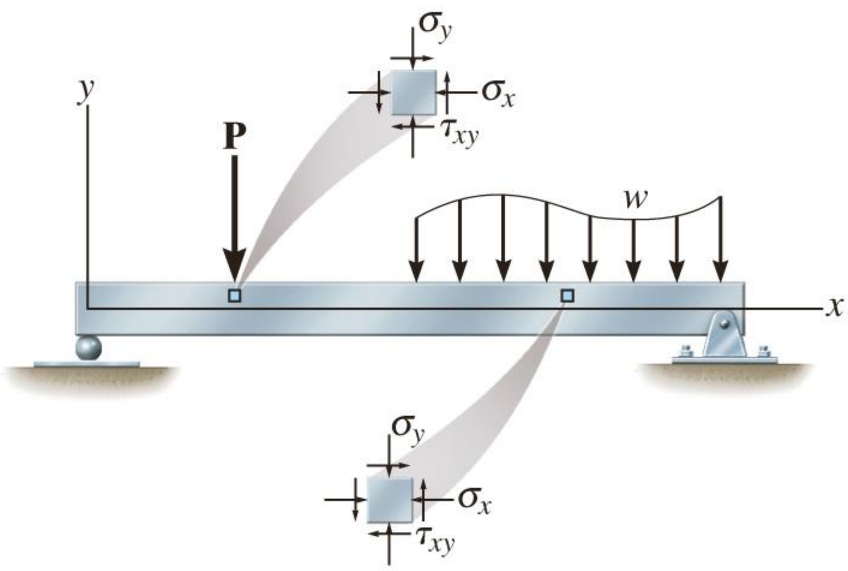
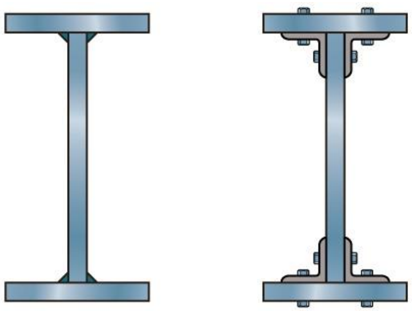
فصل ۱۱- طراحی تیرها (بخش ۱ و ۲)



یادآوری: دو تنش در تیرها مطرح است که با آنها سر و کار داریم:

تنش نرمال σ و تنش برشی τ

طراحی این تیرها چگونه متفاوت است؟

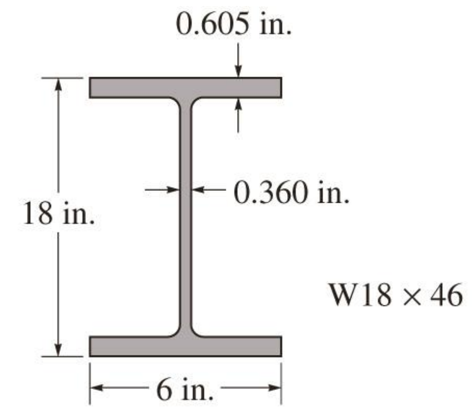


Welded　　Bolted

Steel plate girders

Wooden box beam

(a)



Glulam beam

(b)

خلاصه ای از تیرها

- تیرها عضوهای سازه ای هستد که برای تحمل بارهای عمود بر محور طولی طراحی می شوند.
- اندازه تیرها باید به گونه ای کافی باشد که تنش برشی و تنش خمشی کاهش یابد.
- تنش برشی: تیرهای چوبی، تیرهای کوتاه، تیرهای با سطح مقطع باریک (t کوچک) یا تیرهای چسبی.
- تنش خمشی: تیرهای بلند، تیرهای فلزی و ...

ادامه خلاصه تیرها

– یادآوری:

$$\sigma_{\max} = \frac{Mc}{I}$$

– اگر تعریف کنیم:

S = مدول مقطع = I/c

$$S_{req'd} = \frac{M}{\sigma_{allow}}$$

خلاصه طراحی تیرها

- ترسیم دیاگرام نیروی برشی و ممان خمشی. سپس از روی آن را بیابید. جنس و نوع تیر را مشخص کنید.
- مدول مقطع مورد نیاز را با استفاده از رابطه زیر بدست آورید>

$$S_{req'd} = \frac{M}{\sigma_{allow}}$$

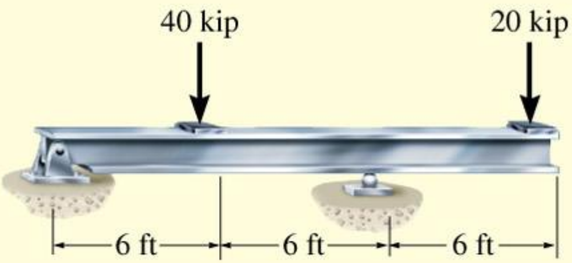
طراحی سطح مقطع!!!!

- تنش برشی ماکزیمم را محاسبه کنید تا مطمئن شوید رابطه زیر برقرار است:

$$\tau = \frac{VQ}{It} < \tau_{allow}$$

EXAMPLE 11.1

A beam is to be made of steel that has an allowable bending stress of $\sigma_{allow} = 24$ ksi and an allowable shear stress of $\tau_{allow} = 14.5$ ksi. Select an appropriate W shape that will carry the loading shown in Fig. 11–5*a*.



(a)

Solution

Shear and Moment Diagrams. The support reactions have been calculated, and the shear and moment diagrams are shown in Fig. 11–5*b*. From these diagrams, $V_{max} = 30$ kip and $M_{max} = 120$ kip·ft.

Bending Stress. The required section modulus for the beam is determined from the flexure formula,

$$S_{req'd} = \frac{M_{max}}{\sigma_{allow}} = \frac{120 \text{ kip} \cdot \text{ft}(12 \text{ in./ft})}{24 \text{ kip/in}^2} = 60 \text{ in}^3$$

Using the table in Appendix B, the following beams are adequate:

W18 × 40	$S = 68.4 \text{ in}^3$
W16 × 45	$S = 72.7 \text{ in}^3$
W14 × 43	$S = 62.7 \text{ in}^3$
W12 × 50	$S = 64.7 \text{ in}^3$
W10 × 54	$S = 60.0 \text{ in}^3$
W8 × 67	$S = 60.4 \text{ in}^3$

The beam having the least weight per foot is chosen, i.e.,

$$W18 \times 40$$

The *actual* maximum moment M_{max}, which includes the weight of the beam, can be computed and the adequacy of the selected beam can be checked. In comparison with the applied loads, however, the beam's weight, $(0.040 \text{ kip/ft})(18 \text{ ft}) = 0.720$ kip, will only *slightly increase* $S_{req'd}$. In spite of this,

$$S_{req'd} = 60 \text{ in}^3 < 68.4 \text{ in}^3 \qquad \text{OK}$$

Shear Stress. Since the beam is a *wide-flange section*, the *average shear stress* within the web will be considered. Here the web is assumed to extend from the very top to the very bottom of the beam. From Appendix B, for a W18 × 40, $d = 17.90$ in, $t_w = 0.315$ in. Thus,

$$\tau_{avg} = \frac{V_{max}}{A_w} = \frac{30 \text{ kip}}{(17.90 \text{ in.})(0.315 \text{ in.})} = 5.32 \text{ ksi} < 14.5 \text{ ksi} \qquad \text{OK}$$

Use a W18 × 40. *Ans.*

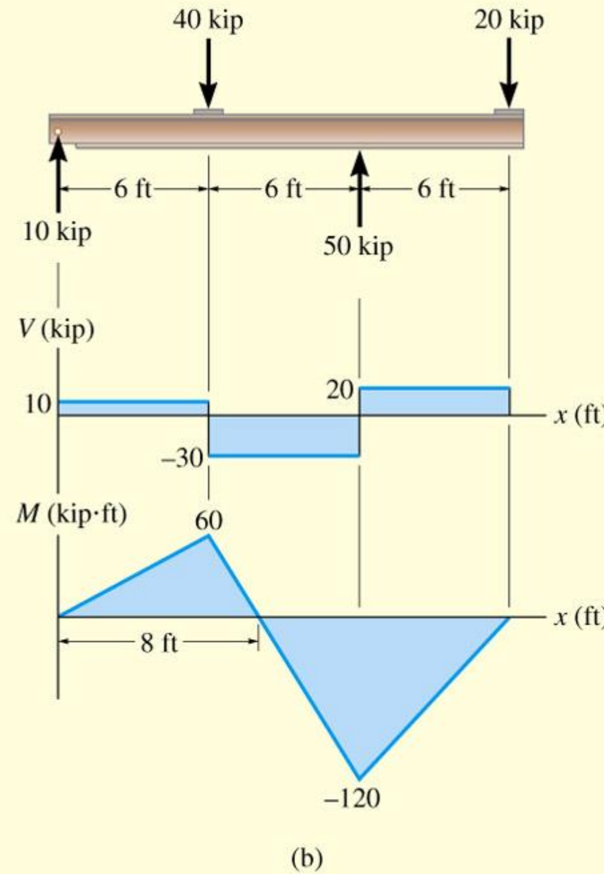
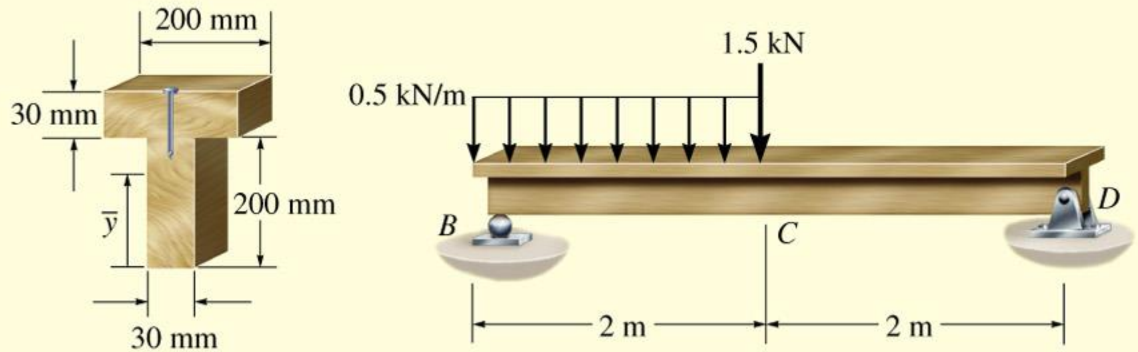


Fig. 11–5

EXAMPLE 11.2

The wooden T-beam shown in Fig. 11–6*a* is made from two 200 mm × 30 mm boards. If the allowable bending stress is $\sigma_{allow} = 12$ MPa and the allowable shear stress is $\tau_{allow} = 0.8$ MPa, determine if the beam can safely support the loading shown.



(a)

Solution

Shear and Moment Diagrams. The reactions on the beam are shown, and the shear and moment diagrams are drawn in Fig. 11–6*b*. Here $V_{max} = 1.5$ kN, $M_{max} = 2$ kN·m.

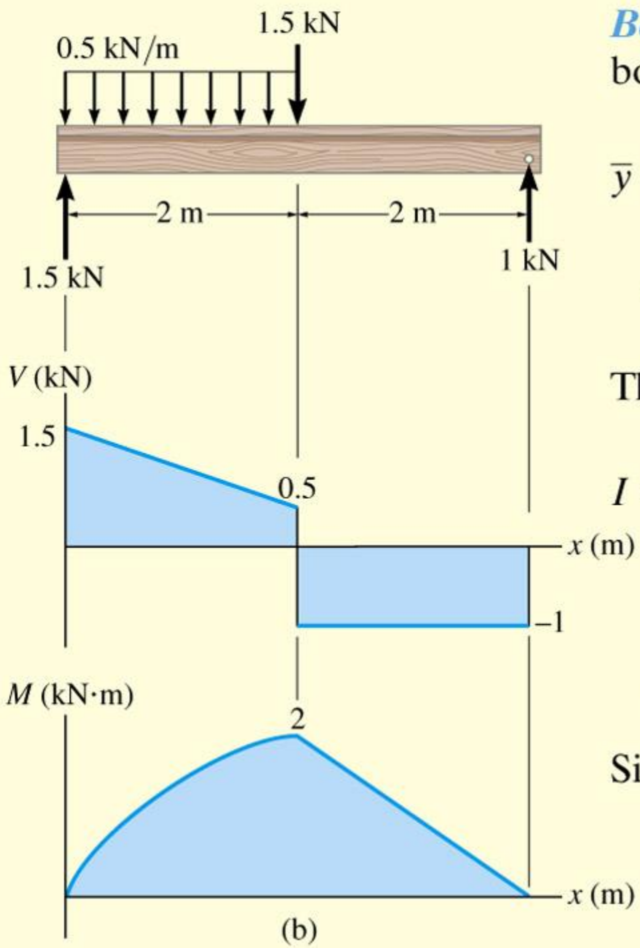


Fig. 11–6

Bending Stress. The neutral axis (centroid) will be located from the bottom of the beam. Working in units of meters, we have

$$\bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A}$$

$$= \frac{(0.1\text{ m})(0.03\text{ m})(0.2\text{ m}) + 0.215\text{ m}(0.03\text{ m})(0.2\text{ m})}{0.03\text{ m}(0.2\text{ m}) + 0.03\text{ m}(0.2\text{ m})} = 0.1575\text{ m}$$

Thus,

$$I = \left[\frac{1}{12}(0.03\text{ m})(0.2\text{ m})^3 + (0.03\text{ m})(0.2\text{ m})(0.1575\text{ m} - 0.1\text{ m})^2\right]$$

$$+ \left[\frac{1}{12}(0.2\text{ m})(0.03\text{ m})^3 + (0.03\text{ m})(0.2\text{ m})(0.215\text{ m} - 0.1575\text{ m})^2\right]$$

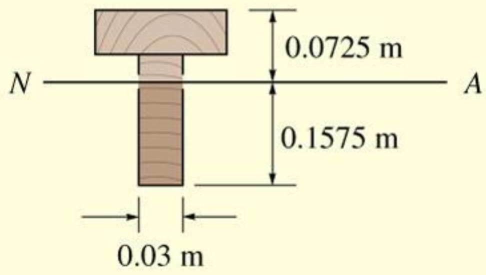
$$= 60.125(10^{-6})\text{ m}^4$$

Since $c = 0.1575$ m (not $0.230\text{ m} - 0.1575\text{ m} = 0.0725\text{ m}$), we require

$$\sigma_{\text{allow}} \geq \frac{M_{\max} c}{I}$$

$$12(10^3)\text{ kPa} \geq \frac{2\text{ kN}\cdot\text{m }(0.1575\text{ m})}{60.125(10^{-6})\text{ m}^4} = 5.24(10^3)\text{ kPa} \qquad \text{OK}$$

Shear Stress. Maximum shear stress in the beam depends upon the magnitude of Q and t. It occurs at the neutral axis, since Q is a maximum there and the neutral axis is in the web, where the thickness $t = 0.03$ m is smallest for the cross section. For simplicity, we will use the rectangular area below the neutral axis to calculate Q, rather than a two-part composite area above this axis, Fig. 11–6c. We have



(c)

$$Q = \bar{y}'A' = \left(\frac{0.1575\text{ m}}{2}\right)[(0.1575\text{ m})(0.03\text{ m})] = 0.372(10^{-3})\text{ m}^3$$

So that

$$\tau_{\text{allow}} \geq \frac{V_{\max}Q}{It}$$

$$800\text{ kPa} \geq \frac{1.5\text{ kN}[0.372(10^{-3})]\text{ m}^3}{60.125(10^{-6})\text{ m}^4(0.03\text{ m})} = 309\text{ kPa} \qquad \text{OK}$$