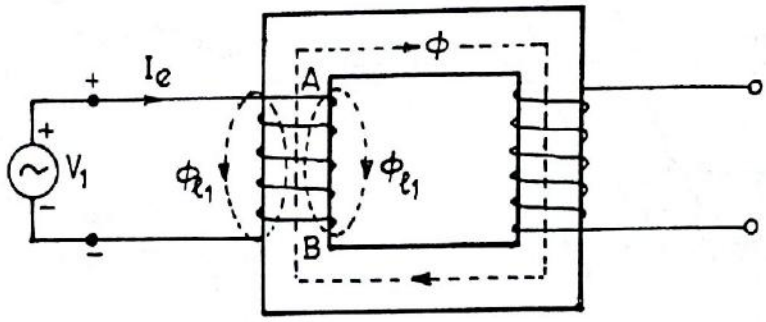


Fig. 1.9. Transformer at no load.

The total voltage drop in primary at no load is $I_e(r_1 + jx_1) = I_e\,z_1$, where z_1 is the primary leakage impedance. Therefore, Fig. 1.8 (*c*) gives the phasor diagram of a transformer at no load, where N_1 is assumed to be equal to N_2. The primary voltage equation at no load can be written as

$$\overline{V}_1 = \overline{V}_1' + \overline{I}_e\,(r_1 + jx_1) \quad \text{...(1.19)}$$

The primary leakage impedance drop shown in Fig. 1.8 (*c*), is drawn to a larger scale, in comparison with V_1' or V_1, just for the sake of clarity. At no load V_1' and V_1 are very nearly equal. Even at full load, primary leakage impedance drop in power transformers, is about 2 to 5% of V_1, so that the magnitude of V_1' or E_1 (and therefore, ϕ as per Eq. (1.4)) does not change appreciably from no load to full load.

It may be noted that the total primary flux is the phasor sum of ϕ_{l1} and ϕ, therefore, its phasor is a little ahead of ϕ.

1.4.2. Transformer phasor diagram under load. The secondary circuit of the transformer is considered first and then the primary circuit, for developing the phasor diagram of a transformer under load.

When switch S is closed, secondary current I_2 starts flowing from terminal n to the load, as described in Art. 1.3 (Fig. 1.4). Assume the load to have a lagging power factor so that I_2 lags

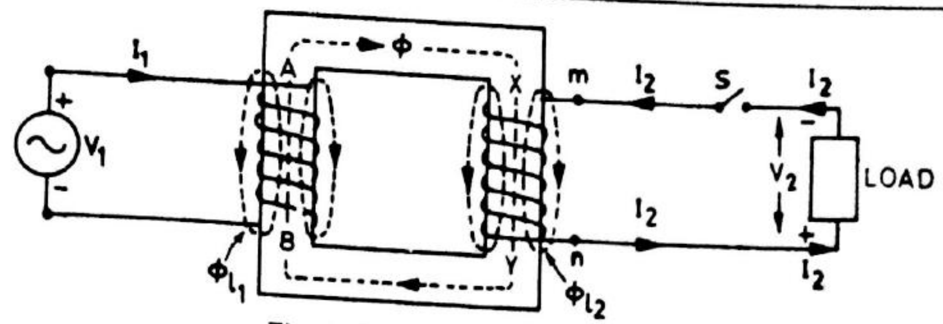


Fig. 1.10. Transformer under load.

secondary load voltage V_2 by an angle θ_2. At first V_2 is drawn with I_2 lagging V_2 by the secondary p.f. angle θ_2, Fig. 1.11 (*a*). The secondary resistance drop is accounted for, by drawing $r_2 I_2$ parallel to I_2. The secondary m.m.f. $I_2 N_2$ gives rise to a leakage flux ϕ_{l2} which links only the secondary and not the primary. The flux ϕ_{l2} is called the secondary leakage flux and is in phase with I_2, for the same reason that ϕ_{l1} is in phase with I_e in Fig. 1.8 (*c*). The secondary leakage flux induces e.m.f. E_{x2} in the secondary winding, lagging ϕ_{l2} by 90°. The secondary no load voltage E_2 must have a component equal and opposite to $-jx_2 \bar{I}_2$. Thus the phasor sum of $\bar{V}_2$, $\bar{I}_2 r_2$ and $j\bar{I}_2 x_2$ gives the secondary induced e.m.f. E_2 as shown in Fig. 1.11 (*a*).

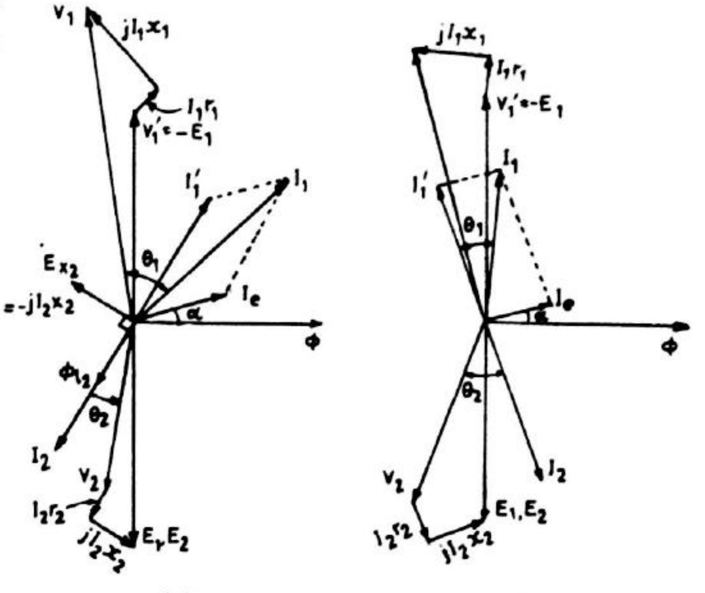


Fig. 1.11. Transformer phasor diagram for (*a*) lagging p.f. load and (*b*) leading p.f. load.

The voltage equation for the secondary circuit can now be written as

$$\bar{E}_2 = \bar{V}_2 + \bar{I}_2 (r_2 + jx_2) = \bar{V}_2 + \bar{I}_2 z_2 \qquad ...(1.20)$$

where z_2 is the secondary leakage impedance of the transformer.

Further the mutual flux ϕ is drawn leading E_2 by 90° and exciting current I_e is drawn leading ϕ by the hysteretic angle α. Note that the phasor V_2 has purposely been taken to the left of vertical line, so that E_2 is vertically downward and the mutual flux ϕ is horizontal.

The component of the primary current which neutralises the demagnetizing effect of I_2 is I_1' $(I_1' N_1 = I_2 N_2)$ and is drawn opposite to I_2. The phasor sum of I_1' and I_e gives the total primary current I_1 taken from the supply mains. The primary leakage impedance drop $\bar{I}_1(r_1 + jx_1)$ is depicted as explained earlier. The voltage equation for the primary circuit under load can be written as

$$\bar{V}_1 = \bar{V}_1' + \bar{I}_1 (r_1 + jx_1) = \bar{V}_1' + \bar{I}_1 z_1 \qquad ...(1.21)$$

where z_1 is the primary leakage impedance of the transformer. Note that the angle θ_1 between V_1 and I_1 is the primary power-factor angle under load.

If the secondary load current I_2 leads the voltage V_2 such that the load p.f. is leading, then the phasor diagram for the transformer is as shown in Fig. 1.11 (*b*). The entire procedure for drawing the phasor diagram is the same as explained for Fig. 1.11 (*a*).

It may be seen that the development of transformer phasor diagram of Fig. 1.11, gives a better physical picture of what happens in the primary and secondary windings of a transformer and its core. But this phasor diagram is helpful only (*i*) when a transformer is to be studied alone and (*ii*) when the internal behaviour of the transformer is to be understood.

When the transformer is a part of the large power system network, the phasor diagram of Fig. 1.11 should not be used. Instead, the transformer equivalent circuit is used.

1.4.3. Leakage flux. In a transformer, as secondary (or load) current is increased, the magnetic potential of point X rises above the magnetic potential of point Y, Fig. 1.10. This results in an increase in the secondary leakage flux ϕ_{l2}. Here points X and Y are on the limb where secondary winding is wound. With increase of secondary current, the primary current also rises and this causes point A to attain a magnetic potential higher than that of point B. As a result, primary leakage flux ϕ_{l1} increases. This shows that leakage fluxes in a transformer are dependent upon the currents in the windings.

Core flux in a transformer depends upon the emf induced in the primary winding. With increase of primary current, $E_1 = V_1 - I_1 (r_1 + jx_1)$ does reduce and likewise core flux is reduced. But this reduction in E_1 and likewise in core flux is quite small. Thus it may be stated that core flux in a transformer depends upon the applied voltage and may be treated as constant from no load to full load.

In the following article, the rating of transformers is discussed. After this, an exact equivalent circuit of the transformer is developed first from which its approximate equivalent circuit is obtained.

1.5. Rating of Transformers

The manufacturer of transformers fixes a name plate on the transformer, on which are recorded the rated output, the rated voltages, the rated frequency etc. of a particular transformer. A typical name plate rating of a single phase transformer is as follows: 20 kVA, 3300/220 V, 50 Hz. Here 20 kVA is the rated output at the secondary terminals.* Note that the rated output is expressed in kilo-volt-amperes (kVA) rather than in kilowatts (kW). This is due to the fact that rated transformer output is limited by heating and hence by the losses in the transformer. The two types of losses in a transformer are core loss and ohmic ($I^2 r$) loss. The core loss depends on transformer voltage and ohmic loss on the transformer current. As these losses depend on transformer voltage (V) and current (I) and are almost unaffected by the load pf, the transformer rated output is expressed in VA ($V \times I$) or in kVA and not in kW. For example, a transformer working on rated voltage and rated current with load pf equal to zero has rated losses and rated kVA output but delivers zero power to load. This shows that transformer rating must be expressed in kVA.

For any transformer :

$$\begin{Bmatrix}\text{(Rated input in kVA at} \\ \text{the primary terminals)} \\ (\cos\theta_1)\end{Bmatrix} = \begin{Bmatrix}\text{(Rated output in kVA at} \\ \text{the secondary terminals)} \\ (\cos\theta_2)\end{Bmatrix} + \text{Losses}$$

Since the transformer operates at a very high efficiency, losses may be ignored. Further, the primary p.f. $\cos\theta_1$ and the secondary p.f. $\cos\theta_2$ are nearly equal. Therefore, the rated kVA marked on the nameplate of a transformer, refers to both the windings, *i.e.* the rated kVA of the primary winding and the secondary winding are equal.

The voltage 3300/220 V refers to the design voltages of the two windings. Either of the two may serve as primary or secondary. If it is a step down transformer, then 3300 V is the rated

* A terminal is that part of an electrical engineering device, which is intended to receive the external connections.

primary voltage and refers to the voltage applied to the primary winding. The voltage of 220 V is the rated secondary voltage and refers to the voltage developed between output terminals at no load, with rated voltage applied to the primary terminals.

Rated primary and secondary currents are calculated from the rated kVA and the corresponding rated voltages. Thus

$$\text{Rated (or full-load) primary current} = \frac{20{,}000}{3300} = 6.06 \text{ A.}$$

$$\text{Rated (or full-load) secondary current} = \frac{20{,}000}{220} = 90.91 \text{ A.}$$

Note that the rated primary and secondary currents refer to the currents for which the windings are designed.

Rated frequency refers to the frequency for which the transformer is designed to operate.

The ratios $\frac{E_1}{E_2}$ and $\frac{N_1}{N_2}$ are called the voltage ratio and turns ratio respectively. These two ratios are equal as seen from Eq. 1.8. At no load, V_1 and E_1 are nearly equal in magnitude for large transformers, therefore, their no-load voltage ratio is

$$\frac{V_1}{E_2} = \frac{N_1}{N_2}, \quad i.e. \quad \frac{\text{Rated primary voltage}}{\text{Rated secondary voltage}} = \frac{N_1}{N_2}.$$

Example 1.1. *The emf per turn for a single phase, 2310/220 V, 50 Hz transformer is approximately 13 volts. Calculate (a) the number of primary and secondary turns and (b) the net cross-sectional area of the core, for a maximum flux density of 1.4 T.*

Solution. Emf per turn $E_t = 13$ volts.

(*a*) Number of secondary turns $= \frac{\text{Secondary voltage}}{E_t}$

$$\therefore \qquad N_2 = \frac{220}{13} = 16.92.$$

Now th^ number of turns can't be a fraction, therefore, $N_2 = 17$ (nearest whole number).

For $N_2 = 17$,

Number of primary turns

$$N_1 = N_2\left(\frac{V_1}{V_2}\right) = 17\left(\frac{2310}{220}\right) = 178.5.$$

This shows that N_2 can't be equal to 17 turns. The other nearest integers are 16 or 18. It is preferable to take $N_2 = 18$.

$$\therefore \qquad N_1 = 18(10.5) = 189 \text{ turns.}$$

Thus the required values of N_1 and N_2 are 189 and 18 turns respectively.

(*b*) New value of e.m.f. per turn $E_t = \frac{220}{18}$ volts.

The net core area can be obtained from the relation,

$$\sqrt{2}\pi f \phi_{max} = E_t$$

or

$$\sqrt{2}\pi f B_m A_i = E_t = \frac{220}{18}$$

Here B_m = **maximum** value of **flux** density in Wb/m^2 or teslas and A_i = Net core area.