useful formulas

name	formula
Euler's formula	$e^{j\,\theta} = \cos(\theta) + j\,\sin(\theta)$
... for cosine	$\cos(\theta) = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$
... for sine	$\sin(\theta) = \dfrac{e^{j\theta} - e^{-j\theta}}{2\,j}$
sinc function	$\mathrm{sinc}(\theta) := \dfrac{\sin(\pi\,\theta)}{\pi\,\theta}$

formulas for continuous-time LTI signals and systems

name	formula
area under impulse	$\int \delta(t)\,dt = 1$
multiplication by impulse	$f(t)\,\delta(t) = f(0)\,\delta(t)$
... by shifted impulse	$f(t)\,\delta(t-t_o) = f(t_o)\,\delta(t-t_o)$
convolution	$f(t) * g(t) = \int f(\tau)\,g(t-\tau)\,d\tau$
... with an impulse	$f(t) * \delta(t) = f(t)$
... with a shifted impulse	$f(t) * \delta(t-t_o) = f(t-t_o)$
transfer function	$H(s) = \int h(t)\,e^{-st}\,dt$
frequency response	$H^f(\omega) = \int h(t)\,e^{-j\omega t}\,dt$
... their connection	$H^f(\omega) = H(j\omega)$ provided $j\omega$-axis $\subset$ ROC

selected Laplace transform pairs

$x(t)$	$X(s)$	ROC
$x(t)$	$\int x(t)\,e^{-st}\,dt$ (def.)	
$\delta(t)$	1	all s
$u(t)$	$\dfrac{1}{s}$	$\mathrm{Re}(s) > 0$
$e^{-a\,t}\,u(t)$	$\dfrac{1}{s+a}$	$\mathrm{Re}(s) > -a$
$\cos(\omega_o t)\,u(t)$	$\dfrac{s}{s^2+\omega_o^2}$	$\mathrm{Re}(s) > 0$
$\sin(\omega_o t)\,u(t)$	$\dfrac{\omega_o}{s^2+\omega_o^2}$	$\mathrm{Re}(s) > 0$
$e^{-a\,t}\cos(\omega_o t)\,u(t)$	$\dfrac{s+a}{(s+a)^2+\omega_o^2}$	$\mathrm{Re}(s) > -a$
$e^{-a\,t}\sin(\omega_o t)\,u(t)$	$\dfrac{\omega_o}{(s+a)^2+\omega_o^2}$	$\mathrm{Re}(s) > -a$

Note: a is assumed real.

Laplace transform properties

$x(t)$	$X(s)$
$a\,x(t) + b\,g(t)$	$a\,X(s) + b\,G(s)$
$x(t) * g(t)$	$X(s)\,G(s)$
$\dfrac{dx(t)}{dt}$	$s\,X(s)$
$x(t-t_o)$	$e^{-s\,t_o}\,X(s)$

selected Fourier transform pairs

$x(t)$	$X^f(\omega)$
$x(t)$	$\int x(t)\, e^{-j\omega t}\, dt$ (def.)
$\frac{1}{2\pi} \int X^f(\omega)\, e^{j\omega t}\, d\omega$	$X^f(\omega)$
$\delta(t)$	1
1	$2\,\pi\,\delta(\omega)$
$u(t)$	$\pi\,\delta(\omega) + \frac{1}{j\omega}$
$e^{j\omega_o t}$	$2\,\pi\,\delta(\omega - \omega_o)$
$\cos(\omega_o\, t)$	$\pi\,\delta(\omega + \omega_o) + \pi\,\delta(\omega - \omega_o)$
$\sin(\omega_o\, t)$	$j\,\pi\,\delta(\omega + \omega_o) - j\,\pi\,\delta(\omega - \omega_o)$
$\frac{\omega_o}{\pi}\ \mathrm{sinc}\left(\frac{\omega_o}{\pi} t\right)$	ideal LPF cut-off frequency ω_o
symmetric pulse width T, height 1	$\frac{2}{\omega}\ \sin\left(\frac{T}{2}\,\omega\right)$
impulse train period T, height 1	impulse train period, height $\omega_o = \frac{2\pi}{T}$

Fourier transform properties

$x(t)$	$X^f(\omega)$
$a\, x(t) + b\, g(t)$	$a\, X^f(\omega) + b\, G^f(\omega)$
$x(a\, t)$	$\frac{1}{\|a\|}\, X\left(\frac{\omega}{a}\right)$
$x(t) * g(t)$	$X^f(\omega)\, G^f(\omega)$
$x(t)\, g(t)$	$\frac{1}{2\pi}\, X^f(\omega) * G^f(\omega)$
$x(t - t_o)$	$e^{-jt_o\omega}\, X(\omega)$
$x(t)\, e^{j\omega_o t}$	$X(\omega - \omega_o)$
$x(t)\ \cos(\omega_o t)$	$0.5\, X(\omega + \omega_o) + 0.5\, X(\omega - \omega_o)$
$x(t)\ \sin(\omega_o t)$	$j\, 0.5\, X(\omega + \omega_o) - j\, 0.5\, X(\omega - \omega_o)$
$\frac{dx(t)}{dt}$	$j\,\omega\, X^f(\omega)$